Music Theory, Mathematics, and Patterns of Innovation in the Saṅgītaratnākara

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Abstract: This paper investigates the relationship between textuality and mathematics in the *Saṅgītaratnākara*, a Sanskrit work on music composed in the thirteenth century by Śārṅgadeva. Within the traditional Sanskrit knowledge system on musicology, the *Saṅgītaratnākara* can be regarded as a seminal work, given the commentaries it has inspired and the innovative features it contains. I shall explore some textual aspects which, in Medieval India, have contributed to establish the authority of this text and whose significance can be traced in later works. Among these are types of verbalization and mathematical procedures whose role, I shall argue, is entirely theoretical. In the *Saṅgītaratnākara*, calculations and diagrams underline an innovative language of musical speculation, as well as the relationship between theory and practice and the shaping influence of other *śāstric* traditions. The set of conventions which are based on a vocabulary and methods shared with other technical literatures, particularly prosody and mathematics, attests the variety of literary practices introduced by Śārṅgadeva. I shall argue that this text builds up a code whose aim and function are not necessarily musicological in character. Although orality clearly retains its special status as the archetype of learning, Śārṅgadeva's contribution manifests the autonomy of literature on s*aṅgīta* as an "art" which constitutes an independent sphere of activity, defining its own rules, and adhering to its own criteria of value.

Keywords: *Saṅgītaratnākara*, *Saṅgītaśāstra*, mathematics, permutations, textuality

1 Introduction

Studies on the mathematics found in Śārṅgadeva's (thirteenth century CE) *Saṅgītaratnākara* (hence SR)[1] have so far concentrated on purely computational

1 This work has been translated into English by Shringy 1978–1989. I follow the SR's verse numbering as found in Shringy's work. See also Subrahmanya Sastri's edition 1943–1953 of the Sanskrit commentaries by Kallinātha and Siṃhabhūpala.

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aspects.[2] This paper investigates the subject from a different perspective: by looking at the way mathematical procedures shape the narrative structure of the text and contribute to establishing its originality. Scholarship has, so far, failed to recognize the role played by mathematics within the broader argumentative movement of the SR. This study aims to understand the reason behind the use of numbers and elaborate calculations found in the SR. The purpose of this discussion is to illustrate that Śārṅgadeva has innovated *saṅgītaśāstra* with ingenious expository techniques, elaborated new forms of knowledge, and proposed a more versatile textuality.[3]

Whether a mathematical theory of music occurs in India in the same way as in the Greek tradition, has been analysed by a number of authors and is hence not further investigated here.[4] The content related to theory of music is mentioned as part of the overall analysis put forward in this paper. My main concern is to elucidate the multifaceted aspects of the species of discourse employed in the SR. My argument is that in Śārṅgadeva's work, mathematics emerges as a literary practice whose purely aesthetic domain has contributed to delineate the unique features of this composition.[5]

Textual authority is concerned with cultural legitimation. Certain texts in history happened to become more authoritative than others, partly due to historical and socio-cultural circumstances which affect their reception. In order to understand the place occupied by the SR within the Sanskrit scholarship on *saṅgīta*,[6] it is crucial to comprehend that Śārṅgadeva introduces unprecedented material and, moreover, the way he models it has provided his work with the status of a new textual paradigm. Beyond the treatment of trends strictly related to traditional and new repertories, two main original discourses

2 A study on permutations and music is Patte 2012 (in French), which, however, explores calculations found in the SR only in relation to rhythmic varieties and mainly from a mathematical point of view. Another paper investigating some aspects of music and mathematics in the SR is by Sridharan et al. 2010. Jairazbhoy 1961 is a study on the enumeration of tonal patterns in the SR with particular reference to its significance in terms of music theory.

3 *Saṅgītaśāstra* denotes the tradition of expert knowledge on *saṅgīta* or "music".

4 See, for instance, Nijenhuis 1992: 7–19 and Rowell 1992 (particularly in the treatment of pitch). Benedetti and Tonietti 2009 is a study (in Italian) which explores the theory of music of ancient India and compares its connection to mathematics with that occurring in other cultures. This paper unfortunately fails, in my opinion, to recognise the meaning of highly elaborate mathematical procedures in Sanskrit Medieval texts. In attempting to discover the mathematical basis of Indian musical theory, these authors discuss neither the *Saṅgītaratnākara* nor the mathematics found therein.

5 By "aesthetic", I mean the sense of "beautiful, elegant, artistic" characterising the form and style of a literary work.

6 The term *saṅgīta* comprises *gīta*, *vādya*, and *nṛtta* or singing, instrumental music, and dance.

are introduced by this author: i) the genesis of human embodiment and its relation to religious-philosophical conception of sound production, and ii) mathematical procedures in relation to tonal patterns (*tāna*) and rhythmic varieties (*tāla*). Both reflect Śārṅgadeva's broad education; he was clearly well-versed in different disciplines. Śārṅgadeva, in fact, belonged to a family acquainted with Sanskrit learning. His family hailed from Kashmir but settled in Deccan, under the patronage of the Yādava dynasty at Devagiri.[7] It seems that his grandfather was the royal physician and his father Soḍhala was the royal accountant in King Bhillama's court, and after his subsequent death, at King Siṅghana's court. It is interesting to note that Śārṅgadeva succeeded his father in the post of royal accountant, which means that he was at ease with numbers. His ability with mathematics is evident in the SR. In this regard, I consider the mathematical calculations found in his work as an ingenious literary practice, since they are not strictly related to the nature of the subject itself but instead to the way of expounding and speculating on music theory. In the SR, mathematics does not have the practical purpose of expanding the territory of musical performance: its power lies in the fact that it opens new possibilities of writing, conceiving, and exploring musicological concepts.

The diagrams, calculations, and the mathematical vocabulary used by the author are a significant testimony to a literary creativity modeled within the boundaries of a newly defined normative form. Here I particularly refer to the domain of textual properties and the set of conventions which establish the literary dimensions of textual works, as well as to the interplay between narrative and stylistic features. However, not all later writers have aspired to or have succeeded in continuing the textual tradition initiated by the SR. For instance, while following a similar topical organization, only a few Sanskrit texts use mathematical procedures as a way of illustrating musicological concepts. In this regard, the *Saṅgītaśiromaṇi* (henceforth SŚ, fourteenth century CE)[8] and the *Saṅgītadarpaṇa* by Dāmodara (henceforth SD, sixteenth century CE)[9] dedicate a

7 I draw on Shringy 1978: xiii here.

8 The *Saṅgītaśiromaṇi* is a late-Medieval Sanskrit work written by a group of experts in musicology at the request of the sultan Malika Śāhi, who governed the districts situated to the West of present day Allahabad during the first half of the 15th century CE. As the text itself explains, the sultan organized a musicological congress in his capital Kaḍa, inviting scholars to write a large textbook on music, to which purpose he had collected a number of older musicological works. Among these texts, which are listed in verses 23–28 of the introductory chapter, is the SR. The *Saṅgītaśiromaṇi* has been translated into English by Nijenhuis 1992, who also provides the Sanskrit Romanised text, which is, however, not critically edited.

9 Dāmodara was the court poet of Tirumalairava of Vijayanagar. See the edition of the SD by Vasudeva Sastri 1989.

long section to mathematical calculations related to varieties of *tāna*s and *tāla*s, also making use of diagrams. These passages are mainly paraphrasing or rearranging topics expounded in Śārṅgadeva's work. The *Saṅgītopaniṣat saroddhāra*[10] by Sudhakalaśa (henceforth SuS, fourteenth century CE), which shows the textual influence of the SR, displays permutations and calculations in a concise but slightly confusing manner.[11] Other well-known works on *saṅgīta* of early and late Medieval India such as the *Saṅgītasamayasāra*[12] (thirteenth century CE), the *Nṛtyaratnakośa*[13] (fifteenth century CE), the *Saṅgītadāmodara*[14] (fourteenth century CE), and the *Saṅgītakalpalatikā*[15] (seventeenth century CE) do not employ mathematical procedures in their treatment of *tāna*s and *tāla*s.

I shall not dwell upon the interpretation of categories of musical thought, which has already been done by a significant number of scholars,[16] nor shall I analyse the calculations found in the SR exclusively from a mathematical point of view and for its own sake, but rather in order to understand the intellectual richness of forms characterising, in this work, the interplay between knowledge and musical discourse. A musicological text is a representation of concepts in music theory: a construction, an artefact having normative standards. My intention is to cast light on certain types of verbalization which distinguish the argumentative movement of Śārṅgadeva's work. An unexpected feature for a Sanskrit text on music is the variety of reasoning and textual forms used. The SR draws upon Indian mythology, yogic literature, literary theory, *āyurveda*, metrics, and mathematics. There, the use of mathematical procedures and techniques represent an artistic necessity, acquired by skilful strategies of formulation, representation, explanation, justification, and example. In my view, the SR exemplifies the dialectic of a tradition which validates attention to textual format.

After having introduced the SR and its treatment of foundational concepts of music theory, I shall investigate the mathematics found in relation to tonal patterns and calculations on rhythmic varieties expounded in the SŚ, a late Medieval musicological work which shows the influence of the SR. Lastly, I shall

10 This text has been edited by Shah 1961 and translated into English by Miner 1998.

11 In this regard, see Miner 1998: xlix.

12 Sanskrit edition by Gaṇapatiśāstrī 1925.

13 Sanskrit edition by Parīkha and Shah 1957.

14 Sanskrit edition by Sastri and Mukhopadhyaya 1960.

15 Sanskrit edition by Panigrahi 1984.

16 See, for instance, Katz 1983, Rowell 1992, the various contributions published by Sharma, and Widdess 1995. For a historical overview of Sanskrit literature on musicology, see Nijenhuis 1977. A recent and comprehensive study on sound and Hinduism is by Wilke and Moebus 2011. A collection of essays on Śārṅgadēva's work is found in Sharma 1998.

discuss issues around textuality and forms of knowledge which attest the originality of Śārṅgadeva's text.

2 Explanatory techniques of permutations of tonal patterns

The originality of the SR manifests itself with a powerful incipit: a long section called *piṇḍotpatti* or the "genesis of human embodiment" begins the work. This includes metaphysical and religious concepts whose sources are classical Āyurvedic texts, works on Haṭhayoga, Tantric and Vedāntic theories, as well as the Purāṇas.[17] In this first section, the SR describes the way in which the cosmic primordial sound, called *nāda*,[18] and voice production come to be in the human body through *prāṇa* or "vital breath" and the system of *cakra*s and *nāḍi*-lines. The Sanskrit term *nāda* denotes the essence of the cosmos and is identified with the basic principle of the universe, *Brahman*.[19] This very same conceptualization is mentioned at the beginning of major works on *saṅgīta*. In this regard, it seems that within Sanskrit musicological literature Mataṅga's *Bṛhaddeśi* (ca. eighth century CE) provides the earliest connection between *Nāda-Brahman* and music.[20] According to Sharma, there is "a clear indication of the Tantric influence on Mataṅga and in extant literature he is the first author to introduce these details".[21]

In the SR, the first topics on music theory are detailed treatments of the classical Indian system of the seven notes (*svara*),[22] the twenty-two smallest audible sounds (*śruti*),[23] the three scales (*grāma*),[24] and their mythological-

17 Kitada 2012 provides a translation and study of the prologue of the SR.

18 SR 3.6 says that the syllable *na* symbolises the vital breath (*prāṇa*), while the syllable *da* represents the fire.

19 Beck 1993 is a study on Hinduism and sacred sound as found in Sanskrit sources.

20 The *Bṛhaddeśī* is also the earliest of the extant treatises to include musical notations and an important source for the tradition of secular song. See the edition and translation by Sharma 1992–1994.

21 Sharma 1971: 59.

22 There are also twelve *vikṛtasvara*s or "altered notes", so that the notes are nineteen in total.

23 Intervals of two, three, or four *śruti*s separate each of the seven basic notes arranged in the octave (Sanskrit *saptaka* or "collection of seven"). These are: *ṣaḍja, ṛṣabha, gāndhāra, madhyama, pañcama, dhaivata*, and *niṣāda*. Rowell 1992: 43 emphasizes that the *śruti*s, despite the literal meaning of the word, are not heard or performed separately, whereas Nijenhuis 1992: 21 defines the system of the *śruti*s as a "linear representation of a complex non-linear sound phenomenon, a simplification resulting from an attempt to visualize musical intervals". The

religious associations as delineated by earlier authorities. Then, the author expounds the *mūrchanā*s, which denote a set of systematic rotations of a given octave, and the *tāna*s or "tonal-patterns".[25] In this respect, the text explains that the *tāna*s are produced by eliminating one or two notes from the heptatonic *mūrchanā*s[26] based on the two scales.[27] In this way, they become *ṣāḍava* or hexatonic (six-notes) and *auḍava* or pentatonic (five-notes) *tāna*s. SR 4. 31 clarifies that in each *grāma* there are forty-nine hexatonic and thirty-five pentatonic *tāna*s, so that the total number of *śuddha* ("pure")[28] *tāna*s is eighty-four. These eighty-four *śuddhatāna*s are represented by a method called *prastāra*: "having drawn eight vertical lines, one should draw the same number horizontally. Then, one should write the notes[29] in the forty-nine squares. By inverting the *mūrchanā*s-series, the order in relation to the pure *tāna*s occurs. One should write one after the other the *mūrchanā*s and the names of their corresponding *tāna*s".[30] The term *prastāra*[31] (lit. "spreading out, extension") denotes the enumeration of the series of the *tāna*s as well as the representation by which all the

names of the *śruti*s are: *tīvrā, kumudvātī, mandā, chandovatī, dayāvatī, rañjanī, ratikā, raudrī, krodhā, vajrikā, prasāriṇī, prīti, mārjanī, kṣitī, raktā, saṃdīpanī, ālāpinī, madantī, rohiṇī, ramyā, ugrā*,and *kṣobhiṇī* (see SR 3.35–38).

24 These are the *ṣaḍjagrāma*, the *madhyamagrāma*, and the *gāndhāragrāma*, which was known in earlier times but is no longer in use. The text clarifies that Brahmā, Viṣṇu, and Śiva are the presiding deities of the three *grāma*s and that these are named after their most important notes (*ṣaḍja*, *madhyama*, and *gāndhāra*). SR 4.2 says that *pañcama* is the *grāma*-indicative note: if it is a four-*śruti*s note, then it is *ṣaḍjagrāma*; if it is a three-*śruti*s note, then it is *madhyamagrāma*.

25 SŚ 5.37 specifies that the term *tāna* derives from the verbal root *tan*-, which is said to mean *vistāra* ("extension").

26 In this paper, I follow Rowell's understanding of the difference between the *grāma*s and the *mūrchanā*s: *grāma* is the basic collection of *svara*s in consecutive order, while *mūrchanā* is a set of systematic rotations of a given octave. See Rowell 1992. Nijenhuis 1992 translates *grāma* as "tone-system" and *mūrchanā* as "scale".

27 Briefly, the *grāma*s are the basic scales; the *mūrchanā*s denote the complete set of rotations of each of the two *grāma*s. Seven *mūrchanās* starting on each note are derived from each *grāma*. It is explained that each of the two scales has seven ascending-descending *mūrchanā*s, each *mūrchanā* has four forms. SR 4. 19–20 says that the total number of *mūrchanā*s is 392, calculated as: 2 × 7 × 4 × 7.

28 The term *śuddha* ("pure") denotes here the fact that this type of the *mūrchanā*s are composed of notes in their standard *śruti*-relation: 4–3–2–4–4–3–2.

29 Sanskrit texts on music widely employ standard abbreviations denoting the notes.

30 See SŚ 5.34–36.

31 Interestingly, *prastāra* is the name of the layer of *kuśa*-grass in which the Vedic sacrificer called *yajamāna* sits during the ceremony of the *Atirātra-agnicayana* ritual. The term *prastāra* is used throughout the text also adjectivally qualifying the noun *saṃkhyā* or "number", so that *prastārikīsaṃkhyā* is the "permutational number". Shringy 1978: 204 translates it as "permutational calculus".

ni	dha	pa	ma	ga	ri	-	*uttaramandrā*	*agniṣṭoma*
dha	pa	ma	ga	ri	-	ni	*rajanī*	*atyagniṣṭoma*
pa	ma	ga	ri	-	ni	dha	*uttarāyatā*	*vājapeya*
ma	ga	ri	-	ni	dha	pa	*śuddhaṣaḍjā*	*ṣoḍaśī*
ga	ri	-	ni	dha	pa	ma	*matsarīkṛt*	*puṇḍarīka*
ri	-	ni	dha	pa	ma	ga	*aśvakrāntā*	*aśvamedha*
-	ni	dha	pa	ma	ga	ri	*abhirudgatā*	*rājasūya*

Figure 1: The *prastāra* of the seven *tāna*s devoid of the note *ṣaḍja* in the *ṣaḍjagrāma*.

possible permutations of their series are enumerated.[32] Figure 1 below represents the *prastāra* from right to left of the seven *tāna*s devoid of the note *ṣaḍja* in the *ṣaḍjagrāma*, with the names of the corresponding seven *mūrchanā*s and the names of the corresponding *tāna*s on the right.[33]

SR 4.60 introduces the treatment of the *kūṭatāna*s or "irregular tonal patterns", as suggested by the word *kūta* ("deceitful") which is used here in contrast with *śuddha* ("pure") of *śuddhatāna*. Briefly, *śuddha* denotes "standard" and *kūṭa* "irregular" tonal patterns. The following verses prescribe the method for ascertaining and then representing all the possible arrangements of the *kūṭatāna* series via a device called *khaṇḍameru*: the fourteen *mūrchanā*s arising from the two *grāmas* should be written and from these the last note should be eliminated, whenever each time it appears. In this manner, *tāna*s of one note, two, three and so forth up to seven notes are produced. The SR explains how to discover the number of permutations of one note, two notes, and so forth of the seven *tāna*s for each *mūrchanā* and how to calculate their basic series (*mūlakrama*).[34]

The procedure is as follows:

in order to obtain the number of the permutations of the *tāna*s up to seven notes, one should write the numbers from one to seven, representing the seven notes, and when each number is multiplied by the preceding, the number of permutations appears (SR 4. 60–61) (Figure 2):

32 The term *prastāra* is found also throughout the earlier Sanskrit text on music *Bṛhaddeśī*.

33 According to SŚ 5. 32–33, *tāna*s are named after Vedic rituals because they are said to have the effect of the sacrifice after which they are named.

34 SR 4.39 specifies that the name of the *kūtāna*s are *ārcikā*, *gāthikā*, *sāmikā, svarāntara, auḍava*, and *ṣāḍava*. Although Mataṅga mentions the *kūtatāna*s giving the totals for all the numbers up to seven notes, his treatment is less detailed and refers neither to the *uḍḍiṣta* and *naṣṭa* problems nor to the *khaṇḍameru*. This point is also emphasised in Jayrazbhoy 1961: 325.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$;

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$;

$5 \times 4 \times 3 \times 2 \times 1 = 120$;

$4 \times 3 \times 2 \times 1 = 24$;

$3 \times 2 \times 1 = 6$;

$2 \times 1 = 2$;

$1 = 1$;

Figure 2: The permutations of the *tāna*s up to seven notes.

In order to obtain the number of the *mūlakrama*s, one should consider that: i) there are 56 *mūrchanā*s in the two *grāma*s,[35] and that: ii) of the 14 *tāna*s of each *mūrchanā*:[36]

a) the two *tāna*s beginning with the note *sa* as well as the two beginning with *ma* have two forms, the other ten *tāna*s are of four kinds, thus: $(4 \times 2) + (10 \times 4) = 48$;
b) six *tāna*s are fourfold, the other eight have two forms, thus: $(6 \times 4) + (8 \times 2) = 40$;
c) two *tāna*s have four forms, the other twelve have two forms, thus: $(2 \times 4) + (12 \times 2) = 32$;
d) among fourteen *tāna*s, two varieties are pure, and the other twelve are of two kinds, thus: $(2 \times 1) + (12 \times 2) = 26$;
e) there are eight *tāna*s having two forms and six having no alternative forms, thus: $(8 \times 2) + (6 \times 1) = 22$;
f) the fourteen *tāna*s of one note have no varieties, thus: 14.

Having obtained the number of the permutations of the *kūṭatāna*s, one should first arrange the notes in the *khaṇḍameru*: draw eight vertical and eight horizontal lines so as to get forty-nine squares, from below eliminate progressively six, five, etc. squares, and finally write the notes in the remaining squares. On the right side, one should write the number of the permutations of the *tāna*s of each *mūrchanā*, while on the left side one should write the basic series of the *tāna*s. In this very way, fourteen *khaṇḍameru*s should be drawn, as the *mūrchanā*s springing from the two *grāma*s number fourteen.[37] The

35 Each note gives rise to a *mūrchanā*, which can be ascending-descending, and of four varieties in each *grāma*, so that: $7 \times 4 \times 2 = 56$.

36 See SR 4.40–49 and SŚ 5.61–80.

37 See the fourteen *khaṇḍameru*s given in SŚ 5.58–60.

56 sa ri ga ma pa dha ni 5040
48 sa ri ga ma pa dha 720
40 sa ri ga ma pa 120
32 sa ri ga ma 24
26 sa ri ga 6
22 sa ri 2
14 sa 1

Figure 3: The *khaṇḍameru* of the *mūlakrama*s and *prastāra* of the seven *tāna*s of the *mūrchanā uttaramandrā*.

khaṇḍameru (see below Figure 3) appears similar to what is known in Western mathematical terms as "Pascal's triangle".

The ŚR clarifies that the numbers on the left represent the permutations of the *tāna*s of each *mūrchanā* (each of the remaining thirteen *khaṇḍameru*s has a different note starting the series),[38] the numbers on the right represent the basic series of the *tāna*s of each *mūrchanā*. The *tāna* varieties having seven notes are called *pūrṇa* ("complete"), whereas all the others (i. e., the hexatonic, pentatonic varieties, and so forth) are called *apūrṇa* ("incomplete").

In addition, Śārṅgadeva elucidates the method for arriving at the total number (*saṃkhyā*) of the *kūṭatāna*s of each variety (SR 4.40-49):

1) in each *mūrchanā*, the number of the permutations of the heptatonic variety is 5,040;[39] when this is multiplied by 56, which is the number of their basic series, it gives 282,240. Thus, 282,240 is the number of the complete *kūṭatāna*s of the heptatonic variety;
2) the number of the permutations of the hexatonic variety is 720; when this is multiplied by 48, which is the number of their basic series, 34,560 is produced. Thus, 34,560 is the number of the complete *kūṭatānas* of the hexatonic variety;
3) the number of the permutations of the pentatonic variety is 120: when multiplied by 40, which is the number of their basic series, gives 4,800. Thus, 4,800 is the number of the complete *kūṭatāna*s of the pentatonic variety;

38 Among the fourteen *khaṇḍameru*s, two start with the note *sa*, two with *ni*, two with *dha*, two with *pa*, two with *ma*, two with *ga*, two with *ri* according to basic forms of the fourteen *mūrchanā*s (seven belonging to the *ṣaḍjagrāma*, seven to the *madhyamagrāma*).

39 Dattila's *Dattilam* (fifth to eighth century CE) verse 39 mentions 5,033 complete *kūtatāna*s for each *grāma*. See the English translation by Nijenhuis 1970.

4) the number of the permutations of the four-note varieties is 24; when this is multiplied by 32, which is the number of their basic series, it gives 768. Thus, 768 is the number of the complete *kūṭatāna*s of the four-notes variety;
5) the number of the permutations of the three-note varieties is 6; when this is multiplied by 26, which is the number of their basic series, 156 is produced. Thus, 156 is the number of the complete *kūṭatāna*s of the three-notes variety;
6) the number of the permutations of the two-note varieties is 2; when this is multiplied by 22, which is the number of their basic forms, it gives 44. Thus, 44 is the number of the complete *kūṭatāna*s of the two-notes variety;
7) in the fourteen *mūrchanā*s the sum of the *kūṭatāna* of one note is fourteen. Thus, 14 is the number of the complete *kūṭatāna*s of the one-note variety.

Hence, each time, the number of the permutations is multiplied by the number of the notes in each of the basic series. Therefore, the total sum of the *kūtatāna*s is 322,582. As this number includes also the number of the original series and many repetitions between the note-series of the two *grāma*s,[40] the text explains how to calculate the repetitive *tāna*s, which should be subtracted from the total sum obtained (see SR 4.50–60). Having taken away the 4,652 repetitive *tāna*s, the sum 317,930 is distributed thus:[41]
281,848 complete *kūṭatāna*s
31,632 hexatonic *kūṭatān*as
3,800 pentatonic *kūṭatāna*s
544 four-note *kūṭatāna*s
94 three-note *kūṭatānas*
12 two-note *kūṭatānas*
0 one-note *kūṭatānas*

3 Musicological puzzles: the Uddiṣṭa and Naṣṭa problems

In the SR, prescriptive language and mathematical emphasis are further shown with the treatment of can be defined as “musicological puzzles”. In fact, having elucidated the calculation of the *kūṭatāna*s, the text presents a method in which

40 For instance, the note *pañcama* distinguishes the *tāna*s of the *ṣaḍjagrāma* from the *tāna*s springing from the *madhyamagrāma*. When this note is missing, the *tāna*s of the *madhyamagrāma* are the same as those springing from the *ṣaḍjagrāma*.
41 See SŚ 5.129–130.

the *khaṇḍameru* is used as an indicator for the solution of *uddiṣṭa* and *naṣṭa* problems. In this context, *uddiṣṭa*, lit. "that which is specified", denotes a given (hence *uddiṣṭa*) note-series, and what is to be determined is its serial number in its permutational extension.[42] *Naṣṭa*, lit. "that which is destroyed, lost" refers to a note-series whose tonal form is to be found out (hence *naṣṭa*), knowing the note-series type (one-note, two-notes, etc.) and its serial number. In Śārṅgadeva's text, these two terms indicate two problems involving mathematical calculations and are, for the first time, applied to music theory. Interestingly, the "unidentified" serial number and the "unidentified" tonal form are discovered by two different procedures and by the use of the *khaṇḍameru*.

Let us first consider *uddiṣṭa*-type problems. One should first arrange the diagram in the following manner:

– eight vertical and horizontal nine lines should be drawn, so to obtain eight rows having seven squares each;
– in the first row, one should write the abbreviations of the seven notes;
– in the row below, in the first square on the left one should write the number one and in each next square one should write the number zero;
– in the six squares with zeros one should place pebbles,[43] representing the notes of the tonal series to be identified;
– in the third row consisting of six squares (in each successive row a square is removed), one should write the numbers one, two, six, twenty-four, one hundred and twenty, seven hundred and twenty.[44] In the row below, one should write the previous numbers multiplied by two, except the number one. One should then multiply by three the numbers of the third row with the exception of the number one and two, and write the results in the row below. The same numbers of the third are now multiplied by four, except one, two, and six, and written in the row below. In the row below, one should multiply the last two numbers (one hundred and twenty and seven hundred and twenty) by five. Then, in the last square the number seven hundred and twenty should be multiplied by six (Figure 4):[45]

42 In 5.130–131, the SŚ says that: "when a specified (*uddiṣṭa*) permutation [of tonal series] is known, that representation (*prastāra*) by which the unspecified (*naṣṭa*) [tonal form] becomes progressively evident in the *khandameru* is considered here".

43 I use the letter x to denote the pebbles. It is interesting that the text mentions the pebbles, thus pointing out the use of moveable objects for identifying *naṣṭa* and *uddiṣṭa* in the *khaṇḍameru*.

44 These numbers represent the permutations of a hexatonic *tāna*, whose calculation has been explained before. See Figure 3

45 See also the Sanskrit passages and diagrams of the SD and of the SŚ in Vasudeva Sastri 1989: 21 and Nijenhuis 1992: 153.

sa	ri	ga	ma	pa	dha	ni
1	0x	0x	0x	0x	0x	0x
-	1	2	6	24	120	720
-	-	4	12	48	240	1440
-	-	-	18	72	360	2160
-	-	-	-	96	480	2880
-	-	-	-	-	600	3600
-	-	-	-	-	-	4320

Figure 4: *khaṇḍameru* for *naṣṭa* and *uddiṣṭa* problems.

The text is concise in explaining the procedures for solving *uddiṣṭa* and *naṣṭa* problems (SR 4.66-71) and provides no example.[46] In order to bring to light the practical application of this ingenious method, I am going to illustrate how to solve musicological problems related first to a specified note-series, and then to a specified serial number.[47]

The procedure for working out the serial number of a specified note-series (*uddiṣṭa*) consists of the following steps:

a) determine the position of the last note of the indicated note-series by referring to the last note in relation to the original order of the notes in the *ṣaḍjagrāma* and by counting the notes from right to left. The last note of the original order indicates the column in which the first pebble is placed, while the square in which it is placed is calculated according to the position of that note in the original order and by counting the notes from right to left. Once performed, the last note of the given note-series is dropped;
b) the last note of the given note-series is performed. This is with reference to the original order of the notes: the column in which the pebble is placed is determined by the number of notes remaining in the given note-series, while the square is determined by the place occupied by that note in relation to the original order and by counting the notes from right to left. Once performed, the note is dropped and the procedure continues in this way until the end;
c) by adding the figures found in the squares in which the pebbles have been moved, the unidentified serial number (*naṣṭa*), to which that specified note-series (*uddiṣṭa*) corresponds in its permutational extension, is arrived at.

46 See the Sanskrit passages modeled on the SR by SŚ 5.130–148 and by the SD in Vasudeva Sastri 1989: 20–22.

47 This description is a revised version of Shringy 1978: 214–223, with changes and corrections.

Having elucidated the main steps of the procedure, I am going now to show how to solve an *uddiṣṭa*-type problem. For instance, let the given note-series be the following five note-series: *pa ma ga sa ri*. We want to know its serial number in its permutational extension, keeping in mind that the permutation of a set of five notes is one hundred and twenty (see Figure 3). According to the *ṣaḍjagrāma* scale, the original order of a five-note series is *sa ri ga ma pa*. The last note of the given note-series *pa ma ga sa ri* is *ri*; the movement of the first pebble begins from the fifth column, which is the column of the note *pa*, the last note of the original order. In this column, the pebble is to be moved into the fourth square, which indicates the number seventy-two, as the note *ri* is the fourth in the original order, counting the notes from right to left (see below Figure 5).[48] Being already computed, the note *ri* is dropped. Thus, the given series is now *pa ma ga sa*, whereas the original order would be *sa ga ma pa*. When counting the notes in the original order from right to left, the note *sa* is the fourth note from the note *pa*: the second pebble is hence moved into the fourth square of the fourth column, now representing the note *sa* a note of a four-note series (*sa ga ma pa)*. The second pebble is thus moved into the square of the number eighteen and being computed the note *sa* drops away. The series now appears as *pa ma ga*, while the original order would be *ga ma pa*. When counting the notes in the original order from right to left, the note *ga* is the third: the third pebble is hence moved into the third square of the column of the note *ga*, where the number four stands, *ga* now being a note of a three-note series (*pa ma ga*). Once the note *ga* is dropped, the series comprises the two-note series *pa ma*,

sa	ri	ga	ma	pa	dha	ni
1x	0x	0x	0x	0x	0	0
-	**1**	2	6	24	120	720
-	-	**4**	12	48	240	1440
-	-	-	**18**	**72**	360	2160
-	-	-	-	96	480	2880
-	-	-	-	-	600	3600
-	-	-	-	-	-	4320

Figure 5: The *khaṇḍameru* showing the solution of the given *uddiṣṭa*-type problem.

48 In Figure 5, I use the letter x to denote the pebbles. These are five as there are five notes in the given note-series of the *uddiṣṭa* problem. In the squares, the numbers in bold denote the squares to which the pebbles have been moved during the procedure. In order to obtain the permutational extension of the given note-series, these numbers should be added.

whose original order is *ma pa*. The last note of the remaining two-note series is the second in the original order and from counting the notes right to left, the pebble is moved into the second square of the second column, where the number one is found. The last pebble remains in its position, which is occupied by the number one in the column of the note *sa*. Therefore, the total number represented by the numbers found in the squares into which the pebbles have been moved is ninety-six, which is the result of the sum of 72 + 18 + 4 + 11. This means that in its permutational extension, the specified five-note series *pa ma ga sa ri* corresponds to the serial number ninety-six. The figure below (Figure 5) shows how to use the *khaṇḍameru* when solving the given *uddiṣṭa* problem.

Having analysed the method for solving the *uddiṣṭa* problem, let us now look at the reverse procedure: how to derive the *naṣṭa*, the unspecified note-series knowing its serial number in its permutational extension. The SR (4. 68–71) explains that:

a) in the horizontal row of the *khaṇḍameru*, below the row with the abbreviations of the seven notes, one should place as many pebbles as there are notes in the series (see Figure 6 below);
b) one should start by deducting the number one from the given serial number and, according to the result obtained, subtract from it the highest possible number found in the *khaṇḍameru*; the first pebble should be moved into the square of that number;
c) this process is repeated, in the same way, until the end;
d) the notes of the missing note-series to be found out are determined according to the position of the squares occupied by the pebbles in each column, counting from top to bottom. The numbers representing these positions indicate the backward order (hence from right to left) of each note in respect to the final note in the original order of the notes;

sa	ri	ga	ma	pa	dha	ni
1x	0**x**	0**x**	0**x**	0	0	0
-	**1**	2	6	24	120	720
-	-	**4**	**12**	48	240	1440
-	-	-	18	72	360	2160
-	-	-	-	96	480	2880
-	-	-	-	-	600	3600
-	-	-	-	-	-	4320

Figure 6: The *khaṇḍameru* showing the solution of the given *naṣṭa*-type problem.

e) once computed, each note is dropped and the original order is to be established again.

For instance, let us solve the following *naṣṭa*-type problem: find out the tonal form of a missing four-note series which corresponds in its permutational extension[49] to the serial number eighteen. According to the *ṣaḍjagrāma*, the original order of a four-note series is: *sa ri ga ma*.

The number one is subtracted from the serial number eighteen: 18–1 = 17. We now have to add up to seventeen, which means that we have to subtract from seventeen the highest possible number found in the *khaṇḍameru*. Here, the highest possible number to subtract from seventeen is the number twelve found in the third square of the fourth column, where the second pebble should be placed. The result of this subtraction is five and this should be deducted from the highest possible number found in the *khaṇḍameru*, which is the four located in the third square of the third column, where the third pebble is to be placed. The result of this subtraction produces the number one. From this, the next number to be subtracted is, thus, the number one found in the second square of the second column, where the last pebble should be placed.

Therefore, according to the position from left to right of each square in their respective columns into which the pebbles have been moved, the numbers produced are 1-2-3-3. These numbers are be associated with the original order of a four-note series, which is *sa ri ga ma*. As these numbers correspond to the backward order of the missing note-series, we should start from a right to left direction. Once its position has been identified, each note is dropped:

i) 3 = *ri*, which is to be considered the last note of the missing four-note series;
ii) 3 = *sa*;
iii) 2 = *ga*;
iv) 1 = *ma*.

By reading these notes *ri sa ga ma* from right to left, the missing note-series (*naṣṭa*) appears as: *ma ga sa ri*.

As a final consideration, it is interesting to observe that the *khaṇḍameru* is not only a device for arriving at serial numbers and missing note-series, but it also informs us of the total number of note-series in each variety such as heptatonic, hexatonic, and so forth. In fact, if we add as many numbers from the first squares on the left in each row as these are notes in a given note-series, we obtain the number of its permutation. For instance, if we want to ascertain

49 The permutation of a set of four-notes is twenty-four. See Figure 3.

the number of the permutation of a three note-series, we have to add three squares: $1+1+4=6$, which is, in fact, the number of permutations of a set of three notes and if, for instance, we want to know the number of the permutation of a four note-series, we have to add four squares: $1+1+4+18=24$, which is indeed the number of permutations of a set of four notes.[50]

4 Metrics and the language of musical time

There are several conclusions to be drawn so far. The significant number of calculations, layouts, and methods for the solution of musicological puzzles emphasises that exploring forms of knowledge through sets of systematic permutations is a connotative feature of Medieval Sanskrit texts on music. In a broader sense, this should not be surprising. In fact, since ancient times in India combinatorial mathematics has attracted a strong interest. Procedures for permutations and combinations are also found in early Jaina philosophical speculations,[51] in the Sanskrit medical treatise *Suśruta Saṃhitā* (third to fourth century CE), in the encyclopaedic work *Bhṛhat Saṃhitā* (sixth century CE), in the mathematical texts by Mahāvirācārya (ninth century CE), Bhāskarācārya (twelfth century CE), Nārāyaṇa (fourteenth century CE), and in other Sanskrit non-mathematical works.[52]

In some earlier Sanskrit non-musicological texts the *naṣṭa* and *uddiṣṭa* problems, as well as the *prastāra* and *khaṇḍameru* procedures and diagrams, also occur. The first use of these terms appears in the text on prosody *Chandaḥsūtra* by Piṅgala (ca. third to second BCE), whose formal theory of Sanskrit meters is considered as the first evidence of procedures of combinatorial mathematics.[53] The use of symbolism and mathematical concepts also distinguishes this Sanskrit technical genre. Metrics or prosody, the science of meters, consists of determining and identifying the various combinations of Sanskrit syllables in a quarter-verse, where the number *n* of syllables is given and each syllable may be either heavy or light. Piṅgala's method uses a highly-mathematical approach. His application of mathematics to metrics includes the first known description of a binary numeral system in connection with the systematic enumeration of meters with fixed patterns of short and long syllables.

50 See Figure 3.
51 See Datta 1935.
52 In this regard, see the studies by Datta and Singh [Revised by Shukla] 1992; Kusuba 1993; and Kusuba and Plofker 2013.
53 See the study by Kulkarni 2008 and Shah 2013.

He gives procedures for listing all possible forms of an *n*-syllable meter and for indexing such a list. Piṅgala also provides an algorithm for determining how many forms have a specified number of short syllables. One of the procedures given for finding out the variations of syllables is called *meruprastāra*, in which the total is obtained by adding numbers arranged so as to resemble the side of the mountain Meru of Hindu mythology. Plofker says that the *meruprastāra* is "just what we know as Pascal's triangle."[54] In the *Chandaḥsūtra*, the term *prastāra* for "permutation, extension of varieties" seems to occur for the first time. This term appears also, in relation to investigations on metrics, in the works by Bharata (ca. first BCE to first century CE), Virahāṅka (ca. sixth to eighth century CE), Janāśraya (ca. seventh century CE), in the mathematical work by Mahāvīrācārya, in Jayadeva's work on metrics (ca. fifth to ninth century CE),[55] in the works by Jayakīrti (ca. eleventh century CE), Kedāra (ca. twelfth century CE), and in Hemacandra's (eleventh century CE) texts.[56] These authors deal with questions involving problems such as: what is the extension of all the possible meters with *n* syllables in a quarter-verse? What is the serial number *m* of any given metrical pattern of *n* syllables within that extension?

Some of the literary practices of Indian musical scholarship reflect the shaping influences of the *śāstric* tradition of metrical theory. This feature reveals a systematic thought in which the conceptualization of music is by no means confined to a unique set of principles. The literary strategies by which the subject of literature on *saṅgīta* was expounded are in fact shared with other domains involving the production of specialized knowledge.

I turn now to investigate briefly the way the SR expounds the system of musical time (*tāla*).[57] For this purpose, I shall refer to the SŚ, a late Medieval Sanskrit musicological text which strongly shows the influence of the SR. In this sense, a precious source of information is the detailed treatment of *tāla* found in the twelfth chapter of the SŚ. In the introductory section (verse 25–28), this text mentions the SR as one of earlier authorities. In *saṅgītasāśtra*, musical metre is treated in the same way as the poetical metre. Like the syllables of words in a poetical passage, the notes of a musical line are arranged according to a scheme of very short (*druta*), short (*laghu*), long (*guru*, taken to be a unit of time), and extra-long (*pluta*) time units. SŚ 12.135 informs us that in musical notation a

54 Plofker 2009: 57.

55 Among Jayadeva's innovations is the use of special marks to indicate the prosodic value of syllables. His system became standard in India. A heavy syllable is represented by a curve, and a light syllable by a straight line. See Brown 1869.

56 These authors are examined in the study on Piṅgala's combinatorics by Shah 2013.

57 In Sanskrit musical treatises, *tāla* is a general term for the entire system of rhythm.

druta time unit is denoted by a circle (0), a *laghu* by a vertical line (|), the *guru* by an (S) and the *pluta* by an (S) with an oblique line on the top (Ś). SS 12.139-140 mentions the presiding deities of these time units and in SŚ 12. 140–141 the mentioned above term *prastāra* occurs in relation to *tāla* varieties, and it is said that "permutation (*prastāra*) means extension, [a process] which is increased by [the extension of rhythmic values such as] *laghu* and so forth." In the treatment of *tāla*, *prastāra* denotes the extension of all possible rhythmic forms with their rhythmic values.[58] SŚ 12.140–146 represents, using the notation for the rhythmic values mentioned above, the permutations of *laghu* and *druta* up to six *druta*s. The next verses explain that the sum total of structural arrangements of *tāla* with two or more *druta*s that can be obtained is called *saṃkhyā*. In the translation of the text by Nijenhuis, one finds a table of *saṃkhyā* numbers illustrating the total sum of rhythmic varieties based on one, two, or more *druta* units according to the procedure explained in verses 148–150: "i. e. by adding up every time, from right to left, the first, second, fourth and sixth preceding numbers".[59] The *naṣṭa* and *uddiṣṭa* problems are applied also to rhythmic varieties. In this context, *naṣṭa* involves finding out the rhythmic structure of a *tāla* variety by knowing its serial number, whereas in the *uddiṣṭa* problem is the serial number to be arrived at by knowing the structure of a particular *tāla*. These procedures are explained in detail in SŚ 12. 155–177 where layouts are reproduced.[60]

The SŚ presents calculations and lists *tāla* varieties up to seventy-two *druta*s so as to arrive at a total of 860497588849848458. In this regard, it is noteworthy that large numbers are expressed by the *bhūtasaṃkhyā* system of notation. This denotes a method developed in India in the early centuries of the CE and in which numbers are expressed by means of symbolic words. In the *bhūtasaṃkhyā* notation, the digits from zero to nine are denoted by certain words reflecting a numerical association and are derived from many areas of Indian culture. This system is the creative attainment of a process in which the flexibility of the Sanskrit language is functional to both the poetic structure and to the method of oral transmission of Sanskrit texts. The *bhūtasaṃkhyā* system allows the retention of the rhythm of the *śloka*s, it avoids unrefined ways to name numbers, and it supports a mnemonic function, linking words with numbers. For instance,

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58 In Nijenhuis 1992: 338, a table of *tāla* varieties is found. Interestingly, *prastāra* charts in relation to *tāla* varieties are found in SuS 2.24 and in the SD. See Vasudeva Sastri 1989: 170–178.

59 Nijenhuis 1992: 339.

60 Cf. the SŚ in Nijenhuis 1992: 349–351 and the SD in Vasudeva Sastri 1989: 170–178. These layouts are different from those previously given in the *naṣṭa* and *uddiṣṭa* problems in relation to tonal patterns. See also the explanation of these procedures in Patte 2012.

according to the *bhūtasaṃkhyā* the expression *candrāgniśara* represents the number 531, since *śara* is "arrow" and the arrows of the Indian cupid are declared to be five; *agni* is "fire" and denotes the number threes as there are three kinds of sacrificial fire (*gārhapatya, āhavanīya, dakṣiṇa*), and *candra* is the number one as it means "moon", of which there is one.[61]

In the elucidation of the *tāla* system the vocabulary, the symbolism, and the explanatory techniques occurring in musicological works resemble those found in the literature on prosody. Rowell emphasises that "Classical Sanskrit metrical theory offered a convenient system for the encoding of musical durations, a code that was adopted by Mataṅga and his successors."[62]

5 Forms of textuality and cultural legitimation

This paper has so far brought attention to the variety of language of musical speculation shown in the SR, as well as in the texts modeled on it: problem solving, regularity, and structure are some of the principles which distinguish the narrative structure of the text. The SR develops a form of textuality based on enumeration, categorization, and literary expressions common to other technical literatures. Within *saṅgītaśāstra*, in the SR the procedures called *prastāra* and *khaṇḍameru* are applied to tonal patterns for the first time. This is emphasised also by Jairazbhoy: "[...] It may well be that the order of sequences discussed in this paper along with the uddiṣṭa, naṣṭa, and khaṇḍameru was Śārṅgadeva's own contribution".[63]

In the SR, the argumentative movement seems to have a theoretical function and to illustrate that the number of attainable forms is unlimited. Śārṅgadeva is able to elucidate patterns and variations by using literary practices and mathematical algorithms as powerful techniques for opening up and exploring new possibilities. In this sense, the SR is an artistic product which should be understood as part of a broader textual culture. However, one must bear in mind that Sanskrit musicological sources are not a homogeneous whole: not only are concepts, repertoires, and the development of forms related to *saṅgīta* (intended as singing, dance, and instrumental music) treated differently, but authors also

61 A study on the *bhūtasaṃkhyā* system and its relation to the Sanskrit language and culture is Petrocchi 2016.
62 Rowell 1992: 217.
63 Jairazbhoy 1961: 325.

have their own literary styles, and a unique way of dealing with the interpretation and production of knowledge.

The SR is the first work, as sources have come down to us, to have manipulated mathematical methods and a related terminology to understand and explain the system of *tāna*s. This practice has not been, however, followed by every author in a demonstration that, together with different regional schools of music, there were different textual traditions too. Despite distinctive characteristics, there was of course a consistency in respect to fundamental notions.[64] Yet, a thorough study of literature on *saṅgīta* brings to light the distinctive ways in which authors deal with their subject matter and with textuality. Questions arise on the long Sanskrit intellectual tradition surrounding music scholarship regarding the production, distribution, and consumption of texts. What was the authors' educational background and who were their readers? Who was commissioning literary compositions on music? Unfortunately, in Sanskrit literature, biographical details are very difficult to come across. Among authors on *saṅgītaśāstra*, some were court poets well-versed in *sāhitya* ("literary composition"), *alaṃkāraśāstra* ("the science of poetics and aesthetics), as well as in other Sanskrit literatures. It was common for Sanskrit scholars (such as poets, musicians, astronomers-mathematicians) to work at royal courts, since kings were patrons of arts, literature, and science.[65]

The intellectual history of the traditional body of knowledge on *saṅgīta* is an unexplored subject, given also the paucity of materials available. Nevertheless, the textual innovations made by the SR indicate an internal development. In this regard, a point which I would like to emphasise is the tension between orality and textuality[66] which affects the domain of textual properties and the set of conventions establishing the literary standards of musicological works, In Śārṅgadeva's text, the interplay between narrative, stylistic features, and textual manipulations of traditional and innovative values exemplifies his contribution, a turning point within the knowledge system of *saṅgīta*. In the SR, the content is presented in a coded form not dissimilar to the style characterizing other Sanskrit technical works, where the demands of economy, memorability, and the metric structure contribute to shape the final design of the texts. Literature in *saṅgītaśāstra* is based upon the same requirements of Sanskrit poetics:

64 Nijenhuis 1977 provides a history of musicological literature.

65 Truschke 2016 points out the role played by Sanskrit scholars, included experts on music, in the Mughal court.

66 On the relationship between orality and textuality characterising music and performance in early Modern North India see the volume edited by Orsini and Schofield 2015.

conciseness, rhythm, and suitability for oral transmission.[67] Nevertheless, the expository techniques investigated in the previous paragraphs such as mathematical procedures, the use of diagrams and of a prescriptive and technical language, highlight the ability of this specialised literature to produce a type of knowledge which is narrative and descriptive. In this sense, the SR testifies a rupture: Śārṅgadeva's work displays richness and profusion of literary forms within a *śāstra* which had been, so far, mainly preoccupied with mythological-religious heritage and practical concerns. The SR lays the foundation of *saṅgīta* as a subject capable of producing comprehensive forms of knowledge. Shringy (1978, xxviii) identifies that later writers adopted the same scheme of arranging topics as that given by the SR and that Śārṅgadeva's compact, elegant style can be compared to that of Mammaṭa (eleventh century CE), the author of *Kāvyaprakāśa*, one of the most influential texts on *alaṃkāraśāstra*.

The paradigm of a flourishing scholarship on music illustrates the vibrant, intellectual life that in Medieval India was remodeling the circulation and consumption of works on music. Increased opportunities, a dynamic environment, fruitful networks, and the exchange of cultural and literary forms must have been crucial in creating the conditions capable of supporting Śārṅgadeva's original work. This author integrates literature and mathematics, tradition and innovation, mythology and metrics, archaic conceptualizations of music and new textual trends into an organic whole. In the introductory section of the first chapter, Śārṅgadeva lists earlier authorities on music such as Bharata (the sage said to be the author of the *Nāṭyaśāstra*, approximately first century BCE to first century CE, a seminal work on the history of Indian theatre, music, dance, and aesthetics), Dattila (author of *Dattilam*, fifth to eighth century CE), Mataṅga (ca. eighth century, author of the *Bṛhaddeśī*), Abhinavagupta (tenth to eleventh century CE, author of the *Abhinavabhāratī*), Someśvara (twelfth century CE, author of the *Mānasollāsa*), and Nānyadeva (eleventh to twelfth century CE, author of the *Bharatabhāṣya*).

A study of previous works such as the *Dattilam* and *Bṛhaddeśī* shows that they pertain to a different phase of *saṅgītaśāstra*. Shringy suggests the division of the history of texts on musicology into four main periods: i) primary and formative (up to 500 CE); ii) expository and expansive (600 CE to 1200 CE); iii) reconciliatory and re-evaluative (1300 CE to 1750 CE), and iv) critical and interpretative (1750 CE onwards).[68] According to this scheme, Shringy points out that

67 For instance, Filliozat 2004 emphasizes that the complex Sanskrit scholarly culture based on orality and memorizing techniques, first developed to transmit the knowledge contained in the *Veda*s, required developing mathematical literature in a specific way.

68 Shringy 1978: xxxii.

the SR occupies a special place in the second period. In my view, the SR undoubtedly represents a shift of the cultural legitimation surrounding literature on music. Śārṅgadeva emancipates, in some ways, literary concerns on music from performative aspects: his way of conceptualizing and expounding musical thought is not limited to aspects merely related to improvisation and performance. In this context, mathematics represents a type of reasoning and expository technique far afield from the actual application of musical values. The SR shows that certain elements of music can be mathematically treated: yet, it can be seen that the way of expounding musical concepts by means of calculations is not intimately linked to the nature of the subject itself. It reflects instead an efficacious, persuasive "aesthetic" exercise displaying beauty, profusion, adaptability, and versatility. This theoretical model is also to be understood as the product of the long-term Indian interest in combinatorics.

Finally, in literature on *saṅgīta*, the opposition between *lakṣya* (lit. "to be marked") and *lakṣaṇa* (lit. "mark"), between teachings and performance, theory and practice, resembles the contrast between *śāstra* (the established body of knowledge) and *prayoga* (the practical application of this knowledge) that characterises other disciplines.[69] According to Rowell, "a hallmark of the early Indian way of thinking about music was to identify and name all possible permutations of the basic elements, but with the realization that only certain melodic constructions can become the basis for actualized music.[...] It was the job of theory to provide the widest selection of possibilities [...]".[70] This statement forgets however to clarify that in "early" Indian musical thought permutations are not found in relation to *tāna*s. The "widest selection of possibilities" is provided with so much elaboration and for the first time as the result of Śārṅgadeva's mathematical and literary abilities. Within *saṅgītaśāstra*, the SR represents a textual paradigm, the emblem of a textual tradition capable of renewing itself.

6 Conclusion

This study has investigated some aspects related to the mathematical calculations found in the Sanskrit medieval work on music *Saṅgītaratnākara* by

69 For instance, the opposition between theory and practice can be noted in the complex relationship to actual practice of the *vāstuśāstra* and *śilpaśāstra*, the canonical Indian texts on architecture and sculpture.
70 Rowell 1992: 154.

Śārṅgadeva (thirteenth century CE). The aim was to bring to light the author's ability to conceive methods and a diversified vocabulary for expounding music theory. It has been shown that mathematical procedures and diagrams visualizing numbers and musical values are found in relation to the treatment of tonal patterns (*tāna*) and rhythmic varieties (*tāla*). In this respect, parallels have been established, particularly with texts showing the influence of the *Saṅgītaratnākara*. The literary practices used by the author adopt procedures and technical terms found in other Sanskrit technical literatures. The technique called *prastāra*, the *khaṇḍameru*, and the use of symbolism found also in metrics convey economy, variety, and recognisability to the elucidation of the subject. I have argued that Śārṅgadeva's contribution towards an innovative textual normativity is modeled upon literary practices (lexical, stylistic, compositional) reflecting a shift in cultural legitimation and in the use and reception of works on *saṅgīta*. Lastly, this study has suggested that the SR's originality testifies to a change in the use, production, and circulation of texts, where performance was not the main concern.

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