Multivariable process identification for robust control

tu/e
Research Report
ISSN 0167-9708
Coden: TEUEDE

Eindhoven
University of Technology
Netherlands

Faculty of Electrical Engineering

Multivariable Process Identification for Robust Control

By
Y.C. Zhu
A.C.P.M. Backx
P. Eykhoff

EUT Report 91-E-249
ISBN 90-6144-249-4

January 1991

Eindhoven University of Technology Research Reports

EINDHOVEN UNIVERSITY OF TECHNOLOGY

Faculty of Electrical Engineering
Eindhoven The Netherlands

ISSN 0167- 9708 Coden: TEUEDE

MULTIVARIABLE PROCESS IDENTIFICATION FOR ROBUST CONTROL

by

Y.C. Zhu
A.C.P.M. Backx
P. Eykhoff

EUT Report 91-E-249
ISBN 90-6144-249-4

Eindhoven
January 1991

适用于鲁棒控制器设计的多变量过程辨识

朱豫才，A.C.P.M. 巴克斯，P. 艾克霍夫

CIP-GEGEVENS KONINKLIJKE BIBLIOTHEEK, DEN HAAG

Zhu, Y.C.

Multivariable process identification for robust control / by Y.C. Zhu, A.C.P.M. Backx and P. Eykhoff. - Eindhoven: Eindhoven University of Technology, Faculty of Electrical Engineering. - (EUT report, ISSN 0167-9708; 91-E-249)
Met lit. opg., reg.
ISBN 90-6144-249-4
SISO 656.2 UDC 519.71.001.3 NUGI 832
Trefw.: systeemidentificatie; procesregeling.

MULTIVARIABLE PROCESS IDENTIFICATION FOR ROBUST CONTROL

Y.C. ZHU*, A.C.P.M. BACKX*,** and P. EYKHOFF**

* IPCOS Group, Datex Industry
Tinnegieterstraat 12, NL-5232 BM 's-Hertogenbosch
The Netherlands
tel. 31-73-412225, fax: 31-73-426055

** Dept. EE, Eindhoven University of Technology
P.O. Box 513, NL-5600 MB Eindhoven
The Netherlands
Tel. 31-40-473300

Contents

MULTIVARIABLE PROCESS IDENTIFICATION FOR ROBUST CONTROL

Y.C. ZHU, A.C.P.M. BACKX and P. EYKHOFF

Abstract—In this work, multi-input multi-output (MIMO) process identification is studied, where the purpose of identification is control system design. An identification procedure is presented by which one can estimate not only a nominal parametric process model, but also an upper bound of the model uncertainty (modelling errors) in the frequency domain. The basic steps of this method consist of high order model estimation and subsequent model reduction. One advantage is that, in this framework, fundamental problems such as *input design* and *model structure selection* can easily be solved. Another advantage of the method is numerical simplicity and reliability. The identified nominal model and the error bound can readily be used for *robust control* system analysis and design. A simulation example is given to illustrate the method.

1 Introduction

Because of our limited knowledge about reality, mathematical models can never give an exact description of the system behavior under study; hence model uncertainties or modelling errors always exist. In the previous decade, robust control theory has been proposed and developed, cf. Zames (1981), Doyle (1984), Vidyasagar (1985) and Morari and Zafiriou (1989). The advantage of robust control is its capability to cope with modelling errors in the analysis and design of control systems. In order to apply robust control theory, one needs not only a nominal process model, but also a suitable description of the modelling errors which are typically in the form of some bounds of either parameter variations or transfer function variations.

Although the topic of linear system identification has been studied extensively in the previous two decades, there are no straightforward methods which can derive error bounds of identified models in the frequency domain. Only very recently some researchers started to study this problem. Cloud and Kouvaritakis (1986) have proposed an error bound for MIMO FIR models when output disturbances are white noises; Kosut (1986) derived an error bound for

SISO prediction error models; Goodwin and Salgado (1989) proposed a method for quantifying uncertainty in the estimation of simplified SISO models; Helmicki et al. (1989) studied the problem in a deterministic setting for SISO processes; Corrêa (1989) presented some discussion about the problem in the framework of prediction error method; Van den Boom et al. (1991) proposed a method of SISO process identification in the frequency domain, where the H_∞-norm is used as error criterion, in a deterministic setting. The work on this topic is as yet so limited that this short list almost completes the story. It seems that identification is not ready yet for its use in robust control, at least not for MIMO processes.

In this work we will present an identification method for linear time-invariant MIMO processes, in which a nominal parametrical model of the process can be estimated together with an upper bound matrix of the modelling errors in the frequency domain. The idea of this method was first proposed in Zhu (1987a, 1989a) for SISO processes, based on a newly developed asymptotic theory of black-box model identification in Ljung (1985) and Ljung and Yuan (1985). Because the asymptotic theory can be extended to the MIMO case and the result has a nice structure (Yuan and Ljung, 1984, and Zhu 1987b,1989b), the method can be generalized for MIMO processes in a straightforward way (Zhu, 1987c, 1990). The method has been applied to analyze the robust stability of a 2-input 2-output glass tube production process (Zhu, 1990, Zhu and Backx, 1991).

First we introduce an asymptotic theory of the MIMO prediction error method (Section 2); then, based on this theory, the identification procedure is presented in Section 3, where identification input design is also discussed. In Section 4 we propose a method for a frequency weighted Frobenius-norm model reduction, which is needed in deriving an optimal reduced order model. The problem of model order determination and structure selection will be treated in Section 5. Section 6 presents a simulation study. Section 7 gives the conclusions of this work.

2 Black-Box Models Identified by Prediction Error Methods

In the previous decade, many sophisticated time domain parametric identification methods have been developed. Most of these techniques can be classified in the set of prediction error methods. Recently, Ljung and Yuan developed an asymptotic theory on the frequency domain properties of prediction error models; cf. Ljung and Yuan (1985), Ljung (1985), and Yuan and Ljung (1984). The MIMO version of the theory (Zhu, 1989b) will briefly be

presented.

Consider a discrete time process with m inputs and p outputs. A general linear time-invariant model for the relationship between inputs and outputs can be written

$$y(t) = \sum_{k=1}^{\infty} G_k \cdot u(t-k) + v(t) \tag{2.1}$$

where: $y(t)$ is the p-dimensional column output vector at time t; $u(t)$ is the m-dimensional column input vector at time t; $\{G_k\}$ is the impulse response of the process, which is a sequence of $p \times m$ matrices; and $\{v(t)\}$ is a p-dimensional stochastic stationary process with zero mean values.

When the unit time delay operator q^{-1} is introduced:

$$q^{-1}u(t) = u(t-1)$$

the model (2.1) can be written as

$$y(t) = G(q)u(t) + v(t) \tag{2.2}$$

where

$$G(q) = \sum_{k=1}^{\infty} G_k \cdot q^{-k} \tag{2.3}$$

is called the transfer operator of the process model.

The transfer function matrix of the model is defined as

$$G(e^{i\omega}) = \sum_{k=1}^{\infty} G_k \cdot e^{-i\omega k} \qquad -\pi \le \omega \le \pi \tag{2.4}$$

For the output disturbances, a common approach is to assume that they are mutually independent and generated as filtered white noises:

$$v(t) = H(q)e(t) \tag{2.5}$$

where

$$H(q) = \sum_{k=0}^{\infty} H_k \cdot q^{-k}$$

H_k is a sequence of $p \times p$ matrices with

$$H_0 = I_p \quad (p \times p \text{ identity matrix})$$

and $\{e(t)\}$ is a p-dimensional white noise vector, mutually independent, with covariance matrix $R = \mathrm{diag}[R_1 \cdots R_p]$, where R_i is the variance of $\{e_i(t)\}$. Both $H(q)$ and $H^{-1}(q)$ are stable. Then $\{v(t)\}$ will be a stationary process with spectral density

$$\Phi_v(\omega) = \sum_{\tau=-\infty}^{\infty} [E\{v(t)v^{\mathrm{T}}(t)\}]e^{-i\omega\tau} = H(e^{i\omega})RH^{\mathrm{T}}(e^{-i\omega}) \tag{2.6}$$

where E means expectation, T means transpose, $H(e^{i\omega})$ is the transfer function matrix of $H(q)$:

$$H(e^{i\omega}) = \sum_{k=0}^{\infty} H_k \cdot e^{-i\omega k} \qquad -\pi \le \omega \le \pi \tag{2.7}$$

which is diagonal.

The identification problem is to estimate an approximate model from observed input-output data. Denote the data sequence by Z^N:

$$Z^N := y(1),\ u(1),\ \ldots\ \ldots,\ y(N),\ u(N) \tag{2.8}$$

where N is the number of samples of the data sequence.

If we have a parametrized model:

$$y(t) = G(q,\theta)u(t) + H(q,\theta)e(t) \tag{2.9}$$

where θ is a d-dimensional parameter vector, we can determine the one step ahead prediction:

$$\hat{y}(t|\theta) = [I_p - H^{-1}(q,\theta)]y(t) + H^{-1}(q,\theta)G(q,\theta)u(t) \tag{2.10}$$

and compute the prediction error

$$\varepsilon(t,\theta) = y(t) - \hat{y}(t|\theta) = H^{-1}(q,\theta)[y(t) - G(q,\theta)u(t)] \tag{2.11}$$

Then a common way to determine the parameters is to minimize the squared sum of the prediction errors

$$\hat{\theta} = \arg \min_{\theta \in D_n} V_N(\theta) \tag{2.12}$$

where

$$V_N(\theta) = \frac{1}{N} \sum_{t=1}^{N} \varepsilon^T(t,\theta)\varepsilon(t,\theta)$$

and $D_n \subset R^d$ is the parameter space. Here the subscript N is to emphasize that N data samples are used for the estimation; the dimension of the parameter space d depends on the numbers of process inputs and outputs, the model order and the model parametrization. The model degree n will be defined in the following.

The general MIMO black-box model is defined as

$$A(q)y(t) = F^{-1}(q)B(q)u(t) + D^{-1}(q)C(q)e(t) \tag{2.13}$$

where $A(q)$, $B(q)$, $C(q)$, $D(q)$ and $F(q)$ are polynomial matrices with dimension $p\times p$, $p\times m$, $p\times p$, $p\times p$ and $p\times p$ respectively:

$$\left.\begin{aligned} A(q) &= I_p + A_1 q^{-1} + \cdots + A_n q^{-n} \\ B(q) &= \quad B_1 q^{-1} + \cdots + B_n q^{-n} \\ C(q) &= I_p + C_1 q^{-1} + \cdots + C_n q^{-n} \\ D(q) &= I_p + D_1 q^{-1} + \cdots + D_n q^{-n} \\ F(q) &= I_p + F_1 q^{-1} + \cdots + F_n q^{-n} \end{aligned}\right\} \qquad (2.14)$$

The assumption that output disturbances are mutually independent implies that $A(q)$, $C(q)$ and $D(q)$ are diagonal matrices.

In order to have a unique representation of a given MIMO process, some structural conditions should be imposed on the general model (2.14). A natural way to do this is by letting $F(q)$ be diagonal matrices. This arrangement decomposes the MIMO process into p MISO (multi-input single-output) sub-processes where the i-th subprocess is given as

$$A_{ii}(q)y(t) = F_{ii}^{-1}(q)B_i(q)u(t) + D_{ii}^{-1}(q)C_{ii}(q)e_i(t) \qquad (2.15)$$

The parameter vector for model (2.14) is

$$\theta = col\ [A_1\ B_1\ C_1\ D_1\ F_1\ \ A_2\ B_2\ C_2\ D_2\ F_2 \cdots\ A_n\ B_n\ C_n\ D_n\ F_n] \qquad (2.16)$$

The *degree* of this model is defined as the degrees of the polynomials which are equal to n. Note that, in general, this *degree* is not the same as the McMillan degree which is defined as the dimension of the minimal state space realization of the model. Generally, the McMillan degree of the model equals $p \cdot n$.

Various model parametrizations used in practice are special cases of this general model:

(1) Box-Jenkins model, $A(q) = I$;
(2) ARMAX model, $F(q) = D(q) = I$;
(3) OE (output error) model, $A(q) = D(q) = C(q) = I$;
(4) ARX (equation error) model, $F(q) = D(q) = C(q) = I$; and
(5) finite impulse response (FIR) or Markov parameter model,
$A(q) = F(q) = D(q) = C(q) = I$

Hence the problem statement (2.9) – (2.16) can cover most of the time domain identification techniques in practice. Specific methods can be obtained by taking a specific model parametrization.

After the parameter estimation, the transfer function estimates are

denoted as:

$$\left.\begin{aligned}\hat{G}_N^n(e^{i\omega}) &= G(\hat{\theta},e^{i\omega})\\ \hat{H}_N^n(e^{i\omega}) &= H(\hat{\theta},e^{i\omega})\end{aligned}\right\}$$

Asymptotic Properties of the Transfer Function Estimates

When the identified model is used in controller design, we are more concerned about the quality of the transfer function estimates than about the accuracy of parameters. The estimation of a transfer function matrix is basically a non-parametric problem. Since the process is viewed as a black-box, the internal parametrization via θ is merely a vehicle to arrive at this estimate. Then, it is natural to let the model order n depend on the number of observed data samples, $n = n(N)$. Typically, in order to have a model set that is large enough to contain the true process dynamics, or to give a good approximation of the true dynamics, we will allow the order n to increase when the number of data samples N increases, but n should be small compared to N. This can be formally expressed by the following assumptions

$$n(N) \to \infty \qquad \text{as } N \to \infty \tag{2.17}$$

and

$$n^2(N)/N \to 0 \qquad \text{as } N \to \infty \tag{2.18}$$

Assume that the true process is described by

$$y(t) = G^o(q)u(t) + H^o(q)e(t) \tag{2.19}$$

where $G^o(q)$ and $H^o(q)$ are the true transfer operators of the process and the disturbance. They are both assumed to be stable filters. Denote the true transfer function matrices as $G^o(e^{i\omega})$ and $H^o(e^{i\omega})$. Then, for an open loop experiment, under some suitable conditions of the inputs, the following results hold:

— $[\hat{G}_N^n(e^{i\omega}),\ \hat{H}_N^n(e^{i\omega})] \to [G^o(e^{i\omega}),\ H^o(e^{i\omega})] \qquad w.p.1 \quad \text{as } N \to \infty$ (2.20)

— $col[\hat{G}_N^n(e^{i\omega}) - E\{\hat{G}(e^{i\omega},\ n)\}] \sim AsN\left(0,\ \frac{n}{N}[\Phi_u^{-T} \otimes \Phi_v(\omega)]\right)$ (2.21)

$$col[\hat{H}_N^n(e^{i\omega}) - E\{\hat{H}(e^{i\omega},\ n)\}] \sim AsN\left(0,\ \frac{n}{N}[R^{-1} \otimes \Phi_v(\omega)]\right) \tag{2.22}$$

as $N \to \infty$

where $col(\cdot)$ denotes the vector operator on a matrix, $E(\cdot)$ denotes mean value, $\Phi_u(\omega)$ and $\Phi_v(\omega)$ are the spectrum matrices of the inputs and disturbances respectively, $\cdot^T$ means inverse and transpose, $\otimes$ denotes the Kronecker product. Result (2.21) and (2.22) are the MIMO extension of the result of Ljung (1985); the proof can be found in Zhu (1989b). Similar results hold for closed loop experiments, which are not shown here.

From (2.20)-(2.22) we can say heuristically that the errors of the transfer function estimates have an asymptotically normal distribution; the covariance matrix of $\hat{G}$ at a given frequency is proportional to the (generalized) noise-to-signal-ratio at that frequency and inversely proportional to the number of data N; the covariance increases with the *model degree* n, not with the number of parameters d. We also find that different model parametrizations will have the same asymptotic properties.

Because the expression of the covariance matrix is remarkably simple, this result is very useful in applications, as will be shown in the following sections.

3 Nominal Model and Error Bounds

In this section, we will propose an identification method which will deliver both a nominal model and an upper bound matrix of the modelling errors, using the asymptotic theory of Section 2. Also the input design problem will be treated.

Because the theory in the previous section is asymptotic both in the number of data samples N and the model order n, the results will be valid if the number of data samples is large and the model order is high. But in practice low order models are used for simulation, prediction and controller design. So our approach is to start with a high order model estimation and then to apply model reduction techniques to arrive at low order models. This approach has its practical background. For industrial process identification and control, we may have a large amount of data; and we have to use high order models to initially fit the complex dynamics of the process, as we in general do not have detailed knowledge of process dynamics. We can avoid numerical problems by combining high order estimation and model reduction, cf. Backx (1987) and Wahlberg (1989). By this approach one also can simplify the step of model order determination and structure selection, which is a hard job for MIMO processes. Input design for high order models is an easy task; cf. Yuan and Ljung (1984).

3.1 An Algorithm for Estimating a Nominal Model with an Error Bound

The asymptotic theory implies that the model parametrization is not a crucial issue for high order models. In this case the ARX model is the most simple parametrization which can supply both the process model and disturbance model. It is well known that the least squares estimation of ARX model parameters is a linear regression problem, which can be solved reliably with simple numerical techniques. Assume that an open loop experiment is performed; also assume that the inputs are mutually independent. The following steps are proposed:

Step 1 Estimate the parameters of the ARX model with a high order, e.g. $n = 20 - 30$. The matrix $A(q)$ has a diagonal form:

$$A(q) = \text{diag}\ [\ A_{11}(q) \cdots A_{pp}(q)].$$

Then we have

$$\hat{G}_N^n(q) = \hat{A}_N^{-1}(q)\hat{B}_N(q) \qquad \hat{H}_N^n(q) = \hat{A}_N^{-1}(q) \tag{3.1}$$

and the spectrum estimates of the disturbances are

$$\hat{\Phi}_v(\omega) = \hat{A}_N^{-1}(e^{i\omega})\hat{R}\ \hat{A}_N^{-T}(e^{-i\omega}) \tag{3.2}$$

where $\hat{R}$ is the estimated covariance of the equation error residuals.

Denote $\hat{G}_{ij}^n(e^{i\omega})$ and $G_{ij}^o(e^{i\omega})$ as the (i,j) element of $\hat{G}_N^n(e^{i\omega})$ and $G^o(e^{i\omega})$ respectively. Then according to (2.20) - (2.22), and remembering that the inputs are mutually independent, we have asymptotically

$$\{G_{ij}^o(e^{i\omega}) - \hat{G}_{ij}^n(e^{i\omega})\} \in AsN\left(0,\ \frac{n}{N}\Phi_{u_j}^{-1}(\omega)\Phi_{v_i}(\omega)\right) \tag{3.3}$$

and we can define the 3σ bound for $\{G_{ij}^o(e^{i\omega}) - \hat{G}_{ij}^n(e^{i\omega})\}$ as

$$\left|G_{ij}^o(e^{i\omega}) - \hat{G}_{ij}^n(e^{i\omega})\right| \le 3\sqrt{\frac{n}{N}\Phi_{u_j}^{-1}(\omega)\Phi_{v_i}(\omega)} \qquad \text{w.p. } 99.99\% \tag{3.4}$$

where $\Phi_{u_j}(\omega)$ is the spectrum of $u_j(t)$, and $\Phi_{v_i}(\omega)$ is the spectrum of $v_i(t)$.

Step 2 Perform a model reduction on $\hat{G}_N^n(q)$ to obtain a low order process model $\hat{G}^l(q)$, and on $\hat{H}_N^n(q) = \hat{A}_N^{-1}(q)$ to obtain a disturbance model $\hat{H}^l(q)$, where l means the *low order*. Model reduction by truncated balanced realization and by optimal Hankel norm approximation are two well known methods (Glover, 1984). When the high order model is obtained by identification, a model

reduction method which can take the properties of the high order model into account will be preferred; cf., Wahlberg (1989). In the next section a method of *frequency weighted Frobenius norm MIMO model reduction* will be proposed which works on polynomial matrices. The order and structure selection for the low order model will be treated in Section 5.

Define the modelling errors as

$$\Delta_{ij}(e^{i\omega}) = G^{o}_{ij}(e^{i\omega}) - \hat{G}^{l}_{ij}(e^{i\omega}) \qquad \forall i,j \tag{3.5}$$

where $\hat{G}^{l}_{ij}(e^{i\omega})$ is the (i,j) element of $\hat{G}^{l}_{N}(e^{i\omega})$. Then

$$\Delta_{ij}(e^{i\omega}) = [\hat{G}^{n}_{ij}(e^{i\omega}) - \hat{G}^{l}_{ij}(e^{i\omega})] + [G^{o}_{ij}(e^{i\omega}) - \hat{G}^{n}_{ij}(e^{i\omega})] \tag{3.6}$$

The first term in (3.6) we call the *bias part* of the modelling errors and the second term the *variance part* of the modelling errors. Now

$$|\Delta_{ij}(e^{i\omega})| \le |\hat{G}^{n}_{ij}(e^{i\omega}) - \hat{G}^{l}_{ij}(e^{i\omega})| + |G^{o}_{ij}(e^{i\omega}) - \hat{G}^{n}_{ij}(e^{i\omega})| \tag{3.7}$$

From (3.4) it follows that,

$$|\Delta_{ij}(e^{i\omega})| \le |\hat{G}^{n}_{ij}(e^{i\omega}) - \hat{G}^{l}_{ij}(e^{i\omega})| + 3\sqrt{\frac{n}{N}\Phi^{-1}_{u_j}(\omega)\Phi_{v_i}(\omega)} \qquad \text{w.p. } \ge 99.99\% \tag{3.8}$$

This is the basis for estimating the upper bound matrix.

Step 3 Define the upper bound matrix

$$\bar{\Delta}(\omega) = \{\bar{\Delta}_{ij}(\omega)\} \tag{3.9}$$

such that

$$|\Delta_{ij}(e^{i\omega})| \le \bar{\Delta}_{ij}(\omega) \qquad \forall i,j \quad \forall \omega \tag{3.10}$$

Finally we estimate $\bar{\Delta}(\omega)$ by

$$\boxed{\bar{\Delta}_{ij}(\omega) = |\hat{G}^{n}_{ij}(e^{i\omega}) - \hat{G}^{l}_{ij}(e^{i\omega})| + 3\sqrt{\frac{n}{N}\hat{\Phi}^{-1}_{u_j}(\omega)\hat{\Phi}_{v_i}(\omega)}} \tag{3.11}$$

where $\hat{\Phi}_{u_j}(\omega)$ is calculated as the sample spectrum:

$$\hat{\Phi}_{u_j}(\omega) = \sum_{\tau=-\tau_u}^{\tau_u} [\frac{1}{N} \sum_{t=1}^{N} u_j(t)u_j(t-\tau)]\ e^{-i\omega\tau}$$

and $\hat{\Phi}_{v_i}(\omega)$ is given in (3.2). ■

Remarks—If a good model structure is selected and a proper model reduction technique is used in Step 2, it is possible to obtain a low order model which is more accurate than the high order model. In this case, the 3σ bounds will be the bounds for the errors of the low order model as well. Then the formula for the upper bound becomes

$$\bar{\Delta}_{ij}(\omega) = 3\sqrt{\frac{n}{N}\hat{\Phi}_{u_j}^{-1}(\omega)\hat{\Phi}_{v_i}(\omega)} \quad . \tag{3.11b}$$

This bound tighter than that is given in (3.11).

— Notice that the probability of 3σ bound of a normal complex variable is greater than that of a normal real variable.

— In formula (3.11) we note that the bias part of the modelling errors is caused by model reduction; the variance part can be affected by the input spectrum, the model order and the number of data samples which are design variables chosen by the user. If at some frequencies the modelling errors are too large for a specific application of the model, we know from (3.11) that we can reduce the modelling errors at those frequencies by: (1) performing another model reduction, (2) modifying the input spectrum (input design), (3) using more data (increase experiment time) and (4) reducing the model degree n. The last choice should be made with caution, because too low an order will introduce more bias.

— This type of algorithm can be applied also to closed loop experiments. In this case, use the expression of the asymptotic covariance matrix for closed loop experiments; see Zhu (1989b).

— This type of algorithm can also be used for a FIR model, see Zhu (1990). Here we prefer an ARX model to FIR model because: (1) for modelling a given process the degree (see the definition in Section 2) and number of parameters of an ARX model are usually less than those of a FIR model; (2) a FIR model does not supply a disturbance model.

3.2 Performing Input Design before any Identification and Control Test

Input design is important in determining the quality of an identified model. Conventional methods of optimal input design use the covariance of the model parameters as the measure of model quality (see Mehra, 1974). This methodology can not address directly the intended use of the model; and the calculation of the optimal input needs the true model which is not known.

The problem of input design is very much related to the intended use of the model. Now let us assume that the model is used for controller design in an *internal model control* (see Morari and Zafiriou, 1989) which is a very suitable scheme for controlling e.g. an industrial process with time delays. In this control scheme, the identified process model is placed in parallel with the process, and the differences between the process outputs and simulated model outputs are fedback to the controller. Therefore a model which can optimally simulate the underlying process for certain given inputs will be most suitable for a internal model control scheme.

Denote $u^s(t)$ as the control input to the process (the simulation input to the model) with spectrum $\Phi_u^s(\omega)$ (which, in general, is different from the spectrum of the input used in the identification experiment). Yuan and Ljung (1985) have shown that in the SISO case, the optimal input which minimizes the mean square of the simulation error for the high order model is given by

$$\Phi_u^{op}(\omega) = \mu\sqrt{\Phi_u^s(\omega)\Phi_v(\omega)} \tag{3.12}$$

where μ is a constant which is adjusted with respect to the constraint of input amplitude or output variance. This result agrees with our engineering intuition: it says that the process should be excited more at those frequencies where the simulation input has more power and where disturbances make more trouble. Notice that this spectrum is *not* related to the process transfer function.

It can be shown (see Lenssen, 1988) that for MIMO processes, under the assumption that the input signals are mutually independent, the optimal input spectrum for the i-th input is given by

$$\Phi_{u_i}^{op}(\omega) = \mu\sqrt{\Phi_{u_i}^s(\omega)\cdot\sum_{j=1}^{p}\Phi_{v_j}(\omega)} \tag{3.13}$$

Similarly, the optimal input spectrum for other uses, such as prediction, pole placement control design, can be derived; see Yuan and Ljung (1985), Gevers and Ljung (1986).

Initially one might think that formula (3.12) or (3.13) is useless

because one does not know $\{u^s(t)\}$ before identifying the process, designing the controller and testing the closed loop control system. Examine a feedback control system in which the main goal of the control is disturbance reduction. Then the control input is the filtered disturbance

$$u^s(t) = T(q)v(t) \tag{3.14}$$

where $T(q)$ is the unknown transfer operator from the disturbance to the input. The optimal input is (for SISO process)

$$\Phi_u^{op}(\omega) = \mu |T(e^{i\omega})| \Phi_v(\omega) \tag{3.15}$$

Because $T(q)$ is unknown, we can simply let $T(q) = 1$. Then we have a first approximation to the optimal input:

$$\Phi_u(\omega) = \mu \Phi_v(\omega) \tag{3.16}$$

If $T(e^{i\omega})$ is flat over the bandwidth of the disturbance $\{v(t)\}$, this will be a very good approximation.

Similarly, for a MIMO process, we can have an approximation to the i-th input

$$\Phi_{u_i}(\omega) = \mu \sum_{j=1}^{p} \Phi_{v_j}(\omega) \tag{3.17}$$

If the number of inputs and the number of outputs of the process are equal and the decoupling technique is used in controller design, $T(e^{i\omega})$ will be nearly diagonal. In this case, the approximation of the i-th optimal input signal is

$$\Phi_{u_i}(\omega) = \mu \sqrt{\Phi_{v_i}(\omega) \cdot \sum_{j=1}^{p} \Phi_{v_j}(\omega)} \tag{3.18}$$

Hence, the nearly optimal input spectrum can be derived without knowing the process model and the controller; the output disturbance $\{v(t)\}$ can be measured from the uncontrolled process when keeping the input constant.

If some readers are still not convinced yet, let us look at the consequence of this input design method for model errors in the SISO case. Substituting (3.16) into (3.4), we obtain a constant 3σ bound of the errors of the high order model as

$$|G^o(e^{i\omega}) - \hat{G}^n(e^{i\omega})| \le 3v\sqrt{\frac{n}{N \cdot \mu}} \qquad \text{w.p. } 99.99\%$$

This means that we will minimize, in a stochastic sense, the L_∞-norm of the errors of the high order model by using (3.16). This is not at all a bad choice.

When taking into account other control requirements, such as setpoint

tracking and robust stability, the following modification of (3.16) is recommended:

$$\Phi_u(\omega) = \mu_1 \Phi_v(\omega) + \mu_2 \phi(\omega) \tag{3.19}$$

Here $\phi(\omega)$ is a spectrum which adds some energy to the input signal at frequency bands which are important for tracking and robust stability; μ_1 and μ_2 are adjusted to meet the constraint on input amplitude or output variance, and to weight the relative importance of the two terms. The spectrum can be realized by filtering a white noise signal or a PRBNS (pseudo random binary noise sequence) signal, with a filter which has the property that the square of the filter transfer function approximates the desired input spectrum given in (3.19). The modification for the MIMO case is the same.

Thus we have a input design method which is not only very simple, but also most economical: the nearly optimal input signal can be determined by only measuring the output of the existing uncontrolled process. This makes the method very applicable.

3.3 On the Use of the Upper Bound Matrix

When a nominal model and an upper bound matrix are obtained by the methods in the previous subsection, the real process $G^o(e^{i\omega})$, which is not completely known, can be described in the following class:

$$\left.\begin{aligned} G^o(e^{i\omega}) &= \hat{G}(e^{i\omega}) + \Delta(e^{i\omega}) \\ |\Delta_{ij}(e^{i\omega})| &\le \bar{\Delta}_{ij}(\omega) \qquad \forall i,j \quad \forall \omega \end{aligned}\right\} \tag{3.20}$$

The upper bound matrix $\bar{\Delta}(\omega) = \{\bar{\Delta}_{ij}(\omega)\}$ gives a *structured* description of the model uncertainty, because it preserves the multivariable nature of the problem: the amplitudes of the errors of each transfer function estimate are upper bounded by the elements of $\bar{\Delta}(\omega)$ in the frequency domain, the only missing information are the phase angles of the errors $\Delta(e^{i\omega})$.

It is known from linear algebra that

$$\bar{\sigma}(\Delta(e^{i\omega})) \le \bar{\sigma}(\bar{\Delta}(\omega)) \tag{3.21}$$

This means that the maximum singular value of the upper bound matrix is an upper bound of unstructured model uncertainty.

Robust Stability Analysis Kouvaritakis and Latchman (1985) have developed a result which is most suitable for the stability test of the process given in (3.20). Denote $C(e^{i\omega})$ as the transfer function matrix of the feedback

controller, then the process defined in class (3.20) will be stabilized by the controller if and only if

$$\min_{L,R} [\bar{\sigma}(L\bar{\sigma}(\omega)R)\cdot\bar{\sigma}(R^{-1}C(I+\hat{G}C)^{-1}L^{-1})] < 1 \qquad \forall\omega \in (0, \pi]$$

where $\bar{\sigma}(\cdot)$ denotes the maximum singular value, and $(e^{i\omega})$ is omitted to save space. Here L and R are diagonal positive nonsingular scaling matrices which need to be determined frequency by frequency by some optimization procedure.

Determining the Weighting Matrix for μ-Synthesis In Doyle's μ-synthesis (Doyle, 1984), the robust stability and robust performance can be analyzed and optimized simultaneously. In this framework, the model error $\Delta(e^{i\omega})$ matrix is rearranged in the following way:

$$G^{o}(e^{i\omega}) - \hat{G}(e^{i\omega}) = \Delta(e^{i\omega}) = W_1\Delta^{e}W_2$$

where Δ^e is a diagonal matrix:

$$\Delta^{e} = \text{diag}[\Delta^{e}_{11}(e^{i\omega}), \Delta^{e}_{21}(e^{i\omega}), \cdots, \cdots, \Delta^{e}_{pm}(e^{i\omega})], \qquad |\Delta^{e}_{ij}(e^{i\omega})| \le 1$$

Notice that $\Delta^{e}_{ij}(e^{i\omega}) \neq \Delta_{ij}(e^{i\omega})$. It can be shown that under this arrangement

$$W_1 = [I \quad I \cdots I], \quad W_2 = \begin{bmatrix} \Delta_1 & & \\ & \Delta_2 & \\ & & \ddots \\ & & & \Delta_m \end{bmatrix}, \quad \Delta_i = \begin{bmatrix} \Delta_{1i} \\ \vdots \\ \Delta_{pi} \end{bmatrix}$$

Now we are ready to use the μ-synthesis.

Robust fault detection A robust fault detection method was proposes by Emami-Naeini et al. (1988) where they use an upper bound of the unstructured model uncertainty in the analysis and design of fault detection systems. The upper bound given in (3.21) can serve their purpose.

4 Model Reduction of the Identified High Order ARX Model

In the procedure proposed in Section 3.1, the low order model is obtained by model reduction. The existing popuiar methods, such as balanced model reduction and model reduction by Hankel norm approximation, are numerically simple, but they do not minimize a criterion .vhich is physically sensible. The properties of these methods in the frequency domain are not clear, except some upper bounds. In this section, a method of model reduction is proposed in which the asymptotic properties of the high order model will be taken into account.

From (2.20)—(2.22) we know that for each high order transfer function

estimate:

$$\{G^{o}_{ij}(e^{i\omega}) - \hat{G}^{n}_{ij}(e^{i\omega})\} \in AsN\left(0, \frac{n}{N}\Phi^{-1}_{u_j}(\omega)\Phi_{v_i}(\omega)\right) \tag{4.1}$$

and for each disturbance model estimate:

$$\{H^{o}_{ij}(e^{i\omega}) - \hat{H}^{n}_{ij}(e^{i\omega})\} \in AsN\left(0, \frac{n}{N}|H^{o}(e^{i\omega})|^2\right) \tag{4.2}$$

We can view the high order ARX model as the noisy observations of the true process (Wahlberg, 1989). Because we know the asymptotic distribution of the high order estimates, it is natural to apply the *maximum likelihood* method to these observations to find the low order model. Denote $G^{l}(q)$ and $H^{l}q)$ as the reduced order process model and disturbance model respectively, then using (4.1) and (4.2), it can be shown (Wahlberg, 1989) that the *asymptotic maximum likelihood* loss function of the process model is:

$$V_1 = \sum_{i=1}^{p}\sum_{j=1}^{m}\frac{1}{2\pi}\int_{-\pi}^{\pi}|\hat{G}^{n}_{ij}(e^{i\omega}) - \hat{G}^{l}_{ij}(e^{i\omega})|^2 \frac{\Phi_{u_j}(\omega)}{|H^{o}_{ii}(e^{i\omega})|^2}\,d\omega \tag{4.3}$$

and the loss function of the disturbance model is

$$V_2 = \sum_{i=1}^{p}\frac{1}{2\pi}\int_{-\pi}^{\pi}|\hat{H}^{n}_{ii}(e^{i\omega}) - \hat{H}^{l}_{ii}(e^{i\omega})|^2 \frac{1}{|H^{o}_{ii}(e^{i\omega})|^2}\,d\omega \tag{4.4}$$

Minimizing these loss functions is equal to the *frequency weighted Frobenius norm model reduction.* Note that the weights in (4.3) and (4.4) are the inverses of asymptotic variances of the high order process model and disturbance model respectively. Hence frequencies where the high order model has small variance (error) will have a large weight in the estimation criterion; this is physically appealing. The direct minimization of these loss functions involves nonlinear optimization which is numerically difficult; we have not yet seen any efficient algorithm for MIMO models.

Conventional model reduction methods calculate the parameters of the reduced model from the parameters of the high order model. The method proposed here takes another approach. We first simulate the high order model to generate the input/output data, and then calculate the reduced order model from the data. This can be called an *identification approach* to model reduction and the advantage of it will become clear soon.

A Reduction of the Process Model

Given the high order ARX model of the process:

$$\hat{G}_N^n(q) = [\hat{A}^n(q)]^{-1}\hat{B}^n(q) \tag{4.5}$$

where

$$\hat{A}^n(q) = \text{diag}[\hat{A}^n_{11}(q) \cdots \hat{A}^n_{pp}(q)] \quad \text{and} \quad \hat{B}^n(q) = \begin{bmatrix} \hat{B}^n_{11}(q) \cdots \hat{B}^n_{1m}(q) \\ \vdots \quad \ddots \quad \vdots \\ \hat{B}^n_{p1}(q) \cdots \hat{B}^n_{pm}(q) \end{bmatrix}$$

As mentioned before this is a diagonal form representation of the process model in which the model is decomposed into p MISO sub-models.

Given the low order model of the process, also in a diagonal form:

$$\hat{G}_N^l(q) = [\hat{A}^l(q)]^{-1}\hat{B}^l(q) \tag{4.6}$$

where

$$\hat{A}^l(q) = \text{diag}[\hat{A}_{11}(q) \cdots \hat{A}_{pp}(q)] \quad \text{and} \quad \hat{B}^l(q) = \begin{bmatrix} \hat{B}_{11}(q) \cdots \hat{B}_{1m}(q) \\ \vdots \quad \ddots \quad \vdots \\ \hat{B}_{p1}(q) \cdots \hat{B}_{pm}(q) \end{bmatrix}$$

Notice that for the low order model there is a degree, denoted by l_i, for each MISO sub-model; in general the degrees of various MISO sub-models are different. Here l only means the *low order* model which is *not* the degree of it.

We propose the following

Procedure 4.1

For the i-th MISO sub-model:

Stage 1 Simulation. Collect the inputs, $\{u(t),\ t = 1, \cdots, N\}$, which have been used in the identification experiment. Filter the inputs by the inverse of the disturbance model $1/\hat{H}^n(q) = \hat{A}^n_{ii}(q)$. Then simulate the i-th MISO high order sub-model using the filtered inputs:

$$\hat{y}_i(t) = \frac{1}{\hat{A}^n_{ii}(q)}[\hat{B}^n_{11}(q) \cdots \hat{B}^n_{1m}(q)]\,[\hat{A}^n_{ii}(q)u(t)] \tag{4.7}$$

This is equivalent to

$$\hat{y}_i(t) = [\hat{B}^n_{11}(q) \cdots \hat{B}^n_{1m}(q)]u(t) \tag{4.8}$$

which is simpler. Thus we obtain the input/output data of the i-th MISO sub-model:

$$Z_i^N := \hat{y}_i(1),\ u_{f,1}(1),\ \cdots,\ u_{f,m}(1),\ \cdots,\ \cdots,\hat{y}(N),\ u_{f,1}(N),\cdots\ u_{f,m}(N)\ .$$

where

$$u_f(t) = \hat{A}_{ii}^n(q)u(t)$$

Stage 2 Steiglitz-McBride iteration. The algorithm was proposed by Steiglitz and McBride (1965), which can be seen as an approximation of the output error method. Denote $\hat{A}_{ii}^k(q)$ and $\hat{B}_{i1}^k(q)$, $\cdots$, $\hat{B}_{im}^k(q)$ as the estimates of the low order i-th MISO sub-model at iteration k. First filter the inputs $u_{f,1}(t),\cdots,u_{f,m}(t)$ and the output $\hat{y}_i(t)$ by $1/\hat{A}_{ii}^k(q)$. Then refined estimates $\hat{A}_{ii}^{k+1}(q)$ and $\hat{B}_{i1}^{k+1}(q)$, $\cdots$, $\hat{B}_{im}^{k+1}(q)$ are obtained by minimizing

$$V_i^{k+1} = \frac{1}{N}\sum_{t=1}^{N}\left\{\hat{A}_{ii}^{k+1}(q)\left[\frac{\hat{y}_i(t)}{\hat{A}_{ii}^k(q)}\right] - \hat{B}_{i1}^{k+1}(q)\left[\frac{u_{f,1}(t)}{\hat{A}_{ii}^k(q)}\right] - \cdots - \hat{B}_{im}^{k+1}(q)\left[\frac{u_{f,m}(t)}{\hat{A}_{ii}^k(q)}\right]\right\}^2 \tag{4.9}$$

Denote $\hat{\theta}_i^{k+1}$ as the parameter vector of $\hat{A}_{ii}^{k+1}(q)$ and $\hat{B}_{i1}^{k+1}(q)$, $\cdots$, $\hat{B}_{im}^{k+1}(q)$:

$$\hat{\theta}_i^{k+1} = \left[\hat{a}_{ii,1}^{k+1},\cdots,\hat{a}_{ii,l_i}^{k+1},\ \hat{b}_{i1,1}^{k+1},\cdots,\hat{b}_{i1,l_i}^{k+1},\cdots,\cdots,\hat{b}_{im,1}^{k+1},\cdots,\hat{b}_{im,l_i}^{k+1}\right]^T$$

and denote the data vector as

$$\varphi(t,\hat{\theta}_i^k) = \left[\frac{-\hat{y}_i(t-1)}{\hat{A}_{ii}^k(q)},\cdots,\frac{-\hat{y}_i(t-l_i)}{\hat{A}_{ii}^k(q)},\ \frac{u_{f,1}(t-1)}{\hat{A}_{ii}^k(q)},\cdots,\frac{u_{f,1}(t-l_i)}{\hat{A}_{ii}^k(q)},\right.$$

$$\left.\cdots,\ \cdots,\ \frac{u_{f,m}(t-1)}{\hat{A}_{ii}^k(q)},\cdots,\frac{u_{f,m}(t-l_i)}{\hat{A}_{ii}^k(q)}\right]$$

Then (4.9) is a *linear least squares* problem and its solution is

$$\hat{\theta}_i^{k+1} = \left[\frac{1}{N}\sum_{t=1}^{N}\varphi(t,\hat{\theta}_i^k)^T\varphi(t,\hat{\theta}_i^k)\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N}\varphi(t,\hat{\theta}_i^k)^T\frac{\hat{y}_i(t)}{\hat{A}_{ii}^k(q)}\right] \tag{4.10}$$

If $i < p$ **go to** Stage 1, **else stop.** ■

The initial estimate

The most simple way of initialization is to perform a normal equation error

least squares on the data. This implies the prefilter $\hat{A}_{ii}^{0} = 1$ in (4.9).

The filter $1/\hat{A}_{ii}^{k}(q)$ will be mostly a lowpass filter, so is $1/\hat{A}_{ii}^{n}(q)$ where $\hat{A}_{ii}^{n}(q)$ is from the high order model. Hence we can initialize the iteration by setting $\hat{A}_{ii}^{0}(q) = \hat{A}_{ii}^{n}(q)$.

It has been experienced that a (frequency weighted) balanced model reduction or a (frequency weighted) Hankel norm approximation can deliver "good" reduced order models. Hence the third choice of initialization is to use the $\hat{A}_{ii}(q)$ polynomial from one of these models.

The convergence properties

The model reduction problem (4.9) and (4.10) is an identification problem in which the data are noise free, but the model order is lower than the order of the process. In the literature there is no theoretical result on the convergence properties of the Steiglitz-McBride method when the model order is lower than that of the process. Our experience shows that the algorithm always converges when used in model reduction. The same finding was reported by Fan and Jenkins (1986) who use the method for adaptive filtering. We feel that these are not just coincidences. Before any theoretical result can be developed, let us propose the following

Conjecture 4.1 Assume that the inputs are persistently exciting, that is, $\sum_{t=1}^{N}\varphi(t,\hat{\theta}_i^k)^T\varphi(t,\hat{\theta}_i^k)$ is nonsingular, and $1/\hat{A}_{ii}^{k}(q)$ is stable for all k. Then the iteration (4.9) and (4.10) converges. ∎

We welcome interested researchers to prove this, or to find a counter example.

The loss function in the frequency domain

Assume that the algorithm converges at iteration k+1. Then the loss function in (4.9) becomes

$$V_i^{k+1} = \frac{1}{N}\sum_{t=1}^{N}\left\{\sum_{j=1}^{m}[\hat{G}_{ij}^{n}(q) - \hat{G}_{ij}^{l}(q)]\hat{A}_{ii}^{n}(q)u_j(t)\right\}^2$$

Letting $N \to \infty$, applying Parseval's identity and remembering that $u_1(t)$, ···, $u_m(t)$ are mutually independent, one can show (see Ljung, 1987) that

$$V_i^{k+1} = \sum_{j=1}^{m} \frac{1}{2\pi} \int_{-\pi}^{\pi} |\hat{G}_{ij}^{n}(e^{i\omega}) - \hat{G}_{ij}^{l}(e^{i\omega})|^2 \frac{\Phi_{u_j}(\omega)}{|\hat{H}_{ii}^{n}(e^{i\omega})|^2} d\omega \qquad (4.11)$$

This is precisely the loss function of the asymptotic maximum likelihood estimation (4.3) for the i-th MISO sub-model, except that $H_{ii}^{o}(e^{i\omega})$ has been replaced by the high order estimate $\hat{H}_{ii}^{n}(e^{i\omega}) = 1/\hat{A}_{ii}^{n}(e^{i\omega})$.

B Reduction of the Disturbance Model

Given the high order disturbance model

$$\hat{H}^n(q) = [\hat{A}^n(q)]^{-1} = \text{diag}[[\hat{A}_{11}^{n}(q)]^{-1} \cdots [\hat{A}_{pp}^{n}(q)]^{-1}]$$

which is diagonal, we want to find the reduced order model in the following form

$$\hat{H}^l(q) = \text{diag}\left[\frac{\hat{C}_{11}(q)}{\hat{D}_{11}(q)}, \cdots, \frac{\hat{C}_{pp}(q)}{\hat{D}_{pp}(q)} \right] \qquad (4.12)$$

where

$$\hat{C}_{ii}(q) = 1 + \hat{c}_{ii,1}q^{-1} + \cdots + \hat{c}_{ii,l_i'}q^{-l_i'}$$

$$\hat{D}_{ii}(q) = 1 + \hat{d}_{ii,1}q^{-1} + \cdots + \hat{d}_{ii,l_i'}q^{-l_i'}$$

with l_i' being the order of the i-th disturbance model which can be different from l_i, the order of the i-th sub-model of the process. Because $\hat{H}^n(q)$ and $\hat{H}^l(q)$ are diagonal, we have p SISO model reduction problems.

For the i-th sub-model:

Stage 1 Simulation. Generate a zero mean white noise sequence $\{e(t), t = 1, \cdots, N\}$. Filter $e(t)$ by the inverse of the disturbance model $1/\hat{H}^n(q) = \hat{A}_{ii}^{n}(q)$. Then simulate the i-th SISO high order sub-model of the disturbance using the filtered white noise

$$\hat{v}_i(t) = \frac{1}{\hat{A}_{ii}^{n}(q)}[\hat{A}_{ii}^{n}(q)e(t)]$$

This is equivalent to

$$\hat{v}_i(t) = e(t)$$

which is simpler. Thus we obtain the input/output data of the i-th SISO sub-model:

$$Z_i^N := \hat{v}_i(1), e_f(1), \cdots, \hat{v}_i(N), e_f(N).$$

where

$$e_f(t) = \hat{A}^n_{ii}(q)e(t)$$

Stage 2 Steiglitz-McBride iteration. This is similar to the iteration for the process model reduction, and we will not repeat it.

Following the same argument as for (4.11), we can show that the proposed model reduction method for the disturbance model minimizes the following loss function

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}|\hat{H}^n_{ij}(e^{i\omega}) - \hat{H}^l_{ij}(e^{i\omega})|^2 \frac{1}{|\hat{H}^n_{ii}(e^{i\omega})|^2}\, d\omega$$

when $N \to \infty$. This is precisely the loss function of the asymptotic maximum likelihood estimation (4.4) for the i-th SISO sub-model of the disturbance, except $H^o_{ii}(e^{i\omega})$ being replaced by the high order estimate $\hat{H}^n_{ii}(e^{i\omega}) = 1/\hat{A}^n_{ii}(e^{i\omega})$.

In this section an asymptotic maximum likelihood model reduction technique is proposed, where the low order models of the process and the disturbance are calculated from the simulated data of the high order models. The advantages of the technique are: (1) it minimizes a criterion which is physically sensible, (2) it is numerically simple, and (3) it can cope with MIMO models. This makes the method suitable not only for model reduction of identified high order models as in this work, but also for other purposes such as model reduction of theoretical (physical) models and controller reduction.

5 Model Structure Determination

Before performing model reduction as proposed in the previous section, the orders of the p MISO sub-models, $\{l_1, \cdots, l_p\}$, need to be determined. This set of indices defines the *model structure* of the MIMO process model (4.6).

Model order and structure selection is a central issue in system identification. Many researchers have put their attention on this topic, and there exists a multitude of methods for model order/structure selection; see Stoica et al. (1986) and Janssen (1988) for recent overviews. Most of the existing methods assume that the true process belongs to a set of candidate model structures and try to find a 'right' structure. This assumption, however, is neither likely to be fulfilled in practice, nor is it necessary

for applications such as simulation, prediction and control. What we want is a model structure which gives a suitable approximation of those process features in which we are interested for the underlying application. Other difficulties of many proposed methods are: they are computationally costly, especially for MIMO processes; and they are numerically not reliable, they can fail to find the best or a 'right' structure when some minimization algorithm stops at a local minimum. Some numerically simple methods exist, but usually they do not use a criterion which is physically sensible.

Here we will propose a method of model structure selection for the low order model (4.6), based on the asymptotic theory in Section 2 and some physical intuition. The advantages of the method are: (1) it uses frequency domain measures in determining the model structure, and (2) it is numerically simple and reliable.

Our idea of order/structure selection is: chose the order such that the variance part and bias part of the errors (see (3.6)) are approximately equal. The reasoning behind this idea can be explained as follows: (loosely speaking) suppose that the high order model has 10% error, it is only possible to find the true model if the low order model can deviate from the high order model with 10%. If we let the variance part and the bias part of the errors to be approximately equal, we get the following:

A MIMO model structure selection rule

For the i-th sub-model: Perform model reduction with various orders and select the lowest order, l_i, such that in the frequency range which is important for control system design:

$$\sum_{j=1}^{m} |\hat{G}^{n}_{ij}(e^{i\omega}) - \hat{G}^{l}_{ij}(e^{i\omega})|^2 \cong \sum_{j=1}^{m} \frac{n}{N}\hat{\Phi}^{-1}_{u_j}(\omega)\hat{\Phi}_{v_i}(\omega) \qquad (5.1)$$

■

The relation (5.1) can be called *bias/variance equivalence* principle. This selection can be done simply by visual inspection. Applying the order selection rule (5.1) to each MISO high order sub-model; then we can determine the structure of the low order MIMO model.

The same idea can be applied for determining the orders of the disturbance model.

Remarks—In this work we look at the identification problem as *approximate modelling*. Hence we do *not* intend to find the 'right' model structure; instead

we want to find a model structure such that the reduced order model will have smallest errors in the important frequency range. The consequence of this philosophy is that the selected order can be either lower or higher than the 'true' order of the process.

— In this selection rule, the order to be selected is related to the noise-to-signal ratio, to the experiment time and to the order of the high order model. This is again physically sensible.

— If one finds that the visual inspection according to (5.1) is not scientific enough, we suggest to use the weighted Frobenius norm; this can be done by integrating the both sides of (5.1) over the frequency range which is important for control system design.

— The final model obtained by our method is given in polynomial matrix description in a diagonal form; and the process is decoupled into MISO sub-processes. This is a Box-Jenkins model. This model can be directly used for controller design by using the polynomial method; see Kucera (1979). The model can also be converted to a state space realization for the use in control system analysis and design. Generally speaking, the McMillan degree of the model equals the sum of the degrees of the MISO sub-models; in this case the diagonal form description is called *irreducible*. It possible, however, that a diagonal form polynomial matrix description is *not* irreducible, i.e., the sum of the degrees of the MISO sub-models is greater than the McMillan degree of the model. If this happens, the minimal realization technique can be used to eliminate the extra states.

6 A Simulation Study

In this section, a simulation study is performed in order to validate the identification method proposed in the previous sections. The model of a glass tube production process is used for the the simulation study. Two variables need to be controlled for this process: diameter of the tube and the wall thickness of the tube; two variables are chosen as the control inputs: drawing speed and pressure of the blowing gas. Thus this is a 2-input 2-output process. The identification and control of this process has been studied in Backx (1987); see also Backx and Damen (1989). The model is given in a state space realization with McMillan degree 6. In this study that model is used as the true process and the input/output data are generated by simulation. Fig. 6.1 shows the transfer functions (amplitudes) of the process. The output disturbances are colored noises which are generated by filtering two white noises by two lowpass filters. The power of the disturbances is 10% of the

power of the correponding outputs The inputs are generated according to the input design method (3.17). 2000 data samples are used in the identification.

High order ARX model estimation. An ARX model with degree 30 is estimated in this step. In Fig. 6.1the transfer functions (amplitudes) are compared with those of the process.

As mentioned before, the errors of this high order model will be called the variance part of the modelling errors. The 3σ bound of the variance part of the errors are calculated according to (3.4), and they are plotted together with the errors in Fig. 6.2. The error is defined as the absolute value of the difference between each true transfer function and that of the model. We see that the 3σ bounds cover the errors almost entirely except some frequency points.

Model order reduction and model structure selection. The model reduction method in Section 4 is used for calculating the low order models. The Steiglitz-McBride iteration converges in the calculations of all the low order models. For the 4th degree model, the iteration converges at iteration 3; for the 5th degree model, the iteration converges at iteration 15.

According the model structure selection rule (5.1), the structure (4, 4) is the best choice. In Fig. 6.3, the errors of the models of structure (4, 4) and (5, 5) are compared. We find that the model quality of the structure (4, 4) is indeed better than that of structure (5, 5).

In Fig. 6.4 the errors of the low order model with structure (4, 4) are compared with the errors of the 30th degree ARX model. We note that the low order model has a better quality.

In Fig. 6.5 the errors of the low order model with structure (4, 4) are compared with the errors of the low order model with structure (4,4) that are obtained by performing the balanced model reduction on each 25th degree MISO models. There is a sharp distinction between the performances of the two methods of model reduction.

Upper bound matrix. Formula (3.11) is used to calculate the upper bound matrix for the model with structure (4, 4); and the upper bounds are plotted together with the errors of the model in Fig. 6.6. We see that the bounds become loser than the 3σ bound for the variance; compare Fig. 6.2. As remarked in Section 3, if a good model structure is selected and a proper model reduction is performed, then the errors of low order model will be in general

smaller than the errors of the high order model; in this case the 3σ bounds can be used as the total bounds. In Fig. 6.7 the errors of the low order model with structure (4, 4) are plotted together with the 3σ bounds. We find that indeed the 3σ bounds can be used as the total bounds, except the transfer function (1, 1) at very low frequencies where the model reduction does not function very well; see Fig. 6.4.

7 Conclusions

Several fundamental problems in MIMO process identification, such as input design, model order and structure selection, and model uncertainty have been treated in this paper. Also an identification approach to model reduction has been proposed. The method of input design presented addresses directly the intended use of the model, and at same time it is very simple and economical. A method has been proposed for identifying a nominal model of a MIMO process, together with an upper bound matrix of the modelling errors in the frequency domain. This is motivated by the development in robust control theory. The identification method is based on recent results on identification combined with our physical intuition. The applicability of our method relies on the fact that in all the steps the method uses criteria which are physically sensible; at the same time the computations needed are simple and reliable. The method has been validated by a simulation study. Note that this method is not only valid for finite dimensional processes. It can also be used for the identification of infinite dimensional processes, if we start with a sufficiently high order model. Now we are ready for real applications.

References

Backx, A.C.P.M. (1987). *Identification of an Industrial Process: A Markov Parameter Approach.* Dr. Dissertation, Dept. EE., Eindhoven University of Technology, The Netherlands.

Backx, A.C.P.M. and A.A.H. Damen (1989). Identification of industrial MIMO processes for fixed controllers—part 1 and part 2. *Journal A*, Vol. 30, pp. 3-12, and pp. 33-43.

Corrêa, C.O. (1989). A system identification problem motivated by robust control. *Int. J. Control*, Vol. 50, No. 2, pp 575-602.

Cloud, D.J. and B. Kouvaritakis (1986). Statistical bounds on multivariable frequency response: an extension of the generalized Nyquist criterion. *IEE Proc.*, Vol. 133, Pt.D., pp. 97-110.

Doyle, J.C. (1984). *Lecture Notes - ONR/Honeywell Workshop on Advances on Multivariable Control.*

Emami-Naeini, A., M.M. Akhter and S.M. Rock (1988). Effect of model uncertainty on failure detection: the threshold selector. *IEEE Trans. Autom. Control*, Vol. AC-33, No. 12, pp. 1106-1115.

Fan, H. and W.K. Jenkins (1986). A new adaptive IIR filter. *IEEE Trans. Autom. Control*, Vol. AC-33, No. 10, pp. 939-947.

Gevers, M. and L. Ljung (1986). Optimal experiment design with respect to the intended model application. *Automatica*, Vol. 22, No. 5, pp. 543-554.

Goodwin, G.C. and M.E. Salgado (1989). A stochastic embedding approach for quantifying uncertainty in the estimation of restricted complexity models. *Int. J. Adaptive Control and Signal Processing*, Vol.3, pp. 333-356.

Glover, K. (1984). All optimal Hankel-norm approximations of linear multivariable systems and their L_∞-error bounds. *Int. J. Control*, Vol. 39, pp. 1115-1193.

Helmicki, A.J., C.A. Jacobson and C.N. Nett (1989). H_∞ identification of stable LSI systems: a scheme with direct application to controller design. *Proc. Amer. of Contr. Conf.*, Pittsburg, pp. 1428-1434.

Janssen, P. (1988). *On Model Parametrization and Model Structure Selection for Identification of MIMO-Systems.* Dr. Dissertation, Dept. EE., Eindhoven University of Technology, The Netherlands.

Kosut, R.L. (1986). Adaptive calibration: an approach to uncertainty modelling and on-line robust control design. *Proc. 25th Conf. Decision and Control*, Athens, Greece, December, 1986, pp. 455-461.

Kouvaritakis, B. and H. Latchman (1985). Necessary and sufficient stability criterion for systems with structured uncertainties: the major principal direction alignment principle. *Int. J. Control*, Vol. 42, No. 3, pp. 575-598.

Kučera, V. (1979). *Discrete Linear Control.* John Wiley & Sons, New York.

Lenssen, M. (1988). *Optimal Input Design for MIMO Processes.* M.Sc. thesis, Dept. EE., Eindhoven University of Technology, The Netherlands.

Ljung, L. (1985). Asymptotic variance expressions for identified black-box transfer function models. *IEEE Trans. Autom. Control*, Vol. AC-30, pp. 834-844.

Ljung, L. (1987). *System Identification: Theory for the User.* Prentice-Hall, Englewood Cliffs, N.J..

Ljung, L. and Z.D. Yuan (1985). Asymptotic properties of black-box identification of transfer functions. *IEEE Trans. Autom. Control*, Vol.

AC-30, pp.514-530.

Hehra, R.K. (1974). Optimal input signals for parameter estimation in dynamic systems—A survey and new results. *IEEE Trans. Autom. Control*, Vol. AC-19, pp. 753-768.

Morari, M. and E. Zafiriou (1989). *Robust Process Control*. Prentice-Hall, New Jersey.

Steiglitz, K. and L.E. McBride (1965). A technique for the identification of linear systems. *IEEE Trans. Autom. Control*, Vol. AC-10, pp. 461-464.

Stoica, P., P. Eykhoff, P. Janssen and T. Söderström (1986). Model-structure selection by cross-validation. *Int. J. Control*, Vol. 43, pp. 1841-1878.

Van den Boom, T., M. Klompstra and A. Damen (1991). System identification for H_∞-robust control. Submitted to *the 9th IFAC/IFORS Symposium on Identification and System Parameter Estimation*, Budapest, July, 1991.

Vidyasagar, M. (1985). *Control System Synthesis: A Factorization Approach*. MIT press.

Wahlberg, B. (1989). Model reduction of high-order estimated models: the asymptotic ML approach. *Int. J. Control*, Vol. 49, No. 1, pp 169-192.

Yuan, Z.D. and L. Ljung (1984). Black-box identification of multivariable transfer functions: asymptotic properties and optimal input design. *Int. J. Control*, Vol. 40, No. 2, pp. 233-256.

Yuan, Z.D. and L. Ljung (1985). Unprejudiced optimal open loop input design for identification of thansfer functions. *Automatica*, Vol. 21, No. 6 pp 697-708.

Zames, G. (1981). Feedback and optimal sensitivity: model reference transformations, multiplicative seminorms, and approximate inverses. *IEEE Trans. Autom. Control*, Vol. AC-26, pp. 301-320.

Zhu, Y.C. (1987a). On a bound of modelling errors of black-box transfer function estimates. *EUT Report 87-E-173*, EE Dept., Eindhoven University of Technology, The Netherlands.

Zhu, Y.C. (1987b). Black-box identification of MIMO transfer functions: asymptotic properties of prediction error models. *EUT Report 87-E-182*, EE Dept., Eindhoven University of Technology, The Netherlands.

Zhu, Y.C. (1987c). On the bounds of modelling errors of black-box MIMO transfer function estimates. *EUT Report 87-E-183*, EE Dept., Eindhoven University of Technology, The Netherlands.

Zhu, Y.C. (1989a). Estimation of transfer functions: asymptotic theory and a bound of model uncertainty. *Int. J. Control*, Vol.49, pp. 2241-2258.

Zhu, Y.C. (1989b). Black-box identification of MIMO transfer functions: asymptotic properties of prediction error models. *Int. J. Adaptive Control and Signal Processing*, Vol.3, pp. 357-373.

Zhu, Y.C. (1990). *Identification and Control of MIMO Industrial Processes: An Integration Approach*. Dr. Dissertation, Dept. EE., Eindhoven University of Technology, The Netherlands.

Zhu, Y.C. and A.C.P.M. Backx (1991). MIMO Process identification for controller design: test signals, norminal model and error bounds. Submitted to *the 9th IFAC/IFORS Symposium on Identification and System Parameter Estimation*, Budapest, July, 1991.

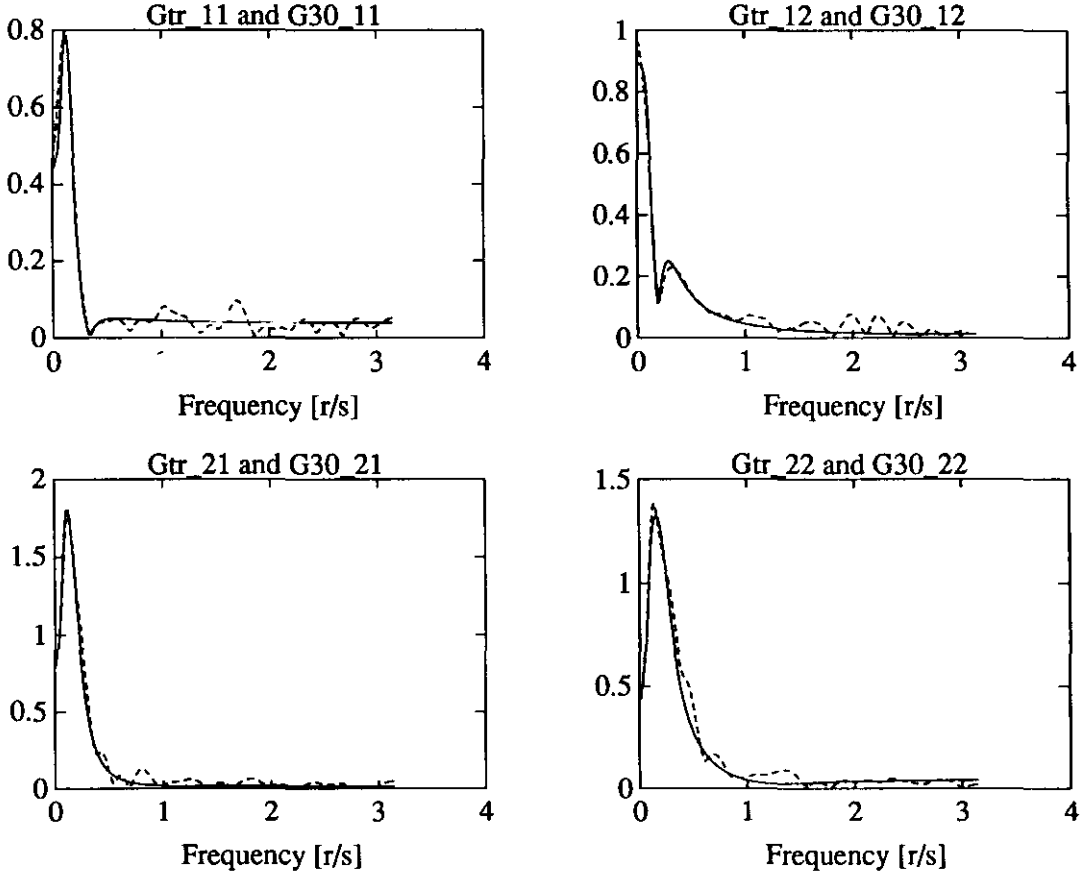

Fig. 6.1 Transfer functions (amplitudes) of the process (solid line) and transfer functions (amplitudes) of the high order model (dashed line).

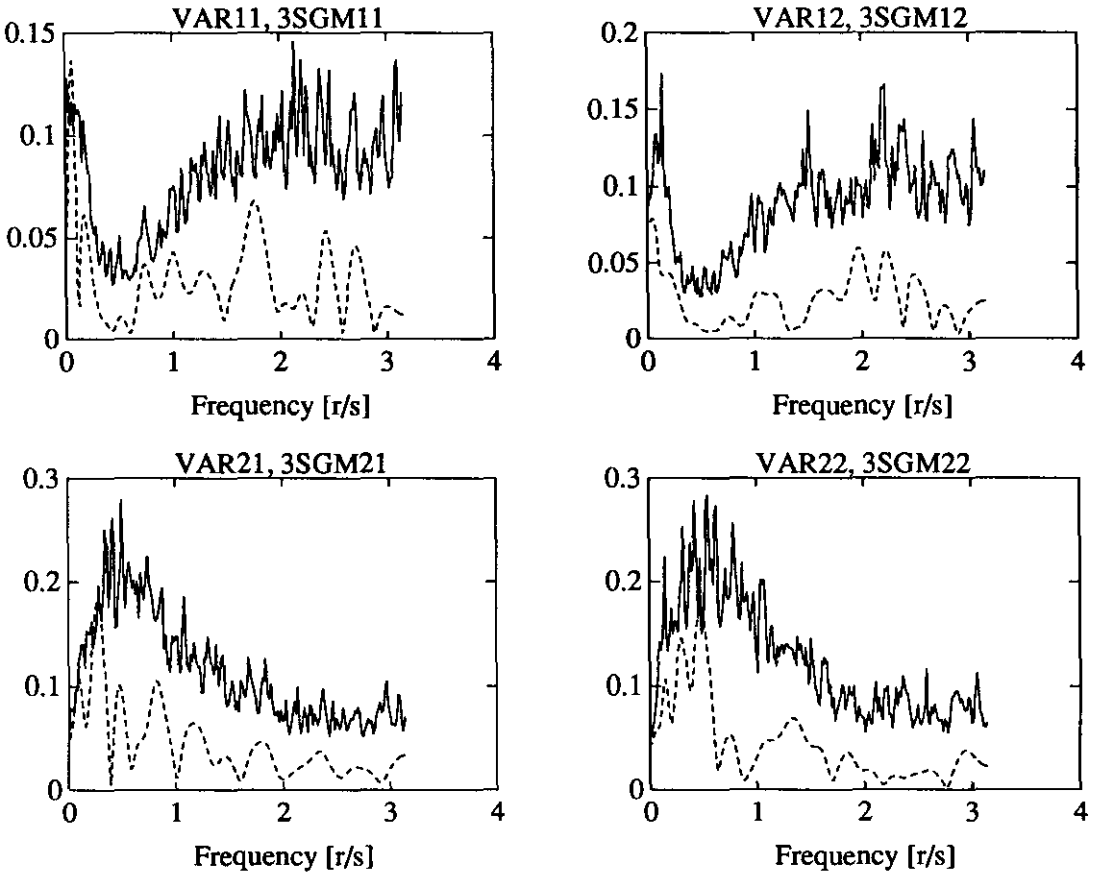

Fig. 6.2 The errors of the high order model (variance part) (dashed line) and the 3σ bounds (solid line).

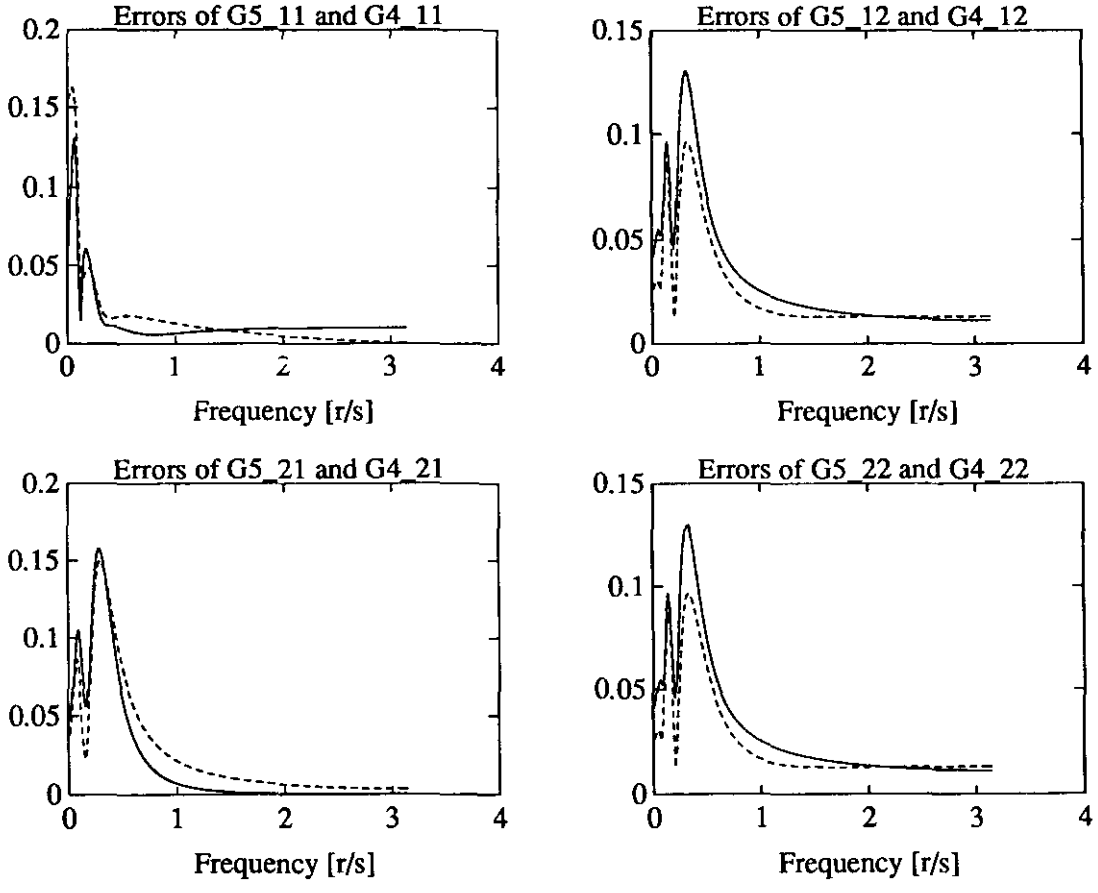

Fig. 6.3 The errors of the model with structure (5,5) (solid line) and the errors of the model with structure (4,4) (dashed line).

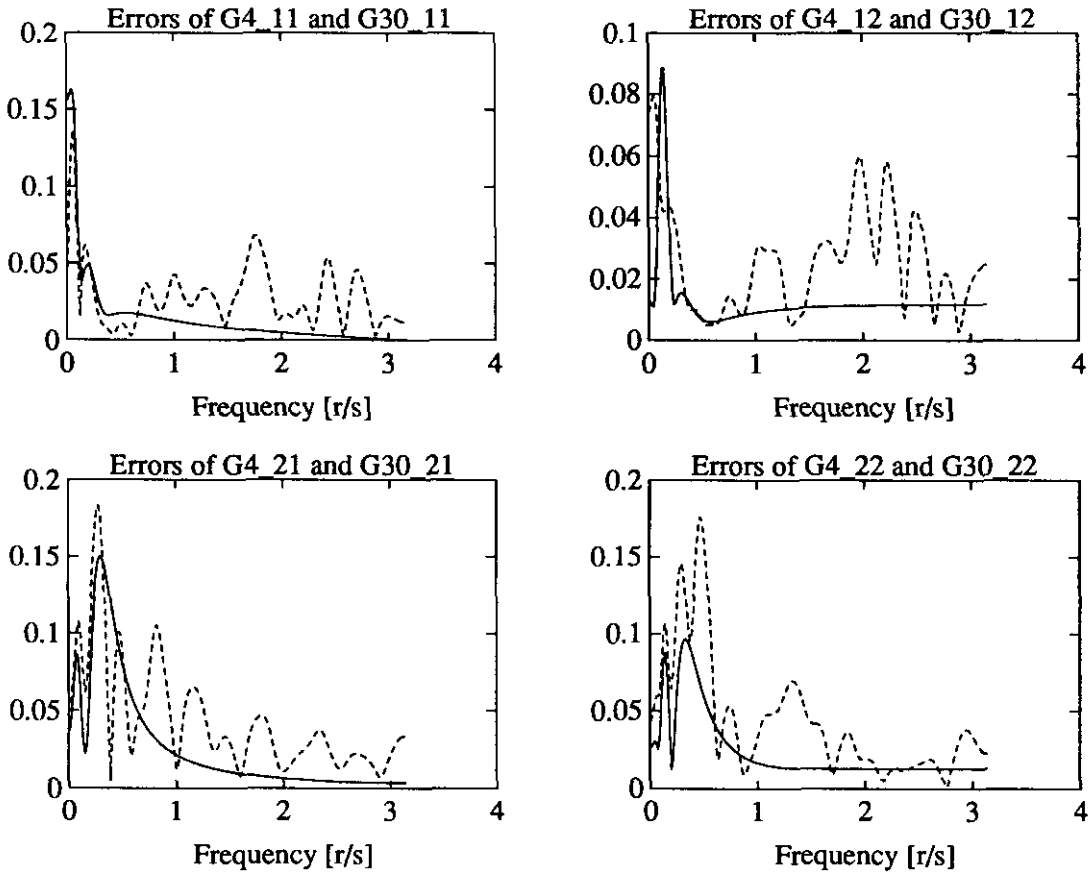

Fig. 6.4 The errors of the model with structure (4,4) (solid line) and the errors of the high order model (dashed line).

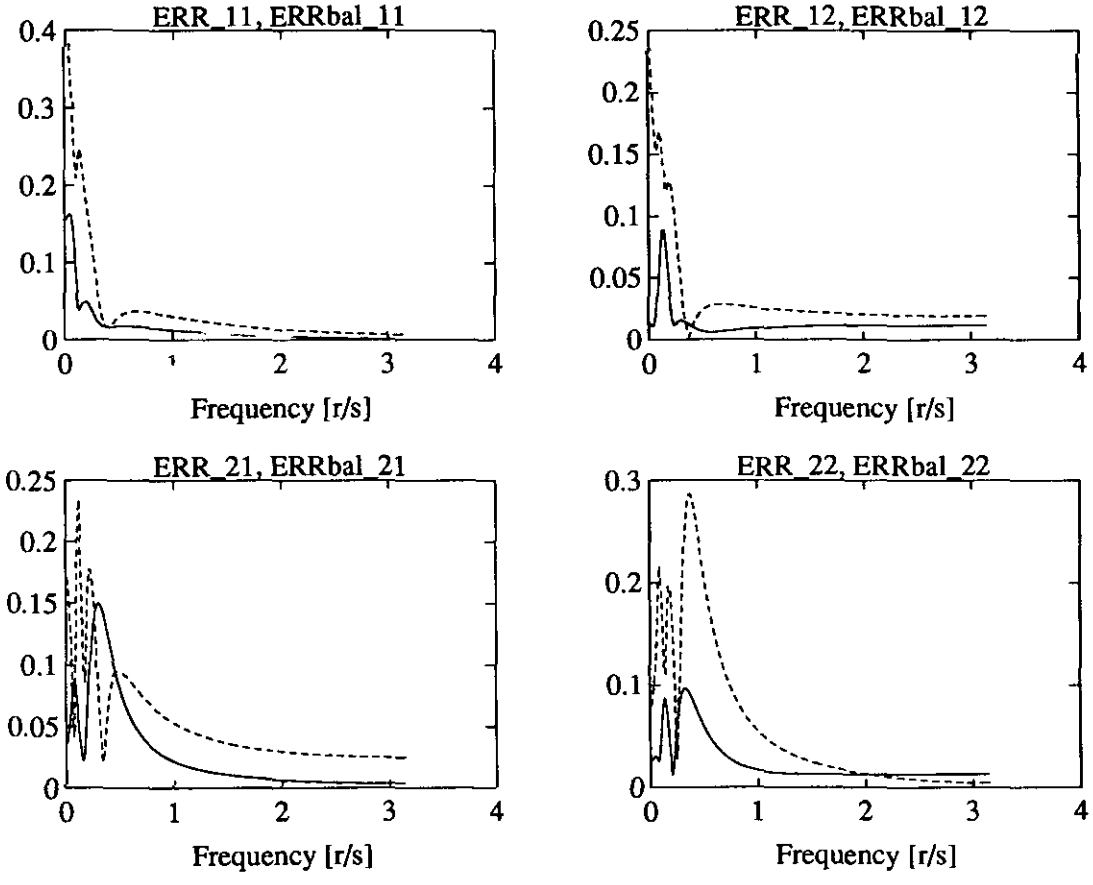

Fig. 6.5 The errors of the model with structure (4,4) (solid line) and the errors of the model with structure (4,4) which is obtained by balanced model reduction (dashed line).

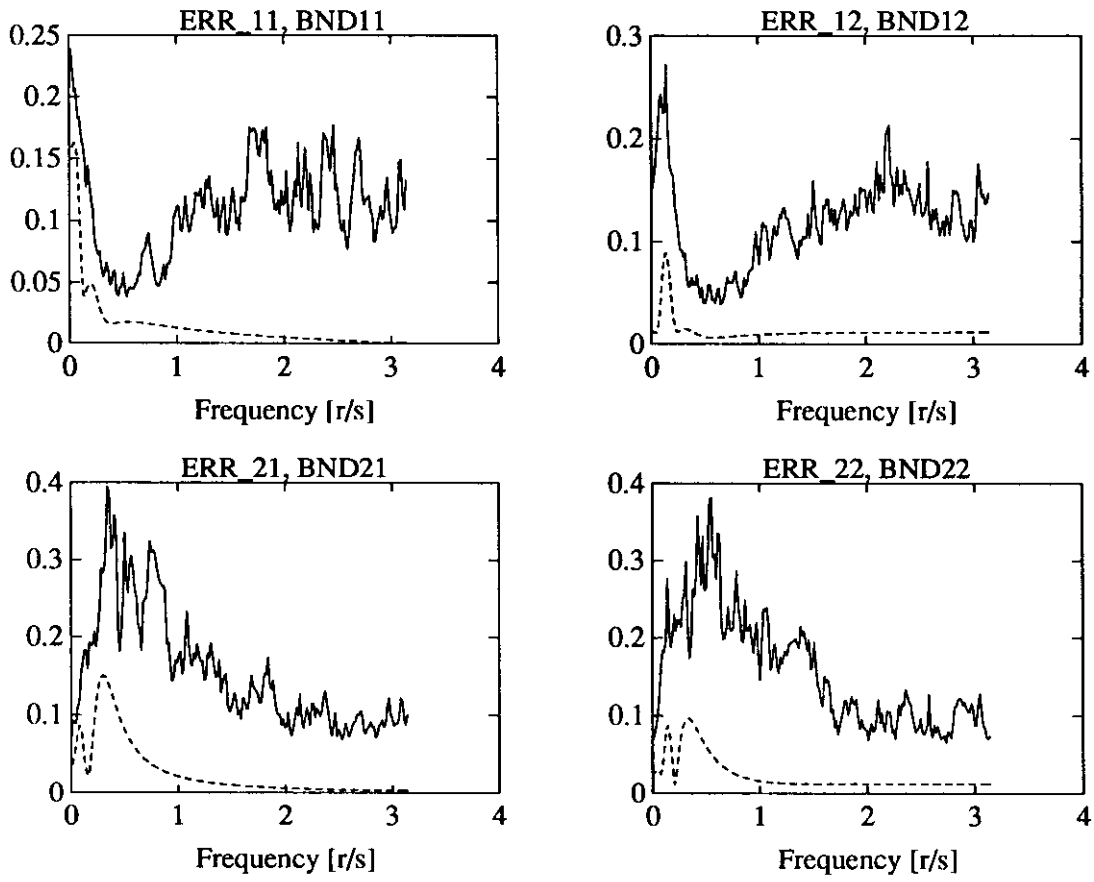

Fig. 6.6 The errors of the model with structure (4,4) (dashed line) and the upper bounds according to (3.11) (solid line).

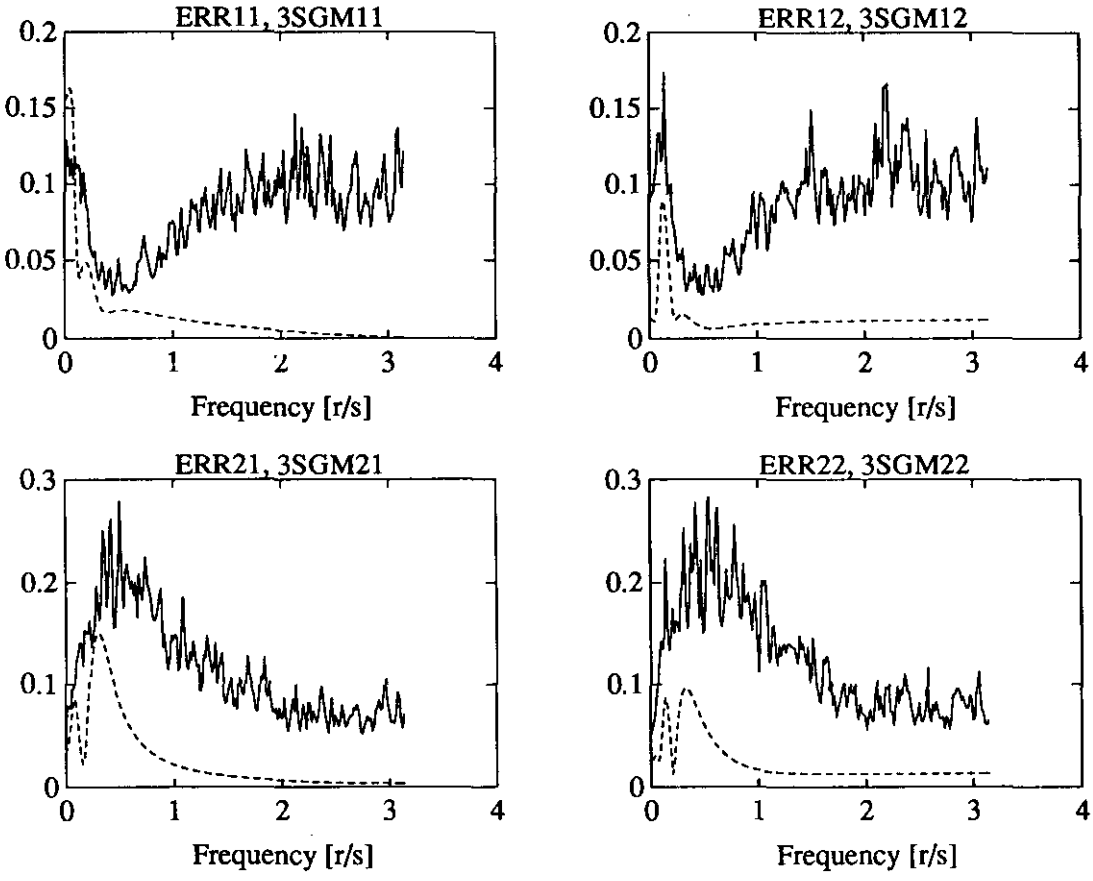

Fig. 6.7 The errors of the model with structure (4,4) (dashed line) and the 3σ upper bounds according to (3.11b) (solid line).

Eindhoven University of Technology Research Reports
Faculty of Electrical Engineering

ISSN 0167-9708
Coden: TEUEDE

(222) Jóźwiak, L.
THE FULL-DECOMPOSITION OF SEQUENTIAL MACHINES WITH THE SEPARATE REALIZATION OF THE NEXT-STATE AND OUTPUT FUNCTIONS.
EUT Report 89-E-222. 1989. ISBN 90-6144-222-2

(223) Jóźwiak, L.
THE BIT FULL-DECOMPOSITION OF SEQUENTIAL MACHINES.
EUT Report 89-E-223. 1989. ISBN 90-6144-223-0

(224) Book of abstracts of the first Benelux-Japan Workshop on Information and Communication Theory, Eindhoven, The Netherlands, 3-5 September 1989.
Ed. by Han Vinck.
EUT Report 89-E-224. 1989. ISBN 90-6144-224-9

(225) Hoeijmakers, M.J.
A POSSIBILITY TO INCORPORATE SATURATION IN THE SIMPLE, GLOBAL MODEL OF A SYNCHRONOUS MACHINE WITH RECTIFIER.
EUT Report 89-E-225. 1989. ISBN 90-6144-225-7

(226) Dahiya, R.P. and E.M. van Veldhuizen, W.R. Rutgers, L.H.Th. Rietjens
EXPERIMENTS ON INITIAL BEHAVIOUR OF CORONA GENERATED WITH ELECTRICAL PULSES SUPERIMPOSED ON DC BIAS.
EUT Report 89-E-226. 1989. ISBN 90-6144-226-5

(227) Bastings, R.H.A.
TOWARD THE DEVELOPMENT OF AN INTELLIGENT ALARM SYSTEM IN ANESTHESIA.
EUT Report 89-E-227. 1989. ISBN 90-6144-227-3

(228) Hekker, J.J.
COMPUTER ANIMATED GRAPHICS AS A TEACHING TOOL FOR THE ANESTHESIA MACHINE SIMULATOR.
EUT Report 89-E-228. 1989. ISBN 90-6144-228-1

(229) Oostrom, J.H.M. van
INTELLIGENT ALARMS IN ANESTHESIA: An implementation.
EUT Report 89-E-229. 1989. ISBN 90-6144-229-X

(230) Winter, M.R.M.
DESIGN OF A UNIVERSAL PROTOCOL SUBSYSTEM ARCHITECTURE: Specification of functions and services.
EUT Report 89-E-230. 1989. ISBN 90-6144-230-3

(231) Schemmann, M.F.C. and H.C. Heyker, J.J.M. Kwaspen, Th.G. van de Roer
MOUNTING AND DC TO 18 GHz CHARACTERISATION OF DOUBLE BARRIER RESONANT TUNNELING DEVICES.
EUT Report 89-E-231. 1989. ISBN 90-6144-231-1

(232) Sarma, A.D. and M.H.A.J. Herben
DATA ACQUISITION AND SIGNAL PROCESSING/ANALYSIS OF SCINTILLATION EVENTS FOR THE OLYMPUS PROPAGATION EXPERIMENT.
EUT Report 89-E-232. 1989. ISBN 90-6144-232-X

(233) Nederstigt, J.A.
DESIGN AND IMPLEMENTATION OF A SECOND PROTOTYPE OF THE INTELLIGENT ALARM SYSTEM IN ANESTHESIA.
EUT Report 90-E-233. 1990. ISBN 90-6144-233-8

(234) Philippens, E.H.J.
DESIGNING DEBUGGING TOOLS FOR SIMPLEXYS EXPERT SYSTEMS.
EUT Report 90-E-234. 1990. ISBN 90-6144-234-6

(235) Heffels, J.J.M.
A PATIENT SIMULATOR FOR ANESTHESIA TRAINING: A mechanical lung model and a physiological software model.
EUT Report 90-E-235. 1990. ISBN 90-6144-235-4

(236) Lammers, J.O.
KNOWLEDGE BASED ADAPTIVE BLOOD PRESSURE CONTROL: A Simplexys expert system application.
EUT Report 90-E-236. 1990. ISBN 90-6144-236-2

(237) Ren Qingchang
PREDICTION ERROR METHOD FOR IDENTIFICATION OF A HEAT EXCHANGER.
EUT Report 90-E-237. 1990. ISBN 90-6144-237-0

Eindhoven University of Technology Research Reports
Faculty of Electrical Engineering

ISSN 0167-9708
Coden: TEUEDE

(238) Lammers, J.O.
THE USE OF PETRI NET THEORY FOR SIMPLEXYS EXPERT SYSTEMS PROTOCOL CHECKING.
EUT Report 90-E-238. 1990. ISBN 90-6144-238-9

(239) Wang, X.
PRELIMINARY INVESTIGATIONS ON TACTILE PERCEPTION OF GRAPHICAL PATTERNS.
EUT Report 90-E-239. 1990. ISBN 90-6144-239-7

(240) Lutgens, J.M.A.
KNOWLEDGE BASE CORRECTNESS CHECKING FOR SIMPLEXYS EXPERT SYSTEMS.
EUT Eeport 90-E-240. 1990. ISBN 90-6144-240-0

(241) Brinker, A.C. den
A MEMBRANE MODEL FOR SPATIOTEMPORAL COUPLING.
EUT Report 90-E-241. 1990. ISBN 90-6144-241-9

(242) Kwaspen, J.J.M. and H.C. Heyker, J.I.M. Demarteau, Th.G. van de Roer
MICROWAVE NOISE MEASUREMENTS ON DOUBLE BARRIER RESONANT TUNNELING DIODES.
EUT Report 90-E-242. 1990. ISBN 90-6144-242-7

(243) Massee, P. and H.A.L.M. de Graaf, W.J.M. Balemans, H.G. Knoopers, H.H.J. ten Kate
PRELIMINARY DESIGN OF AN EXPERIMENTAL (5-10 MWt) DISK MHD FACILITY AND PROSPECTS OF COMMERCIAL (1000 MWt) MHD/STEAM SYSTEMS.
EUT Report 90-E-243. 1990. ISBN 90-6144-243-5

(244) Klompstra, Martin and Ton van den Boom, Ad Damen
A COMPARISON OF CLASSICAL AND MODERN CONTROLLER DESIGN: A case study.
EUT Report 90-E-244. 1990. ISBN 90-6144-244-3

(245) Berg, P.H.G. van de
ON THE ACCURACY OF RADIOWAVE PROPAGATION MEASUREMENTS: Olympus propagation experiment.
EUT Report 90-E-245. 1990. ISBN 90-6144-245-1

(246) Maagt, P.J.L. de
A SYNTHESIS METHOD FOR COMBINED OPTIMIZATION OF MULTIPLE ANTENNA PARAMETERS AND ANTENNA PATTERN STRUCTURE.
EUT Report 90-E-246. 1990. ISBN 90-6144-246-X

(247) Jóźwiak, L. and T. Spassova-Kwaaitaal
DECOMPOSITIONAL STATE ASSIGNMENT WITH REUSE OF STANDARD DESIGNS: Using counters as sub-machines and using the method of maximal adjacensies to select the state chains and the state codes.
EUT Report 90-E-247. 1990. ISBN 90-6144-247-8

(248) Hoeijmakers, M.J. and J.M. Vleeshouwers
DERIVATION AND VERIFICATION OF A MODEL OF THE SYNCHRONOUS MACHINE WITH RECTIFIER WITH TWO DAMPER WINDINGS ON THE DIRECT AXIS.
EUT Report 90-E-248. 1990. ISBN 90-6144-248-6