

## $\mathcal{N} = 3$ four dimensional field theories

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**ABSTRACT:** We introduce a class of four dimensional field theories constructed by quotienting ordinary  $\mathcal{N} = 4$   $U(N)$  SYM by particular combinations of R-symmetry and  $SL(2, \mathbb{Z})$  automorphisms. These theories appear naturally on the worldvolume of D3 branes probing terminal singularities in F-theory, where they can be thought of as non-perturbative generalizations of the O3 plane. We focus on cases preserving only 12 supercharges, where the quotient gives rise to theories with coupling fixed at a value of order one. These constructions possess an unconventional large  $N$  limit described by a non-trivial F-theory fibration with base  $AdS_5 \times (S^5/\mathbb{Z}_k)$ . Upon reduction on a circle the  $\mathcal{N} = 3$  theories flow to well-known  $\mathcal{N} = 6$  ABJM theories.

**KEYWORDS:** F-Theory, Supersymmetric gauge theory, Brane Dynamics in Gauge Theories, M-Theory

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## 1 Introduction

One of the fascinating properties of string theory lies in its ability to geometrize various deep and subtle field theory phenomena, often giving insight not available in any other known way.

A particularly fruitful geometric context in which to study four dimensional field theories is F-theory [1]. It can be understood as a geometric rewriting of IIB string theory with an axio-dilaton varying over a base, but crucially also as a particular limit of M-theory on the Calabi-Yau fourfold defined by the axio-dilaton fibration. Questions about the field theory translate into questions about the geometry of the fourfold, and can often be dealt with using algebraic geometry methods.

A crucial point is that most of the interesting phenomena (for a field theorist using M-theory as a computing tool) arise when the geometry develops singularities. Much of the recent work dealing with F-theory model building (starting with [2–5]), for instance, is concerned with the construction of appropriately singular geometries in order to model features of the standard model, and of its supersymmetric and grand unified extensions.

While the physics of interest happens on the singular locus, we do not have much control directly over M-theory on singular spaces, so in practice one constructs a family of smooth Calabi-Yau spaces parameterized by some parameter  $t$ , such that when  $t \rightarrow 0$  the Calabi-Yau develops the singularity of interest, but the representatives for  $t \neq 0$  are all smooth. By following which cycles in the geometry vanish as we approach  $t = 0$  we can read off which BPS states become massless at the singular point [6], and thus reconstruct much of the low energy physics.

In this note we deal with singularities in the M-theory description of certain four dimensional theories that cannot be approached in this way: one cannot construct a family of smooth Calabi-Yau spaces abutting the singular Calabi-Yau. Nevertheless, as we will discuss below, examples of singularities in the M-theory fourfold that cannot be smoothed out in a supersymmetric way are both rather common — the ordinary O3 plane gives rise to such a structure, and the local geometry of the M-theory backgrounds that appear in our construction is precisely the one in the ABJM construction [7, 8] — and quite interesting from a field theoretic point of view. We focus on the simplest hitherto unknown examples, which happen to preserve twelve (but not sixteen) supercharges in four dimensions, which we refer to as  $\mathcal{N} = 3$  theories in the customary fashion. It is well known that four dimensional supergravity Lagrangians preserving just twelve supercharges exist, but to our knowledge this is the first time that  $\mathcal{N} = 3$  examples outside that class have been constructed.

The theories under analysis here have a number of amusing properties. The most characteristic one — and the one that allows them to evade well known results [9] stating that perturbative  $\mathcal{N} = 3$  theories necessarily have  $\mathcal{N} = 4$  supersymmetry — is that their associated SCFTs do not have a marginal deformation associated with taking the gauge coupling to a perturbative regime.

This fact will in fact appear rather naturally from the construction: when formulated in field theory terms the  $\mathcal{N} = 3$  theories in this paper arise as quotients of ordinary  $\mathcal{N} = 4$   $U(N)$  SYM theory by a symmetry involving a non-trivial action of the  $SL(2, \mathbb{Z})$  duality group, together with an action of the  $SU(4)$  R-symmetry group. (Quotients obtained by gauging a subgroup of the  $SU(4)$  R-symmetry group can be seen to describe the theory of D3 branes at Calabi-Yau orbifold singularities in perturbative IIB string theory [10], our work can be thought of as a generalization of this viewpoint to F-theory, where extending the orbifold action to the fiber is natural to consider.) For generic values of the Yang-Mills coupling the  $SL(2, \mathbb{Z})$  duality group is not a symmetry of the theory, and it only becomes so for specific self-dual values of the coupling. Deformations away from this point should then be projected out from the quotient, and indeed we show in section 3.3 that this is the case.

This construction is most naturally motivated from the M-theory viewpoint, where our construction comes from orbifold actions on  $\mathbb{R}^3 \times \mathbb{C}^3 \times T^2$  that only make sense for specific values of the complex structure of the torus fiber. This was in fact our original motivation for approaching this problem: as reviewed in section 2 the ordinary O3 plane in IIB string theory can be understood as an orbifold of IIB theory in flat space by an orbifold generator involving the action of the  $-1 \in SL(2, \mathbb{Z})$  element of the duality group. It is fairly natural to ask if this can be generalized in the F-theory context to orbifolds involving duality elements acting non-trivially on the IIB axio-dilaton, and if so what is the physics of probe D3 branes on top of the resulting singularity. The purpose of this note is to answer the existence question in the affirmative, and initiate the study of the resulting field theories.

**Note added.** As we were finishing the contents of this note [11] appeared, giving a field theoretical derivation of some properties of  $\mathcal{N} = 3$  SCFTs, under the assumption that such

theories exist. The properties they find seem to be in agreement with those of the theories we construct below.

## 2 D3 branes on the O3 plane

In order to motivate our construction, we now revisit a familiar system, the theory of D3 branes on top of an O3 plane. This system is most commonly studied from the point of view of the worldsheet CFT, a construction which we briefly review now (for a more exhaustive review see for instance [12]). In this context the theory on the D3 branes on top of O3 planes is defined as the quotient of the theory of open strings moving in flat space with D3 boundary conditions, by the orientifold action  $\mathcal{I}(-1)^{F_L}\Omega$ . Here  $\Omega$  reverses the orientation of the worldsheet,  $(-1)^{F_L}$  acts as  $-1$  on RNS and RR states, and  $\mathcal{I}$  acts as reflection on the three complex directions transverse to the O3

$$\mathcal{I}: (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, -z_3) \tag{2.1}$$

leaving the four real dimensions along the O3 plane invariant. Before quotienting by the orientifold action the low energy description of the system is given by four dimensional  $\mathcal{N} = 4$  U(N) theory (forgetting about the ten dimensional dynamics in the bulk, which decouples at low energies), arising from open strings with ends on the D3 stack. The orientifold preserves all the supersymmetry of the original D3 stack, but projects down the gauge group to a subgroup. For concreteness we locate the D3 stack on top of the O3 plane, i.e. at  $z_1 = z_2 = z_3 = 0$ . The nature of the preserved subgroup depends on the choice of the representation of the orientifold action on the Chan-Paton factors. There are two inequivalent choices for this representation: we either end up with the algebra  $\mathfrak{so}(N)$  (we will discuss the global form of the gauge group momentarily), or  $\mathfrak{usp}(N)$ . In this latter case we need to restrict to  $N \in 2\mathbb{Z}$  for consistency.

We now want to discuss this construction from two alternative viewpoints, in order to motivate the generalization presented in the next section. Consider first the description of the system in F-theory, obtained by taking the zero size limit of the fiber for an M-theory compactification down to three dimensions on a torus fibered Calabi-Yau fourfold. The basic necessary fact for describing the O3 in this language is that  $(-1)^{F_L}\Omega$  lifts to an inversion of the torus fiber, i.e. a monodromy matrix

$$\mathcal{M}_{(-1)^{F_L}\Omega} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.2}$$

A simple derivation of this fact can be obtained by looking to the action of  $(-1)^{F_L}\Omega$  on the IIB spacetime fields. For instance, in the CFT language it is easy to see that both the NSNS two-form  $B_2$  and the RR two-form  $C_2$  get an intrinsic minus sign under  $(-1)^{F_L}\Omega$ , while in the F/M-theory language they come from the reduction of  $C_3$  along two independent one-cycles of the torus fiber. We immediately conclude that  $(-1)^{F_L}\Omega$  acts as inversion of the torus.

The F-theory lift of a stack of  $N$  D3 branes in flat space (the U(N) theory) is given by a stack of  $N$  M2 branes on  $\mathbb{C}^3 \times T^2$ . The fibration is trivial in this case, with the

complex structure of the torus arbitrary. This arbitrariness maps to the existence of the marginal deformation of  $\mathcal{N} = 4$  U( $N$ ) changing the value of the complexified coupling. The orientifolded system can then be constructed in F-theory by taking the quotient

$$\sigma: (z_1, z_2, z_3, u) \rightarrow (-z_1, -z_2, -z_3, -u) \tag{2.3}$$

with  $u$  the flat coordinate of the  $T^2$ . That is,  $T^2$  is the quotient of  $\mathbb{C}$ , parameterized by  $u$ , by some lattice  $\mathbb{L} = \{ae_1 + be_2\}$  with  $a, b \in \mathbb{Z}$ . This involution clearly exists for any complex structure of the torus, since a change of sign maps any integer lattice  $\mathbb{L}$  to itself.

The resulting geometry  $X = (\mathbb{C}^3 \times T^2)/\sigma$  has four fixed points at  $z_1 = z_2 = z_3 = 0$  and  $u = -u \bmod \mathbb{L}$ . (For instance, if we take  $e_1 = 1, e_2 = \tau$ , the fixed points are at  $\{0, \frac{1}{2}, \frac{1}{2}\tau, \frac{1}{2}(\tau+1)\}$ .) Close to any of these fixed points we have a  $\mathbb{C}^4/\mathbb{Z}_2$  geometry, with the  $\mathbb{Z}_2$  inverting all coordinates of the  $\mathbb{C}^4$ . Notice that the  $\mathbb{C}^4/\mathbb{Z}_2$  singularity is terminal [13, 14]: it admits no supersymmetric resolution or deformation into a smooth fourfold. This is in good agreement with the fact that there are no twisted sectors that could smooth out the O3 plane.

We will come back to the F/M description momentarily, but let us first consider the field theory description of the orientifold operation. Such a description in terms of the low energy EFT must exist, since the  $\mathcal{N} = 4$  theories before and after orientifolding are consistent truncations of the full string theory. It will be illuminating to consider the simplest case: the theory on one mobile D3 brane on top of an O3<sup>-</sup> plane. As is well known, this is given by the  $\mathcal{N} = 4$  theory with  $\mathfrak{so}(2)$  algebra. At this level the theory is identical to the U(1) theory arising from a D3 on flat space, without considering the orientifold. There is a difference when one considers the global structure of the gauge group, though. This is perhaps best seen by comparing the moduli spaces. For the U(1) theory we expect the moduli space to be simply  $\mathbb{C}^3$ , while for the  $\mathfrak{so}(2)$  theory we would rather expect  $\mathbb{C}^3/\mathbb{Z}_2$  (with the  $\mathbb{Z}_2$  acting with a minus sign on all  $\mathbb{C}^3$  coordinates). The moduli space arises from the vevs of the scalars on the  $\mathcal{N} = 4$  vector multiplet, which take values in the adjoint. A way to achieve our goal, while keeping supersymmetry, is then to take a quotient by a  $\mathbb{Z}_2$  acting as  $t \rightarrow -t$  on the algebra generators. For the cases of orientifolds giving rise to SO( $2N + 1$ ) and USp( $2N$ ) groups the expected  $\mathbb{Z}_2$  quotient of moduli space is always present as part of the Weyl group, but the Weyl group is trivial for  $\mathfrak{so}(2)$ . Rather, the sign action is precisely the  $\mathbb{Z}_2$  outer automorphism of the SO(2) group enhancing it to O(2), so we learn that the expected form of the gauge group is  $\mathbb{Z}_2 \ltimes \text{SO}(2) = O(2)$ .<sup>1</sup>

There is a way of understanding this  $\mathbb{Z}_2$  automorphism that connects nicely to the F/M description, and which immediately suggests the generalization that is the main topic of this note. The  $\mathcal{N} = 4$  theory with gauge group U(1) has a global symmetry which includes the R-symmetry group SU(4)<sub>R</sub>, that has a natural interpretation as the rotation group in

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<sup>1</sup>This conclusion for the global form of the gauge group can be reached in many additional ways. For example, upon compactification of the theory on a circle, and T-dualization, the global form must be O(2) so that we can construct the component containing two  $\widetilde{O2}^-$  planes. Or simply by looking to the perturbative symmetry group: as opposed to the type I case, in this case there are no finite action non-BPS instantons spoiling the conclusion.

the  $\mathbb{R}^6$  orthogonal to the stack. We wish to focus on the element  $r \in \text{SU}(4)_R$  acting as

$$r: (A_\mu, \lambda_\alpha^a, \phi^I) \rightarrow (A_\mu, \sqrt{-1} \lambda_\alpha^a, -\phi^I) \tag{2.4}$$

on the components of the  $\mathcal{N} = 4$  vector multiplet, corresponding to  $\text{SO}(6)$  rotations in the  $\mathbb{R}^6$  acting as an overall sign  $z_i \rightarrow -z_i$ . Here  $a$  and  $I$  are indices in the spinor and vector representations of  $\text{SU}(4)_R$ . This action does not preserve supersymmetry by itself, in agreement with the fact that  $\mathbb{C}^3/\mathbb{Z}_2$  is not a Calabi-Yau space. Notice also that  $r^2 \neq 1$ .

There is, in addition, a subgroup of the  $\text{SL}(2, \mathbb{Z})$  duality group that acts nontrivially as a symmetry of the theory for any value of the coupling  $\tau$ , it is given by  $s = -1 \in \text{SL}(2, \mathbb{Z})$ . It leaves  $\tau$  invariant, but it acts non-trivially on the space of states. In particular, it sends a BPS state with electric and magnetic charges  $(p, q)$  to another state with charges  $(-p, -q)$ , so it acts with a minus sign on the gauge boson  $A_\mu$ . The full action of  $s$  on the components of the  $\mathcal{N} = 4$  multiplet is

$$s: (A_\mu, \lambda_\alpha^a, \phi^I) \rightarrow (-A_\mu, \sqrt{-1} \lambda_\alpha^a, \phi^I). \tag{2.5}$$

(For a field theory derivation of the action on the gauginos see for example [15].) Notice again that  $s^2 \neq 1$ . The inversion of the  $(p, q)$  charges has a natural interpretation as the  $\mathbb{Z}_2$  action on the F-theory torus, since light strings come from M2 branes wrapped on one-cycles.

We now have an intrinsic definition of the  $\mathbb{Z}_2$  orientifold action in the field theory: it is simply given by the quotient by  $r \cdot s$ , which acts as a global  $-1$  on the whole vector multiplet:

$$O(2)_{\mathcal{N}=4} = \frac{U(1)_{\mathcal{N}=4}}{r \cdot s}. \tag{2.6}$$

One may worry about the self-duality of the  $O3^-$  under S-duality, given that the gauge group is  $O(2N)$  rather than  $\text{SO}(2N)$ . We postpone a discussion of this point to appendix A.

**Interacting theories.** Consider the  $O(2N)$  theory, with  $N > 1$ . In order to construct this theory field theoretically, starting with a theory without orientifold, we need to generalize (2.6) slightly. The first way of doing this is familiar from the worldsheet description of the orientifold. Start with the theory in the double cover of the geometry and in the absence of orientifold, having twice the amount of mobile branes, giving rise to a gauge group  $U(2N)$ . In this doubled theory, in addition to specifying the  $r \cdot s$  quotient, we specify a Chan-Paton action on the gauge bosons

$$A \rightarrow -A^t \tag{2.7}$$

in such a way that only antisymmetric gauge matrices survive, giving rise to the  $\mathfrak{so}(2N)$  gauge algebra.

Nevertheless, for the non-perturbative generalizations that we have in mind a different description is perhaps more adequate. We work directly on the quotient geometry, where we have  $N$  mobile branes. Away from the singularity in the geometry, the low energy gauge group is  $U(N)$ . As the stack of branes hits the singularity some light modes become

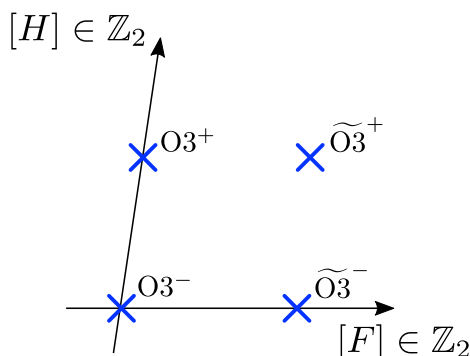
massless, and enhance the algebra to  $\mathfrak{so}(2N)$ . In the M-theory description these light modes arise from BIon excitations on the probe M2 brane stack. More precisely, the massless modes transform in the  $\square \oplus \bar{\square}$  representation of the  $U(N)$  group. Let us separate slightly the M2 branes in the stack for ease of visualization, and consider the topology of the quotient geometry, removing the origin of the base  $\mathbb{C}^3$  for simplicity. The projection of the orbifold to the base has topology  $\mathbb{R} \times (S^5/\mathbb{Z}_2)$ , which for the purposes of homology is the same as  $S^5/\mathbb{Z}_2 = \mathbb{RP}^5$ . There is a non-trivial cycle in this  $\mathbb{RP}^5$  arising in its  $S^5$  double cover from the path between one point and its image. The action on the  $T^2$  fiber homology as we go around this closed path is changing the sign of the generators: a  $(p, q)$  cycle goes to  $(-p, -q)$ . So, between any two distinct M2 branes on the stack, generating a Cartan subgroup  $U(1)^2$ , there are two non-trivial paths which differ by the non-trivial torsional element in the base. Due to the non-trivial monodromy in the fiber, if one gives a state with  $(1, 1)$  charges under the  $U(1)^2$  Cartan, the other gives a state with  $(1, -1)$  charges. We recognize these states as the generators of the adjoint and antisymmetric representations of  $U(N)$ , and together they enhance  $U(1)^N$  to  $O(2N)$ .

We could have also considered states going from one brane on the stack to itself, wrapping the non-trivial cycle in  $\mathbb{RP}^5$ . Whether they exist or not depends on the NSNS charge of the O3 plane: they are absent for  $O3^-$  and they are present for  $O3^+$ . Had we included them, the antisymmetric would have been replaced by a symmetric representation, giving rise to an enhancement to  $USp(2N)$  when the stack hits the singularity. How do we explain this distinction in the resulting projection without resorting to CFT language? One suggestive observation is that the  $O3^-$  and  $O3^+$  can be transformed into each other by dropping a NS5 wrapped on the nontrivial (twisted) two-cycle of  $\mathbb{RP}^5$  [16, 17]. The NS5 brane and the fundamental string are electric-magnetic duals, so this suggests that the origin of the distinction, from the target space point of view, may come from an unsatisfied Dirac quantization condition on the F1/M2 worldvolume if we drop an even number of NS5/M5 branes on the  $O3^-$  plane. It would be very interesting to develop this viewpoint further as a possible tool in order to study the spectrum of BPS states in the more exotic configurations we construct later in the paper.

**Flux classification of O3 types.** It is a familiar fact that there are various kinds of O3 planes, as was implicit in the discussion above. A convenient classification is in terms of the discrete fluxes  $[H]$  and  $[F]$  for the NSNS and RR two-forms around the O3 plane [16]. Both kinds of torsion are  $H^3(S^5/\mathbb{Z}_2, \tilde{\mathbb{Z}}) = \mathbb{Z}_2$  valued, so we have four distinct possibilities, shown in figure 1 together with conventional names for each orientifold type. We emphasize that the statement that we have four different kinds of O3 planes in IIB is perturbative: once we take the  $SL(2, \mathbb{Z})$  duality of the theory into account the four orientifolds organize themselves into a singlet of  $SL(2, \mathbb{Z})$  (the  $O3^-$ ) and a triplet (the  $\widetilde{O3}^-$ ,  $O3^+$  and  $\widetilde{O3}^+$ ). Accordingly, we expect that there is no distinction between the M-theory lifts of the orientifolds in the triplet, but that they are distinct from the M-theory lift of the  $O3^-$ .

The discrete fluxes  $[H]$  and  $[F]$  lift to the M-theory description as discrete fluxes for the  $C_3$  form. More precisely, around each  $\mathbb{C}^4/\mathbb{Z}_2$  local singularity in the M-theory dual of the O3 plane, we can turn on a nontrivial  $C_3$  represented by the torsional generator of  $H^4(S^7/\mathbb{Z}_2, \mathbb{Z}) = \mathbb{Z}_2$ .





**Figure 1.** The four types of O3 planes, classified based on their discrete torsion [16]. A positive superscript denotes non-trivial NSNS torsion, while a tilde denotes nontrivial RR torsion.

Alternatively [16, 17], we can start with the  $O3^-$  plane, and generate the discrete  $[H]$  and  $[F]$  torsions by dropping an NS5 or D5 wrapped on the non-trivial  $\mathbb{Z}_2$  generator of  $H_2(S^5/\mathbb{Z}_2, \tilde{\mathbb{Z}})$ . In the M-theory language, we change the flux on the  $\mathbb{C}^4/\mathbb{Z}_2$  singularity by dropping a M5 wrapped on the non-trivial generator of  $H_3(S^7/\mathbb{Z}_2) = \mathbb{Z}_2$ .

In either description, the net result is that we have  $2^4 = 16$  discrete choices for the discrete torsion on the M-theory lift of the orientifold. (There are actually only five inequivalent configurations in M-theory, once we account for possible torus redefinitions.) The fact that we have a larger number of discrete choices in M-theory than in IIB reflects the fact that some of these configurations can become trivial or equivalent once we take the F-theory limit [18].

This is most easily seen in the current case by performing an intermediate step in the duality between IIB and M-theory: in IIA string theory — obtained by reducing M-theory on one of the cycles of the  $T^2$ , or T-dualizing on the circle in which we reduce IIB — we also have four types of O2 planes, distinguished by the flux of the NSNS three-form and the flux of the RR four-form. We denote the different planes by  $O2^\pm$  and  $\tilde{O}2^\pm$  in analogy with the O3 case. The lift of the different O2 planes to M-theory is given by M-theory on  $(\mathbb{R}^7 \times S^1)/\mathbb{Z}_2$ , with the  $\mathbb{Z}_2$  acting as a reflection on all directions. The reflections on the  $\mathbb{R}^7$  base are inherited from IIA, while the fact that the M-theory  $S^1$  gets reflected can be understood from the fact that D0 branes transform with an intrinsic minus sign under a O2, and D0 branes lift to momentum along the M-theory circle. Over the origin in  $\mathbb{R}^7$  where the O2 is located the M-theory  $S^1$  degenerates to a segment, and the local geometry around the endpoints of the segments is  $\mathbb{C}^4/\mathbb{Z}_2$ . For future convenience we denote the  $\mathbb{C}^4/\mathbb{Z}_2$  geometry with  $f = \{0, 1\}$  units of flux by  $OM2_2^f$ . Thus, there should be a one-to-one map between types of O2 planes and pairs of OM2 planes. The map can be easily worked out by computing the charges under  $C_3$ , see for instance [18], with the result displayed in table 1.

We now come back to the issue of equivalence of M-theory configurations under the F-theory limit. Consider for example the case of one mobile D3 brane probing a  $O3^-$  plane. The low energy dynamics is described by a  $O(2)$  group. There are two components in moduli space once we reduce on a circle, depending on the Wilson line along the circle: in one we have (after T-duality along the circle) two  $O2^-$  planes and a mobile D2, and



O2 <sup>-</sup>	$\widetilde{\text{O2}}^-$	O2 <sup>+</sup>	$\widetilde{\text{O2}}^+$
(0,0)	(1,1)	(1,0)	(0,1)

**Table 1.** Lift to M-theory of the different types of O2 planes in IIA. The labels in the bottom row denote the discrete flux on the two OM2<sub>2</sub> planes present in the lift, with 0 denoting trivial flux.

in the other we have two  $\widetilde{\text{O2}}^-$  planes with no mobile brane. The M-theory lift of these two configurations is different, but the distinction disappears in the F-theory limit, where the Wilson line becomes irrelevant. One can also see in this way that two of the other orientifolds give rise to shift orientifolds in IIB acting on the compactification circle, and thus become trivial orientifolds in the strict F-theory limit.

These remarks complete our discussion of the O3 plane. As we see, while the object was originally introduced in CFT language, it can be described intrinsically both in M-theory and in effective field theory (although it would be good to develop both of these viewpoints further). Unlike the CFT viewpoint, these generalize to non-classical configurations, so we now proceed to discuss some of these generalizations.

### 3 F-theory at $\mathcal{N}_{4d} = 3$ singularities

We will be interested in the F-theory limit of M-theory on abelian orbifolds of the form  $\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$  where  $k \in \{2, 3, 4, 6\}$ . The action of  $\mathbb{Z}_k$  on the complex coordinates  $(z^1, z^2, z^3, z^4)$  of  $\mathbb{C}^3 \times T^2$  is given by<sup>2</sup>

$$z^i \longrightarrow e^{2\pi i v_k^i} z^i \tag{3.1}$$

with  $v_k = (1, -1, 1, -1)/k$  and where  $z^4 = x + \tau y$  with  $\tau$  the complex structure of  $T^2$ . These values for  $k$  are the only possibilities that are well-defined on the torus.<sup>3</sup> The case  $k = 2$  corresponds to the O3 plane studied in the last section (and which preserves sixteen supercharges) so we will focus on the other possibilities, which preserve just twelve out of the sixteen supercharges. (A proof of this fact will be given in section 3.2.)

Notice that the  $\mathbb{Z}_k$  symmetry can only act as an involution of the torus for certain fixed values of the complex structure  $\tau$ , which we list in table 2. Since the complex structure of the torus in M-theory corresponds to the axio-dilaton when we take the F-theory limit, these backgrounds are intrinsically non-perturbative in ten dimensions. The situation is similar to F-theory compactifications on certain singular limits of K3 [19]. However, in those cases one can smoothly move in complex structure moduli space of the K3 to a perturbative configuration. As emphasized in the introduction, this is not possible for the orbifolds we are considering.

These orbifolds, or generalized orientifolds, can also be understood in field theory. Recall that the R-symmetry in the worldvolume of a stack of D3 branes can be understood

<sup>2</sup>This convention is such that  $\mathbb{Z}_k$  lies in SU(4), which makes the orbifold a Calabi-Yau. The choice  $z^i \rightarrow e^{2\pi i/k} z^i$  is physically equivalent [7] but we find our convention more convenient for our purposes.

<sup>3</sup>See [13, 14] for classifications of codimension four terminal Gorenstein quotient singularities. The cases we consider fall into this classification so they are terminal.

	$\tau$	$\chi$	$Q(\text{OM}2_{k,0})$	$Q(\text{OF}3_{k,0})$
$k = 2$	any	384	$-1/16$	$-1/4$
$k = 3$	$\exp(\pi i/3)$	216	$-1/9$	$-1/3$
$k = 4$	$i$	240	$-5/32$	$-3/8$
$k = 6$	$\exp(\pi i/3)$	240	$-35/144$	$-5/12$

**Table 2.** Data for the  $\mathcal{N} \geq 3$  orientifolds analyzed in this paper. D3 charges for the  $\text{OF}3_{k,0}$  planes are given in units of mobile D3 branes.

as rotations on the six directions transverse to the worldvolume, while  $\text{SL}(2, \mathbb{Z})$  duality is associated with large diffeomorphisms of the torus fiber in the F-theory description. When we take the  $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$  quotient in (3.1), we are taking a quotient of  $\mathcal{N} = 4$  by the combined action of a R-symmetry generator  $r_k$ , and an  $\text{SL}(2, \mathbb{Z})$  symmetry generator  $s_k$ . With the inclusion of appropriate massless sectors (which we do not describe in this paper, but follow from the M-theory description, in analogy with the extra massless two-index tensors in the O3 case described above), this defines a theory by the analog of formula (2.6). Our goal in the rest of the note will be to describe some properties of this theory.

### 3.1 Orientifold variants

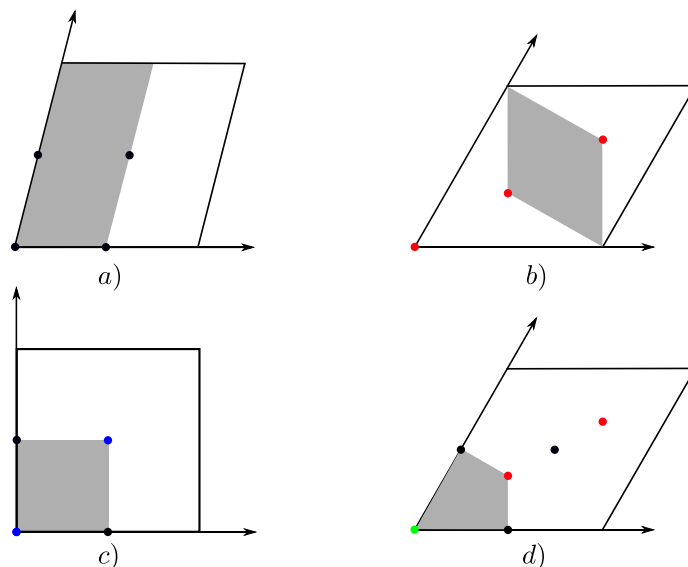
A basic quantity to compute for any BPS object in string theory is its charge. In the M-theory formulation the contribution to the charge under  $C_3$  of a  $\mathbb{C}^4/\mathbb{Z}_k$  singularity comes from the curvature coupling  $-\int C_3 \wedge I_8(R)$ , where  $\int I_8(R) = \chi/24$ . For the M-theory configurations of interest to us in this paper, we typically have more than one fixed point. More precisely, the topology of the singular fiber at the origin of the  $\mathbb{C}^3$  base is of the form displayed in figure 2 (see for example [20, 21] for details), namely:

- $T^2/\mathbb{Z}_2$  is a sphere with four  $\mathbb{Z}_2$  orbifold points.
- $T^2/\mathbb{Z}_3$  is a sphere with three  $\mathbb{Z}_3$  orbifold points.
- $T^2/\mathbb{Z}_4$  is a sphere with two  $\mathbb{Z}_4$  and one  $\mathbb{Z}_2$  orbifold points.
- $T^2/\mathbb{Z}_6$  is a sphere with a  $\mathbb{Z}_6$ ,  $\mathbb{Z}_3$  and  $\mathbb{Z}_2$  orbifold points.

In the F-theory limit, the D3 charge of the 4d object  $\text{OF}3_{k,0}$  (the limit of M-theory on  $\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$ , with no torsional flux) is given by the sum of the contribution of each fixed point.

The charge under the M-theory three-form  $C_3$  of the orbifold  $\mathbb{C}^4/\mathbb{Z}_k$ , in turn, can be conveniently computed by taking a compact Calabi-Yau  $X_k = T^8/\mathbb{Z}_k$ , computing  $\chi(X_k)$  and then dividing the result by the number of fixed points — taking into account that the resulting fixed points may be of different types. This computation was done in [22], with the result

$$\chi(\mathbb{C}^4/\mathbb{Z}_k) = k - \frac{1}{k} \tag{3.2}$$



**Figure 2.**  $T^2/\mathbb{Z}_k$  for a)  $k = 2$ , b)  $k = 3$ , c)  $k = 4$  and d)  $k = 6$ . The black, red, blue and green dots represent the  $\mathbb{Z}_k$  fixed points for  $k = 2, 3, 4$  and  $6$  respectively. The grey regions denote the fundamental domains in each case.

or in terms of the M2 charge of the  $OM_{k,0}$  orbifold (the  $\mathbb{C}^4/\mathbb{Z}_k$  singularity with no torsion)

$$Q(OM_{k,0}) = -\frac{1}{24} \left( k - \frac{1}{k} \right). \tag{3.3}$$

The results are collected in table 2. Adding up the induced charges of the fixed points on the orbifolded  $T^2$  fibration, according to the fixed point topology described above, we obtain the D3 charge for the  $OF3_{k,0}$  fixed point, as in table 2.

**Discrete torsion.** In the same way that the O3 plane comes in different flavors depending on the value of the discrete fluxes, one may expect that the generalized OF3 planes also come in different flavors, distinguished by discrete flux data.

Consider first the neighborhood of an  $OM2_{k,0}$  plane. Since  $H^4(S^7/\mathbb{Z}_k, \mathbb{Z}) = \mathbb{Z}_k$ , we can add  $p = 0, 1, \dots, k - 1$  units of discrete torsion to  $OM2_{k,0}$  to make an  $OM2_{k,p}$ . The contribution to the charge coming from the discrete torsion can be computed from the coupling  $-\frac{1}{6} \int C_3 \wedge G_4 \wedge G_4$  appearing in the M-theory action. The charge is then [23]

$$Q(OM2_{k,p}) = Q(OM2_{k,0}) + pq_k \tag{3.4}$$

with

$$q_k = -\frac{1}{2} \int_{S^7/\mathbb{Z}_k} \frac{C_3}{2\pi} \wedge \frac{G_4}{2\pi} = \frac{1}{2k}. \tag{3.5}$$

This is

$$Q(OM2_{k,p}) = -\frac{1}{24} \left( k - \frac{1 + 12p}{k} \right). \tag{3.6}$$

Let us compute the D3 charge of the  $OF3_3$  for all choices of flux. Looking at figure 2 we see that the the M-theory lift is constructed out of three  $OM2_3$  points. Each one has

Orientifold	Charges
OF3 <sub>2</sub>	$-\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$
OF3 <sub>3</sub>	$-\frac{1}{3}, -\frac{1}{6}, 0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$
OF3 <sub>4</sub>	$-\frac{3}{8}, -\frac{1}{4}, -\frac{1}{8}, 0, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}$
OF3 <sub>6</sub>	$-\frac{5}{12}, -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{6}, -\frac{1}{12}, 0, \frac{1}{12}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{5}{12}, \frac{1}{2}, \frac{7}{12}$

**Table 3.** D3 charges (in F/M-theory conventions, i.e. counting mobile branes) for the different OF3<sub>k</sub> planes for all possible choices of torsion, starting with the configuration with no torsion. We have included all possibilities allowed from the M-theory perspective, even if some are known or expected to lead to unorientifolded IIB backgrounds in the F-theory limit.

a  $\mathbb{Z}_3$ -valued flux associated to it, for a total of 27 possibilities. Out of these some are equivalent: we can choose any of the three OM2<sub>3</sub> planes to lie at the origin of the unit cell, and an overall reflection of the choice of unit cell does not change the M-theory geometry. This reduces the number of possibilities to 10. The D3 charge of the different OF3<sub>3</sub> planes depends only on the sum of the fluxes, by enumeration we find that the possible charges in the set are given by  $\{-\frac{1}{3}, -\frac{1}{6}, 0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}\}$ .

The exercise can be repeated for the other OF3<sub>k</sub> planes, with the result shown in table 3. There is a clear pattern to the result: the possible OF3<sub>k</sub> planes can be labeled by an integer  $p \in \{-k + 1, -k + 2, \dots, k - 1, k, k + 1\}$ . The charge of the OF3<sub>k</sub><sup>p</sup> is then simply  $\frac{p}{2k}$ . Notice that contrary to the OF3<sub>2</sub> case, we do not necessarily expect that the different OF3<sub>k</sub> planes are distinguished by their D3 charge only (up to  $SL(2, \mathbb{Z})$ ), so there may be more than one inequivalent OF3<sub>k</sub> plane with the same D3 charge. It would be very interesting to clarify this point, but we keep the notation for convenience in any case.

As an example, in the perturbative case we have  $OF3_2^{-1} = O3^-$  and  $OF3_2^1 = O3^+$  (or any of its  $\widetilde{O3}^\pm SL(2, \mathbb{Z})$  duals). It is interesting to note that various special cases in the known OF3<sub>2</sub> setting seem to persist for higher  $k$ . For instance, note that the difference between the smallest and largest amount of charge is one unit of D3 charge for all  $k$ . A natural conjecture is then that  $OF3_k^{k+1}$  comes from a disconnected component of the moduli space of a mobile D3 probing an OF3 without discrete torsion, when compactified on a circle. Similarly,  $OF3_2^0$  and  $OF3_2^2$  both lift to unorientifolded IIB backgrounds in the F-theory limit (in the presence of a D3 in the latter case), so it seems plausible that in general  $OF3_k^0$  and  $OF3_k^k$  also become trivial in the F-theory limit.

### 3.2 Preserved supercharges

We have made the claim above that the OF3<sub>k</sub> planes, for  $k = 3, 4, 6$ , preserve twelve supercharges. We now justify this claim from the point of view of the field theory (we will reproduce the same conclusions below by analyzing the string construction directly, but it may be interesting to give a purely field theoretical derivation). We start with  $\mathcal{N} = 4 U(N)$  SYM theory. The charges of the fields under the different symmetries of the theory are shown in table 4. The subscript in the representation under  $SL(2, \mathbb{Z})$  denotes the charge

	$SU(4)_R$	$SL(2, \mathbb{Z})$	$SU(2)_L \times SU(2)_R$	$\Delta$
$\phi^I$	<b>6</b>	<b>1<sub>0</sub></b>	$(0, 0)$	1
$\lambda_\alpha^a$	<b>4</b>	<b>1<sub>1/2</sub></b>	$(\frac{1}{2}, 0)$	$\frac{3}{2}$
$\begin{pmatrix} F_{\mu\nu} \\ \star F_{\mu\nu} \end{pmatrix}$	<b>1</b>	<b>2<sub>0</sub></b>	$(1, 0) \oplus (0, 1)$	2
$Q_{\alpha a}$	<b><math>\bar{4}</math></b>	<b>1<sub>1/2</sub></b>	$(\frac{1}{2}, 0)$	$\frac{1}{2}$

**Table 4.** Charges of the different fields on  $\mathcal{N} = 4$  SYM in four dimensions.

under the  $U(1)$  bundle with transition functions [15, 24]

$$\gamma = \frac{c\tau + d}{|c\tau + d|}. \tag{3.7}$$

As explained in section 2, an O3 plane is associated with a quotient by the product of  $r$  and  $s$ . Under  $r$ , the supercharges transform as

$$r: Q_{\alpha a} \rightarrow -\sqrt{-1}Q_{\alpha a} \tag{3.8}$$

whereas under  $s$  they behave as

$$s: Q_{\alpha a} \rightarrow \sqrt{-1}Q_{\alpha a}. \tag{3.9}$$

Thus, under the combined action of R-symmetry and  $SL(2, \mathbb{Z})$  they remain invariant and therefore the quotient preserves sixteen supercharges, as is familiar.

For higher  $k$ , we have that the R-symmetry rotation  $r_k$  corresponds to a rotation of  $2\pi/k$  along every 2-plane, in the spinorial representation. Concretely, it is given by (as an element in  $SO(6)_R$ )

$$R_k = \begin{pmatrix} \hat{R}_k & 0 & 0 \\ 0 & \hat{R}_k & 0 \\ 0 & 0 & \hat{R}_k^{-1} \end{pmatrix} \tag{3.10}$$

with  $\hat{R}_k$  a  $2 \times 2$  matrix corresponding to a  $2\pi/k$  rotation. Thus, the action on a spinor of negative helicity (such as  $Q_{\alpha a}$ ) is

$$\begin{aligned} (-, -, -) &\rightarrow e^{-i\pi/k}(-, -, -) \\ (-, +, +) &\rightarrow e^{-i\pi/k}(-, +, +) \\ (+, -, +) &\rightarrow e^{-i\pi/k}(+, -, +) \\ (+, +, -) &\rightarrow e^{3i\pi/k}(+, +, -) \end{aligned} \tag{3.11}$$

so its action on the supercharges is

$$r_k: \begin{aligned} Q_{\alpha A} &\rightarrow e^{-\pi i/k}Q_{\alpha A} \\ Q_{\alpha 4} &\rightarrow e^{3\pi i/k}Q_{\alpha 4} \end{aligned} \tag{3.12}$$

where the index  $A$  runs from 1 to 3. Furthermore, the action of the S-duality generator  $s_k$  is the same on all supercharges

$$s_k : Q_{\alpha a} \rightarrow e^{\pi i/k} Q_{\alpha a}. \tag{3.13}$$

The product  $r_k \cdot s_k$  only leaves  $Q_{\alpha A}$  invariant, so the theory preserves twelve supercharges.

### 3.3 $\mathcal{N} = 3$ and CPT invariance

The field theories discussed in the previous section have  $\mathcal{N} = 3$  supersymmetry in four dimensions. This is surprising since there is a well known argument [9] which states that, in the absence of gravity, perturbative four dimensional theories with twelve supercharges are actually  $\mathcal{N} = 4$ . We review such argument in the following stressing the fact that it only applies to theories with a perturbative limit. Since the cases we are studying are intrinsically non-perturbative, the argument does not apply. Furthermore, we will explicitly show that the operator spectrum of these theories does not fall into representations of  $SU(4)_R$ , so there is no possibility for the supersymmetry to enhance to  $\mathcal{N} = 4$  (which could have happened in principle, even if the standard argument for enhancement does not apply).

Consider an  $\mathcal{N} = 3$  supersymmetric QFT in 4d Minkowski space. This means that the  $\mathcal{N} = 3$  supersymmetry algebra has a well-defined action on the space of operators that define the theory, so these can be arranged in irreducible representations of the supersymmetry algebra. As usual, we want CPT to be a symmetry of the theory so it should also act on the space of operators. Then, under CPT an irreducible representation of supersymmetry may be mapped to itself or it can happen that two of them are mapped to each other.

Now let us assume that the theory has a Lagrangian description. This means that there are certain preferred operators (elementary fields) which are the ones that appear in the Lagrangian and in the measure of the path integral. The Lagrangian may depend on some parameters and we assume there is a point in such parameter space where the theory becomes free (the Lagrangian is quadratic in the elementary fields). At that point, the operator algebra can be recovered from the elementary fields and the Hilbert space obtained from canonical quantization. There are various free theories that one can consider at this point:

- The elementary fields include only massless vector multiplets. As it happens, there are two kinds of  $\mathcal{N} = 3$  vector multiplets that get mapped into each other under CPT and the two of them together make a vector multiplet of  $\mathcal{N} = 4$ . All the possible actions involving these fields turn out to be invariant under  $\mathcal{N} = 4$  [9]. Notice that since the whole space of operators is built from the elementary fields (which are in an  $\mathcal{N} = 4$  representation) we have that the action of the  $\mathcal{N} = 4$  algebra extends to all of them. Now one can argue that since the number of supersymmetries is discrete, it cannot change as we move continuously towards the strong coupling regime. This is the argument that in gauge theories,  $\mathcal{N} = 3$  supersymmetry is actually  $\mathcal{N} = 4$ .

- The next option is to include a gravity multiplet (and vector multiplets possibly). Again, there are two multiplets which get mapped into each other so we need to include them both at the same time. However, in this case the direct sum of the two does not admit the action of the full  $\mathcal{N} = 4$  algebra. Therefore, we find genuine  $\mathcal{N} = 3$  (supergravity) theories.

In a non-perturbative theory, there is no argument that suggests that there is an enhancement from  $\mathcal{N} = 3$  to  $\mathcal{N} = 4$  for non-gravitational theories which preserve CPT. Then, if there is an operator in a representation of  $\mathcal{N} = 3$  which cannot combine with others to form a representation of  $\mathcal{N} = 4$ , the theory only has  $\mathcal{N} = 3$ . As we argue in the following, this can be explicitly checked for the theories we are considering and there are indeed operators which only admit the action of  $\mathcal{N} = 3$ .

For simplicity, we start with the free theories obtained as a  $\mathbb{Z}_k$  quotient of  $U(1)$   $\mathcal{N} = 4$  SYM. The chiral primaries in the parent theory correspond to symmetric traceless products of  $n$  scalars  $\phi^I$ , which has conformal dimension 1 (see for instance [25]). Let us look at the case  $n = 2$  and check which operators survive the quotient. We have that  $\mathbf{6} \times \mathbf{6} = \mathbf{1} \oplus \mathbf{15} \oplus \mathbf{20}'$  where the symmetric traceless piece corresponds to the  $\mathbf{20}'$ . This is the only chiral primary of conformal dimension  $\Delta = 2$ .

Since  $\mathbb{Z}_k$  (for  $k > 2$ ) breaks the R-symmetry  $SU(4)_R \rightarrow S(U(3) \times U(1)_P)$ , we split the scalar fields as follows,

$$\mathbf{6} \rightarrow \mathbf{3}_{1,1} \oplus \bar{\mathbf{3}}_{-1,-1}. \tag{3.14}$$

The first subindex denotes the charge under  $U(1)_P$  (in some normalization) and the second is the charge under  $\mathbb{Z}_k$  (which is an integer mod  $k$ ). All we have to do now is take the  $\mathbf{20}'$  we had in the original theory, decompose it as in (3.14) and keep only the states invariant under  $\mathbb{Z}_k$ . This gives

$$\mathbf{20}' \rightarrow \mathbf{8}_{0,0} \oplus \mathbf{6}_{2,2} \oplus \bar{\mathbf{6}}_{-2,-2}, \tag{3.15}$$

so the last two factors are not invariant under  $\mathbb{Z}_k$  (for  $k > 2$ ) and are projected out. The remaining representation  $\mathbf{8}_{0,0}$  clearly does not admit the action of  $SU(4)_R$  so we cannot have an enhancement to  $\mathcal{N} = 4$ .

This argument is in fact also valid for interacting theories since the conformal dimension is protected for chiral primaries, so we conclude that there is no enhancement to  $\mathcal{N} = 4$  for any of the theories with  $k > 2$ .

As a final comment, notice that the fact that the marginal operator associated with changes in the Yang-Mills coupling is projected out can also be understood from this viewpoint. The relevant marginal operator is of the form  $F \wedge \star F + iF \wedge F + \dots$ , given by a component with conformal dimension 4 in the  $n = 2$  multiplet. It is a singlet of  $SU(4)_R$ , but transforms nontrivially under  $SL(2, \mathbb{Z})$  for  $k > 2$  (for instance, for  $k = 4$  we have  $(F, \star F) \rightarrow (\star F, -F)$ , so the multiplet gets an overall minus sign), so it gets projected out in the quotient  $\mathcal{N} = 3$  theory.

### 3.4 Large $N$ limit

One can place an arbitrary number of D3 branes on top of the OF3 planes, so a natural question is whether the system admits a dual holographic description at large  $N$ . The



orbifold description of the theory suggests that the answer is positive: start with the holographic dual of  $U(N)$   $\mathcal{N} = 4$  SYM, and quotient by  $r_k \cdot s_k$ . The  $r_k$  generator becomes a freely acting  $\mathbb{Z}_k$  action on the  $S^5$ , while  $s_k$  maps to the  $SL(2, \mathbb{Z})$  duality of IIB string theory, as usual.

As in previous cases, the  $k = 2$  case [16] may be illuminating. We have that the orientifold projection maps to the  $\mathbb{Z}_2$  involution  $\sigma: S^5 \rightarrow \mathbb{RP}^5$ . In addition to the geometric action, it acts with  $(-1)^{FL}\Omega$  on the theory, which is encoded in a reflection of the torus fiber, if we represent the background in F-theory.

An equivalent but perhaps clearer description is to start with the F-theory description of the system, as  $AdS_5 \times S^5 \times T^2$ , and taking a freely acting quotient by a  $\mathbb{Z}_k$  symmetry acting simultaneously on the  $S^5$  and  $T^2$  factors.

Either way, it is clear that the axio-dilaton of the the holographic system will be projected out, in accordance with the fact that the dual  $\mathcal{N} = 3$  theory on the boundary has no marginal deformation associated with the complexified gauge coupling. This fact makes the dual gravity description somewhat subtle, but it may still be approachable in the M-theory picture.

It is also interesting to compute the amount of supersymmetry preserved by the holographic dual. We do so by first looking at  $N$  D3 branes probing an  $OF3_k$  and then taking the near horizon limit. The computation runs parallel to that in section 3.2, and it is in fact more standard (see for example [7, 26]), so we will be brief.

Type IIB has two supercharges  $Q_{10}^+$  (Weyl spinors of positive helicity) in ten dimensions, which are decomposed under  $SO(1, 9) \rightarrow SO(1, 3) \times SO(6)$  as

$$Q_{10}^+ = (Q_4^+ \otimes Q_6^+) \oplus (Q_4^- \otimes Q_6^-), \tag{3.16}$$

where the first (second) term is a positive (negative) helicity Weyl spinor in both four and six dimensions. As explained earlier,  $r_k$  corresponds to an R-symmetry rotation, which is realized in the ten dimensional perspective as a rotation in the six dimensions transverse to the  $OF3_k$ . Thus, we have that  $Q_6^-$  transforms as (3.11) and analogously for  $Q_6^+$ . Furthermore we have that  $Q_6^\pm$  has charge  $\pm \frac{1}{2}$  under the S-duality  $U(1)$  bundle (3.7) so we find that  $s_k$  is given by

$$s_k : Q_6^\pm \rightarrow e^{\pm \pi i/k} Q_6^\pm, \tag{3.17}$$

and under the combined action  $r_k \cdot s_k$  only twelve supercharges survive for  $k > 2$ . Here we see again that it is crucial to include a non-trivial action under  $SL(2, \mathbb{Z})$  to preserve any supersymmetry.

Finally, the presence of  $N$  parallel D3 branes does not break supersymmetry further so the full system is indeed  $\mathcal{N} = 3$  from a four dimensional viewpoint. This is still true once we take the near horizon limit which gives Type IIB on  $AdS_5 \times (S^5/\mathbb{Z}_k)$  with a non-trivial  $SL(2, \mathbb{Z})$  bundle over  $S^5/\mathbb{Z}_k$ .

## 4 Conclusions

In this paper we have argued for the existence of interesting nontrivial theories arising from D3 branes probing what are essentially non-perturbative F-theory generalizations

of the O3 plane. For the cases that we have studied, the existence of these generalized orientifold planes projects out the axio-dilaton, so the resulting theories have no marginal deformations associated to the coupling.

The simplest case to study, beyond the well understood O3 plane, are the OF $3_k$  planes with  $k \in \{3, 4, 6\}$ . We have seen that they preserve twelve supercharges, providing the first examples (beyond supergravity) of  $\mathcal{N} = 3$  theories in four dimensions. These theories have to be necessarily somewhat exotic in order to avoid well-known theorems about enhancement to  $\mathcal{N} = 4$ , and we have shown how indeed it seems to be the case that our theories evade the assumptions of the no-go results.

The F-theory construction provides an intuitive way of understanding these field theories as exotic ‘‘S-duality orbifolds’’ of ordinary  $\mathcal{N} = 4$  theories, which explain readily some of their most puzzling properties. When it comes to further developments, it may be easier to study the string theory realization of the theories instead, and in this paper we have given some concrete steps in this direction, providing an explicit M-theory realization.

**Further directions.** Our analysis has been focused on arguing for the existence of these theories, and finding some of their most elementary properties. There are clearly a large number of further avenues of study, we will highlight here a few.

A first observation is that if we compactify the system on a circle, it admits a dual description as a stack of M2 branes moving on a background with singularities of the type  $\mathbb{C}^4/\mathbb{Z}_k$ . So the compactified theories flow at low energies to ABJM [7, 8] at certain loci of their moduli spaces. We should thus be able to gain quite a bit of insight into these  $\mathcal{N} = 3$  theories by studying their flow to the well understood (by comparison) ABJM theories. As an example, one can hope to gain information about the four dimensional theories by studying the superconformal index of appropriate ABJM theories. The  $k = 1, 2$  cases have in fact been approached in a related way [27], and it would be rather interesting to generalize this analysis in order to learn more about the class of theories introduced in this note.

Also, we have been strongly guided by the IIB string construction, but this comes at the risk of missing possible consistent theories. It would be desirable to sharpen the purely field theoretical description of our construction, in order to have a purely field theoretical understanding of which orbifolds are allowed and which extra massless particles one must include for each choice of field theory orbifold. For instance: we started with  $U(N)$  theories, coming from the D3 branes, but this is certainly not the only known example of  $\mathcal{N} = 4$  theories. Perhaps other  $\mathcal{N} = 3$  theories, beyond the ones discussed here, can be constructed starting from  $\mathcal{N} = 4$  theories with other gauge groups. The harmonic superspace formulation of  $\mathcal{N} = 3$  SYM may be helpful in this regard [28–34].

Conversely, the string picture clearly shows that there are other, less supersymmetric, generalized orientifold planes that one can construct in F-theory. There are certainly a number of less supersymmetric M-theory orbifolds we could have taken, and we could also try to study other non-orbifold elliptic fibrations with complex codimension four singularities. Often it will not be possible to deform or resolve the resulting geometries into a neighboring smooth Calabi-Yau, so they are out of reach of conventional F-theory tech-

niques (with possibly the remarkable exception of [35]). Nonetheless the close connection between these generalized orientifolds and ABJM-like theories — a field in which much progress has been achieved in the last years — gives a promising window into this interesting class of constructions.

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## A Self-duality of $O3^-$

On the one hand, we claim that the theory associated to  $N$  physical D3-branes probing an  $O3^-$  has gauge group  $O(2N)$  rather than  $SO(2N)$ . On the other, there are arguments suggesting that such a theory should map to itself under S-duality (but with different coupling generically). It is well known that the theory with gauge group  $SO(2N)$  is indeed self-dual and we would like to show in the following that the same is true for  $O(2N)$ . We do so for the case  $N = 1$ , since it corresponds to a free theory and the duality can be performed explicitly at the level of the path integral (see for instance [36]). However, we expect the same to be true also for arbitrary  $N$ , we give a heuristic argument for this below.

The partition function of the theory (in Euclidean signature) is given by

$$Z = \sum_E \int \mathcal{D}\phi \mathcal{D}\lambda \mathcal{D}A e^{-S_E} \tag{A.1}$$

where  $S_E$  is the classical action for  $\mathcal{N} = 4$  SYM with gauge algebra  $\mathfrak{so}(2)$ . The global structure of the gauge group enters in two different places. First, in the sum over topologically distinct gauge bundles  $E$ . For gauge group  $O(2)$ , one must sum over all bundles with transition functions valued in  $O(2)$  which are, in general, not in  $SO(2)$ .<sup>4</sup> Second, as explained in the main text, the  $\mathbb{Z}_2$  in  $O(2)$  which is not in  $SO(2)$  acts on the different fields with a minus sign. Thus, two field configurations related by such  $\mathbb{Z}_2$  should be considered equivalent and only included once in the path integral.

Since the theory is free, the dynamics of the scalars, fermions and vectors decouples and we can focus just on the duality for the vectors, which already shows the relevant point. Thus, consider the path integral

$$\hat{Z} = \sum_E \int \mathcal{D}A e^{-\hat{S}_E} \tag{A.2}$$

with

$$\hat{S}_E = \int_{\mathbb{R}^4} \frac{1}{2e^2} F \wedge \star F - \frac{i\theta}{8\pi^2} F \wedge F \tag{A.3}$$

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<sup>4</sup>When the spacetime is simply-connected we actually have that every  $O(2)$  bundle is equivalent to an  $SO(2)$  bundle [37]. However, we will also be interested in the theory on  $\mathbb{R}^3 \times S^1$  where this becomes important. See for instance the comment in footnote 1.

where we took spacetime to be  $\mathbb{R}^4$  for simplicity. Notice that  $\hat{S}_E$  is invariant under the action of  $\mathbb{Z}_2$  which flips the sign of the field strength  $F$ . Following [36] we can rewrite this integral as

$$\hat{Z} = \int \mathcal{D}F \mathcal{D}B e^{-\hat{S}_E + i \int B \wedge dF} \tag{A.4}$$

where the integration over the one-form  $B$  is included so that once we integrate over it we effectively restrict ourselves to closed field strengths. For the case in which spacetime in  $\mathbb{R}^4$ , this is indeed equivalent to (A.2). If, on the other hand, we integrate over  $F$ , we end up with the dual description of the theory in which  $B$  is the gauge potential and the gauge coupling is  $-1/\tau$ . The crucial point is that in order to make the extra term  $B \wedge dF$  invariant under  $\mathbb{Z}_2$ , we must declare that  $B$  is odd under it. Thus, we see that that the  $\mathbb{Z}_2$  that acts on the electric variables must act *at the same time* on the magnetic ones. In other words, the dual theory is again given by an  $O(2)$  theory rather than  $SO(2)$ . This can also be seen from the realization of the  $\mathbb{Z}_2$  as the product of R-symmetry and  $SL(2, \mathbb{Z})$ , as explained in section 2.

There is a slightly different way to see this which applies to  $O(2N)$  for any  $N$ . One can regard the  $O(2N)$  theory as arising from a  $\mathbb{Z}_2$  quotient of  $SO(2N)$ , where the  $\mathbb{Z}_2$  acts on both the electric and magnetic descriptions in the same way. Thus, since the original  $SO(2N)$  theory is self-dual, we expect the same is true for  $O(2N)$ .

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