Multiuser and Multirelay Cognitive Radio Networks Under Spectrum-Sharing Constraints

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***Abstract*—In this paper, the outage behavior of dual-hop multiuser multirelay cognitive radio networks under spectrum-sharing constraints is investigated. In the proposed cognitive radio network, the secondary network is composed of one secondary-user (SU) source that communicates with one out of L destinations through a direct link and also via the help of one out of N relays by using an efficient relay–destination selection scheme. Additionally, a selection combining (SC) scheme to select the best link (direct or dual-hop link) from the SU source is employed at the selected SU destination. Adopting an underlay approach, the SU communication is performed accounting for an interference constraint, where the overall transmit power is governed by the interference at the primary-user (PU) receiver, as well as by the maximum transmission power available at the respective nodes. Closed-form expressions for the outage probability are derived, from which an asymptotic analysis reveals that the diversity order of the considered system is not affected by the interference and is equal to $N + L$ for both decode-and-forward (DF) and amplify-and-forward (AF) relaying protocols. The analytical results are corroborated by Monte Carlo simulations, and insightful discussions are provided.**

***Index Terms*—Cooperative diversity (CD), multiuser diversity, outage analysis, selection schemes, underlay approach.**

I. Introduction

During the last decade, cooperative diversity (CD) [1], [2] has received considerable attention from the wireless community due to the enormous performance gains obtained with its use and without the need for multiple antennas implemented at the terminals. The key idea behind CD is to allow single-antenna devices to share their antenna to mimic a physical multiple-antenna array. From this concept, the same gains obtained in multiple-antenna systems can be also attained in single-antenna CD systems. Basically, there are two main relaying protocols in CD systems, namely, decode-and-forward (DF) and amplify-and-forward (AF), which have been extensively studied in the technical literature. Another concern that is currently being studied by the wireless community is the need for more efficient use of spectrum resources. In this sense, cognitive radio has arisen as a promising technique [3] to alleviate the underutilization spectrum problem [4]. Motivated by the promising gains acquired with the use of CD and cognitive spectrum-sharing concepts, several works have investigated the joint use of these two promising aforementioned technologies, and such papers are briefly described as follows.

In [5], the outage analysis of simple dual-hop cooperative spectrum-sharing systems (CSSSs) with an interference constraint in Nakagami-m fading channels was analyzed. In [6], a CSSS consisting of one secondary-user (SU) source, multiple SU relays, one SU destination, and one primary-user PU receiver was considered. Neglecting the presence of the direct link, the authors of [6] studied the outage performance of the secondary network in which an appropriate relay selection criterion was employed. Following a different approach, the authors of [7] assumed the presence of the direct link between the SU source and SU destination and employed a selection combining (SC) technique to choose the best link (direct or dual-hop link) from the SU source. Considering a Nakagami-m fading scenario, in [8], the outage performance of CSSSs was investigated. This work has been recently extended in [9] assuming the presence of the direct link. In [10], the outage performance of CSSSs with multiple SU relays and in the presence of the direct link was analyzed, adopting that the overall transmit power is governed solely by the interference at the PU receiver.

In this paper, different from all previous works and with the aim of analyzing a more general CSSS, we study the outage performance of multirelay cognitive networks in multiuser spectrum-sharing systems, composed of one SU source, N SU relays, L available SU destinations, and one PU receiver. In our analysis, both DF and AF relaying protocols are considered. Focusing on the SU communication, an efficient relay–destination selection scheme is employed. Briefly speaking, the SU source selects the best SU destination node based on the channel quality of the direct links and then selects the best SU relay that yields the best path from the SU source to the selected SU destination. Adopting an underlay approach due to the spectrum-sharing environment, the transmit power values of the SU nodes are governed by both the interference at the PU receiver and their maximum respective transmission power values. After the communication process is performed, the selected SU destination employs an SC technique to choose the best link (direct or dual-hop link) from the SU source. Closed-form expressions for the outage probability are derived, and asymptotic analysis is carried out, revealing that the diversity order of the considered system is not affected by the interference and is equal to $N + L$ for both DF and AF relaying protocols. The proposed formulations are validated by means of Monte Carlo simulations, and insightful discussions are provided. Throughout this paper, $f_Z(\cdot)$ and $F_Z(\cdot)$ denote the probability density function and cumulative distribution function of an arbitrary random variable (RV) Z, respectively, and $E[\cdot]$ stands for expectation.

II. System Model

Consider a dual-hop CSSS composed of one SU source S, N SU relays $R_n (n = 1, \ldots, N)$, L SU destinations $D_l (l = 1, \ldots, L)$, and one PU receiver P, as shown in Fig. 1. All nodes have a single antenna and operate in a half-duplex mode. The PU transmitter is assumed to be far away from the SU nodes so that it does not interfere with the selection process of the relay and destination nodes. It is also assumed that the SU source has a line-of-sight with all the SU destinations. The channel coefficients h_{MT} experience Rayleigh quasi-static fading, with M and T denoting two arbitrary nodes, and all noise terms are additive white Gaussian noise signals with mean power N_0. Due the presence of the PU receiver, maximum tolerable interference I generated by the SU transmitters at the PU receiver is established such that the primary communication will not be affected by the secondary transmission. Let P_S and P_{R_n} be the maximum transmit power values

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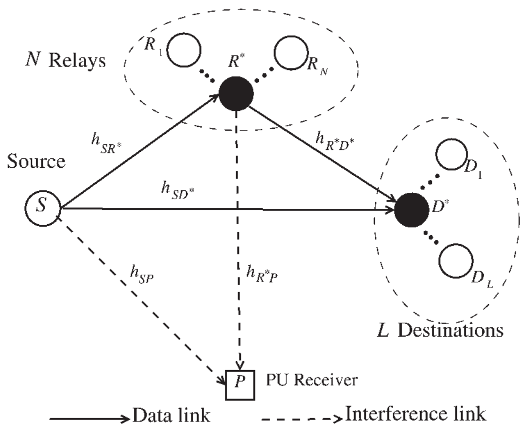

Fig. 1. System model.

at the SU source and nth SU relay, respectively. Thus, making use of an underlay approach,[1] the transmit power values at S and R_n can be written, respectively, as $\overline{P}_S = \min((I/|h_{SP}|^2), P_S)$ and $\overline{P}_{R_n} = \min((I/|h_{R_nP}|^2), P_{R_n})$. Focusing on the SU communication, a time-division multiple-access strategy is employed, which is performed in two phases. However, before the communication process starts, an efficient relay–destination selection scheme is carried out. More specifically, the best SU destination D^* is first selected based on the channel quality of the direct links, i.e., $D^* = \arg\max_l[\gamma_{SD_l}]$, where $\gamma_{SD_l} = \min((I/|h_{SP}|^2), P_S)(d_{SD_l}^{-\rho}|h_{SD_l}|^2/N_0)$, d_{MT} is the distance between two arbitrary nodes M and T, and ρ represents the path loss coefficient.[2] After the SU destination is selected, the relay selection process is performed in such a way that the chosen relay R^* will maximize the end-to-end SNR from the SU source to the selected SU destination.[3] In what follows, the relay selection process and the end-to-end SNR will be formulated for both DF and AF relaying protocols.

A. DF Relaying Protocol

As stated previously, the communication process comprises two phases. In phase I, the SU source broadcasts its information to both R^* and D^* with transmission power $\overline{P}_S$. In particular, for DF relays, R^* is given by

$$R^* = \arg\max_n \left[\min\left[\gamma_{SR_n}, \gamma_{R_nD^*}\right]\right] \tag{1}$$

[1]In practice, the channel state information (CSI) of the links between the secondary and primary nodes can be obtained through a direct feedback from the PU or through an indirect feedback by a band manager that mediates the exchange of information between the primary and secondary networks.

[2]Note that, if we consider the path loss for the link between the SU and PU, it would be hard for the former to determine the distance from the PU to calculate its respective transmit power. For instance, the transmit power of the SU source would be $\overline{P}_S = \min((I/d_{SP}^{-\rho}|h_{SP}|^2), P_S)$ instead of $\overline{P}_S = \min((I/|h_{SP}|^2), P_S)$. Thus, due to practical implementation issues, we opted to not include the path loss for the link between the SU and PU nodes.

[3]Although the scheduling policy is suboptimal from the outage performance point of view, it will be shown that this strategy achieves the same diversity order of the optimal node selection strategy, i.e., $L + N$. Furthermore, the amount of CSI required in the proposed scheme is $L + 2N + 2$, whereas it would be $LN + L + N + 2$ if an optimal joint relay-destination scheme were employed. In addition, our proposed scheme only needs to compare $L + N$ potential links in each transmission process, whereas for a joint relay-destination selection scheme it would require $L(N + 1)$ potential links. Thus, one can notice that, for large-scale multirelay multidestination cooperative spectrum-sharing systems, the proposed scheme can significantly reduce the amount of overhead when compared with the optimal strategy, in addition to achieving the same diversity order.

where $\gamma_{SR_n} = \min((I/|h_{SP}|^2), P_S)(d_{SR_n}^{-\rho}|h_{SR_n}|^2/N_0)$, and $\gamma_{R_nD^*} = \min((I/|h_{R_nP}|^2), P_{R_n})(d_{R_nD^*}^{-\rho}|h_{R_nD^*}|^2/N_0)$. In phase II, assuming that the selected SU relay is always able to fully decode the received signal, R^* forwards it to D^* with transmission power $\overline{P}_{R_n}$. At the end of this two-phase transmission, an SC strategy is performed by the selected SU destination. In this case, the path with the highest instantaneous SNR is chosen between the direct and the selected dual-hop link so that the end-to-end SNR can be written as

$$\gamma_{\text{end}}^{\text{DF}} = \max\left[\max_l[\gamma_{SD_l}], \max_n\left[\min\left[\gamma_{SR_n}, \gamma_{R_nD^*}\right]\right]\right]. \tag{2}$$

B. AF Relaying Protocol

For the AF case, the first phase occurs similarly to the DF case, with the SU source broadcasting its information to D^* and R^*, where the latter is now given by

$$R^* = \arg\max_n\left[\frac{\gamma_{SR_n}\gamma_{R_nD^*}}{1 + \gamma_{SR_n} + \gamma_{R_nD^*}}\right]. \tag{3}$$

In the second phase, the selected SU relay amplifies[4] the received signal from the SU source and forwards it to the selected SU destination. After the second phase, an SC strategy is employed at the selected SU destination so that the end-to-end SNR can be written as

$$\gamma_{\text{end}}^{\text{AF}} = \max\left[\max_l[\gamma_{SD_l}], \max_n\left[\frac{\gamma_{SR_n}\gamma_{R_nD^*}}{1 + \gamma_{SR_n} + \gamma_{R_nD^*}}\right]\right]. \tag{4}$$

In (2) and (4), note that the two terms inside the $\max[., .]$ operator are not statistically independent due to the presence of the common RV $|h_{SP}|^2$. Next, we will use the similar approach, as presented in [7], to study the system's outage performance.

III. Outage Probability Analysis

A. DF

The outage probability is defined as the probability that the instantaneous end-to-end SNR, i.e., $\gamma_{\text{end}}^{\text{DF}}$, falls below a given threshold γ_{th}. Due to the common term $|h_{SP}|^2$, as previously mentioned, the conditional outage probability can be written as

$$\begin{aligned}\Pr\left(\gamma_{\text{end}}^{\text{DF}} < \gamma_{\text{th}}|h_{SP}\right) &= \overbrace{\Pr\left(\max_l[\gamma_{SD_l}] < \gamma_{\text{th}}|h_{SP}\right)}^{\theta} \\ &\quad \times \underbrace{\Pr\left(\max_n\left[\min\left[\gamma_{SR_n}, \gamma_{R_nD^*}\right]\right] < \gamma_{\text{th}}|h_{SP}\right)}_{\Psi}. \end{aligned} \tag{5}$$

Since all the links from S to D_l are statistically independent, θ can be expressed as

$$\begin{aligned}\theta &= \prod_{l=1}^{L}\Pr(\gamma_{SD_l} < \gamma_{\text{th}}|h_{SP}) = \prod_{l=1}^{L}F_{\gamma_{SD_l}}(\gamma_{\text{th}}|h_{SP}) \\ &= \prod_{l=1}^{L}(1 - e^{-\gamma_{\text{th}}\,\beta_{SD_l}}) \end{aligned} \tag{6}$$

[4]In this paper, we consider only variable-gain relays in which the amplification factor is determined by the instantaneous channel statistics of the source-relay links.

where $\beta_{SD_l} \triangleq 1/E[\gamma_{SD_l}]$. According to the total probability theorem [11], Ψ in (5) can be written as

$$\Psi = \Pr\left(\max_n\left[\min\left[\gamma_{SR_n}, \gamma_{R_nD^*}\right]\right] < \gamma_{\text{th}}|h_{SP}\right)$$
$$= \sum_{l=1}^{L} \Pr(D^* = D_l)\underbrace{\Pr\Big(\max_n\Big[\min\Big[\gamma_{SR_n}, \gamma_{R_nD_l}\Big]\Big] < \gamma_{\text{th}}|h_{SP}\Big)}_{\Phi}. \tag{7}$$

By its turn, knowing that $\Pr(\min[W, V] < \gamma_{\text{th}}) = F_W(\gamma_{\text{th}}) + F_V(\gamma_{\text{th}}) - F_W(\gamma_{\text{th}}) \times F_V(\gamma_{\text{th}})$ [11], Φ can be reexpressed as

$$\Phi = \prod_{n=1}^{N}\Big[(1 - e^{-\gamma_{\text{th}}\,\beta_{SR_n}}) + (1 - e^{-\gamma_{\text{th}}\,\beta_{R_nD_l}}) - (1 - e^{-\gamma_{\text{th}}\,\beta_{SR_n}})(1 - e^{-\gamma_{\text{th}}\,\beta_{R_nD_l}})\Big] \tag{8}$$

where $\beta_{SR_n} \triangleq 1/E[\gamma_{SR_n}]$, and $\beta_{R_nD_l} \triangleq 1/E[\gamma_{R_nD_l}]$. To determinate the value of $\Pr(D^* = D_l)$, we make use of the results presented in [12], yielding

$$\Pr(D^* = D_l) = 1 + \sum_{k=1}^{L-1}\sum_{\substack{A_k\subseteq\{1,2,\ldots,l-1,l+1,\ldots,L\}\\ |A_k|=k}} \times(-1)^k\frac{\beta_{SD_l}}{\beta_{SD_l} + \sum_{j\in A_k}, \beta_{SD_j}}. \tag{9}$$

Now, let $X = |h_{SP}|^2$, $Y = |h_{R_nP}|^2$, $\gamma_{SD^*} = \max_l[\gamma_{SD_l}]$, and $\gamma_{SR^*D^*} = \max_n[\min[\gamma_{SR_n}, \gamma_{R_nD^*}]]$. Thus, a general expression for the outage probability can be found by solving the following integral [11]:

$$P_{\text{out}} = \int_0^\infty\int_0^\infty F_{\gamma_{SD^*}}(\gamma_{\text{th}}|X)\,F_{\gamma_{SR^*D^*}}(\gamma_{\text{th}}|X,Y) \times f_X(x)\,f_Y(y)\,dx\,dy \tag{10}$$

where $F_{\gamma_{SD^*}}(\gamma_{\text{th}}|X) = \prod_{l=1}^{L}(1 - e^{-\gamma_{\text{th}}\,\beta_{SD_l}})$, and $F_{\gamma_{SR^*D^*}}(\gamma_{\text{th}}|X, Y) = \sum_{l=1}^{L}\Pr(D^* = D_l) \times \Phi$. To determine the integral in (10), it is important to see that

$$\min\left(\frac{I}{X}, P_S\right) = \begin{cases} P_S, & \text{when } X \le I/P_S\\ I/X, & \text{when } X > I/P_S\end{cases}$$
$$\min\left(\frac{I}{Y}, P_{R_n}\right) = \begin{cases} P_{R_n}, & \text{when } Y \le I/P_{R_n}\\ I/Y, & \text{when } Y > I/P_{R_n}.\end{cases} \tag{11}$$

Thus, the outage probability can be rewritten as a sum of four terms, i.e., $P_{\text{out}}^{\text{DF}} = \xi_1 + \xi_2 + \xi_3 + \xi_4$, where

$$\xi_1 = \int_0^{I/Q}\int_0^{I/Q} F_{\gamma_{SD^*}}(\gamma_{\text{th}}|X)\,F_{\gamma_{SR^*D^*}}(\gamma_{\text{th}}|X,Y) \times f_X(x)\,f_Y(y)dx\,dy \tag{12}$$

$$\xi_2 = \int_0^{I/Q}\int_{I/Q}^{\infty} F_{\gamma_{SD^*}}(\gamma_{\text{th}}|X)\,F_{\gamma_{SR^*D^*}}(\gamma_{\text{th}}|X,Y) \times f_X(x)\,f_Y(y)dx\,dy \tag{13}$$

$$\xi_3 = \int_{I/Q}^{\infty}\int_0^{I/Q} F_{\gamma_{SD^*}}(\gamma_{\text{th}}|X)\,F_{\gamma_{SR^*D^*}}(\gamma_{\text{th}}|X,Y) \times f_X(x)\,f_Y(y)dx\,dy \tag{14}$$

$$\xi_4 = \int_{I/Q}^{\infty}\int_{I/Q}^{\infty} F_{\gamma_{SD^*}}(\gamma_{\text{th}}|X)\,F_{\gamma_{SR^*D^*}}(\gamma_{\text{th}}|X,Y) \times f_X(x)\,f_Y(y)dx\,dy. \tag{15}$$

Without any loss of generality, let $P_{R_n} = P_S = Q$. Thus, performing the appropriate substitutions in (12), it follows that

$$\xi_1 = \int_0^{I/Q}\int_0^{I/Q}\prod_{l=1}^{L}\left(1 - e^{-\gamma_{\text{th}}\beta^Q_{SD_l}}\right)\sum_{l=1}^{L}\Pr(D^* = D_l) \times \prod_{n=1}^{N}\left(1 - e^{-\gamma_{\text{th}}\left(\beta^Q_{SR_n}+\beta^Q_{R_nD_l}\right)}\right) \times \beta_{SP}e^{-x\beta_{SP}}\beta_{R_nP}e^{-y\beta_{R_nP}}dx\,dy \tag{16}$$

where $\beta_{SP} \triangleq 1/E[X]$, $\beta_{R_nP} \triangleq 1/E[Y]$, and $\beta^Q_{MT} \triangleq 1/(E[Q\,d_{MT}^{-\alpha}|h_{MT}|^2/N_0])$, with $M \in \{S, R_n\}$ and $T \in \{R_n, D_l\}$. By integrating the previous equation, (16) results in

$$\xi_1 = \prod_{l=1}^{L}\left(1 - e^{-\gamma_{\text{th}}\beta^Q_{SD_l}}\right)\sum_{l=1}^{L}\Pr(D^* = D_l) \times \prod_{n=1}^{N}\left(1 - e^{-\gamma_{\text{th}}\left(\beta^Q_{SR_n}+\beta^Q_{R_nD_l}\right)}\right)(1 - e^{-\frac{I}{Q}\beta_{SP}}) \times (1 - e^{-\frac{I}{Q}\beta_{R_nP}}). \tag{17}$$

In a similar manner, by performing the appropriate substitutions in (13), we have

$$\xi_2 = \int_0^{I/Q}\int_{I/Q}^{\infty}\prod_{l=1}^{L}\left(1 - e^{-\gamma_{\text{th}}\,\beta^Q_{SD_l}}\right)\sum_{l=1}^{L}\Pr(D^* = D_l) \times \prod_{n=1}^{N}\left(1 - e^{-\gamma_{\text{th}}\left(\beta^Q_{SR_n}+y\beta^I_{R_nD_l}\right)}\right) \times \beta_{SP}e^{-x\beta_{SP}}\beta_{R_nP}e^{-y\beta_{R_nP}}dx\,dy \tag{18}$$

where $y\beta^I_{R_nD_l} \triangleq 1/E[(I/y)(d_{R_nD_l}^{-\alpha}|h_{R_nD_l}|^2/N_0)]$. Now, using identity $\prod_{k=1}^{K}(1 - x_k) = \sum_{k=0}^{K}(-1)^k/k!\sum_{n_1,\ldots,n_k}^{K}\prod_{t=1}^{k}x_{n_t}$, it follows that

$$\xi_2 = \prod_{l=1}^{L}\left(1 - e^{-\gamma_{\text{th}}\beta^Q_{SD_l}}\right)\sum_{l=1}^{L}\Pr(D^* = D_l)(1 - e^{-\frac{I}{Q}\beta_{SP}}) \times \int_{I/Q}^{\infty}\beta_{R_nP}e^{-y\beta_{R_nP}}\sum_{k=0}^{K}\frac{(-1)^k}{k!}\sum_{n_1,\ldots,n_k}^{K} \times \prod_{t=1}^{k}e^{-\gamma_{\text{th}}\left(\beta^Q_{SR_{n_t}}+y\beta^I_{R_{n_t}D_l}\right)}dy. \tag{19}$$

In the sequel, we assume that the links from S to R_n undergo independent identically distributed (i.i.d.) Rayleigh fading, which implies the same average SNR (i.e., $\beta_{SR_n} = \beta_{SR}, \forall n$).[5] The same assumption can be made for the SU destinations, where the links from S and R_n to D_l are also i.i.d. However, it is worth noting that the channels pertaining to different hops experience distinct fading conditions from each other. Thus, (19) can be rewritten as

$$\xi_2 = \prod_{l=1}^{L}\left(1 - e^{-\gamma_{\text{th}}\beta^Q_{SD_l}}\right)\sum_{l=1}^{L}\Pr(D^* = D_l)(1 - e^{-\frac{I}{Q}\beta_{SP}}) \times \beta_{R_nP}\sum_{n=0}^{N}\binom{N}{n}(-1)^n \times \exp\left(-\gamma_{\text{th}} n\left(\beta^Q_{SR_n} + \frac{I}{Q}\beta^I_{R_nD_l}\right)\right)\frac{e^{-\frac{I}{Q}\beta_{R_nP}}}{\gamma_{\text{th}} n\beta^I_{R_nD_l} + \beta_{R_nP}}. \tag{20}$$

Applying the same procedure, (14) and (15) can be derived in closed form, respectively, as

$$\xi_3 = \sum_{m=0}^{L}\binom{L}{m}(-1)^m\sum_{l=1}^{L}\Pr(D^* = D_l) \times \sum_{n=0}^{N}\binom{N}{n}(-1)^n\beta_{S_P}(1 - e^{-\frac{I}{Q}\beta_{R_nP}})\, e^{(-\frac{I}{Q}\beta_{SP})} \times \exp\left(-\gamma_{\text{th}}\left(m\frac{I}{Q}\beta^I_{SD_m} + n\left(\frac{I}{Q}\beta^I_{SR_n} + \beta^Q_{R_nD_m}\right)\right)\right) \times \frac{1}{\gamma_{\text{th}}\left(m\beta^I_{SD_m} + n\beta^I_{SR_n}\right) + \beta_{SP}} \tag{21}$$

$$\xi_4 = \sum_{m=0}^{L}\binom{L}{m}(-1)^m\sum_{l=1}^{L}\Pr(D^* = D_l) \times \sum_{n=0}^{N}\binom{N}{n}(-1)^n\beta_{SP}\beta_{R_nP} \times \exp\left(-\gamma_{\text{th}}\frac{I}{Q}\left(m\beta^I_{SD_m} + n\beta^I_{SR_n} + n\beta^I_{R_nD_m}\right)\right) \times \frac{e^{\left(-\frac{I}{Q}(\beta_{SP} + \beta_{R_nP})\right)}}{\left[\gamma_{\text{th}}(m\beta^I_{SD_m} + n\beta^I_{SR_n}) + \beta_{SP}\right]\left[\gamma_{\text{th}} n\beta^I_{R_nD_m} + \beta_{R_nP}\right]} \tag{22}$$

where $\beta^I_{SR_n} \triangleq 1/E[I\, d^{-\alpha}_{SR_n}|h_{SR_n}|^2/N_0]$, and $\beta^I_{SD_l} \triangleq 1/E[I\, d^{-\alpha}_{SD_l}|h_{SD_l}|^2/N_0]$. Finally, by substituting (17), (20)–(22) into $P^{\text{DF}}_{\text{out}} = \xi_1 + \xi_2 + \xi_3 + \xi_4$, a closed-form expression for the outage probability of a DF relaying scenario is derived.

B. AF

For AF relays, because the two terms inside (4) are not statistically independent, we adopt a similar approach previously used for DF relays. In this case, the conditional outage probability can be written as

$$\Pr\left(\gamma^{\text{AF}}_{\text{end}} < \gamma_{\text{th}}|h_{SP}\right) = \Pr\left(\max_l[\gamma_{SD_l}] < \gamma_{\text{th}}|h_{SP}\right) \times \underbrace{\Pr\left(\max_n\left[\frac{\gamma_{SR_n}\gamma_{R_nD^*}}{1 + \gamma_{SR_n} + \gamma_{R_nD^*}}\right] < \gamma_{\text{th}}|h_{SP}\right)}_{\Omega}. \tag{23}$$

[5] Such an assumption relies on the fact that the SU relays are clustered relatively close together and are selected by a long-term routing process (see [13] and references therein).

The term $\Pr(\max_l[\gamma_{SD_l}] < \gamma_{\text{th}}|h_{SP})$ is given by (6). In addition, Ω can be calculated as

$$\Omega = \Pr\left(\max_n\left[\frac{\gamma_{SR_n}\gamma_{R_nD_l}}{1 + \gamma_{SR_n} + \gamma_{R_nD_l}}\right] < \gamma_{\text{th}}|h_{SP}\right) = \sum_{l=1}^{L}\Pr(D^* = D_l) \times \prod_{n=1}^{N}\underbrace{\Pr\left(\left[\frac{\gamma_{SR_n}\gamma_{R_nD_l}}{1 + \gamma_{SR_n} + \gamma_{R_nD_l}}\right] < \gamma_{\text{th}}|h_{SP}\right)}_{\Theta} \tag{24}$$

where $\sum_{l=1}^{L}\Pr(D^* = D_l)$ is given by (9), and Θ can be determined as [12]

$$\Theta = \int_0^\infty \Pr\left(\max_n\left[\frac{\gamma_{SR_n}\gamma_{R_nD_l}}{1 + \gamma_{SR_n} + \gamma_{R_nD_l}}\right] < \gamma_{\text{th}}|h_{SP}\right) \times f_{\gamma_{SR_n}}(\gamma_{SR_n})d\gamma_{SR_n} = 1 - \beta_{SR_n}e^{-\gamma_{\text{th}}(\beta_{SR_n} + \beta_{R_nD_l})}2\sqrt{\frac{z}{\beta_{SR_n}}}K_1\left(2\sqrt{z\beta_{SR_n}}\right) \tag{25}$$

where $z = \gamma_{\text{th}}(\gamma_{\text{th}} + 1)\beta_{R_nD_l}$, and $K_1(\cdot)$ denotes the first-order modified Bessel function of the second kind [14, Eq. (8.432.6)]. By replacing (25) and (9) into (24) and plugging this latter and (6) into (23), a closed-form expression for the conditional outage probability is obtained. Then, unconditioning such expression with respect to h_{SP}, the outage probability can be determined as in (10) by making the appropriate substitutions of the required statistics. In this case, for AF relays, $F_{\gamma_{SD^*}}(\gamma_{\text{th}}|X) = \prod_{l=1}^{L}(1 - e^{-\gamma_{\text{th}}\beta_{SD_l}})$ and $F_{\gamma_{SR^*D^*}}(\gamma_{\text{th}}|X, Y) = \sum_{l=1}^{L}\Pr(D^* = D_l) \times \prod_{n=1}^{N}\Theta$. Finally, using the same rationale employed to DF relays, the outage probability can be written as $P^{\text{AF}}_{\text{out}} = \vartheta_1 + \vartheta_2 + \vartheta_3 + \vartheta_4$, where ϑ_i is determined in the same way as ξ_i, $i = 1, 2, 3, 4$, by substituting the appropriate statistics for the AF relaying case. In particular, it can be shown that

$$\vartheta_1 = \int_0^{I/Q}\int_0^{I/Q}\prod_{l=1}^{L}\left(1 - e^{-\gamma_{\text{th}}\beta^Q_{SD_l}}\right)\sum_{l=1}^{L}\Pr(D^* = D_l) \times \prod_{n=1}^{N}\left(1 - \beta^Q_{SR_n}e^{-\gamma_{\text{th}}(\beta^Q_{SR_n} + \beta^Q_{R_nD_l})} \times 2\sqrt{\frac{z}{\beta_{SR_n}}}K_1\left(2\sqrt{z\beta_{SR_n}}\right)\right) \times \beta_{SP}e^{-x\beta_{SP}}\beta_{R_nP}e^{-y\beta_{R_nP}}\,dx\,dy \tag{26}$$

which results in

$$\vartheta_1 = \prod_{l=1}^{L}(1 - e^{-\gamma_{\text{th}}\beta^Q_{SD_l}}) \times \sum_{l=1}^{L}\Pr(D^* = D_l)(1 - e^{-\frac{I}{Q}\beta_{SP}})(1 - e^{-\frac{I}{Q}\beta_{R_nP}}) \times \prod_{n=1}^{N}\left(1 - \beta^Q_{SR_n}e^{-\gamma_{\text{th}}\left(\beta^Q_{SR_n} + \beta^Q_{R_nD_l}\right)}2\sqrt{\frac{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)\beta^Q_{R_nD_l}}{\beta^Q_{SR_n}}} \times K_1\left(2\sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)\beta^Q_{R_nD_l}\beta^Q_{SR_n}}\right)\right). \tag{27}$$

By its turn, ϑ_2 is expressed as

$$\vartheta_2 = \int_0^{I/Q}\int_{I/Q}^{\infty}\prod_{l=1}^{L}(1-e^{-\gamma_{\rm th}\beta^Q_{SD_l}})\sum_{l=1}^{L}\Pr(D^*=D_l) \times \prod_{n=1}^{N}\left(1-\beta^Q_{SR_n}e^{-\gamma_{\rm th}\left(\beta^Q_{SR_n}+y\beta^I_{R_nD_l}\right)} \times 2\sqrt{\frac{z_I}{\beta^Q_{SR_n}}}K_1\left(2\sqrt{z_I\beta^Q_{SR_n}}\right)\right) \times \beta_{SP}e^{-x\beta_{SP}}\beta_{R_nP}e^{-y\beta_{R_nP}}\,dx\,dy \quad (28)$$

where $z_I = y\gamma_{\rm th}(\gamma_{\rm th}+1)\beta^I_{R_nD_l}$. To evaluate (28) in closed form, we make use of the following approximation[6] $K_1(\zeta)\approx 1/\zeta$ [14, Eq. (9.6.9)]. Thus

$$\vartheta_2 \approx \int_0^{I/Q}\int_{I/Q}^{\infty}\prod_{l=1}^{L}(1-e^{-\gamma_{\rm th}\beta^Q_{SD_l}})\sum_{l=1}^{L}\Pr(D^*=D_l) \times \prod_{n=1}^{N}\left(1-e^{-\gamma_{\rm th}(\beta^Q_{SR_n}+y\beta^I_{R_nD_l})}\right) \times \beta_{SP}e^{-x\beta_{SP}}\beta_{R_nP}e^{-y\beta_{R_nP}}\,dx\,dy \quad (29)$$

which results in (20). In the same way, ϑ_3 and ϑ_4 can be well approximated by (21) and (22), respectively. Finally, by substituting (27), (20)–(22) into $P^{\rm AF}_{\rm out} = \vartheta_1+\vartheta_2+\vartheta_3+\vartheta_4$, an accurate closed-form approximation for the system outage probability with AF relays is attained.

IV. Asymptotic Analysis

To provide further insights from the attained expressions, an asymptotic analysis (high-SNR regime) is now carried out for both DF and AF relaying protocols. Without loss of generality, let $\overline{\gamma} \triangleq 1/N_0$ be the system SNR and assume that $I/Q = \mu$, where μ is a positive constant. As $\overline{\gamma}\to\infty$, note that $\beta_{SP} \gg \gamma_{\rm th}/\overline{\gamma}$ and $\beta_{R_nP} \gg \gamma_{\rm th}/\overline{\gamma}$. Thus, using the Maclaurin series of exponential functions, we have $e^{-\overline{\gamma}x} \simeq 1-\overline{\gamma}x$. Based on this approximation, making use of $K_1(\zeta)\simeq 1/\zeta$ for the AF case and performing the appropriate substitutions in addition to some algebraic manipulations, we arrive at the following asymptotic expressions:

1) DF

$$P^{\xi_1}_{\rm out} \simeq \prod_{m=1}^{L}\left(\gamma_{\rm th}\beta^Q_{SD_m}\right) \times \sum_{l=1}^{L}\Pr(D^*=D_l)(1-e^{(-\mu\beta_{SP})})(1-e^{(-\mu\beta_{R_nP})}) \times \prod_{n=1}^{N}\left[\gamma_{\rm th}\left(\beta^Q_{SR_n}+\beta^Q_{R_nD_m}\right)\right] \propto \left(\frac{1}{\overline{\gamma}}\right)^{L+N} \quad (30)$$

$$P^{\xi_2}_{\rm out} \simeq \prod_{m=1}^{L}\left(\gamma_{\rm th}\beta^Q_{SD_m}\right) \times \sum_{l=1}^{L}\Pr(D^*=D_l)(1-e^{\mu\beta_{SP}})e^{-\mu\beta_{R_nP}} \times \prod_{n=1}^{N}\left[\gamma_{\rm th}\left(\beta^Q_{SR_n}+\mu\beta^I_{R_nD_m}\right)\right] \propto \left(\frac{1}{\overline{\gamma}}\right)^{L+N} \quad (31)$$

$$P^{\xi_3}_{\rm out} \simeq \prod_{m=1}^{L}\left(\mu\gamma_{\rm th}\beta^I_{SD_m}\right) \times \sum_{l=1}^{L}\Pr(D^*=D_l)(1-e^{-\mu\beta_{R_nP}})e^{-\mu\beta_{SP}} \times \prod_{n=1}^{N}\left[\gamma_{\rm th}\left(\mu\beta^I_{SR_n}+\beta^Q_{R_nD_m}\right)\right] \propto \left(\frac{1}{\overline{\gamma}}\right)^{L+N} \quad (32)$$

$$P^{\xi_4}_{\rm out} \simeq \prod_{l=1}^{L}\left(\mu\gamma_{\rm th}\beta^I_{SD_l}\right)\Pr(D^*=D_l)(1-\mu\beta_{SP}-\mu\beta_{R_nP}) \times \prod_{n=1}^{N}\left[\mu\gamma_{\rm th}\left(\beta^I_{SR_n}+\beta^I_{R_nD_l}\right)\right] \propto \left(\frac{1}{\overline{\gamma}}\right)^{L+N} \quad (33)$$

2) AF

$$P^{\vartheta_1}_{\rm out} \simeq \prod_{m=1}^{L}\left(\gamma_{\rm th}\beta^Q_{SD_m}\right) \times \sum_{l=1}^{L}\Pr(D^*=D_l)(1-e^{(-\mu\beta_{SP})})(1-e^{(-\mu\beta_{R_nP})}) \times \prod_{n=1}^{N}\left[\gamma_{\rm th}\left(\beta^Q_{SR_n}+\beta^Q_{R_nD_m}\right)\right] \propto \left(\frac{1}{\overline{\gamma}}\right)^{L+N} \quad (34)$$

$$P^{\vartheta_2}_{\rm out} = P^{\xi_2}_{\rm out} \quad P^{\vartheta_3}_{\rm out} = P^{\xi_3}_{\rm out} \quad P^{\vartheta_4}_{\rm out} = P^{\xi_4}_{\rm out}. \quad (35)$$

From the previous equation, it is easy to see that the system under consideration achieves full diversity, with the diversity order being equal to $L+N$ for both DF and AF relaying protocols. In addition, note that this gain is not affected by the interference constraint, and it is only determined by the number of SU relays and destinations.

V. Numerical Plots and Discussions

To evaluate the outage performance of the considered dual-hop cognitive multirelay network, representative numerical examples are now presented. As will be observed, all the cases investigated revealed excellent agreement between analytical (exact $\rightarrow$ DF relays; approximate $\rightarrow$ AF relays) and simulation results. For the plots, without loss of generality, the statistical average of the channel gains is determined by the distance among the nodes, the threshold $\gamma_{\rm th}$ is set to 3 dB, and the path loss coefficient ρ is set to 4. The network is generated in a 2-D plane, where the SU source is located at (0, 0), the SU destinations are clustered together and collocated at (1, 0), the SU relays are also clustered together and collocated at (0.5, 0), and the PU receiver is located at (0, 1).

Figs. 2 and 3 show the outage probability against the system SNR for four different combinations of SU relays and destinations, as well as adopting DF and AF relays, respectively. Note that the asymptotic curves are shown to be very tight with the analytical curves at high-SNR regions, which confirms the correctness of our analysis. It is shown that the diversity order is given by $L+N$ for both DF and AF relays, as expected. In addition, in both figures, considering a diversity order equal to 3, the case with $\{N=2, L=1\}$ outperforms the case with $\{N=1, L=2\}$. In other words, for the same diversity order, the outage performance is higher when the number of SU relays surpasses the number of SU destinations. This shows that the use of CD is much more beneficial for the system performance than the use of multiuser diversity, which motivates the use of the former.

[6]Interestingly, such approximation will exhibit good tightness with the simulated results for all ranges of system SNRs.

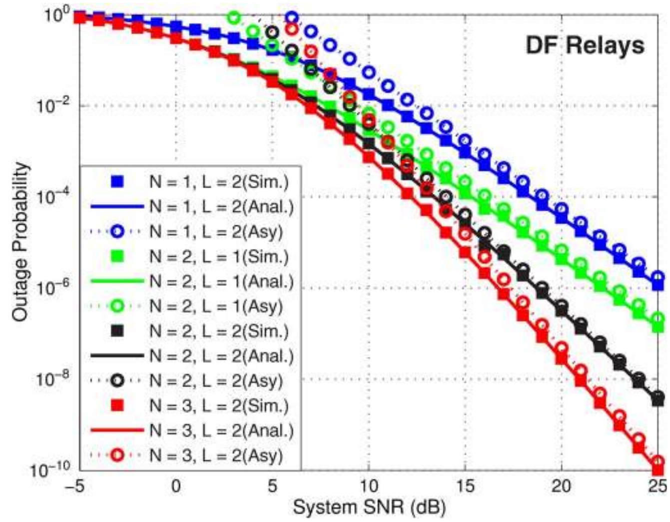

Fig. 2. Outage probability and asymptotic behavior versus system SNR using DF strategy for different numbers of SU relays and destinations ($Q = I = 0.5$).

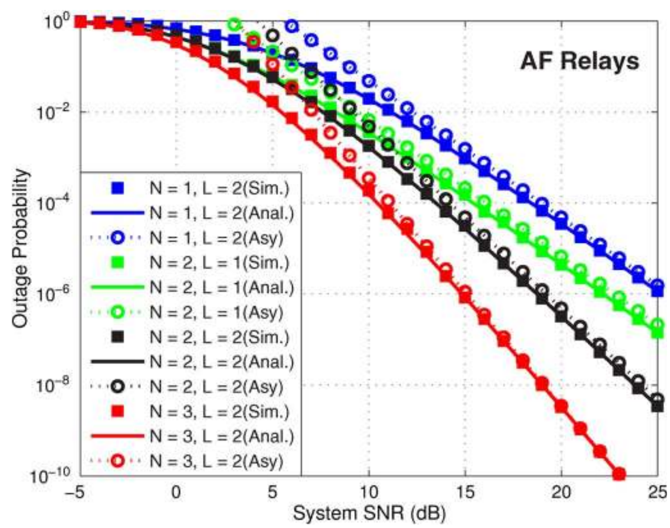

Fig. 3. Outage probability and asymptotic behavior versus system SNR using AF strategy for different numbers of SU relays and destinations ($Q = I = 0.5$).

Figs. 4 and 5 show the impact of interference temperature constraints on the outage probability for DF relays and AF relays, respectively, and assuming $N = L = 2$. Note that, due to the interference constraint, the outage probability of the system becomes saturated because the maximum allowed power for transmission is reached, as similarly observed in [5]. Moreover, as I gets larger, the outage performance improves, approaching the no-interference case.

In Fig. 6, a comparative outage analysis between the two relaying strategies is performed assuming the same diversity order (i.e., 6). To make the figures clearer, simulation data have been omitted. As expected, the outage performance for the DF relaying case is better than that for the AF relaying case. Note also that, when the number of SU relays increases, the gap between DF and AF becomes higher at low SNRs. This means that, when the CD gets higher, choosing the DF relaying strategy is even more preferable. Finally, in Fig. 7, a comparative analysis between DF and AF relaying protocols is carried out assuming that the outage probability becomes saturated. As expected, the outage performance increases for higher values of diversity order. In addition, as shown in Fig. 6, in this figure, the

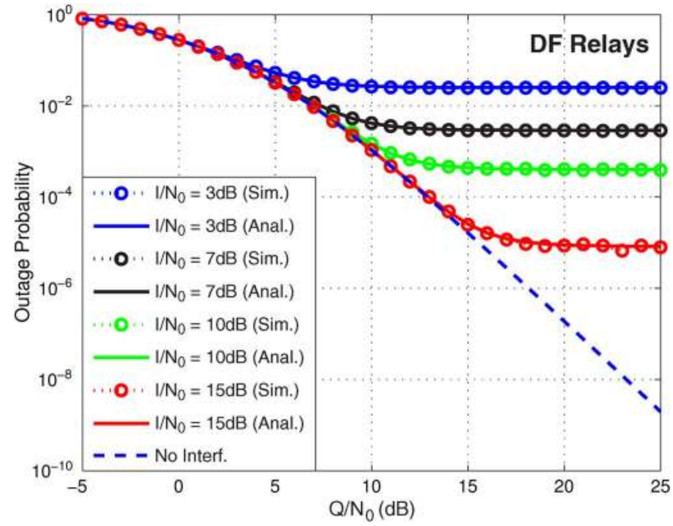

Fig. 4. Impact of interference constraints on the outage performance assuming DF relays ($N = L = 2$).

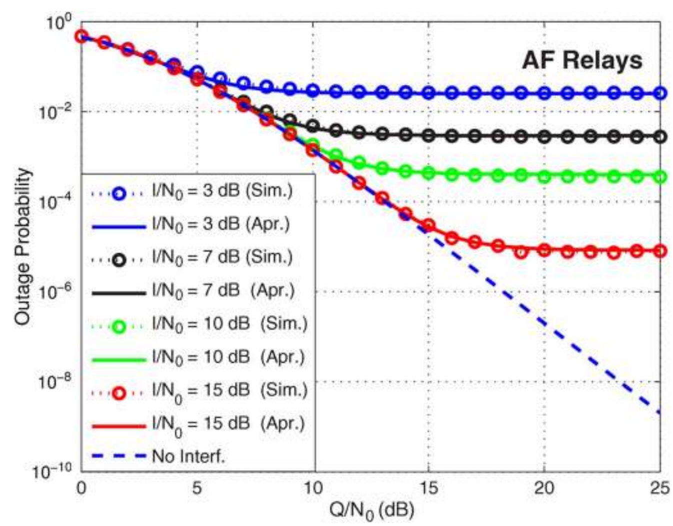

Fig. 5. Impact of interference constraints on the outage performance assuming AF relays ($N = L = 2$).

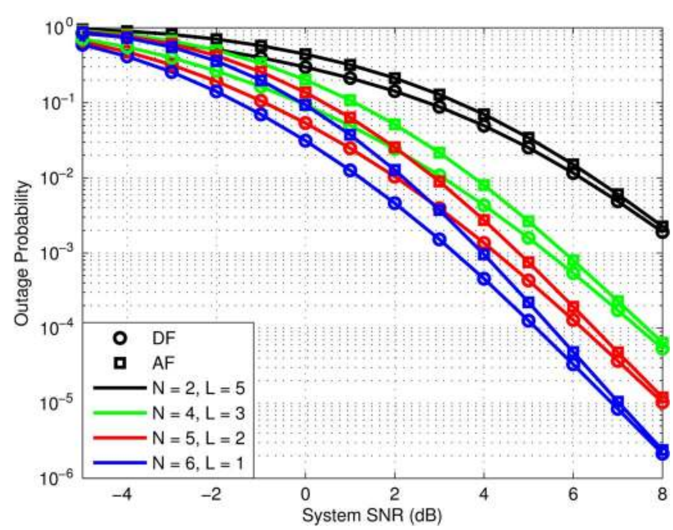

Fig. 6. Comparative outage analysis between DF and AF relaying protocols when $Q = I = 0.5$.

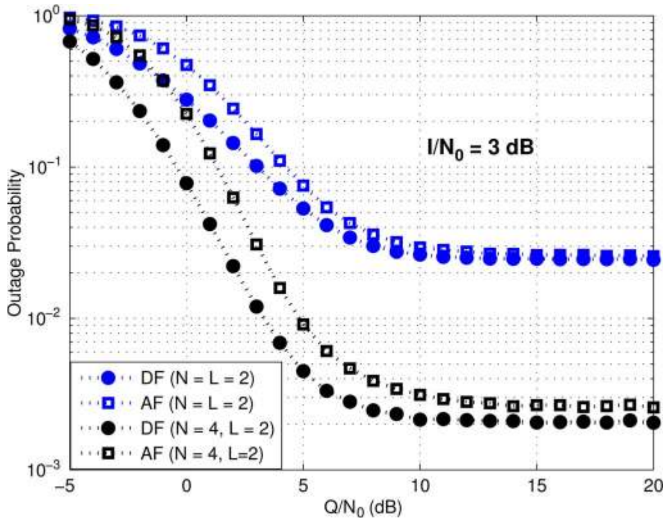

Fig. 7. Comparative analysis between DF and AF relaying protocols when the outage probability becomes saturated.

performance gap between DF and AF increases when the CD gets higher (i.e., when N increases).

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Embedded Iterative Semi-Blind Channel Estimation for Three-Stage-Concatenated MIMO-Aided QAM Turbo Transceivers

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***Abstract*—The lack of accurate and efficient channel estimation (CE) for multiple-input–multiple-output (MIMO) channel state information (CSI) has long been the stumbling block of near-MIMO-capacity operation. We propose a semi-blind joint CE and three-stage iterative detection/decoding scheme for near-capacity MIMO systems. The main novelty is that our decision-directed (DD) CE exploits the *a posteriori* information produced by the MIMO soft demapper within the inner turbo loop to select a "just sufficient number" of high-quality detected soft bit blocks or symbols for DDCE, which significantly improves the accuracy and efficiency of DDCE. Moreover, our DDCE is naturally embedded into the iterative three-stage detection/decoding process, without imposing an additional external iterative loop between the DDCE and the three-stage turbo detector/decoder. Hence, the computational complexity of our joint CE and three-stage turbo detector/decoder remains similar to that of the three-stage turbo detection/decoding scheme associated with the perfect CSI. Most significantly, the mean square error (MSE) of our DD channel estimator approaches the Cramér–Rao lower bound (CRLB) associated with the optimal-training-based CE, whereas the bit error rate (BER) of our semi-blind scheme is capable of achieving the optimal maximum-likelihood (ML) performance bound associated with the perfect CSI.**

***Index Terms*—Cramér–Rao lower bound (CRLB), joint channel estimation and three-stage turbo detection/decoding, multiple-input–multiple-output (MIMO) systems, near-capacity.**

I. INTRODUCTION

Under idealized conditions, coherent multiple-input–multiple-output (MIMO) systems are capable of achieving substantial diversity and/or multiplexing gains. However, the challenge is the acquisition of accurate MIMO channel state information (CSI) without imposing excessive pilot overhead, which would erode the system's throughput too much, and without resulting in potentially excessive channel estimation (CE) complexity. The current state of the art [1]–[16] typically combines the decision-directed (DD) CE (DDCE) with powerful iterative detection/decoding schemes to form semi-blind joint CE and turbo detection/decoding, where only a small number of training symbols are employed to generate an initial least squares channel estimate (LSCE). The turbo detection/decoding operation then commences with the initial LSCE. After the convergence of the turbo detector and decoder, the detected data are fed into the DDCE for the CE update. The DDCE and the turbo detector/decoder iterate a number of times until the channel estimate converges. The most effective schemes [10]–[13], [15], [16] employ soft-decision-aided channel estimators, which are more robust against error propagation than the hard-decision-aided CE schemes. Consequently, these joint soft-decision-based CE and turbo

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