

NATURAL AND MARANGONI CONVECTIONS IN A TWO-DIMENSIONAL RECTANGULAR OPEN BOAT

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The natural and Marangoni convections formed spontaneously in the melt inside a two-dimensional rectangular open boat were investigated by means of an order-of-magnitude evaluation and a numerical analysis according to the finite difference method. A quantitative evaluation was made of the Grashof number, Marangoni number, Prandtl number and melt depth, all of which affect the interfacial velocity and the velocity distribution of the melt convection. It was concluded that Marangoni convection as well as natural convection is important when the melt is shallow.

Introduction

Single crystals of semiconductors, compound semiconductors and oxides are very important as device materials in the electronics industry. When single crystals are grown from a melt, natural convection due to density differences and Marangoni convection due to an interfacial tension gradient at the free interface of the melt are formed spontaneously in the melt. These convections affect the quality of single crystals. Therefore, to grow high-quality single crystals it is necessary to clarify and control the melt convections. With this in view, much work has been carried out related to melt convection processes, both theoretical^{9,11,12)} and experimental.^{6,7,10,18)} However, many points remain unclear. In particular, it has been suggested^{1,3,21)} that Marangoni convection may be important as melt convection, not only in micro-gravity environments but also under normal gravity, but these investigations are not complete. Also, when the melt convections are controlled by Lorentz force due to the application of a magnetic field^{4,5,20)} or by the formation of forced convection due to crystal rotation,^{13-15,19)} it is extremely important to clarify the effect of natural and Marangoni convections on both the velocity and distribution of melt convections in order to determine the correct magnitude for the applied magnetic field and also the appropriate crystal rotation rate.

In the present study, the natural and Marangoni convections in the melt inside a two-dimensional rectangular open boat were investigated by means of an order-of-magnitude evaluation and a numerical analysis according to the finite difference method.

This yielded a quantitative evaluation of the effects of the Grashof number, Marangoni number, Prandtl number and melt depth on the interfacial velocity at the free melt interface and the velocity distribution within the melt.

1. Analysis

The theoretical model is shown in Fig. 1. The model considers a two-dimensional rectangular open boat with a free interface which is heated from one side (T_H) and cooled from the other (T_C). The model included the following assumptions: (i) steady state, (ii) an incompressible and Newtonian fluid, (iii) a flat interface, (iv) an adiabatic bottom wall and free interface, and (v) constant values of all physical properties except the interfacial tension in the stress balance equation for the free melt interface and the density in the buoyancy force term.

The basic equations and boundary conditions are described by the following equations:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Navier-Stokes equations

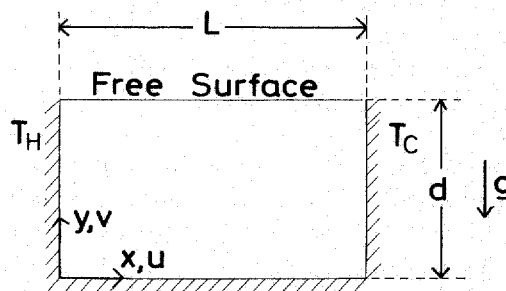


Fig. 1. Configuration considered by analysis

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$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta\Delta T \quad (3)$$

Energy equation

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

Boundary conditions

(a) along the free interface; $0 \leq x \leq L$, $y = d$.

$$\mu \frac{\partial u}{\partial y} = -\frac{\partial \sigma}{\partial T} \cdot \frac{\partial T}{\partial x} \quad (5)$$

$$v = 0, \quad \partial T / \partial y = 0 \quad (6)$$

(b) along the bottom wall; $0 \leq x \leq L$, $y = 0$.

$$u = v = 0, \quad \partial T / \partial y = 0 \quad (7)$$

(c) along the hot wall; $x = 0$, $0 \leq y \leq d$.

$$u = v = 0, \quad T = T_H \quad (8)$$

(d) along the cold wall; $x = L$, $0 \leq y \leq d$.

$$u = v = 0, \quad T = T_C \quad (9)$$

A steady-state solution was obtained from Eqs. (1)–(9) by means of an order-of-magnitude evaluation¹⁷⁾ and a numerical analysis according to the finite difference method. In carrying out the numerical analysis, a 21×21 grid system was used in the range of $Gr_d = 1 - 10^6$ and $Ma_L = 1 - 10^4$.

2. Results and Discussion

2.1 The effect of natural and Marangoni convections on velocity distribution in the melt

The effect of natural and Marangoni convections on velocity distribution in the melt obtained by the present numerical analysis are shown in Fig. 2. The arrows in the figure show the velocity vector of the convection at each point in the melt. The results shown in column (a) of Fig. 2 are for the situation in which only natural convection exists; column (b) shows the results for the coexistence of both natural and Marangoni convections; and column (c) shows the results obtained when only Marangoni convection exists. The melt convection caused by natural convection circulates around the whole of the melt, the velocity of the convection decreasing as the depth becomes smaller (Fig. 2(a)). In the case of melt convection caused by Marangoni convection, there is a circulating flow near the free interface when the melt is deep, but in the vicinity of the bottom wall the flow velocity is low. When the melt is shallow, however, the Marangoni convection circulates throughout the

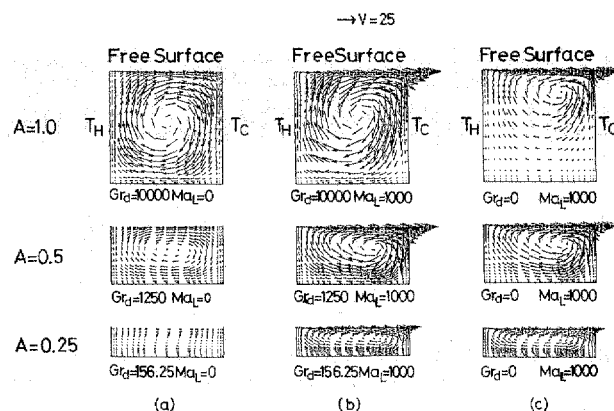


Fig. 2. Effect of natural and Marangoni convections on velocity distribution in melt (Arrows show velocity vector at each point in the melt.)

(a) only natural convection presents

(b) both natural and Marangoni convections present

(c) only Marangoni convection presents

melt, and the melt convection becomes large in comparison with those observed for a deep melt (Fig. 2(c)). With Marangoni convection, therefore, when the melt is deep the existence of the bottom wall has no effect on melt convection; but when the melt is shallow, the shear stress of the bottom wall does have an effect on melt convection, and the flow velocity decreases as the melt depth decreases.

When $A = 1$, the distribution of flow velocity in Fig. 2(b) appears similar to that of Fig. 2(a), which represents the situation in which only natural convection exists. On the other hand, when $A = 0.25$, the distribution in Fig. 2(b) resembles that in Fig. 2(c), which represents the situation in which only Marangoni convection exists. This means that when natural and Marangoni convections coexist, if the melt is deep then natural convection is dominant, whereas in the case of a shallow melt natural convection is suppressed and Marangoni convection becomes dominant. The results shown in Fig. 2 suggest the possibility that Marangoni convection is important when the melt is shallow, not only in microgravity environments but also under normal gravity.

2.2 The effect of natural convection, Marangoni convection and Prandtl number on interfacial velocity

It is possible to classify melt convection processes according to whether $Re_s \gg o[1]$ (i.e. $o[\text{inertial force}] = o[\text{viscous force}]$) or $Re_s \ll o[1]$ (i.e. $o[\text{inertial force}] \ll o[\text{viscous force}]$). Here, the symbol “ o ” has the meaning of “the order of”.

(a) When $Re_s \ll o[1]$

When $Re_s \ll o[1]$, since $o[\text{inertial force}] \ll o[\text{viscous force}]$,

in Eq. (2),

$$o \left[u \frac{\partial u}{\partial x} \right], \quad o \left[v \frac{\partial u}{\partial y} \right] \ll o \left[\nu \frac{\partial^2 u}{\partial x^2} \right], \quad o \left[\nu \frac{\partial^2 u}{\partial y^2} \right] \quad (10)$$

and in Eq. (3),

$$o\left[u\frac{\partial v}{\partial x}\right], o\left[v\frac{\partial v}{\partial y}\right] \ll o\left[v\frac{\partial^2 v}{\partial x^2}\right], o\left[v\frac{\partial^2 v}{\partial y^2}\right] \quad (11)$$

Here, u and ∂u in Eqs. (2) and (3) are replaced by u_0 , v and ∂v by v_0 , ∂t by t , and $(T_H - T_C)$ by ΔT . Also, in Eq. (2), ∂x and ∂y are replaced by L and δ_y respectively, and in Eq. (3) ∂x and ∂y by δ_x and d respectively. Here δ_x is the thickness of the velocity boundary layer along the hot-side wall, and δ_y is the thickness of the velocity boundary layer along the free interface.

Since $t \rightarrow \infty$ under steady state,

$$\begin{cases} \partial u / \partial t = o[u_0/t] \rightarrow 0 \\ \partial v / \partial t = o[v_0/t] \rightarrow 0 \end{cases} \quad (12)$$

Also, when $Re_s \ll o[1]$, the viscous force influences the entire melt, so that in Eq. (2),

$$o\left[v\frac{\partial^2 u}{\partial x^2}\right] \left(=o\left[\frac{vu}{L^2}\right]\right) = o\left[v\frac{\partial^2 u}{\partial y^2}\right] \left(=o\left[\frac{vu}{\delta_y^2}\right]\right) \quad (13)$$

Therefore

$$o[\delta_y] = o[L] \quad (14)$$

Similarly in Eq. (3),

$$o[\delta_x] = o[d] \quad (15)$$

Also, from Eqs. (1), (14) and (15),

$$u_0 = o[dv_0/L] \quad (16)$$

When a melt convection is formed as a result of buoyancy force, (i.e. when natural convection is dominant in the melt), the following relationship is obtained from Eqs. (3), (15) and (16):

$$v_0 = o[g\beta\Delta T d^2/\nu] \quad (17)$$

Furthermore, since from Eq. (5), $(\partial u / \partial y)_{y=d} = 0$, then $u_s = o[u_0]$. The following relationships are obtained from Eqs. (16) and (17):

$$u_s = o[(g\beta\Delta T d^2/\nu)d/L] \quad (18)$$

and

$$Re_s = o[Gr_d] \quad (19)$$

When a melt convection is formed as a result of thermocapillary force (i.e. when Marangoni convection is dominant in the melt), the following relationships are obtained by replacing ∂T in Eq. (5) by ΔT :

$$\begin{aligned} u_s &= o[|\partial\sigma/\partial T| \Delta T \delta_y / (L \cdot \mu)] \\ &= o[|\partial\sigma/\partial T| \Delta T / \mu] \end{aligned} \quad (20)$$

and

$$Re_s = o[Ma_L] \quad (21)$$

(b) When $Re_s > o[1]$

When a melt convection is formed as a result of buoyancy force, the thickness of the thermal boundary layer is taken to be the boundary layer thickness.²⁾ For the vicinity of the heated vertical wall, the terms u and ∂u in Eqs. (1)–(4) are replaced by u_0 ; similarly, v and ∂v are replaced by v_0 , ∂t by t , and ∂T by ΔT . Also, in Eqs. (1), (3) and (4), ∂x and ∂y are replaced by δ_{Tx} and d respectively.

In Eq. (1),

$$\begin{cases} \partial u / \partial x = o[u_0/\delta_{Tx}] \\ \partial v / \partial y = o[v_0/d] \end{cases} \quad (22)$$

From Eqs. (1) and (22), the following relationship is obtained for the vicinity of the heated vertical wall:

$$u_0 = o[v_0 \delta_{Tx} / d] \quad (23)$$

In Eq. (4), the order of magnitude of convective heat transfer is:

$$\begin{cases} u(\partial T / \partial x) = o[u_0 \Delta T / \delta_{Tx}] = o[v_0 \Delta T / d] \\ v(\partial T / \partial y) = o[v_0 \Delta T / d] \end{cases} \quad (24)$$

On the other hand, the order of magnitude of conductive heat transfer is:

$$\begin{aligned} \alpha(\partial^2 T / \partial x^2) &= o[\alpha \Delta T / \delta_{Tx}^2] \\ \alpha(\partial^2 T / \partial y^2) &= o[\alpha \Delta T / d^2] \end{aligned} \quad (25)$$

Here, since $\delta_{Tx} \ll d$,

$$o[\alpha \Delta T / \delta_{Tx}^2] \gg o[\alpha \Delta T / d^2] \quad (26)$$

For the thermal boundary layer since $o[\text{convective heat transfer}] = o[\text{conductive heat transfer}]$, then from Eq. (4), and Eqs. (24)–(26), the thickness of the thermal boundary layer can be described as follows:

$$\delta_{Tx}^2 = o[\alpha d / v_0] \quad (27)$$

In Eq. (3) the order of magnitude of the inertial force is given by

$$\begin{cases} u(\partial v / \partial x) = o[u_0 v_0 / \delta_{Tx}] = o[v_0^2 / d] \\ v(\partial v / \partial y) = o[v_0^2 / d] \end{cases} \quad (28)$$

and the order of magnitude of the viscous force, considering Eq. (27), by

$$\begin{aligned} \nu(\partial^2 v / \partial x^2) &= o[\nu v_0 / \delta_{Tx}^2] = o[Pr(v_0^2/d)] \\ \nu(\partial^2 v / \partial y^2) &= o[\nu v_0 / d^2] \end{aligned} \quad (29)$$

Here, in the same way as for Eq. (26),

$$o[\nu v_0 / \delta_{Tx}^2] \gg o[\nu v_0 / d^2] \quad (30)$$

Thus from Eqs. (28)–(30), it is known that for $Pr \gg 1$,

$o[\text{inertial force}] \ll o[\text{viscous force}]$ and for $Pr \ll 1$, $o[\text{inertial force}] \gg o[\text{viscous force}]$.

When $Pr \gg 1$, $o[\text{viscous force}] = o[\text{buoyancy force}]$, which leads to the following relationship:

$$v_0 = o[(g\beta\Delta T d)^{1/2} Pr^{-1/2}] \quad (31)$$

From a consideration of the mass balance within the melt,

$$u = o[v_0(\delta_{Tx}/\delta_{Ty})] \quad (32)$$

Here δ_{Ty} is the thickness of the thermal boundary layer along the free interface. By replacing ∂x and ∂y in Eq. (4) by L and δ_{Ty}

$$\delta_{Ty}^2 = o[\alpha L/u_0] \quad (33)$$

Thus the following relationships can be obtained from Eqs. (31)–(33):

$$u_s = o[u_0] = o\left[\left(\frac{\alpha g \beta \Delta T d^3}{\nu}\right)^{1/2} \cdot \frac{1}{L}\right] \quad (34)$$

and

$$Re_s = o[Gr_d^{1/2} Pr^{-1/2}] \quad \text{for } Pr \gg 1 \quad (35)$$

On the other hand, when $Pr \ll 1$, $o[\text{inertial force}] = o[\text{buoyancy force}]$, which leads to the following relationship:

$$v_0 = o[(g\beta\Delta T d)^{1/2}] \quad (36)$$

Similarly, the following relationships can be obtained for $Pr \ll 1$:

$$u_s = o[u_0] = o[(g\beta\Delta T d^3)^{1/2} \cdot 1/L] \quad (37)$$

and

$$Re_s = o[Gr_d^{1/2}] \quad \text{for } Pr \ll 1 \quad (38)$$

The results of Eqs. (19), (35) and (38) are in agreement with those obtained by Ostrach.¹⁶⁾

When a melt convection is formed as a result of thermocapillary force, Eq. (5) yields the following relationship:

$$o[\mu u_s/\delta] = o[|\partial\sigma/\partial T| \Delta T/L] \quad (39)$$

Here, δ is the reference thickness of the boundary layer. When $Pr \gg 1$, by putting $\delta = \delta_{Ty}$, because $\delta_y \gg \delta_{Ty}$, the following relationships can be obtained from Eqs. (33) and (39):

$$u_s = o\left[\left(\frac{|\partial\sigma/\partial T|^2 \Delta T^2 \alpha}{\mu^2 L}\right)^{1/3}\right] \quad (40)$$

and

$$Re_s = o[Ma_L^{2/3} Pr^{-1/3}] \quad \text{for } Pr \gg 1 \quad (41)$$

On the other hand, when $Pr \ll 1$, by putting $\delta = \delta_y$, because $\delta_{Ty} \gg \delta_y$, and replacing ∂y in Eq. (2) by δ_y , the following relationship is obtained:

$$\delta_y^2 = o[\nu L/u_0] = o[\nu L/u_s] \quad (42)$$

Therefore, from Eqs. (39) and (41):

$$u_s = o[(|\partial\sigma/\partial T|^2 \Delta T^2 \nu/(\mu^2 L))^{1/3}] \quad (43)$$

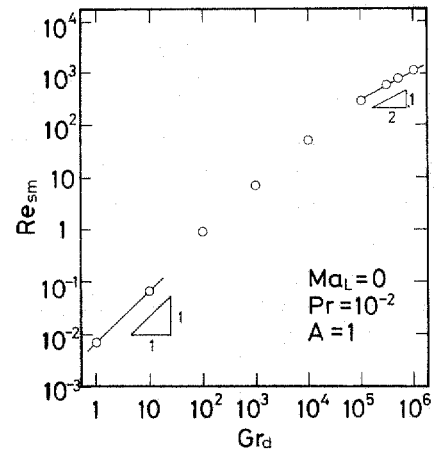


Fig. 3. Effect of natural convection on Reynolds number based on interfacial velocity

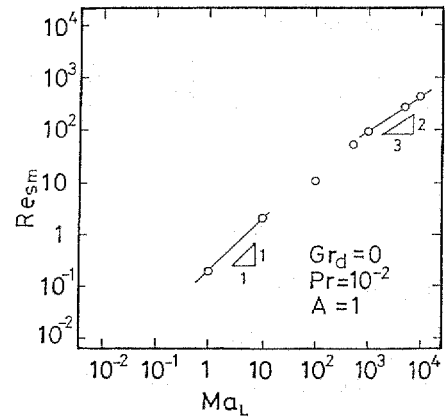


Fig. 4. Effect of Marangoni convection on Reynolds number based on interfacial velocity

and

$$Re_s = o[Ma_L^{2/3}] \quad \text{for } Pr \ll 1 \quad (44)$$

The results obtained by the present numerical analysis are shown in Figs. 3–6; these show good agreement with Eqs. (19), (21), (35), (38), (41) and (44).

2.3 The effect of melt depth on interfacial velocity

The effect of melt depth on interfacial velocity due to natural convection obtained by the present numerical analysis are shown in Fig. 7. From Fig. 7, when $Re_s \gg o[1]$,

$$Re_s = o[A^{-1/2} Gr_d^{1/2}] \quad (45)$$

and when $Re_s < o[1]$,

$$Re_s = o[A^{-5/2} Gr_d] \quad (46)$$

Here, $A = d/L$, and describes the aspect ratio.

Figure 8 shows the results obtained by the present numerical analysis on the effect of melt depth on interfacial velocity due to Marangoni convection.

When $Re_s < o[1]$,

$$Re_s = o[A^{1/2} Ma_L] \quad (47)$$

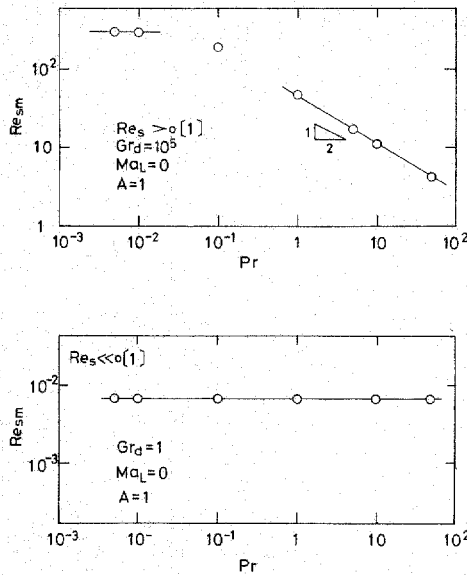


Fig. 5. Effect of Prandtl number on Reynolds number based on interfacial velocity in natural convection-dominant condition

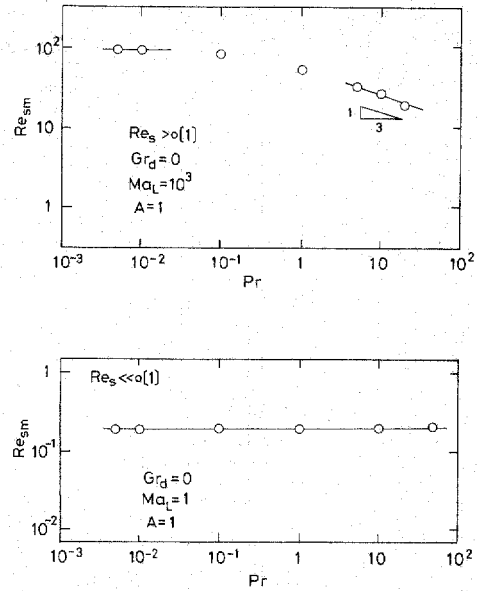


Fig. 6. Effect of Prandtl number on Reynolds number based on interfacial velocity in Marangoni convection-dominant condition

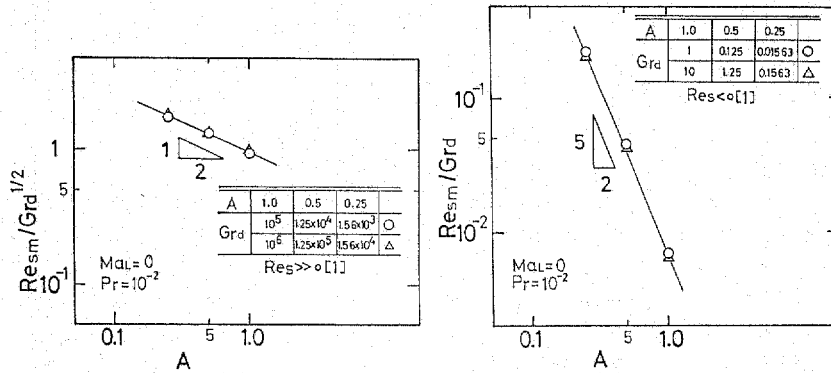


Fig. 7. Effect of melt aspect ratio on Reynolds number based on interfacial velocity in natural convection-dominant condition

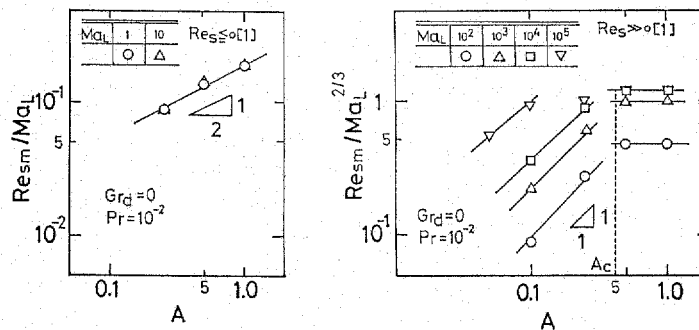


Fig. 8. Effect of melt aspect ratio on Reynolds number based on interfacial velocity in Marangoni convection-dominant condition

On the other hand, when $Re_s \gg o[1]$, there is a critical aspect ratio, A_c . When $A > A_c$, the interfacial velocity does not depend on the melt depth, but is described by the following relationship:

$$Re_{sm} = o[Ma_L^{2/3}] \quad (48)$$

When $A < A_c$, the interfacial velocity decreases as the melt depth decreases, and is described by

$$Re_{sm} = o[A \cdot Ma_L^{1/6} \cdot Ma_L^{2/3}] \quad (49)$$

When $A_c > A$, as described in Section 2.1, the shear stress at the bottom wall has an effect on the melt

Table 1. Effect of Grashof number, Marangoni number, Prandtl number and aspect ratio of melt ($0 < A \leq 1$) on Reynolds number based on interfacial velocity

Re_s	$Re_s \leq o[1]$ $o[\text{Inertial force}] \ll o[\text{Viscous force}]$		$Re_s \gg o[1]$ $o[\text{Inertial force}] \approx o[\text{Viscous force}]$	
	Natural convection dominant condition	Marangoni convection dominant condition	Natural convection dominant condition	Marangoni convection dominant condition
Estimated parameters	$\frac{A^3 \cdot Ma_L}{Gr_d} \ll o[1]$	$\frac{A^3 \cdot Ma_L}{Gr_d} \gg o[1]$	$\frac{f(A, Ma_L) \cdot Ma_L^{2/3}}{Gr_d^{1/2}} \ll o[1]$	$\frac{f(A, Ma_L) \cdot Ma_L^{2/3}}{Gr_d^{1/2}} \gg o[1]$
$Pr \ll o[1]$ (Si, Ge, GaAs)	$Re_s = o[A^{-2/5} \cdot Gr_d]$	$Re_s = o[A^{1/2} \cdot Ma_L]$	$Re_s = o[A^{-1/2} \cdot Gr_d^{1/2}]$	$Re_s = o[Ma_L^{2/3}]$ ($A \geq A_c$) $Re_s = o[A \cdot Ma_L^{1/6} \cdot Ma_L^{2/3}]$ ($A < A_c$)
Estimated parameters	$\frac{A^3 \cdot Ma_L}{Gr_d} \ll o[1]$	$\frac{A^3 \cdot Ma_L}{Gr_d} \gg o[1]$	$\frac{f(A, Ma_L) \cdot Pr^{1/6} \cdot Ma_L^{2/3}}{Gr_d^{1/2}} \ll o[1]$	$\frac{f(A, Ma_L) \cdot Pr^{1/6} \cdot Ma_L^{2/3}}{Gr_d^{1/2}} \gg o[1]$
$Pr \gg o[1]$ (Oxides)	$Re_s = o[A^{-2/5} \cdot Gr_d]$	$Re_s = o[A^{1/2} \cdot Ma_L]$	$Re_s = o[A^{-1/2} \cdot Pr^{-1/2} \cdot Gr_d^{1/2}]$	$Re_s = o[Pr^{-1/3} \cdot Ma_L^{2/3}]$ ($A \geq A_c$) $Re_s = o[A \cdot Pr^{-1/3} \cdot Ma_L^{1/6} \cdot Ma_L^{2/3}]$ ($A < A_c$)
* When $A \geq A_c$, $f(A, Ma_L) = A^{1/2}$. When $A < A_c$, $f(A, Ma_L) = A^{3/2} \cdot Ma_L^{1/6}$.				

convection, with the effect thought to increase as the melt depth decreases.

The above results are shown in **Table 1**. In Table 1, $A^3 \cdot Ma_L / Gr_d$, $f(A, Ma_L) Ma_L^{2/3} / Gr_d^{1/2}$, and $f(A, Ma_L) \cdot Pr^{1/6} Ma_L^{2/3} / Gr_d^{1/2}$ are parameters used to judge whether natural or Marangoni convection is the dominant convection in the melt. These parameters are not a simple ratio of the Marangoni to Grashof number, which had previously been generally thought; they are actually more complex, and also involve the aspect ratio.

To evaluate the above parameters, it is necessary to know the values of the physical properties of the melt.

In this study it was also assumed that the interfacial tension gradient (which is the cause of Marangoni convection) is formed only as a result of the temperature gradient at the free interface of the melt. However, the relationship between the interfacial tension gradient and the temperature and concentration gradients at the melt's free interface is complex, and when impurities, etc. adhere to the free interface, Marangoni convection is to a considerable degree suppressed (the interfacial contamination phenomenon). In future it is hoped that an assessment of Marangoni convection can be performed that takes into account the effects of the interfacial contamination phenomenon.

Conclusions

A theoretical consideration of the natural and Marangoni convections in a melt inside a two-dimensional rectangular open boat using an order of

magnitude evaluation and a numerical analysis according to the finite-difference method led to the following conclusions:

- 1) When the melt is shallow, Marangoni convection as well as natural convection is important, not only in microgravity environments but also under normal gravity.
- 2) Interfacial velocity due to natural and Marangoni convections is a function of the Grashof number, Marangoni number, Prandtl number and melt depth.

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Nomenclature

A	= aspect ratio ($= d/L$)	[—]
A_c	= critical aspect ratio	[—]
d	= melt depth	[m]
g	= gravitational acceleration	[m/s ²]
Gr_d	= Grashof number ($= g\beta\Delta T d^3/\nu^2$)	[—]
L	= length of free interface	[m]
Ma_L	= Marangoni number ($= \partial\sigma/\partial x \Delta T L / (\nu \cdot \mu)$)	[—]
o	= order	[—]
p	= pressure	[Pa]
Pr	= Prandtl number ($= \nu/\alpha$)	[—]
Re_s	= Reynolds number based on interfacial velocity ($= u_s L / \nu$)	[—]
Re_{sm}	= Reynolds number based on interfacial	[—]

	velocity at $x=L/2$ ($=u_{sm}L/v$)	[—]
T	= temperature	[K]
t	= time	[s]
u	= velocity parallel to interface	[m/s]
u_s	= interfacial velocity	[m/s]
u_{sm}	= interfacial velocity at $x=L/2$	[m/s]
u_0	= reference velocity parallel to interface	[m/s]
V	= dimensionless velocity ($=uL/v$ or vL/v)	[—]
v	= velocity normal to interface	[m/s]
v_0	= reference velocity normal to interface	[m/s]
x	= coordinate parallel to interface	[m]
y	= coordinate normal to interface	[m]
α	= thermal diffusivity	[m ² /s]
β	= thermal expansion coefficient	[1/K]
ΔT	= $T_H - T_C$	[K]
δ	= reference thickness of boundary layer	[m]
δ_{Tx}	= thickness of thermal boundary layer along hot-side wall	[m]
δ_{Ty}	= thickness of thermal boundary layer along free interface	[m]
δ_x	= thickness of velocity boundary layer along hot-side wall	[m]
δ_y	= thickness of velocity boundary layer along free interface	[m]
μ	= viscosity	[Pa · s]
ν	= kinematic viscosity	[m ² /s]
ρ	= density	[kg/m ³]
σ	= interfacial tension	[N/m]
<Subscripts>		
C	= cold	
H	= hot	

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