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Multivariable MRAC with State Feedback for Output Tracking

Jiaxing Guo, *Yu Liu* and *Gang Tao*

Department of Electrical and Computer Engineering
University of Virginia
Charlottesville, VA 22904

Abstract— **This paper revisits the multivariable MRAC problem, by studying adaptive state feedback control for output tracking of multi-input and multi-output (MIMO) systems. With such a control scheme, the plant-model matching condition is much less restrictive than those for state tracking, while the controller enjoys a simpler structure than that of an output feedback design with the guarantee of the asymptotic tracking of multiple outputs. Such a control scheme is useful for applications when the plant-model matching condition for state tracking cannot be satisfied. A stable adaptive control scheme is developed based on LDS decomposition of high frequency gain matrix, which ensures closed-loop stability and asymptotic output tracking. A simulation study is conducted for an aircraft model, with desired simulation results presented.**

Keywords: Multivariable model reference adaptive control, state feedback, output tracking, high frequency gain matrix, LDS decomposition.

I. INTRODUCTION

Model reference adaptive control (MRAC) is one of the most important adaptive control methods, which provides feedback controller structures and adaptive laws for control of systems to ensure closed-loop signals boundedness and asymptotic tracking of independent reference signals by the system signals, despite of the uncertainties of the system parameters. Much effort has been devoted, such as [2], [3], [6], and [9]. While MRAC theory has evolved into a mature control theory, refined and new designs of MRAC schemes are still needed for many applications, especially, for systems with multiple inputs and multiple outputs, such as aircraft systems, where some important issues remain open.

MRAC can be designed using either state feedback or output feedback. In many applications, such as flight control, system states are available and state feedback control is commonly used due to its simpler structure (as compared with compensator based output feedback designs) and powerful functions. State feedback control systems can be designed for either state tracking or output tracking with different design conditions. To develop an adaptive state feedback controller, it is necessary to solve the related nonadaptive control problem assuming the plant parameters are known, thus an ideal fixed state feedback controller can be obtained. This ideal (nominal) controller will be used as *a priori* knowledge in the design of the adaptive control scheme. The existence of such a nominal controller is equivalent to a set of matching equations. A state feedback controller for state tracking has a restrictive matching condition which can only be satisfied for system matrices in certain canonical forms, which largely confines its applications in control problems. State feedback for output tracking, on the other hand, while keeping the simple controller structure, needs a less restrictive matching condition, and does not require system matrices in canonical forms. Therefore, state feedback for output tracking adaptive control has high potential for important applications such as aircraft flight control when the matching conditions for state tracking are often difficult to satisfy due to system parameter uncertainties.

Research in adaptive state feedback control for output tracking has been reported in the literature. In [4], state feedback output tracking control is studied for certain classes of nonlinear systems. In [7], a state feedback output tracking MRAC scheme for single input single output (SISO) systems is derived. Technical issues including design conditions, plant-model matching conditions, controller structures, adaptive laws and stability analysis are addressed in detail, with extensions to adaptive disturbance rejection. In [8], adaptive state feedback output tracking designs for actuator failure compensation are developed and applied to aircraft flight control. However, state feedback output tracking MRAC for MIMO systems still needs to be solved. It is the goal of this paper to propose and study a multivariable state feedback output tracking MRAC scheme, for systems with multiple inputs and outputs. Such an adaptive control scheme is designed based on the LDS decomposition of the system's high frequency gain matrix to reduce its knowledge needed for implementing the adaptive laws. A comparison study will be given to show the advantages of the state feedback output tracking approach over state feedback state tracking as well as output feedback output tracking. Controller structure, plant-model matching conditions, adaptive law design, and stability analysis are addressed. A simulation study of the developed adaptive control scheme is done on a transport aircraft model, with illustration of simulation results.

The paper is organized as follows. The control problem is formulated in Section II where a comparison between different model reference adaptive control designs is also discussed. The adaptive controller structure is given in Section III, for which the relaxed plant-model matching equation is introduced and established under some much weaker system conditions (than those for state tracking). The adaptive scheme for updating the controller parameters is developed in Section IV, together with its stability analysis. The simulation results are presented in Section V.

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II. Problem Statement

Consider an M-input and M-output linear time-invariant system described by

$$\dot{x}(t) = Ax(t) + Bu(t), \; y(t) = Cx(t) \tag{1}$$

with $A \in R^{n\times n}$, $B \in R^{n\times M}$ and $C \in R^{M\times n}$ being unknown and constant parameter matrices, and $x(t) \in R^n$, $u(t) \in R^M$ and $y(t) \in R^M$ being the system state, input and output vector signals.

The control object is to design a state feedback control signal $u(t)$ to make all signals in the closed-loop system bounded and the output vector signal $y(t)$ asymptotically track a given reference vector signal $y_m(t)$, which is generated from the reference model system

$$y_m(t) = W_m(s)[r](t), \; W_m(s) = \xi_m^{-1}(s) \tag{2}$$

where $r(t) \in R^M$ is a bounded reference input signal, and $\xi_m(s)$ is a modified left interactor matrix of the system transfer matrix $G(s) = C(sI - A)^{-1}B$, which has a stable inverse, such that $W_m(s)$ is stable [7].

The reason why we study the multivariable MRAC with *state feedback for output tracking* is that, for many important applications, there are some shortcomings in the existing two types of multivariable MRAC schemes: the one using state feedback for state tracking and the one using output feedback for output tracking, as analyzed next.

State feedback for state tracking. For a *state feedback for state tracking* design, the controller structure is

$$u(t) = K_1^T(t)x(t) + K_2(t)r(t) \tag{3}$$

where $K_1(t) \in R^{n\times M}$ and $K_2(t) \in R^{M\times M}$ are parameter matrices updated from some adaptive laws, so that the plant state vector signal $x(t)$ can asymptotically track a reference state vector signal $x_m(t)$ generated from a chosen reference system

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \tag{4}$$

where $A_m \in R^{n\times n}$ is stable and $B_m \in R^{n\times M}$. For such an adaptive control design, the matching conditions

$$A + BK_1^{*T} = A_m, BK_2^* = B_m \tag{5}$$

need to be satisfied, for some constant matrices $K_1^* \in R^{n\times M}$ and $K_2^* \in R^{M\times M}$ [7].

To satisfy the matching conditions (5), the system matrices A and B need to be in certain restrictive canonical forms. In many applications, the controlled plants are not in canonical forms and the matching conditions cannot be satisfied.

Consider the linearized lateral motion dynamic model of a large transport airplane [8] described by

$$\dot{x} = Ax + Bu, \; x = [v_b, p_b, r_b, \phi, \psi]^T, \; u = [d_r, d_a]^T, \tag{6}$$

where v_b is the lateral velocity, p_b is the roll rate, r_b is the yaw rate, ϕ is the roll angle, ψ is the yaw angle, d_r is the rudder position, and d_a is the aileron position. From the reference [8], the parameter matrices A and B are

$$A = \begin{bmatrix} -0.13858 & 14.326 & -219.04 & 32.167 & 0 \\ -0.02073 & -2.1692 & 0.91315 & 0.000256 & 0 \\ 0.00289 & -0.16444 & -0.15768 & -0.00489 & 0 \\ 0 & 1 & 0.00618 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.15935 & 0.00211 \\ 0.01264 & 0.021326 \\ -0.012879 & 0.00171 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{7}$$

Since many parameters in the matrices A and B are in fact uncertain as they depend on different operation conditions [1], we should consider the general forms of A and B:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 \\ 0 & 1 & a_{43} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{8}$$

where $a_{ij}, i = 1, 2, 3, j = 1, 2, 3, 4$, a_{43} and $b_{ij}, j = 1, 2, 3, j = 1, 2$, are unknown parameters. For the parameter matrices K_1^* and K_2^* of the form

$$K_1^* = \begin{bmatrix} k_{111}^* & k_{112}^* \\ k_{121}^* & k_{122}^* \\ k_{131}^* & k_{132}^* \\ k_{141}^* & k_{142}^* \\ k_{151}^* & k_{152}^* \end{bmatrix}, K_2^* = \begin{bmatrix} k_{211}^* & k_{212}^* \\ k_{221}^* & k_{222}^* \end{bmatrix}, \tag{9}$$

substituting (8) and (9) in the state tracking plant-model matching equations (5), we have

$$A + BK_1^{*T} = \begin{bmatrix} a_{m11} & a_{m12} & a_{m13} & a_{m14} & a_{m15} \\ a_{m21} & a_{m22} & a_{m23} & a_{m24} & a_{m25} \\ a_{m31} & a_{m32} & a_{m33} & a_{m34} & a_{m35} \\ 0 & 1 & a_{43} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{10}$$

where $a_{mij} = a_{ij} + b_{i1}k_{1j1}^* + b_{i2}k_{1j2}^*, i = 1, 2, 3, j = 1, 2, 3, 4$, $a_{mi5} = b_{i1}k_{151}^* + b_{i2}k_{152}^*, i = 1, 2, 3$, and

$$BK_2^* = \begin{bmatrix} b_{11}k_{211}^* + b_{12}k_{221}^* & b_{11}k_{212}^* + b_{12}k_{222}^* \\ b_{21}k_{211}^* + b_{22}k_{221}^* & b_{21}k_{212}^* + b_{22}k_{222}^* \\ b_{31}k_{211}^* + b_{32}k_{221}^* & b_{31}k_{212}^* + b_{32}k_{222}^* \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{11}$$

We see that the matrices $A + BK_1^{*T}$ and BK_2^* in (10) and (11) may not be made as a stable matrix A_m and a matrix B_m independent of the plant parameters in A and B which are not in canonical forms. Thus, the reference model of the form (4) may not be specified for designing a state feedback controller for the plant (6).

Output feedback for output tracking. If the control objective is to achieve closed-loop signal boundedness and output tracking of a reference signal, we can use a multivariable MRAC scheme with *output feedback for output tracking* to avoid the strict matching conditions. It is well-known that such a controller structure is

$$u(t) = \Theta_1^T\omega_1(t) + \Theta_2^T\omega_2(t) + \Theta_{20}y(t) + \Theta_3 r(t), \quad (12)$$

where $\Theta_1 \in R^{(\nu-1)M\times M}$, $\Theta_2 \in R^{(\nu-1)M\times M}$, with ν being the observability index of the plant (1), $\Theta_{20} \in R^{M\times M}$ and $\Theta_3 \in R^{M\times M}$, and

$$\omega_1(t) = F(s)[u](t), \omega_2(t) = F(s)[y](t), \quad (13)$$

$$F(s) = \frac{A_0(s)}{\Lambda(s)}, A_0(s) = [I_M, sI_M, \ldots, s^{\nu-2}I_M]^T, \quad (14)$$

for a monic and stable polynomial $\Lambda(s)$ of degree $\nu-1$. It is clear that such a controller structure with filter $F(s)$ is much more complex than the state feedback controller structure (3).

Thus, from the above illustrations, we see that there are some shortcomings for the above two popular multivariable MRAC schemes. In many applications, the system state variables are readily available and the control objective is to achieve output tracking, such as aircraft flight control. For such applications, an effective and simple controller structure is desirable. It is the goal of this paper to develop a new multivariable MRAC scheme, which uses a state feedback to achieve asymptotically output tracking of reference signals. This multivariable MRAC with state feedback for output tracking can avoid both the strict matching conditions and the complicated output feedback controller structure.

Assumptions. To design a multivariable state feedback MRAC scheme, we make the standard assumptions:

(A1) all zeros of $G(s)$ have negative real parts;

(A2) $G(s)$ is strictly proper, has full rank and its modified left interactor matrix $\xi_m(s)$ is known; and

(A3) all leading principal minors $\Delta_i, i = 1, 2, \ldots, M$, of the high frequency gain matrix K_p are nonzero and their signs are known.

III. Controller Structure

As we explained above, for achieving the stated output tracking control objective when the state vector $x(t)$ is available, we use the simple state feedback controller structure

$$u(t) = K_1^T(t)x(t) + K_2(t)r(t) \quad (15)$$

where $K_1(t) \in R^{n\times M}$ and $K_2(t) \in R^{M\times M}$ are the adaptive estimates of the unknown constant parameters $K_1^* \in R^{n\times M}$ and $K_2^* \in R^{M\times M}$ which are defined to satisfy

$$C(sI - A - BK_1^{*T})^{-1}BK_2^* = W_m(s), K_2^{*-1} = K_p, \quad (16)$$

where $K_p = \lim_{s\to\infty} \xi_m(s)G(s)$ is the high frequency gain matrix of $G(s)$.

The existence of K_1^* and K_2^* is guaranteed by Lemma 1, under the nominal system condition:

(A4) (A, B) is stabilizable and (A, C) is observable.

Lemma 1: *There exist parameter matrices $K_1^* \in R^{n\times M}$ and $K_2^* \in R^{M\times M}$ to meet the plant-model matching equations (16), for which the pole-zero cancellations are stable, that is, $A + BK_1^{*T}$ is a stable matrix.*

This result is stated in [5] and can be proved using the method of [10]. In [5], the equation (16) is further parameterized to be used to derive an output feedback adaptive controller structure for multivariable systems. In this paper, we use it to directly derive a state feedback adaptive controller which has a simpler controller structure.

Tracking error equation. Substituting the control law (15) in the plant (1), we have

$$\begin{aligned}\dot{x}(t) &= (A + BK_1^{*T})x(t) + BK_2^*r(t)\\ &\quad + B((K_1^T(t) - K_1^{*T})x(t) + (K_2(t) - K_2^*)r(t)),\\ y(t) &= Cx(t). \end{aligned} \quad (17)$$

In view of the reference model (2), matching equations (16) and (17), the output tracking error $e(t) = y(t) - y_m(t)$ is

$$e(t) = W_m(s)K_p[\tilde{\Theta}^T\omega](t) + Ce^{(A+BK_1^{*T})t}x(0), \quad (18)$$

where $Ce^{(A+BK_1^{*T})t}x(0)$ converges to zero exponentially fast due to the stability of $A + BK_1^{*T}$, and

$$\tilde{\Theta}(t) = \Theta(t) - \Theta^*, \quad (19)$$

$$\Theta(t) = \left[K_1^T(t), K_2(t)\right]^T, \quad (20)$$

$$\Theta^* = \left[K_1^{*T}, K_2^*\right]^T, \quad (21)$$

$$\omega(t) = \left[x^T(t), r^T(t)\right]^T. \quad (22)$$

IV. Adaptive Scheme

In this section, we present the design and analysis of an adaptive scheme based on the LDS decomposition of the high frequency gain matrix K_p.

A. *Design Based on the LDS Decomposition*

To design an adaptive parameter update law, it is crucial to develop an error model in terms of some related parameter errors and the tracking error $e(t) = y(t) - y_m(t)$.

Error model development. Ignoring the term $Ce^{(A+BK_1^{*T})t}x(0)$, and from (18) and (2), we obtain

$$\xi_m(s)[e](t) = K_p\tilde{\Theta}^T(t)\omega(t). \quad (23)$$

To deal with the uncertainty of the high frequency gain matrix K_p, we use its LDS decomposition

$$K_p = L_sD_sS \quad (24)$$

where $S \in R^{M\times M}$ with $S = S^T > 0$, L_s is an $M\times M$ unit lower triangular matrix, and

$$\begin{aligned}D_s &= \mathrm{diag}\{s_1^*, s_2^*, \ldots, s_M^*\}\\ &= \mathrm{diag}\{\mathrm{sign}[\Delta_1]\gamma_1, \ldots, \mathrm{sign}[\frac{\Delta_M}{\Delta_{M-1}}]\gamma_M\}\end{aligned} \quad (25)$$

such that $\gamma_i > 0$, $i = 1, \ldots, M$, may be arbitrary [7].

Substituting (24) in (23), we obtain

$$L_s^{-1}\xi_m(s)[e](t) = D_sS\tilde{\Theta}^T(t)\omega(t). \quad (26)$$

To parameterize the unknown matrix L_s, we introduce

$$\Theta_0^* = L_s^{-1} - I = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \theta_{21}^* & 0 & \cdots & 0 \\ \theta_{31}^* & \theta_{32}^* & \cdots & 0 \\ & \cdots & \cdots & \\ \theta_{M-1\,1}^* & \cdots & 0 & 0 \\ \theta_{M\,1}^* & \cdots & \theta_{M\,M-1}^* & 0 \end{bmatrix}. \quad (27)$$

Then we have

$$\xi_m(s)[e](t) + \Theta_0^* \xi_m(s)[e](t) = D_s S \tilde{\Theta}^T(t)\omega(t). \quad (28)$$

We introduce a filter $h(s) = 1/f(s)$, where $f(s)$ is a stable and monic polynomial of degree equals to the degree of $\xi_m(s)$, Operating both sides of (28) by $h(s)I_M$ leads to

$$\begin{aligned} &\bar{e}(t) + [0, \theta_2^{*T}\eta_2(t), \theta_3^{*T}\eta_3(t), \ldots, \theta_M^{*T}\eta_M(t)]^T \\ &= D_s\, S\, h(s)[\tilde{\Theta}^T\omega](t), \end{aligned} \quad (29)$$

where

$$\bar{e}(t) = \xi_m(s)h(s)[e](t) = [\bar{e}_1(t), \ldots, \bar{e}_M(t)]^T, \quad (30)$$

$$\eta_i(t) = [\bar{e}_1(t), \ldots, \bar{e}_{i-1}(t)]^T \in R^{i-1}, i = 2, \ldots, M, \quad (31)$$

$$\theta_i^* = [\theta_{i1}^*, \ldots, \theta_{ii-1}^*]^T,\ i = 2, \ldots, M. \quad (32)$$

Based on this parameterized error equation, we introduce the estimation error signal

$$\begin{aligned} \epsilon(t) = &[0, \theta_2^T(t)\eta_2(t), \theta_3^T(t)\eta_3(t), \ldots, \theta_M^T(t)\eta_M(t)]^T \\ &+ \Psi(t)\xi(t) + \bar{e}(t), \end{aligned} \quad (33)$$

where $\theta_i(t), i = 2, 3, \ldots, M$ are the estimates of θ_i^*, and $\Psi(t)$ is the estimate of $\Psi^* = D_s\, S$, and

$$\xi(t) = \Theta^T(t)\zeta(t) - h(s)[\Theta^T\omega](t), \quad (34)$$

$$\zeta(t) = h(s)[\omega](t). \quad (35)$$

From (29)–(35), we can derive that

$$\begin{aligned} \epsilon(t) = &[0, \tilde{\theta}_2^T(t)\eta_2(t), \tilde{\theta}_3^T(t)\eta_3(t), \ldots, \tilde{\theta}_M^T(t)\eta_M(t)]^T \\ &+ D_s\, S\tilde{\Theta}^T(t)\zeta(t) + \tilde{\Psi}(t)\xi(t), \end{aligned} \quad (36)$$

where $\tilde{\theta}_i(t) = \theta_i(t) - \theta_i^*$, $i = 2, 3, \ldots, M$, and $\tilde{\Psi}(t) = \Psi(t) - \Psi^*$ are the related parameter errors.

Adaptive laws. With the estimation error model (36), we choose the adaptive laws

$$\dot{\theta}_i(t) = -\frac{\Gamma_{\theta i}\epsilon_i(t)\eta_i(t)}{m^2(t)}, i = 2, 3, \ldots, M \quad (37)$$

$$\dot{\Theta}^T(t) = -\frac{D_s\epsilon(t)\zeta^T(t)}{m^2(t)} \quad (38)$$

$$\dot{\Psi}(t) = -\frac{\Gamma\epsilon(t)\xi^T(t)}{m^2(t)} \quad (39)$$

where the signal $\epsilon(t) = [\epsilon_1(t), \epsilon_2(t), \ldots, \epsilon_M(t)]^T$ is computed from (33), $\Gamma_{\theta i} = \Gamma_{\theta i}^T > 0$, $i = 2, 3, \ldots, M$, and $\Gamma = \Gamma^T > 0$ are adaptation gain matrices, and

$$m(t) = (1 + \zeta^T(t)\zeta(t) + \xi^T(t)\xi(t) + \sum_{i=2}^{M} \eta_i^T(t)\eta_i(t))^{1/2} \quad (40)$$

is a standard normalization signal.

Complexity analysis. From the Lemma 1, we have a much less restrictive plant-model matching condition than that needed for a state feedback for state tracking scheme. Next, we will show the state feedback for output tracking scheme is simpler than the output feedback for output tracking scheme.

Firstly, the state feedback controller structure (15) is less complex than the output feedback controller structure (12) . Since there exists a matrix filter $F(s)$ in the output feedback controller (12), it needs to solve extra differential equations to calculate the signals $\omega_1(t)$ and $\omega_2(t)$ in (12), which increases the computing time of the output feedback control signal (12), while the state feedback control signal (15) can be directly calculated from the state signal $x(t)$ and the reference input signal $r(t)$.

Another comparison is about the number of updated parameters and the number of filtered signals:

Proposition 1: *The state feedback for output tracking scheme is simpler than the output feedback for output tracking scheme, in terms of the number of updated parameters and the number of filtered signals in the adaptive laws.*

Proof: The number of parameters updated in the state feedback for output tracking adaptive laws (37)–(39) is

$$N_s = \frac{M^2 - M}{2} + (n + M)M + M^2, \quad (41)$$

and the number of parameters in the output feedback for output tracking adaptive laws is

$$N_o = \frac{M^2 - M}{2} + (2\nu + 1)M^2. \quad (42)$$

Based on the definition of observability index ν [7], we have

$$\nu M \geq n. \quad (43)$$

Then, we can derive that

$$N_o - N_s = 2\nu M^2 - nM - M^2 \geq (n - M)M \geq 0. \quad (44)$$

That is the state feedback for output tracking has less parameters needed to be updated than that of the output feedback for output tracking in most cases (only when $\nu = 1$ and $n = M$, the numbers are equal).

The number of filtered signals $\bar{e}(t)$, $\xi(t)$, and $\zeta(t)$ used in state feedback for output tracking is

$$N_{fs} = 3M + n, \quad (45)$$

while the number of filtered signals in output feedback for output tracking is

$$N_{fo} = 2\nu M + 2M. \quad (46)$$

From $\nu M \geq n$, we have

$$N_{fo} - N_{fs} = 2\nu M - n - M \geq n - M \geq 0. \quad (47)$$

So the state feedback for output tracking has less filtered signals than that of the output feedback for output tracking in most cases (only when $\nu = 1$ and $n = M$, the numbers are equal).

Then, we can conclude that the state feedback for output tracking scheme is simpler. ∇

B. *Stability Analysis*

For the adaptive laws (37)–(39), we have the following desired stability properties.

Lemma 2: *The adaptive laws (37)–(39) ensure that*

(i) $\theta_i(t) \in L^\infty$, $i = 2, 3, \ldots, M$, $\Theta(t) \in L^\infty$, $\Psi(t) \in L^\infty$, *and* $\frac{\epsilon(t)}{m(t)} \in L^2 \cap L^\infty$*; and*

(ii) $\dot{\theta}_i(t) \in L^2 \cap L^\infty$, $i = 2, 3, \ldots, M$, $\dot{\Theta}(t) \in L^2 \cap L^\infty$, *and* $\dot{\Psi}(t) \in L^2 \cap L^\infty$.

Proof: Consider the positive definite function

$$V = \frac{1}{2}\left(\sum_{i=2}^{M} \tilde{\theta}_i^T \Gamma_{\theta i}^{-1} \tilde{\theta}_i + \mathrm{tr}[\tilde{\Psi}^T \Gamma^{-1} \tilde{\Psi}] + \mathrm{tr}[\tilde{\Theta} S \tilde{\Theta}^T]\right). \quad (48)$$

From (37)–(39), we derive the time-derivative of V

$$\begin{aligned}\dot{V} &= -\sum_{i=2}^{M} \frac{\tilde{\theta}_i^T \epsilon_i(t) \eta_i(t)}{m^2(t)} - \frac{\xi^T(t) \tilde{\Psi}^T \epsilon(t)}{m^2(t)} - \frac{\zeta^T(t) \tilde{\Theta} S D_s \epsilon(t)}{m^2(t)} \\ &= -\frac{\epsilon^T(t)\epsilon(t)}{m^2(t)} \leq 0. \end{aligned} \quad (49)$$

From (49), we can derive that $\theta_i(t) \in L^\infty$, $i = 2, 3, \ldots, M$, $\Theta(t) \in L^\infty$, $\Psi(t) \in L^\infty$, $\frac{\epsilon(t)}{m(t)} \in L^2 \cap L^\infty$, $\dot{\theta}_i(t) \in L^2 \cap L^\infty$, $i = 2, 3, \ldots, M$, $\dot{\Theta}(t) \in L^2 \cap L^\infty$, and $\dot{\Psi}(t) \in L^2 \cap L^\infty$. ∇

Based on these properties, we have the desired closed-loop system properties as summarized as:

Theorem 1: *The multivariable MRAC scheme with the state feedback control law (15) updated by the adaptive laws (37)–(39), when applied to the plant (1), guarantees the closed-loop signal boundedness and asymptotic output tracking:* $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$*, for any initial conditions.*

Although this is a state feedback control design, a direct Lyapunov stability analysis is not applicable because the error model (36) involves the filtered tracking error $\bar{e}(t)$ not a state error which is not available in this output tracking case. The proof of Theorem 1 needs to be carried out, using a small gain theory applied to adaptive control, as described in [7] for multivariable MRAC using output feedback (see (12)). A key step of such an analysis procedure is to express a filtered version of the plant output $y(t)$ in a feedback framework which has a small gain due to the L^2 properties of $\dot{\Theta}(t)$, $\dot{\theta}_i(t)$ and $\frac{\epsilon(t)}{m(t)}$. Since the state feedback control signal $u(t)$ depends on the plant state variables $x(t)$, we need to express it in terms of the output $y(t)$ (and the input $u(t)$ itself through a dynamic block). This can be done using a state observer representation of the plant $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t)$: $\dot{x}(t) = (A - LC)x(t) + Bu(t) + Ly(t)$ for a gain matrix $L \in R^{n\times M}$ such that $A - LC$ is stable (which is possible because (A, C) is observable). Then, the analysis procedure in [7] can be used to conclude the closed-loop signal boundedness and asymptotic output tracking: $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$ for the state feedback case.

V. An Aircraft Control Example

In this section, we present the simulation results of an application of the above multivariable MRAC scheme to an aircraft flight control system example.

A. *Aircraft System Description*

We choose the lateral dynamics model of a transport aircraft model [8] as the controlled plant which, as partially described in (6), is

$$\begin{aligned}&\dot{x} = Ax + Bu,\ x = [v_b, p_b, r_b, \phi, \psi]^T,\ u = [d_r, d_a]^T, \\ &y = Cx. \end{aligned} \quad (50)$$

The five state variables are the lateral velocity v_b, the roll rate p_b, the yaw rate r_b, the roll angle ϕ, and the yaw angle ψ. The control inputs are the rudder position d_r and the aileron position d_a. The basic units used in this model are foot, radian, and second. We choose the lateral velocity v_b and the yaw angle ψ as plant outputs, so that C is

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (51)$$

We use this linearized model to test our proposed adaptive control scheme.

B. *Adaptive Controller Design*

In this subsection, we will verify the assumptions (A1)–(A3) in Section II for the considered plant model, and choose the design parameters for the adaptive laws.

Verification of design conditions. From the nominal aircraft model (C, A, B) in (51) and (7), we can calculate the zeros of $G(s) = C(sI - A)^{-1}B$ as

$$z_1 = -11.79, z_2 = -2.69 \quad (52)$$

which both have negative real parts, and $G(s)$ is strictly proper and has full rank. The interactor matrix $\xi_m(s)$ can be chosen as

$$\xi_m(s) = \begin{bmatrix} s+1 & 0 \\ 0 & (s+1)^2 \end{bmatrix}, \quad (53)$$

so that the high frequency gain matrix is

$$K_p = \lim_{s\to\infty} \xi_m(s)G(s) = \begin{bmatrix} 0.1593 & 0.0021 \\ -0.0129 & 0.0017 \end{bmatrix} \quad (54)$$

which is finite and non-singular. Therefore, the reference system transfer matrix $W_m(s)$ can be chosen as

$$W_m(s) = \xi_m^{-1}(s) = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{(s+1)^2} \end{bmatrix} \quad (55)$$

which is stable and strictly proper. In this case, the leading principal minors of K_p are

$$\Delta_1 = 0.1593, \Delta_2 = 0.000298 \quad (56)$$

which are nonzero, and the signs of the leading principal minors are positive and are to be used in the adaptive laws.

Now we have verified that all the assumptions in Section II can be satisfied for the aircraft plant described in (50).

Design parameters. For the adaptive laws (37)–(39), We choose $\Gamma_{\theta 2} = 1$, $D_s = \Gamma = \mathrm{diag}\{1, 1\}$, and $h(s) = \frac{1}{(s+1)^2}$.

C. Simulation Results

For numerical study, the reference input $r(t)$ is selected as (i) constant reference inputs $r(t) = [1, 0.1]^T$, and (ii) varying reference inputs $r(t) = [\sin(0.014t), 0.1\sin(0.014t)]^T$. From the results we can see that these two choices can make the aircraft follow realistic flying courses. Simulation results are presented as below.

Case I: Constant reference inputs $r(t) = [1, 0.1]^T$. In Figure 1, the dotted lines represent the reference outputs and the solid lines represent the aircraft outputs. From Figure 1, we can have that the aircraft outputs track the reference outputs to 1 ft/sec and 5.7 deg respectively. The responses of control input signal $u(t) = [d_r, d_a]^T$ are also bounded in some reasonable regions, which are not shown since the space limitation.

Case II: Varying reference inputs $r(t) = [\sin(0.014t), 0.1\sin(0.014t)]^T$. In Figure 2, the dotted lines represent the reference outputs and the solid lines represent the aircraft outputs. Figure 2 shows that the output tracking errors converge to zero. The responses of control input signal $u(t) = [d_r, d_a]^T$ are also bounded in some reasonable regions, which are not shown since the space limitation.

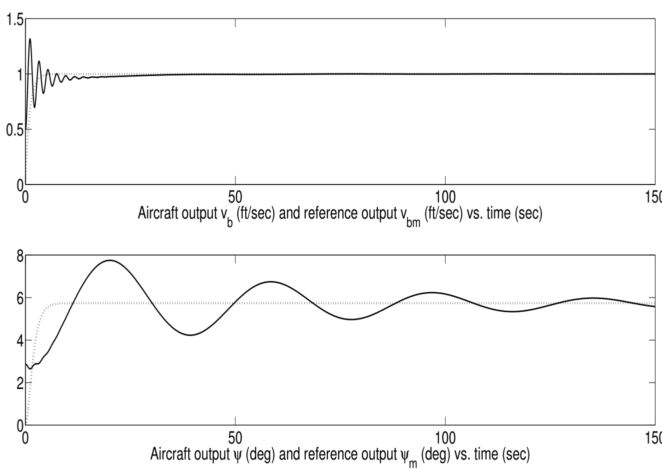

Fig. 1. Aircraft outputs (solid) vs. reference outputs (dotted) in Case I.

VI. Conclusions

In this paper, we have shown the development of a state feedback output tracking multivariable MRAC scheme. Such a scheme needs less restrictive plant-model matching conditions than a state tracking scheme, while with a simpler controller structure than an output feedback scheme. It is an addition to the collection of multivariable MRAC designs, and has high potential for output tracking applications in which system states are available but the state tracking matching conditions cannot be satisfied, such as aircraft flight control. Like other multivariable MRAC schemes, the state feedback output tracking MRAC scheme can be designed based on different decompositions of the plant high frequency gain matrix, and in this paper it is designed based on an LDS decomposition. Relaxed plant-model matching conditions and desired stability and asymptotic tracking properties have been established in theory and verified by simulation results from a study of application to a linearized transport aircraft dynamic model.

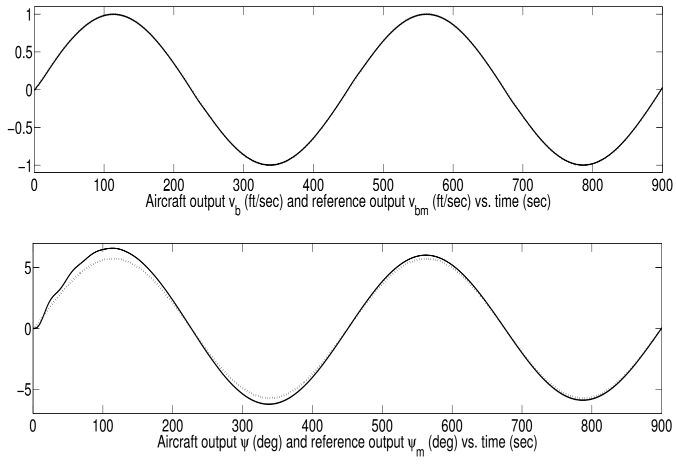

Fig. 2. Aircraft outputs (solid) vs. reference outputs (dotted) in Case II.

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References

[1] Bryson Jr., A. E., *Control of Spacecraft and Aircraft*, Princeton University Press, Princeton, NJ, 1994.

[2] Goodwin, G. C. and K. S. Sin, *Adaptive Filtering Prediction and Control*, Prentice-Hall, Englewood Cliffs, NJ, 1984.

[3] Imai, A. K., R. R. Costa, L. Hsu, G. Tao and P. V. Kokotović, "Multivariable MRAC using high frequency gain matrix factorization," *IEEE Transactions on Automatic Control*, vol. 49, no. 7, pp. 1152-1157, 2004.

[4] Krstić, M., I. Kanellakopoulos and P. V. Kokotović, *Nonlinear and Adaptive Control Design*, John Wiley & Sons, New York, 1995.

[5] Morse, A. S., "Parametrizations for Multivariable Adaptive Control," *Proc. 20th IEEE CDC*, pp. 970–972, San Diego, CA, 1981.

[6] Ortega, R., L. Hsu and A. Astolfi, "Adaptive control of multivariable systems with reduced prior knowledge," *Proceedings of the 40th IEEE Conference on Decision and Control*, pp. 4198–4203, Orlando, FL, 2001.

[7] Tao, G., *Adaptive Control Design and Analysis*, John Wiley and Sons, New York, 2003.

[8] Tao, G., Chen, S., Tang, X., and Joshi, S. M., *Adaptive Control of Systems with Actuator Failures, Springer-Verlag*, London, 2004.

[9] Weller, S. R. and G. C. Goodwin, "Hysteresis switching adaptive control of linear multivariable systems," *IEEE Trans. on Automatic Control*, vol. AC-39, no. 7, pp. 1360–1375, 1994.

[10] Wolovich, W. A., *Linear Multivariable Systems*, Springer-Verlag, New York, 1974.