Multivariable Feedback Control

Analysis and Design

Sigurd Skogestad
Norwegian University of Science and Technology

Ian Postlethwaite
University of Leicester

JOHN WILEY & SONS
Chichester · New York · Brisbane · Toronto · Singapore

CONTENTS

PREFACE ix

1 INTRODUCTION 1
- 1.1 The process of control system design 1
- 1.2 The control problem 2
- 1.3 Transfer functions 3
- 1.4 Scaling 5
- 1.5 Deriving linear models 8
- 1.6 Notation 11

2 CLASSICAL FEEDBACK CONTROL 15
- 2.1 Frequency response 15
- 2.2 Feedback control 21
- 2.3 Closed-loop stability 24
- 2.4 Evaluating closed-loop performance 27
- 2.5 Controller design 39
- 2.6 Loop shaping 40
- 2.7 Shaping closed-loop transfer functions 54
- 2.8 Conclusion 62

3 INTRODUCTION TO MULTIVARIABLE CONTROL 63
- 3.1 Introduction 63
- 3.2 Transfer functions for MIMO systems 64
- 3.3 Multivariable frequency response analysis 68
- 3.4 Control of multivariable plants 79
- 3.5 Introduction to multivariable RHP-zeros 84
- 3.6 Condition number and RGA 86
- 3.7 Introduction to MIMO robustness 91
- 3.8 General control problem formulation 98
- 3.9 Additional exercises 110
- 3.10 Conclusion 112

4 ELEMENTS OF LINEAR SYSTEM THEORY 113
4.1 System descriptions 113
4.2 State controllability and state observability 122
4.3 Stability 127
4.4 Poles 127
4.5 Zeros 130
4.6 More on poles and zeros 132
4.7 Internal stability of feedback systems 137
4.8 Stabilizing controllers 142
4.9 Stability analysis in the frequency domain 144
4.10 System norms 151
4.11 Conclusion 157

5 LIMITATIONS ON PERFORMANCE IN SISO SYSTEMS 159
5.1 Input-Output Controllability 159
5.2 Perfect control and plant inversion 163
5.3 Constraints on S and T 164
5.4 Ideal ISE optimal control 172
5.5 Limitations imposed by time delays 173
5.6 Limitations imposed by RHP-zeros 174
5.7 Non-causal controllers 182
5.8 Limitations imposed by RHP-poles 184
5.9 Combined RHP-poles and RHP-zeros 185
5.10 Performance requirements imposed by disturbances and commands 187
5.11 Limitations imposed by input constraints 189
5.12 Limitations imposed by phase lag 193
5.13 Limitations imposed by uncertainty 195
5.14 Controllability analysis with feedback control 196
5.15 Controllability analysis with feedforward control 200
5.16 Applications of controllability analysis 201
5.17 Conclusion 212

6 LIMITATIONS ON PERFORMANCE IN MIMO SYSTEMS 213
6.1 Introduction 213
6.2 Constraints on S and T 214
6.3 Functional controllability 218
6.4 Limitations imposed by time delays 220
6.5 Limitations imposed by RHP-zeros 221
6.6 Limitations imposed by RHP-poles 224
6.7 RHP-poles combined with RHP-zeros 224
6.8 Performance requirements imposed by disturbances 226
6.9 Limitations imposed by input constraints 228
6.10 Limitations imposed by uncertainty 234

6.11 Input-output controllability 246
6.12 Conclusion 252

7 UNCERTAINTY AND ROBUSTNESS FOR SISO SYSTEMS 253
7.1 Introduction to robustness 253
7.2 Representing uncertainty 255
7.3 Parametric uncertainty 257
7.4 Representing uncertainty in the frequency domain 259
7.5 SISO Robust stability 270
7.6 SISO Robust performance 276
7.7 Examples of parametric uncertainty 283
7.8 Additional exercises 289
7.9 Conclusion 290

8 ROBUST STABILITY AND PERFORMANCE ANALYSIS 291
8.1 General control configuration with uncertainty 291
8.2 Representing uncertainty 294
8.3 Obtaining P, N and M 301
8.4 Definitions of robust stability and robust performance 303
8.5 Robust stability of the $M\Delta$-structure 304
8.6 RS for complex unstructured uncertainty 306
8.7 RS with structured uncertainty: Motivation 309
8.8 The structured singular value 311
8.9 Robust stability with structured uncertainty 319
8.10 Robust performance 322
8.11 Application: RP with input uncertainty 326
8.12 μ-synthesis and DK-iteration 335
8.13 Further remarks on μ 344
8.14 Conclusion 347

9 CONTROLLER DESIGN 349
9.1 Trade-offs in MIMO feedback design 349
9.2 LQG control 352
9.3 $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control 362
9.4 $\mathcal{H}_\infty$ loop-shaping design 376
9.5 Conclusion 396

10 CONTROL STRUCTURE DESIGN 397
10.1 Introduction 397
10.2 Optimization and control 399
10.3 Selection of controlled outputs 402
10.4 Selection of manipulations and measurements 408
10.5 RGA for non-square plant 410

10.6 Control configuration elements 413
10.7 Hierarchical and partial control 422
10.8 Decentralized feedback control 432
10.9 Conclusion 448

11 MODEL REDUCTION 449
11.1 Introduction 449
11.2 Truncation and residualization 450
11.3 Balanced realizations 451
11.4 Balanced truncation and balanced residualization 452
11.5 Optimal Hankel norm approximation 454
11.6 Two practical examples 456
11.7 Reduction of unstable models 465
11.8 Model reduction using MATLAB 466
11.9 Conclusion 467

12 CASE STUDIES 469
12.1 Introduction 469
12.2 Helicopter control 470
12.3 Aero-engine control 480
12.4 Distillation process 490
12.5 Conclusion 496

A MATRIX THEORY AND NORMS 497
A.1 Basics 497
A.2 Eigenvalues and eigenvectors 500
A.3 Singular Value Decomposition 503
A.4 Relative Gain Array 510
A.5 Norms 514
A.6 Factorization of the sensitivity function 526
A.7 Linear fractional transformations 528

B PROJECT WORK and SAMPLE EXAM 533
B.1 Project work 533
B.2 Sample exam 534

BIBLIOGRAPHY 539

INDEX 548