Nash equilibrium in multiparty competition with "stochastic" voters*

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Theoretical spatial models of electoral voting tend to predict either convergence to an electoral mean (when voting is probabilistic) or chaos (when voting is deterministic). Here, we construct an empirical model of voting for the Israeli Knesset in 1992 (based on a large electoral sample and on analysis of party declarations). The probabilistic voting model so estimated fits the known election results. We then use the same model to simulate the effect of expected vote maximization by the parties. Contrary to the usual results, there is no unique convergent Nash equilibrium under this objective function. We do infer, however, that the two large parties are "Downsian", in the sense that they maximize expected vote (up to the margin of error of the model). We suggest that the empirical results are compatible with a hybrid model of utility maximization, where each party computes the effects of its policy declaration both in terms of electoral response and of post-election coalition negotations.

1. Spatial coalition theory

The last three elections in Israel (in 1988, 1992 and May 1996) provide an opportunity both to develop spatial coalition theory and to test models of "rational" voter behavior as well as Downsian [7] models of party response.

The spatial model used in this paper is based on an electoral sample (of size over 1000) collected by Arian and Shamir [5] at the time of the 1992 election. As discussed in appendix A, this sample allowed us to infer that policy in Israel is based on two dimensions: the first dimension – "National Security" – describes attitudes to the PLO and the "occupied territories" (where a low value can be interpreted as "pro-Arab"). The second dimension – "Religion" – describes beliefs about the proper

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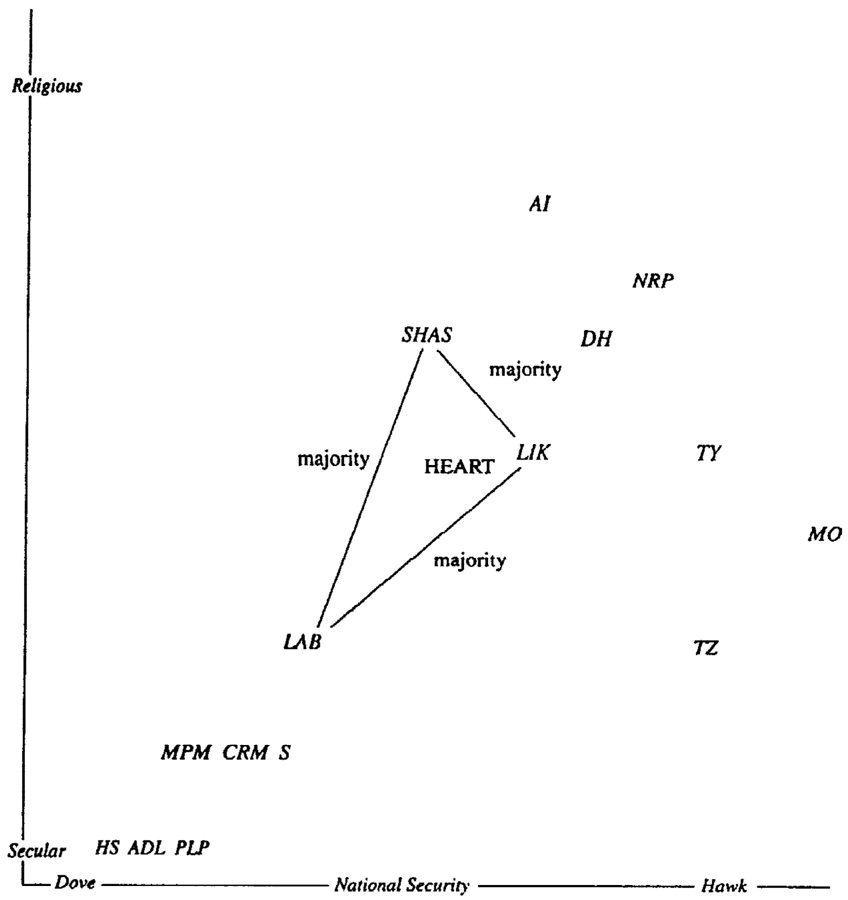

Figure 1. The Knesset in 1988. (Reprinted with permission from Schofield [28].)

relationship between the State and the Jewish religion (where a high value is associated with a belief in a strong link between the two). Using these two dimensions, we can construct "policy maps" representing the positions of the parties, as in figures 1 and 2. These figures present the approximate "location" of the parties in the Israeli Knesset after the two elections of 1988 and 1992. The acronyms used for the parties are given in table 1 together with the number of seats controlled by the parties after the elections of 1988, 1992 and 1996. The "positions" of the parties at the time of the 1992 election were obtained by examining party manifestos or pre-election declarations in order to ascertain how the parties would respond to the questions presented in the 1992 electoral survey. The 1988 positions represented in figure 1 are based either on the 1992 manifesto analysis or on a reasonable guess as to party positions. For example, the three small parties (Mapan, Shinu and CRM) contested the 1988 election separately, while in 1992 they coalesced to form Meretz. Their positions in 1988 are represented

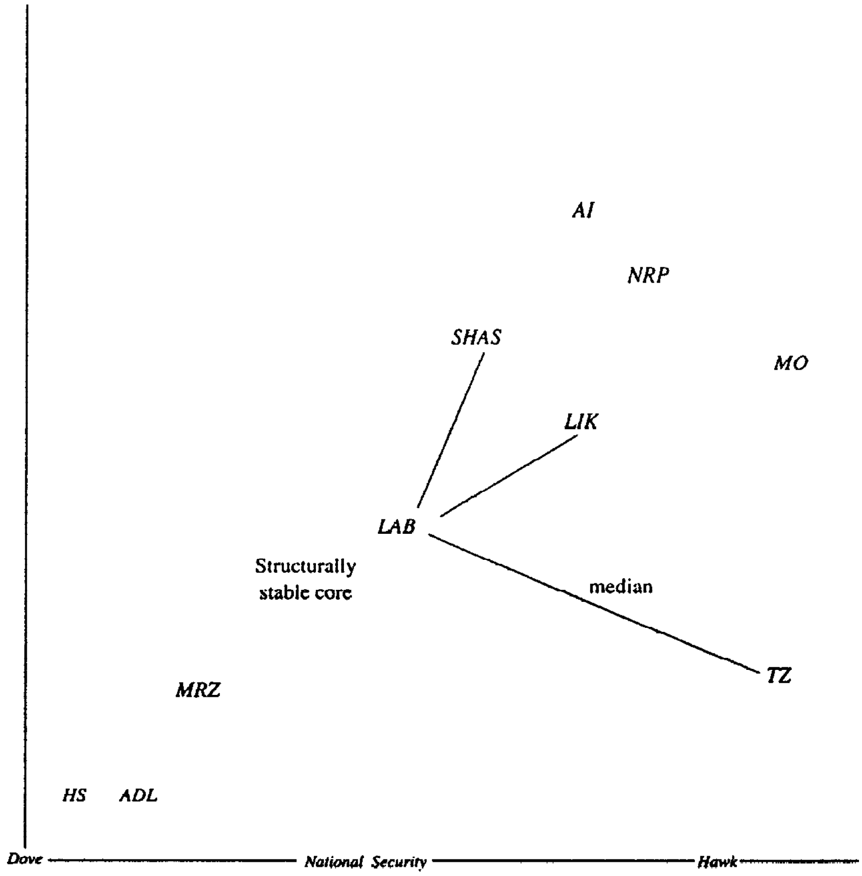

Figure 2. The Knesset in 1992. (Reprinted with permission from Schofield [28].)

as close to the 1992 Meretz position. In 1996, Tzomet formed a pre-election pact with Likud, and did not contest the election as an independent party. On the other hand, in 1996, a faction of Labor called "The Third Way" broke off from Labor and gained 4 seats, while a new party, Israel Ba'aliya (a Center "Russian" party), also contested the election and gained 7 seats. Analysis of 1996 manifestos by these two small parties reported elsewhere (Nachmias and Sened [18]) puts them close to the Likud Position.

Spatial coalition theory, as it has been developed in the past decade (Schofield et al. [29], Laver and Schofield [13], Schofield [26]), assumes that each party (j) is committed to a particular policy position (φ_j) or "bliss point" in the policy space W, and that the "utility" of the party for a policy position, x, say, is a monotonically decreasing function of a distance measure $\delta(x, \varphi_j)$ between x and φ_j.

For example, suppose that the distance measure is simply the Euclidean distance $\|x - \varphi_j\|$ between x and φ_j. After an election, the "compromise set" associated with

Table 1

Three elections in Israel.

Party (acronym)	Knesset seats		
	1988	1992	1996
Labor (LAB)	39	44	34
Meretz (MRZ)	–	12	9
CRM	5	–	–
Mapan (MPM)	3	–	–
Shinu (S)	2	–	–
PLP	1	–	–
Daroushe (ADL)	1	2	4
Hadash (HS)	4	3	5
Subtotal	55	61	52
Shas (SHAS)	6	6	10
Likud (LIK)	40	32	32
Aguda (AI)	5	4	4
Mafdal (NRP)	5	6	9
Molodet (MO)	2	3	2
Techiya (TY)	3	–	–
Degel Hatora (DH)	2	–	–
Tzomet (TZ)	2	8	–
Subtotal	65	59	57
Israel Ba'aliya (RUS)	–	–	7
Third Way (TW)	–	–	4
Total	120	120	120

any winning coalition, M, say, will simply be the convex hull $W(M)$ of bliss points of the members of M. The *core* of the post-election coalition game (if it exists) is the intersection of the family $\{W(M)\}$ across all winning coalitions. To illustrate this notion, consider figure 2 representing the situation after the 1992 election. With 120 seats in the Knesset, a majority coalition needs 61 to win; it is clear that Likud together with the religious parties (Aguda, Mafdal and Molodet) and Tzomet control only 59 seats. For this five-party coalition to attain a majority it must include another party, either Meretz or Labor. But then the convex hull of the bliss points of the coalition members will include the Labor position.[1] The conclusion is that the Labor position

[1] Another way to determine the core is to draw in the median lines (those lines through the party positions with majorities on each side). Three typical median lines are drawn in figure 2, all intersecting in the Labor position. Since this intersection is stable under perturbation of the party positions, the core so obtained is called "structurally stable" (Schofield [24]).

is the core of the two-dimensional coalition game. The theoretical inference drawn in the earlier work was that a party at the core position would be able to form a minority government. In fact, Yitzhak Rabin, the leader of Labor (until his assassination in November 1995), initially formed a majority coalition with Shas and Meretz, controlling 62 seats. The coalition was able to pursue a peace accord with the PLO. Although Shas resigned from the government in the summer of 1993, Rabin was able to face numerous votes of no confidence (with the support of Meretz alone until December 1994 and with the further support of three members of Tzomet from early 1995). Effectively, Labor formed a minority government until the May 1996 election.

To better understand the outcome of the 1996 contest between Labor (under Shimon Peres) and Likud (under Benjamin Netanyahu), it will prove useful to go back to the 1988 election.

As table 1 shows, after the 1988 election the Likud coalition, including Shas, controlled 65 seats, while the Labor coalition, excluding Shas, controlled 55. However, Shas was *pivotal* in the sense that neither the Likud nor Labor coalition could win without the support of Shas. Figure 1 (for 1988) illustrates the fact that the three median lines (through the Shas, Labor and Likud positions) do not intersect. Consequently, the core is empty. The theoretical prediction is that any one of three possible winning coalitions could occur: either a grand coalition of {Labor, Likud}, a {Likud, Shas} grouping with the support of some of the religious parties, or a {Labor, Shas} grouping with the support of the smaller “liberal” and Arab parties. Since Likud was the larger of the two major parties in 1988, it was able to move first, forming a coalition with Labor until 1990 and then one with Shas until 1992. Clearly, relatively small changes in electoral support between 1988 and 1992 led to major changes in the possible government configurations.

The situation in 1996 was even more complicated because of the simultaneous electoral competition for prime minister. Netanyahu won this election by about 30,000 votes (out of 3.12 million, with 148,000 blank or invalid ballots). Although Labor was the larger party, Netanyahu was declared Prime Minister, and constructed a majority coalition of Likud, Shas, Aguda, Mafdal, Israel Ba’aliya and the Third Way.

Figure 3 gives a representation of the spatial relationship in 1996 between Shas, Labor, Likud and the other smaller parties, based on our analysis of party manifestos. The point to note is that (as in 1988), Labor (and its “left wing” allies) can form a majority with Shas, or with Likud. Moreover, to form a majority, Likud (with the religious parties) requires either Israel Ba’aliya or “The Third Way” to construct a majority. Unlike 1992, but like 1988, the “coalitional” core in 1996 is empty.

The earlier theoretical work has suggested that, in complicated situations with an empty core, coalitional bargaining will lead into a set known as the “heart” (Schofield [25]). In the 1996 situation, as in 1988, the heart is the convex set whose boundaries comprise the three median lines through the pairs {Labor, Likud}, {Likud, Shas} and {Labor, Shas}.

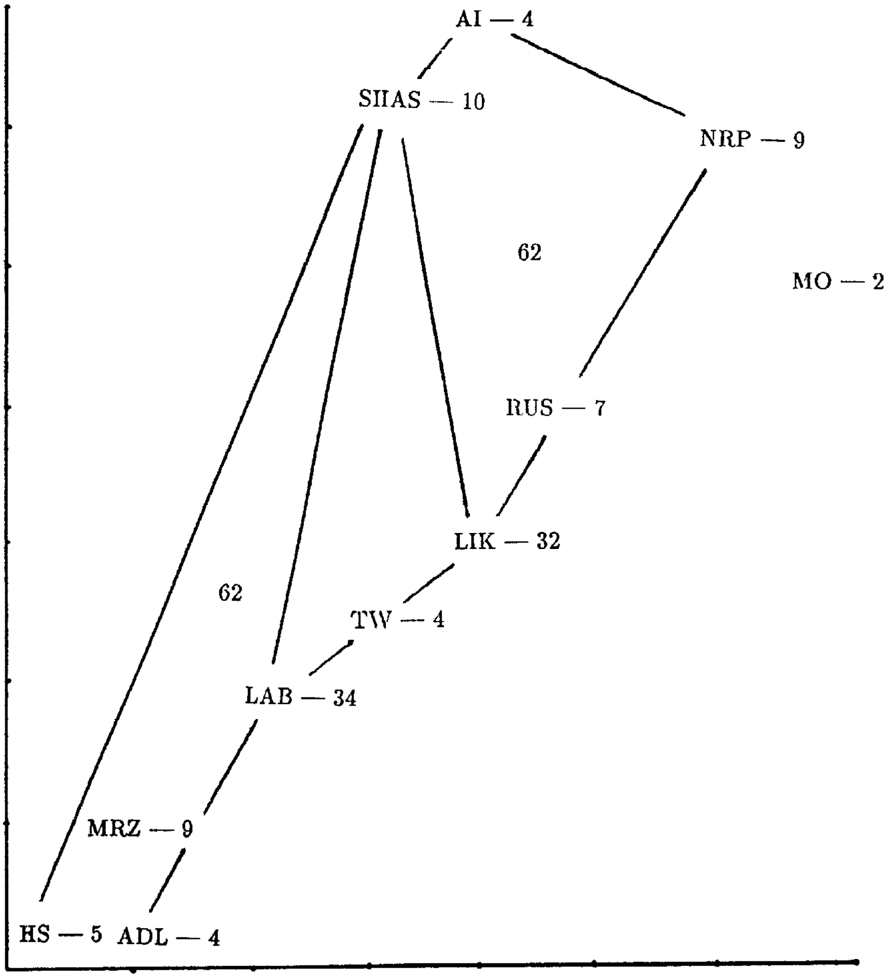

Figure 3. The Knesset in 1996, showing coalition weights.

Since Netanyahu had been appointed Prime Minister, it was not surprising that Likud (by moving first) was able to construct a majority coalition. The point to note is that Shas was pivotal, in the sense that it could break the Likud-based coalition by switching to Labor. Nachmias and Sened [18] have emphasized that the change in electoral rules for 1996 may have enhanced the bargaining power of Shas and the other small parties. In the post-election coalition bargaining between Shas and Likud, one would expect Shas to be able to extract high policy and portfolio payoffs. In particular, one may note that the Shas position is somewhat more "pro-Arab" than Likud. It is probably for this reason that Netanyahu was obliged to continue with the peace accords, and the agreement over Hebron, even against the opposition of many of the Likud members of his cabinet. Similarly, to retain Mafdal, Nethanyahu had to offer the important Ministry of Education (controlling over 40% of the budget). The analysis of the post-1996 election bargaining game provided by Nachmias and Sened [18] gives substantial support for the theoretical idea of the heart as well as the model of bargaining over both policy and payoffs as proposed in Sened [31,32].

However, these models are essentially derived from theories of committee bargaining (Schofield [23]), where the agents (parties) are assumed to have policy preferences that are common knowledge. In this paper, we instead suppose that each party j makes a declaration φ_j to the electorate with φ_j chosen for strategic reasons. We seek to understand the basis of this choice.

2. Spatial models of electoral behavior

The post-election spatial coalition model described above takes party positions and weights (seats) as given, but is mute on how the party positions and weights are obtained. In this section of the paper, we present an electoral model that attempts to derive the weights (or votes) obtained by the parties in the 1992 Knesset election. Having constructed a statistical model of voter response, we then go on in section 3 to consider models of how parties may choose their policy positions.

Table 2 gives the 1992 data on voter choice for the sample of 1192 respondents used by Arian and Shamir [5]. Of the responses, 172 gave no information on voting, but the other 1020 votes generate a sample distribution that is close to the national vote for the parties. Since the electoral system is based on proportional representation, the seat shares and national vote shares are very close. Unfortunately, no sample voters chose either of the two Arab parties (Daroushe and Hadash).

The survey itself involved six questions on security issues, four on religion and four on economics. Factor analysis of the responses gave two dominant factors (S and R, say). The four questions on economic topics led to no obvious strong factor. Factor analysis of all questions gave eigenvalues of 4.07 (for S), 1.52 (for R) and only 0.78 and 0.72 for the economic factors. For this reason, it was decided to proceed with only two policy dimensions (S and R). To represent the beliefs of voter i, the factor loadings of the six security questions on S and of the four religious questions on R were combined with i's responses to give a "position" $(s_i, r_i) = x_i$. Such a point we term an "ideal point" for voter i. These positions were rescaled so that the average response on each scale was 0. On security, the scale ranged from –2 to 1.5 and on religion from –1.5 to 3. Figure 4 presents our estimate of the underlying density function of these voter "beliefs". Clearly there is some "correlation" of beliefs across the two dimensions, and this accounts for the correlation coefficient of 0.4 between the factors. For convenience, however, we have drawn the dimensions in figure 4 as orthogonal. Implicitly, we assume the distance between two points x_1, x_2, say, in this policy space, is given by the Euclidean metric

$$\| x_1 - x_2 \| = ((x_{11} - x_{21})^2 + (x_{12} - x_{22})^2)^{1/2}.$$

Table 2

Election data for 1992.

Party (acronym)	Actual sample vote	Vote shares (%) (from 1020 samples)	National vote (%)	Knesset seats (%)	
LABOR BLOC					
Labor (LAB)	358	35.1	36.0	44	37.0
Meretz (MRZ)	128	12.5	10.0	12	10.0
Daroushe (ADL)	0	0	1.7	2	1.7
Hadash (HS)	0	0	2.5	3	2.5
Subtotal				61	
LIKUD BLOC					
Likud (LIK)	299	29.3	26.0	32	27.0
Tzomet (TZ)	87	8.5	6.7	8	7.0
Mafdal (NRP)	41	4.0	5.0	6	5.0
Molodet (MO)	42	4.0	2.5	3	2.5
Shas (SHAS)	20	2.0	5.0	6	5.0
Aguda (AI)	20	2.0	3.3	4	3.3
Techiya (TY)	14	1.4	*	*	
Modaei	5	0.5	*	*	
Russian Party	5	0.5	*	*	
Hamit Kadenet	1	0	*	*	
Undecided/No vote	172	not counted			
Subtotal				59	
Total	1192			120	

Much of the analysis that follows was repeated using a metric taking into account non-orthogonality of the axes. No discernible difference was detected.

The positions of the ten parties contesting the 1992 election were determined in analogous fashion, by examining their manifestos and attempting to deduce each party's response to the same questions presented to the electoral sample.

Table 3 presents the estimated "position" of each of the ten parties on these two dimensions. On each dimension, and for each party, j, the column labeled "voter" gives the average position on that dimension of the set of voters who chose party j. It is clear that there is a close correlation between each party's position on the two dimensions and the average "position" of the voters for the respective parties.

The only apparent slight anomaly is that the average voters for Aguda (AI) and Shas typically have a position closer to Likud on the security dimension. However, such an average voter also has a position far from Likud on the religious dimension. This gives some indirect evidence that it is the Euclidean distance in the two dimensional policy space that determines voter choice.

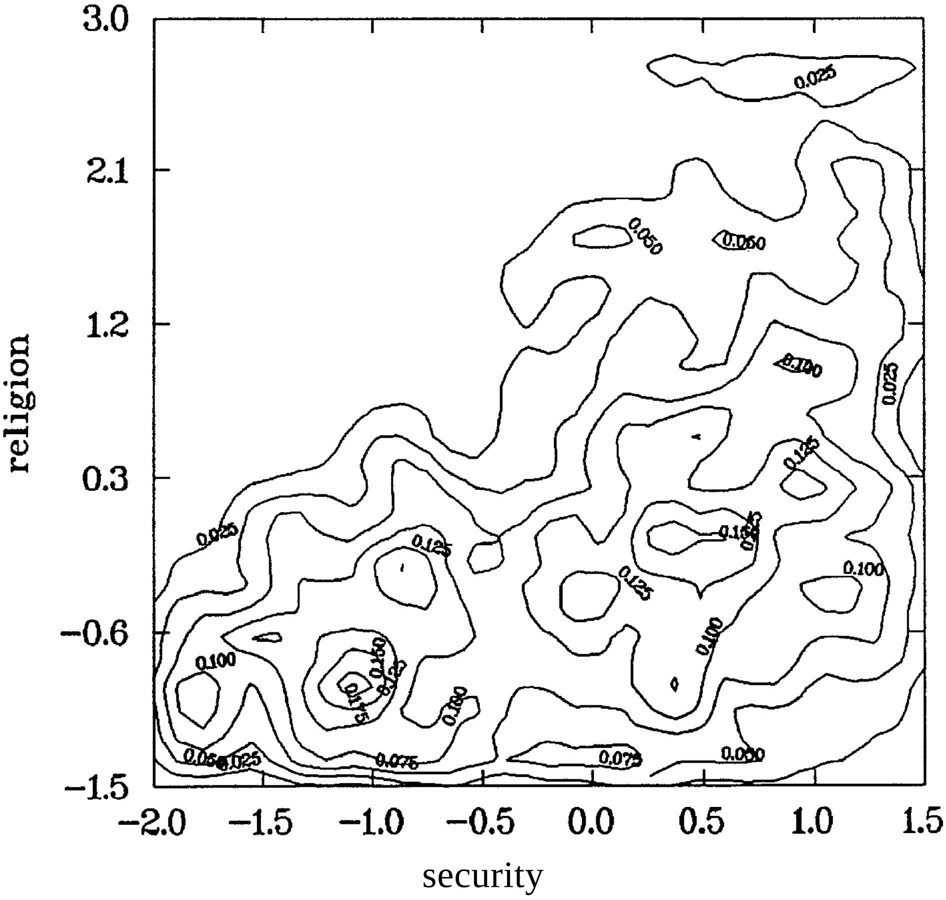

Figure 4. The estimated density distribution of electoral ideal points in 1992.

Table 3

Party positions and average voter positions.

Party (acronym)	Security		Religion	
	Party	Voter	Party	Voter
ADL	– 1.8	–	– 1.5	–
HS	– 2.0	–	– 1.5	–
MRZ	– 1.61	– 1.23	– 0.75	– 0.76
LAB	– 0.63	– 0.55	– 0.37	– 0.47
LIK	0.33	0.77	0.38	0.41
TZ	0.91	0.82	– 0.75	– 0.02
NRP	1.11	0.82	2.24	1.91
AI	– 0.05	0.79	3.0	2.4
MO	1.5	1.13	1.5	0.66
SHAS	– 0.44	0.85	2.62	2.11

To describe a formal, spatial voting model involving K parties, let $\varphi = (\varphi_1,\ldots,\varphi_j,\ldots,\varphi_K)$ be the positions of the parties and let $x = (x_1, x_i,\ldots,x_n)$ be the voter positions. This determines a $K \times n$ matrix (δ_{ij}) of distances given by $\delta_{ij} = \|x_i - \varphi_j\|$. A theoretical voting model is a matrix (χ_{ij}) based on (δ_{ij}), chosen to be close to the matrix (y_{ij}) of actual voting. Here, $y_{ij} = 1$ if voter i states that party j was chosen, and is zero otherwise. For example, a *deterministic* spatial voting model can be based on the assumption that voter i's utility for the policy choice φ_j is of the form $\bar{u}_i = -\beta\delta_{ij}$ so that the voter choice is given by $\chi_{ij} = 1$ (voter i chooses j) iff $\delta_{ij} < \delta_{ik}$ for all $k \neq j$. Alternatively a *probabilistic* voting model can be based on the assumption that the realized utility is $u_i = \bar{u}_i + \varepsilon_j = \varepsilon_j - \beta\delta_{ij}$, where ε_j is an error term associated with party j. It is usual to assume that the components of $\varepsilon = (\varepsilon_1,\ldots,\varepsilon_K)$ are independently distributed, each ε_j having variance σ_j^2. In this case, the probability χ_{ij} that i chooses j is the probability that $\varepsilon_j - \beta\delta_{ij}$ exceeds $\varepsilon_k - \beta\delta_{ik}$ for all $k \neq j$. Indeed, a typical assumption made in theoretical voting models (see for example, Lin et al. [14]) is that the errors ε_j are normally, as well as independently, distributed. Then the probability χ_{ij} that voter i chooses party j is given by the probability $\Pr([\varepsilon_k - \varepsilon_j) < \beta(\delta_{ik} - \delta_{ij})]_{k\neq j})$, where $e_j = (\varepsilon_k - \varepsilon_j)_{k\neq j}$ is a $(K-1)$ variate with normal distribution, whose moments are easily calculated.

If the vector $\varphi = (\varphi_1,\ldots,\varphi_K)$ of party positions is known, then it is possible to compute, in such theoretical models, the *expected share of the vote* of party j. The share can be expressed as $\pi_j(\chi)(\varphi) = (1/n)\sum_1^n \chi_{ij}(\varphi)$. Here, $\chi_{ij} : W^K \to \Delta$ will be a continuous function from vectors of party positions to the expected vote share (a point in the simplex, Δ) derived from the underlying stochastic model.

In empirical work, both conditional logit or multinomial probit models can be used. In conditional logit (CL), the errors $(\varepsilon_1,\ldots,\varepsilon_K)$ are assumed to be independently and identically distributed (iid). The maximum likelihood estimator (Maddala [15]) for χ_{ij} then takes the form

$$\chi_{ij} = \upsilon_{ij} \Big/ \sum_{k=1}^{K} \upsilon_{ik}, \qquad (1)$$

where $\upsilon_{ij} = e^{-\bar{u}_{ij}}, \bar{u}_{ij} = -\beta\delta_{ij} + \psi_j$, and ψ_j is a party specific constant.

It would also be possible to incorporate individual specific terms, α_i, say, but we ignore such a possibility. (See McKelvey and Palfrey [16] for further discussion of such a quantal, or logistic, reponse function.)

Given the information on the voting behavior (y_{ij}) of the sample, it is then possible to estimate the coefficients $\{\beta; \psi_1,\ldots,\psi_K\}$ under relevant assumptions on the errors. Note that, in the conditional logit model, the ratio χ_{ij}/χ_{ik} will be independent of other possible choices. Appendix B reports the result of a Hausman test [11] on this "independence of irrelevant alternative" or IIA property. The test indicates that the model fails IIA. (This implies that the errors in the estimation are correlated.) In contrast, the multinomial probit model (MNP) assumes that the error vector $e = (\varepsilon_K - \varepsilon_1,\ldots,\varepsilon_K - \varepsilon_{K-1})$ is a $(K-1)$-dimensional normal variate with covariance matrix Σ. Unfortunately, maximum likelihood estimation of χ_{ij} in this model depends on computing a $(K-1)$-dimensional integral of the underlying probability density function of e.

Appendix B briefly reports MNP estimation of a voter response model involving six of the ten parties. The results indicate that the β coefficient in the latent utility $\bar{u}_{ij} = -\beta\delta_{ij}$ under MNP estimation is statistically significant (non-zero).

Since this gave evidence that a latent utility model of the described form could be used to analyze voter behavior, we proceeded to use conditional logit to compute a number of models using various assumptions on the form of the latent utility $\bar{u}_{ij}$, given in equation (1). These models are derived from the data matrix (δ_{ij}) obtained from our factor analysis of voter positions (x_i) and party positions (φ_j).

Model 1. This is a pure probabilistic spatial CL model, based on estimated positions of ten parties, but assuming that the constant term $\psi_j = 0$ for all j. In this estimation, $\beta = +1.618$, with t-score 24.5. Given this estimation and the data matrix (δ_{ij}), we compute the probability matrix (χ_{ij}) for all $i = 1,\ldots,1192$ and $j = 1,\ldots,10$. By summation, we compute the expected vote for each of the ten parties, based on the vector φ, of party positions as described by table 3.

Table 4 lists the estimates so obtained for the expected vote share under Model 1, and compares these estimates with that of national vote share and sample vote shares. It is obvious that Model 1 seriously underestimates the vote share for the two large parties (Labor and Likud) and overestimates the vote share for Tzomet and Molodet.

To indicate why this might occur, contrast figure 4 with figure 5. Figure 4 presents an estimate of the density function of the voter positions in the sample, while figure 5 presents the estimated party positions (taken from table 3). Notice that the Tzomet position is relatively isolated in the bottom right-hand quadrant of figure 5. A voter at a position near to (0.70, –0.3) is approximately equidistant between the Labor, Likud, and Tzomet positions. Such a voter is assumed by Model 1 to vote for these three parties with similar probabilities. In fact, it would be more realistic to suppose that such a voter might tend to vote for one of the two larger parties. The basis of this supposition is that the coalition game is dominated by Labor and Likud, so that a vote for Tzomet is, to some degree, wasted. “Strategic voting” of this kind by the electorate would tend to increase the expected vote by the larger parties.

Model 2. In this model, the party specific constants ψ_j are included, but only for a choice set of six parties. We restrict ourselves to six parties in this model because we shall examine strategic behavior by the parties in the next section, and computational difficulties allowed us to deal with no more than six.

Table 4 lists the party specific constants obtained from CL estimation of Model 2, together with their t-values. For convenience, the constant term for Shas is set to zero.

Table 4

1992 election in Israel.

Party	Seats	National vote (%)	Sample vote (%)	Model 1 vote (%)	Model 2 (ψ_k)	Model 2 t-score	Model 2 (%)	Model 3 (%)	Model 4 (%)
LABOR BLOC									
Labor (LAB)	44	36.0	35.1	19.4	2.35	(7.92)	37.8	24.7	26.3
Meretz (MRZ)	12	10.0	12.5	11.0	1.67	(5.30)	12.8	17.8	14.0
Daroushe (ADL)	2	1.7	–	7.3	–	–	–	–	
Hadash (HS)	3	2.5	–	5.7	_	–	–	–	
	(61)	(50.2)	(47.6)	(43.4)			(50.6)	(42.5)	(40.3)
LIKUD BLOC									
Likud (LIK)	32	26.0	29.3	18.2	2.03	(6.99)	32.4	22.3	22.3
Tzomet (TZ)	8	6.7	8.5	16.9	0.89	(2.84)	9.6	19.7	22.0
Mafdal (NRP)	6	5.0	4.0	6.8	0.04	(0.125)	5.3	10.5	12.5
Shas (SHAS)	6	5.0	2.0	3.6	0	(2.1)	2.1	5.0	2.9
Molodet (MO)	3	2.5	4.0	8.0	–	–	–	–	–
Aguda (ERP)	4	3.3	2.0	3.1	–	–	–	–	–
	(59)	(48.5)	(49.8)	(56.6)			(49.4)	(57.5)	(59.7)
Russian Party	0	< 0.5	0.5	–	–	–	–	–	–
Modaei	0	< 0.5	0.5	–	–	–	–	–	–
β				1.618		1.739		1.628	
(t-score)				(24.5)		(21.8)			

All the constant terms except for Mafdal (NRP) are significant. As expected, the constant terms for Labor and Likud are quite large. Just to illustrate the significance of these constant terms in Model 2, a voter who is one unit distant from both Tzomet and Likud will vote for these parties with probabilities in a ratio about 1:2, respectively, rather than in the ratio 1:1 as in Model 1. The estimated β coefficient increases (from 1.618 to 1.739). The t-value for β is still very high, so the distance matrix (δ_{ij}) is statistically significant in determining voting behavior. The estimated expected vote shares of these six parties are now very close to the sample vote shares. In particular, the vote share of the "Labor bloc" exceeds 50%, while that of the Likud bloc is below 50%. The six-party Model 2 thus gives quite a good picture of the overall balance of power between the two blocs. Notice that, in the subgame defined by Model 2, the Labor party would be at the core of the coalitional spatial game.

Models 3 and 4. The other two models reported in table 4 will be utilized in a later section. Model 3 was based on a multinomial probit re-estimation of β with six parties (see appendix B.) This figure for β was then used in equation (1) to re-estimate the

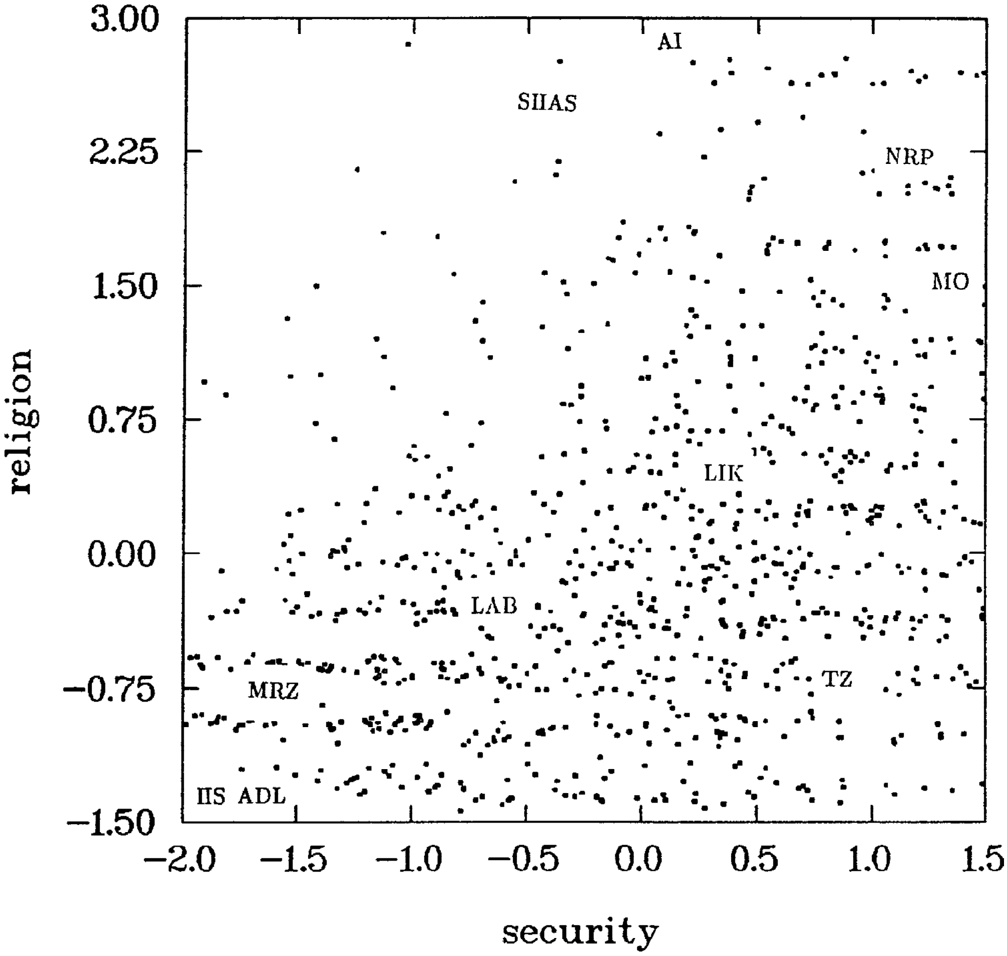

Figure 5. Estimated party positions in 1992 in the electorally-based policy space. Each dot represents a voter's ideal point.

vote response function. In this model the party specific terms were set to zero. Model 4 is an approximation to a deterministic model, under the assumption that $\beta \to \infty$. If the probabiity estimates χ_{ij} are computed from equation (1), with $\beta \to \infty$ and $\psi_j = 0$, then this is equivalent to a deterministic, or non-stochastic model, where voter i chooses party j if and only if $\delta_{ij} < \delta_{ik}$ for all $k \neq j$. Note that the estimates of vote shares for these two models are very close.

The results presented in table 4 indicate that Model 2 does give a close approximation to overall voter behavior in the 1992 election for the Knesset. The constant terms measure the non-policy attractiveness of the parties, and may be the effect of electoral strategic voting.

3. A "Downsian" theory of party competition

Spatial models of pre-election party behavior derived from the early work of Downs [7] have concentrated on two-party competition. It has been natural in these models to assume that each party has an incentive to choose a policy position so as to maximize the number of seats it obtains. In electoral systems based on proportional representation, this incentive can be interpreted in terms of vote maximization. Deterministic models of electoral voting, even in two dimensions, indicate that there will in general exist no Nash equilibrium policy pair (ψ_1, ψ_2), say, for two-party vote maximization (Schofield [22], McKelvey and Schofield [17]). The same phenomenon has been found by simulation when there are three or more parties (Eaton and Lipsey [8]). See Ordeshook [20] for a recent exposition of such spatial models of party competition.

On the other hand, existence of Nash equilibrium is more readily demonstrated in two-party models based on probabilistic voting. In theoretical models, the voter probability functions $\chi_{ij}(\varphi)$ are typically continuous in the vector φ of party positions. Moreover, the usual assumptions of the model are enough to guarantee that each expected vote function $\pi_j(\chi)(\varphi)$ is a concave function in the party position, φ_j. If π_j is identified with the payoff of party *j*, then the Nash equilibrium, φ^*, is that pair of positions $(\varphi_1^*, \varphi_2^*)$, say, such that neither party may increase expected vote. The usual result is that *strong convergence* occurs, namely that $\varphi_1^* = \varphi_2^*$ are both at the mean of the distribution of voter positions.

It might be observed of such models that a more natural party objective would be to maximize the probability of winning a majority of the vote. However, such probabilistic models are usually based on the assumption that voter behavior is *independent*, i.e., for two voters *i* and *j*, the probability that *i* and *j* both choose party *k* is simply $\chi_{jk}\chi_{ik}$. Under the independence assumption, maximization of expected vote and maximization of probability of winning give identical Nash equilibria (Aranson et al. [4]). This result may be regarded as a version of the Condorcet Jury Theorem (see Schofield [27]). However, if the stochastic errors are correlated, then these two party objectives may give very different equilibria (see Ladha [12] for a discussion).

If there are more than two parties, then the objective for a party to win a clear majority is generally impracticable. Even if it is assumed that voter behavior is independent, it is not self-evident that vote maximization is an obvious strategy for a party in a multi-party ($K \geq 3$) situation. Vote maximizing models implicitly assume that a party can, by increasing its size, also increase its power. However, if, in attempting to increase its vote, a party must change its declared policy position away from its preferred policy position, then the indirect effect on the eventual policy outcome may more than offset any advantage from increased size.

A second question is whether "Downsian" parties engaged in expected vote maximization will tend to converge towards the mean of the voter distribution. Theoretical work on the multi-party probabilistic model ($K \geq 3$) by Lin et al. [14] implies that *if* each expected vote function $\pi_j(\chi)(\varphi)$ is concave in each φ_j, then there will be a strong convergence (with φ_j^* equal for all parties at a position which minimizes the average distance, $\bar{\delta}$, between the voters and the party position). However, Lin et al. also found that concavity of π_j depended on the values of the variances $(\sigma_1^2, \ldots, \sigma_K^2)$ used to construct the model. For low values of the variances, it was possible for concavity to fail. As a consequence, non-convergent Nash equilibria could be found, where parties adopt different policy positions. These simulations also showed that, with very small values for the variance, it was possible for the mean voter position to be associated with minimizing expected vote. This result suggests that it is possible under some configurations for the parties to adopt positions at the extremes of the voter distribution, even when they do attempt to maximize expected vote.

Another simulation exercise by Adams [1] examined the effect of the β-coefficient in equation (1) on Nash equilibrium under vote maximization. His analysis shows that with a coefficient β close to zero, then the Nash equilibrium is unique and strongly convergent (φ_j^* equal for all parties, and at a point near the center of the voter distribution). As we have noted, if $\beta \to \infty$, then there may in fact be no Nash equilibrium. Moreover, there appears to be a critical value, β^*, say, such that vote maximization for the logistic equation (1), with $\beta > \beta^*$, gives non-convergent Nash equilibria.

In the next section of the paper, we use our estimation of voter response in Israel to determine how "Downsian" parties would behave in this particular situation. It turns out that two of the parties can, for good reason, be regarded as "Downsian". We then go on to suggest a basis for "non-Downsian" behavior by the other parties.

We also simulated "Downsian" behavior by six of the parties, under the objective of expected vote maximization, using an unbiased estimate of β for the probabilistic voter response. We found Nash equilibria that were not strongly convergent. This suggests that empirical electoral systems are neither as "chaotic" as the theoretical deterministic models, nor as "convergent" as the theoretical probabilistic models.

4. Empirical estimates of vote maximizing in the Knesset

The theoretical results mentioned above do not give a clear prediction about the nature of Nash equilibria under vote maximization. However, the empirical models presented in section 2 allow us to determine party expected vote for different party positions. Using Model 3 (with constant terms zero), for example, we can graph the expected vote for each party, given the positions of the other parties. Figure 6 presents such a graph of the expected vote for the Labor party given the positions of the other parties as fixed at the positions described in table 3. The two-dimensional coordinates represent the plane of possible positions for the Labor party, holding the other party positions fixed. The graph of expected vote is that of a quasi-concave function, not a concave function. Since this illustration is generic, we can use standard theorems in game theory (Glicksberg [10]) to infer that there will exist a Nash equilibrium under expected vote maximization. However, we should not expect such equilibria to be unique or convergent.

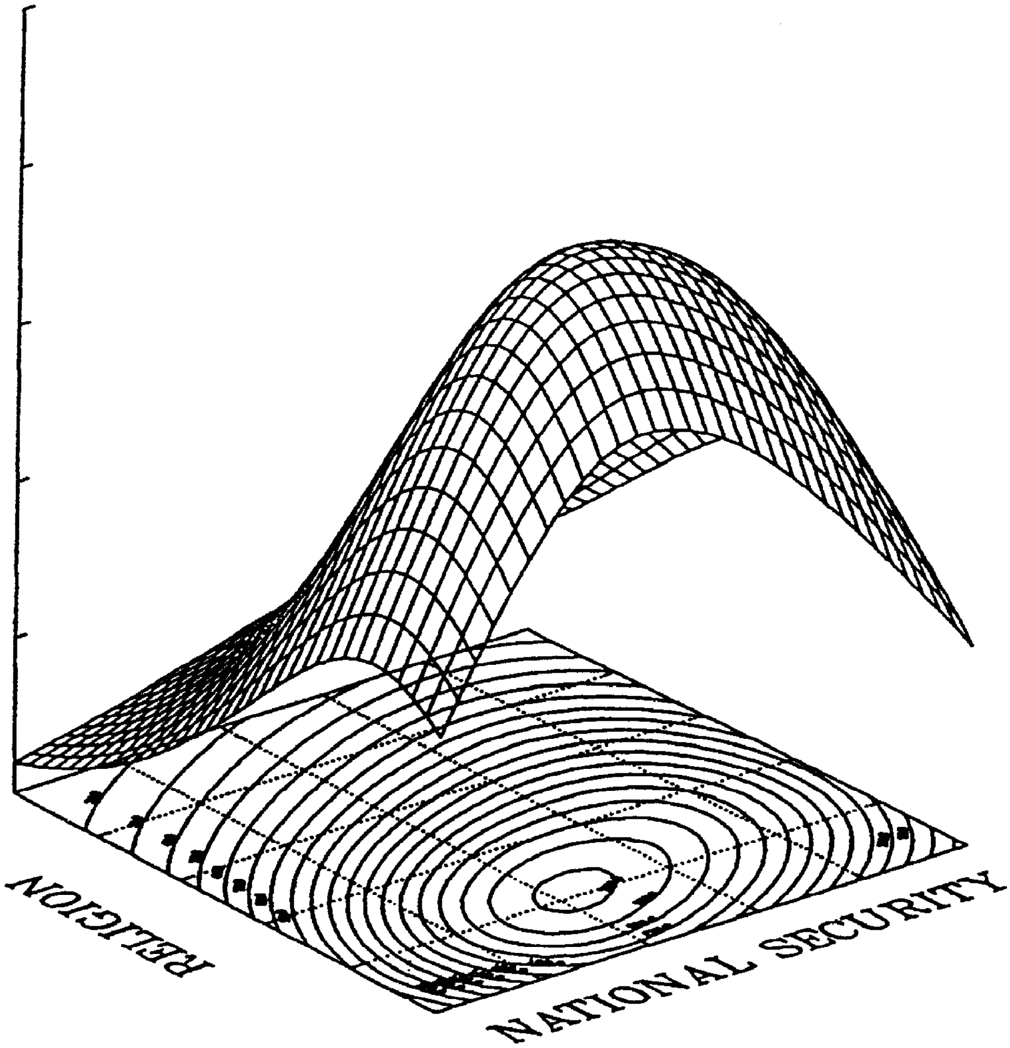

Figure 6. Labor's expected vote (based on the sample size of 1192 and using conditional logit estimation). The independent variables are Labor's positions on security and religion; other parties are fixed at positions given in table 3.

Suppose that the vector of party positions described in table 3 is a Nash equilibrium. Then no party may modify its position to increase its expected vote. Table 5 reports the effect on the expected vote of each party if it moves to that position which maximizes its expected vote, assuming the other parties remain fixed. These estimates for such "one-step optimization" are based on the voter behavior, χ, described in Model 2. (Obviously, the vote share figures in the last column of this table do not sum to 100%.)

Table 5

One-step optimization for six parties using Model 2.

Party	Old position	Original vote share (%)	Optimal position	Optimal share (%)
Labor	(– 0.63, – 0.37)	37.8	(– 0.41, – 0.33)	38.2
Meretz	(– 1.61, – 0.75)	12.8	(– 0.85, – 0.58)	18.2
Likud	(0.33, 0.38)	32.4	(0.40, 0.08)	33.0
Tzomet	(0.91, – 0.75)	9.6	(0.75, 0.15)	11.3
Mafdal	(1.11, 2.24)	5.3	(1.0, 1.43)	6.3
Shas	(– 0.44, 2.62)	2.9	(0.80, 1.09)	5.0

Notice that these "best responses to the status quo" described in table 5 move the parties towards the center of the voter distribution, and can have quite a dramatic effect on vote share for a small party. However, the changes in vote shares for Labor and Likud are negligible. We may infer that both Labor and Likud are at positions which are best response (in terms of expected vote) to the positions of the other parties.

We can continue the optimization process for the parties, from random initial positions of the parties.[2)] This does not guarantee that all Nash equilibria can be found. Nonetheless, two distinct Nash equilibria were found. These are presented in table 6.

Table 6

Nash equilibria under expected vote maximization in Model 2.

Party	Initial position	Nash position A	Nash position B
Labor	(– 0.63, – 03.7)	(– 0.60, – 0.45)	(– 0.65, – 0.48)
Meretz	(– 1.61, – 0.75)	(– 0.60, – 0.45)	(– 0.65, – 0.48)
Shas	(– 0.44, 2.62)	(– 0.60, – 0.45)	(1.03, 1.37)
Mafdal	(1.11, 2.24)	(0.898, 1.30)	(1.03, 1.37)
Likud	(0.33, 0.38)	(0.898, 1.30)	(0.88, 1.27)
Tzomet	(0.91, – 0.75)	(0.898, 1.30)	(0.96, 1.32)

In Nash Equilibrium A, Labor, Meretz, and Shas converge to a position fairly close to the initial Labor position, while Mafdal, Likud and Tzomet converge to a compromise position. This compromise position is close to the Tzomet position on the security dimension, but is moderately extreme on the religious dimension. In equilibrium B, Labor and Meretz coalesce as do Shas and Mafdal. Likud and Tzomet approach very close to one another at a position which is more extreme than Likud's original position.

[2)] By this we mean constant iteration of one-step optimization by all parties, until no increase in expected vote, as predicted by Model 2, is found.

While these two estimated Nash equilibria must be viewed as somewhat tentative, because of the difficulty of the simulation, nonetheless they do give a perspective on the underlying pattern of party alliance in the 1992 Knesset. Not only is Shas pivotal in the post-coalition game in the way we have described, but the bifurcation in the two Nash equilibria appears to depend on the strategy by Shas: viz. Shas can either "converge" towards Labor or move towards a more extreme position on the security dimension. Secondly, were Tzomet to move to increase expected vote, then it would face the prospect of being absorbed by Likud. Thirdly, were the religious parties (Shas and Mafdal) to encroach on Likud's position to increase expected vote, then Likud would have to move to face them. The result would be a greater degree of polarization between the eventual Labor and Likud positions.

Contrary to the usual results in probabilistic voting models, expected vote maximization by the parties leads to polarization rather than convergence to a mean electoral position. Notice that the initial Likud position on the Security axis is close to one of the local maxima of the density function of the voter distribution. In our model of voter behavior, we have used Euclidean distance, and this implies that the two cleaveages (Security and Religion) are weighed equally. It is possible that if the cleavages were weighed differently in the model, then the resulting Nash equilibrium under vote maximization would not exhibit such a degree of polarization. Nonetheless, it would seem to be the case that vote maximization would still pull Labor and Likud apart, *if the other smaller parties also engaged in vote maximization*.

We also used the iteration procedure for six parties under both Model 3 (with zero constant terms and $\beta = 1.628$) and Model 4 (with $\beta \to \infty$). In these two cases, Nash equilibria under vote maximization were found to be very similar to the Model 2 case just discussed. The existence of non-convergent Nash equilibria is therefore not a consequence of specific features of the model (such as strategic voting) but appears instead to be a fundamental aspect of probabilistic voting in this electorate. To relate this to the results of Lin et al. [14] and Adams [1], it seems that the variance terms of the empirical probabilistic model are too low and the estimated β coefficient too high to generate unique, convergent Nash equilibria. It is moreover surprising that the "deterministic" Model 4 (for the limiting case $\beta \to \infty$ also generates Nash equilibria. This suggests that deterministic spatial models with $K \geq 3$ under vote maximization in two dimensions may be quite different from the case $K = 2$. Further research on the relation between these results and those of Eaton and Lipsey [8] is planned.

The most important inference to be drawn from these results is that the two large parties, Labor and Likud, appear to have adopted policy positions in 1992 that maximized their expected vote (or were best response in terms of such an objective function) given the positions of the other parties. We may therefore refer to these two parties as "Downsian". The evidence also suggests that the other parties were not "Downsian", since the distance between their declared and optimal positions exceeds any likely measurement error in our model.

"Downsian" behavior by Labor and Likud has an obvious rationale. As the discussion in section 1 makes clear, if Labor could gain enough votes in 1992 to deny the Likud grouping a majority, then it could, in principle, operate a minority government, gain all the perquisites of power and implement its chosen policy. Both parties thus have an incentive to maximize their vote. However, the empirical model that we have estimated shows that the expected vote for these parties depends not just on their positions, but on the party specific constants (ψ_j). It is plausible that these constants measure not just strategic voting by the electorate, but also the level of credibility of the policy platforms presented by the parties. We return to this point in the next section when discussing the 1996 election. It is relevant to note here that the high constant term ($\psi = 2.35$) for Labor may be due to the credibility of Rabin as leader of Labor in the 1992 election.

The calculations by the smaller parties over policy choices to declare to the electorate can be quite subtle. Schofield [25] has suggested that a formal model should be based on expected utility analysis based on the probabilities of different electoral results and coalition outcomes. The most interesting case to examine is that of Shas. As the discussion in section 1 suggests, Shas must surely have estimated that it could be pivotal between a Labor led and a Likud led coalition. Given uncertainty about electoral response, its policy declaration could effect the probabilities of these two coalition outcomes. Moreover, the party may plausibly believe that its declaration will be interpreted as a benchmark which will affect the policy negotiations after the election.

A formal model proposed by Schofield and Parks [30] suggests that even if a pivotal party, like Shas, does have a true preferred policy position (say φ_j), it may nevertheless choose to declare a policy position (φ_j', say) which is more extreme than φ_j. By committing to φ_j' in post election negotations, it brings about a more desired compromise policy than if it had committed to φ_j.

In contrast, a small party like Tzomet whose preferred position (φ_k, say) is close to its likely coalition partner (Likud) may declare a position φ_k', say, which is even closer to Likud's position. By so doing, it induces a higher probability of membership of the Likud-led coalition. This may account for the pre-election pact between Tzomet and Likud, prior to the 1996 election.

Finally, the notion of the heart suggests that the policy declarations of smaller religious parties (such as Mafdal and Molodet) will have little or no effect on the eventual policy bargain within a Likud-led coalition.

A plausible inference is that such parties will declare their "true" preferences. By this we mean that the elites of these parties will adopt policies that accord with the beliefs of their committed supporters. Presumably, such elites expect to influence outcomes not through policy negotiation but by control of cabinet positions.

5. The election of 1996

Table 7 gives the national vote and number of seats for the eleven parties in the Knesset after the 1996 election. This table also gives the overall vote shares for the

Table 7

The 1996 election in Israel.

Party	Seats	National % vote	Groupings	Estimated % Model 1	Groupings
LABOR BLOC					
Labor	34	27.6		19.4	
Meretz	9	7.6		11.0	
Daroushe	4	3.0		7.3	
Hadash	5	4.3		5.7	
	(52)		42.5		43.4
CENTER					
Third Way	4	3.2			
Russian Party	7	5.9			
	(11)		9.1		
LIKUD BLOC					
Likud				18.2	
(+Tzomet)	32	25.9	25.9	16.9	35.1
Shas	10	8.8		3.6	
Molodet	2	2.4		8.0	
Mafdal	9	8.0		6.8	
Aguda	4	3.3	22.5	3.1	21.5
	(57)				
Total	120	100.0	100.0		

Prime ministerial election:

Netanyahu (Likud)	50.5% of valid votes
Peres (Labor)	49.5% of valid votes
Invalid votes	4 % of total

two prime ministerial contenders, Netanyahu and Peres, together with estimated party vote shares under Model 1, based on the 1992 data discussed above. As mentioned above, Model 1 involves no party specific constants. It is our contention that Model 1 is a pure spatial voting model based on the assumption that the electorate does not engage in strategic voting. We now compare the Model 1 estimates for 1992 with the national vote shares for 1996. In calculating National vote shares, we have re-normalized so that the vote shares of the 11 parties in the Knesset sum to 100%. (A national vote of 3% went to other parties not represented in the Knesset.) The Labor bloc (including Meretz, Daroushe and Hadash) gained 42.5%, while our Model 1 estimate for this groups is 43.4% of the vote. These two figures become quite compatible if it is assumed that approximately 1% switched from voting Labor in 1992 to voting for the Third Way in 1996. Similarly, the Model 1 estimate for the combined vote of the four religious parties is 21.5% compared with their actual national vote of 22.5% in 1996. While the combined vote shares are very close, our estimation underpredicts the vote for Shas (only 3.6% compared to 8.8%). However, if we are correct in emphasizing the degree to which Shas is pivotal, then strategic voting for Shas by supporters of Molodet makes sense. The combined national vote for Shas and Molodet is 11.2% compared with our Model 1 estimate of 11.6%. Finally, the combined national vote for Likud, the Third Way and Israel Be'aliya is 35% compared with the Model 1 estimate for Likud and Tzomet of 35.1%.

The very close correspondence between these figures suggests that Model 1 can be used to understand the 1996 election. We also infer that voters chose sincerely, or non-strategically, in 1996, when weighing the choice between large and small parties. We can also infer that Likud and Tzomet supporters (from 1992) split their vote in 1996 between Likud and the Center parties. The main difference between these two elections was the prime ministerial election, won by Netanyahu, by about 1% of the vote. Since 4% of the ballots were invalid, we infer that some Arab voters chose not to vote for Peres and some supporters of the religious parties chose not to vote for Netanyahu. More importantly, supporters of the small parties appear to have refused to switch, for strategic reasons, to one of the two large parties. Two reasons come to mind to account for this. The voters may have viewed the prime ministerial election as the appropriate context in which to choose strategically. Alternatively, we may interpret the constant terms in Model 2 as the weight given by the electorate to non-policy considerations, such as the importance of party leaders. As suggested earlier, these effects may also be related to the credibility of the party leaders.

As Nachmias and Sened [18] have emphasized, the unintended consequence of the change in electoral rules was an increase in the fractionalization of the party configuration of the Knesset. A final point can be made about the impact of the prime ministerial election. Under the post-1996 rules, a vote of no confidence supported by 61 members would require a new election. Shas could therefore ally with the Labor bloc to bring down the government. As Nachmias and Sened [18] point out, Shas was therefore in a very strong bargaining position in the post-1996 negotiations with Likud over cabinet positions.

6. Conclusion

Models of electoral competition based on the work of Downs [7] have tended to assume that parties seek office-related payoffs, and thus choose policy positions which maximize their own political power (measured by the expected number of votes or seats). In contrast, models of multiparty coalition behavior have either focused on office-related payoffs and the size of coalitions (Riker [21]), or on policy-related payoffs and the existence or otherwise of a core in the policy space (Laver and Schofield [13]).

In this paper, we have presented the outline of a policy-based theory of electoral competition in which parties adopt policy positions to declare to the electorate, based on the objective of maximizing their expected return from the eventual coalition outcomes.

For the particular situation studied in Israel, we have argued that the results are compatible with the pursuit of power by the two dominant parties. The change in the electoral law between 1992 and 1996, which was intended to increase the power of one or the other of the leaders of these two parties, may have had the indirect effect of reducing the importance of strategic voting in the electorate. Both Labor and the Likud–Tzomet alliance lost votes as a consequence.

The results we have obtained are compatible with the inference that the smaller Israeli parties chose not to maximize expected vote, but to declare "true" policy preferences, and to concentrate on influencing government policy after the election. Paradoxically these parties may have benefitted from the change in electoral law.

The analysis suggests that political behavior in multiparty systems based on proportional representation (PR) is much more subtle than assumed by "Downsian" models. In particular, the empirical analysis suggests a model of utility maximization by parties. Such a model does not result in Downsian convergence to an electoral mean.

Indeed since policy convergence does not occur in any known multiparty system, we expect to find that a hybrid model of the kind proposed here can be used to understand any PR political system.

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Appendix A: The survey

There were six questions on the security dimension on the following topics.

S1. negotiating with the PLO, before or after the PLO recognition of Israel.

S2. agreement to the establishment of a Palestinian state in Gaza, Judah and the occupied territories.

S3. settlement of occupied territories.

S4. solutions to the conflict.

S5. left or right on security issues.

S6. left or right in politics.

There were four questions on religion on the following topics:

R1. Jewish tradition and the government.

R2. mixed bathing.

R3. the Sabbath.

R4. keeping Jewish religious tradition.

There were four questions on economic issues:

E1. socialism versus capitalism.

E2. closer to Labor or Likud?

E3. average size of family expense.

E4. socio-economic status.

On security issues, four factors emerged with eigenvalues 3.48, 0.74, 0.60, 0.53. The factor loadings of questions S1 to S6 on the first factor were 0.60, 0.75, 0.77, 0.70, 0.86, 0.86. These loadings and the responses to these questions gave a security scale. Cronbach's α (Carmines and Zeller [6]) for this scale was 0.83, exceeding the required consistency value of 0.75.

The four religious questions generated four factors with eigenvalues 2.44, 0.61, 0.50, 0.46. On the first factor, the loadings were 0.80, 0.74, 0.79, 0.80. Again, a scale was constructed with Cronbach's $\alpha = 0.779$.

On economic questions, two factors emerged with eigenvalues 1.20 and 1.14. The loadings for the two factors were in the range between –0.6 and +0.7. Cronbach's $\alpha = -0.15$. No single scale was evident. In particular, middle class respondents seemed to have socialist views but to support a free market economy.

Factor analysis of all responses gave eigenvalues of 4.07 (for S), 1.52 (for R), with 0.78 and 0.72 for E. Contrary to the usual assumptions concerning the significance of a left–right economic dimension, in Israel this dimension is subsumed into the security dimension.

Similar questions on security, religious and economic issues were answered on the basis of the manifestos of the parties.

Appendix B

The conditional logit (CL) model implicitly assumes an independence property (IIA) that an estimate of β based on a set of K parties should be similar to that for β in any subset. Hausman [11] gives a test-statistic based on chi-square with 1 d.o.f. and critical value of 7.88. Excluding subsets of parties in Model 3 (based on six parties) resulted in re-estimates of β as low as 1.38 (with chi-square equal to 48.8) and as high as 2.13 (with chi-square equal to 39.9). Since IIA could be rejected, a more appropriate model was multinomial probit. (See Alvarez and Nagler [3] for a comparison of CL and MNP.) A technique known as Gibbs sampling (Gelfand and Smith [9]) together with data augmentation (Albert and Chib [2]) was employed. MNP estimation resulted in a β value of 1.085. Since MNP estimates are usually 1.5 times CL estimates, this gives an (approximate) unbiased estimate of 1.628 for the CL figure for β. This figure was used in the simulation to find one-step optimization (or best response) and Nash equilibria, based on the logistic response function given in equation (1), and described in section 4.

Further details of both the factor scaling and MNP estimation can be found in Nixon et al. [19].

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