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Review Article

Mutual Coupling in Phased Arrays: A Review

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The mutual coupling between antenna elements affects the antenna parameters like terminal impedances, reflection coefficients and hence the antenna array performance in terms of radiation characteristics, output signal-to-interference noise ratio (SINR), and radar cross section (RCS). This coupling effect is also known to directly or indirectly influence the steady state and transient response, the resolution capability, interference rejection, and direction-of-arrival (DOA) estimation competence of the array. Researchers have proposed several techniques and designs for optimal performance of phased array in a given signal environment, counteracting the coupling effect. This paper presents a comprehensive review of the methods that model and mitigate the mutual coupling effect for different types of arrays. The parameters that get affected due to the presence of coupling thereby degrading the array performance are discussed. The techniques for optimization of the antenna characteristics in the presence of coupling are also included.

1. Introduction

A phased array comprises of definitely arranged, finite sized antenna elements, which are fed by an appropriate feed network. In such an array, the fields radiated from one antenna might be received by the other elements. Furthermore, this signal might get reflected, reradiated, or scattered. The properties of these signals depend on the power of the signal, reflection coefficients, and the additional electrical phase introduced due to the propagation delay from one element to the other. This kind of interaction between the antenna elements will lead to coupling effect and hence can alter the array characteristics.

In other words, in a phased array, the electromagnetic (EM) characteristics of a particular antenna element influence the other elements and are themselves influenced by the elements in their proximity. This interelement influence or mutual coupling between the antennas is dependent on various factors, namely, number and type of antenna elements (A), interelement spacing, relative orientation of elements, radiation characteristics of the radiators, scan angle, bandwidth, direction of arrival (DOA) of the incident signals, and the components of the feed network (Figure 1), that is, phase shifters (P) and couplers (C).

The presence of coupling in an array changes the terminal impedances of the antennas, reflection coefficients, and the array gain. These being the fundamental properties of the array have a greater influence on their radiation characteristics, output signal-to-interference plus noise ratio (SINR) and radar cross section (RCS). Furthermore, it affects the steady state response, transient response, speed of response, resolution capability, interference rejection ability, and DOA estimation competence of the array.

Several researchers have studied the effect of mutual coupling on different types of adaptive arrays. These include Yagi array [1], LMS and Applebaumarrays [2], power inversion array [3], circular array of isotropic elements and semicircular array of printed dipoles [4], microstrip patch antenna arrays [5], linear arrays of dipole, sleeve dipole and spiral antennas [6], conformal dipole arrays [7], helical arrays [8] and arrays of arbitrary geometry [9].

The parameters governing the array performance are obtained using various techniques like method of moments (MoM), multiple signal classification (MUSIC), estimation of signal parameters via rotational invariance techniques (ESPRIT), scheme for spatial multiplexing of local elements (SMILE), and direct data domain (DDD) algorithms. The algorithms used to estimate the coupling can be extended

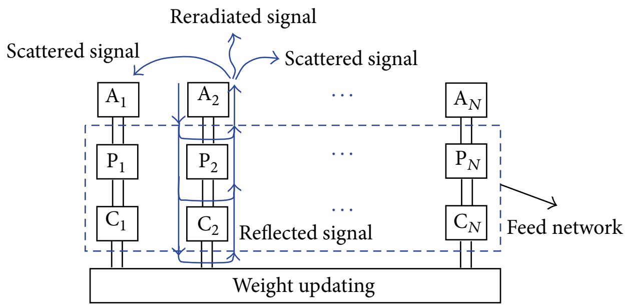


Figure 1: Schematic of adaptive antenna array.

towards the compensation of its effect. Moreover optimization techniques such as genetic algorithm (GA), particle swarm optimization (PSO), and linear programming (LP) can be used in conjuncture with these techniques towards the enhancement of array efficiency. This paper presents a unified review of the techniques and the designs proposed for mitigating mutual coupling effect in phased arrays. The performance parameters that get affected due to the coupling effect in antenna array are discussed. The work reported in open domain towards efficient array design mitigating mutual coupling effect is reviewed and compared.

The subsequent sections describe the analysis and compensation of mutual coupling effect in phased arrays using these techniques. The effect of mutual coupling on the array parameters such as antenna impedance and steering vector, which further affects the radiation pattern, resolution and interference suppression ability, DOA estimation, output SINR, response speed, and RCS is described in Section 2. Section 3 reviews the methods for efficient antenna design and the use of optimization techniques to counteract the adverse effects of mutual coupling on the array performance. In Section 4, the mutual coupling effects and the techniques to compensate it for the phased arrays conforming to the surface are discussed. Section 5 presents the cases where the presence of coupling proves to be advantageous. This is followed by a brief summary on the review of mutual coupling in Section 6.

2. Parameters Affected due to Mutual Coupling Effect

2.1. Antenna Impedance. The antenna radiation pattern depends mainly on the impedance at the antenna terminals. However, the antenna impedance of phased array is significantly different in comparison to that of an isolated element. This variation in impedance is due to the presence of coupling between the array elements. In general, for an array of N-elements, the impedance matrix is given by [2]

$$Z = \begin{bmatrix} Z_{11}+Z_L & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22}+Z_L & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN}+Z_L \end{bmatrix}, \quad (1)$$

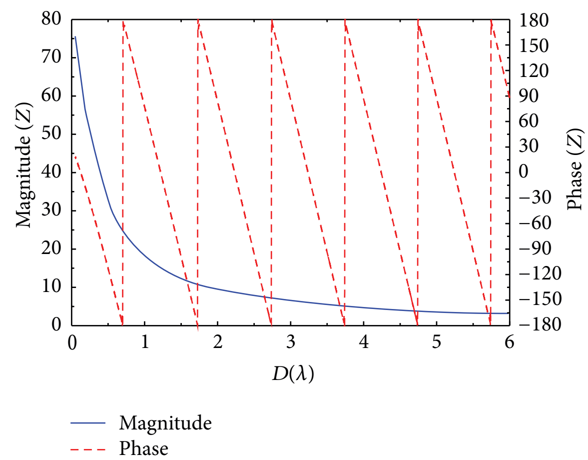


Figure 2: Variation of mutual impedance between two half-wavelength, center-fed dipoles with interelement spacing.

where Z_{mn} represents the self ($m = n$) and mutual ($m \neq n$) impedances of the antennas and Z_L is the terminating load. This impedance matrix yields the net impedance at the antenna terminal [10]; that is,

$$Z_x = \sum_{y=1}^{N} Z_{x,y} \left(\frac{I_y}{I_x} \right), \quad (2)$$

where I_p represents the current at the terminals of pth antenna element and $Z_{x,y}$ is the impedance between xth and yth antenna elements.

In general, the mutual impedance matrix is a square matrix with the order corresponding to the array size. This implies that the computations involved in arriving at the matrix coefficients increases with the size of array. However, it is possible to exploit certain properties of coupling for reducing the computation complexity. One such property is the inverse dependence of coupling coefficients on the distance between the array elements [11], as shown in Figure 2. It is apparent that farther the elements in the array, least will be the mutual impedance and hence the coupling effect. In uniform linear arrays, the coupling matrix possesses rotational symmetry and hence has a symmetric Toeplitz matrix structure. Similarly for a uniform circular array, the coupling matrix is circulant.

Furthermore, the self- and mutual impedances of (1) are dependent on the type and configuration of the antenna array. The estimation of these impedances is essentially a problem of finding the current distribution on the antenna surface. Among the available methods for estimation of self- and mutual impedances of wire-type antennas, the integral equation-moment method [12] and the induced electromotive force (EMF) method [13] are the most popular. Both of these methods are based on the integral forms of induced current and voltages at the antenna terminals. The method of induced EMF is advantageous over the integral equation-moment method, as it facilitates the simplified/easier design of antennas by providing the closed form solutions. However, it is applicable only to the simplified models of straight wire antennas with smaller radii and the antenna arrays with typical geometries. Moreover, this method does not account for the radius of the antenna wire and the gaps at the feed of the antennas accurately. On the other hand, integral equation-moment method is valid for both larger radii dipoles and for the arrays with complex geometries, including skewed arrangements of elements.

Carter [13] determined the mutual impedance between two half-wave dipoles in echelon configuration using induced EMF method. King [14] extended the method for two antennas of arbitrary and unequal lengths; however, the method was found computationally extensive. This drawback was overcome by using a concise formula by Hansen [15], assuming a sinusoidal current distribution for the dipoles in echelon configuration. It was shown that the convergent iterative algorithms could be used to analyze the coupling effects in linear dipole array [16, 17].

Ehrlich and Short [18] analyzed the effect of coupling in slot array on a finite ground plane fed by two resonant waveguides. It was shown that the input slot admittance of the driven slot changes due to the presence of matched terminated parasitic slot waveguide. An experimental method of determining the coupling between the microstrip antennas was presented by Jedlicka et al. [19].

The mutual impedance between the microstrip dipoles of arbitrary configuration, printed on a grounded substrate was determined using integral equation-based method [20]. It was shown that the mutual coupling depends on the magnitude of surface wave, and hence on the substrate thickness, for different configurations. The surface waves enhance the mutual coupling to a greater extent in a collinear arrangement of the printed dipoles as compared to the broadside configuration. Furthermore, for a single propagating mode, the mutual coupling in broadside arrangement was found to be mainly due to the direct, higher order, and leaky waves. In broadside configuration, the coupling effect is mainly due to TE mode of surface waves. while in collinear configuration, it is due to TM mode.

Inami et al. [21] estimated the mutual impedances between the rectangular slot antennas located on the concave side of a spherical surface. The surface fields excited by a horizontal magnetic source located on a conducting concave spherical surface were formulated using the combination of ray-optical fields and simplified integrals. The mutual coupling between the slots was determined based on the assumption that aperture field is the dominant mode field in the waveguides connected to slots. It is shown that the mutual coupling decays rapidly on the convex side of the surface. On the other hand, its magnitude exhibits an oscillatory behaviour when the arc distance is increased, on the concave side.

Similar integral equation-based method was employed by Eleftheriades and Rebeiz [22] to estimate the mutual impedance of arbitrarily placed, parallel-aligned narrow slots on a semi-infinite dielectric substrate. The electric vector potential was determined in terms of the equivalent magnetic currents in the slots. These currents were expanded using fast converging basis functions. The admittance matrix was obtained by Galerkin's method in the spectral domain along the slot length and point matching across their widths. Porter and Gearhart [23] extended this technique to calculate the mutual coupling in arbitrarily placed, perpendicularly aligned slots on an infinite dielectric substrate. In this method, the admittance matrix was used to determine the coupling between the slots present in different dielectrics namely, free-space, fused quartz and GaAs. The copolarized and cross-polarized radiation patterns were obtained using the Fourier transform of the electric fields in the slots. The peak cross-polarization levels were found strongly correlated to the level of coupling, which in turn depends on both dielectric constant and the slot geometry. This MoM-based technique is useful to determine the mutual impedance between slot antennas, operating in different polarizations.

Pozar [24] proposed the technique of computing the input and mutual impedances of rectangular microstrip antennas for various configurations. The exact Green's function for an isotropic grounded dielectric substrate was evaluated using the moment method. The analysis considered the effects of both surface waves and the coupling due to nearby antennas. For similar antenna structures, the mutual impedance can be obtained by solving the reaction integral equation using MoM [25]. Another technique is based on transmission line model [26], which neglects the effect of surface waves and approximates each rectangular resonator in terms of two equivalent radiating slots.

Hansen and Patzold [27] used spectral domain approach to analyze both input and mutual impedance of a rectangular microstrip antenna with a dielectric superstrate. This method is based on the Richmond's reaction and assumes isotropic substrates and superstrates, with the conducting patches located on the same dielectric substrate. The technique of computing the mutual coupling between the microstrip dipoles in multielement arrays using the dyadic Green's function was presented [28]. The dipoles were excited by an electromagnetically coupled transmission line in the isotropic substrates.

The reaction theorem was used to calculate the mutual impedance between two printed antennas of a grounded isotropic dielectric substrate [29]. The impedances of a microstrip array were calculated by replacing the elements by equivalent magnetic current sources. Similar approach of replacing the edge aperture field by an equivalent magnetic line source has been used [30]. The method accounts for the effect of both isotropic substrate and superstrate on the edge-fed rectangular microstrip antennas placed on the same dielectric layer. Terret et al. [31] analyzed the mutual coupling in stacked microstrip antennas using a combination of spectral domain Green's function [24] and the reciprocity theorem. This approach is unique; it considers the coupling between two printed antennas located on different isotropic layers.

The mutual coupling in arbitrarily located parallel cylindrical dipoles [32] was determined using a numerical technique based on the method of auxiliary sources (MAS). In this method, a set of fictitious sources, like infinitesimal dipoles, were assumed to be located inside the spatially overlapped unequal sized dipoles. The EM fields generated from the fictitious sources were subjected to boundary conditions to obtain the current distribution over the dipole elements, and then the self- and mutual admittances of array elements can be estimated. Although increasing the number of fictitious sources enhances the accuracy of the proposed method, it also has an adverse effect. It cannot be denied that the method proposed is simple and easily extendable to complex antenna arrays, such as curtain arrays and Yagi-Uda antennas, or for predicting coupling phenomena in closely spaced transceivers. Moreover, the method is computationally less intense when compared to MoM, which involves single or double numerical integrations.

The fuzzy modelling technique can be employed to calculate the input impedance of two coupled monopole antennas [33] or coupled dipoles [34]. First, the knowledge base for the two-coupled dipoles in parallel and collinear configurations was obtained using MoM. This knowledge base together with the concept of spatial membership functions was used to predict the input impedance of two coupled dipole antennas in echelon form. This fuzzy-inference-based method is faster than MoM, where the computational complexity increases with the array size and the accuracy desired.

The mutual coupling in a nonlinearly loaded antenna array [35] was determined using power series expansion [36, 37]. The method of nonlinear currents [38] was used to treat every port of a microwave circuit, describing the antenna input terminals of an array [39]. This method is advantageous as it provides physical interpretations of the scattering mechanisms within the array structure at different harmonics. The method has no restriction on the antenna type and also accounts for higher order coupling, establishing that the higher order couplings have negligible effect.

2.2. Steering Vector. The response of an array towards the incident signal is expressed mathematically as steering vector. This vector is dependent on the antenna element positions, inter-element spacing, radiation characteristics of each element, and the polarization of the incident wave. The steering vector of an array can be obtained by direct measurement of complex array element patterns; however, it proves to be difficult. Moreover, it demands for a huge memory to save the measured data, and hence does not present a practical approach [40]. The popular method to obtain the steering vector, without storing the data of all searching directions is to use numerical techniques like MoM [41]. Furthermore, the limitation of huge memory requirement can be overcome by using the distortion matrix with a limited size [42]. The distortion matrix is derived by comparing the actual array manifold (steering vector) with the ideal one in limited number of directions.

The presence of coupling between the array elements affects the steering vector, and hence the array response [43]. The analysis of the steering vector of an array in the presence of coupling can be done analytically [44]. Kato and Kuwahara [45] studied the effect of mutual coupling on a planar dipole antenna array of vertically polarized, identical elements with a back reflector. Assuming reciprocal antenna elements, the steering vector of nth driven element is equivalent to the complex array element pattern, provided all

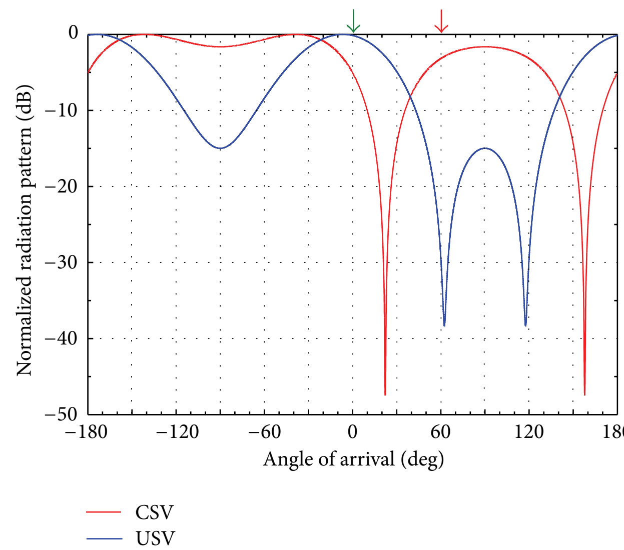


Figure 3: Synthesized pattern of a 2-element dipole array. Desired signal (green arrow): 0°, 40 dB; 1 jammer (red arrow): 60°, 0 dB.

other elements are short circuited (zero voltage). The mutual coupling effect on the array performance was analyzed using both conventional steering vector (CSV) and induced EMF method. Mutual coupling effects in the vertical plane were shown to be less than that in the horizontal plane, for an inter-element spacing of more than half wavelength.

As the CSV does not account for coupling effects, the compensation of received voltages becomes mandatory for the analysis of a practical array. To overcome this difficulty, Yuan et al. [42] proposed the method of obtaining the array manifold directly using universal steering vector (USV).

The CSV and USV of an array are related to each other by the following relation:

$$\begin{aligned} \left[A^{u}(\theta,\Phi)\right] &= \left[C(\theta,\Phi)\right]^{-1}\left[A^{c}(\theta,\Phi)\right], \\ \text{with } \left[C(\theta,\Phi)\right]^{-1} &= Z_L\,[Y]\,[T(\theta,\Phi)], \end{aligned} \tag{3}$$

where A^u and A^c represent USV and CSV of the array, respectively, T is the transformation matrix, Y is the admittance matrix, and Z_L is the termination impedance of the antenna. The matrix C in (3) depends on the angle and polarization under consideration, unlike the impedance matrix of Gupta and Ksienski [2].

The presence of coupling in between the elements changes the impedance and hence the radiation pattern of the phased array. This indicates that the accurate calculation of the radiation pattern of an array is feasible only if (i) CSV with an appropriate compensation technique or (ii) USV is used for the calculation. This is apparent from Figure 3. The USV is shown to provide the desired radiation pattern by directing the main beam in the direction of desired signal (at 0°) and simultaneously nulling the jammer at 50°.

Zhang et al. [46] used MoM for analyzing the performance of a power inversion adaptive array in the presence of coupling. The quiescent array patterns and the output

SINR depend on the accurate choice of steering vector. The weighting function in the proposed technique is expressed as

$$[W] = \{[U] + g\,[\Phi]\}^{-1}\,[W_0], \quad (4)$$

where g is the loop gain and $[W_0]$ is the steering vector.

In order to form a desired receiving array pattern, proper weights with and without mutual coupling are needed. For an array of omnidirectional antenna elements, the steering vector in the absence of mutual coupling is given by

$$[W_{00}] = [1, 0, 0 \ldots 0]^T. \quad (5)$$

However, when mutual coupling is considered, the steering vector is modified as

$$[W_0] = [Z_L]^{-1}\,[Z]\,[W_{00}], \quad (6)$$

where Z and Z_L indicate the impedance and load matrices, respectively.

Liao and Chan [47] proposed an adaptive beamforming algorithm based on an improved estimate of calibrated signal steering vector with a diagonally loaded robust beamformer. The angularly independent mutual coupling was modeled in terms of angularly dependent array gain and phase uncertainties. The mutual coupling matrix used was in the form of a banded symmetric Toeplitz matrix [48] for a uniform linear array and a circulant matrix for a uniform circular array. The inverse relationship between the mutual coupling between two sensor elements and their relative separation was used.

2.3. Radiation Pattern. The effect of coupling on the pattern synthesis of an array can be analyzed using classical techniques (like pattern multiplication), numerical techniques (like MoM), and active element patterns (AEP) method [49]. The choice of a particular pattern synthesis method for a given array type needs a tradeoff depending on (i) the type of array element, (ii) spacing between the elements, (iii) array size, (iv) level of acceptable accuracy, and (v) the resources available for the computation. In particular, the radiation characteristic of an array is highly influenced by the current fed at the antenna terminals.

An adaptive antenna is expected to produce a pattern, which has its main beam towards the desired signal, and nulls towards the undesired signals. It is also required that patterns have sufficiently low sidelobe level (SLL). Although the standard pattern synthesis methods [50] yield lower SLL in the antenna pattern, the elements were assumed to be isotropic in nature. Moreover, the mutual coupling effect was ignored in the pattern synthesis, which leads to pattern errors in practical situations. The corrections to the pattern errors for a dipole array were proposed [51] incorporating the mutual coupling effect. Two methods, namely, (i) characteristic mode approach, and (ii) array mode with point matching, were presented. The characteristic mode-based technique exploits the orthogonality properties of characteristic modes to compute the array pattern. This method provides volumetric correction at the expense of complex computations. This demerit may be overcome by using point matching for the array modes, which need not be orthogonal. These methods are valid for both uniform and nonuniform arrays, provided that array can be expressed in the form of moment method matrix.

The principle of pattern multiplication uses the knowledge of currents at the feed terminals of individual elements to arrive at the complete array pattern. This classical technique is applicable only to the array with similar elements as it assumes identical element patterns for all individual array elements. The radiation pattern of an array is expressed as the product of an element and array factor. However, in a practical antenna array, the presence of coupling results in the variation of individual element patterns. Moreover for an array with electrically large elements, which differ in size, shape and/or orientation, the patterns of the individual elements differ considerably. This introduces a noise floor in the array pattern, thereby increasing SLL and degrading the quality of the result.

Such practical situations for which classical approaches become unsuitable can be analyzed using the numerical techniques such as MoM or FDTD. These numerical techniques directly estimate the current distributions over the antenna elements. The voltage and current expansion coefficients are related in terms of self- and mutual impedance matrices. These numerical techniques yield accurate results; however the size of the matrices increases with the array size, and beam scanning. Moreover, these numerical techniques cannot be used for the arrays that are located in highly complex inhomogeneous media.

Kelley and Stutzman [49]proposed the method of active element patterns (AEP) for analyzing the array pattern in the situations where both classical and numerical techniques fail. In this method, the pattern of complete array is obtained using the calculated or measured individual element patterns. AEP method is faster than the other methods, provided the data of individual element pattern is available in advance. AEP techniques can be broadly classified as nonapproximate AEP methods (unit-excitation AEP method and phase-adjusted unit-excitation AEP method) and approximate AEP methods (average AEP method and hybrid AEP method).

In unit excitation AEP method, the individual elemental patterns are computed assuming the elements excited by a feed voltage of unit magnitude. These active element patterns represent the pattern of entire array, considering direct excitation of a single element and parasitic excitation of others. Furthermore, these individual patterns are superimposed/summed-up and scaled by a factor of complex-valued feed voltage applied at the terminals to arrive at the complete array pattern. This method is advantageous, as it needs to compute the pattern of individual array elements only once. Moreover, this method is valid for arrays of both similar and/or dissimilar elements, located in inhomogeneous linear media.

The dependence of the element pattern on the array geometry can be explicitly mentioned using an exponential term. Such approach in which individual element patterns vary due to the presence of additional spatial phase information factor is called phase-adjusted unit-excitation AEP method. This method considers the spatial translation of the array elements, unlike the unit-excitation element pattern, which refers only to the origin of the array coordinate system. Although the phase-adjusted element patterns differ for different array geometries, the concept is useful in the development of approximate array analysis methods.

The computational complexity of both unit-excitation and phase-adjusted unit-excitation methods increases with the array size due to the need of the active element pattern data for every array element. However, if the uniform array is infinitely large, then the phase-adjusted active element patterns of individual elements will become identical. In such scenarios, the complete array pattern can be expressed in the form of an average active element pattern, which will be the active element pattern of a typical interior element [52]. Although this method yields accurate results for very large, equally spaced arrays, it fails for arrays with smaller size. This is because the individual elemental patterns of a small array vary widely due to edge and mutual coupling effects.

Hybrid active element pattern method, proposed by Kelley and Stutzman [49], can be used for array with moderate number of elements. In this method, the array is divided into two groups, namely, an interior element group and an edge element group. The pattern of the interior group is determined using average AEP method; while the pattern of edge element group is calculated using phase-adjusted unit-excitation AEP method. The total antenna pattern is the sum of the two patterns obtained. This method is advantageous, as it requires less memory than the exact methods. Moreover, it yields accurate results for arrays of any antenna type.

In a large array, almost all the elements experience similar EM environment, unlike small array with prominent edge effects. This causes a considerable difference in the individual element patterns of the array, leading to higher SLL. Moreover, small arrays require exceedingly fine control over both magnitude and phase of each element for accurate beam steering. Darwood et al. [53] analyzed the mutual coupling effect in a small linear dipole array using the adaptive array-based technique [54]. The pattern synthesis yields low SLL in both sum and difference beams, in the presence of coupling effect. In small array [55], voltages at the antenna terminals (with coupling), V_c can be expressed in terms of voltages (no coupling), V_n and coupling matrix, C as

$$V_c = CV_n. \tag{7}$$

The mutual coupling compensation can be done by multiplying the inverse of coupling matrix and V_c as

$$V_n = C^{-1}V_c. \tag{8}$$

This process is simple for an array with single mode elements; the coupling compensation matrix, C^{-1}, is scan independent. However, an array of multimode elements requires the scan-dependent coupling compensation. Although the determination of the compensation matrix is a difficult task, it is easily realized in a digital beamforming (DBF) antenna system. The performance of large arrays can be predicted from the mutual admittance matrices of small arrays of similar lattice [56–58].

The coupling matrix can be estimated [55] using either Fourier decomposition of the measured element patterns or coupling measurements between the array ports. Fourier decomposition method is based on the fact that the coupling coefficients, c_{mn}, are the Fourier coefficients of the complex voltage patterns of the array elements, $g_m(u)$, and the isolated element pattern, $f^i(u)$ as

$$c_{mn} = \frac{1}{2\pi}\int_{-\pi/kd}^{\pi/kd} \frac{g_m(u)}{f^i(u)} e^{-jnkdu} du. \tag{9}$$

This solution is based on the assumptions that $f^i(u)$ is independent of nulls in the integration interval and that the inter-element spacing is larger than half-wavelength. This method is applicable only for the nonreciprocal antenna systems as it requires the antennas to be driven in a single mode, either transmit or receive. Moreover, the derived coupling coefficient matrix accounts for channel imbalances like the differences in insertion amplitude and phase between the aperture and the output terminal of the antenna element.

The second method for calculating coupling coefficients is based on the scattering matrix S of an array of uniformly spaced waveguide elements fed by matched generators. The scattering matrix as measured from a reference plane, coinciding with the plane of element apertures is given by

$$C = I + S, \tag{10}$$

where I denotes the identity matrix. In general, the scattering matrix is measured from a reference plane, which is at a certain distance away from the aperture plane. Transmission lines with different insertion loss are included between the planes of aperture and reference. Assuming these feed lines to be matched and reciprocal, the modified scattering matrix S' is given by

$$S' = TST, \tag{11}$$

where T is the diagonal matrix of transmission coefficients. This yields the modified coupling matrix C' at the reference plane as

$$C' = T + S'T^{-1}. \tag{12}$$

The measurement of the network parameters becomes difficult when the feed lines between the element apertures and output terminals are not matched. Furthermore, this method requires each element to be driven in both transmit and receive modes. The information about the reference plane corresponding to the phase center of each radiating element is required, which is not feasible for a real array, composed of nonideal elements and complex feed network [59].

Darwood et al. [60] showed that the method of Steyskal and Herd [55] is unsuitable for small planar arrays of arbitrary geometry. The compensation of mutual coupling effect was done by multiplying active element pattern with the inverse coupling matrix. This improves the performance of small planar dipole array by reducing SLL and increasing the directivity of sum and difference beams. This method wins over the Fourier decomposition method as it yields a more robust compensation for the coupling effects. Moreover, the estimation of the coupling matrix is to be carried on only once for a given array geometry.

The mutual coupling compensation requires the knowledge of coupling coefficients, which can be waived by using the experimental method of applying retrodirective beams [61]. This technique yields low SLL patterns by applying determined set of complex weights to each antenna element. Su and Ling [62] compared the approaches that model mutual coupling as coupling matrix. In Gupta and Ksienski [2] approach, the terminal voltage of an isolated antenna is taken to be the same as that in an array provided all other elements are open circuited. This assumption is impractical, as the open circuit condition does not imply zero current on the antenna elements. This approach is valid only if very small current is induced on half-wave dipole elements. On the other hand, Friedlander and Weiss [11] approach is valid only if the relationship between the active element pattern and the stand-alone element pattern is angle independent. This condition is achieved when (i) all antennas are vertical wires with all incident directions having the same elevation angle [63], and (ii) array elements operate near resonance, that is, when the shape of stand-alone current distribution is the same for all incident angles. Although this method outperforms the approach of Gupta and Ksienski [2], it fails for complex structures. Su and Ling [62] employed an extended approach of coupling matrix formulation for a Yagi-Uda array. This technique includes the coupling effect due to both active and parasitic elements. However, this method is bound by the conditions of standard approach for parasitic elements and requires the knowledge of large number of incident angles for unique solution.

In general, the coupling matrix-based methods assume that the coupling matrix is an averaged effect of the angle-dependent relationship between the active element patterns and the stand-alone element patterns. In such scenario, the minimum mean-square error (MMSE) matching of the two pattern sets for a few known incident angle yields the coupling matrix [64, 65]. The coupling matrix method, though capable of modeling both coupling and calibration effects, fails to determine the array pattern accurately [66]. This is because these methods neglect the effect of structural scattering, which is significant especially for an antenna conforming to the surface. Apart from structural scattering, the array pattern gets affected due to the calibration errors. The effects of such calibration issues can be dealt by using the autocalibration methods [67–70]. These techniques are based on the common criteria that every source of error can be represented through coupling matrix. The calibration technique proposed by Hung [70] was verified experimentally for a narrow-band array in the presence of coupling [71]. This robust auto-calibration method is shown to improve the array performance by compensating for the sources of errors.

In certain scenarios, the antenna arrays are loaded with nonlinear devices in view of protection from the external power. The analysis of such arrays is complex due to the nonlinear characteristics of each array element. For such arrays, Lee [72] approximated the mutual coupling effects by the infinite periodic array method [73–75]. This method is suitable only for large periodic arrays due to infinite periodic Green's function. Poisson sum technique was employed to reduce the analysis of an infinite array into that of a single antenna element. Moreover, this approach ignores the edge effects.

The mutual coupling in a finite array of printed dipoles fed by a corporate feed network was studied by Lee and Chu [76]. The analysis was based on the variation in the mismatches within the feed network, array pattern, and gain due to presence of coupling. The self- and mutual impedances, for a given excitation, were determined using the spectral domain technique [77]. This method is applicable for both forced and free excitations; that is, radiation impedances with and without feed network can be obtained. It was shown that the edge effects gain more and more prominence as the array size decreases causing greater variations in the impedance values. This in turn leads to the enhanced multiple reflections within the feed network, causing ripples in the amplitude and phase distributions across the antenna aperture. The array performance gets degraded, especially for large scan angles.

The effects of coupling for a microstrip GSM phased array fed by a Butler feed network were analyzed [78]. The coupling distorts the array pattern with higher SLL, due to increase in the reflected power towards the feed network [79].

2.4. Resolution. The presence of mutual coupling amongst the array elements affects the array resolution adversely. Manikas and Fistas [80] proposed a complex mutual coupling matrix (MCM) given by

$$C = A \Theta L \Theta e^{j\Phi} \Theta G \Theta e^{j\pi D}, \quad (13)$$

where Θ denotes Hadamard product, matrix A represents the rms values of direct and reradiated signals, L is the free space propagation loss matrix, Φ represents the random phases introduced to the re-radiated signal by the elements, G is the matrix of gain and phase of the elements, and D is the matrix of inter-element distances. Alternatively, the MCM can be expressed as

$$C = \left(1 \cdot b^T - \operatorname{diag}\left(\operatorname{diag}\left(1 \cdot b^T\right)\right) + I\right) \Theta G \Theta \left(S_r + I\right), \quad (14)$$

where S_r is the matrix whose columns are source position vectors (SPV) of the re-radiated signals, and b is the vector obtained by pre- and postprocessing of the matrix ($C \cdot S \cdot S^H \cdot C^H$). The computed MCM is independent of angle of incidence. However, it depends on the geometry and the electrical characteristics of array. The associated signal covariance matrix is given by

$$R_{xx} = C \cdot S \cdot S^H \cdot C^H + \sigma^2 \cdot I. \quad (15)$$

The mutual coupling worsens the array performance further if the signals are wideband. This is because the array loses its ability to match the desired signals or to null the jammers [6, 81] over a broad frequency range. The coupling is compensated by correcting the actual voltage matrix using effective weights, computed based on the terminal impedance matrix, derived from MoM impedance matrix. The proposed method fails to compensate the loss in antenna gain due to wideband signals.

The resolution capability of an array is also affected due to array calibration errors, similar to that of its radiation pattern. Such effects in eigenstructure-based method, MUSIC, presented by Friedlander [82], are seen to be independent of signal-to-noise ratio (SNR). Pierre and Kaveh [68] showed that the array fails to resolve the signal sources when prone to calibration errors in spite of having high SNR.

2.5. Interference Suppression. An adaptive array is expected to accurately place sufficiently deep nulls towards the impinging unwanted signals. The presence of mutual coupling between the antenna elements affects both the positioning and the depth of the nulls. The interference rejection ability depends on array geometry, direction of arrivals (DOA) of the signals, and weight adaptation. Riegler and Compton [83] analyzed the performance of an adaptive array, prone to mutual coupling, in rejecting the interfering signals using minimum mean-square error technique. It is shown that for high power incoming or desired signals, the array responds readily as the corresponding weight vector is updated faster, minimizing the error signal.

Adve and Sarkar [84] proposed a numerical technique to account for and/or to eliminate the coupling effects in linear array in the presence of near field scatterers. Similar MoM-based approach was used [85] to analyze the coupling effect on the interference suppression of direct data domain (DDD) algorithms. The reported results present an insight of the signal recovery problems in case of a linear array of equispaced, centrally loaded thin half-wavelength dipoles. Array analysis in contrast to Gupta and Ksienski's approach [2] uses multiple basis functions for each element. This method successfully nulls the strong interferences and is computationally simple and efficient. However, it requires the incoming elevation angles of the signals and interferences to be equal and known *a priori*.

An improvement over the technique of open circuit voltage method so as to include the scattering effect of antenna elements was presented [86]. The estimated mutual impedance matrix was shown to improve the performance of DDD techniques in nulling the interference. Another technique to suppress the interfering signals at the base stations of mobile communication system using normal-mode helical antenna array was proposed by Hui et al. [87]. This method neither requires the current distributions over the antenna elements [6] nor the incoming elevation angle of the desired and interfering signals [85]. Single estimated current distribution for every antenna element is required to estimate the mutual coupling matrix. This method can reduce the coupling effect to an extent, but not eliminating it completely.

The current distribution of a small helical antenna is shown to be independent of azimuth angle of the incident field, if it impinges from horizontal direction [87]. This yields a reasonably accurate estimate of the current distributions on the antenna elements and hence the mutual impedance terms. This new technique shows an improvement in the adaptive nulling capability for an array of helical antenna. The improvement of adaptive nulling in dipole array was presented by Hui [88]. The calculation of mutual impedances was based on an estimated current distribution with phase corrections instead of an equal-phase sinusoidal current distribution. This method performs satisfactorily for strong interfering signals and for the elevation angles (of the signals and interferences), which are not too far from the horizontal direction. This method is less sensitive to the variation in elevation angle and the intensity of signal of interest (SOI). It was further used to compensate the mutual coupling effect in uniform circular array [89] estimating the DOA using maximum likelihood (ML) algorithm. Although optimization of log-likelihood function of the ML method overcomes its complexity, this method is used mostly for ULAs. Moreover, this method does not assure global convergence in general cases [90, 91].

Some special techniques were proposed for small and ultrawideband arrays. Darwood et al. [60] used MoM to compute the coupling coefficients of small planar array of printed dipoles. The depth of nulls was improved by mutual coupling compensation. The performance of LMS array of dipoles for wideband signal environment was analyzed by Zhang et al. [92]. The array size was shown to aid the effective suppression of the wideband interfering signal. Adaptive nulling in small circular and semicircular arrays in the presence of coupling and edge effects was presented [4]. Broader nulls were achieved incorporating multiple constraints over a small angular region. However, the constraints increase SLL in the array pattern, as more number of degrees of freedom was used in null placement. The ability of linear and circular dipole array to reject the interfering signal in the presence of mutual coupling effect was studied by Durrani and Bialkowski [93]. The signal-to-interference ratio (SIR) was shown to improve with array size up to certain limit. However, SIR of linear array gets degraded as one move, from the broad side to the end fire unlike circular array. This is because of the coupling effect, which is more pronounced at the broadside of a linear array than that in the case of a circular array of half-wavelength spaced dipoles.

2.6. Direction of Arrival (DOA). An adaptive array needs to estimate accurately the emitter location (DOA) and other details so as to suppress it effectively. DOA estimation depends on the array parameters determined by various techniques. These techniques are either spectral based or parametric based [94]. In spectral-based approach, the locations of the highest peaks of the spectrum are recorded as the DOA estimates. On the other hand, parametric techniques perform simultaneous multidimensional search for all parameters of interest.

MUSIC and ESPRIT algorithms are among the popular methods for DOA estimation. The sensitivity of the MUSIC algorithm to the system errors in the presence of coupling was studied by Friedlander [82]. The effect of phase errors is less in linear arrays, while the gain errors affect the sensitivity of both linear and nonlinear arrays identically. The linear arrays do not fail to resolve the sources due to merging of spectral peaks. The sensitivity of MUSIC algorithm in case of a non-linear array is inversely proportional to the source separation. The optimal design of an array requires a tradeoff between the cost required for accurate calibration and the cost incurred due to an increase in the array aperture. Friedlander and Weiss [11] proposed the coupling matrix inclusion in DOA estimation. This eigenstructure-based method uses only signals of opportunity to yield the calibrated array parameters. A banded Toeplitz coupling matrix for linear arrays and banded circulant coupling matrix for circular arrays were used. This method neither requires the knowledge of array manifold nor the locations of the elements *a priori*. Roller and Wasylkiwskyj [95] attempted to analyze the effect of both coupling and terminal impedance mismatches on the DOA estimation capacity of an isotropic array. This approach unlike Friedlander and Weiss's approach [11] does not change the MUSIC algorithm. The improvement in the angle of arrival (AOA) estimation is achieved by varying the impedances at the antenna terminations. This indicates that the array recalibration might not be required if the error is essentially due to mutual coupling between the antenna elements.

A preprocessing technique for accurate DOA estimation in coherent signal environment was proposed for uniform circular array [43]. This method in conjunction with spatial smoothing is capable of tackling the effects of coupling and array geometry imperfections in narrowband signal environment. The received signal vector of the array is transformed to a virtual vector on which spatial smoothing technique is applied. The eigenstructure of the signal covariance matrix varies by a factor of inversed normalized impedance matrix due to the presence of coupling. The coupling effect can be compensated either by multiplying the search vector by inverse of the normalized impedance matrix or by resolving the coherent signals using spatial smoothing technique [63]. A similar method of transforming search vector to nullify coupling effect is proposed for a circular dipole array [96].

The coupling affects the phase vectors of radiation sources, which in turn varies the signal covariance matrix and its eigenvalues, affecting the array performance [80]. The direct application of spatial smoothing schemes is not feasible when the coupling effects are prominent. This is because, in such situations, the phase vectors at the source and subarray do not differ only in phase. This necessitates additional computations to reconstruct the signal and noise subspaces and hence to improve the estimation capability.

Pasala and Friel [6] analyzed the accuracy of DOA estimation using MUSIC algorithm for signals distributed over a broad frequency range. Results were presented for linear arrays comprising of dipole, sleeve dipole, and spiral antenna. The method of moments was used for calculating the induced current and the actual voltages. The coupling effect is least for spiral antenna as compared to the dipole and the sleeve dipole. Although the spiral antenna element can mitigate the coupling effect, it fails in accurate DOA estimation owing to its grating lobes.

The coupling effect on the performance of ESPRIT algorithm for a uniform linear array was studied by Himed and Weiner [97] using modified steering vector. Swindlehurst and Kailath [98] employed statistical approach to study the first-order effects of gain and phase perturbations, sensor position errors, mutual coupling effects, and channel perturbations on MUSIC algorithm. A similar performance analysis was carried for multidimensional subspace-fitting algorithms like deterministic ML, multidimensional MUSIC, weighted subspace fitting (WSF), and ESPRIT [99].

Fletcher and Darwood [4] analyzed the beam synthesis and DOA estimation capacity of small circular and semicircular arrays. It is shown that the smaller circular arrays are less affected due to the effect of mutual coupling when compared to that of small linear or planar or semicircular arrays. This is because the linear or planar or semicircular arrays have edge effects which lead to the failure of conventional beamforming algorithms.

The coupling effect in DOA estimation capacity of a smart array of dipoles was studied using Numerical Electromagnetics Code (NEC). The NEC simulation considers the coupling effect by using compensated steering vectors [41], before applying any DOA estimation algorithm such as Bartlett or MUSIC. Su et al. [100] employed coupling matrix method for DOA estimation in circular arrays. This approach makes use of both full-wave electromagnetic solver NEC and MUSIC algorithm for direction finding. The technique is effective for simple antenna structures, provided the array calibration data is accurate.

Inoue et al. [40] analyzed the performance of MUSIC and ESPRIT algorithms for the cases of uniformly spaced dipole and sleeve antenna arrays. It is shown that the adverse effects of coupling and position errors gain more and more prominence for larger angles of arrival. Similar to uniform arrays, the performance of non-uniform arrays also degrades due to mutual coupling [101]. The conventional adaptive algorithms for analyzing these non-uniform arrays are inadequate [68, 102].

Another approach based on the concept of interpolated arrays was proposed [103] for non-linear arrays. A semicircular array composed of half-wave, thin wire, centrally loaded dipole antennas, prone to coupling, near-field scatterers, coherent jammers, clutter, and thermal noise was considered. The approach used Galerkin method in conjunction with MoM. The method transforms non-uniform array into a virtual array of omnidirectional isotropic sources before applying direct data domain least squares algorithm.

Accurate DOA estimation in the presence of coupling for a normal mode helical antenna and dipole array was presented [88, 104]. This method is based on receiving mutual impedance [87] and does not rely on the current distributions [6] or the elevation angles of the incoming signals [85]. The uncoupled voltage vector is related to the coupled voltage vector by the relation [105]

$$\begin{bmatrix} V_{nc_1} \\ V_{nc_2} \\ \vdots \\ V_{nc_N} \end{bmatrix} = \begin{bmatrix} 1 & -\dfrac{Z_t^{12}}{Z_L} & \cdots & -\dfrac{Z_t^{1N}}{Z_L} \\ -\dfrac{Z_t^{21}}{Z_L} & 1 & \cdots & -\dfrac{Z_t^{2N}}{Z_L} \\ \vdots & \vdots & \ddots & \vdots \\ -\dfrac{Z_t^{N1}}{Z_L} & -\dfrac{Z_t^{N2}}{Z_L} & \cdots & 1 \end{bmatrix} \begin{bmatrix} V_{c_1} \\ V_{c_2} \\ \vdots \\ V_{c_N} \end{bmatrix}, \quad (16)$$

where $[V_{nc}]$ and $[V_c]$ represent uncoupled and coupled voltage vectors, respectively, Z_t^{mn} is the receiving mutual impedance between the mth and nth antenna elements, and Z_L represents the terminating load. This equation is readily comparable with the expression given by Gupta and Ksienski [2] that relates the open circuited voltages, V_o, with the coupled voltage vectors as

$$\begin{bmatrix} V_{o_1} \\ V_{o_2} \\ \vdots \\ V_{o_N} \end{bmatrix} = \begin{bmatrix} 1+\frac{Z_{11}}{Z_L} & \frac{Z_{12}}{Z_L} & \cdots & \frac{Z_{1N}}{Z_L} \\ \frac{Z_{21}}{Z_L} & 1+\frac{Z_{22}}{Z_L} & \cdots & \frac{Z_{2N}}{Z_L} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{Z_{N1}}{Z_L} & \frac{Z_{N2}}{Z_L} & \cdots & 1+\frac{Z_{NN}}{Z_L} \end{bmatrix} \begin{bmatrix} V_{c_1} \\ V_{c_2} \\ \vdots \\ V_{c_N} \end{bmatrix}, \tag{17}$$

where Z_{mn} represents the conventional mutual impedance between mth and nth elements.

The mutual impedance terms in (16) are calculated in terms of phase-corrected current distribution instead of equal-phase sinusoidal currents used in (17). This enables the proposed method to overcome the over-simplified assumption of current distribution [2] and arrive at a closer estimate of compensated voltage. However, the method is valid only for receiving antennas due to the immature technology in case of transmitters.

As already mentioned, an array response towards the incident field is accurately expressed by USV and not by CSV. Thus the accuracy of DOA estimation algorithms can be improved using USV [42]. The method employs MoM to arrive at USV, and MUSIC algorithms for DOA estimation. The results were shown for the arrays of dipoles, monopoles, and planar inverted-F antenna (PIFA) mounted on mobile handsets. The method does not require any further compensation for the received voltages to remove the coupling effects.

Any conventional method assumes a ULA for mutual coupling compensation. Lindmark [59] proposed the extension of the method of Gupta et al. [66] to account for both co- and crosscoupling in dual polarized arrays. The array response using Vandermonde structure was expressed as

$$A_{\mathrm{ULA}}(\phi) = [a_{\mathrm{ULA}}(\phi_1), a_{\mathrm{ULA}}(\phi_2), \ldots, a_{\mathrm{ULA}}(\phi_d)] \tag{18a}$$

with

$$a_{\mathrm{ULA}}(\phi) = e^{-jkd(N-1)\sin(\phi/2)} \begin{bmatrix} 1 \\ e^{jkd\sin\phi} \\ \vdots \\ e^{jkd(N-1)\sin\phi} \end{bmatrix}, \tag{18b}$$

where N is the number of array elements, k is wave number, ϕ is the angle of incidence, and d is the inter-element spacing.

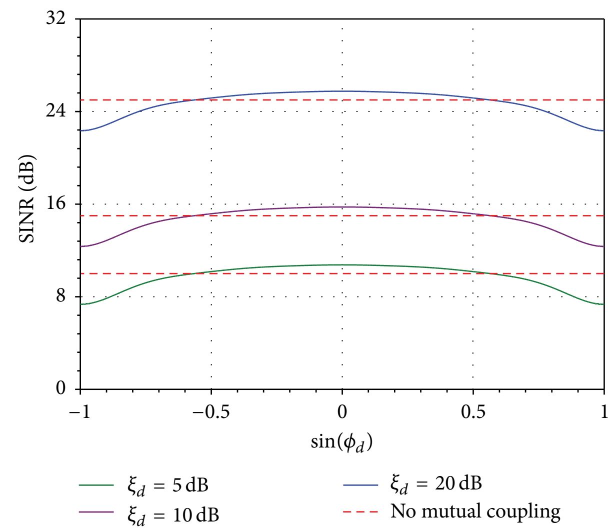


Figure 4: Effect of ξ_d on output SINR of a 6-element array of half-wavelength, center-fed dipoles.

The array response with nonunit amplitudes was used to mimic the behavior of a real array; that is,

$$a(\phi) = \cos^n(\phi)\, a_{\mathrm{ULA}}(\phi), \tag{19}$$

where n is an exponent chosen to best fit the elements in use. A mutual coupling compensation technique is employed for an array, rotated around a point, off its phase center.

The techniques of coupling compensation proposed by Coetzee and Yu [106] and Chua and Coetzee [107] are based on decoupling the input ports of the feeding networks. Yu and Hui [105] presented the design of coupling compensation network for a small and compact-size receiving monopole array. The proposed method provides the coupling-free voltages at the antenna terminals and is more suitable for applications such as beamforming and direction finding.

2.7. Output SINR and Response Speed. The presence of coupling between the antenna elements affects both steady state and transient response of an array. In general, the output SINR represents the steady-state performance of the array, while the transient response is expressed in terms of speed of array response. Figure 4 shows the change in output SINR (steady-state performance) of 6-element uniform dipole array with and without mutual coupling effect. The effect of ratio of the desired signal power to thermal noise power, ξ_d on the array performance is analyzed.

The output SINR of a least mean square (LMS) adaptive array in the presence of multiple interfering signals is given by [2, 108]

$$\mathrm{SINR} = \xi_d U_d^T R_n^{-1} U_d^*, \tag{20}$$

where R_n is the covariance matrix of the undesired signals (interference signals and thermal noise), ξ_d the ratio of desired signal power to thermal noise power, T denotes transpose, and U_d is the desired signal vector ofthe array.

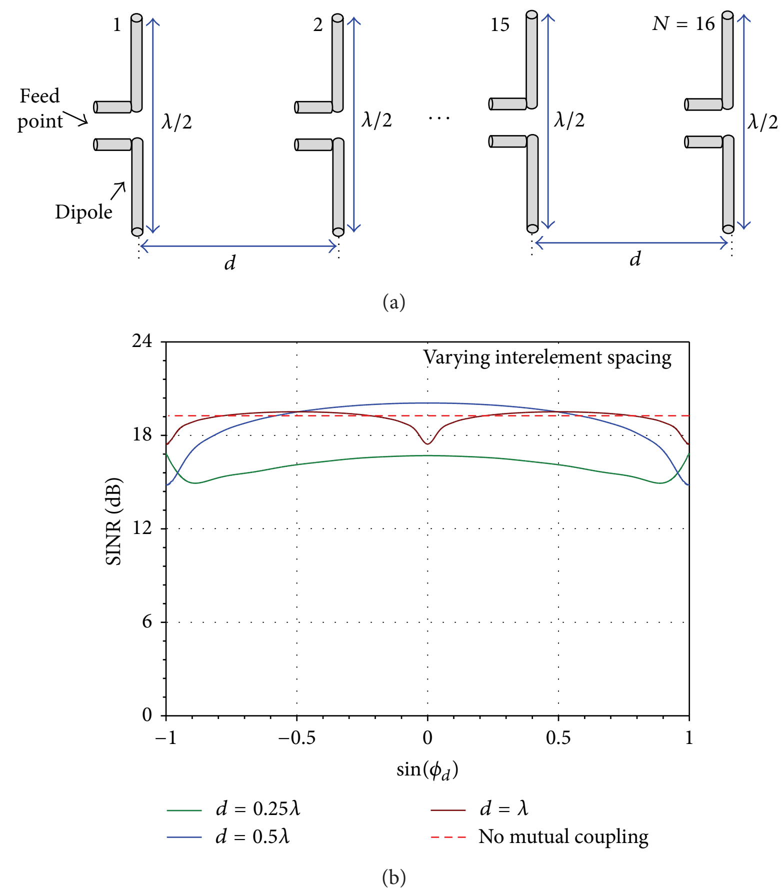


FIGURE 5: Effect of inter-element spacing on output SINR of a 16-element array of half-wavelength, center-fed dipoles; $\xi_d = 10$ dB, $\theta_d = 90^\circ$. (a) Schematic of dipole array (b) Output SINR.

For narrowband signals, uniformly distributed over $(0, 2\pi)$, R_n and the steady state weight vector are expressed as [108]

$$R_n = I + \sum_{k=1}^{m} \xi_{ik} U_{ik}^{*} U_{ik}^{T}, \quad W = K R_n^{-1} U_d^{*}, \tag{21}$$

where K is the constant, I is an identity matrix of array-size, m is the number of jammers, ξ_{ik} is the ratio of the kth jammer power to the thermal noise power, and U_{ik} is the kth jammer vector. The coupling between the elements changes the covariance matrix and the weight vector as [2]

$$R_n = Z_0^{*} Z_0^{T} + \sum_{k=1}^{m} \xi_{ik} U_{ik}^{*} U_{ik}^{T}, \quad W = K Z_0^{T} R_n^{-1} U_d^{*}, \tag{22}$$

where Z_0 is the normalized characteristic impedance obtained from (1). Equations (20) and (22) give insight of the output SINR in the presence of mutual coupling. Figure 5 shows the dependence of output SINR on inter-element spacing and hence the coupling factor. A reduction in inter-element spacing will reduce the antenna aperture and hence the incident energy due to the desired signal. However, the noise being internal to the receiver array will not be affected, causing reduced output SINR. Moreover, the performance of output SINR degrades due to the addition of more elements into the fixed array aperture (Figures 6 and 7). This is because, in such cases, total thermal noise of the array increases while the available signal power remains constant. Further when mutual coupling is considered, the output SINR of an array would also depend on incident angle of the desired signal (Figure 4). However, this is not observed for no mutual coupling case.

The output SINR is proportional to the gain of adaptive system based on its input SINR. The presence of coupling affects both input and output SINRs, especially if the inter-element spacing is less [109]. Although the gain of the system reduces as both input and output SINRs degrade due to coupling, the adaptive processing remains unaffected. Moreover, the insertion of invertible compensation matrix would not improve output SINR, as it has no effect on the output signal.

One of the desired characteristics of the adaptive array is its ability to adapt to the changes in signal environment instantly. This requires a quick updating process of the weight vector of an array, which in turn is a function of feedback loop

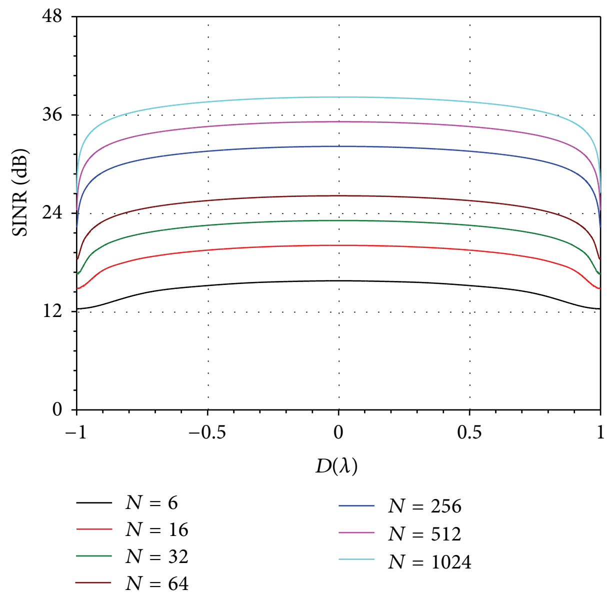


FIGURE 6: Comparison of output SINR in the presence of mutual coupling by varying the number of antenna elements.

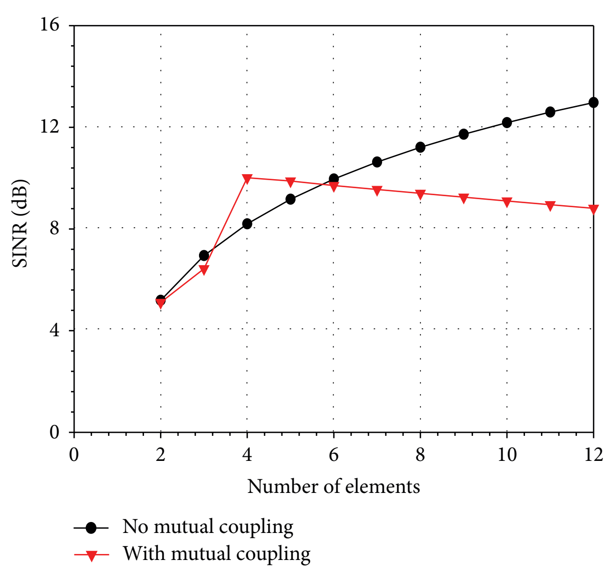


FIGURE 7: : Variation of output SINR of an array of half-wavelength, center-fed dipoles of fixed aperture with array size. ξ_d = 5 dB, $Z_L = Z_{ii}^*$, $(\theta_d, \phi_d) = (90^\circ, 0^\circ)$, total aperture = 2λ.

gain, steering vector, and signal covariance matrix. The signal covariance matrix of an adaptive array is expressed as [2]

$$R_{xx} = \sigma^2 \left[I + \sum_{k=1}^{m} \xi_{ik} \left(Z_o^{-1} U_{ik} \right) * \left(Z_o^{-1} U_{ik} \right)^T + \xi_d \left(Z_o^{-1} U_d \right) * \left(Z_o^{-1} U_d \right)^T \right], \quad (23)$$

where σ^2 is the thermal noise power, ξ_d is the ratio of desired signal power to the thermal noise power, ξ_{ik} represent, the ratio of kth jammer power to the thermal noise power, U_d and U_{ik} are the desired and kth jammer vectors, and m represents the number of jammers.

Since the covariance matrix depends on the impedance of antenna elements, the mutual coupling affects the transient response of the array. The coupling changes the eigenvalues of the signal covariance matrix. Smaller inter-element spacing causes greater coupling effect, lowering the eigenvalues, and hence longer transients. This reduces the speed of response of an array, resulting in delayed suppression of jammers. The performance analysis of an adaptive array in the presence of coupling by Dinger [110], Gupta and Ksienski [2], Leviatan et al. [1], and Zhang et al. [3] holds only for narrow-band signals. The effect of mutual coupling on the array performance in wideband signal environment was analyzed using MoM [92]. The output SINR in the presence of mutual coupling shows more oscillations than in no mutual coupling case. Although the coupling effect is similar to that for narrow band signals, it is more pronounced in small arrays. This indicates that the coupling effect can be mitigated by increasing the array size.

2.8. Radar Cross Section (RCS). The major focus for strategic applications is towards the reduction of radar cross section (RCS) of antenna array while maintaining an adequate array functionality in terms of gain, beam steering, and interference rejection. This necessitates the analysis and compensation of mutual coupling in array system. The RCS of an array is affected by coupling effects; mutual coupling changes the terminal impedance of the antenna elements and hence the reflection coefficients within the feed network. The coupling effect depends on the type of antenna element, array geometry, scan angle, and the nature of feed network. Figure 8 shows the schematic of series-fed dipole array, which can have different configurations, namely, collinear, parallel-in-echelon, and side-by-side configurations. Figures 9 through 11 show that the coupling effect is least with collinear configuration of series-fed dipole array as compared to side-by-side and parallel-in-echelon arrays. Beam scanning over the large angle has greater coupling effect on the RCS pattern, irrespective of its configuration.

Abdelaziz [111] tried to improve the performance of a microstrip patch antenna array by using an absorbing radar cover. The mutual coupling between the microstrip patch antennas is due to both space and surface waves. In particular, the surface wave contributes to the coupling and scattering [112, 113]. It is shown that the surface waves is reduced considerably by using a radar absorbing cover. The coupling factor between the elements is given by

$$C_{mn} = \frac{Y_{mn}}{Y_{nn} + Y_g}, \quad (24)$$

where Y_{mn} is the mutual admittance between mth and nth antenna, elements, Y_{nn} is the self admittance of nth antenna and Y_g is the generator or feed line admittance.

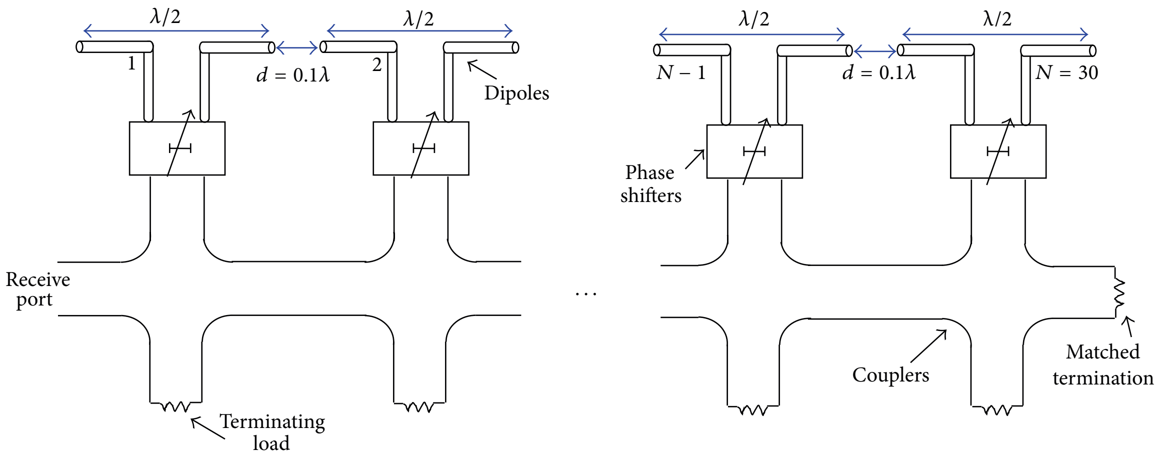


FIGURE 8: Schematic for 30-element series-fed dipole array.

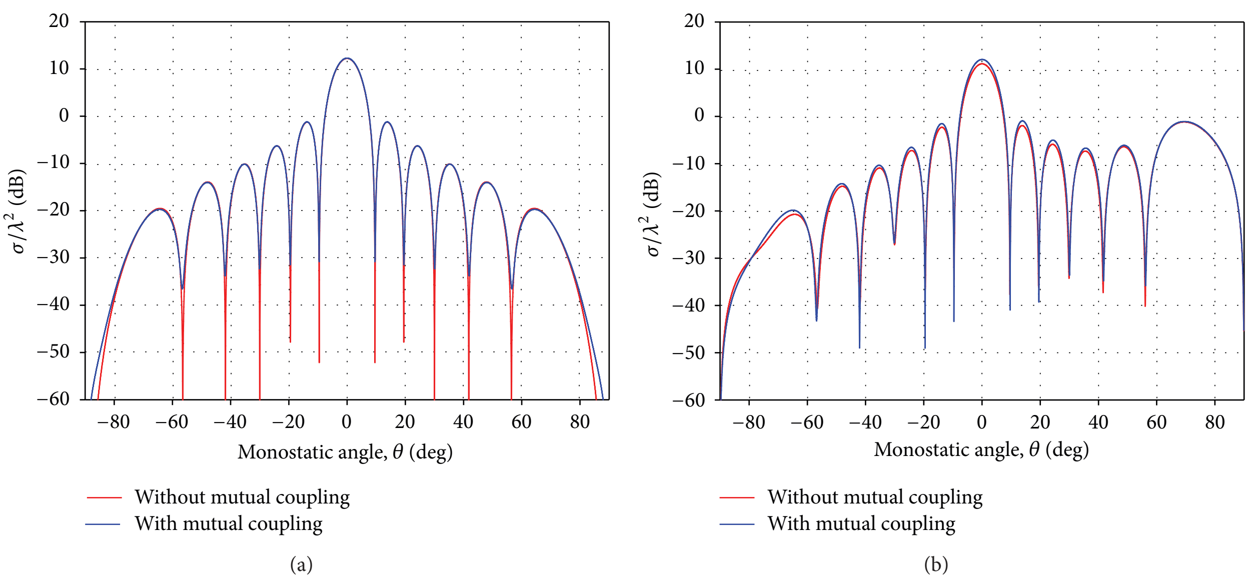


FIGURE 9: Effect of mutual coupling on RCS of series-fed linear collinear dipole array of $N = 30$, $\psi = \pi/2$, $d = 0.1\lambda$, $l = 0.5\lambda$, $a = 10^{-5}\lambda$, $Z_0 = 75\,\Omega$, and $Z_l = 150\,\Omega$; unit amplitude uniform distribution. (a) $\theta_s = 0^\circ$ (b) $\theta_s = 85^\circ$.

Knowing the coupling factor, actual excited voltages at the antenna terminals are obtained as

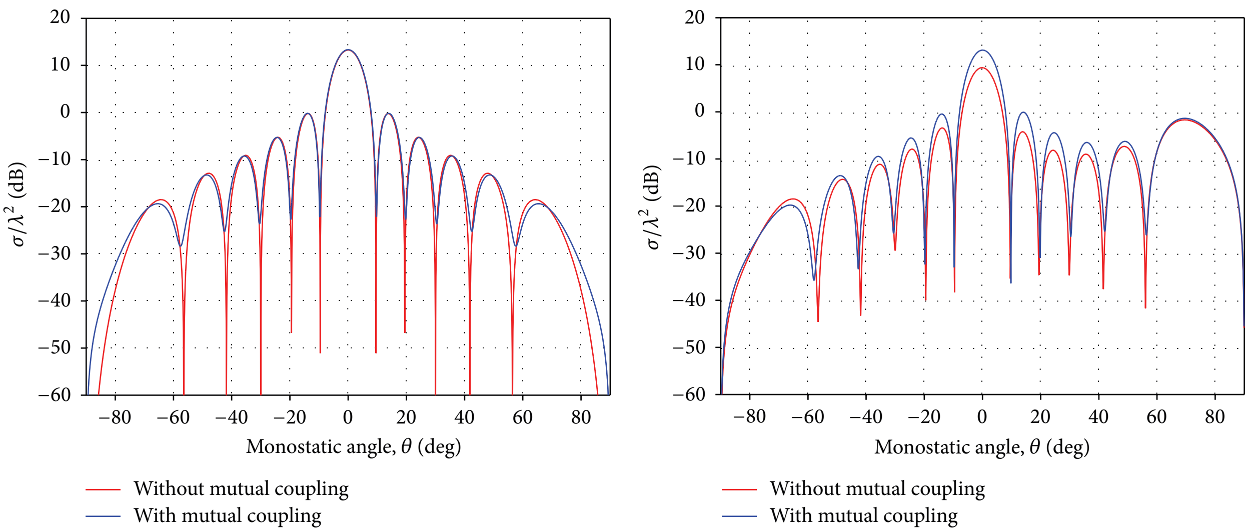
$$V_n = V_n^{\mathrm{app}} - \sum_{\substack{m=1 \\ m \neq n}}^{N} C_{mn} V_n^{\mathrm{app}}, \tag{25}$$

where V_n^{app} is the applied voltage and N is the number of elements in the array. The proposed method shows reduction in mutual coupling as well as RCS over a wide band of frequencies, without affecting the antenna parameters.

Zhang et al. [114, 115] presented both the radiation and scattering patterns of the linear dipole array in the presence of coupling. The radiation and scattered fields are determined in terms of self- and mutual impedance and terminal load impedance. The array performance is improved by optimizing the position of array elements so as to have a low sidelobe radiation and scattering pattern. It does not account for the effect of secondary scattering. The method can be used for planar arrays as well.

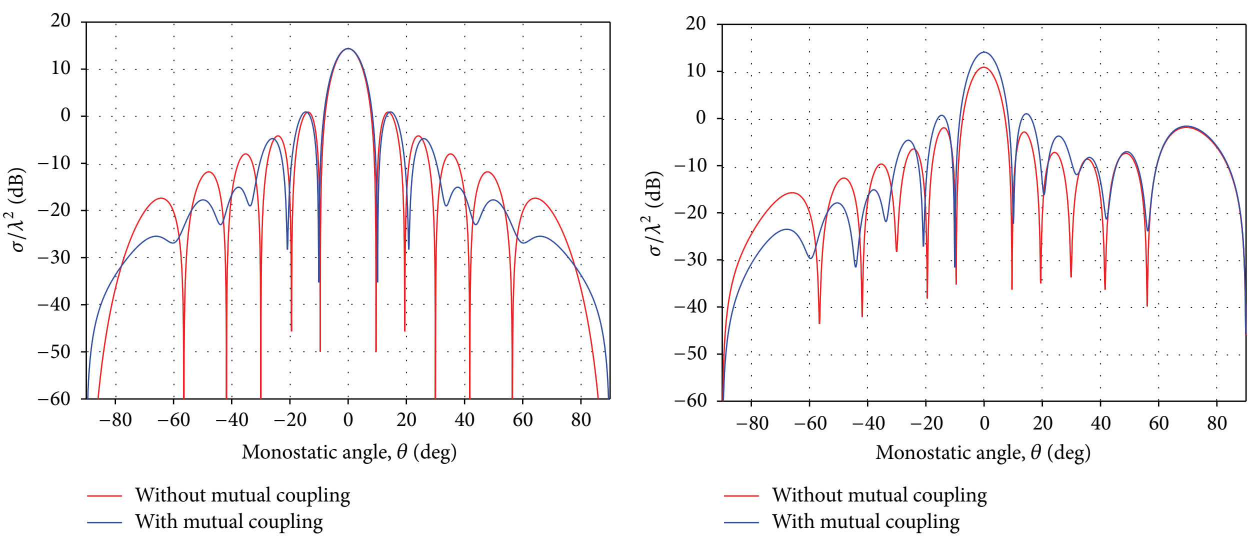
3. Optimization Techniques for Reduction in Coupling Effect

The array performance can be further improved by optimizing the array parameters. These optimization techniques can be either global [55, 71] or local in nature. The global optimization techniques optimize uniformly over all the directions. As a result, they will be inefficient in calibrating every direction properly. While the array suppresses the errors in pilot signal directions, the residual errors increase in other directions. On the other hand, the local optimization techniques are based on the application of array calibration for each and every direction separately.



(a) (b)

Figure 10: Effect of mutual coupling on RCS of series-fed linear parallel-in-echelon dipole array of $N = 30$, $\psi = \pi/2$, $d = 0.1\lambda$, $l = 0.5\lambda$, $a = 10^{-5}\lambda$, $Z_0 = 125\,\Omega$, and $Z_l = 235\,\Omega$; unit amplitude uniform distribution. (a) $\theta_s = 0^\circ$, (b) $\theta_s = 85^\circ$.



(a) (b)

Figure 11: Effect of mutual coupling on RCS of series-fed linear side-by-side dipole array of $N = 30$, $\psi = \pi/2$, $d = 0.1\lambda$, $l = 0.5\lambda$, $a = 10^{-5}\lambda$, $Z_0 = 150\,\Omega$, and $Z_l = 280\,\Omega$; unit amplitude uniform distribution. (a) $\theta_s = 0^\circ$ (b) $\theta_s = 85^\circ$.

3.1. Array Design. In the preceding sections, the effects of the coupling on the array performance and their compensation were discussed. It should be noted that the source of errors that hinder the array performance are due to improper antenna designs. In other words, a careful and efficient design of an antenna system can effectively minimize the mutual coupling between the array elements.

Lindmark et al. [116] proposed a design of a dual-polarized 12 × 12 planar array for a spatial division multiple access (SDMA) system. The performance ability in terms of DOA estimation was analyzed using least squares estimation of signal parameters via rotational invariance techniques (TLS-ESPRIT). The proposed array design showed better results than that of Pan and Wolff [117]. The improvement was due to the minimization of coupling effects in corrugated array design. However, cross polarization was more and the design was cost-inefficient [59].

Another design technique to mitigate the effects of coupling is to use dummy columns terminated with matched loads on each side of the array. This is effective as it pseudo equalizes the environment around the outer columns of the array to that at its inner columns. Although such an array design shows an improved performance [118], it is not cost-effective.

A wideband folded dipole array in the presence of mutual coupling was analyzed [119] using MoM and closed form of Green's function. The proposed method is valid only for thin substrates or the substrates with a relative permittivity and permeability close to unity. All the metallic surfaces are taken as perfect electric conductors (PEC). The array performance improves by using metallic walls to prevent inter-element coupling via parallel-plate waveguide modes.

In general, patch antenna is designed on a thick substrate for wideband performance and higher data rates. However, this enhances the coupling effect, as thicker substrate supports higher amount of current flow in the form of surface waves. Fredrick et al. [79] presented the coupling compensation for such smart antenna fed by a corporate feed network. The method is based on the fact that the reduction in flow of surface current on the adjacent elements reduces the coupling effect. The magnitude of surface currents on the antenna elements depends on the terminating load impedances of the elements. These terminal impedances are varied from a matched load condition to an open circuit condition using a switching PIN diode multiplexer. The coupling effect on the array performance is compensated by locating the switch at a proper location along the feed line of the element.

Blank and Hutt [120] presented an empirical optimization algorithm, in two versions. The method considered the effect of both mutual coupling and scattering between the array elements and nearby environment. The method is based on the measured or calculated element-pattern data and optimizes the design using an iterative technique. The first version of the proposed optimization algorithm is used if inter-element spacing is less than half-wavelength and the coupling effects do not vary rapidly as a function of element locations. Although this version considers the effect of passive elements in the vicinity of the array, it cannot optimize their locations. In the second version, induced EMF method is used to compute the admittance matrix of the array considering the effect of both active and passive elements. This helps to arrive at the active element scan impedances and is applicable to an array of arbitrary geometry. Moreover it can optimize the array in terms of both inter-element spacing and element excitations and has a higher rate of convergence.

Many attempts have been made to compensate the effect of coupling in microstrip antennas. The finite difference time domain (FDTD) method was used to analyze the array of electromagnetic band gap structures, composing printed antennas on a single isotropic dielectric substrate [121]. The analysis of mutual coupling between a two-element array of circular patch antennas on an isotropic dielectric substrate is presented by Chair et al. [122].

Yousefzadeh et al. [123] designed a linear array of uniformly fed microstrip patch antennas by iterating the fractal geometries. The performance of such an array surpasses that of an array with ordinary rectangular microstrip patches as both mutual coupling effects and the return loss at the input of each patch get reduced. The array performance is dependent on the fractal characteristics (type, size, and relative spacing), feed point location, and the number of parasitic elements. Buell et al. [124] proposed the mutual coupling compensation between the elements in a densely packed array by using metamaterial isolation walls. The proposed design enhances the array performance in controlled beam scanning and null steering. The mutual coupling in an array operating in receive mode differ from that of a transmitting array. A least square approach is used to compensate the coupling effect [125, 126]. The method is valid only when the number of emitting sources is greater than the number of elements in the receiving array.

Yang and Rahmat-Samii [121] proposed the technique of reducing the mutual coupling between two collinear, orthogonal or parallel planar inverted-F antennas (PIFAs) above a single ground plane with air substrate. It is based on the concept of suppressing the surface wave propagation by using mushroom-like electromagnetic band gap (EBG) structures. The fabrication of such structure is complicated. On the other hand, the slitted ground plane structure suggested by Chiu et al. [127] is simple, economical and can be fabricated easily. The slitted ground plane reduces the coupling between the radiators and thus improves the isolation between them. The method proposed is applicable for nonplanar radiating elements and large number of array elements. The size of slits and strips required to reduce mutual coupling differs based on the resonant frequency and the type of antennas in use. Other decoupling networks comprise of transmission line decouplers [128] and capacitively loaded loop (CLL) magnetic resonators [129].

Bait-Suwailam et al. [130] used single-negative magnetic (MNG) metamaterials to suppress the electromagnetic coupling between closely spaced high-profile monopoles. In this method, the single-negative magnetic inclusions realized using broadside coupled split-ring resonators (SRRs) act as antenna decouplers. These structures satisfy the property that their mutual impedance should be purely reactive at the resonance frequency in order to decouple the antenna elements. Arrays of such structures when properly arranged and excited with a specific polarization mimic the behavior of magnetic dipole arrays and have a negative magnetic permeability over a frequency range. This prevents the existence of real propagating modes within the MNG metamaterials, thus avoiding the coupling between the array elements. The analysis of the proposed method in terms of scattering parameters is shown to increase the performance of MIMO system by reducing the correlation between its array elements. Moreover, the proposed technique increases the system gain in the desired direction by reducing back radiation and thus helps in achieving quasiorthogonal patterns.

3.2. Other Optimization Techniques. Digital beamforming (DBF) of an array is preferred over analog beamforming, owing to low sidelobe beamforming, adaptive interference cancellation, high-resolution DOA estimation, and easy compensation for coupling and calibration errors [131, 132]. A technique of combined optimization method (COM), based on the concept of space equalization, was proposed [133]. This method is a combination of global and local optimisation and hence minimizes local errors in the desired signal directions while maintaining the global errors at a tolerable level. The performance analysis of COM calibrated uniform linear array shows better performance than the one calibrated using global optimization technique. Moreover, COM eliminates the coupling errors, channel (both gain and phase) errors over a broad angular region and compensates for errors due to both manufacture and nonuniform antenna material.

Demarcke et al. [134] presented a technique of accurate beamforming for a uniform circular microstrip patch antenna, subjected to mutual coupling and platform effects. This DOA-based method of beamforming using AEPs uses the received information to construct a transmitting weight vector. Basically, it concentrates the energy in the desired main beam direction(s), minimizing the total radiated power. This maximizes the gain in the specified main beam direction(s) and hence yields maximum SIR at the receiver. Here the beamforming is treated as a constrained minimization problem, with complex valued steering vector being its argument.

In general, beamforming is viewed as a constrained optimization problem. Thus the evolutionary algorithms and related swarm-based techniques, useful for solving unconstrained optimization problems, are not applicable readily for beamforming. The improvement can be achieved by optimizing the system parameters using algorithms like particle swarm optimization (PSO). Basu and Mahanti [135] used modified PSO for reconfiguring the beam of a linear dipole array, with or without ground plane. In the dual-beam switching technique, the self and mutual impedances of parallel half-wavelength dipoles are obtained using induced EMF method. Multiple beams are generated by switching through real excitation voltages, leading to a simple design of the feed network.

4. Conformal Array

The mutual coupling in conformal arrays is dependent on the curvature of surface on which antennas are mounted. A majority of techniques used to analyze the conformal arrays [136–138], attempt to reduce/avoid or simplify coupling between the antenna and platform on which it is mounted. This is not feasible in every scenario as it is not possible to isolate array from its platform or from the environment of near-field scatterers. Moreover, current minimization does not always assure an optimal solution, as the performance improves by reinforcing the currents induced due to coupling. Thus it is necessary to consider rather than neglect the effect of platform coupling.

Pathak and Wang [139] used uniform theory of diffraction (UTD) to compute coupling between the slots on conducting convex surface. This method relies on the surface geodesics obtained by ray tracing. The coupling between the antennas mounted on general paraboloid of revolution (GPOR) can also be determined using geometrical theory of diffraction (GTD). This ray-based method, however, requires the geometric parameters like arc length, Fock parameter, and so forth associated with all the geodesics to be known *a priori* [140]. This approach can be extended for quadric cylinders, ellipsoids, paraboloids, and for composite quadric surfaces like cone cylinder, parabolic cylinder, and so forth [141].

Wills [142] analyzed the effects of creeping fields and coupling on double curved conformal arrays of waveguide elements. The theoretical far-field patterns were compared with the measured results for an ellipsoidal array. Persson et al. [143] used a hybrid UTD-MoM method to calculate the mutual coupling between the circular apertures on a singly and doubly curved perfectly conducting surface [144]. The mutual coupling between the circular waveguide fed apertures on the curved surface is shown to be heavily dependent on the polarization. This hybrid method deals with isolated coupling values. The accuracy of the parameters involved in the UTD formulation relies on the geodesic constant and can be further improved by including higher order modes.

The radiation pattern of a dipole array mounted on a real complex conducting structure [145] was obtained by including coupling between elements, coupling due to near-field objects, and the coupling between the array and the surface and the feed network. The excitation coefficients of the impedance and radiation matrices were obtained using a MoM-based electromagnetic code NEC-2. An optimization procedure, based on the pattern synthesis algorithm [146], was employed to obtain radiation pattern with low sidelobe level.

Obelleiro et al. [147] analyzed the conformal monopole antenna array considering both the mutual coupling between the array elements and their interaction with the mounting platform. This method modeled the currents induced on the platform and the antennas using the surface-wire MoM formulation. A global-optimization procedure was used to arrive at the optimal excitation coefficients for the array elements. This approach is applicable for both PEC and dielectric platforms.

A method of finite element-boundary integral (FE-BI) was used to determine the mutual impedance between conformal cavity-backed patch antennas [148] mounted on PEC cylinder. The vector-edge-based elliptic-shell element basis functions were used to describe the field within the cavity region. The field external to the cylinder was represented by elliptic-cylinder dyadic Green's function. The method of weighted residuals was used to enforce the field continuity across the cavity aperture and obtain a matrix equation for the basis function amplitudes. The fields within the cavity region were obtained using bi-conjugate gradient method. The self- and mutual impedances of the patches were determined using induced EMF method. The H-plane coupling exists due to TE-surface wave and is inversely proportional to the surface curvature. The E-plane coupling is stronger than H-plane coupling due to TM surface-wave mode and is maximum for a specific curvature.

A finite array of microstrip patch antennas loaded with dielectric layers on a cylindrical structure was studied by Vegni and Toscano [149]. The mutual coupling coefficients were determined using method of lines (MoL) in terms of cylindrical coordinates. The effect of inter-element spacing and the superstrates on the coupling was analyzed. It was shown that the mutual coupling does not decay monotonically with the increasing patch separations. Instead, it exhibits oscillatory behaviour depending on the dielectric layer. The superstrates used can efficiently reduce the effect of mutual coupling in the antenna array. The proposed method can be readily extended to multiple, stacked layers and patches with superstrates with an acceptable increase in computational complexity, unlike MoM.

The effect of human coupling on the performance of textile antenna, like a log periodic folded dipole array (LPFDA) antenna, was analyzed [150]. The coupling due to human operator affects the antenna input impedance by shifting its resonant frequency and by increasing the return loss. These adverse effects can be compensated by designing an antenna in free space that exhibits a good impedance match over a bandwidth, say a linear array of wire folded dipoles.

The mutual coupling between the apertures of dielectric-covered PEC circular cylinders was determined in terms of tangential magnetic current sources of the waveguide-fed aperture antennas/arrays [151]. The method involves closed form of Green's function (CFGF) representations and two-level generalized pencil of function (GPOF). The method is valid for both electrically small and large cylinders over wider source-field regions. Yang et al. [152] synthesized the pattern of a vertically polarized rectangular microstrip patch conformal array by decomposing the embedded element patterns as a product of a characteristic matrix and a Vandermonde structured matrix. The weights of the modes were optimized using a modified PSO technique. The synthesized pattern showed low SLL copolarization beam with a mainlobe constraint. The method being simple in terms of both memory/storage requirement and computation, can be easily implemented for any antenna array, provided their embedded element patterns differ greatly.

The discussion presented so far shows that the conformal arrays can be analyzed using various techniques, each with their own merits and demerits. This indicates that a few of them when carefully chosen and combined [24, 153] might result in a hybrid technique with most of the merits. Sipus et al. [154] presented such a hybrid technique that combines spectral domain method with UTD to analyze a waveguide array embedded in a multilayer dielectric structure. It is robust method and capable of analyzing multilayer electrically large structures.

Raffaelli et al. [155] analyzed an array of arbitrarily rotated perfectly conducting rectangular patches embedded in a multilayered dielectric grounded circular cylinder. The cylinder was assumed to be of infinite length in the axial direction with infinitesimally thin patches. The method involves 2D Fourier transform to model fields and currents and MoM to solve the electric field integral equations (EFIE). The coupling effect was analyzed in terms of frequency and edge spacing between the array elements. The embedded element pattern varies more in E-plane than in H-plane due to the stronger mutual coupling along E-plane. This method can deal with the structures with larger radii, unlike the spectral approach.

A conformal conical slot array was optimally synthesized taking coupling effect into account [156]. The synthesis aimed at maximizing the directivity with minimum SLL and nulling the interfering signal considering least number of active element patterns. This constrained optimization problem can be readily extended to quadratic constraints, other than just linear ones, such as the constraints on main beam radiation efficiency and power in the cross-polarization component. The effect of coupling is compensated using the technique of Steyskal and Herd [55].

Thors et al. [157] presented the scattering properties of dielectric coated waveguide aperture antennas mounted on circular cylinders. The proposed hybrid technique is a combination of MoM and asymptotic techniques. It relies on MoM to solve the integral equation for the aperture fields and on asymptotic techniques to compute the coupling. The mutual coupling between the rectangular patch antennas in nonparaxial region was computed using the asymptotic solution [143, 158]. The self- and the mutual coupling terms in the paraxial region were determined using spectral domain technique. This constrains the ability of the technique to consider smaller radii cylinders.

5. Advantages of Mutual Coupling

The performance of the phased array deteriorates due to the presence of coupling; however, this cannot be generalized. The presence of coupling is reported to be advantageous in certain cases. The coupling has positive effect on the channel capacity of multiple element antenna (MEA) systems [159]. The channel capacity in terms of coupling matrices of receiver and transmitter is expressed as

$$H' = C_R H C_T, \tag{26}$$

where H' and H are the channel capacity with and without mutual coupling, C_R, and C_T are the coupling matrices at the receiver and transmitter, respectively. The coupling effect reduces the correlation between the channel coefficients for closely spaced MIMO systems. It also compensates the propagation phase difference of the frequencies lying within the range [93]. This increases the channel capacity, which is contradictory to the statements [160]. Mutual coupling is also reported to increase the efficiency by decreasing the bit-error rate of a Nakagami fading channel. Nonradiative coupling between high-frequency circuits within their near-field zone is found to aid in the efficient transfer of power in wireless applications. This kind of biofriendly power transfer is free from electromagnetic interference (EMI) caused by scattering and atmospheric absorption, unlike radiative EM energy transfer [161].

The presence of coupling between the array elements is shown to be desirable, if arrays are to be made self-calibrating [162]. Furthermore, Yuan et al. [109] showed that the presence of coupling improves the convergence of adaptive algorithm. The rate of convergence of an algorithm is shown to decrease if the eigenvalues of the array covariance matrix differ widely. On the other hand, the convergence becomes faster when the ratio of the maximum eigenvalue to the minimum eigenvalue of the covariance matrix decreases. This is due to the fact that closer array elements, and hence high coupling, yield a smaller maximum eigenvalue with approximately the same minimum eigenvalue. This indicates an improvement in the eigenvalue behavior for an array in the presence of coupling even when coherent sources are considered [163]. However, in most of the cases, the compensation of the mutual coupling effects is required to obtain optimal array performance.

6. Summary

This paper presents an overview of the methods that model mutual coupling effect in terms of impedance matrix for different arrays. Researchers have extended the conventional methods based on self- and mutual impedance matrix to include the effects of calibration errors and near field scatterers. These methods aimed at compensating the effects of mutual coupling by including the coupling matrix in the pattern generation. There are autocalibration methods which mitigate the effects of structural scattering along with the mutual coupling mitigation; such methods facilitate the analysis of conformal phased arrays.

The trend moved towards developing the techniques, which estimate the parameters affecting the performance of real-scenario arrays accurately in extreme conditions including that for coherent signals with minimum number of inputs. This reduced the computational complexity and facilitated easy experimental verification and parametric analysis. It has been shown that the accurate array pattern synthesis is feasible only if the effect of coupling on the array manifold is considered. Thus the USV is used instead of CSV in the coupling analysis of phased arrays mounted over the platform, such as aircraft wings or mobile phones.

Further hybrid techniques, like UTD-MoM, spectral domain method with UTD, were shown to be better in terms of both accuracy and computation. Most of these techniques are suitable only for uniform and infinitely large arrays in narrowband scenarios. Therefore, adaptive array based technique was developed to deal with the small non-uniform planar and circular arrays operating over a wide frequency range.

The effect of mutual coupling on the parameters like terminal impedances and eigenvalues has been discussed considering the radiation pattern, steady state response, transient response, and the RCS of the array. Many compensation techniques are analyzed to mitigate the adverse effects of coupling on array performance issues such as high resolution, DOA estimation, and interference suppression. These have been further simplified due to the optimization of antenna design parameters. It is suggested that a good and efficient design of an antenna system would compensate for the mutual coupling effects. However, in few cases the presence of coupling has been proved advantageous as well.

References

[1] Y. Leviatan, A. T. Adams, P. H. Stockmann, and D. R. Miedaner, "Cancellation performance degradation of a fully adaptive Yagi array due to inner-element coupling," *Electronics Letters*, vol. 19, no. 5, pp. 176–177, 1983.

[2] I. J. Gupta and A. A. Ksienski, "Effect of mutual coupling on the performance of adaptive arrays," *IEEE Transactions on Antennas and Propagation*, vol. 31, no. 5, pp. 785–791, 1983.

[3] Y. Zhang, K. Hirasawa, and K. Fujimoto, "Performance of power inversion adaptive array with the effect of mutual coupling," in *Proceedings of the International Symposium on Antennas and Propagation*, pp. 803–806, Kyoto, Japan, August 1985.

[4] P. N. Fletcher and P. Darwood, "Beamforming for circular and semicircular array antennas for low cost wireless LAN data communication systems," *IEE Proceedings on Microwave Antennas and Propagation*, vol. 145, pp. 153–158, 1998.

[5] M. M. Dawoud and M. K. Amjad, "Analytical solution for mutual coupling in microstrip patch antenna arrays," *Arabian Journal for Science and Engineering*, vol. 31, no. 1, pp. 47–60, 2006.

[6] K. M. Pasala and E. M. Friel, "Mutual coupling effects and their reduction in wideband direction of arrival estimation," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 30, no. 4, pp. 1116–1122, 1994.

[7] J. L. Guo and J. Y. Li, "Pattern synthesis of conformal array antenna in the presence of platform using differential evolution algorithm," *IEEE Transactions on Antennas and Propagation*, vol. 57, no. 9, pp. 2615–2621, 2009.

[8] H. T. Hui, K. Y. Chan, and E. K. N. Yung, "Compensating for the mutual coupling effect in a normal-mode helical antenna array for adaptive nulling," *IEEE Transactions on Vehicular Technology*, vol. 52, no. 4, pp. 743–751, 2003.

[9] B. Strait and K. Hirasawa, "Array design for a specified pattern by matrix methods," *IEEE Transactions on Antennas and Propagation*, vol. 17, pp. 237–239, 1969.

[10] C. A. Balanis, *Antenna Theory, Analysis and Design*, John Wiley & Sons, Hoboken, NJ, USA, 3rd edition, 2005.

[11] B. Friedlander and A. J. Weiss, "Direction finding in the presence of mutual coupling," *IEEE Transactions on Antennas and Propagation*, vol. 39, pp. 273–284, 1991.

[12] G. A. Thiele, "Analysis of Yagi-Uda type antennas," *IEEE Transactions on Antennas and Propagation*, vol. 17, pp. 24–31, 1969.

[13] P. S. Carter, "'Circuit relations in radiating systems and applications to antenna problems," *Proceedings of IRE*, vol. 20, pp. 1004–1041, 1932.

[14] H. E. King, "Mutual impedance of unequal length antennas in echelon," *IRE Transactions on Antennas Propagation*, vol. 5, pp. 306–313, 1957.

[15] R. C. Hansen, "Formulation of echelon dipole mutual impedance for computer," *IEEE Transactions on Antennas and Propagation*, vol. 20, pp. 780–781, 1972.

[16] J. A. G. Malherbe, "Calculator program for mutual impedance," *Microwave Journal*, vol. 27, no. 2, p. 82, 1984.

[17] J. A. G. Malherbe, "Analysis of a linear antenna array including the effects of mutual coupling," *IEEE Transactions on Education*, vol. 32, no. 1, pp. 29–34, 1989.

[18] M. J. Ehrlich and J. Short, "Mutual coupling considerations in linear-slot array design," *Proceedings of the IRE*, vol. 42, pp. 956–961, 1954.

[19] R. P. Jedlicka, M. T. Poe, and K. R. Carver, "Measured mutual coupling between microstrip antennas," *IEEE Transactions on Antennas and Propagation*, vol. 29, no. 1, pp. 147–149, 1981.

[20] N. G. Alexopoulos and I. E. Rana, "Mutual impedance computation between printed dipoles," *IEEE Transactions on Antennas and Propagation*, vol. 29, no. 1, pp. 106–111, 1981.

[21] K. Inami, K. Sawaya, and Y. Mushiake, "Mutual coupling between rectangular slot antennas on a conducting concave spherical surface," *IEEE Transactions on Antennas and Propagation*, vol. 30, no. 5, pp. 927–933, 1982.

[22] G. V. Eleftheriades and G. M. Rebeiz, "Self and mutual admittance of slot antennas on a dielectric half-space," *International Journal of Infrared and Millimeter Waves*, vol. 14, no. 10, pp. 1925–1946, 1993.

[23] B. G. Porter and S. S. Gearhart, "Theoretical analysis of coupling and cross polarization of perpendicular slot antennas

on a dielectric half-space," *IEEE Transactions on Antennas and Propagation*, vol. 46, no. 3, pp. 383–390, 1998.

[24] D. M. Pozar, "Input impedance and mutual coupling of rectangular microstrip antennas," *IEEE Transactions on Antennas and Propagation*, vol. 30, no. 6, pp. 1191–1200, 1982.

[25] E. H. Newman, J. H. Richmond, and B. W. Kwan, "Mutual impedance computation between microstrip antennas," *IEEE Transactions on Microwave Theory and Techniques*, vol. 31, no. 11, pp. 941–945, 1983.

[26] E. H. Van Lil and A. R. van de Capelle, "Transmission line model for mutual coupling between microstrip antennas," *IEEE Transactions on Antennas and Propagation*, vol. 32, no. 8, pp. 816–821, 1984.

[27] V. Hansen and M. Patzold, "Input impedance and mutual coupling of rectangular microstrip patch antennas with a dielectric cover," in *Proceedings of the 16th European Microwave Conference*, pp. 643–648, Dublin, Ireland, October 1986.

[28] P. B. Katehi, "Mutual coupling between microstrip dipoles in multielement arrays.," *IEEE Transactions on Antennas and Propagation*, vol. 37, no. 3, pp. 275–280, 1989.

[29] A. H. Mohammadian, N. M. Martin, and D. W. Griffin, "Theoretical and experimental study of mutual coupling in microstrip antenna arrays," *IEEE Transactions on Antennas and Propagation*, vol. 37, no. 10, pp. 1217–1223, 1989.

[30] A. Benalla, K. C. Gupta, and R. Chew, "Mutual coupling between rectangular microstrip patches covered with a dielectric cover layer," in *Proceedings of the Antennas and Propagation Society International Symposium, Merging Technologies for the 90's' Digest*, pp. 358–361, May 1990.

[31] C. Terret, S. Assailly, K. Mahdjoubi, and M. Edimo, "Mutual coupling between stacked square microstrip antennas fed on their diagonal," *IEEE Transactions on Antennas and Propagation*, vol. 39, no. 7, pp. 1049–1051, 1991.

[32] P. J. Papakanellos and C. N. Capsalis, "Study of two arbitrarily located parallel cylindrical dipoles based on an auxiliary sources technique," *Electromagnetics*, vol. 23, no. 5, pp. 399–416, 2003.

[33] M. Tayarani and Y. Kami, "Fuzzy inference in engineering electromagnetics: an application to conventional and angled monopole-antenna," *IEICE Transactions on Electronics*, vol. E83, no. 1, pp. 85–95, 2000.

[34] S. R. Ostadzadeh, M. Soleimani, and M. Tayarani, "A fuzzy model for computing input impedance of two coupled dipole antennas in the echelon form," *Progress in Electromagnetics Research*, vol. 78, pp. 265–283, 2008.

[35] K. C. Lee and T. H. Chu, "Mutual coupling mechanisms within arrays of nonlinear antennas," *IEEE Transactions on Electromagnetic Compatibility*, vol. 47, no. 4, pp. 963–970, 2005.

[36] D. Westreich, "Evaluating the matrix polynomial $\mathrm{I}+\mathrm{A}+\mathrm{A}^2+\ldots+\mathrm{A}^{N-1}$," *IEEE Transactions on Circuits and Systems*, vol. 36, no. 1, pp. 162–164, 1989.

[37] K. C. Lee and T. H. Chu, "A circuit model for mutual coupling analysis of a finite antenna array," *IEEE Transactions on Electromagnetic Compatibility*, vol. 38, no. 3, pp. 483–489, 1996.

[38] S. A. Maas, *Nonlinear Microwave Circuits*, Artech House, Norwood, Mass, USA, 1988.

[39] K. C. Lee and T. H. Chu, "Analysis of injection-locked antenna array including mutual coupling effects," *IEEE Transactions on Antennas and Propagation*, vol. 52, no. 11, pp. 2885–2890, 2004.

[40] Y. Inoue, K. Mori, and H. Arai, "DOA estimation in consideration of the array element pattern," in *Proceedings of the IEEE 55th Vehicular Technology Conference (VTC '02)*, vol. 2, pp. 745–748, May 2002.

[41] K. R. Dandekar, H. Ling, and G. Xu, "Effect of mutual coupling on direction finding in smart antenna applications," *Electronics Letters*, vol. 36, no. 22, pp. 1889–1891, 2000.

[42] Q. Yuan, Q. Chen, and K. Sawaya, "Accurate DOA estimation using array antenna with arbitrary geometry," *IEEE Transactions on Antennas and Propagation*, vol. 53, no. 4, pp. 1352–1357, 2005.

[43] M. Wax and J. Sheinvald, "Direction finding of coherent signals via spatial smoothing for uniform circular arrays," *IEEE Transactions on Antennas and Propagation*, vol. 42, no. 5, pp. 613–620, 1994.

[44] Y. Kuwahara, "Steering vector of MSPA array taking mutual coupling effects into consideration," in *Proceedings of the International Symposium on Antennas and Propagation*, pp. 1614–1617, ACROS Fukuoka, Japan, August 2000.

[45] A. Kato and Y. Kuwahara, "Steering vector taking mutual coupling effects into consideration," in *Proceedings of the IEEE Antennas and Propagation Society International Symposium*, vol. 2, pp. 918–921, Salt Lake City, Utah, USA, July 2000.

[46] Y. Zhang, K. Hirasawa, and K. Fujimoto, "Analysis of power inversion adaptive array performance by moment method," *Transactions of IEICE*, vol. 71, pp. 600–606, 1988.

[47] B. Liao and S. C. Chan, "Adaptive beamforming for uniform linear arrays with unknown mutual coupling," *IEEE Antennas and Wireless Propagation Letters*, vol. 11, pp. 464–467, 2012.

[48] Z. Ye and C. Liu, "Non-sensitive adaptive beamforming against mutual coupling," *IET Signal Processing*, vol. 3, no. 1, pp. 1–6, 2009.

[49] D. F. Kelley and W. L. Stutzman, "Array antenna pattern modeling methods that include mutual coupling effects," *IEEE Transactions on Antennas and Propagation*, vol. 41, no. 12, pp. 1625–1632, 1993.

[50] R. S. Elliot, *Antenna Theory and Design*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1st edition, 1981.

[51] Y. W. Kang and D. M. Pozar, "Correction of error in reduced sidelobe synthesis due to mutual coupling," *IEEE Transactions on Antennas and Propagation*, vol. 33, no. 9, pp. 1025–1028, 1985.

[52] W. Wasylkiwskyj and W. K. Kahn, "Element patterns and active reflection coefficient in uniform phased arrays," *IEEE Transactions on Antennas and Propagation*, vol. 22, no. 2, pp. 207–212, 1974.

[53] P. Darwood, P. M. Fletcher, and G. S. Hilton, "Pattern synthesis in small phased arrays using adaptive array theory," *Electronics Letters*, vol. 33, no. 4, pp. 254–255, 1997.

[54] E. C. Dufort, "Pattern synthesis based on adaptive array theory," *IEEE Transactions on Antennas and Propagation*, vol. 37, no. 8, pp. 1011–1018, 1989.

[55] H. Steyskal and J. S. Herd, "Mutual coupling compensation in small array antennas," *IEEE Transactions on Antennas and Propagation*, vol. 38, no. 12, pp. 1971–1975, 1990.

[56] S. Karimkashi and A. A. Kishk, "Focused microstrip array antenna using a Dolph-Chebyshev near-field design," *IEEE Transactions on Antennas and Propagation*, vol. 57, no. 12, pp. 3813–3820, 2009.

[57] A. A. Kishk and S. L. Ramaraju, "Design of different configurations of finite microstrip arrays of circular patches," *International Journal of Microwave and Millimeter-Wave Computer-Aided Engineering*, vol. 4, no. 1, pp. 6–17, 1994.

[58] A. A. Kishk, "Prediction of large array characteristics from small array parameters," in *Proceedings of the 2nd European Conference on Antennas and Propagation (EuCAP '07)*, pp. 584–588, Edinburgh, UK, November 2007.

[59] B. Lindmark, "Comparison of mutual coupling compensation to dummy columns in adaptive antenna systems," *IEEE Transactions on Antennas and Propagation*, vol. 53, no. 4, pp. 1332–1336, 2005.

[60] P. Darwood, P. N. Fletcher, and G. S. Hilton, "Mutual coupling compensation in small planar array antennas," *IEE Proceedings on Microwave Antennas and Propagation*, vol. 145, pp. 1–6, 1998.

[61] P. N. Fletcher and M. Dean, "Application of retrodirective beams to achieve low sidelobe levels in small phased arrays," *Electronics Letters*, vol. 32, no. 6, pp. 506–508, 1996.

[62] T. Su and H. Ling, "On modeling mutual coupling in antenna arrays using the coupling matrix," *Microwave Optical Technologies Letters*, vol. 28, pp. 231–237, 2001.

[63] C. C. Yeh, M. L. Leou, and D. R. Ucci, "Bearing estimations with mutual coupling present," *IEEE Transactions on Antennas and Propagation*, vol. 37, no. 10, pp. 1332–1335, 1989.

[64] C. M. S. See, "A method for array calibration in parametric sensor array processing," in *Proceedings of the IEEE Singapore International Conference on Communications Systems*, pp. 915–919, Singapore, November 1994.

[65] A. N. Lemma, E. F. Deprettere, and A. J. Veen, "Experimental analysis of antenna coupling for high-resolution DOA estimation algorithms," in *Proceedings of the IEEE Workshop on Signal Processing Advances in Wireless Communications*, pp. 362–365, Annapolis, Md, USA, May 1999.

[66] I. J. Gupta, J. R. Baxter, S. W. Ellingson, H. G. Park, H. S. Oh, and M. G. Kyeong, "An experimental study of antenna array calibration," *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 3, pp. 664–667, 2003.

[67] A. J. Weiss and B. Friedlander, "Array shape calibration using sources in unknown locations—a maximum likelihood approach," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 37, no. 12, pp. 1958–1966, 1989.

[68] J. Pierre and M. Kaveh, "Experimental performance of calibration and direction-finding algorithms," in *Proceedings of the International Conference on Acoustics, Speech, and Signal Processing (ICASSP '91)*, pp. 1365–1368, Toronto, Canada, May 1991.

[69] C. M. S. See, "Sensor array calibration in the presence of mutual coupling and unknown sensor gains and phases," *Electronics Letters*, vol. 30, no. 5, pp. 373–374, 1994.

[70] E. K. L. Hung, "Computation of the coupling matrix among the elements of an array antenna," in *Proceedings of the International Conference on Radar (Radar '94)*, pp. 703–706, Paris, France, May 1994.

[71] E. K. L. Hung, "Matrix-construction calibration method for antenna arrays," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 36, no. 3, pp. 819–828, 2000.

[72] K. C. Lee, "Two efficient algorithms for the analyses of a nonlinearly loaded antenna and antenna array in the frequency domain," *IEEE Transactions on Electromagnetic Compatibility*, vol. 42, no. 4, pp. 339–346, 2000.

[73] A. A. Oliner and R. G. Malech, "Mutual coupling in infinite scanning arrays," in *Microwave Scanning Antennas*, R. C. Hansen, Ed., chapter 3, pp. 195–335, Academic Press, New York, NY, USA, 1966.

[74] D. M. Pozar and D. H. Schaubert, "Scan blindness in infinite phased arrays of printed dipoles," *IEEE Transactions on Antennas and Propagation*, vol. 32, no. 6, pp. 602–610, 1984.

[75] L. Stark, "Radiation impedance of a dipole in an infinite planar phased array," *Radio Science*, vol. 1, pp. 361–377, 1966.

[76] K. M. Lee and R. S. Chu, "Analysis of mutual coupling between a finite phased array of dipoles and its feed network," *IEEE Transactions on Antennas and Propagation*, vol. 36, no. 12, pp. 1681–1699, 1988.

[77] D. M. Pozar, "Analysis of finite phased arrays of printed dipoles," *IEEE Transactions on Antennas and Propagation*, vol. 33, no. 10, pp. 1045–1053, 1985.

[78] F. Najib, R. Mohamad, and V. Patrick, "Effects of mutual coupling in phased arrays with butler network feed," in *Proceedings of the 17th International Conference on Applied Electromagnetics and Communications*, pp. 374–379, Dubrovnik, Croatia, October 2003.

[79] J. D. Fredrick, Y. Wang, and T. Itoh, "Smart antennas based on Spatial Multiplexing of Local Elements (SMILE) for mutual coupling reduction," *IEEE Transactions on Antennas and Propagation*, vol. 52, no. 1, pp. 106–114, 2004.

[80] A. Manikas and N. Fistas, "Modelling and estimation of mutual coupling between array elements," in *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP '94)*, vol. 4, pp. 553–556, April 1994.

[81] R. A. Monzingo and T. W. Miller, *Introduction to Adaptive Arrays*, John Wiley & Sons, 1980.

[82] B. Friedlander, "A sensitivity analysis of the MUSIC algorithm," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 38, no. 10, pp. 1740–1751, 1990.

[83] R. L. Riegler and R. T. Compton Jr., "An adaptive array for interference rejection," *Proceedings of the IEEE*, vol. 61, no. 6, pp. 748–758, 1973.

[84] R. S. Adve and T. K. Sarkar, "Elimination of the effects of mutual coupling in an adaptive nulling system with a look direction constraint," in *Proceedings of the IEEE Antennas and Propagation Society International Symposium*, vol. 2, pp. 1164–1167, July 1996.

[85] R. S. Adve and T. K. Sarkar, "Compensation for the effects of mutual coupling on direct data domain adaptive algorithms," *IEEE Transactions on Antennas and Propagation*, vol. 48, no. 1, pp. 86–94, 2000.

[86] H. T. Hui, "Reducing the mutual coupling effect in adaptive nulling using a re-defined mutual impedance," *IEEE Microwave and Wireless Components Letters*, vol. 12, no. 5, pp. 178–180, 2002.

[87] H. T. Hui, H. P. Low, T. T. Zhang, and Y. L. Lu, "Antenna Designer's Notebook: receiving mutual impedance between two normal-mode helical antennas (NMHAs)," *IEEE Antennas and Propagation Magazine*, vol. 48, no. 4, pp. 92–96, 2006.

[88] H. T. Hui, "A practical approach to compensate for the mutual coupling effect in an adaptive dipole array," *IEEE Transactions on Antennas and Propagation*, vol. 52, no. 5, pp. 1262–1269, 2004.

[89] T. T. Zhang, Y. L. Lu, and H. T. Hui, "Compensation for the mutual coupling effect in uniform circular arrays for 2D DOA estimations employing the maximum likelihood technique," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 44, no. 3, pp. 1215–1221, 2008.

[90] P. Stoica and A. B. Gershman, "Maximum-likelihood DOA estimation by data-supported grid search," *IEEE Signal Processing Letters*, vol. 6, no. 10, pp. 273–275, 1999.

[91] T. Zhang and W. Ser, "Robust beampattern synthesis for antenna arrays with mutual coupling effect," *IEEE Transactions on Antennas and Propagation*, vol. 59, pp. 2889–2895, 2011.

[92] Y. Zhang, K. Hirasawa, and K. Fujimoto, "Signal bandwidth consideration of mutual coupling effects on adaptive array performance," *IEEE Transactions on Antennas and Propagation*, vol. 35, no. 3, pp. 337–339, 1987.

[93] S. Durrani and M. E. Bialkowski, "Effect of mutual coupling on the interference rejection capabilities of linear and circular arrays in CDMA systems," *IEEE Transactions on Antennas and Propagation*, vol. 52, no. 4, pp. 1130–1134, 2004.

[94] H. Krim and M. Viberg, "Two decades of array signal processing research: the parametric approach," *IEEE Signal Processing Magazine*, vol. 13, no. 4, pp. 67–94, 1996.

[95] C. Roller and W. Wasylkiwskyj, "Effects of mutual coupling on super-resolution DF in linear arrays," in *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing*, vol. 5, pp. 257–260, 1992.

[96] J. Litva and M. Zeytinoglu, "Application of high-resolution direction finding algorithms to circular arrays with mutual coupling present," Final Report, Part II, Defence Research Establishment, Ottawa, Canada, 1990.

[97] B. Himed and D. D. Weiner, "Compensation for mutual coupling effects in direction finding," in *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP '90)*, vol. 5, pp. 2631–2634, Albuquerque, NM, USA, April 1990.

[98] A. L. Swindlehurst and T. Kailath, "A performance analysis of subspace-based methods in the presence of model errors—I: the MUSIC algorithm," *IEEE Transactions on Signal Processing*, vol. 40, no. 7, pp. 1758–1774, 1992.

[99] A. L. Swindlehurst and T. Kailath, "Performance analysis of subspace-base methods in the presence of model errors: part II—multidimensional algorithms," *IEEE Transactions on Signal Processing*, vol. 41, pp. 1277–1308, 1993.

[100] T. Su, K. Dandekar, and H. Ling, "Simulation of mutual coupling effect in circular arrays for direction-finding applications," *Microwave and Optical Technologies Letters*, vol. 26, pp. 331–336, 2000.

[101] K. R. Dandekar, H. Ling, and G. Xu, "Experimental study of mutual coupling compensation in smart antenna applications," *IEEE Transactions on Wireless Communications*, vol. 1, no. 3, pp. 480–487, 2002.

[102] B. Friedlander, "The root-MUSIC algorithm for direction finding with interpolated arrays," *Signal Processing*, vol. 30, no. 1, pp. 15–29, 1993.

[103] K. Kim, T. K. Sarkar, and M. S. Palma, "Adaptive processing using a single snapshot for a nonuniformly spaced array in the presence of mutual coupling and near-field scatterers," *IEEE Transactions on Antennas and Propagation*, vol. 50, no. 5, pp. 582–590, 2002.

[104] H. T. Hui, "Improved compensation for the mutual coupling effect in a dipole array for direction finding," *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 9, pp. 2498–2503, 2003.

[105] Y. Yu and H. T. Hui, "Design of a mutual coupling compensation network for a small receiving monopole array," *IEEE Transactions on Microwave Theory and Techniques*, vol. 59, pp. 2241–2245, 2011.

[106] J. C. Coetzee and Y. Yu, "Port decoupling for small arrays by means of an eigenmode feed network," *IEEE Transactions on Antennas and Propagation*, vol. 56, no. 6, pp. 1587–1593, 2008.

[107] P. T. Chua and J. C. Coetzee, "Microstrip decoupling networks for low-order multiport arrays with reduced element spacing," *Microwave and Optical Technology Letters*, vol. 46, no. 6, pp. 592–597, 2005.

[108] I. J. Gupta and A. A. Ksienski, "Prediction of adaptive array performance," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 19, no. 3, pp. 380–388, 1983.

[109] Q. Yuan, Q. Chen, and K. Sawaya, "Performance of adaptive array antenna with arbitrary geometry in the presence of mutual coupling," *IEEE Transactions on Antennas and Propagation*, vol. 54, no. 7, pp. 1991–1996, 2006.

[110] R. J. Dinger, "Reactively steered adaptive array using microstrip patch elements at 4 MHz," *IEEE Transactions on Antennas and Propagation*, vol. 32, no. 8, pp. 848–856, 1984.

[111] A. A. Abdelaziz, "Improving the performance of an antenna array by using radar absorbing cover," *Progress In Electromagnetics Research Letters*, vol. 1, pp. 129–138, 2008.

[112] G. Dubost, "Influence of surface wave upon efficiency and mutual coupling between rectangular microstrip antennas," in *Proceedings of the IEEE International Symposium Digest on Antennas and Propagation*, vol. 2, pp. 660–663, Dallas, Tex, USA, May 1990.

[113] G. E. Antilla and N. G. Alexopoulos, "Surface wave and related effects on the RCS of microstrip dipoles printed on magnetodielectric substrates," *IEEE Transactions on Antennas and Propagation*, vol. 39, no. 12, pp. 1707–1715, 1991.

[114] S. Zhang, S. X. Gong, Y. Guan, J. Ling, and B. Lu, "A new approach for synthesizing both the radiation and scattering patterns of linear dipole antenna array," *Journal of Electromagnetic Waves and Applications*, vol. 24, no. 7, pp. 861–870, 2010.

[115] S. Zhang, S. X. Gong, Y. Guan, and B. Lu, "Optimized element positions for prescribed radar cross section pattern of linear dipole arrays," *International Journal of RF and Microwave Computer-Aided Engineering*, vol. 21, pp. 622–628, 2011.

[116] B. Lindmark, S. Lundgren, J. R. Sanford, and C. Beckman, "Dual-polarized array for signal-processing applications in wireless communications," *IEEE Transactions on Antennas and Propagation*, vol. 46, no. 6, pp. 758–763, 1998.

[117] S. G. Pan and I. Wolff, "Computation of mutual coupling between slot-coupled microstrip patches in a finite array," *IEEE Transactions on Antennas and Propagation*, vol. 40, no. 9, pp. 1047–1053, 1992.

[118] S. Lundgren, "Study of mutual coupling effects on the direction finding performance of ESPRIT with a linear microstrip patch array using the method of moments," in *Proceedings of the IEEE Antennas and Propagation Society International Symposium Digest*, pp. 1372–1375, Baltimore, Md, USA, July 1996.

[119] M. C. V. Beurden, A. B. Smolders, and M. E. J. Jeuken, "Design of wide-band phased array antennas," in *Perspectives on Radio Astronomy: Technologies for Large Antenna Arrays*, Proceedings of the Conference, pp. 64–70, ASTRON Institute, Dwingeloo, The Netherlands, 1999.

[120] S. J. Blank and M. F. Hutt, "On the empirical optimization of antenna arrays," *IEEE Antennas and Propagation Magazine*, vol. 47, no. 2, pp. 58–67, 2005.

[121] F. Yang and Y. Rahmat-Samii, "Microstrip antennas integrated with electromagnetic band-gap (EBG) structures: a low mutual coupling design for array applications," *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 10, pp. 2936–2946, 2003.

[122] R. Chair, A. A. Kishk, and K. F. Lee, "Comparative study on the mutual coupling between different sized cylindrical dielectric resonators antennas and circular microstrip patch antennas," *IEEE Transactions on Antennas and Propagation*, vol. 53, no. 3, pp. 1011–1019, 2005.

[123] N. Yousefzadeh, C. Ghobadi, and M. Kamyab, "Consideration of Mutual Coupling in a microstrip patch array using fractal elements," *Progress in Electromagnetics Research*, vol. 66, pp. 41–49, 2006.

[124] K. Buell, H. Mosallaei, and K. Sarabandi, "Metamaterial insulator enabled superdirective array," *IEEE Transactions on Antennas and Propagation*, vol. 55, no. 4, pp. 1074–1085, 2007.

[125] H. S. Lui and H. T. Hui, "Improved mutual coupling compensation in compact antenna arrays," *IET Microwaves, Antennas and Propagation*, vol. 4, no. 10, pp. 1506–1516, 2010.

[126] H. S. Lui and H. T. Hui, "Mutual coupling compensation for direction-of-arrival estimations using the receiving-mutual-impedance method," *International Journal of Antennas and Propagation*, vol. 2010, Article ID 373061, 7 pages, 2010.

[127] C. Y. Chiu, C. H. Cheng, R. D. Murch, and C. R. Rowell, "Reduction of mutual coupling between closely-packed antenna elements," *IEEE Transactions on Antennas and Propagation*, vol. 55, no. 6, pp. 1732–1738, 2007.

[128] S. C. Chen, Y. S. Wang, and S. J. Chung, "A decoupling technique for increasing the port isolation between two strongly coupled antennas," *IEEE Transactions on Antennas and Propagation*, vol. 56, no. 12, pp. 3650–3658, 2008.

[129] P. J. Ferrer, J. M. González-Arbesú, and J. Romeu, "Decorrelation of two closely spaced antennas with a metamaterial AMC surface," *Microwave and Optical Technology Letters*, vol. 50, no. 5, pp. 1414–1417, 2008.

[130] M. M. Bait-Suwailam, M. S. Boybay, and O. M. Ramahi, "Electromagnetic coupling reduction in high-profile monopole antennas using single-negative magnetic metamaterials for MIMO applications," *IEEE Transactions on Antennas and Propagation*, vol. 58, no. 9, pp. 2894–2902, 2010.

[131] R. Schmidt, "Multiple emitter location and signal parameter estimation," in *Proceedings of the RADC Spectrum Estimation Workshop*, pp. 243–258, 1979.

[132] T. J. Shan, M. Wax, and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 33, no. 4, pp. 806–811, 1985.

[133] Y. Wang and S. Xu, "Mutual coupling calibration of DBF array with combined optimization method," *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 10, pp. 2947–2952, 2003.

[134] P. Demarcke, H. Rogier, R. Goossens, and P. De Jaeger, "Beamforming in the presence of mutual coupling based on constrained particle swarm optimization," *IEEE Transactions on Antennas and Propagation*, vol. 57, no. 6, pp. 1655–1666, 2009.

[135] B. Basu and G. K. Mahanti, "Beam reconfiguration of linear array of parallel dipole antennas through switching with real excitation voltage distribution," *Annals of Telecommunications*, vol. 67, pp. 285–293, 2012.

[136] O. M. Bucci, A. Capozzoli, and G. D. Elia, "Reconfigurable conformal array synthesis with near-field constraints," in *Proceedings of the Progress in Electromagnetics Research Symposium (PIERS '00)*, pp. 864–870, Cambridge, Mass, USA, July 2000.

[137] L. Landesa, F. Obelleiro, J. L. Rodríguez, J. A. Rodríguez, F. Ares, and A. G. Pino, "Pattern synthesis of array antennas with additional isolation of near field arbitrary objects," *Electronics Letters*, vol. 34, no. 16, pp. 1540–1542, 1998.

[138] H. Steyskal, "Synthesis of antenna patterns with imposed near-field nulls," *Electronics Letters*, vol. 30, no. 24, pp. 2000–2001, 1994.

[139] P. H. Pathak and N. Wang, "Ray analysis of mutual coupling between antennas on a convex surface," *IEEE Transactions on Antennas and Propagation*, vol. 29, pp. 911–922, 1981.

[140] R. M. Jha, V. Sudhakar, and N. Balakrishnan, "Ray analysis of mutual coupling between antennas on a general paraboloid of revolution (GPOR)," *Electronics Letters*, vol. 23, no. 11, pp. 583–584, 1987.

[141] R. M. Jha and W. Wiesbeck, "Geodesic constant method: a novel approach to analytical surface-ray tracing on convex conducting bodies," *IEEE Antennas and Propagation Magazine*, vol. 37, no. 2, pp. 28–38, 1995.

[142] R. W. Wills, "Mutual coupling and far field radiation from waveguide antenna elements on conformal surfaces," in *Proceedings of the International Conference (RADAR '87)*, pp. 515–519, London, UK, October 1987.

[143] P. Persson, L. Josefsson, and M. Lanne, "Investigation of the mutual coupling between apertures on doubly curved convex surfaces: theory and measurements," *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 4, pp. 682–692, 2003.

[144] P. Persson and L. Josefsson, "Calculating the mutual coupling between apertures on a convex circular cylinder using a hybrid UTD-MoM method," *IEEE Transactions on Antennas and Propagation*, vol. 49, no. 4, pp. 672–677, 2001.

[145] L. Landesa, J. M. Taboada, F. Obelleiro, and J. L. Rodriguez, "Design of on-board array antennas by pattern optimization," *Microwave and Optical Technology Letters*, vol. 21, pp. 446–448, 1999.

[146] L. Landesa, F. Obelleiro, J. L. Rodríguez, and A. G. Pino, "Pattern synthesis of array antennas in presence of conducting bodies of arbitrary shape," *Electronics Letters*, vol. 33, no. 18, pp. 1512–1513, 1997.

[147] F. Obelleiro, L. Landesa, J. M. Taboada, and J. L. Rodríguez, "Synthesis of onboard array antennas including interaction with the mounting platform and mutual coupling effects," *IEEE Antennas and Propagation Magazine*, vol. 43, no. 2, pp. 76–82, 2001.

[148] C. W. Wu, L. C. Kempel, and E. J. Rothwell, "Mutual coupling between patch antennas recessed in an elliptic cylinder," *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 9, pp. 2489–2492, 2003.

[149] L. Vegni and A. Toscano, "The method of lines for mutual coupling analysis of a finite array of patch antennas on a cylindrical stratified structure," *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 8, pp. 1907–1913, 2003.

[150] H. J. Visser and A. C. F. Reniers, "Textile antennas, a practical approach," in *Proceedings of the 2nd European Conference on Antennas and Propagation (EuCAP '07)*, pp. 1–8, Edinburgh, UK, November 2007.

[151] M. S. Akyuz, V. B. Erturk, and M. Kalfa, "Closed-form green's function representations for mutual coupling calculations between apertures on a perfect electric conductor circular cylinder covered with dielectric layers," *IEEE Transactions on Antennas and Propagation*, vol. 59, pp. 3094–3098, 2011.

[152] K. Yang, Z. Zhao, Z. Nie, J. Ouyang, and Q. H. Liu, "Synthesis of conformal phased arrays with embedded element pattern decomposition," *IEEE Transactions on Antennas and Propagation*, vol. 59, pp. 2882–2888, 2011.

[153] V. B. Ertürk and R. G. Rojas, "Efficient analysis of input impedance and mutual coupling of microstrip antennas mounted on large coated cylinders," *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 4, pp. 739–749, 2003.

[154] Z. Sipus, P. Persson, and M. Bosiljevac, "Modeling conformal antennas using a hybrid spectral domain-UTD method," in *Proceedings of the Nordic Antenna Symposium (Antenn '06)*, Linkoping, Sweden, May 2006.

[155] S. Raffaelli, Z. Sipus, and P. S. Kildal, "Analysis and measurements of conformal patch array antennas on multilayer circular cylinder," *IEEE Transactions on Antennas and Propagation*, vol. 53, no. 3, pp. 1105–1113, 2005.

[156] T. E. Morton and K. M. Pasala, "Performance analysis of conformal conical arrays for airborne vehicles," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 42, no. 3, pp. 876–890, 2006.

[157] B. Thors, L. Josefsson, and R. G. Rojas, "The RCS of a cylindrical array antenna coated with a dielectric layer," *IEEE Transactions on Antennas and Propagation*, vol. 52, no. 7, pp. 1851–1858, 2004.

[158] P. Persson and R. G. Rojas, "High-frequency approximation for mutual coupling calculations between apertures on a perfect electric conductor circular cylinder covered with a dielectric layer: nonparaxial region," *Radio Science*, vol. 38, no. 4, pp. 181–1814, 2003.

[159] T. Svantesson and A. Ranheim, "Mutual coupling effects on the capacity of multielement antenna systems," in *Proceedings of the IEEE Conference on Acoustics, Speech and Signal Processing (ICASSP '01)*, pp. 2485–2488, Salt Lake City, Utah, USA, May 2001.

[160] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311–335, 1998.

[161] Y. Urzhumov and D. R. Smith, "Metamaterial-enhanced coupling between magnetic dipoles for efficient wireless power transfer," *Physical Review B*, vol. 83, no. 20, Article ID 205114, 2011.

[162] H. M. Aumann, A. J. Fenn, and F. G. Willwerth, "Phased array antenna calibration and pattern prediction using mutual coupling measurements," *IEEE Transactions on Antennas and Propagation*, vol. 37, no. 7, pp. 844–850, 1989.

[163] C. Yang and Y. Z. Ruan, "Eigenvalues of covariance matrix of adaptive array with mutual coupling and two correlated sources present," in *Proceedings of the IEEE Antennas and Propagation Society International Symposium*, pp. 706–709, July 1993.