Naked Exclusion: Comment

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The ability of an incumbent firm to deter entry by writing exclusionary contracts with customers has been a subject of contention in the antitrust literature. The courts' concern with such exclusionary contracts has been challenged by those who argue that an incumbent, faced with buyers whose interest is to promote entry and competition, would have to pay buyers more for the inclusion of exclusionary provisions than it could possibly gain from exclusion.

In a provocative article, Eric B. Rasmusen et al. (1991) (henceforth, RRW) have argued that an incumbent may in fact be able to exclude rivals profitably using such contractual provisions because it can exploit buyers' lack of coordination. In essence, if buyers expect other buyers to sign such provisions, then they may see little reason not to do so themselves. The RRW argument is potentially an important one because most alleged instances of entry deterrence through exclusionary contracts with customers occur in situations with multiple buyers.[1,2]

Unfortunately, however, RRW's two main results contain errors. In this Note, we reconsider the RRW model, providing correct characterizations of the likelihood and cost of exclusion for an incumbent firm. Our results indicate that while the intuition suggested by RRW is valid, the equilibrium likelihood and cost of exclusion differ from what RRW derive. Moreover, our analysis illuminates some further aspects of exclusionary contracting with multiple buyers. Among these issues, we focus on how an incumbent can use discriminatory offers to exploit the externalities that exist among buyers in the provision of competition.[3]

In Section I, we review the basic assumptions of the RRW model. In Sections II and III we then consider, in turn, the cases of simultaneous and sequential offers studied by RRW. For the simultaneous model, we distinguish between settings in which the incumbent can and cannot discriminate in its offers to different buyers. (This distinction is the source of the problem in RRW's analysis: their simultaneous-offer model assumes no discrimination, but the proof of their Proposition 2 assumes—at times—that discrimination is feasible.) We show that absent the ability to discriminate, the incumbent can exclude profitably only when buyers fail to coordinate on their most preferred continuation equilibrium. In contrast, we show that when discrimination is possible, the incumbent need not rely on a lack of buyer coordination to

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[1] For example, a recent Department of Justice investigation concerned the use of exclusive contracts by the leading provider of computerized ticketing services, Ticketmaster. Most major cities have several large concert/sports venues. In many cities, Ticketmaster has exclusive contracts with a very large share of these venues.

[2] The other leading response to the argument that contracts with buyers cannot be a profitable method of exclusion is given in Philippe Aghion and Patrick Bolton (1987 Section I). One factor limiting the applicability of the Aghion and Bolton theory, however, is that it involves *partial* exclusion: the contracts signed are not fully exclusionary (they involve finite stipulated damages) and the profitability of the strategy arises from extracting rents from an entrant *when entry occurs.* Indeed, in the Aghion and Bolton model, contracts that fully exclude offer no benefits over writing no contract at all. In Section III of their paper, Aghion and Bolton also discuss a two-buyer version of their model in which externalities arise across buyers. In contrast to the analysis here and in RRW, they allow the incumbent to make an offer to a buyer i that is conditional on the acceptance decision of the other buyer.

[3] R. Innes and R. J. Sexton (1994) also discuss the use of discriminatory offers in the context of a model in which buyers can form coalitions with the entrant.

exclude profitably: discrimination allows the incumbent to successfully exploit the externalities that exist across buyers.

For the sequential model, we provide a correct characterization of equilibrium play (RRW's analysis failed to complete the necessary backward induction). Our results confirm RRW's finding that the incumbent's ability to deal with buyers sequentially strengthens its ability to exclude. However, our result for the sequential model also has important differences from the characterization given in RRW. We show, for example, that as the number of buyers becomes large, the externalities across buyers become so severe that the incumbent is always able to exclude for free.

In Section IV we introduce the possibility for the incumbent to offer partially exclusionary contracts (stipulated damages) as in Aghion and Bolton (1987). We show that although this lowers the likelihood of total exclusion, the basic thrust of our previous analysis remains unchanged. Section V concludes.

I. The Model

The model has three sets of agents: an incumbent firm (I), a potential rival (R), and a set of N buyers. There are three basic periods to the game: In period 1, the incumbent offers buyers exclusionary contracts. (The precise structure of this period will be specified in later sections.) Following RRW, we assume that an exclusionary contract commits the buyer to purchasing only from the incumbent.[4] In period 2, firm R decides whether to enter or not. In period 3, active firms name prices.[5] The incumbent is able to discriminate between those buyers who have signed an exclusive contract and those who have not (the "free" buyers). The former are offered a (unit) price p_s, while the latter are offered a price p_f. The rival, if it has entered, is able to make offers only to free buyers. It offers them a price p_r. Each buyer has a demand function $q(\cdot)$ with $q'(\cdot) < 0$. We denote the number of buyers who have signed an exclusionary contract by S. We also define $CS(p) = \int_p^\infty q(s)\, ds$ to be a buyer's surplus at price p.

RRW assume that the incumbent and the rival have the same average cost function $c(\cdot)$, with $c(Q) = \bar{c}$ if $Q \geq Q^*$, and $c'(Q) < 0$ at all $Q < Q^*$. Firm R enters in period 2 if and only if it can make nonnegative profits splitting the free market at a price of $\bar{c}$ and, if firm R enters, the prices offered to the free buyers in period 3 are $p_f = p_r = \bar{c}$.[6] Following RRW, we define $p^m = \arg\max_p (p - \bar{c})q(p)$ to be firm I's optimal price to a buyer who has signed an exclusionary contract, as well as to those who have not in the event of no entry by firm R. We also let $\pi = (p^m - \bar{c})q(p^m)$ denote firm I's monopoly profit per buyer, and $x^* = CS(\bar{c}) - CS(p^m)$ denote the extra consumer surplus enjoyed by a free buyer in the event of entry. The difference between x^* and π is due to the deadweight "triangle" loss from monopoly pricing; we will assume that this loss is strictly positive (with the exception of our analysis in Section IV), hence $\pi < x^*$.[7]

The presence of scale economies means that there is an integer N^* such that firm R enters if and only if the number of buyers that have signed exclusive contracts (S) is less than N^*. Specifically, letting $\lceil z \rceil$ denote the smallest integer greater than or equal to z, we have $N^* = \lceil N - 2Q^*/q(\bar{c}) \rceil$. When N^* equals N, firm I can exclude firm R only by signing all buyers to exclusionary contracts. In this case, RRW's

[4] We consider the possibility of partially exclusionary contracts (i.e., stipulated damages for purchasing from firm R) in Section IV.

[5] RRW also allow for sales by firm I in period 1, but we drop these because they play no role in the analysis.

[6] There are some problems with RRW's specific formulation of periods 2 and 3 (e.g., firm R's optimal entry and pricing strategies are actually indeterminate). Here we simply adopt the outcome of periods 2 and 3 used by RRW. In Section IV, however, we provide an alternative model that leads to these same period 2 and 3 outcomes.

[7] If the incumbent were able to write arbitrary contracts with buyers, it would be able to eliminate the distortion under an exclusionary contract (i.e., we would have $x^* = \pi$). To rule this out, we assume that (1) contracts in period 1 can incorporate an exclusivity provision, but no agreement on the price of the good in period 3, and (2) offers in period 3 can specify only a per unit price. The first assumption can be justified in circumstances in which the precise nature of the good to be delivered in period 3 is not known in period 1. The second assumption can be justified formally when each buyer wants at most one unit of the good and has a random reservation price [$q(p)$ is then the probability that the consumer buys at price p].

"triangle loss argument" applies and profitable exclusion is impossible: since every buyer is pivotal for whether entry occurs, each requires a payment of at least x^* to sign an exclusive contract. But, if so, exclusion is unprofitable for the incumbent since it earns at most π in period 3 from each buyer it signs to an exclusive contract.[8] In the remainder of this Note, we focus on cases in which $N^* \in (0, N)$.

II. Simultaneous Offers

In this section, we study the potential for exclusion when period 1 consists of firm I simultaneously announcing a set of offers to buyers, and the N buyers then simultaneously accepting or rejecting the offers made. Formally, we let $x_i \geq 0$ denote the amount of money firm I offers to pay buyer i for signing an exclusive contract, and we let $s_i \in \{0, 1\}$ denote buyer i's response, with $s_i = 1$ denoting acceptance and $s_i = 0$ denoting rejection. The number of buyers who accept is $S = \sum_i s_i$.

In the RRW model the incumbent makes a single (nondiscriminatory) offer x to all buyers.[9] We begin by considering this case, which amounts to the restriction that $x_i = x_j$ for all $i \neq j$:

PROPOSITION 1: *When the incumbent makes simultaneous offers to buyers and is unable to discriminate, period* 1 *play in a subgame-perfect equilibrium can take the following forms:*

(*i*) EXCLUSION EQUILIBRIA: $x \in [0, \pi]$ *and* $S > N^*$, *with* $S = N$ *whenever* $x > 0$;
(*ii*) NO EXCLUSION EQUILIBRIA: $x \in [0, x^*]$ *and* $S = 0$.

PROOF:

Consider the continuation play following an offer of x. If $x > x^*$, the unique continuation equilibrium has all buyers accept firm I's offer since the most a buyer could ever gain by rejecting is x^* (this occurs when entry follows the buyer's rejection). When $x \leq x^*$, we get multiple continuation equilibria, with exclusion succeeding in some continuation equilibria, and failing in others. In particular, we have the following set of buyer acceptances in continuation equilibria:

Offer (x)	Number accepting (S)
$x = x^*$	$S \in [0, N^*)$; $S = N$
$x \in (0, x^*)$	$S = 0$; $S = N$
$x = 0$	$S = 0$; $S \in (N^*, N]$.

Note that for any $x \leq x^*$, there is always a continuation equilibrium in which $S = 0$.

In any subgame-perfect equilibrium in which exclusion succeeds we must have $x \leq \pi$, since otherwise firm I would earn negative profits (it can assure itself nonnegative profits by offering $x = 0$). Also, in any subgame-perfect equilibrium in which exclusion fails, we must have $x \leq x^*$. This establishes the bounds on x given in the two types of equilibria described in the proposition. The number of buyers who can accept in these equilibria follows from the table above [the fact that $S = 0$ in any NO EXCLUSION equilibrium follows because we can have $S \in (0, N^*)$ only if $x = x^*$, in which case firm I would be better off offering $x = 0$]. Finally, to verify that the configurations of period 1 play described in the proposition all constitute subgame-perfect equilibrium play, note that firm I earns $N(\pi - x) \geq 0$ in an EXCLUSION equilibrium, and zero in a NO EXCLUSION equilibrium. Thus, all of these equilibria can be sustained by having the continuation equilibrium following any offer $\hat{x} \neq x$ with $\hat{x} \in [0, x^*]$ be such that $S = 0$ (in which case firm I earns zero).

According to Proposition 1, when offers must

[8] This result can be overturned with other models of *ex post* competition. Suppose, for example, that there is a single buyer and that stage 3 competition is such that firm R's entry in the absence of an exclusive contract is socially inefficient [as in N. Gregory Mankiw and Whinston (1986)]. In that case, firm I and the buyer have an incentive to sign an exclusive since their joint surplus must fall whenever firm R finds entry profitable. Here these issues do not arise because firm R's entry generates a positive aggregate externality. Similar issues arise when the buyer can form a coalition with firm R, as Innes and Sexton (1994) have shown.

[9] As we mentioned earlier, there is actually some ambiguity here, because although RRW's model assumes no discrimination, the proof of their Proposition 2 assumes, at some points, that discrimination is possible [specifically, in establishing that exclusion must occur under their condition (3), RRW allow firm I to make the exclusionary contract offer to only a subset of buyers, a form of discrimination].

be nondiscriminatory, there are always both exclusionary and nonexclusionary equilibria. Note, however, that since the payment in an exclusionary equilibrium is bounded above by $\pi < x^*$, all buyers would be better off if all instead rejected firm I's offer. Thus, it appears that when offers must be nondiscriminatory firm I is able to successfully exclude only if buyers fail to coordinate on their most preferred continuation equilibrium.

This point can be made more formally by restricting attention to equilibria in which, following firm I's offer, subsets of buyers are able to make nonbinding agreements among themselves concerning their accept/reject decisions. This idea is captured by the concept of a perfectly coalition-proof Nash equilibrium (PCPNE) [B. Douglas Bernheim et al. (1987)]. This concept requires that equilibria be immune to self-enforcing coalitional deviations. Formally, we then have the following.

PROPOSITION 2: *When the incumbent makes simultaneous offers to buyers and is unable to discriminate, exclusion cannot occur in any perfectly coalition-proof Nash equilibrium.*

PROOF:

We argue that any PCPNE must be a NO EXCLUSION subgame-perfect equilibrium. Any PCPNE must be a subgame-perfect equilibrium and must involve coalition-proof Nash equilibrium behavior in the continuation subgames following firm I's offer [see Bernheim et al. (1987)]. Since $x < x^*$ in any EXCLUSION equilibrium, every buyer must receive a payoff strictly less than $CS(\bar{c})$. Thus, a joint deviation in which every buyer rejects firm I's offer [and thereby earns exactly $CS(\bar{c})$], would strictly increase every buyer's payoff. Moreover, since this is every buyer's maximal possible payoff in the subgame following firm I's offer, this deviation is necessarily self-enforcing. Hence, only NO EXCLUSION equilibria can be perfectly coalition-proof.

Thus, once we allow buyers to coordinate their responses to firm I's exclusionary offers (in a nonbinding manner), exclusion never succeeds if offers must be nondiscriminatory.

Typically, however, we would expect discrimination to be feasible whenever informational conditions are such that firm I can enforce exclusivity provisions (at least absent any laws prohibiting discrimination). In the remainder of this section we explore how allowing discriminatory offers affects this result. Once we allow for discrimination, a question arises as to whether a buyer can observe the offers made to other buyers prior to making its decision (in the absence of discrimination, a buyer can infer this directly from its own offer). In what follows, we focus on the case of observable offers; after deriving results for this case, we comment on how these results change when offers are unobservable.

PROPOSITION 3: *When the incumbent makes simultaneous offers to buyers and is able to discriminate, period 1 play in a subgame-perfect equilibrium can take the following forms.*[10]

Case A: $N\pi > N^*x^*$.

(*i*) $S > N^*$, $\sum_i s_i x_i \le N^*x^*$, $x_i = 0$ *whenever* $s_i = 0$.
(*ii*) $S = N^*$, $x_i = 0$ *when* $s_i = 0$, $x_i = x^*$ *otherwise.*

Case B: $N\pi < N^*x^*$.

(*i*) $S > N^*$, $\sum_i s_i x_i \le N\pi$, $x_i = 0$ *whenever* $s_i = 0$.
(*ii*) $S = 0$, $x_i \in [0, x^*]$ *for all* i.

Moreover, only the period 1 *equilibrium plays described in Cases A(ii) and B(ii) arise in a perfectly coalition-proof Nash equilibrium.*

PROOF:

We begin with classifying all the continuation equilibria following firm I's offers $(x_1, \ldots, x_N)$ into three types:

1. $S < N^*$ (exclusion fails). Then for every buyer i we can have $s_i = 1$ only if $x_i \ge x^*$

[10] When $N\pi = N^*x^*$, all of the listed equilibria are subgame-perfect equilibria, and the set of perfectly coalition-proof equilibria consists of the equilibria described in Cases A(ii) and B(ii).

(otherwise it could profitably deviate by rejecting), and $s_i = 0$ only if $x_i \leq x^*$ (otherwise it could profitably deviate by signing). Observe that this continuation equilibrium can only arise when $|\{i : x_i > x^*\}| < N^*$. Firm I's payoff in this equilibrium is $\sum_i s_i(\pi - x_i) \leq S(\pi - x^*) \leq 0$.

2. $S = N^*$ (exclusion occurs). Then every buyer i with $s_i = 1$ is pivotal: if it rejects, exclusion fails. For every such buyer we must have $x_i \geq x^*$, since otherwise it could profitably deviate by rejecting. Also, every buyer i with $s_i = 0$ must have $x_i = 0$, since otherwise it could profitably deviate by signing. Firm I's payoff in this equilibrium is $N\pi - \sum_i s_i x_i \leq N\pi - N^*x^*$.
3. $S > N^*$ (exclusion occurs). Then every buyer i with $s_i = 0$ must have $x_i = 0$, since otherwise it could profitably deviate by signing.

Now we can show that any play described in the proposition can indeed arise in a subgame-perfect equilibrium. To see this, consider an equilibrium in which buyers respond to any firm I's deviation $(\hat{x}_1, \ldots, \hat{x}_N) \neq (x_1, \ldots, x_N)$ by playing a type 1 (nonexclusive) continuation equilibrium if $|\{i : \hat{x}_i > x^*\}| < N$, and playing a type 3 continuation equilibrium otherwise. Given that buyers use such strategies, a deviation never brings firm I more than $\max\{0, N\pi - N^*x^*\}$. But the firm earns at least as much in all the plays described in the proposition, hence any such play can be sustained in a subgame-perfect equilibrium.

We next argue that no other period 1 subgame-prefect equilibrium play exists in the two cases given in the proposition:

Case A: In this case firm I can earn a profit arbitrarily close to $N\pi - N^*x^* > 0$ by offering $x^* + \varepsilon$ to N^* buyers for $\varepsilon > 0$ sufficiently small, which forces buyers to play an exclusive continuation equilibrium. This rules out type 1 (nonexclusive) equilibria, and implies that $\sum_i s_i x_i \leq N^*x^*$. The remaining type 2 and type 3 equilibria satisfy Cases A(ii) and A(i) respectively.

Case B: Since firm I is always assured of earning 0 by offering $x_i = 0$ for all i, this rules out type 2 equilibria, in which firm I's payoff does not exceed $N\pi - N^*x^* < 0$. Type 1 and type 3 equilibria in which firm I earns nonnegative profits satisfy Cases B(ii) and B(i) respectively.

Finally, consider perfectly coalition-proof equilibria. Here we shall argue only that play following firm I's equilibrium offers in Cases A(i) and B(i) is not coalition-proof; hence, these equilibria are not PCPNEs; the argument showing that the outcomes described in Cases A(ii) and B(ii) are supportable as PCPNEs is contained in our working paper [Segal and Whinston (1996)]. Note that in both Case A(i) and Case B(i) fewer than N^* buyers are offered $x_i \geq x^*$. Since exclusion succeeds in both cases, each of the buyers who is offered $x_i < x^*$ receives a payoff strictly less than $CS(\bar{c})$. If these buyers arrange a joint deviation in which they all reject, exclusion will fail and each will receive a payoff of exactly $CS(\bar{c})$. Since this is each of these buyers' greatest possible payoff in the subgame (holding fixed firm I's offers and the other buyers' responses), this deviation is necessarily self-enforcing.

Proposition 3 shows that when discrimination is feasible exclusion necessarily occurs in a subgame-perfect equilibrium in Case A, and may or may not occur in Case B (depending on whether buyers coordinate). The result makes clear that when firm I can make discriminatory offers, it need not rely (in Case A) on a lack of buyer coordination to exclude profitably. Rather, it can turn buyers against one another, offering an exclusionary contract to only a subset of the buyers, who then impose the externality of no entry on the other buyers. This is profitable if $N\pi$, the amount earned from the N buyers under exclusion, exceeds N^*x^*, the minimum amount that must be paid (with buyer coordination) to exclude.

Comparing our Propositions 2 and 3 to RRW's Proposition 2, we can see elements of both of our results in their Proposition 2. Their bounds on payoffs coincide with those we derive in Proposition 2, because their model assumes that offers are nondiscriminatory. But their Proposition 2, like our Proposition 3, also asserts that exclusion necessarily occurs in any subgame-perfect equilibrium when $N\pi > N^*x^*$, because at one point in their proof they implicitly allow the incumbent to discriminate.

Propositions 2 and 3 are also similar to the findings of Innes and Sexton (1994 Section II, subsection A). Instead of using a coalitional refinement to model buyers' coordination on their preferred equilibrium, they obtain the same result by letting buyers make their acceptance decisions sequentially rather than simultaneously. They find that inefficient exclusion may occur if and only if the incumbent firm is allowed to make discriminatory offers.

Finally, we note that the results of Proposition 3 change very little when a buyer does not observe other buyers' offers: the only alterations are that we must have $x_i = 0$ for all i in Cases A(i) and B(i). The formal analysis of this case can be found in our working paper [Segal and Whinston (1996)].

III. Sequential Offers

In this section we analyze the game in which the incumbent approaches buyers sequentially in period 1 with offers of exclusionary contracts. Each buyer's acceptance decision is permanent, and it is observed by all other players in the game. Following RRW, we number the N stages of the period 1 game in reverse order by $T = 1, \ldots, N$, where T is the stage at which T buyers remain to be offered contracts. Firm I's decision whether to exclude at stage T depends on the comparison of continuation benefits and continuation costs at this stage. Both numbers depend on the number S of buyers who have already signed exclusionary contracts by stage T.

The continuation cost of exclusion to firm I at stage T depends on buyers' willingness to accept exclusionary contracts, which is in turn determined by their expectation of the likelihood of exclusion if they reject. The simplest case to consider is the one in which each remaining buyer is crucial for exclusion, i.e., $S + T = N^*$. Then each remaining buyer will not accept an exclusionary offer for less than x^*. Thus, the continuation cost of exclusion is $(N^* - S)x^*$. The continuation benefit of exclusion to firm I is $(N - S)\pi$, since the S "captured" customers will be charged the monopoly price in period 3 whether or not exclusion occurs. Firm I will choose to exclude if and only if the continuation cost of exclusion does not exceed the continuation benefit:[11]

$$(1) \qquad Tx^* = (N^* - S)x^* \le (N - S)\pi.$$

To develop some intuition for this condition, observe that it can be rewritten as

$$(2) \qquad (N\pi - N^*x^*) + S(x^* - \pi) \ge 0.$$

The first term is firm I's net benefit of exclusion from the *ex ante* perspective, taking into account that each buyer signing an exclusionary contract will be paid x^*. The second term is the net "sunk cost" of exclusion, assuming that S buyers have already been paid x^* for signing exclusionary contracts, and firm I will earn π on each of them whether or not exclusion actually occurs. This condition demonstrates that when S is large, a substantial portion of the cost of exclusion is already sunk, and firm I is more likely to proceed with exclusion. Therefore, firm I may be able to commit to exclusion by sinking some of its cost in the early stages of the game.

It turns out, moreover, that firm I's commitment to exclusion may allow it to exclude at zero continuation cost. For example, consider the situation in which there is one more buyer left than is necessary for exclusion, i.e., $S + T = N^* + 1$, and where $S < N^*$. If $S \ge (N^*x^* - N\pi)/(x^* - \pi)$, then condition (1) is satisfied, and the next remaining buyer knows that even if it rejects an exclusionary contract, firm I will proceed with exclusion. Therefore, this buyer is willing to sign an exclusionary contract for free. The same reasoning will apply to subsequent buyers, so firm I will achieve exclusion at a continuation cost of zero.

In contrast, if $S < (N^*x^* - N\pi)/(x^* - \pi)$, then condition (1) is violated, and the next remaining buyer knows that his rejection of an exclusionary contract prevents exclusion. This buyer will not sign an exclusionary contract for less than x^*. In order to exclude, firm I needs to sign up sufficiently many buyers for x^* each so

[11] For simplicity we assume in what follows that, at any stage in the game, firm I chooses to exclude if it is indifferent between excluding and not.

that condition (1) becomes satisfied. Once it is satisfied, the remaining buyers sign for free. Depending on how many buyers firm I needs to sign up for x^* each, it may or may not choose to exclude.

The main result of this section extends these observations to the general case. We find that, in general, the continuation cost of exclusion depends on two numbers: one is the number S of "captured" buyers, and the other is the number $k = T + S - N^*$ of buyers left in excess of that necessary for exclusion. As we have seen, under some conditions firm I will be able to exclude at zero continuation cost. Our analysis will demonstrate that the minimum number of captured buyers required for such costless exclusion at a stage where there are k more buyers left than necessary for exclusion is given by a sequence $\{S_k\}_{k=0}^{\infty}$ that can be recursively defined as follows:

$$S_0 = N^*,$$

$$S_{k+1} = \left\lceil \frac{S_k x^* - N\pi}{x^* - \pi} \right\rceil \qquad \text{for } k \geq 0.$$

In the simplest case where $k = 0$ each remaining buyer is crucial for exclusion, so free exclusion is impossible unless $S \geq N^* = S_0$. Our previous observations demonstrate that when $k = 1$, free exclusion can be achieved if and only if S satisfies condition (1), which is equivalent to $S \geq S_1$. Before we proceed to establish the general result, we establish some useful properties of the sequence S_k.

LEMMA 1: *For all* $k \geq 1$, *if*

(3) $$N - N^* \geq x^*/\pi - 1,$$

then $S_k < S_{k-1} \leq N^*$; *otherwise* $S_k = S_{k-1} = N^*$.[12]

[12] Condition (3) can be interpreted as saying that when $N^* - 1$ buyers have already signed exclusionary contracts, the incumbent is willing to pay x^* for the last buyer to sign [to see this, substitute $S = N^* - 1$ in condition (2)]. Holding the ratio N^*/N fixed, this condition is satisfied when N is sufficiently large.

PROOF:

By induction on k. For $k = 1$ we can write

$$S_1 - S_0 = \left\lceil \frac{N^* - N}{x^*/\pi - 1} \right\rceil.$$

This implies that $S_1 \leq S_0 + \lceil -1 \rceil < S_0 = N^*$ when $N - N^* \geq x^*/\pi - 1$, and $S_1 = S_0 = N^*$ otherwise. This establishes the inductive statement for $k = 1$.

Suppose the statement is true for $k \geq 1$, then for $k + 1$ we can write

$$S_{k+1} - S_k = \left\lceil \frac{S_k - N}{x^*/\pi - 1} \right\rceil \leq \left\lceil \frac{N^* - N}{x^*/\pi - 1} \right\rceil.$$

This implies that $S_{k+1} \leq S_k + \lceil -1 \rceil < S_k \leq N^*$ when $N - N^* \geq x^*/\pi - 1$, and $S_{k+1} = S_k = N^*$ otherwise. This establishes the inductive statement for $k + 1$.

We now establish the main result of this section.

PROPOSITION 4: *When the incumbent makes offers to buyers sequentially, exclusion occurs in a subgame-perfect equilibrium if and only if* $S_{N-N^*+1} \leq 0$. *If exclusion occurs, the cost of exclusion is* $\max\{x^* S_{N-N^*}, 0\}$. *Otherwise, no buyer signs an exclusionary contract.*

PROOF:

The proof is based on the following lemma.

LEMMA 2: *Suppose there are* $T \geq 0$ *stages left in the game,* $S \geq 0$ *buyers have signed, and* $S + T \geq N^*$. *Then exclusion occurs in a subgame-perfect equilibrium if and only if* $S \geq S_{S+T-N^*+1}$. *If exclusion occurs, the continuation cost of exclusion is* $\max\{(S_{S+T-N^*} - S)x^*, 0\}$. *Otherwise, no buyer signs an exclusionary contract.*

PROOF OF LEMMA 2:

By induction on $S + T \geq N^*$. We have established above that when each remaining buyer is crucial for exclusion (i.e., $S + T = N^*$), the continuation cost of exclusion is

$(N^* - S)x^* = (S_0 - S)x^* = (S_{S+T-N^*} - S)x^* \geq 0$. Exclusion occurs in this case if and only if condition (1) holds, which is equivalent to $S \geq S_1 = S_{S+T-N^*+1}$. If this condition fails, exclusion will not occur, and no new exclusionary contracts will be signed. This verifies the inductive statement for $S + T = N^*$.

Suppose the inductive statement is known to be true for $S + T = n - 1 \geq N^*$, and consider the situation where $S + T = n$. There are two cases to consider:

(a) $S \geq S_{n-N^*} = S_{(n-1)-N^*+1}$. Then the next buyer knows that even if it rejects the contract (which reduces the value of $S + T$ to $n - 1$), according to the inductive hypothesis exclusion would still occur. Thus, it is willing to sign for free. If it signs, $S + T$ will stay constant and the same reasoning will apply for the next remaining buyer, etc. Thus, in any continuation equilibrium firm I achieves exclusion at zero continuation cost.[13]

(b) $S < S_{n-N^*} = S_{(n-1)-N^*+1}$. Then the next buyer is "pivotal": if it refuses to sign the contract, then $S + T$ is reduced to $n - 1$, and according to the inductive hypothesis exclusion does not occur. Thus, the next buyer will only be willing to sign for x^*. In order to exclude, firm I needs to sign up $(S_{n-N^*} - S)$ buyers in a row for x^* each. Once the threshold is crossed, we will switch to case (a), and the remaining buyers sign for free. On the other hand, when firm I chooses not to exclude, it will not sign any additional buyers (it makes at most $x^* - \pi < 0$ on each buyer it signs). Thus, the continuation cost of exclusion is $(S_{n-N^*} - S)x^*$. Firm I will choose to exclude if and only if the continuation benefit of exclusion, $(N - S)\pi$, is greater than or equal to the continuation cost:

$$(S_{n-N^*} - S)x^* \leq (N - S)\pi.$$

Since S is an integer, this inequality can be rewritten as $S \geq \lceil (S_{n-N^*}x^* - N\pi)/(x^* - \pi) \rceil = S_{n-N^*+1} = S_{S+T-N^*+1}$. If this condition holds, exclusion will occur at the continuation cost of $(S_{n-N^*} - S)x^* > 0$, otherwise it will fail and no new exclusionary contracts will be signed.

Combining the two cases, we see that the inductive statement is true for $S + T = n$.

The proposition follows by applying Lemma 2 for $S = 0$, $T = N$.

The exclusionary condition obtained in Proposition 4 differs from the incorrect condition obtained by RRW, whose Proposition 3 states that exclusion occurs in the sequential game if and only if $N^*/N \leq (\pi/x^*)[2 - (\pi/x^*)]$.[14] The difference can be seen in the following numerical example considered by RRW.

Example 1: Suppose that $N = 100$, $\pi = 10$, $x^* = 14$. The condition obtained by RRW says that exclusion will occur if and only if $N^* < 92$. Take $N^* = 94$. Then we have $S_0 = 94$, $S_1 = 79$, $S_2 = \lceil 53/2 \rceil = 27$, $S_3 = \lceil -311/2 \rceil < 0$. This implies that $S_{N-N^*} = S_6 \leq S_3 < 0$, so our Proposition 4 predicts that exclusion does occur in this case, and it is costless to firm I.

Unfortunately, it is impossible to obtain a closed-form solution for S_k, and so the exclusionary condition obtained in Proposition 4 cannot be expressed in terms of primitive variables. However, by investigating the properties of the sequence $\{S_k\}$ we are able to establish some qualitative properties of our exclusionary condition. While different from the condition obtained by RRW, our exclusionary condition shares some of its qualitative properties. For

[13] Different continuation equilibria may differ in which of the remaining T customers sign exclusionary contracts, as long as at least $N^* - S$ of them do so.

[14] While the derivation of RRW's Proposition 3 begins correctly, they cut the inductive argument after the second step. Indeed, ignoring the integer problem (and this is a second problem with their argument), RRW's exclusionary condition is equivalent to $S_2 \leq 0$. The error is contained in the last statement of their "Situation 6." The first buyers are not necessarily safe in refusing to sign: if $T > T^{**} \geq (N^* - S) + 1$, each remaining buyer is not crucial and will sign for free. When $N > N^* + 1$, firm I can achieve this situation by signing sufficiently many buyers for x^*. This possibility brings us to the next induction step, and it makes exclusion more likely than RRW describe.

example, our exclusionary condition, like that of RRW, is weaker than the condition $N^*x^* - N\pi \leq 0$ which is necessary for exclusion in a perfectly coalition-proof equilibrium with simultaneous offers (to see this, note that the last condition is equivalent to $S_1 \leq 0$). Also, both conditions say that given N and N^*, the likelihood of exclusion is a function of the ratio π/x^*. Our exclusionary condition, just like that of RRW, implies that the closer this ratio is to one (i.e., the less distortionary exclusion is), the more likely we are to observe exclusion.

COROLLARY 1: *For a fixed N and a fixed $N^* \in (0, N)$, there exists $\beta \in (0, 1)$ such that exclusion occurs if and only if $\pi/x^* \geq \beta$.*

PROOF:
In the Appendix.

To get a feeling for how our result qualitatively *differs* from that of RRW, we can obtain an approximation to S_k by ignoring integer problems. Define $\tilde{S}_k$ by:

$$\tilde{S}_0 = N^*,$$

$$\tilde{S}_{k+1} = \frac{\tilde{S}_k x^* - N\pi}{x^* - \pi} \qquad \text{for } k \geq 0.$$

Then successive substitution enables us to write

$$(4) \quad \tilde{S}_k = N\left[1 - \left(1 - \frac{N^*}{N}\right)\left(\frac{x^*}{x^* - \pi}\right)^k\right] \qquad \text{for } k \geq 0.$$

This expression allows us to consider the following experiment. Start with a model with $\hat{N}$ buyers, and consider subdividing each buyer into m identical smaller buyers, so that the total demand does not change: each new buyer demands $q_m(p) = q(p)/m$ at any price p, and the total number of buyers is $N_m = m\hat{N}$. Denote by N_m^* the number of buyers firm I needs to sign to exclude when there are N_m buyers. Then in the RRW model we have $N_m^* = \lceil \alpha N_m \rceil$ for all m, where $\alpha = 1 - 2Q^*/(\hat{N}q(\bar{c}))$. (Note that $N_m^*/N_m \to \alpha$ as $N \to \infty$.) Substituting $N = N_m = m\hat{N}$ and $N^* = N_m^* = \lceil \alpha m\hat{N} \rceil$ in expression (4), we can see that since $x^*/(x^* - \pi) > 1$, we must have $\tilde{S}_{N_m - N_m^*} < 0$ for m large enough. If $\tilde{S}_k$ is a good enough approximation to S_k, we can therefore use Proposition 4 to establish that exclusion occurs for free for m large enough. The following result establishes that this is indeed the case.

COROLLARY 2: *As each buyer is subdivided into m identical buyers keeping total demand constant, for m sufficiently large the incumbent excludes for free.*

PROOF:
In the Appendix.

According to RRW's condition, when $\alpha = 2Q^*/(\hat{N}q(\bar{c}))$ (which equals the limit of $1 - N_m^*/N_m$ as buyers are subdivided) is small enough, exclusion does not occur even as buyers become infinitely small relative to the market. In contrast, our Corollary 2 establishes that for any level of α exclusion occurs costlessly when buyers are sufficiently small.

To understand the result intuitively, observe that as buyers become very small relative to the market, the number of buyers in excess of those necessary for exclusion goes to infinity. The first buyer offered an exclusionary contract in this situation knows for sure that it is not pivotal, so it will accept such a contract for free. This acceptance will keep the number of buyers in excess of those necessary for exclusion unchanged. The next remaining buyer will then also realize that it is not pivotal, so it will also accept an exclusionary contract for free. Continuing this reasoning, we see that firm I excludes for free.

The analysis of this section shows that sequential offering of contracts exacerbates the free-rider problem among buyers. However, it leaves unclear whether the incumbent can commit to make buyers choose sequentially if all buyers are always present in the market. In the sequential model, a buyer turns down an exclusionary offer for $x \in (0, x^*)$ only if it expects that this decision prevents exclusion. If, however, firm I could instead make further offers to this buyer, the buyer might have another reason to turn down such an offer: after the rejection, firm I may be tempted to make this buyer an exclusionary offer on better terms. In our working paper [Segal and Whinston (1996)], we analyze a game with T stages, where in each stage the incumbent can make exclusive offers to all the buyers who have not yet signed. We show that this temptation does not undermine the result of the sequential model; rather, the opportunity to approach all buyers in each of the N stages actually helps the incumbent to exclude. Intuitively, the reason is that when firm I is restricted to approach buyers sequentially, there are situations in which firm I is no longer able to exclude because many buyers have already rejected its offers, and this may put early buyers in a position to prevent exclusion by rejecting firm I's offers. This is less likely when firm I can reapproach buyers.

IV. Partially Exclusionary Contracts

Up to this point, we have assumed that firm I's contract offers take the form of a purely ("naked") exclusionary contract. One might wonder to what extent our conclusions are robust to the possibility that firm I could use partially exclusionary contracts and benefit from the presence of the entrant, as in Aghion and Bolton (1987). In this section we explore this issue in the context of the simultaneous-offer model, assuming that buyers are able to coordinate.

To consider this issue we need to introduce a model of period 3 competition in which firm R is more efficient than firm I (to provide a possible rent-extraction motive for partial exclusion). In what follows, we focus on a model in which firm R has a constant marginal production cost of $\underline{c}$ and there is a sunk cost of entry f that firm R must pay in period 2 if it decides to enter. Firm I, in contrast, has a constant marginal cost of $\bar{c} > \underline{c}$ (it has already paid its sunk cost) satisfying the condition that $(\bar{c} - \underline{c})q(\bar{c}) > (p - \underline{c})q(p)$ for all $p \in [\underline{c}, \bar{c})$. Price offers in period 3 following entry are made simultaneously. With these assumptions, absent any penalties, following entry firm R makes all sales to free buyers at price $\bar{c}$. Assuming that firm R enters if indifferent, all of our previous analysis of fully exclusionary contracts would then still apply with respect to $N^* \equiv N - \lceil f/[(\bar{c} - \underline{c})q(\bar{c})] \rceil + 1$.[15]

We now allow firm I to make contract offers of the form (t_i, x_i) to buyer i, where t_i is the penalty that buyer i pays firm I if it buys from firm R in period 3 and, as before, x_i is the payment that firm I makes to buyer i for signing the contract. (Fully exclusionary contracts correspond to $t_i = \infty$.) We assume that if firm R enters, it is able to discriminate in period 3 in its price offers to different buyers (allowing firm R to discriminate among free buyers would have had no effect on the results in previous sections).

To keep matters as simple as possible, in what follows we shall focus on the special case in which each buyer has a demand for at most one unit, and has a known reservation value $v > \bar{c}$ (our assumption of linear pricing in period 3 is fully justified in this case; see footnote 7). In this case, $q(p) = 1$ if $p \leq v$ and $= 0$ if $p > v$. Note that we now have $\pi = x^* = v - \bar{c}$, and so absent the ability to engage in partial exclusion, firm I would always exclude as long as $N^* < N$. We shall see, however, that when partial exclusion is feasible, firm I will sometimes (but not always) choose to accommodate firm R's entry in order to extract some of the surplus firm R brings to the market, much as in Aghion and Bolton (1987).

Consider first the competition for buyer i in period 3 after firm R enters. If $t_i \leq v - \bar{c}$, firm I's equilibrium price offer to buyer i will be $\bar{c} + t_i$ (at this price firm I is indifferent between winning and losing the buyer's business), firm R's equilibrium price will be $\bar{c}$, and the buyer will buy from firm R. The buyer's total cost of purchase will be $\bar{c} + t_i$.[16] Firm I and firm R's profits from buyer i are then t_i and $\bar{c} - \underline{c}$

[15] Note that in this model firm R's optimal entry and pricing strategies are well defined (recall footnote 6). Notice also that by considering a sequence of cases $\{\underline{c}_m, f_m\}_{m=1}^{\infty}$ in which $\underline{c}_m \rightarrow \bar{c}$, $f_m \rightarrow 0$, and $N - \lceil f_m/[(\bar{c} - \underline{c}_m)q(\bar{c})] \rceil + 1 = N^*$ for all m, we can allow the efficiency difference between the firms to grow arbitrarily small while maintaining the same threshold number of free buyers necessary for entry, in essence approaching the case considered by RRW.

[16] This is not the unique equilibrium. Just as in the standard Bertrand model with differing costs, there are other equilibria in which firm R wins the buyer's business at a

respectively. If, instead, we have $t_i \in [v - \underline{c}, v - \bar{c})$, then firm R's equilibrium price to buyer i will be $v - t_i$, firm I's price to this buyer will be v, and buyer i will buy from firm R. Note that in this case firm R's price is forced below $\bar{c}$ and its profit is less than $\bar{c} - \underline{c}$, its profit on a free buyer. Finally, if $t_i > v - \underline{c}$, then firm I wins buyer i's business at a price of v and firm R earns 0.

Now consider firm R's entry decision. Suppose that the vector of damage penalties for the N buyers is $(t_1, \ldots, t_N)$ (where a buyer who has rejected firm I's offer is taken to have $t_i = 0$). Then firm R enters if and only if

$$\sum_i \max\{0, \min\{\bar{c}, v - t_i\} - \underline{c}\} \geq f.$$

Note that if buyer i expects entry to occur, it is willing to accept any contract (t_i, x_i) that gives it a surplus of $v - \bar{c}$. It is immediate, therefore, that firm I can earn an amount arbitrarily close to $N(\bar{c} - \underline{c}) - f$ by offering every buyer i a contract $(\hat{t}, \hat{x})$ with penalty $\hat{t} = (v - \underline{c}) - f/N$ and payment $\hat{x} = v - \bar{c}$ (note that $\hat{t} > v - \bar{c}$ by virtue of the fact that $\bar{c} - \underline{c} > f/N$ and that, given these offers, entry is assured regardless of buyers' acceptance decisions, and so all buyers accept). This is precisely the point made by Aghion and Bolton (1987), but here in a nonstochastic setting: by being a first mover in contracting with the buyers, firm I can appropriate the efficiency gain that firm R brings to the market.

However, firm I also has another possible strategy that it can follow: it can attempt to exclude firm R. The key point is that with buyer coordination it still costs firm R exactly N^*x^* to do so (this is established formally in the proof of Proposition 5). In particular, we then get the following result.[17]

PROPOSITION 5: *Suppose that firm I can offer partially exclusionary contracts and that all buyers have a reservation value v. Then in any perfectly coalition-proof Nash equilibrium exclusion occurs if and only if*

$$N\pi - N^*x^* = (N - N^*)(v - \bar{c}) \geq N(\bar{c} - \underline{c}) - f.$$

If exclusion occurs, firm I offers N^* buyers contracts with $t_i \geq v - \underline{c}$ and a payment of $x_i = v - \bar{c}$, and earns exactly $N\pi - N^*x^* = (N - N^*)(v - \bar{c})$. If exclusion does not occur, firm I earns exactly $N(\bar{c} - \underline{c}) - f$, firm R earns 0, and every buyer has a surplus of $CS(\bar{c}) = v - \bar{c}$.

PROOF:
In the Appendix.

To understand Proposition 5's condition for exclusion, note that ignoring the integer problem (alternatively, assuming that firm R is almost indifferent about entering when N^* buyers have signed fully exclusionary contracts), we have $N^* \approx N - f/(\bar{c} - \underline{c})$, and the condition in Proposition 5 is approximately

$$\frac{v - \bar{c}}{v - \underline{c}} \geq \frac{N^*}{N} = 1 - \frac{f}{N(\bar{c} - \underline{c})}.$$

The condition shows that firm R is excluded when its fixed cost f is sufficiently high (correspondingly, N^* is sufficiently low). Thus, in our simple case the availability of partially exclusionary contracts does not eliminate the possibility that firm I will exclude firm R. Note that this may happen even though the joint welfare of firm I and the buyers is maximized by appropriating firm R's efficiency advantage through partially exclusive contracts [yielding them a joint surplus of $N(v - \underline{c}) - f$]. As before, it is the presence of externalities across buyers that may make exclusion a profitable strategy for firm I.[18]

price below $\bar{c}$. These equilibria involve firm I pricing below $\bar{c} + t_i$, which is a weakly dominated choice for firm I.

[17] Note that in the limiting case described in footnote 15 in which we let $\underline{c} \to \bar{c}$ and $f \to 0$, while keeping N^* constant, the profits obtainable by firm I from allowing entry approach 0, and the exclusionary condition identified in Proposition 5 becomes identical to that in Proposition 3.

[18] With more general demand functions $q(\cdot)$, exclusion is detrimental to the joint welfare of firm I and buyers

V. Conclusion

In this Note we put the RRW insight on a firmer foundation, offering correct results for their model and illuminating some further aspects of exclusionary contracting with multiple buyers. We showed that when an incumbent firm can make discriminatory offers of exclusive contracts to buyers, it need not rely on buyers' disorganization to successfully exclude. Rather, by exploiting the externalities present among buyers, an incumbent firm can often profitably exclude potential rivals.

The RRW argument can be nicely related to the history of debates on the use of exclusionary contracts. Suppose that we denote by $U(\mathbf{s})$ the incumbent's gross payoff given buyers' response profile $\mathbf{s} = (s_1, \ldots, s_N)$, and by $u_i(\mathbf{s})$ buyer i's gross payoff given this profile. Then, in the absence of buyer coordination, to generate buyers response profile $\mathbf{s}$ with simultaneous offers, the incumbent must pay each buyer i precisely $u_i(0, \mathbf{s}_{-i}) - u_i(\mathbf{s})$. Given this fact, the incumbent will induce buyer responses that solve

$$\max_{\mathbf{s} \in \{0,1\}^N} U(\mathbf{s}) - \sum_i \{u_i(0, \mathbf{s}_{-i}) - u_i(\mathbf{s})\},$$

or with a slight rewriting:

$$\max_{\mathbf{s} \in \{0,1\}^N} \left[U(\mathbf{s}) + \sum_i u_i(\mathbf{s}) \right] - \sum_i u_i(0, \mathbf{s}_{-i}).$$

The pre-Coasian/pre-Chicago view can be thought of as saying that the incumbent will exclude in order to maximize $U(\cdot)$. The Chicago School, however, correctly pointed out that the incumbent would have to compensate the buyers for signing, and asserted that this would imply that surplus is maximized, which is precisely the expression in square brackets above. But we see that the last term, $\sum_i u_i(0, \mathbf{s}_{-i})$, is not accounted for by this argument. It represents the sum of buyers' reservation utilities when faced with the incumbent's offers and it arises precisely because of the externality that buyers impose on one another: when many other buyers sign exclusionary contracts, buyer i's achievable utility will be low and the incumbent will not have to pay much to get this buyer to sign also. Indeed, when each buyer sees himself as nonpivotal for exclusion, $u_i(s_i, \mathbf{s}_{-i}) = u_i(0, \mathbf{s}_{-i})$ for $s_i \in \{0, 1\}$, and so the seller can exclude for free. In this case, he will exclude whenever exclusion maximizes $U(\mathbf{s})$—the pre-Coasian result! With buyer coordination, firm I may need to pay more than $\sum_i \{u_i(0, \mathbf{s}_{-i}) - u_i(\mathbf{s})\}$ to generate response profile $\mathbf{s}$, but, as we have seen, using a "divide-and-conquer" strategy, firm I is still able to exploit the externalities across buyers to raise its payoff.

This feature of the exclusionary contracting problem is in fact shared by many other settings in which a single party contracts with many other agents among whom externalities exist, such as problems of takeovers, licensing, mergers, and debt bailouts. In a recent paper, Segal (1999) addresses this issue in a more systematic way.

because of the deadweight loss from monopoly pricing considered in the previous section, as well as because of the loss of firm R's appropriable surplus considered in this section. While the qualitative conclusions of this section continue to be true in this case, the analysis of period 3 pricing in the general case is substantially more difficult (in particular, firm R may receive a strictly positive surplus in a nonexclusionary equilibrium).

Appendix: Proofs of Corollary 1, Corollary 2, and Proposition 5

PROOF OF COROLLARY 1:

The proof is based on the following three claims.

Claim 1: For any $k \geq 0$, S_k is a nonincreasing and continuous from the right function of $\pi/x^* \in (0, 1)$.

PROOF:

By induction on k. The statement is trivially true for $S_0 \equiv N^*$. Suppose the statement is true for a certain $k \geq 0$. Write $S_k(r)$ to denote S_k as a function of $r = \pi/x^*$. Then we can write

$$\text{(A1)} \qquad S_{k+1}(r) = \left\lceil N - \frac{N - S_k(r)}{1 - r} \right\rceil.$$

Since $S_k(\cdot)$ is nonincreasing by the inductive hypothesis, the fraction $[N - S_k(r)]/(1 - r)$ is nondecreasing in r, which implies that $S_{k+1}(\cdot)$ is a nonincreasing function. Take any sequence $r_n \searrow \hat{r} \in (0, 1)$. Since by the inductive hypothesis $S_k(\cdot)$ is nonincreasing and continuous from the right, we must have $S_k(r_n) \nearrow S_k(\hat{r})$. This implies that $N - [N - S_k(r_n)]/(1 - r) \nearrow N - [N - S_k(\hat{r})]/(1 - r)$. Using (A1) and the fact that $\lceil \cdot \rceil$ is continuous from the left, we have $S_{k+1}(r_n) = \lceil N - [N - S_k(r_n)]/(1 - r) \rceil \nearrow S_{k+1}(\hat{r})$. Thus, $S_{k+1}(\cdot)$ is also continuous from the right.

Claim 2: For any $k \geq 0$, $S_k = N^*$ when $\pi/x^* \in [0, 1/(N - N^* + 1)]$.

PROOF:
See Lemma 1.

Claim 3: For any $k \geq 1$, $S_k \to -\infty$ as $\pi/x^* \to 1$.

PROOF:
From (A1) we see that $S_1(r) = \lceil N - (N - N^*)/(1 - r) \rceil \to -\infty$ as $r \to 1$. Since by Lemma 1 we have $S_k \leq S_1$ for $k \geq 1$, the statement follows.

As Proposition 4 shows, exclusion occurs if and only if $S_{N - N^* + 1} \leq 0$. Claims 1–3 show that S_{N-N^*+1} is a nonincreasing and continuous from the right function of $\pi/x^* \in (0, 1)$, that it is positive for π/x^* close enough to zero, and that it is negative for π/x^* close enough to one. This implies the statement of Corollary 1.

PROOF OF COROLLARY 2:
Let S_k^m and $\tilde{S}_k^m$ be the sequences S_k and $\tilde{S}_k$ defined for the model where each of the $\hat{N}$ original buyers is subdivided into m, i.e., where $N = N_m = m\hat{N}$, $N^* = N_m^* = \lceil \alpha m\hat{N} \rceil$.

We show that for a given k, $\tilde{S}_k^m$ is a "good enough approximation" to S_k^m when m is sufficiently large. Namely, we establish that for any fixed k, we have $(S_k^m - \tilde{S}_k^m)/m \to 0$ as $m \to \infty$. We do this by induction. The statement is trivially true for $k = 0$. Suppose it is true for a certain $k \geq 0$. Using the fact that $\lceil x \rceil \in [x, x + 1)$, we can then write

$$\frac{S_{k+1}^m - \tilde{S}_{k+1}^m}{m} = \frac{1}{m}\left\lceil \frac{S_k^m x^* - N_m \pi}{x^* - \pi} \right\rceil - \frac{1}{m}\frac{\tilde{S}_k^m x^* - N_m \pi}{x^* - \pi}$$
$$\in \left[\frac{S_k^m - \tilde{S}_k^m}{m}\frac{x^*}{x^* - \pi}, \frac{1}{m} + \frac{S_k^m - \tilde{S}_k^m}{m}\frac{x^*}{x^* - \pi} \right).$$

By the inductive hypothesis both bounds go to zero as $m \to \infty$, so we must have $(S_{k+1}^m - \tilde{S}_{k+1}^m)/m \to 0$. Therefore, the inductive statement is true for all $k \geq 0$.

Substituting $N = N_m = m\hat{N}$, $N^* = N_m^* = \lceil \alpha m\hat{N} \rceil$ into (4), we obtain

$$\frac{1}{m}\tilde{S}_k^m = \hat{N}\left[1 - \left(1 - \frac{\lceil \alpha m\hat{N} \rceil}{m\hat{N}}\right)\left(\frac{x^*}{x^* - \pi}\right)^k\right].$$

Observe that $\lceil \alpha m\hat{N} \rceil/(m\hat{N}) \to \alpha \in (0, 1)$ as $m \to \infty$, therefore we can find $\delta \in (0, 1)$ and $\hat{m} \geq 0$ such that $\lceil \alpha m\hat{N} \rceil/(m\hat{N}) \leq \delta$ for any $m \geq \hat{m}$.

Using the above expression for $(1/m)\tilde{S}_k^m$ and the fact that $x^*/(x^* - \pi) > 1$, for any $m \geq \hat{m}$ we can write

$$\frac{1}{m}\tilde{S}_k^m \leq \hat{N}\left[1 - \delta\left(\frac{x^*}{x^* - \pi}\right)^k\right] \to -\infty \quad \text{as} \quad k \to \infty.$$

Hence, $(1/m)\tilde{S}_k^m \to -\infty$ uniformly on $m \geq \hat{m}$ as $k \to \infty$. In particular, there exists $\hat{k} \geq 0$ such that $(1/m)\tilde{S}_{\hat{k}}^m \leq -1$ for all $m \geq \hat{m}$. Since $(S_{\hat{k}}^m - \tilde{S}_{\hat{k}}^m)/m \to 0$ as $m \to \infty$, there exists $\bar{m} \geq 0$ such that $(1/m)S_{\hat{k}}^m \leq (1/m)\tilde{S}_{\hat{k}}^m + 1$ for all $m \geq \bar{m}$. Combining the two inequalities, we see that $(1/m)S_{\hat{k}}^m \leq 0$ for all $m \geq \max\{\hat{m}, \bar{m}\}$. Using in addition Lemma 1, for all $m \geq \max\{\hat{m}, \bar{m}, [(\hat{k} + 1)/(1 - \alpha)\hat{N}]\}$ we can write:

$$S_{N_m - N_m^*}^m = S_{m\hat{N} - \lceil \alpha m\hat{N} \rceil}^m \leq S_{(1-\alpha)m\hat{N} - 1}^m \leq S_{\hat{k}}^m \leq 0.$$

Applying Proposition 4, we see that the incumbent excludes costlessly for all m large enough.

PROOF OF PROPOSITION 5:

It follows from the discussion in the text that firm I can earn $N(\bar{c} - \underline{c}) - f$ by allowing entry. Note that it cannot earn more than this if firm R enters: aggregate surplus when firm R enters is at most $N[CS(\bar{c}) + (\bar{c} - \underline{c})] - f = N(v - \underline{c}) - f$ (since the cost of purchase is at least $\bar{c}$ for each buyer) and, if entry is occurring, each buyer can assure itself at least $CS(\bar{c}) = v - \bar{c}$ by rejecting firm I's offer.

Now suppose that there is a PCPNE in which entry does not occur and firm I pays less than $N^*x^* = N^*(v - \bar{c})$ in aggregate to buyers who sign contracts. Let $S \geq N^*$ denote the set of buyers who sign and let $S^* = \{i \in S : x_i \geq v - \bar{c}\}$. Then we must have $|S^*| < N^*$ and so $(N - |S^*|)(\bar{c} - \underline{c}) \geq f$. Now, since

$$(N - |S|)(\bar{c} - \underline{c}) + \sum_{i \in S} \max\{0, \min\{\bar{c}, v - t_i\} - \underline{c}\} - f < 0,$$

there exists a set of buyers $J \subset S \backslash S^*$ such that

$$(N - |S| + |J|)(\bar{c} - \underline{c}) + \sum_{i \in S \backslash J} \max\{0, \min\{\bar{c}, v - t_i\} - \underline{c}\} - f \geq 0$$

and such that for all $J' \subset J$ we have

$$(N - |S| + |J'|)(\bar{c} - \underline{c}) + \sum_{i \in S \backslash J'} \max\{0, \min\{\bar{c}, v - t_i\} - \underline{c}\} - f < 0.$$

But buyers in set J have a self-enforcing Pareto-improving deviation in which they all reject firm I's offer. By doing so, they each earn $CS(\bar{c}) = v - \bar{c}$, which is the most they can earn given other buyers' acceptance decisions, and which strictly exceeds their payoffs in the purported equilibrium (which is an amount strictly less than $v - \bar{c}$). This yields a contradiction: we conclude that firm I's cost of exclusion must be at least $N^*(v - \bar{c})$ in any PCPNE in which firm R does not enter. That firm I can exclude for a total cost of $N^*(v - \bar{c})$ follows from the same arguments as in Proposition 3.

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