Progress of Theoretical Physics, Vol. 40, No. 4, October 1968

Muon Capture in C^{12} and Nuclear Structure

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(Received May 27, 1968)

We have calculated the muon capture rate for $C^{12}+\mu^- \to B^{12}$(ground state)$+\nu$, using general p-shell wave functions which were obtained by considering the dynamical properties in the $A=12$ system. Our theoretical formula for the capture rate is of such a form that the effect of the finite size of the nucleus on the muon wave functions is easily taken into account. The experimental ft value of B^{12}, which in all previous theories is used to normalize the muon capture rate, has not been adopted in our analysis. An excellent fit to the experimental data for the capture rate as well as for the ft value has been obtained with the B^{12} wave function proposed by Kurath to explain the magnetic moment of B^{12} and a C^{12} wave function introduced by us. These wave functions have slightly different values of configuration mixing parameters from those originally given by Cohen and Kurath. The value for the oscillator strength parameter which we use is $b=1.64$ f; this is in conformity with the elastic and most of the inelastic electron scattering data. On the other hand, should b be taken as 1.90 f, the value derived from the analysis of the inelastic electron scattering data associated with the 15.1 MeV level in C^{12}, it is relatively difficult to have a simultaneous fit to the data for the capture rate and the ft value. Our results are based on the assumptions of 1) universality of the V-A interaction in muon capture and beta decay, 2) the validity of the conserved vector current theory, and 3) the induced pseudoscalar coupling constant estimated by Goldberger and Treiman.

§ 1. Introduction

As is well known, understanding muon capture reactions involves two problems. One concerns the law of the weak interactions itself, and the other is the problem of nuclear structure. These two problems are not clearly separable. For example, our knowledge of weak interaction coupling constants is limited by uncertainties in both nuclear matrix elements and lepton wave functions. Fortunately, these uncertainties are being reduced by more precise experiments and continuing theoretical investigations in this field of physics.[1] A typical example of muon capture is the reaction in which C^{12} in the ground state absorbs a muon yielding the ground states of B^{12}. Measurements of this partial transition rate have been repeatedly performed (see Table I) and the latest data of the Carnegie group[2] is

$$\mathcal{W}=\left(6.75\begin{matrix}+0.30\\-0.75\end{matrix}\right)\times 10^3\ \mathrm{sec}^{-1}.$$

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Theoretical work in this problem was first performed by Fujii and Primakoff[3] using a single particle *j-j* coupling shell model with Primakoff's original theory of the muon capture.[4] A similar calculation was also made by Wolfenstein.[5] Later, the theory of muon capture was formulated in a way similar to the theory of allowed and forbidden transitions of the beta decay.[6] This theory includes

Table I. Experimental data for muon capture rate in C^{12} in units of 10^3 sec^{-1}.

Year	Muon capture rate	Reference
1953	5.9 ± 1.5	*a*
1958	9.18 ± 0.5	*b*
1959	9.05 ± 0.95	*c*
1959	6.8 ± 1.5	*d*
1960	5.8 ± 1.3	*e*
1960	6.8 ± 1.1	*f*
1961	6.31 ± 0.24	*g*
1963	6.6 ± 0.9	*h*
1964	$6.75 {+0.30 \atop -0.75}$	*i*

a) T.N.K. Godfrey, Phys. Rev. **92** (1953), 512.
b) J.O. Burgman, J. Fischer, B. Leontic, A. Lundby, R. Meunier, J.P. Stroot, and J.D. Teja, Phys. Rev. Letters **1** (1958), 469.
c) H.V. Argo, F.B. Harrison, H.W. Kruse, and A.D. McGuire, Phys. Rev. **114** (1959), 626.
d) W. Love, S. Marder, I. Nadelhaft, R. Siegel, and A.E. Taylor, Bull. Am. Phys. Soc. **4** (1959), 81.
e) B.L. Bloch, Thesis, Carnegie Institute of Technology Report NYO-9280 (1960).
f) J.G. Fetkovich, T.H. Fields, and R.L. McIlwain, Phys. Rev. **118** (1960), 319.
g) E.J. Maier, B.L. Bloch, R.M. Edelstein, and R.T. Siegel, Phys. Rev. Letters. **6** (1961), 417.
h) G.T. Reynolds, D.B. Scarl, R.A. Swanson, J.R. Waters, and R.A. Zdanis, Phys. Rev. **129** (1963), 1790.
i) E.J. Maier, R.M. Edelstein, and R.T. Siegel, Phys. Rev. **133** (1964), B663.

the relativistic terms correctly. Whereas these relativistic terms are only corrections to the main terms in allowed or unique forbidden transitions, they make an essential contribution to the capture rate in the case of nonunique transitions. Morita and Fujii incorporated these terms and calculated[6] the partial muon capture rate in C^{12} (which can be classified as an allowed transition). They found that the relativistic corrections change the capture rate by about 10%. Their theory was again based on the single particle *j-j* coupling shell model. In these three theories, the calculated muon capture rate and the beta decay rate of the ground state of B^{12} are about five times larger than the experimental data. Therefore a normalization has been adopted so that the calculated beta decay rate coincides with the experimental one. Flamand and Ford[7] considered the muon capture in a slightly different manner. They evaluated nuclear matrix elements in the intermediate coupling shell model, where the strength parameter a/K was adjusted so as to give the correct beta decay rate. They included the effect of the nuclear finite size on the muon wave functions in the K-orbit. The effect was

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about -6%. In another approach Foldy and Walecka[8] expressed the Gamow-Teller matrix ([1 0 1] in Eq. (6) below) in terms of the ft value of the beta decay of the ground state of B^{12} and of the transverse magnetic dipole operator in the inelastic electron scattering[9] to the 15.1 MeV 1^+ level in C^{12}. This level of C^{12} is the isobaric analog of the ground state of B^{12}. The contribution of the smaller nuclear matrix elements was evaluated in the j-j and L-S coupling shell model. The muon capture rate in their theory is, therefore, nearly nuclear model independent.

An entirely new approach to calculate the muon capture rate was developed by Kim and Primakoff,[10] who treated the nuclei of B^{12} and C^{12} as elementary particles. This theory is based on the hypothesis of the conserved polar vector hadron weak current and the hypothesis of the partially conserved axial vector hadron weak current. The theory does not use the impulse approximation which was adopted by all previous authors. Instead the momentum transfer dependence of the weak nuclear form factors is found from an analysis of suitable empirical nuclear structure data.

The above theories (including the one by Kim and Primakoff[10]) are all in substantial agreement with the experimental data $\mathcal{W}$, *if* the normalization for the nuclear matrix elements or form factors is chosen so as to give the correct beta decay rate of the ground state of B^{12}. In this paper, we will try to give a correct muon capture rate in C^{12} without introducing such normalization constant. We will do it using general p-shell wave functions with configuration mixing parameters obtained by dynamical properties of the $A=12$ system, as described by Cohen, Kurath, and Soga.[11] As a matter of fact, these wave functions with the original configuration mixing parameters reproduce 88% of the muon capture and beta decay rates, and 76% of the magnetic moment[12] of the ground state of B^{12}. Recently, Kurath suggested a slight modification of the B^{12} wave function[13] in order to reproduce the correct value of the magnetic moment of B^{12}. We examined this wave function, too, and we found the fact that this modification does not change the muon capture rate at all. Finally, we make a similar modification of the C^{12} wave function and find an excellent fit to the experimental data on the muon capture rate as well as the beta decay rate. To perform this investigation, we use an explicit formula of the muon capture rate with well-defined nuclear matrix elements which include the large and small components of the muon wave functions, (§ 2). This formula can, in principle, be derived from Eqs. (40), (41), and Table I in reference 6). For the weak coupling constants, we are in a better position than the authors of the previous works. We adopt the revised values of the vector and axial vector coupling constants, obtained from a new calculation of the ft value of the O^{14} beta decay by Behrens and Bühring,[14] and a new measurement of the neutron half-life by the Copenhagen group.[15] The universality of the V-A interaction in beta decay and muon capture has also been confirmed in a recent measurement of the muon

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capture in atomic protons,[16] if one accepts the conjectures concerning renormalization and induced couplings.[4),17),18] These are summarized in § 3. In § 4 the reduced nuclear matrix elements are explicitly given in terms of general p-shell wave functions. In § 5 muon wave functions are obtained numerically as the solutions of the Dirac equation where the finite nuclear size is included by using harmonic oscillator approximation in conformity with elastic electron scattering data.[19] The results of our analysis are given in detail in § 6. There nuclear wave functions, which reproduce all available experimental data on the muon capture, beta decay, level scheme, magnetic moment, and electron scattering, are found. It is also proved that the weak coupling constants usually accepted in the theory of the muon capture are all consistent with the data of the muon capture rate in C^{12}.

§ 2. Interaction Hamiltonian for muon capture reaction and capture rate

The lepton bare-nucleon coupling is assumed to be via vector and axial vector interactions of the Fermi-type. Thus the most general interaction Hamiltonian density for the reaction, $\mu^- + p \rightarrow n + \nu$, is

$$H_{\text{int}} = (\bar{\phi}_\nu \gamma_\alpha \psi_n)(\bar{\psi}_n [f_V \gamma_\alpha - i g_V \sigma_{\alpha\beta} p_\beta - i h_V p_\alpha] \psi_p) + (\bar{\phi}_\nu i \gamma_\alpha \gamma_5 \psi_n)(\bar{\psi}_n [i f_A \gamma_\alpha \gamma_5 - g_A p_\alpha \gamma_5 + h_A \sigma_{\alpha\beta} \gamma_5 p_\beta] \psi_p) \tag{1}$$

with

$$\sigma_{\alpha\beta} = \tfrac{1}{2}[\gamma_\alpha, \gamma_\beta], \qquad \phi_\nu = (1+\gamma_5)\psi_\nu/\sqrt{2}\,.$$

Here, the four momentum transfer p_β is the difference of the initial proton momentum and the final neutron momentum. In order to evaluate the nuclear matrix elements, we introduce the nonrelativistic approximation for nucleons. H_{int} becomes

$$H_{\text{int}} = u_n^\dagger \boldsymbol{H} u_p \tag{2}$$

with

$$\begin{aligned}\boldsymbol{H} = {} & C_V 1 \cdot L(1) + C_A \boldsymbol{\sigma} \cdot L(\boldsymbol{\sigma}) \\ & + (C_V/2M)[2L(\boldsymbol{\alpha}) \cdot \boldsymbol{p} + \boldsymbol{p} \cdot L(\boldsymbol{\alpha}) + i\boldsymbol{\sigma} \cdot \boldsymbol{p} \times L(\boldsymbol{\alpha})] \\ & + (C_A/2M)[2L(\gamma_5)\boldsymbol{\sigma} \cdot \boldsymbol{p} + \boldsymbol{\sigma} \cdot \boldsymbol{p} L(\gamma_5)] \\ & - (C_P/2M)\boldsymbol{\sigma} \cdot \boldsymbol{p} L(\beta\gamma_5) + (\mu_p - \mu_n)(C_V/2M)[i\boldsymbol{\sigma} \cdot \boldsymbol{p} \times L(\boldsymbol{\alpha})] \\ & - C_S 1 \cdot L(\beta) - C_T \boldsymbol{\sigma} \cdot L(\boldsymbol{\sigma}) - (C_T/W_0)\boldsymbol{\sigma} \cdot \boldsymbol{p} L(\gamma_5),\end{aligned} \tag{3}$$

and the lepton currents $L(\boldsymbol{\sigma})$ given by

$$L(\boldsymbol{\sigma}) = \psi_\nu^\dagger (1+\gamma_5) \boldsymbol{\sigma} \psi_\mu / \sqrt{2}\,, \quad \text{etc.} \tag{4}$$

The u's are the large components of nucleon wave functions. The coupling

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constants are introduced as follows:

$$C_V=f_V\,,\quad g_V=(\mu_p-\mu_n)(C_V/2M)\,,\quad C_S=-m_\mu h_V\,,$$
$$C_A=f_A\,,\quad C_P=m_\mu g_A\,,\quad C_T=-h_A W_\circ\,,\tag{5}$$

where $W_\circ$ is the maximum energy of electron in the beta decay, $J_f\rightarrow J_i$. The form factors g_V, g_A, h_V, and h_A refer to the weak magnetism, induced pseudoscalar, induced scalar, and induced Konopinski-Uhlenbeck-type interactions, respectively. A relation between g_V and C_V is given by the conserved vector current theory[20] for which $h_V=0$. This relation does not necessarily hold in cases where $h_V\neq 0$. The last line in Eq. (3) gives the induced interaction under the assumption of possible G-parity nonconservation. All quantities f, g, and h are dependent on p^2, and they are real under the assumption of time-reversal invariance.

The muon capture rate has been investigated for cases in which the final states have definite spin and parity.[6] The explicit formulas were, however, given under the assumption that the small component of the muon wave function is negligible. Formulas with no such assumption are derived from Eq. (39) and Table I of the reference;[6] these will be published elsewhere. A complete expression of the capture rate is given below in the case of the $0^+\rightarrow 1^+$ transition which corresponds to the muon capture in C^{12}.

$$\begin{aligned}\mathcal{W}=&[1-q(m_\mu+AM)^{-1}]q^2(C_A/C_A{}^\beta)^2(C_A{}^\beta/C_V{}^\beta)^2G^2(mc^2/\hbar)\\
&\times\Big(\{-\sqrt{2}[1-(C_T/C_A)][1\,0\,1,1]+\sqrt{1/3}(2C_V/C_AM)[1\,1\,1\,p,1]\\
&+(qC_V/3C_AM)\mu(\sqrt{2}[1\,0\,1,2]-[1\,2\,1,2])\\
&-\sqrt{2}(1/M)[0\,1\,1\,p,1]\\
&-\sqrt{1/2}(q/3M)[1-(2MC_T/W_\circ C_A)]([1\,0\,1,3]+\sqrt{2}[1\,2\,1,3])\\
&+\sqrt{1/2}(qC_P/3C_AM)([1\,0\,1,2]+\sqrt{2}[1\,2\,1,2])\}^2\\
&+\{-\sqrt{2}[1-(C_T/C_A)][1\,2\,1,1]-\sqrt{2/3}(C_V/C_AM)[1\,1\,1\,p,2]\\
&-\sqrt{2}(qC_V/6C_AM)\mu(\sqrt{2}[1\,0\,1,4]-[1\,2\,1,4])\\
&-(2/M)[0\,1\,1\,p,2]\\
&-(q/3M)[1-(2MC_T/W_\circ C_A)]([1\,0\,1,5]+\sqrt{2}[1\,2\,1,5])\\
&+(qC_P/3C_AM)([1\,0\,1,4]+\sqrt{2}[1\,2\,1,4])\}^2\Big),\end{aligned}\tag{6}$$

with

$$q=(m_\mu-W_\circ)[1-\tfrac{1}{2}m_\mu(m_\mu+AM)^{-1}],$$
$$mc^2/\hbar=0.7763\times 10^{21}\ \text{sec}^{-1},$$

and

$$\mu=1+\mu_p-\mu_n=4.706\,.$$

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All quantities are given in units where $\hbar=c=m=1$. The symbols m, m_μ, and M refer to the electron, muon, and nucleon mass, respectively. A is the mass number of the nucleus, and W_0 is the maximum energy of electron in the beta decay which is an inverse reaction of muon capture, $1^+\to 0^+$. G-parity invariance is assumed in this paper, and we put $C_T=0$ hereafter. The coupling constants, G, $C_V{}^\beta$, $C_A{}^\beta$, for beta decay will be explained in the next section.

The reduced nuclear matrices are defined by

$$[S\,L\,J,\,n]\,(J_iJM_iM|J_fM_f)$$
$$=\int u_{J_f}^{M_f*}\sum_{k=1}^{A}\boldsymbol{O}(\boldsymbol{r})\,\delta(\boldsymbol{r}-\boldsymbol{r}_k)\,\tau_-^{(k)}u_{J_i}^{M_i}d\boldsymbol{r}_1d\boldsymbol{r}_2\cdots d\boldsymbol{r}_A\,. \tag{7}$$

Here $u_{J_i}^{M_i}$ and $u_{J_f}^{M_f}$ are nuclear wave functions of the initial and final states specified by the spin J and its projection M. $\tau_-^{(k)}$ and $\boldsymbol{O}(\boldsymbol{r})\,\delta(\boldsymbol{r}-\boldsymbol{r}_k)$ are the isospin and the interaction operator for the kth nucleon. S, L, and J in the brackets are the resultant spin, the effective orbital angular momentum, and the resultant total angular momentum of the lepton system (or equivalently, those of the proton-neutron system), respectively. J is equal to the rank of the matrix elements. The fourth number n in the brackets specifies different matrices whose interaction operators are slightly different each other. The matrices $[S\,L\,J\boldsymbol{p},\,n]$, which include the differential operator $\boldsymbol{p}$ acting on the nuclear wave function may be defined by an equation similar to Eq. (7). The operators $\boldsymbol{O}(\boldsymbol{r})$ in Eq. (7) are summarized in Table II where the vector harmonics are defined by

Table II. Operators $\boldsymbol{O}(\boldsymbol{r})$ in Eq. (7). The spherical Bessel functions $j_L(qr)$ are abbreviated as j_L. The upper and lower signs in the expression refer to the upper and lower numbers in the brackets of the matrix, respectively. The radial parts in the curly brackets correspond to the symbols $O(r)$ in Eqs. (26)-(28). The prime means the derivative with respect to $\rho=qr$.

Matrix element	$\boldsymbol{O}(\boldsymbol{r})$
$[1\ 0\ 1,\ 1]$	$\{j_0g+(1/3)j_1f\}\boldsymbol{Y}_{101}^m(\hat{r},\boldsymbol{\sigma})$
$\left[1\ 0\ 1,\ \begin{matrix}2\\3\end{matrix}\right]$	$\{j_0g+j_1g'\pm[j_1-(2/\rho)j_0]f\mp j_0f'\}\boldsymbol{Y}_{101}^m(\hat{r},\boldsymbol{\sigma})$
$\left[1\ 0\ 1,\ \begin{matrix}4\\5\end{matrix}\right]$	$\{j_0g+j_1g'\pm[j_1-(1/\rho)j_2]f\pm j_2f'\}\boldsymbol{Y}_{101}^m(\hat{r},\boldsymbol{\sigma})$
$[0\ 1\ 1\,p,\ 1]$	$i\{j_1g+j_0f\}\boldsymbol{Y}_{011}^m(\hat{r})\boldsymbol{\sigma}\cdot\boldsymbol{p}$
$[0\ 1\ 1\,p,\ 2]$	$i\{j_1g-j_2f\}\boldsymbol{Y}_{011}^m(\hat{r})\boldsymbol{\sigma}\cdot\boldsymbol{p}$
$[1\ 1\ 1\,p,\ 1]$	$i\{j_1g-j_0f\}\boldsymbol{Y}_{111}^m(\hat{r},\boldsymbol{p})$
$[1\ 1\ 1\,p,\ 2]$	$i\{j_1g+j_2f\}\boldsymbol{Y}_{111}^m(\hat{r},\boldsymbol{p})$
$[1\ 2\ 1,\ 1]$	$\{j_2g+(1/3)j_1f\}\boldsymbol{Y}_{121}^m(\hat{r},\boldsymbol{\sigma})$
$\left[1\ 2\ 1,\ \begin{matrix}2\\3\end{matrix}\right]$	$\{j_2g-j_1g'\mp[j_1+(1/\rho)j_0]f\pm j_0f'\}\boldsymbol{Y}_{121}^m(\hat{r},\boldsymbol{\sigma})$
$\left[1\ 2\ 1,\ \begin{matrix}4\\5\end{matrix}\right]$	$\{j_2g-j_1g'\mp[j_1-(4/\rho)j_2]f\mp j_2f'\}\boldsymbol{Y}_{121}^m(\hat{r},\boldsymbol{\sigma})$

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$$\boldsymbol{Y}^m_{0vu}(\hat{r}) = (1/4\pi)^{1/2} Y_{vm}(\hat{r})\,\delta_{vu}\,,$$

$$\boldsymbol{Y}^m_{kvu}(\hat{r},\boldsymbol{\sigma}) = \sum_{m'm''} (kvm'm''|um)\, Y_{vm''}(\theta,\varphi)\,\boldsymbol{Y}_{km'}(\boldsymbol{\sigma})\,, \tag{8}$$

with

$$\boldsymbol{Y}_{00}(\boldsymbol{\sigma}) = (1/4\pi)^{1/2}, \qquad \boldsymbol{Y}_{10}(\boldsymbol{\sigma}) = (3/4\pi)^{1/2}\sigma_z\,, \text{ etc.} \tag{9}$$

$\boldsymbol{Y}^m_{kvu}(\hat{r},\boldsymbol{p})$ is obtained from Eq. (8) by replacing $\boldsymbol{\sigma}$ by the differential operator $\boldsymbol{p}$. In particular,

$$\boldsymbol{Y}^m_{011}(\hat{r}) = (1/4\pi)^{1/2} Y_{1m}(\hat{r})\,,$$

and

$$\boldsymbol{Y}^m_{101}(\hat{r},\boldsymbol{\sigma}) = (1/4\pi)^{1/2}\,\boldsymbol{Y}_{1m}(\boldsymbol{\sigma})\,.$$

The spherical Bessel functions which appear in Table II are the radial wave functions of the neutrino. The radial wave functions of the bound muon in the K-orbit are denoted by g and f, and have the following forms for a point nucleus,

$$g = (2Z/a_0)^{3/2}[(1+\gamma)/2\Gamma(2\gamma+1)]^{1/2} e^{-Zr/a_0}(2Zr/a_0)^{\gamma-1},$$

$$f = -[(1-\gamma)/(1+\gamma)]^{1/2} g \qquad \text{with } \gamma = [1-(\alpha Z)^2]^{1/2}.$$

Here a_0 is the Bohr radius of the mu-mesic atom, Z the atomic number of the parent nucleus, and α the fine structure constant. In § 5, we will, however, find more accurate forms of the functions g and f, by taking into account the charge distribution in the nucleus explicitly.

§ 3. Coupling constants in weak interactions

The weak coupling constants C for muon capture are derived from C^β for beta decay, by taking into account the p^2 dependence. We first adopt the following relations given by Fujii and Primakoff[3] in connection with the electromagnetic form factor of the proton and the dispersion theoretic conjecture:

$$C_V = 0.972 C_V{}^\beta = 0.972 G \frac{\hbar^3}{m^2 c}\,, \tag{10a}$$

and

$$C_A = 0.999 C_A{}^\beta. \tag{10b}$$

The vector coupling constant for beta decay can be obtained from the ft values of the $0^+\rightarrow0^+$ positron decays, the end point energies and half-lives of which have been measured with remarkable accuracy in the last few years. Several authors calculated ft values under different assumptions. The results are, however, in good agreement. Furthermore, all reported ft values for $0^+\rightarrow0^+$ transitions are consistent, if experimental errors are taken into consideration.[1] Using the end point energy 1812 ± 1.4 keV and half-life 71.36 ± 0.09 sec measured

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by the Pasadena group,[21] Behrens and Bühring computed the ft value of the O^{14} decay and coupling constant:[14]

$$ft(O^{14}) = 3142 \pm 11 \text{ sec}, \tag{11}$$

and

$$C_V{}^{\beta} = (1.3986 \pm 0.0024) \times 10^{-49} \text{ erg cm}^3, \tag{12}$$

or equivalently

$$G = 2.966(1 \pm 0.17\%) \times 10^{-12}.$$

The axial vector coupling constant is usually derived by considering the ft values of neutron decay and of O^{14}. Bhalla computed the ft value of neutron,[22]

$$ft(n) = 1213.4 \pm 35 \text{ sec}, \tag{13}$$

from the neutron half-life[23]

$$t = 11.7 \pm 0.3 \text{ min}. \tag{14}$$

Using his calculated value of $ft(O^{14}) = 3127.3 \pm 31$ sec, he obtained

$$|C_A{}^{\beta}/C_V{}^{\beta}| = 1.18 \pm 0.02. \tag{15}$$

Recently, a new measurement of the neutron half-life was published by the Copenhagen group,[15]

$$t = 10.80 \pm 0.16 \text{ min}., \tag{16}$$

which corresponds to*)

$$C_A{}^{\beta}/C_V{}^{\beta} = -1.23 \pm 0.01\,. \tag{17}$$

The revised value of $C_A{}^{\beta}/C_V{}^{\beta}$ is in better agreement with the old value derived from the angular distributions of the electron and neutrino in the beta decay of polarized neutrons, measured by the Argonne group.[24] That value is

$$C_A{}^{\beta}/C_V{}^{\beta} = -1.25 \pm 0.05\,. \tag{18}$$

An induced term, g_V, associated with weak magnetism, is important in the region of higher momentum transfer. It is proportional to the magnetic moment μ. In the theory of the conserved vector current,[20] μ is the difference of the magnetic moments for proton and neutron,

$$\mu = 1 + \mu_p - \mu_n = 4.706\,. \tag{19}$$

(Strictly speaking, $(\mu_p - \mu_n)$ comes from the g_V term, and 1 comes from the f_V term. This is seen in Eq. (5).) Alternatively, μ may be adjusted as unknown parameter in the numerical calculations.

The induced pseudoscalar coupling, C_P, has been estimated in a dispersion

*) One of the authors (M.M.) found $|C_A{}^{\beta}/C_V{}^{\beta}| = 1.24$ (or 1.23) from $ft(O^{14}) = 3142$ (or 3127) and $ft(n) = 1213.4 \times (10.8/11.7)$.

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theoretic calculation[25] as

$$C_P/C_A = 7\sim 8\,. \tag{20}$$

This value is subject to be examined in the experimental muon capture rates.

All of the relations (10), (12), (15), (17), (19), and (20) have been verified to be consistent with the recent experimental data of the atomic muon capture rates.[8] In particular the experimental value is

$$\mathcal{W}_{\mu p}^{\text{singlet}} = 640 \pm 70\ \text{sec}^{-1}, \tag{21a}$$

while the theoretical values, e.g., given by Ohtsubo and Fujii,[18] are roughly:

$$\begin{aligned} \mathcal{W}_{\mu p}^{\text{singlet}} &= 646\sim 668\ \text{sec}^{-1} \qquad \text{for } C_A{}^{\beta}/C_V{}^{\beta} = -1.19 \\ &= 679\sim 722\ \text{sec}^{-1} \qquad \text{for } C_A{}^{\beta}/C_V{}^{\beta} = -1.25\,. \end{aligned} \tag{21b}$$

§ 4. Reduced nuclear matrix elements and nuclear wave functions of C^{12} and B^{12}

For the ground state wave functions of C^{12} and B^{12}, we will take the most general p-shell configurations studied by Cohen and Kurath.[11] In this framework, the wave functions of C^{12} and B^{12} are expressed by the following five and eight components, respectively:

$$\begin{aligned} \psi(C^{12}, T=0, J=0) = & C_1 |(p_{1/2})^4 (p_{3/2})^4, T=0, J=0\rangle \\ & + C_2 |(p_{1/2})^3 (p_{3/2})^5, T=0, J=0\rangle \\ & + C_3 |(p_{1/2})^2 (p_{3/2})^6 \alpha, T=0, J=0\rangle \\ & + C_4 |(p_{1/2})^2 (p_{3/2})^6 \beta, T=0, J=0\rangle \\ & + C_5 |(p_{3/2})^8, T=0, J=0\rangle, \end{aligned} \tag{22}$$

$$\begin{aligned} \psi(B^{12}, T=1, J=1) = & B_1 |(p_{1/2})^4 (p_{3/2})^4, T=1, J=1\rangle \\ & + B_2 |(p_{1/2})^3 (p_{3/2})^5 \alpha, T=1, J=1\rangle \\ & + B_3 |(p_{1/2})^3 (p_{3/2})^5 \beta, T=1, J=1\rangle \\ & + B_4 |(p_{1/2})^3 (p_{3/2})^5 \gamma, T=1, J=1\rangle \\ & + B_5 |(p_{1/2})^2 (p_{3/2})^6 \alpha, T=1, J=1\rangle \\ & + B_6 |(p_{1/2})^2 (p_{3/2})^6 \beta, T=1, J=1\rangle \\ & + B_7 |(p_{1/2})^2 (p_{3/2})^6 \gamma, T=1, J=1\rangle \\ & + B_8 |(p_{1/2})^1 (p_{3/2})^7, T=1, J=1\rangle, \end{aligned} \tag{23}$$

where all components are ortho-normal, and the coefficients, $C_i\,(i=1, 2, \cdots, 5)$ and $B_i\,(i=1, 2, \cdots, 8)$, are determined by dynamical properties of the system.

The reduced nuclear matrix elements for the muon capture reactions are essentially those of the single particle operator between the above two wave

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functions. They are evaluated by the standard technique of the nuclear shell model as a linear combination of the reduced nuclear matrix elements between different single particle states. For example,

$$[S\,L\,J,\,n]$$

$$=\int \psi^*(\mathrm{B}^{12},\,T=1,\,J=1)\,(\sum_{k=1}^{A}\boldsymbol{O}(\boldsymbol{r})\,\delta(\boldsymbol{r}-\boldsymbol{r}_k)\,\tau_-^{(k)})$$

$$\times\psi(\mathrm{C}^{12},\,T=0,\,J=0)\,d\boldsymbol{r}_1 d\boldsymbol{r}_2\cdots d\boldsymbol{r}_A$$

$$=\sum_{i,j} B_i C_j \langle (p_{1/2})^{ni}(p_{3/2})^{mi},\,T=1,\,J=1\|\boldsymbol{O\tau}\|(p_{1/2})^{nj}(p_{3/2})^{mj},\,T=0,\,J=0\rangle$$

$$\text{with } n_i+m_i=n_j+m_j=8$$

$$=\sum_{i,j} B_i C_j \sum_{j_p,j_n}\langle j_p\,1/2\,\|\boldsymbol{O\tau}\|\,j_n\,1/2\rangle$$

$$\times\sum_{J_0,T_0,\beta_0}(8/3)\langle 0\,0\,j\{|J_0\,T_0\,\beta_0,\,j_p\rangle\langle 1\,1\,i\{|J_0\,T_0\,\beta_0,\,j_n\rangle\delta_{J_0,j_p}\delta_{T_0,1/2} \tag{24a}$$

$$=\sum_{i,j}\sum_{j_p,j_n} B_i C_j D(i,\,j,\,j_n,\,j_p)\,[S\,L\,J,\,n]_{j_n j_p}\,, \tag{24b}$$

where the coefficient D is the sum in the second line of Eq. (24a). The reduced matrix elements $\langle j_p\,1/2\|\boldsymbol{O\tau}\|j_n\,1/2\rangle$ for a single nucleon are dependent on the total angular momenta, j_p and j_n, of the proton and neutron. Accordingly such reduced matrix elements are abbreviated by $[S\,L\,J,\,n]_{j_n j_p}$. In this notation, we write Eq. (24) as

$$[S\,L\,J,\,n]=\sum_{j_n,j_p} A(j_n,\,j_p)\,[S\,L\,J,\,n]_{j_n j_p}\,. \tag{25}$$

In the numerical calculations, $A(j_n,\,j_p)$ and $[S\,L\,J,\,n]_{j_n j_p}$ are separately computed, and finally we take the sum over $j_n=1/2$, $3/2$, and $j_p=1/2$, $3/2$. The factors $A(j_n,\,j_p)$ can be obtained from computer programs used in previous work by Cohen and Kurath.[11] The reduced nuclear matrix elements $[S\,L\,J,\,n]_{j_n j_p}$ have the following forms:

$$[S\,L\,J,\,n]_{j_n j_p}=(-)^{J+l_p+l_n+1}[3(2S+1)/4\pi]\,[(2L+1)\,(2l_p+1)\,(2j_n+1)]^{1/2}$$

$$\times(l_p L\,0\,0|l_n\,0)\begin{pmatrix} j_p & j_n & J \\ l_p & l_n & L \\ \frac{1}{2} & \frac{1}{2} & S\end{pmatrix}\int\phi(n_n l_n)\,O(r)\,\phi(n_p l_p)\,r^2 dr\,, \tag{26}$$

$$[0\,J\,J\,p,\,n]_{j_n j_p}=\sum_{l_p'=l_p\pm1}(-)^{j_p+l_p+1/2}[3(2l_p'+1)/4\pi]\,[3(2j_n+1)]^{1/2}$$

$$\times(l_p'\,J\,0\,0|l_n\,0)\,W(l_p' j_p l_n j_n,\,\tfrac{1}{2}\,J)\,W(l_p j_p\,1\,\tfrac{1}{2},\,\tfrac{1}{2}\,l_p')$$

$$\times\int\phi(n_n l_n)\,O(r)\,(D_{l_p,l_p'}\phi(n_p l_p))\,r^2 dr\,, \tag{27}$$

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$$[1\,L\,J\,p,\,n]_{j_n j_p} = \sum_{l_p'=l_p\pm1} (-)^{j_p+l_p+1/2}[3(2J+1)(2l_p'+1)/4\pi(2l_n+1)]$$
$$\times[(3/2)(2L+1)(2j_n+1)]^{1/2}(l_p'L\,0\,0|l_n\,0)$$
$$\times W(l_p j_p l_n j_n, \tfrac{1}{2}J)\,W(l_p\,1\,l_n L,\,l_p'J)$$
$$\times\int\phi(n_n l_n)\,O(r)\,(D_{l_p,l_p'}\phi(n_p l_p))\,r^2dr\,, \tag{28}$$

with differential operators

$$D_{l,l+1} = [(l+1)/(2l+3)]^{1/2}(d/dr - l/r),$$
$$D_{l,l-1} = -[l/(2l-1)]^{1/2}[d/dr + (l+1)/r]. \tag{29}$$

In the above formulas, the operators $O(r)$ are the radial parts of the operators $\boldsymbol{O}(\boldsymbol{r})$ in Table II, which are given in the curly brackets. Notice that the fourth symbol n in $[S\,L\,J,\,n]$ is just a serial number. In the reaction, the proton in the n_p, j_p, l_p orbit transforms to the neutron in the n_n, j_n, l_n orbit. The radial part ϕ of the nucleon wave function is approximated by the harmonic oscillator functions. In our particular problem, we assume the purely p-shell wave functions so that $n_n = n_p = l_n = l_p = 1$ and

$$\phi = Nr\exp(-r^2/2b^2) \qquad \text{with } N^2 = (8/3)\pi^{-1/2}b^{-5}. \tag{30}$$

Muon wave functions will be discussed in the next section.

§ 5. Exact muon wave functions

In the initial state for the muon capture reaction, the muon is bound in the K-orbit which has the quantum number $\kappa = -1$, corresponding to the $1s_{1/2}$ state. The radial wave function is affected by the Coulomb field of the extended nuclear source of C^{12}. We find the small and large components of the radial wave function, f and g, as the solutions of the Dirac equation with a Coulomb potential of the finite size nucleus, for which the shape of the nuclear charge is assumed to be the harmonic oscillator type,

$$\rho(r) = (1/3\pi^{3/2}b^3)\,[1 + (4/3)(r/b)^2]\exp[-(r/b)^2], \tag{31}$$

with

$$\langle r^2\rangle^{1/2} = 2.40\text{ f} \quad (\text{or equivalently}^{*)}\ b = 1.63\text{ f}).$$

This type of radial dependence has been adopted by Crannell in conformity with electron scattering data.[19] The calculated values of f and g by the Runge-Kutta method are shown in Fig. 1 as functions of qr where q is the neutrino energy given in Eq. (6). We have also computed numerical values of the derivatives of f and g which are, however, not shown here. Using the nuclear charge distribution in Eq. (31), two of the present authors[26] computed the energy of the

*) We can easily prove a relation, $\langle r^2\rangle^{1/2} = (13/6)^{1/2}b$.

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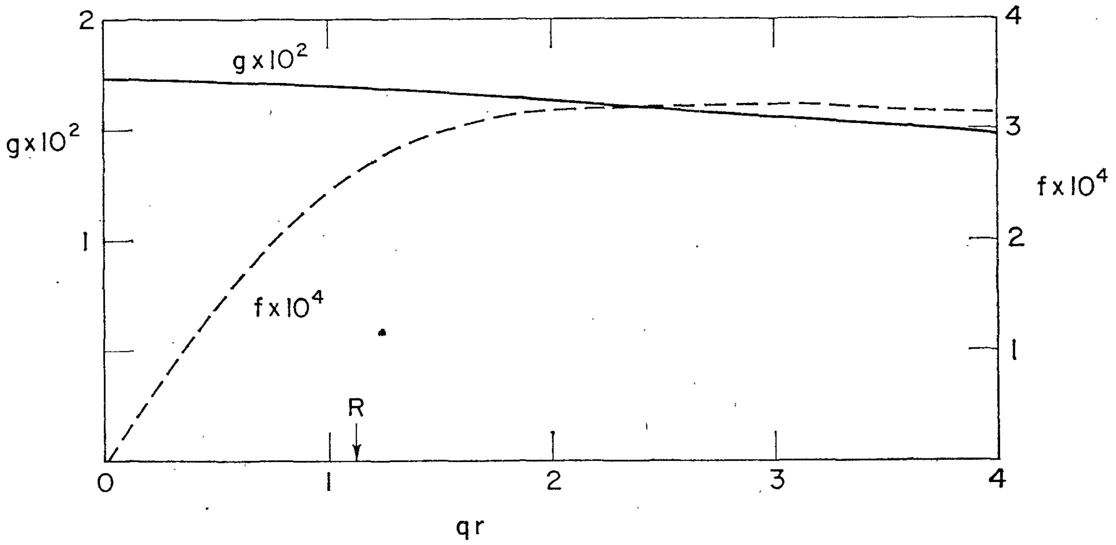

Fig. 1. Radial wave functions of the muon in the K-orbit. The large and small components, g and f, are obtained as the solutions of the Dirac equation with a Coulomb potential of the finite size nucleus for which the shape of the nuclear charge is assumed as Eq. (31). $g\times10^2$ and $f\times10^4$ are plotted against qr, where the neutrino energy is $q=178.59$, in the left-hand and right-hand ordinates, respectively. The arrow R corresponds to nuclear surface.

muonic X-ray ($2p_{3/2}\rightarrow1s_{1/2}$) to be 75.28 keV, which is consistent with the experimental value,[27] 75.25 ± 0.15 keV.

It is interesting to note that the elastic electron scattering and also the inelastic scattering due to excitation of the 4.43 MeV and 9.64 MeV nuclear levels in C^{12} have been studied by Crannell experimentally,[19] and all data are in agreement with the root mean square radius,

$$\langle r^2\rangle^{1/2}=2.40\pm0.02\,\mathrm{f}\,. \tag{32}$$

That is, the data are in favor of the oscillator strength parameter $b=1.63$ f. On the other hand, in the analysis[28] of the inelastic electron scattering data due to the excitation of the 15.1 MeV nuclear level in C^{12} (which is the analog of the ground state in B^{12}), the best fit is obtained with $b=1.90$ f.

We did not calculate the variation of f and g due to the change of b. Instead, we adopt the numerical values of f and g in Fig. 1 throughout in § 6. We have, however, calculated the muon wave functions, f and g, under the assumption of a Fermi distribution of the nuclear charge, which reproduces $\langle r^2\rangle^{1/2}=2.40$ f. No significant change of the muon wave functions have obtained.

§ 6. Results

In the preceding sections, we have discussed the formula for the muon capture rate, the weak coupling constants, and the nuclear and muon wave functions. Now in order to find the muon capture rate, we compute the nuclear

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matrix elements in Eq. (25) with our nuclear and muon wave functions, and we then evaluate Eq. (6) with those nuclear matrix elements. In this section, we summarize the results of our theoretical analysis of the muon capture rate. These results are given as a function of the parameters involved in the nuclear wave functions, Eqs. (22) and (23), and of the weak coupling constants.

In Tables III and IV, the coefficients, C_i and B_i, defined in Eqs. (22) and

Table III. Coefficients C_i for the nuclear wave functions with $T=J=0$ in C^{12}. The lowest state ψ_1 is expressed by Eq. (22) with the set of C_i in the first column. The first excited state is ψ_2, and so on. The last column refers to the ground state wave function proposed by us, which corresponds to Eq. (37) with $\alpha=0.03$.

	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	$\psi(C^{12})$
C_1	0.3190	−0.5217	0.3088	−0.7190	0.1173	0.3031
C_2	0.2548	−0.3948	−0.1898	0.1800	−0.8433	0.2429
C_3	0.2610	−0.1235	−0.8963	−0.1288	0.3108	0.2572
C_4	0.6246	−0.2204	0.2516	0.6045	0.3642	0.6177
C_5	0.6124	0.7132	0.0435	−0.2619	−0.2141	0.6335

Table IV. Coefficients B_i for the nuclear wave functions with $T=J=1$ in B^{12}. The lowest state ψ_1 is expressed by Eq. (23) with the set of B_i in the first column. The first excited state is ψ_2, and so on. The last column refers to the ground state wave function proposed by Kurath,[30] which reproduces the experimental value of the magnetic moment of B^{12}.

	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7	ψ_8	$\psi(B^{12})$
B_1	0.0272	−0.0548	−0.2148	−0.3097	−0.1124	0.1740	−0.0640	0.8985	0.0393
B_2	0.3572	−0.0437	0.4112	−0.5734	−0.5358	0.0429	−0.2066	−0.2028	0.3284
B_3	0.1435	−0.3513	−0.0805	−0.2666	0.0679	0.1660	0.8581	−0.0994	0.1375
B_4	−0.2314	0.0530	0.4194	−0.0513	−0.0472	−0.7891	0.2702	0.2590	−0.2563
B_5	−0.2552	−0.3138	−0.4683	−0.5482	0.2513	−0.3470	−0.2798	−0.2336	−0.2339
B_6	0.1894	0.2755	0.4013	−0.3219	0.7706	0.1485	−0.0642	0.0591	0.1715
B_7	−0.0232	−0.8289	0.3914	0.2403	0.1530	0.0886	−0.2404	0.1113	−0.0740
B_8	−0.8342	0.0860	0.2624	−0.1993	−0.1145	0.4090	0.0599	−0.0647	−0.8463

(23), are given. These coefficients were chosen so as to fit the level scheme for the $T=J=0$ and $T=J=1$ states in $A=12$ system and also to fit the available data on the gamma transitions. In the case of C^{12}, the set of C_i given in the first column determines the wave function of the lowest energy state with $T=J=0$. This wave function, which is then a best approximation to the ground state in C^{12}, is denoted by $\psi_1(C^{12})$. The set of C_i in the second column determines the wave function of the first excited state with $T=J=0$, denoted by $\psi_2(C^{12})$, and so on. Similar expressions hold for the $T=J=1$ wave functions in B^{12}, if we use the coefficients B_i in Table IV and Eq. (23).

In the early stage of our investigation, we assumed the ground state wave functions of C^{12} and B^{12} to be

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$$\psi(C^{12}) = \psi_1(C^{12}) \tag{33a}$$

and

$$\psi(B^{12}) = \psi_1(B^{12}). \tag{33b}$$

This is the original scheme for the ground states in the work by Cohen and Kurath.[11] In this case, the calculated muon capture rate in C^{12} and the ft value of the beta decay of B^{12} are as follows:

$$\mathcal{W} = 6.01 \times 10^3\ \text{sec}^{-1},$$
$$ft = 1.32 \times 10^4\ \text{sec}, \tag{34}$$

where we have assumed $C_P/C_A = 8$, $C_A{}^\beta/C_V{}^\beta = -1.24$, $b = 1.64$ f, and $\mu = 4.706$. The experimental values are[2),29)]

$$\mathcal{W} = \left(6.75 \begin{matrix} +0.30 \\ -0.75 \end{matrix}\right) \times 10^3\ \text{sec}^{-1},$$
$$ft = (1.18 \pm 0.007) \times 10^4\ \text{sec}. \tag{35}$$

As is seen in the above results, the theory gives about 88% of both the muon capture and beta decay rates, under a reasonable assumption for the weak coupling constants.

Recently, Kurath noted[30] that his wave function $\psi_1(B^{12})$ gives 0.76 nm for the magnetic moment of the ground state in B^{12} in disagreement with the experimental value[12] 1.003 ± 0.001 nm. He emphasized, however, that the coefficients B_i in Table IV should not be accepted too seriously, because a slight change in numerical values of B_i can remedy the disagreement between theory and experiment on the magnetic moment without changing the level scheme appreciably. In fact, such an example proposed by him is[13]

$$\psi(B^{12}) = 0.9974\ \psi_1(B^{12}) + 0.0308\ \psi_2(B^{12}) - 0.0646\ \psi_3(B^{12}), \tag{36}$$

for which B_i's are tabulated in the last column for B^{12} in Table IV. We also computed the muon capture rate and ft value with this B^{12} wave function and the C^{12} wave function in Eq. (33a), and we have found practically no change of the calculated values from Eq. (34). We might expect a better fit to the muon capture rate by adjusting the pseudoscalar coupling,

$$\mathcal{W} = 6.60 \times 10^3\ \text{sec}^{-1} \qquad \text{for } C_P/C_A = 2$$

or

$$\mathcal{W} = 6.78 \times 10^3\ \text{sec}^{-1} \qquad \text{for } C_P/C_A = 34,$$

with those wave functions. The ft value is, however, independent of C_P, and it remains at the value shown in Eq. (34).

In order to explain the ft value and the muon capture rate simultaneously, we finally modify the C^{12} wave function as follows:

$$\psi(C^{12}) = \sqrt{1-\alpha^2}\ \psi_1(C^{12}) + \alpha\psi_2(C^{12}). \tag{37}$$

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The magnitude of α should be reasonably small, and we set tentatively the limit, $\alpha^2 \lesssim 0.01$, so that any change of the level scheme should be negligible. We now give the muon capture rates with the C^{12} and B^{12} wave functions*) in Eqs. (37) and (36). The results are shown in Figs. 2~12, where we have assumed five

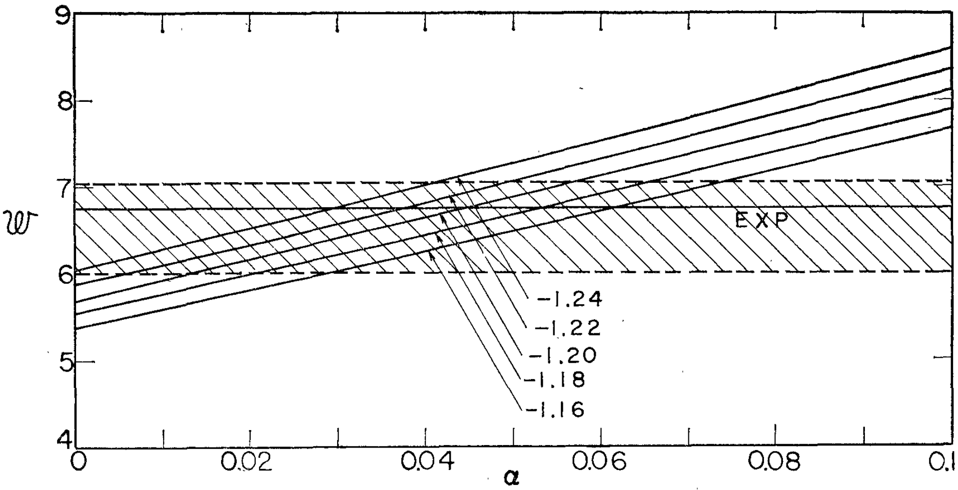

Fig. 2. Experimental and theoretical values of the muon capture rate for $C^{12}+\mu^- \to B^{12}+\nu$ plotted against the parameter α in the C^{12} wave function, Eq. (37), assuming that $C_V=0.972\,C_V{}^\beta$, $C_A=0.999C_A{}^\beta$, and $C_P/C_A=8$, and that the CVC theory is valid. The oscillator strength parameter b is chosen to be 1.64 f. Five different values for $C_A{}^\beta/C_V{}^\beta$ are adopted and indicated by the arrows. $\mathcal{W}$ is in units of $10^3\,\mathrm{sec}^{-1}$.

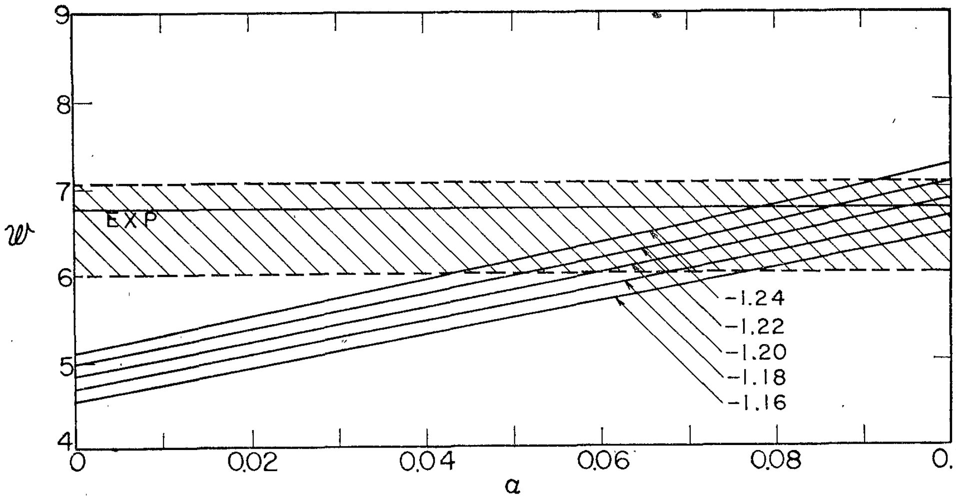

Fig. 3. Experimental and theoretical values of the muon capture rate for $C^{12}+\mu^- \to B^{12}+\nu$ plotted against the parameter α in the C^{12} wave function. Eq. (37). The oscillator strength parameter b is chosen to be 1.90 f. The other assumptions are the same as those in Fig. 2. $\mathcal{W}$ is in units of $10^3\,\mathrm{sec}^{-1}$.

---

*) For these wave functions, the coefficients, Eq. (25), take the following values:

$$A(3/2, 3/2) = -0.0604\sqrt{1-\alpha^2} - 0.0066\alpha\,,$$
$$A(3/2, 1/2) = 0.2200\sqrt{1-\alpha^2} - 0.1870\alpha\,,$$
$$A(1/2, 1/2) = -0.0047\sqrt{1-\alpha^2} + 0.0124\alpha\,,$$
$$A(1/2, 3/2) = 0.6589\sqrt{1-\alpha^2} + 0.4985\alpha\,.$$

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different values for $C_A{}^\beta/C_V{}^\beta$, -1.16, -1.18, -1.20, -1.22, -1.24 indicated by the arrows. We have also assumed $C_P/C_A=8$ in all Figures but 7 and 8, and the validity of the CVC theory unless it is mentioned particularly.

Now, the muon capture rate is plotted against the configuration mixing parameter α in Eq. (37), in Figs. 2 and 3 for the oscillator strength parameter $b=1.64$ f and 1.90 f, respectively. From these two Figures, we see that the parameter α is dependent on the choice of b, and the capture rate is almost linear in α for a fixed value of b. For a good fit, we have

$$0.02 \lesssim \alpha \lesssim 0.05 \qquad \text{for } b=1.64\,\text{f}\,,$$

or

$$0.06 \lesssim \alpha \lesssim 0.09 \qquad \text{for } b=1.90\,\text{f}\,. \tag{38}$$

Next, the muon capture rate is plotted against the oscillator strength parameter b in the cases $\alpha=0$, 0.04, and 0.08 in Figs. 4, 5, 6, respectively. We see from Fig. 4 that the lowest C^{12} wave function $\psi_1(C^{12})$ is not appropriate for the ground state wave function of C^{12} unless b is unreasonably small. The muon capture rate is almost linear in b for a fixed value of α. For a good fit, we have

$$1.60\,\text{f} \lesssim b \lesssim 1.75\,\text{f} \qquad \text{for } \alpha=0.04\,,$$

or

$$1.80\,\text{f} \lesssim b \lesssim 1.95\,\text{f} \qquad \text{for } \alpha=0.08\,. \tag{39}$$

The following two typical choices of the parameters, b and α, are examined by computing the ft value of the B^{12} beta decay:

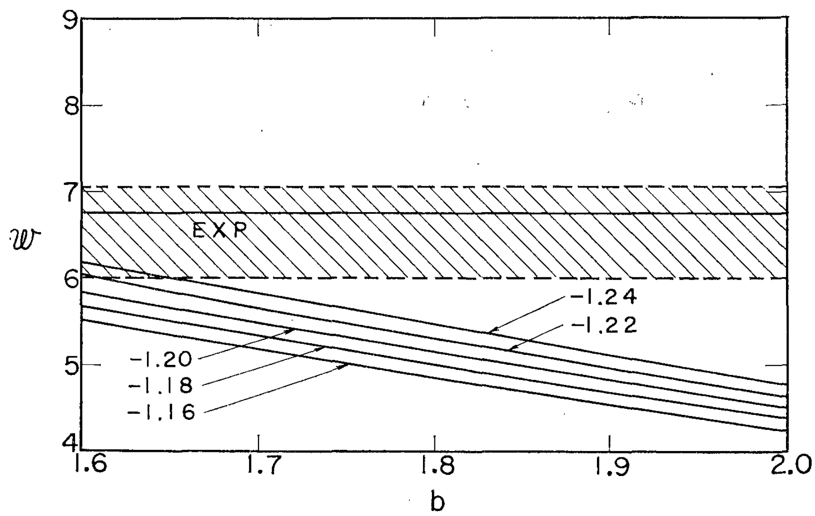

Fig. 4. Experimental and theoretical values of the muon capture rate for $C^{12}+\mu^-\rightarrow B^{12}+\nu$ plotted against the oscillator strength parameter b, assuming that $C_V=0.972C_V{}^\beta$, $C_A=0.999\,C_A{}^\beta$, and $C_P/C_A=8$, and that the CVC theory is valid. The parameter α in the C^{12} wave function, Eq. (37), is chosen to be zero. Five different values for $C_A{}^\beta/C_V{}^\beta$ are adopted and indicated by the arrows. $\mathcal{W}$ and b are in units of $10^3\,\text{sec}^{-1}$ and f, respectively.

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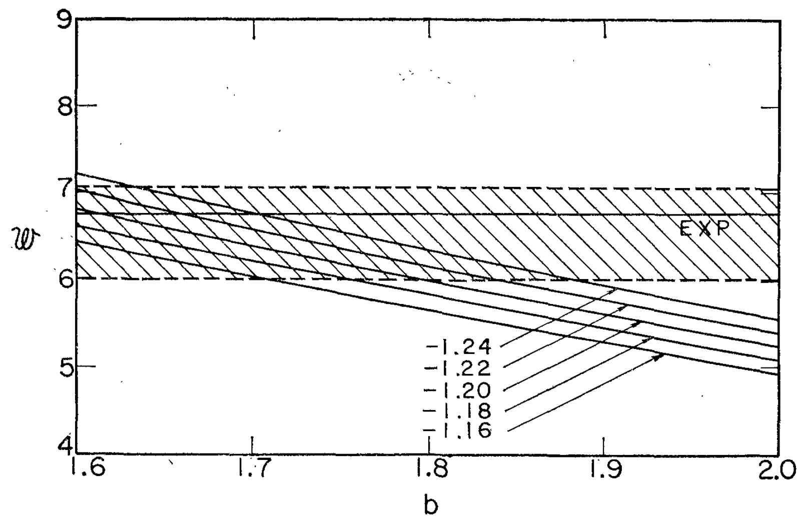

Fig. 5. Experimental and theoretical values of the muon capture rate for $C^{12}+\mu^-\rightarrow B^{12}+\nu$ plotted against the oscillator strength parameter b, assuming that $\alpha=0.04$. The other assumptions are the same as those in Fig. 4. $\mathcal{W}$ and b are in units of $10^3\,\mathrm{sec}^{-1}$ and f, respectively.

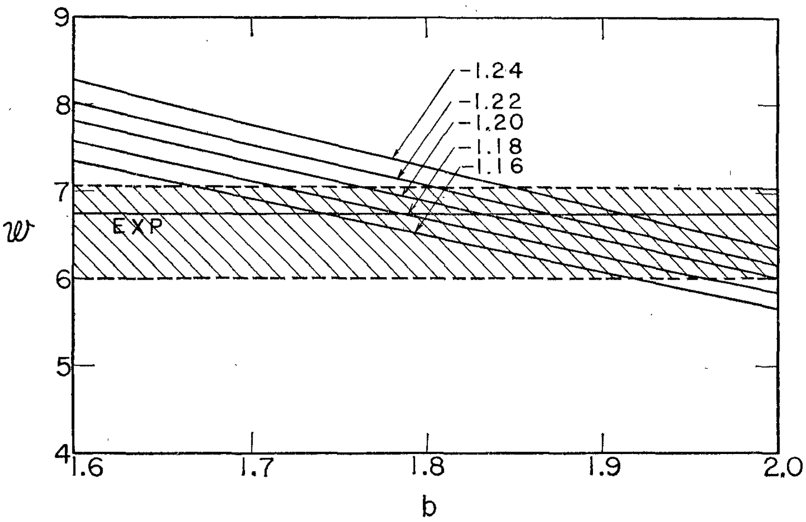

Fig. 6. Experimental and theoretical values of the muon capture rate for $C^{12}+\mu^-\rightarrow B^{12}+\nu$ plotted against the oscillator strength parameter b, assuming that $\alpha=0.08$. The other assumptions are the same as those in Fig. 4. $\mathcal{W}$ and b are in units of $10^3\,\mathrm{sec}^{-1}$ and f, respectively.

Set I: $b=1.64\,\mathrm{f}$, $\alpha=0.03$, $C_P/C_A=8$, $C_A^\beta/C_V^\beta=-1.24$.

$$\mathcal{W}=6.67\times10^3\,\mathrm{sec}^{-1},$$

$$ft=1.18\times10^4\,\mathrm{sec}. \tag{40}$$

Here we put $\alpha=0.03$ instead of 0.04, since it gives a better fit.

Set II: $b=1.90\,\mathrm{f}$, $\alpha=0.08$, $C_P/C_A=8$, $C_A^\beta/C_V^\beta=-1.24$.

$$\mathcal{W}=6.71\times10^3\,\mathrm{sec}^{-1},$$

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$$ft = 0.99 \times 10^4 \text{ sec.} \tag{41}$$

Generally speaking, a consistent explanation of the beta decay and muon capture can be obtained with smaller values of b and α. This is in favor of the set I. As is shown in set II, a larger value of b with an appropriate value of α which fit to the data of the muon capture rate does not give a reasonable ft value.

The second part of our analysis is to investigate the effect of the various coupling constants on the muon capture rate. Since the reaction in C^{12} is a Gamow-Teller transition, the capture rate is roughly proportional to the square of the axial vector coupling constant C_A, and consequently, it is sensitive to the assumed value of C_A^β/C_V^β. This is seen in all of Figs. 2~12. The muon capture rate is plotted against C_P/C_A for the set, $b=1.64$ f and $\alpha=0.04$, and the set, $b \doteq 1.90$ f and $\alpha=0.08$, in Figs. 7 and 8, respectively. The dependence on

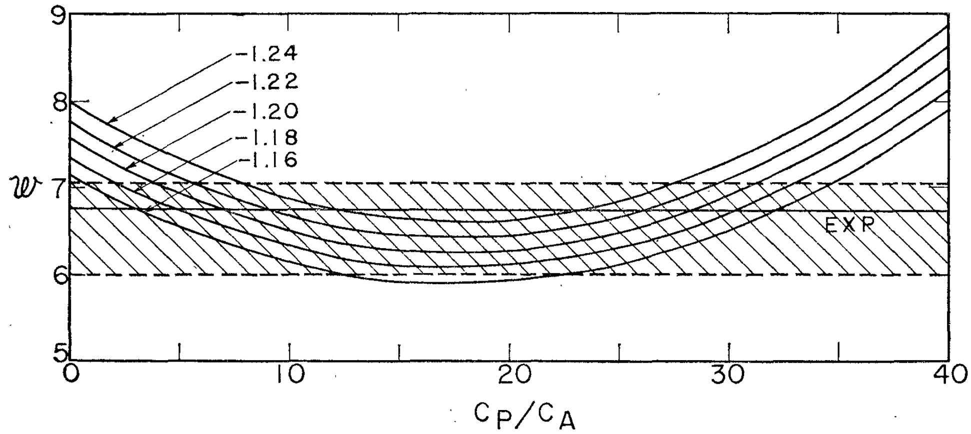

Fig. 7. Experimental and theoretical values of the muon capture rate for $C^{12}+\mu^- \to B^{12}+\nu$ plotted against C_P/C_A, assuming that $b=1.64$ f, $\alpha=0.04$, $C_V=0.972C_V^\beta$, and $C_A=0.999C_A^\beta$, and that the CVC theory is valid. Five different values for C_A^β/C_V^β are adopted and indicated by the arrows. $\mathcal{W}$ is in units of 10^3 sec^{-1}.

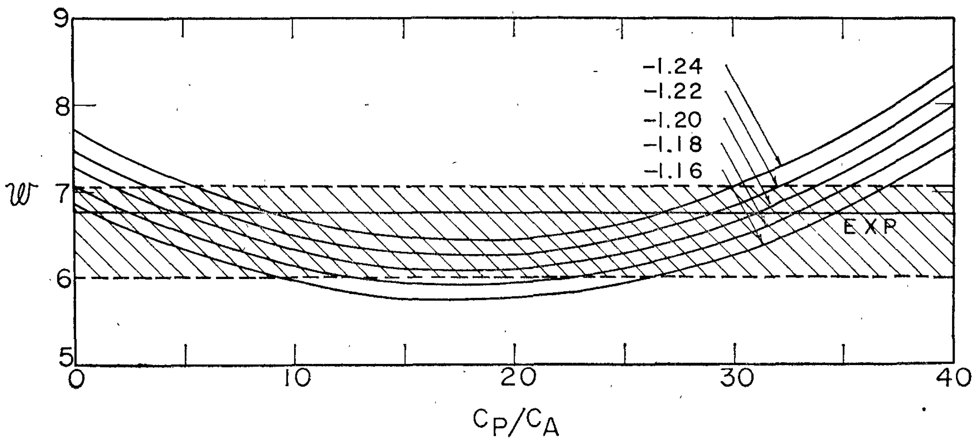

Fig. 8. Experimental and theoretical values of the muon capture rate for $C^{12}+\mu^- \to B^{12}+\nu$ plotted against C_P/C_A, assuming that $b=1.90$ f and $\alpha=0.08$. The other assumptions are the same as those in Fig. 8. $\mathcal{W}$ is in units of 10^3 sec^{-1}.

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C_P/C_A is relatively weak and we have

$$5 \lesssim C_P/C_A \lesssim 30 \,. \tag{42}$$

The theoretical value expected in the dispersion theory[25] is within this restriction.

Until now we have assumed

$$C_A/C_A{}^\beta = 0.999 \,. \tag{10b}$$

We have studied the validity of this assumption in Figs. 9 and 10 for the set,

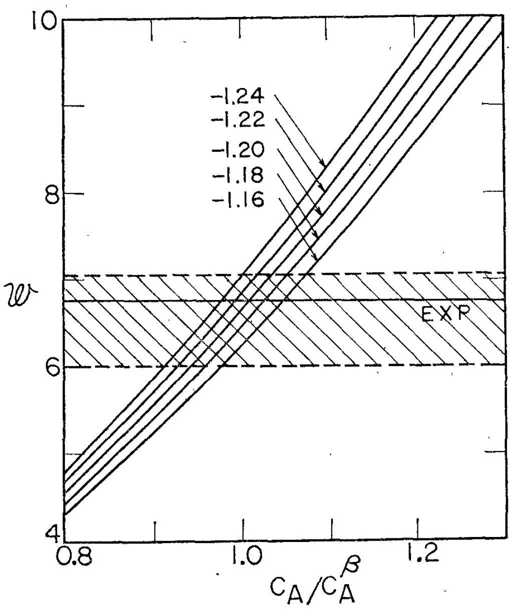

Fig. 9. Experimental and theoretical values of the muon capture rate for $C^{12}+\mu^- \to B^{12}+\nu$ plotted against $C_A/C_A{}^\beta$, assuming that $b=1.64$ f, $\alpha=0.04$, $C_V=0.972C_V{}^\beta$, and $C_P/C_A=8$, and that the CVC theory is valid. Five different values for $C_A{}^\beta/C_V{}^\beta$ are adopted and indicated by the arrows. $\mathcal{W}$ is in units of $10^3\,\text{sec}^{-1}$.

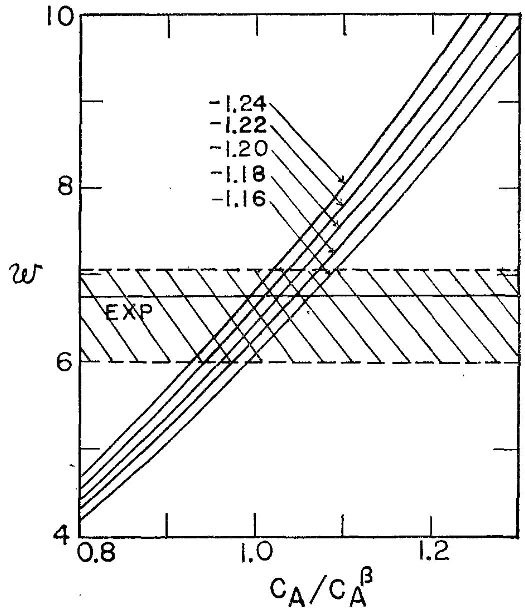

Fig. 10. Experimental and theoretical values of the muon capture rate for $C^{12}+\mu^- \to B^{12}+\nu$ plotted against $C_A/C_A{}^\beta$, assuming that $b=1.90$ f and $\alpha=0.08$. The other assumptions are the same as those of Fig. 9. $\mathcal{W}$ is in units of $10^3\,\text{sec}^{-1}$.

$b=1.64$ f and $\alpha=0.04$, and the set, $b=1.90$ f and $\alpha=0.08$, respectively. To do this, muon capture rate is plotted against $C_A/C_A{}^\beta$. We have found the assumption (10b) is consistent with the experiment. A similar investigation has also performed for checking the validity of the CVC theory, for which the magnetic moment parameter μ is given by

$$\mu = 4.706 \,. \tag{43}$$

The muon capture rate is plotted against μ for the set, $b=1.64$ f and $\alpha=0.04$, and the set, $b=1.90$ f and $\alpha=0.08$, in Figs. 11, and 12, respectively. From these Figures, we can see that the validity of the conserved vector current theory is well confirmed.

In conclusion, the general p-shell wave functions of B^{12} in Eq. (36) and of

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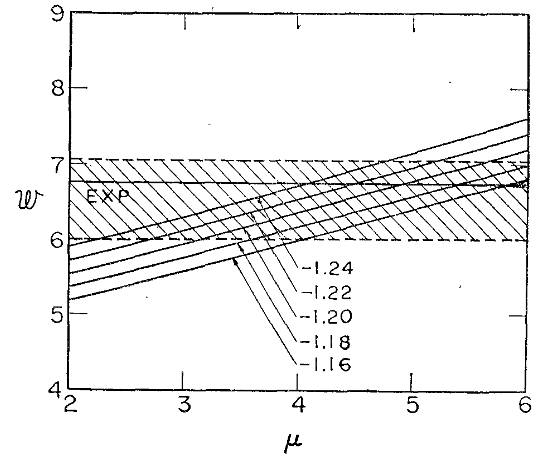

Fig. 11. Experimental and theoretical values of the muon capture rate for $C^{12}+\mu^- \to B^{12}+\nu$ plotted against μ, assuming that $b=1.64$ f, $\alpha=0.04$, $C_V=0.972 C_V{}^\beta$, $C_A=0.999 C_A{}^\beta$, and $C_P/C_A=8$. Five different values for $C_A{}^\beta/C_V{}^\beta$ are adopted and indicated by the arrows. $\mathcal{W}$ is in units of $10^3\,\text{sec}^{-1}$.

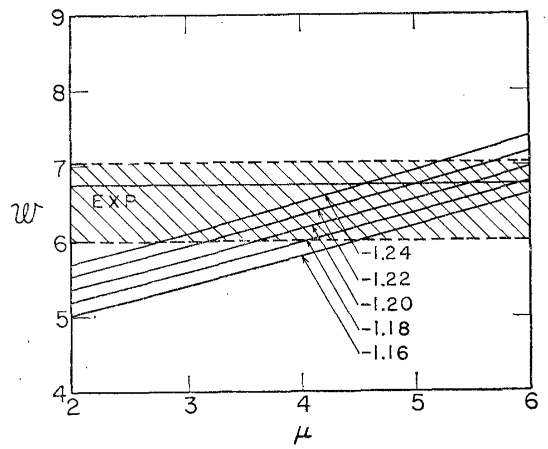

Fig. 12. Experimental and theoretical values of the muon capture rate for $C^{12}+\mu^- \to B^{12}+\nu$ plotted against μ, assuming that $b=1.90$ f and $\alpha=0.08$. The other assumptions are the same as those of Fig. 11. $\mathcal{W}$ is in units of $10^3\,\text{sec}^{-1}$.

C^{12} in Eq. (37) can explain both the muon capture rate in C^{12} and the ft value of the beta decay in B^{12} excellently, if we choose the configuration mixing parameter $\alpha=0.03$ and the oscillator strength parameter $b=1.64$ f. (The coefficients, C_i and B_i for these wave functions are given in the last columns of Tables III and IV, respectively.) All other available data concerning the $A=12$ system are also consistently explained by these nuclear wave functions. Considering the variation of the muon capture rate with α in the C^{12} wave function, we may expect the theoretical uncertainty involved in the capture rate to be about 10%. The other choice of the parameters, $\alpha=0.08$ and $b=1.90$ f which is favored by the analysis of the inelastic electron scattering due to the 15.1 MeV, can also reproduce the muon capture rate. In this case the beta decay rate is about 10% over the experimental value. In this connection, there are still some remaining questions. For example, the introduction of two different values of b for the $p_{1/2}$ and $p_{3/2}$ states or the effect due to possible f-state mixing should be studied in the future investigation.

Acknowledgements

The authors would like to thank Dr. D. Kurath for his stimulating communications. The computational work was performed at the Computer Centers of Columbia University, New York University, the University of Tokyo, Osaka University, and the IBM Company in Tokyo. The authors are greatly indebted to the staffs of these Computer Centers for their work, and also to Professor H. Yukawa for supporting an annual research project of the Research Institute for Fundamental Physics, Kyoto University. A part of this work was done when

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one of the authors (M.M.) was at Columbia University, New York. He would like to express his sincere thanks to Professor R. Serber, Professor T. D. Lee, Professor C. S. Wu, and Professor I. I. Rabi, for their hospitality.

A part of this work was done while the one of the authors (M.S.) was at Bartol Research Foundation of the Franklin Institute, Swarthmore, Pennsylvania, where he was supported by the U. S. Atomic Energy Commission under Contract AT (30-1)-3525. His present address is Department of Physics, Western Michigan University, Kalamazoo, Michigan.

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