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Multivariable Adaptive Controller for a Turbogenerator

S. M. Z. Sharaf
B. W. Hogg
O. H. Abdalla
M. L. El-Sayed

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S.M.Z. Sharaf, M.Sc., Ph.D., Prof. B.W. Hogg, Ph.D., C.Eng., F.I.E.E., Sen.Mem.I.E.E.E., O.H. Abdalla, M.Sc.(Eng.), Ph.D., Sen.Mem.I.E.E.E., and Prof. M.L. El-Sayed, M.Sc., Ph.D.

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Abstract: The paper describes the design and application of an adaptive controller which overcomes the nonlinearity of the turbogenerator system. The controller tracks the operating conditions and updates an optimal controller as conditions change. Modelling of the turbogenerator employs an output prediction equation and recursive least-squares algorithm. The design uses identified input/output models, obtained over a range of operating conditions in the *PQ*-plane. Optimal controllers are designed offline for each zone of the *PQ*-plane and stored in online memory as a look-up table. The computer monitors the operating conditions of the generator and selects the corresponding optimal controller. The paper presents a practical comparison between the fixed-gain and the look-up-table controllers over a wide range of operating conditions, with various fault conditions, including transient stability boundaries of each controller. The results show that a very high quality of control may be achieved using this adaptive controller.

List of symbols

P	= real power
Q	= reactive power
V	= voltage
δ	= rotor angle
p	$= \frac{d}{dt}$
U	= control input
ψ	= flux linkage
I	= current
X	= reactance
R	= resistance
T	= torque
H	= inertia constant
μ	= steam flow
τ	= time constant
F	= fraction of mechanical power
ρ	= boiler steam pressure
γ	= valve position
ω	= angular frequency
K	= sampling instant

Subscripts

T	= at generator terminals
ex	= exciter
g	= turbine governor
a	= armature
d, q	= direct, quadrature axis circuits
fd	= field winding
kd, kq	= d and q damper circuits
ad	= mutual, armature/field
fkd, fkq	= mutuals, field/damper circuits
akd, akq	= mutuals, armature/damper circuits
e	= electrical
m	= mechanical
hp, ip, lp	= high-, intermediate- and low-pressure stages of turbine
rh	= reheater
mv	= main turbine valve
iv	= turbine interceptor valve
t	= deviation of terminal quantity from steady-state value

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Dr. Sharaf and Prof. Hogg are with The Queen's University of Belfast, Department of Electrical & Electronic Engineering, Ashby Building, Stranmillis Road, Belfast BT9 5AH, United Kingdom. Dr. Abdalla and Prof. El-Sayed are with the Helwan University, 7 Moderiet El-Tahrir St., Garden City, Cairo, Egypt

1 Introduction

The increasing complexity of modern power systems, with high transmission voltages and large ratings for individual generating units, has prompted a substantial effort towards the development of improved methods of operation and control. One area of considerable interest is the application of modern control techniques to turbogenerators, and there have been numerous publications on this topic. The design of such controllers is usually based on linearised models of plant dynamics, and one of the fundamental problems is that of obtaining accurate models. This may be overcome by system identification [1]. A further difficulty arises owing to the nonlinearity of the system, which limits the validity of linear models to small perturbations about a particular set of operating conditions. It is therefore necessary to employ wide-range controllers [2], which are suboptimal, or to use some form of adaptive control [3–6].

This paper describes an adaptive controller which tracks the system operating point and updates an optimal controller as conditions change. The design employs identified input/output models [7], obtained over a range of operating conditions in the *PQ*-plane, and is implemented as a digital adaptive controller in the form of a look-up table. Optimal controllers are designed offline for each zone of the *PQ*-plane, and stored in online memory. The computer monitors the operating conditions of the generator, and selects the corresponding optimal controller; this occurs at each sampling instant, and so the controller is updated even during large transient disturbances. The scheme has been thoroughly tested on a laboratory model of a turbogenerator, and results are presented which show that it performs well.

2 System modelling

2.1 Systems equations

The system considered in this paper consists of a turbogenerator unit connected to an infinite busbar through a transformer and two transmission lines in parallel. The synchronous generator is described by a 7th-order nonlinear mathematical model. The transmission lines are represented by lumped series resistance and reactance, which can be combined with the transformer impedance. A thyristor exciter is assumed, to give fast control action, and the generator is driven by a 3-stage steam turbine with single reheater. The turbine is represented by a 6th-order model [8], in which each stage is described by a single time constant. The reheater and valve servomechanisms are also described by 1st-order transfer functions. The complete 13th-order system equations are given in Appendix 9.1.

2.2 Output prediction equation

The standard discrete-time state-space formulation of a linear time-invariant multivariable system is specified by the following equations:

$$X(K+1) = PX(K) + DU(K) \quad (1)$$

and

$$Y(K+1) = CX(K+1) \quad K = 0, 1, 2, \ldots \quad (2)$$

The derivation of the discrete prediction equation appears in Reference 9. This takes the form

$$Y(K+1) = AZ(K) + BV(K) \quad (3)$$

where

$$Z(K) = |\, Y(K)^T, Y(K-1)^T, \ldots, g(K-N+1)^T|^T$$

$$V(K) = |\, U(K)^T, U(K-1)^T, \ldots, U(K-N+1)^T|^T$$

and A and B are the system parameter matrices.

This output-prediction equation predicts the $(p \times 1)$ output vector $Y(K+1)$ at time $t = (K+1)T$, in terms of N previous output measurement vectors $Y(K)$ back to a $g(K-N+1)$ and N previous input control vectors $U(K)$ back to $U(K-N+1)$. For an observable system with n states, the minimum number of stages N is n/p, rounded up to the nearest integer. The output vector $g(K-N+1)$ represents the first r rows of $Y(K-N+1)$, such that $p(N-1) + r = n$, and thus the dimension of $Z(K)$ is $(n \times 1)$. Eqn. 3 defines the dynamics of a linear system completely, without reference to the state vector. Standard least-squares identification techniques may be applied to derive the parameter matrices A and B of this prediction equation (7).

2.3 Realisation of identified input/output turbogenerator model

The selected variables for the output vector Y are terminal power P_t, speed deviation $p\delta$ and terminal voltage V_t, respectively. These represent the principal generator quantities to be controlled, and are easily measured. The lower-case subscript t denotes deviations from steady-state values P_T and V_T. A 5th-order model of the form of eqn. 3 can be obtained, having a model structure with three outputs and two inputs, exciter voltage and governor valve position [7]. The output vector is

$$Y(K) = |\, P_t, p\delta, V_t|^T \quad (4)$$

The model employs values of output and control quantities obtained at the current sampling instant K, and also at the preceding sampling instant $(K-1)$, as defined by the vectors

$$Z(K) = |(P_t, p\delta, V_t)(K) \vdots (P_t, p\delta)(K-1)|^T \quad (5)$$

and

$$V(K) = |(U_e, U_g)(K) \vdots (U_e, U_g)(K-1)|^T \quad (6)$$

in which $(P_t, p\delta, V_t)(K)$ indicates values of these quantities obtained at the Kth sampling instant etc.

2.4 Simulation

The mathematical model of the system was used in preliminary simulation studies, and to investigate suitable model structures for identification. The initial development and evaluation of controllers was undertaken using the nonlinear mathematical model. Fig. 1 shows a comparison

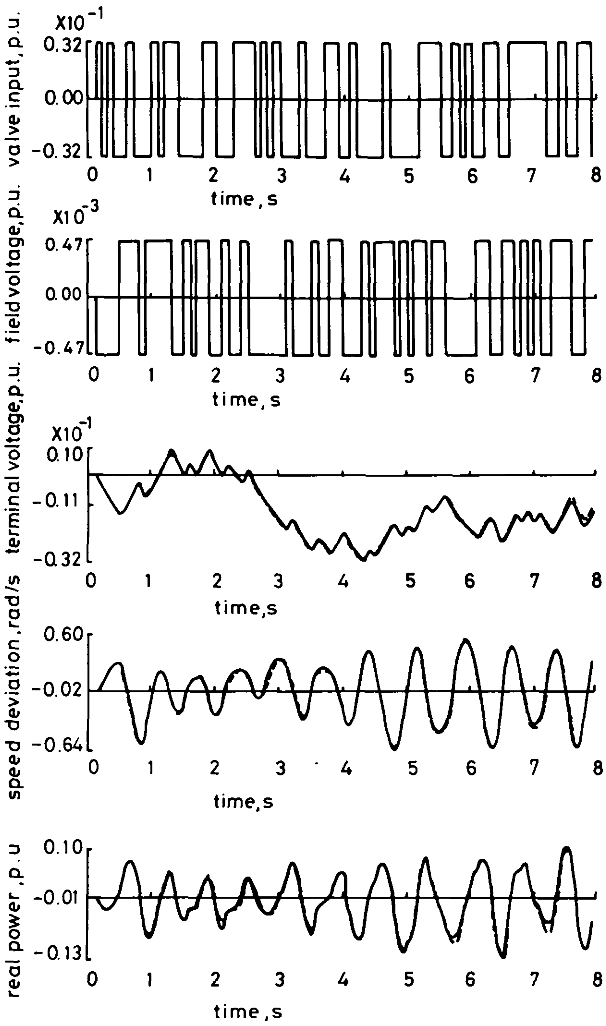

Fig. 1 *Comparison of 13th-order nonlinear model and identified model responses*

——— nonlinear model
– – – – identified model
$P = 0.8, Q = 0.0$

of the responses of the nonlinear model and a 5th-order identified input/output model, estimated using the unbiassed recursive least-squares approach which is described in Reference 10. The graphs show the pseudorandom binary sequence (PRBS) disturbances applied to the exciter and governor references, and the superimposed responses of the nonlinear model and the identified linear model. These are illustrated in terms of the deviations from the steady operating values. The sampling period is 20 ms, and the period of PRBS is 100 ms.

3 Laboratory system

3.1 Micromachine and computer hardware

The micromachine is a laboratory model turbogenerator, which has been described elsewhere [1, 2]. It consists of a 3 kVA 3-phase microalternator, driven by a separately-excited DC motor, and connected to the laboratory busbar through a transformer and two transmission lines. A 3-stage steam turbine with single reheater is represented by analogue simulation, and the output of the simulation determines the shaft torque produced by the motor. This is achieved by suitable thyristor control of the motor armature and field currents.

A modular instrument computer (MINC) has been employed to generate the disturbances, synthesise the forced inputs to the micromachine, and monitor up to 24 quantities for identification and control; the system is shown in Fig. 2. It consists of an LSI 11/23 processor with 256 K bytes of RAM, a floating point processor, two 512 K bytes floppy disc drives, an IEEE bus, analogue-to-digital (A/D) and digital-to-analogue (D/A) convertors. The analogue-to-digital module consists of a single convertor and a 16-channel multiplexer, and the dual multiplexer (MPX) preceding the A/D module increases the capacity to 32 A/D channels. There is no sample-and-hold facility on any of these channels, and so a unit was constructed to provide simultaneous sampling of all channels. The line clock module generates a hold pulse at regular intervals, and interrupts the computer. On receipt of this interrupt, the computer executes an interrupt service routine which converts, and stores in memory, the current instantaneous values of the monitored signals. The digital-to-analogue module contains four independently addressable convertors, and is used to transmit transient control signals or random disturbances from the computer to the micromachine.

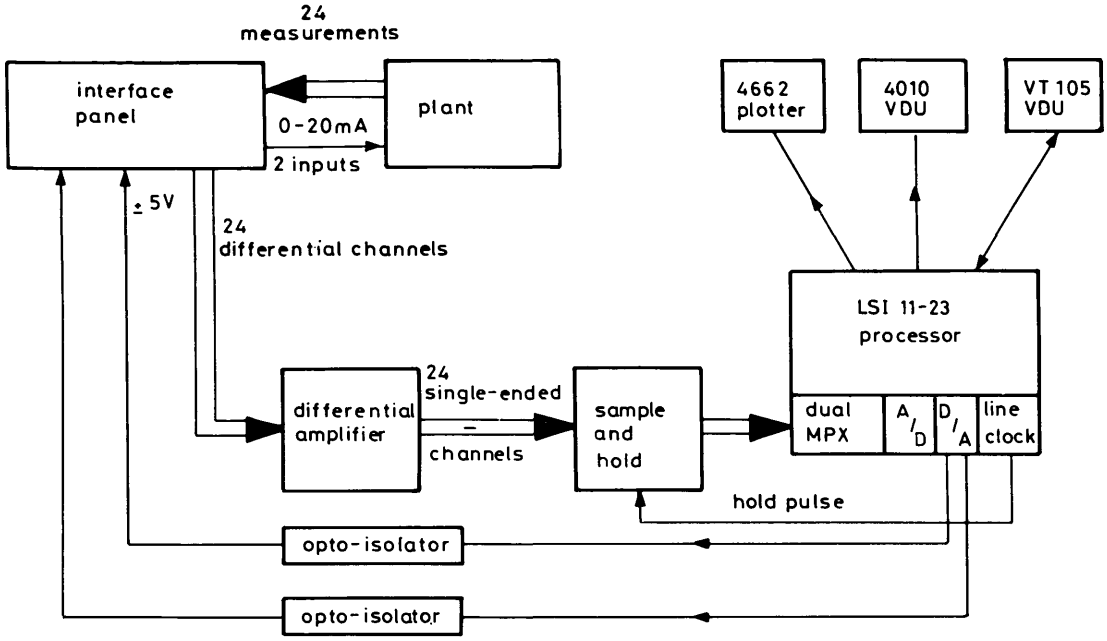

Fig. 2 *MINC/micromachine*
MPX = multiplexer

3.2 Software
The micromachine software package consists of the Fortran mainline code, a fast graphics assembler language routine, and several real-time assembler language routines, and is run under the RT–11 operating system. It generates the disturbances and the forced inputs to the micromachine, monitors the desired quantities, filters the raw data, converts the filtered data to engineering units, holds the results in memory during the test, and stores the data on floppy at the end of each test. It also provides for the simultaneous graphical display of eight channels of data during each test on a Tektronix 4010 VDU, and for hard copies (Tektronix 4662 plotter) at the end of each test.

The Fortran program is used for the question-and-answer dialogue with user. Before each test begins the scaling factors and filter time constants for each monitored quantity, along with the type, level and period of the disturbances are obtained by questioning the user, or from a previously compiled user disc file. During the tests, the dialogue is mainly related to the graphical display (e.g. channels to be displayed, time period of display etc.)

The real-time assembler-language subroutines form an interrupt service routine, which is entered each time the line clock sends a hold pulse to the sample-and-hold unit. The assembler-language routines are as follows:

(i) *Analogue-to-digital conversion and filtering*

(ii) *Scaling routine:* The scaling factors are obtained from the Fortran program, and the raw data is converted to engineering units by the floating point processing unit

(iii) *Memory storage:* The LSI 11/23 processor can only directly address up to 32 K words of memory, but by means of a memory management unit (MMU), a further 96 K words of memory can be accessed. This further 96 K words of memory is used by this routine to store the scaled data. As two words are required to store a floating point value, 48 K floating point numbers can be stored. Therefore, when storing 26 channels, 24 measurements and two inputs, 1890 samples of each channel can be stored

(iv) *Disturbance generation:* This subroutine generates up to 4 distinct pseudorandom binary sequences with the desired periods and amplitudes, for use in system identification tests.

At the end of each test the data held in memory is transferred to floppy disc and hard copies of graphs can be obtained.

3.3 Micromachine identification
The laboratory was operated at various steady-state conditions, defined in terms of the active power P_T and reactive power Q_T at the generator terminals, PRBS disturbances were applied simultaneously to the exciter and turbine governor references, and the system state deviations were measured. This data was processed by an unbiassed recursive least-squares algorithm [7] to achieve identified models of the system.

The quality of the identified models may be assessed by comparing the responses of the micromachine and those of the models, at the same operating conditions and with the same inputs. Fig. 3 shows a typical set of results.

4 Optimal controller

The identified model parameter matrices of the turbogenerator system are estimated, and discrete optimal controllers can be designed based on the model parameters by minimising a quadratic performance index (Appendix 9.2). The choice of the weighting matrices becomes very important, and these have to be selected carefully to achieve good performance for both steady-state and transient conditions.

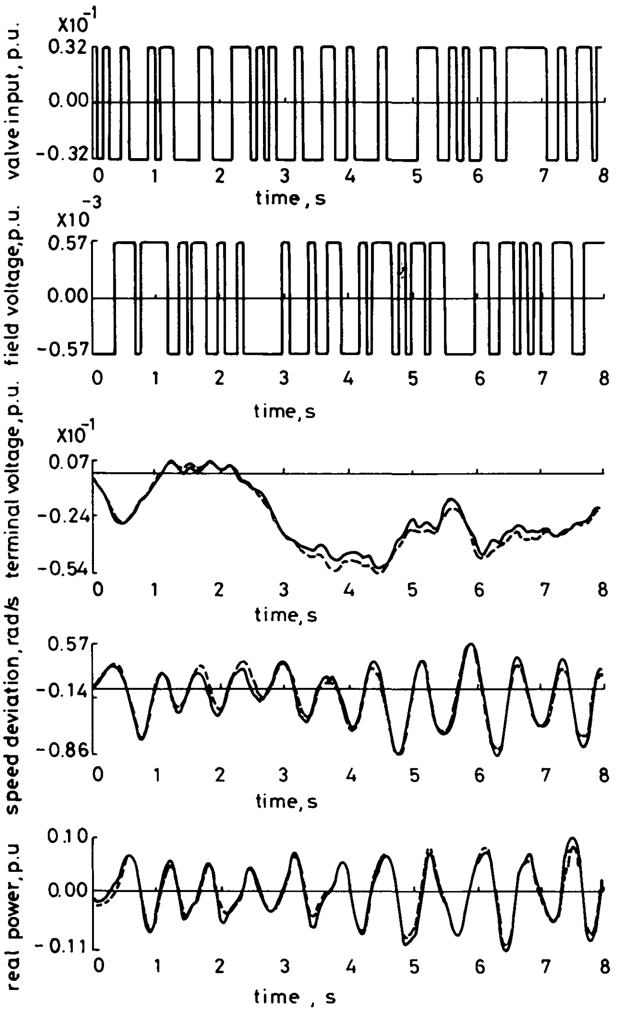

Fig. 3 *Comparison of micromachine and identified model responses*
——— micromachine
- - - - model

No suitable analytical procedure appears to be available for this purpose, and an interactive trial-and-error approach was adopted in the context of computer-aided design.

The closed-loop dynamic behaviour of the turbogenerator system becomes

$$U(K) = KpW(K) \tag{7}$$

where

$$W(K) = | P_t, p\delta, V_t)(K), (P_t, p\delta)(K-1), (U_e, U_g)(K-1)|^T$$

Using the dynamic programming procedure, and with knowledge of the parameter matrices at a certain operating point, the optimal feedback gain matrix KF is calculated [7].

The gains of an optimal controller designed using eqn. 7 depend on the parameters of the turbogenerator model, which vary with the operating conditions, and a fixed-gain controller is suboptimal at all but one set of conditions. The effect of the performance of the system has been investigated by comparing the responses of the micromachine at various operating points, using a fixed-gain controller and controllers which are optimal at each point. Fig. 4 shows responses at $P_T = 0.8$ p.u., $Q_T = 0.0$ p.u., when a 3-phase short circuit is applied at the generator terminals for 120 ms. One controller has been designed for an identified model obtained at $P_T = 0.8$, $Q_T = 0.0$, and the other for a model at $P_T = 0.8$, $Q_T = 0.6$.

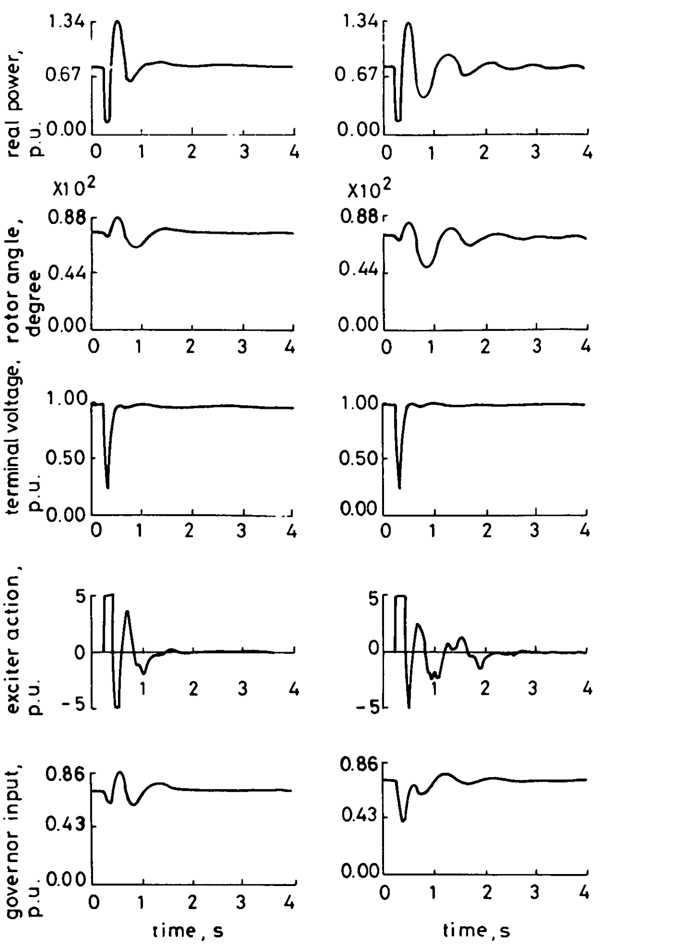

Fig. 4 *Controllers comparison at $P_T = 0.8$ and $Q_T = 0.0$*
The first is designed at $P_T = 0.8$, $Q_T = 0.0$, and the second is designed at $P_T = 0.8$, $Q_T = 0.6$

From a large number of similar results, it was evident that the performance of fixed-gain optimal controllers deteriorated significantly as conditions changed, and that some form of adaptive scheme is desirable. Ideally, this should minimise online computation, to achieve the essential high-speed control and maximum reliability.

5 Look-up-table adaptive controller

5.1 Introduction

A practical method of dealing with this problem is to employ a look-up-table technique, based on the system identification. The design employs identified input/output models, obtained over a range of operating conditions in the *PQ*-plane, and is implemented as a digital adaptive controller in the form of a look-up table. Optimal controllers are designed offline for each small zone of the *PQ*-plane and stored in the online computer memory. The computer monitors the operating conditions of the generator and selects the corresponding controller. This occurs at each sampling instant, so that the controller is updated even during transient disturbances.

5.2 Look-up-table controller planning and design

The operating region of the generator in the PQ-plane was divided into 17 zones, and an optimal controller was designed for an operating point at the centre of each zone. The zones are defined by a rectangular grid, and the number was selected by engineering judgment, to achieve a reasonable approximation of continuous adaptive control, consistent with keeping the number of controllers to be designed and stored to a reasonable limit. The controller gains associated with each zone are stored in online memory in the form of a look-up table. The computer tracks the operating point of the generator by measuring the outputs at each sampling instant; i.e. active and reactive power, rotor angle, speed deviation, governor valve position and exciter input voltage. It then selects the nearest operating point in the look-up table, and introduces the corresponding controller gains.

The block diagram in Fig. 5 shows the arrangement for

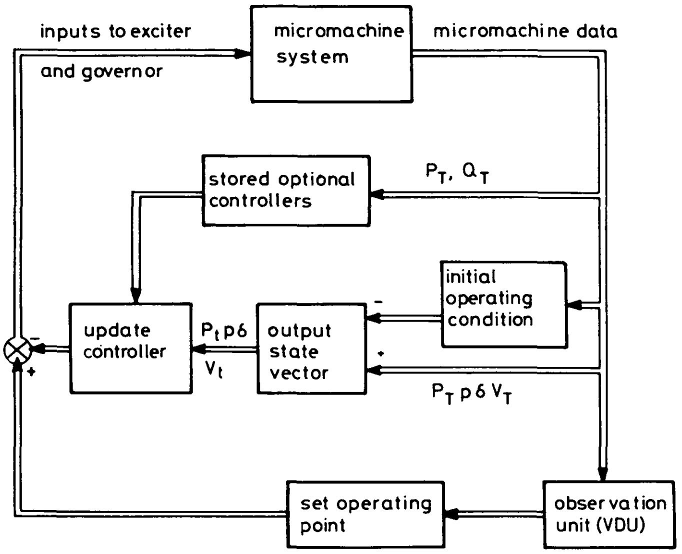

Fig. 5 *Schematic diagram of adaptive controller*

implementing the controller. The initial steady-state conditions are set by adjusting the governor valve position and exciter voltage to the required levels. The computer measures the inputs and outputs every sampling instant, and the sampling period T is 20 ms. Just before the computer goes into closed-loop control, it stores the initial values of P_T and V_T. Once closed-loop control begins, the control program continuously reads the output values, and subtracts the initial conditions of each from the read values. The resultant values are representative of the output vector $| P_t, p\delta, V_t |^t$ at each sample.

According to the active and reactive power, the control program will choose the corresponding optimal controller from the online look-up-table memory. Then the computer will calculate the exciter and governor inputs according to eqn. 7 by simple multiplications, and the inputs are updated accordingly. The calculation time is 7.5 ms, which is much less than the chosen sampling period of 20 ms. In contrast, a fully adaptive scheme, based on reduced-order models obtained by identification, requires a calculation time of 38 ms, with a corresponding sampling period of 40 ms [11].

6 Experimental results

This Section provides a comparison between the look-up-table controllers and a fixed-gain controller whose design is based on identified model parameters estimated at the operating point $P_T = 0.8$ p.u. and $Q_T = 0.0$. An objective comparison is achieved by using each controller with the laboratory micromachine system at the same operating point with the same constraints. An assessment has been undertaken over a wide range of operating conditions, and typical results are presented.

Fig. 6 shows the performance of the system when a

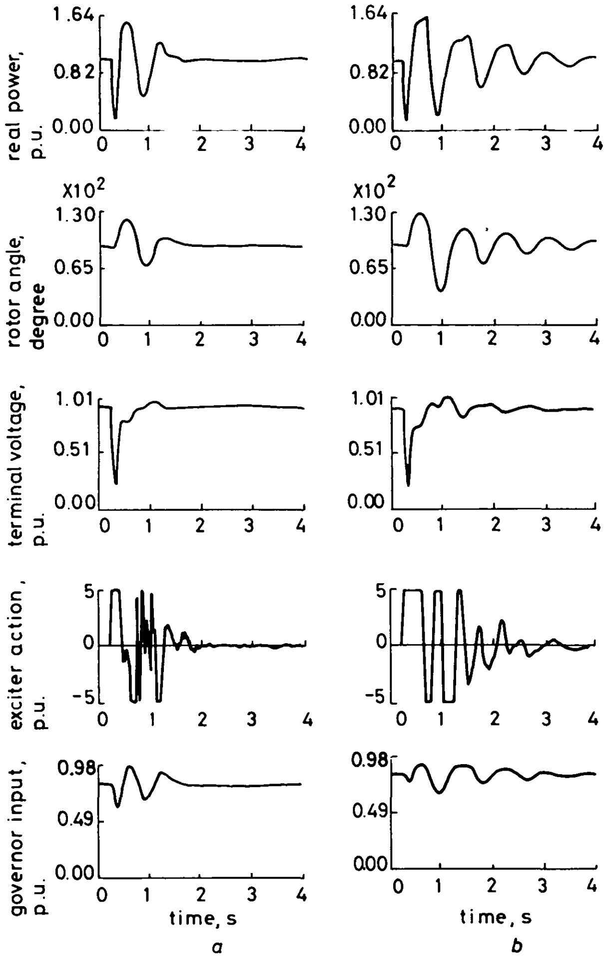

Fig. 6 *Comparison of adaptive and fixed-gain controllers*
a Adaptive *b* Fixed-gain

3-phase short circuit of duration 120 ms is applied at the sending end of the transmission lines. The steady-state operating point is $P_T = 1.0$ p.u., $Q_T = 0$.

Figs. 7–9 illustrate the behaviour of the system with the look-up-table controller under very severe transient conditions, which render it unstable with a fixed-gain controller. The initial operating point is $P_T = 0.8$ p.u., $Q_T = 0.4$ p.u., and the fault period is 480 ms. Fig. 7 shows the response of the system, Fig. 8 shows the variation of the controller gains ($Kp(1, 1)$, $Kp(1, 2)$, $Kp(1, 3)$, $Kp(1, 6)$, $Kp(1, 7)$, $Kp(2, 1)$, $Kp(2, 2)$, $Kp(2, 3)$, $Kp(2, 6)$ and $Kp(2, 7)$, referring to eqn. 7, as the operating point changes, and Fig. 9 depicts the trajectory in the PQ-plane.

The transient stability limits for look-up table and fixed-gain controllers are shown in Fig. 10. These define the range of operating conditions for which the generator remains in synchronism, when a 3-phase short circuit of duration t_f occurs. Boundaries are determined by holding P_T constant (at 0.8 p.u. in this case) and selecting a range of values of Q_T. At each of these values, the short circuit is applied with increasing durations, until instability occurs. The corresponding values of Q_T and fault time define a point on the boundary. The boundaries change with different values of P_T, but provide a good indication of the effectiveness of controllers. The stable region is below the boundary lines.

7 Conclusions

The paper describes a practical application of optimal control techniques, which overcomes the disadvantages of fixed-gain controllers for nonlinear turbogenerator systems. Although based on identified plant models, the controllers depend on monitoring plant operating conditions, and avoid online computation of controller gains. The look-up-table arrangement, in conjunction with high-speed measurement techniques, constitutes a reliable and high-performance controller for all operating conditions.

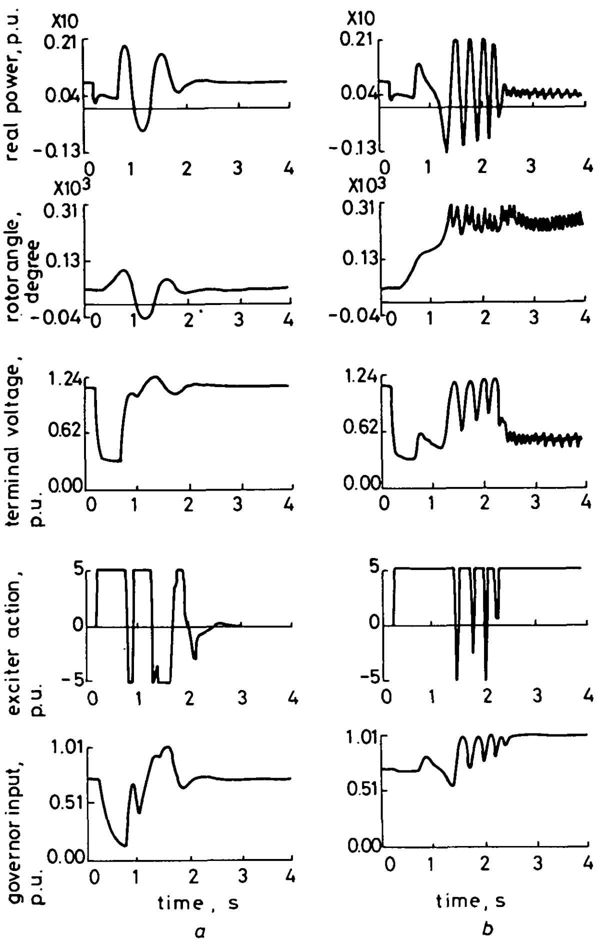

Fig. 7 *Comparison of adaptive and fixed-gain controllers*
$P_T = 0.8$, $Q_T = 0.4$ and $t_f = 480$ ms
a Adaptive
b Fixed-gain

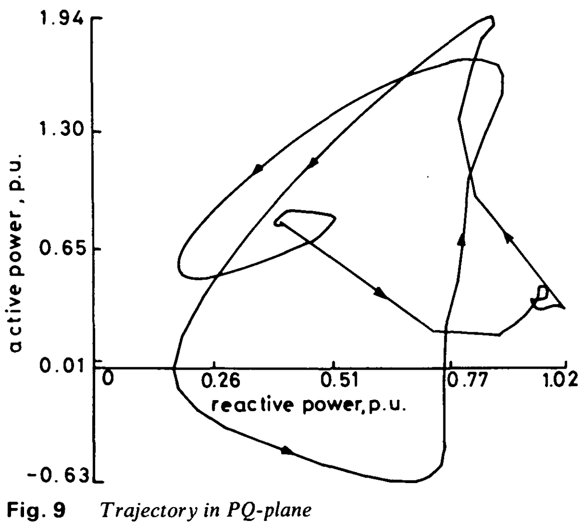

Fig. 9 *Trajectory in PQ-plane*
$P_T = 0.8$, $Q_T = 0.4$ and $t_f = 480$ ms

The investigation of an approximate form of adaptive control also served to justify the further development of continuous adaptive controllers for this application [11].

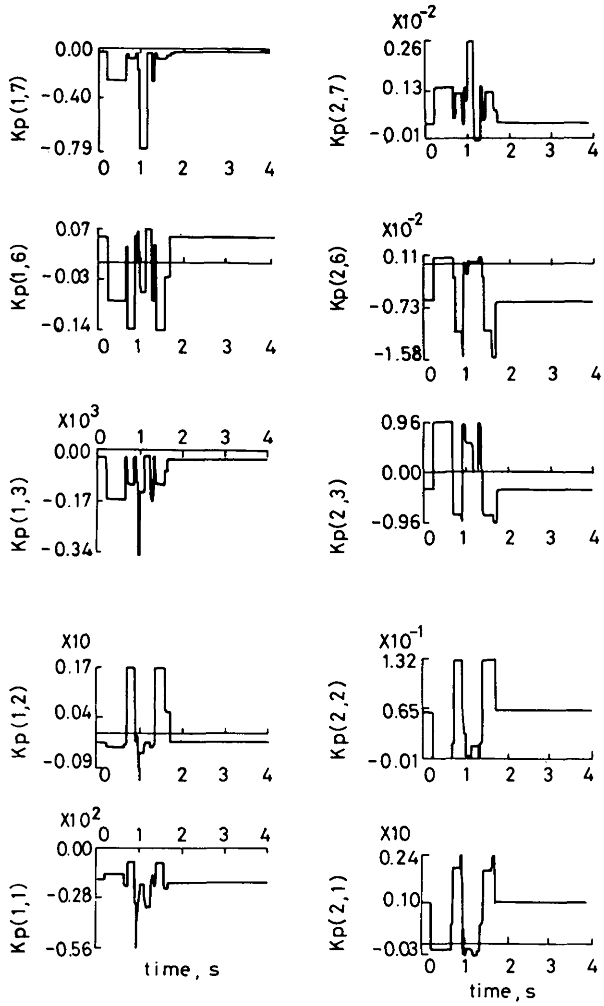

Fig. 8 *Variation of gain during transient*
$P_T = 0.8$, $Q_T = 0.4$ and $t_f = 480$ ms

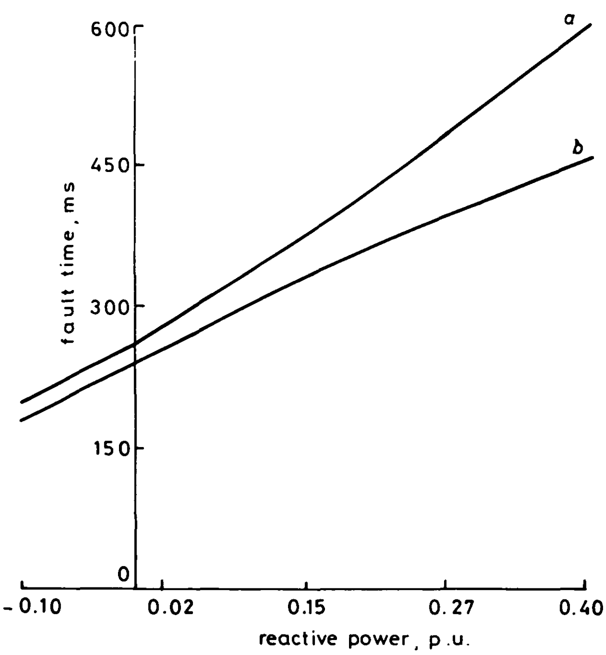

Fig. 10 *Transient stability boundaries*
a Adaptive and *b* Fixed-gain
($P_T = 0.8$ p.u.)

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9 Appendixes

9.1 System equations

The synchonous generator and transmission system equations are given in per-unit as follows [7, 8]:
The direct flux linkage equations are

$$\psi_{fd} = X_{fd} I_{fd} - X_{ad} I_d + X_{fkd} I_{kd} \tag{8}$$

$$\psi_d = X_{ad} I_{fd} - (X_d + X_e) I_d + X_{akd} I_{kd} \tag{9}$$

$$\psi_{kd} = X_{fkd} I_{fd} - X_{akd} I_d + X_{kd} I_{kd} \tag{10}$$

The quadrature flux linkage equations are

$$\psi_q = -(X_q + X_e) I_q + X_{akq} I_{kg} \tag{11}$$

$$\psi_{kq} = -X_{akq} I_q + X_{kq} I_{kq} \tag{12}$$

The voltage equations are

$$V_{fd} = (1/Wo) p\psi_{fd} + R_{fd} I_{fd} \tag{13}$$

$$V_d = (1/Wo) p\psi_d - R_a I_d - (W/Wo)\psi_q \tag{14}$$

$$V_{kd} = (1/Wo) p\psi_{kd} - R_{kd} I_{kd} = 0 \tag{15}$$

$$V_q = (1/Wo) p\psi_q - R_a I_q + (W/Wo)\psi_d \tag{16}$$

$$V_{kq} = (1/Wo) p\psi_{kq} + R_{kq} I_{kq} = 0 \tag{17}$$

The electric torque T_e equation is

$$T_e = \psi_d I_q - \psi_q I_d \tag{18}$$

The mechanical equation is

$$\left(\frac{2H}{Wo}\right) p^2\delta = T_m - T_e - Kp\delta \tag{19}$$

The machine terminal equations are

$$V_{tb} = V_b \sin(\delta) + R_e I_d - X_e I_q \tag{20}$$

$$V_{tq} = V_b \cos(\delta) + R_e I_q + X_e I_d \tag{21}$$

$$V_t^2 = V_{td}^2 + V_{tq}^2 \tag{22}$$

$$I_t^2 = I_d^2 + I_q^2 \tag{23}$$

$$P_t = V_{td} I_d + V_{tq} I_q \tag{24}$$

Steam turbine equations:

$$p\mu_{hp} = (\rho Y_{mv} - \mu_{hp})/\tau_{hp} \tag{25}$$

$$p\mu_{rh} = (\mu_{hp} - \mu_{rh})/\tau_{rh} \tag{26}$$

$$p\mu_{ip} = (\mu_{rh} Y_{iv} - \mu_{ip})\tau_{ip} \tag{27}$$

$$p\mu_{lp} = (\mu_{ip} - \mu_{lp})/\tau_{lp} \tag{28}$$

$$p\gamma_{mv} = (U_g - \gamma_{mv})/\tau_{mv} \tag{29}$$

$$p\gamma_{iv} = (U_g - \gamma_{iv})/\tau_{iv} \tag{30}$$

$$T_m = F_{hp}\mu_{hp} + F_{ip}\mu_{ip} + F_{lp}\mu_{lp} \tag{31}$$

9.2 Optimal control law

A discrete optimal controller can be designed based on the discrete linear state-space model of a turbogenerator system.

Given a discrete turbogenerator prediction equation

$$W(K+1) = P_1 W(K) + D_1 U(K) \tag{32}$$

An optimal controller can be designed for this system by minimising the quadratic performance index

$$J = \tfrac{1}{2} \sum_{k=0}^{S} | W^T(K) Q W(K) + U^T(K) R U(K) | \tag{33}$$

where S is the number of iterations after which the controller parameters become time-invariant, and

$$W(K) = | P_t(K) p\delta(K) V_t(K) P_t(K-1) p\delta(K-1) U_e \times (K-1) U_g(K-1) |^T \tag{34}$$

$$U(K) = | U_e(K) U_g(K) |^T$$

Q is the square positive semidefinite symmetriic matrix, and R is a square positive definite symmetric matrix. These matrices may be selected to weight the system vectors.
By using the Riccati transformation,

$$U(K) = -| R + D_1^T B_1(K+1) D_1 |^{-1} D_1^T B_1(K+1) P_1(K+1) W(K) \tag{35}$$

and

$$B_1(K) = P_1^T | B_1(K+1) - B_1(K+1) D_1 \times (R + D_1^T B_1(K+1) D_1)^{-1} \times D_1^T B_1(K+1) | P_1 + Q \tag{36}$$

The initial values of $B_1(S)$ may be equal to Q, and eqn. 34 must be solved backwards with the initial values of B_1.

To solve for the control vector U use eqn. 35,

$$U(K) = -Kp W(K) \tag{37}$$

where

$$Kp = | R + D_1^T B_1(K+1) D_1 |^{-1} D_1^T B_1(K+1) P_1 \tag{38}$$

Kp is the fixed-gain optimal controller of the turbogenerator system.