

Natural Convection as a Heat Engine: A Theory for CAPE

NILTON O. RENNÓ* AND ANDREW P. INGERSOLL

Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, California

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ABSTRACT

On many planets there is a continuous heat supply to the surface and a continuous emission of infrared radiation to space by the atmosphere. Since the heat source is located at higher pressure than the heat sink, the system is capable of doing mechanical work. Atmospheric convection is a natural heat engine that might operate in this system. Based on the heat engine framework, a simple theory is presented for atmospheric convection that predicts the buoyancy, the vertical velocity, and the fractional area covered by either dry or moist convection in a state of statistical equilibrium. During one cycle of the convective heat engine, heat is taken from the surface layer (the hot source) and a portion of it is rejected to the free troposphere (the cold sink) from where it is radiated to space. The balance is transformed into mechanical work. The mechanical work is expended in the maintenance of the convective motions against mechanical dissipation. Ultimately, the energy dissipated by mechanical friction is transformed into heat. Then, a fraction of the dissipated energy is radiated to space while the remaining portion is recycled by the convecting air parcels. Increases in the fraction of energy dissipated at warmer temperatures, at the expense of decreases in the fraction of energy dissipated at colder temperatures, lead to increases in the apparent efficiency of the convective heat engine. The volume integral of the work produced by the convective heat engine gives a measure of the statistical equilibrium amount of convective available potential energy (CAPE) that must be present in the planet's atmosphere so that the convective motions can be maintained against viscous dissipation. This integral is a fundamental global number qualifying the state of the planet in statistical equilibrium conditions. For the earth's present climate, the heat engine framework predicts a CAPE value of the order of 1000 J kg^{-1} for the tropical atmosphere. This value is in agreement with observations. It also follows from our results that the total amount of CAPE present in a convecting atmosphere should increase with increases in the global surface temperature (or the atmosphere's opacity to infrared radiation).

1. Introduction

Many types of convection are observed in the earth's atmosphere. They range from boundary layer convection, which might be topped by shallow non-precipitating cumulus, to deep convective systems that produce intense precipitation. The non-precipitating convection determines, in part, the structure and depth of the boundary layer in regions of undisturbed weather. It has a significant effect on the planet's heat budget. The deep convection, occurring in convectively disturbed regions, determines the structure and depth of the free troposphere. It has a crucial influence on the large-scale fields of heat, moisture, mass, momentum, and radiation.

In numerical modeling of planetary atmospheres, it is always necessary to compute (from given averaged fields) the transports due to subgrid-scale phenomena. This is done by using physical models that relate the subgrid-scale processes to the averaged fields. In classical turbulence theories, as well as in some models of atmospheric convection, it has often been assumed that the transports are related to the local gradient of the grid-scale fields. This is generally not true in convective atmospheres where convective plumes can extend through the entire depth of the atmosphere and are not controlled by local gradients at any one level (Betts 1974). In this paper, convection is assumed to be controlled by large-scale entropy gradients and atmospheric convection is studied from a global perspective.

A global theory for convection has eluded atmospheric scientists. It is generally agreed that convection tends to adjust the atmosphere toward an adiabatic state. However, the nature of the mean state of the adjusted atmosphere is controversial. Currently, in numerical models of the earth's atmosphere, it is assumed that dry convection adjusts superadiabatic temperature lapse rates to dry adiabatic ones. In general, those models also assume that moist convection adjusts moist unstable temperature lapse rates to moist neutral ones.

* Current affiliation: Department of Atmospheric Sciences, PAS Building #81, The University of Arizona, Tucson, AZ 85721.

Corresponding author address: Dr. Nilton O. Rennó, Department of Atmospheric Sciences, The University of Arizona, PAS Building Room #81, Tucson, AZ 85721.
E-mail: renno@air.atmo.arizona.edu

Little consideration is given to the physical process by which convection adjusts unstable atmospheres. In this study, we look at planetary convection from a large-scale perspective, that is, in statistical equilibrium. We focus on the thermodynamical process by which convection adjusts unstable atmospheres by describing atmospheric convection as a natural heat engine. Our objective is to present a framework useful for the basic conceptual understanding of the equilibrium state of convecting atmospheres.

On many planets there is a continuous heat supply to the surface and a continuous emission of heat to space by the colder troposphere. Since in this case the heat source is located at higher pressure than the heat sink, the system is capable of doing mechanical work. Sverdrup (1917), Brunt (1926), Lettau (1954), Oort (1964), and Lorenz (1967) used the heat engine concept to explain the earth's general circulation. They showed that the efficiency of the general circulation heat engine is of the order of one percent. Lorenz argued that the accurate determination and explanation of this efficiency constitutes the fundamental observational and theoretical problem of atmospheric energetics. Shuleikin (1953) used the heat engine framework to explain the monsoon circulations. Riehl (1950), Kleinschmidt (1951), and Emanuel (1986) in a classic paper, used the heat engine framework to explain the maintenance of the steady circulation of hurricanes. They pointed out that the energy source of hurricanes is in the isothermal expansion of the near-surface air moving inward into the storm center. In this study, we formulate atmospheric convection as a simple heat engine operating in planetary atmospheres. We show that, in statistical equilibrium conditions, substantial convective available potential energy (CAPE $\sim 1000 \text{ J kg}^{-1}$) is necessary to maintain the earth's convective circulations against mechanical dissipation.

If an air parcel is not in thermodynamical equilibrium with its surroundings, it will either expand or contract. Work will be performed against the environment. To first order, except for mechanical dissipation, this work of expansion is the only kind of work that must be considered in atmospheric systems. In convecting regions, the expanding hot air does work against the environment when heat is taken from the hot surface during its high temperature expansion near the surface. Further work is done at the expense of the thermal energy of the updraft air during its moist adiabatic expansion. Then, energy is expended in compressing the mixture of cooled air and condensate, while a smaller amount of heat is rejected to the environment. The net result of this convective cycle is that heat is taken from the surface layer (the hot source) and a portion of it is rejected to the free troposphere (the cold sink) while the balance is transformed into mechanical work. This mechanical work is expended in the maintenance of the convective motions against mechanical dissipation. Ultimately, the energy dissipated by mechanical friction

is transformed into heat. Then, a fraction of the dissipated energy is radiated to space while the remaining portion is recycled by the convecting air parcels.

On the large-scale, or in statistical equilibrium conditions, the convective motions are just strong enough so that the net work done by the convective heat engine is used exclusively to overcome the mechanical dissipation of energy. On the convective scale, the energy available for atmospheric convection is proportional to the local value of the convective available potential energy (CAPE). Arakawa and Schubert (1974) showed evidences that, on the large-scale, an ensemble of atmospheric convection might be considered to be in statistical equilibrium with the large-scale forcing. In statistical equilibrium conditions, there is a near equality between the rate of production of CAPE by the large-scale processes and its consumption by the ensemble of convective systems. The amount of CAPE present in a convecting atmosphere, in statistical equilibrium conditions, is a measure of the amount of mechanical dissipation of energy present in the convecting atmosphere. Thus, the volume integral of the work produced by the convective heat engine gives a measure of the statistical equilibrium amount of CAPE that must be present in a planet's atmosphere so that the convective circulations can be maintained against dissipation.

2. Boundary layer convection as a heat engine

A heat engine is defined as any device that transforms heat into mechanical work. Therefore, atmospheric convection is a natural heat engine. In this study we assume that natural convection is a simple Carnot Engine. In a Carnot Engine the maximum thermal efficiency of a heat engine is attained. For a real heat engine to approach the Carnot efficiency, the thermodynamic process must be thermally and mechanically reversible, which does not occur in natural convection. However, since unstable systems drift toward local states of maximum efficiency (Glansdorff and Prigogine 1971), nature probably strives toward the Carnot efficiency.

In a simple Carnot cycle, the work done by the adiabatic expansion is thermodynamically constrained to be canceled by the adiabatic compression (since the work done by external forces, reversibly and adiabatically on a system, is independent of the path). Thus, in a Carnot cycle, both the adiabatic expansion and the adiabatic compression have to be either dry or moist adiabatic. Therefore, a Carnot cycle should not be used in the idealization of a cycle in which the ascent is moist adiabatic and the descent is dry adiabatic.

Observations show that in non-precipitating boundary layer convection, below the cloud base, the updrafts and downdrafts are approximately adiabatic (Fig. 1). Above cloud base, the updrafts and downdrafts might also be considered adiabatic, since most of the mixing in shallow clouds occurs at the cloud's top (Squires

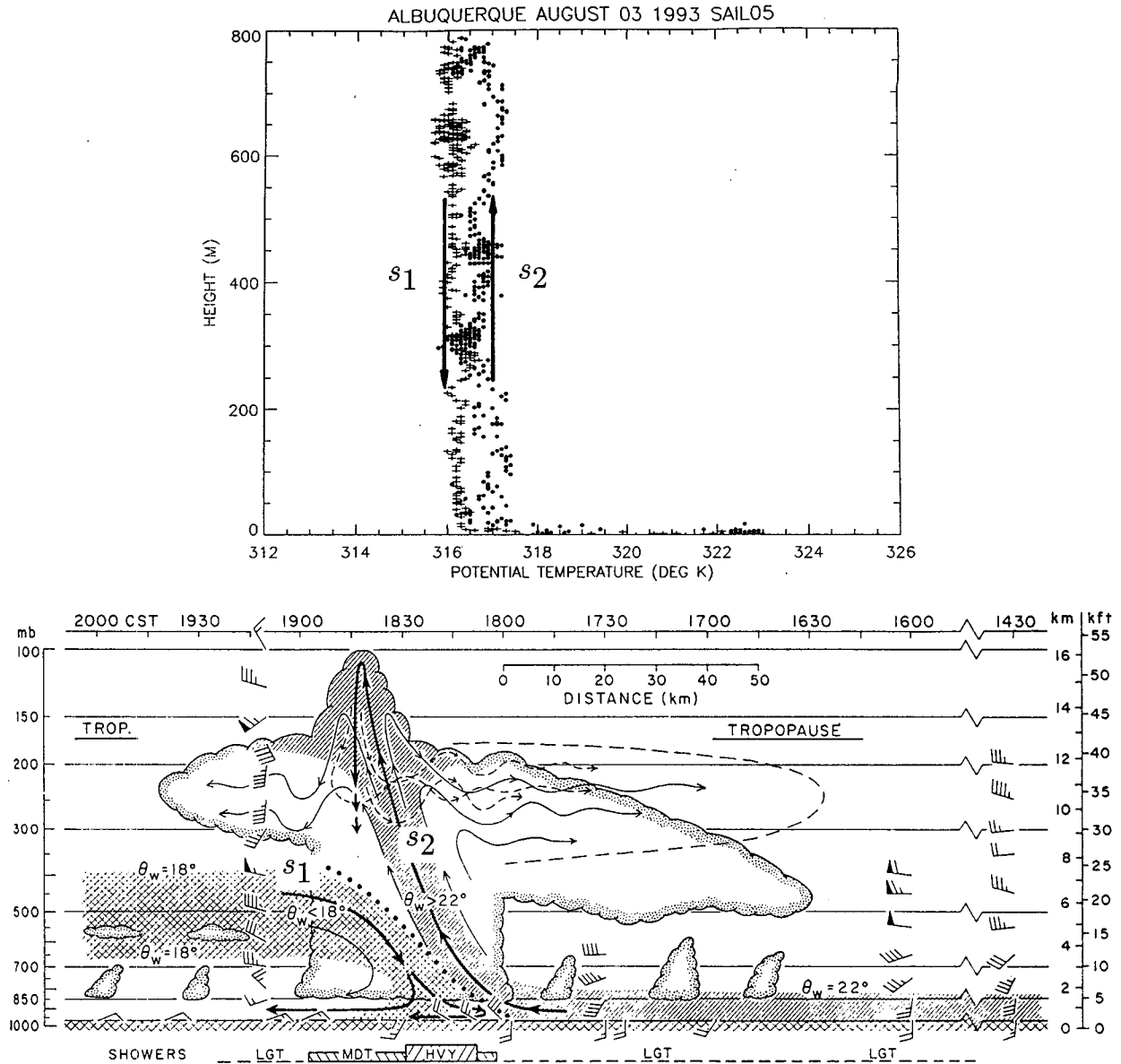


FIG. 1. Sketch of the heat engine circulation superimposed on the observations of the thermodynamic properties of the updraft and downdraft air of convective systems. The axes of the main drafts are represented by heavy arrows. The top figure (after Rennó and Williams 1995) is a sketch for a boundary layer convective system. The observations in the updraft are represented by dots, and the observations in the downdraft are represented by crosses. The temperatures well above 318 K are spurious surface temperatures obtained when the sensor gets in contact with the ground after the RPV's landing. The bottom figure (after Newton 1966) is a sketch for a deep convective system. Hatching indicates the high entropy updraft air (wet-bulb potential temperature in excess of 22°C), and cross-hatching indicates the low entropy downdraft air (wet-bulb potential temperature less than 18°C).

1958; Paluch 1979). Here, we assume that the mechanical dissipation of energy occurs only near the surface and at the top of the convecting layer where the small-scale eddies are more intense. Since the updrafts and downdrafts are approximately adiabatic and the absorption and rejection of heat are approximately isothermal, the simple Carnot cycle might be a good approximation for non-precipitating boundary layer convection.

We assume that convection is in statistical equilibrium, and we follow the convecting air parcel around a streamline (in a closed cycle). Since the closed steady circulation might exist only in a statistical sense, we might imagine that we are following a hypothetical air parcel around an averaged streamline representing a collection of convective drafts in statistical equilibrium. The energy cycle of a heat engine can be obtained by integrating Bernoulli's equation and the first law of

thermodynamics around a streamline (Emanuel 1986, 1989); that is

$$Tds - d\left(\frac{1}{2}|\mathbf{v}|^2 + c_p T + L_v r + gz\right) - \mathbf{f} \cdot d\mathbf{l} = 0, \quad (1)$$

where T is the absolute temperature, s the specific entropy, \mathbf{v} the vector velocity, c_p the heat capacity at constant pressure per unit mass, L_v the latent heat of vaporization of water per unit mass, r the water vapor mixing ratio, g the gravity acceleration, z the height above a reference level, \mathbf{f} the frictional force per unit mass, and $d\mathbf{l}$ the incremental distance along the streamline.

Note that Eq. (1) neglects the heat capacity of water substance and the effects of water substance on the density of the moist air. Emanuel (1988) derives a more precise equation and estimates the error in Eq. (1) to be of the order of 5%. We also assume the processes to be thermodynamically reversible. In particular, we neglect the diffusive and radiative heat fluxes in the updrafts and the downdrafts, which are assumed to be adiabatic. The assumptions of steadiness and closed circulation are irrelevant to the energetics of the heat engine. The assumptions of reversibility and of either, moist or dry up-downdraft couplet is necessary only for the idealization of the heat engine cycle as a Carnot cycle (they are not necessary in a general heat engine cycle). We make these assumptions because here we wish both to isolate the essential physics to the energy cycle of convective systems, as well as to study the statistical equilibrium state of convecting atmospheres. Our main goal is to present a simple model that captures in mathematical terms the essence of convective systems. Furthermore, we intend to show that substantial convective available potential energy (CAPE $\sim 1000 \text{ J kg}^{-1}$) is necessary to maintain the convective circulations against mechanical dissipation.

The essential feature of a heat engine is the fact that heat must be absorbed by the working fluid at a higher temperature than heat is rejected. The fact that heat must be absorbed by the working fluid at a higher temperature than heat is rejected also means that, in convecting atmospheres, heat must be absorbed by the working fluid at a higher pressure than heat is rejected. Furthermore, in a heat engine the flow does not have to be steady, and the working fluid can exchange mass with the environment. Indeed, this is what happens in most engineering heat engines (e.g., gas turbines and internal combustion engines). The Stirling Cycle Engine is one of the few examples of closed cycle engineering heat engines. This engine employs the alternate heating and cooling of an enclosed working fluid (hydrogen) for the production of mechanical energy (Considine and Considine 1989). The justification for the use of a hypothetical closed thermodynamic cycle in the study of a real, noncyclic open engine, is that the main constituent of the working fluid—dry air (satu-

rated air in our idealization of deep moist convection)—remains virtually unchanged in the mixing processes.

Integrating Eq. (1) around a closed cycle, we get

$$\oint Tds - \oint \mathbf{f} \cdot d\mathbf{l} = 0, \quad (2)$$

which states that in statistical equilibrium, the friction is balanced by the net heat input.

Integrating the first law of thermodynamics, we get

$$\oint Tds = \oint p d\alpha, \quad (3)$$

where p is the pressure, and α is the specific volume. Thus, the first term in Eq. (2) represents the net work done by the heat engine cycle.

Since the net work done by one cycle of the heat engine is equal to the total mechanical energy available for convection over one cycle, we define

$$\text{TCAPE} \equiv \oint Tds, \quad (4)$$

where TCAPE is the total convective available potential energy from a *reversible* heat engine over one cycle. This is the energy that can be converted to kinetic energy by a *reversible* heat engine. It includes the available energy that is converted to kinetic energy by both the updraft and the downdraft.

In a thermodynamic diagram, Eq. (4) integrated around a Carnot cycle represents the area enclosed by the hot and the cold adiabats (s_2 and s_1), and the hot and the cold isotherms (T_h and T_c), respectively, at the bottom and the top of a "Carnot convective circulation." This area, in turn, represents the total amount of work done by the buoyancy forces in moving an air parcel around the convective cycle. Brunt (1941) showed that the area enclosed by the hot adiabat, the ambient temperature sounding, and the top and the bottom of the convective layer represents the total amount of work done by the buoyancy forces in moving an air parcel along the updraft (that is CAPE). In the appendix, we show that TCAPE is equivalent to the "standard" meteorological definition of *total* CAPE. Since for nonprecipitating boundary layer convection most of the mixing occurs at the cloud's top (Squires 1958; Paluch 1979), T_c is approximately equal to the temperature at this level (T_c for deep convection is defined in section 3). Since, in general, the entropy excess of the updrafts over the environment is of the same order of magnitude as the entropy excess of the environment over the downdrafts, we argue that to first order $\text{TCAPE} \approx 2 \times \text{CAPE}$. The *total* CAPE—that is TCAPE—is the energetically meaningful quantity, *not* CAPE.

It follows from Eqs. (2) and (4) that for a general convective heat engine we must have

$$\text{TCAPE} - \oint \mathbf{f} \cdot d\mathbf{l} = 0. \quad (5)$$

This states that, in statistical equilibrium, friction is balanced by TCAPE. Therefore, a zero total CAPE value is impossible in convecting atmospheres, except in a hypothetical planet in which the convective motions are inviscid.

We define a dimensionless ‘‘coefficient of dissipation of mechanical energy’’ along the integration path as

$$\mu \equiv \frac{\oint \mathbf{f} \cdot d\mathbf{l}}{w^2}, \quad (6)$$

where w is the magnitude of the convective velocity.

Thus, from Eqs. (5) and (6), we get

$$w = (\mu^{-1} \text{TCAPE})^{1/2}. \quad (7)$$

Note that large μ implies large dissipation and, therefore, small vertical velocities.

The dissipation of mechanical energy is mainly due to the turbulent viscosity $\nu_{\text{turb}} \sim l\Delta w$, where l and Δw are the eddy’s characteristic length and velocity scale. Thus, the coefficient of dissipation of mechanical energy can be written as

$$\mu \sim \frac{(\nu_{\text{turb}} \nabla^2 w) l_{\text{path}}}{w^2} \sim \frac{(\Delta w)^2 l_{\text{path}}}{w^2 l}. \quad (8)$$

In homogeneous and isotropic turbulence, the magnitude of the velocity fluctuations and of the scale of the largest eddies corresponds, respectively, to the amplitude of the velocity change across the fluid and to the size of the unstable region. Assuming homogeneous and isotropic turbulence, the turbulent eddies are as energetic as the convective drafts. Thus, the characteristic velocity scale is $\Delta w \sim 2w$. Moreover, assuming that the length of the integral path is $l_{\text{path}} \sim 4l$ —a lower bound for a shallow atmosphere—the coefficient of dissipation of mechanical energy is given by $\mu \sim 16$.

In a rotating planet, the length of the integral path must be a function of the Rossby radius of deformation. In this study we have ignored the effects of rotation. We could have used a more sophisticated parameterization for μ , but since our present objective is an order-of-magnitude calculation, the above estimative is sufficient. Consequently, μ is the potentially tunable parameter of our scaling analysis.

3. Deep convection as a heat engine

In the deep convection occurring in convectively disturbed regions, the bulk of the vertical transport is accomplished by subcloud-scale drafts (Stommel 1947; Squires 1958; Paluch 1979; Raymond and Wilkening 1985). These subcloud-scale drafts consist of undiluted updrafts, mixed-air updrafts, and downdrafts. Their

spatial scale goes down to the smallest detectable scale with current instruments—about 10 m (Blyth and Latham 1990). In these deep convective systems, most of the convective-scale cooling—through mixing and the subsequent evaporation of condensate—occurs at the middle troposphere where the moist entropy (equivalent potential temperature) presents a minimum value (Browning 1964; Newton 1966; Zipser 1977) and a strong convergence is forced by the release of latent heat of fusion (Fig. 1b). Additional cooling occurs at higher levels by the emission of infrared radiation to space.

Observations and numerical modeling (Browning 1964; Newton 1966; Klemp and Wilhelmson 1978) show that the fact that the main updraft is isolated from the main downdraft is the dynamically important feature of intense thunderstorms (see Figs. 1 and 2). Moreover, the intense updraft is a consequence of almost undiluted ascent in the moist unstable atmosphere. Observations also show that the strong downdraft is a consequence of the melting and evaporation of precipitation, as well as of the evaporation of condensate in the mixing of cloud material with the cold and dry middle tropospheric air. Since in severe storms the downdrafts and updrafts are able to coexist, they continuously gain energy from the convective heat engine. In steady state, this energy is used to balance the mechanical dissipation of energy by the convective system.

Since the warm moist adiabatic updraft and the cold moist adiabatic downdraft (that includes the falling precipitation) are the most important features of the deep convective systems, which frequently reach a quasi-steady state, these systems can also be idealized as Carnot heat engines in statistical equilibrium conditions. Note that we assume that both the updrafts and the downdrafts follow a reversible saturated adiabatic process. This way, the model discussed in the previous

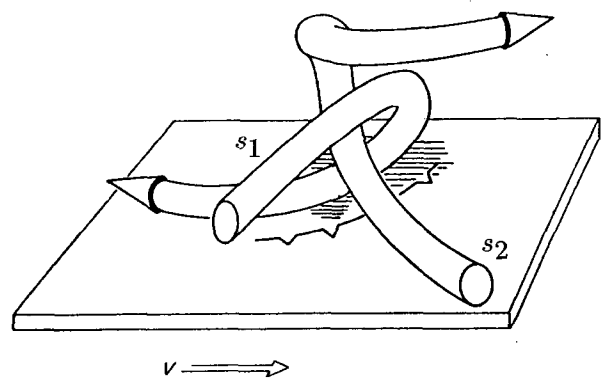


FIG. 2. Three-dimensional sketch of the updraft and downdraft, relative to the storm, in a severe thunderstorm. Here, s_2 and s_1 refer to the updraft and downdraft, respectively. Also shown are the approximate extent of precipitation at the surface (hatched area) and the position of the surface gust front. Note the exaggeration of the vertical scale (after Browning 1964).

section is also valid for deep moist convection, with the caveat that now the temperature of the cold source, T_c , is the entropy-weighted mean temperature between the top of the intense moist adiabatic updrafts, and the root of the intense moist adiabatic downdrafts [see Eq. (18)]. The temperature of the cold source for an ensemble of convective drafts is rigorously defined in section 4. Figure 3 illustrates deep convection as a heat engine. Because of the large complexity of deep precipitating convection, the simple Carnot engine might not be as good an idealization for deep convection as it is for boundary-layer convection. In another study we intend to present a more rigorous model for deep precipitating convection.

As in the previous section, we assume that convection is in statistical equilibrium, and we follow the convecting air parcel around a streamline (in a closed cycle). Again, since the closed circulation might exist only in a statistical sense, we imagine that we are following a hypothetical air parcel around an averaged streamline representing a collection of convective drafts in statistical equilibrium (Figs. 1, 2, and 3).

4. Quasi-equilibrium CAPE in convecting atmospheres

The quasi-equilibrium principle states that the rate of generation of kinetic energy by an ensemble of convective clouds is approximately equal to its rate of dissipation by the circulations associated with the ensemble. That is, it states that an ensemble of convective clouds is approximately in statistical equilibrium with the large-scale forcing. This principle was originally suggested by Arakawa (1969), was applied to shallow nonprecipitating convection by Betts (1973), and was systematically applied to atmospheric convection by Arakawa and Schubert (1974).

The quasi-equilibrium principle is based on the fact that the convective timescale is appreciably shorter than that of the large-scale processes with which it interacts. Thus, for an ensemble of convective clouds there is a near equality between the rate of production of total convective available potential energy (CAPE) by the large-scale processes, its rate of conversion to kinetic energy by the convective systems, and the rate of dissipation of kinetic energy by mechanical friction. At quasi-equilibrium the convective circulations are just strong enough so that all the energy available from the heat engine is used to overcome the mechanical dissipation.

Equation (5) is valid for an i th subensemble of convective up-draft couplets convecting between the surface layer and the i th atmospheric layer of temperature T_c^i . Here, we derive an equation for the entire ensemble of convective drafts existing between the surface layer and the troposphere's top. Multiplying Eq. (5) by the i th convective draft mass flux, M^i , and summing over all drafts, we get an equation for the entire

ensemble of convective drafts in quasi-equilibrium conditions

$$\sum_i \left[M^i \left(\overline{\text{TCAPE}}^i - \oint \mathbf{f}^i \cdot d\mathbf{l}^i \right) \right] = 0$$

$$\overline{M\text{TCAPE}} - F_d = 0, \quad (9)$$

where

$$M = \sum_i (M^i) \quad (10)$$

is the *total* convective mass flux,

$$\overline{\text{TCAPE}} = \frac{1}{M} \sum_i (M^i \text{TCAPE}^i) \quad (11)$$

is the *mass-flux-weighted mean* TCAPE value, and

$$F_d = \sum_i \left(M^i \oint \mathbf{f}^i \cdot d\mathbf{l}^i \right) \quad (12)$$

is the *total* flux of energy mechanically dissipated by the ensemble of convective drafts. Assuming that both the radiative cooling rate and the temperature lapse rate are constant throughout the troposphere (this implies constant convective mass flux throughout the troposphere), the above equations simplify to their regular mean values. In this study we assume $\overline{\text{TCAPE}} \approx \text{TCAPE} \approx 2 \times \text{CAPE}$.

At quasi-equilibrium, the flux of energy dissipated by convection is nearly equal to the flux of energy available for the heat engine; that is,

$$F_d = F_{av}, \quad (13)$$

where F_{av} is the heat flux available for mechanical work by the heat engine. We assume that all the energy available for the convective heat engine is transformed into mechanical energy, and subsequently consumed by mechanical friction. For a general thermodynamically reversible heat engine, we have that

$$F_{av} = F_{in} - F_{out}, \quad (14)$$

where F_{in} and F_{out} are, respectively, the heat fluxes in and out of the convective heat engine. Here F_{av} can be written as

$$F_{av} = \eta F_{in}, \quad (15)$$

where η is the thermodynamic efficiency of the heat engine. The thermodynamic efficiency of a general heat engine is defined as

$$\eta \equiv \left(\frac{\int_{in} T ds - \int_{out} T ds}{\int_{in} T ds} \right), \quad (16)$$

where the subscripts *in* and *out* represent, respectively, integration at the heat intake and at the heat rejection

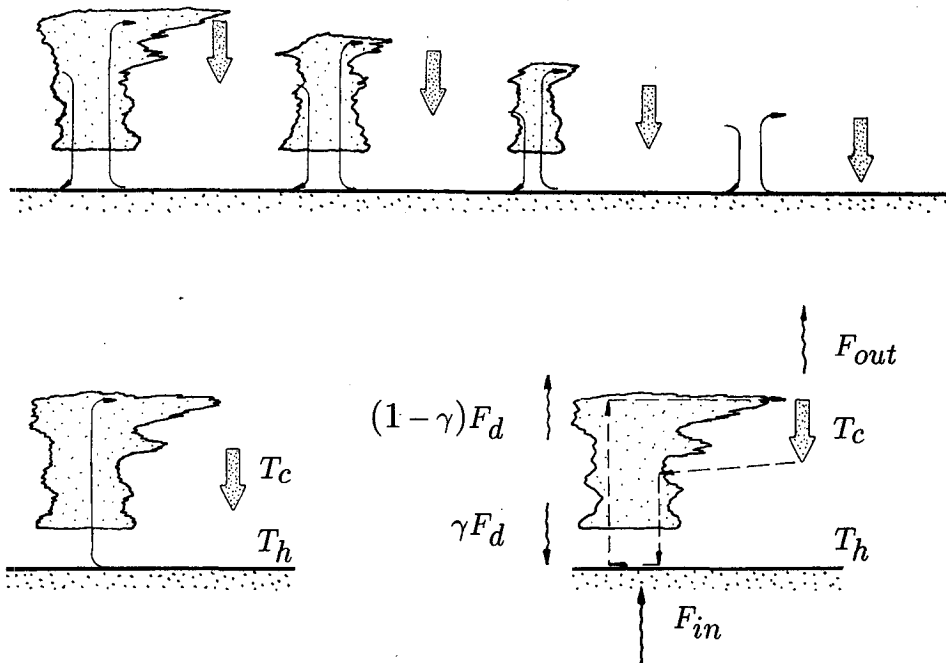


FIG. 3. Idealization of atmospheric convection: (i) An ensemble of convective systems in radiative-convective equilibrium (top); (ii) net result of an ensemble of convective systems in radiative-convective equilibrium (bottom-left); and (iii) a single convective system as a heat engine—a unit of the ensemble (bottom right). The convective system is defined as the volume of air containing the convective-scale up-draft and down-draft couplet. The up-draft circulation (adiabatic branches) together with the subsidence region (heat rejection branch—at T_c) and the near-surface expansion (heat intake branch—at T_h) form the Carnot cycle (dashed line). Here F_{in} , F_{out} , and F_d are, respectively, the input, the output, and the dissipated heat flux. A fraction of the dissipated heat flux, γF_d , is recycled by the heat engine while the remaining portion, $(1 - \gamma)F_d$, is radiated to space.

$$\eta \approx \left(\frac{T_h \Delta s - \int_{out} T ds}{T_h \Delta s} \right)$$

$$\eta \approx \left(\frac{T_h - T_c}{T_h} \right), \quad (17)$$

where

$$T_c = \frac{1}{\Delta s} \int_{out} T ds, \quad (18)$$

$\Delta s = \int_{out} ds = s_2 - s_1$ is the entropy excess of the updrafts over the downdrafts, and T_c is the entropy-weighted mean temperature of the layer emitting infrared radiation. It follows from the above that the temperature of the cold source for the entire ensemble of convective drafts in radiative-convective equilibrium is equal to the entropy-weighted-mean tropospheric temperature (the region rejecting heat).

In a semi-opaque planet the total heat input from the hot reservoir to the near surface air, at temperature T_h , is

$$F_{in} = F_S + F_L + F_R + F_d^h, \quad (19)$$

where F_S , F_L , and F_R are, respectively, the sensible, latent, and net radiative heat fluxes into the near surface air, and F_d^h is the irreversible heat flux due to kinetic energy dissipated near the surface (the hot source). We show in section 5 that increases in the fraction of energy dissipated near the hot source, at the expense of decreases in the fraction of energy dissipated at colder temperatures, lead to increases in the apparent efficiency of the convective heat engine.

Using Eqs. (9), (13), and (15) we get that for a general natural heat engine

$$\text{TCAPE} \approx \left(\frac{1}{M} \right) \eta F_{in}. \quad (20)$$

It is important to note that increases in the heat engine efficiency, or in the heat input to the convective heat engine, lead to increases in TCAPE. This happens because, in this case, more energy is available to be converted into mechanical energy. Thus, the equilibrium state is one of strong convective circulations and intense dissipations. Then, a large TCAPE value is necessary to maintain the intense circulations.

The heat input to the convective heat engine can be written as $F_{in} = M(c_p \Delta T + L_v \Delta r)$, where ΔT and Δr

are, respectively, the temperature and moisture excess of the convective updrafts over the downdrafts. Thus, using Eq. (20) it follows that

$$\text{TCAPE} \approx \eta(c_p \Delta T + L_v \Delta r). \quad (21)$$

Equation (21) shows that only a fraction η of the heat input to natural convection is available to be transformed into kinetic energy. The second term on the right-hand side of Eq. (21) shows that only a fraction η of the "latent heat released" by moist convection is available to be transformed into kinetic energy.

The convective mass flux can be written as

$$M = \rho \sigma w, \quad (22)$$

where ρ is the updraft air density, σ is the fractional area covered by updrafts, and w is the magnitude of the convective velocity. Using Eqs. (7) and (22), and the perfect gas law, the convective mass flux at the top of the superadiabatic surface layer—at the root of the convective updrafts (Figs. 1 and 2)—is approximately given by

$$M \approx \left(\frac{\sigma}{\mu^{1/2} R} \right) \left(\frac{p_h}{T_h} \right) \text{TCAPE}^{1/2}, \quad (23)$$

where p_h is the surface pressure, and R is the specific gas constant for air.

Thus, using Eqs. (20) and (23) we get that for a general natural heat engine

$$\text{TCAPE}^{3/2} \approx \left(\frac{\mu^{1/2} R}{\sigma p_h} \right) (T_h - T_c) F_{in}; \quad (24)$$

alternatively we can write

$$\text{TCAPE}^{3/2} \approx \left(\frac{\mu^{1/2} \eta}{\sigma} \right) \left(\frac{F_m}{\rho} \right). \quad (25)$$

Equations (25) and (24) give the total amount of CAPE that should be present in a convecting atmosphere in order for the net work done by the convective heat engine to be balanced by dissipation. The $\text{TCAPE}^{(3/2)}$ value is proportional to the square root of the mechanical dissipation of energy parameter μ , to the total heat input at the surface, as well as to the difference between the temperature at the root of the updrafts, T_h , and the temperature at the root of the downdrafts, T_c [note that $(T_h - T_c) = \eta T_h$ is a measure of the thermal efficiency of the convective heat engine]. It is also inversely proportional to the fractional area covered by updrafts, σ , and to the surface pressure, p_h . The fact that the TCAPE value is inversely proportional to the fractional area covered by updrafts has important consequences to the equilibrium state of convecting atmospheres (see section 8).

5. Effects of mechanical dissipation on thermodynamic efficiency

Here, we show that increases in the fraction of energy dissipated near the heat source, at the expense of

decreases in the fraction of energy dissipated near the heat sink, lead to increases in the apparent efficiency of the convective heat engine.

Let γ be the fraction of the mechanical energy dissipated at the hot source (see Fig. 4), thus

$$F_{in} = F_{ext} + \gamma F_d, \quad (26)$$

and

$$F_{out} = F_{ext} - (1 - \gamma) F_d, \quad (27)$$

where F_{in} and F_{out} are, respectively, the heat input and the heat output to the heat engine; F_{ext} is the "external" heat input to the heat engine (e.g., the net flux of solar radiation); and, F_d is the flux of mechanical energy irreversibly dissipated by the heat engine (at equilibrium $F_d = F_{av} = F_{in} - F_{out}$).

Figure 4 indicates that for an atmosphere in radiative-convective equilibrium $F_{in} > F_{out}$, otherwise there would be no energy available to maintain the convective motions against viscous dissipation (the irreversible heat source). Note that $F_{in} = F_{out}$ implies $\eta = 0$. However, the external heat flux to the convecting atmosphere is balanced by the sum of the reversible heat output from the convective heat engine, F_{out} , and the fraction of the irreversible heat flux rejected to the cold reservoir, $(1 - \gamma) F_d$. That is, $F_{ext} = F_{out} + (1 - \gamma) F_d$.

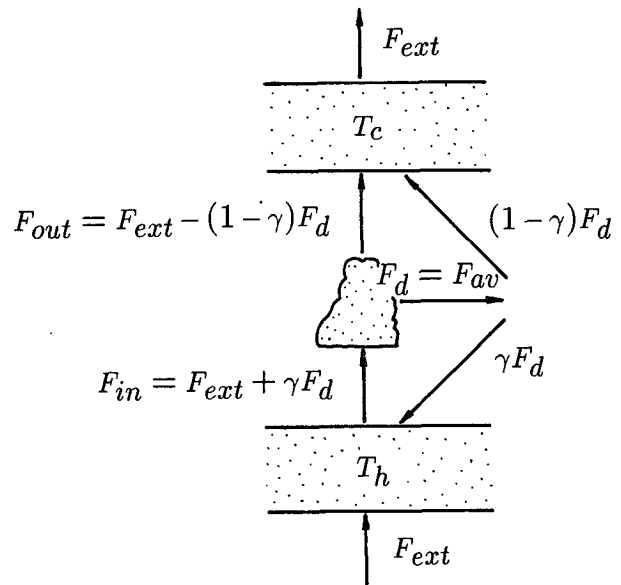


FIG. 4. Sketch of the energy fluxes throughout the convective heat engine in quasiequilibrium conditions. The first law of thermodynamics requires that the net heat flux across the heat source be equal to the net heat flux across the heat sink (F_{ext}). The second law of thermodynamics requires that the flux of energy available for mechanical work from a reversible heat engine be equal to the difference between the input and output heat fluxes ($F_m = F_{in} - F_{out} = F_d$). The fraction of the mechanical energy dissipated by viscous friction (γF_d) that is recycled by the convecting air parcels increases the heat input (F_{in}) to the convective heat engine.

From the standard definition of thermodynamic efficiency of heat engines, we have

$$\eta \equiv \frac{F_{av}}{F_{in}}. \quad (28)$$

Defining the apparent thermodynamic efficiency of the convective heat engine as that based on the external heat flux only, we have

$$\eta_{app} \equiv \frac{F_{av}}{F_{ext}}. \quad (29)$$

It follows from Eqs. (28) and (29) that

$$\eta_{app} = \frac{\eta}{(1 - \gamma\eta)}. \quad (30)$$

Thus, increases in the fraction of energy dissipated near the hot source (γ) lead to increases in the apparent efficiency of the heat engine. Tozer (1965) and Hewitt et al. (1975) arrived to a similar conclusion in studies of astrophysical and mantle convection.

6. A simple theory for the fractional area covered by convection

Observations of moist convection on earth (Wyngaard 1990; Williams et al. 1992), as well as linear stability analysis (Bjerknes 1938; Lilly 1960; Kuo 1961) suggest that the fractional area covered by updrafts decreases with increases in the air temperature near the surface. This happens because increases in the air temperature lead to increases in the moisture content, which, in turn, lead to larger differences between moist and dry adiabatic temperature lapse rates. In this section, we use physical arguments to propose a simple theory for the fractional area covered by an ensemble of convective updrafts in quasi-equilibrium conditions, σ , valid for either dry or moist convection.

Integrating the entropy budget equation (Betts 1983, 1989) throughout a radiating slab (subscript Δp) in quasi-equilibrium conditions, we get

$$\int_{\Delta p} \left(\rho g w_R \frac{\partial s}{\partial p} \right) dp = \int_{\Delta p} \frac{g}{T_c} \left(\frac{\partial N}{\partial p} \right) dp$$

$$\rho g w_R \Delta s = \frac{g}{\bar{T}_c} \Delta N$$

$$\rho g w_R \Delta s = \left(\frac{\Delta s_R}{\tau_R} \right) \Delta p, \quad (31)$$

where g is the gravity acceleration; w_R is the magnitude of the mean vertical velocity of the ensemble's subsiding air parcels; N is the net radiative flux; \bar{T}_c is a mean with respect to the divergence of the net radiative flux (the radiative cooling); Δs and $\Delta s_R = (c_p \Delta T_R / \bar{T}_c)$ are, respectively, the ensemble mean entropy excess of the updrafts over the downdrafts and the ensemble

mean entropy excess of the subsiding parcels over the slab's local radiative equilibrium entropy; ΔT_R is the ensemble mean temperature excess of the subsiding parcels over the slab's local radiative equilibrium temperature; τ_R is the radiative timescale; and, Δp is the pressure thickness of the radiating slab. We have approximated the mean radiative cooling of the atmospheric slab to $(\Delta T_R / \tau_R) = g / c_p (\Delta N / \Delta p)$, as well as neglected the small increase in entropy due to the nonequilibrium thermodynamics of the mixing process at the root of the convective downdrafts. Moreover, we have neglected the heating of the radiating slab by mechanical dissipation of energy.

In quasi-equilibrium conditions, the entropy excess of the ensemble of convective updrafts over the downdrafts must be lost by the emission of infrared radiation to space by the subsiding air parcels (in between the top of convective updrafts and the root of the convective downdrafts). Furthermore, the departure of the slab's entropy from the local radiative equilibrium value is forced by convection. Thus, we have that $(\Delta s_R / \Delta s) \approx 1$ and Eq. (31) is simplified to

$$w_R \approx \left(\frac{\Delta p}{\rho g \tau_R} \right). \quad (32)$$

A rough estimate of τ_R can be obtained by considering a slab of atmosphere with pressure thickness Δp and uniform density radiating like a graybody (Houghton 1986); in this case we get

$$\tau_R \approx \left(\frac{c_p \Delta p}{8 g \epsilon \sigma_R T_c^3} \right), \quad (33)$$

where ϵ is the slab's emissivity and σ_R is the Stefan-Boltzmann constant. The factor $(1/8)$ is due to the fact that the slab is radiating both upward and downward ($2\sigma_R T_c^4$). Consistently with section 4, the radiative cooling term must be computed using the entropy-weighted mean temperature of the subsidence region. Thus, T_c is the entropy-weighted mean temperature of the subsidence region. Note that for the entire ensemble of convective drafts in quasi-equilibrium conditions, Δp is the troposphere's thickness, and T_c is the entropy-weighted mean tropospheric temperature [see Eq. (18), section 4].

From Eqs. (20) and (22), we get

$$\eta F_{in} \approx \rho \sigma w \text{TCAPE}. \quad (34)$$

The mass continuity equation can be written as

$$\rho \sigma w = \rho_R (1 - \sigma) w_R$$

$$w_R \approx \sigma w, \quad (35)$$

where we have assumed $\sigma \ll 1$, and that the density of the subsiding air parcel $\rho_R \approx \rho$.

From Eqs. (7), (34), and (35), we get

$$\begin{aligned}\eta F_{in} &\approx \rho w_R (\mu w^2) \\ \eta F_{in} &\approx \rho \mu \left(\frac{w_R^3}{\sigma^2} \right).\end{aligned}\quad (36)$$

Substituting Eq. (32) into Eq. (36), we get

$$\sigma \approx \left(\frac{\mu}{\eta} \right)^{1/2} \left(\frac{\Delta p}{\rho g \tau_R} \right)^{3/2} \left(\frac{F_{in}}{\rho} \right)^{-1/2}.\quad (37)$$

For tropical values of the various parameters ($\eta \approx 0.1$, $F_{in} \approx F_L \approx 155 \text{ W m}^{-2}$, $\Delta p \approx 8 \times 10^4 \text{ Pa}$, $\rho \approx 1 \text{ kg m}^{-3}$, $g = 9.8 \text{ m s}^{-2}$, $\tau_R \approx 1.3 \times 10^6 \text{ s}$, and $\mu \sim 16$), we get $\sigma \approx 5.0 \times 10^{-4}$. This is a reasonable value, since about 3% of the Tropics is covered by active deep convection, and only about 1% of the fractional area covered by deep convection is covered by undiluted drafts (Cotton and Anthes 1989; Black et al. 1994). With the above parameters, Eq. (25) gives $\text{TCAPE} \approx 2500 \text{ J kg}^{-1}$, thus $\text{CAPE} \approx 1250 \text{ J kg}^{-1}$, which is of the order of magnitude of the observed value (see section 8).

Defining the convective (updraft) timescale as $\tau_u \equiv (\Delta p / \rho g w)$, we get

$$\sigma \approx \frac{\tau_u}{\tau_R},\quad (38)$$

where for physically possible solutions we must have $0 < \sigma < 1$. Since in the derivation of the above equation we assumed $(1 - \sigma) \approx 1$, Eq. (38) is valid only for $\sigma \ll 1$ (that is for $\tau_u \ll \tau_R$). Note that our theory predicts decreases in the fractional area covered by convective drafts with decreases in the convective timescale and increases in the radiative timescale.

7. A simple theory for quasi-equilibrium convection

Based on the results described in sections 4–6, we propose a simple theory for atmospheric convection in radiative–convective equilibrium conditions. Substituting Eq. (37) into Eq. (25), we get

$$\text{TCAPE} \approx \left(\frac{g \tau_R}{\Delta p} \right) \eta F_{in}.\quad (39)$$

Alternatively, for an opaque atmosphere radiating as a graybody from a layer of temperature T_c , one can use Eq. (33), getting

$$\text{TCAPE} \approx \left(\frac{c_p}{8 \epsilon \sigma_R T_c^3} \right) \eta F_{in}.\quad (40)$$

In this case, one should also use Eqs. (33) and (37), getting

$$\sigma \approx \left(\frac{\mu}{\eta} \right)^{1/2} \left(\frac{8 \epsilon \sigma_R T_c^3}{\rho c_p} \right)^{3/2} \left(\frac{F_{in}}{\rho} \right)^{-1/2}.\quad (41)$$

Equation (39) suggests that increases in the radiative timescale, or decreases in the pressure thickness of the layer radiating to space (a Chapman layer in an optically thick atmosphere), leads to increases in TCAPE. This happens because they both lead to decreases in the fractional area covered by convective drafts. Thus, larger TCAPE values are necessary to maintain a given convective heat flux.

Equation (40) shows that increases in the atmosphere's heat capacity lead to linear increases in the equilibrium TCAPE value. This happens because increases in the atmosphere's heat capacity leads to decreases in the fractional area covered by convective drafts through increases in the radiative timescale. It also shows that decreases in the atmosphere's emission temperature, T_c , lead to both a linear increase in the equilibrium TCAPE value (through increases in the heat engine efficiency) and a strong nonlinear increase in the equilibrium TCAPE value (through decreases in the emission of thermal radiation to space). This happens because increases in η lead to increases in the energy available for convection, and both increases in η and T_c lead to decreases in the fractional area covered by convective drafts.

Substituting Eq. (7) into Eq. (40), we get an expression for the magnitude of the convective velocity

$$w \approx \left[\left(\frac{c_p}{8 \epsilon \sigma_R T_c^3} \right) \frac{\eta F_{in}}{\mu} \right]^{1/2}.\quad (42)$$

Using Eqs. (40), (41), and (42), as well as typical values of the various parameters (Peixoto and Oort 1991; Wolfe and Zissis 1985), valid for the tropical atmosphere ($\eta \approx 0.1$, $\epsilon \approx 0.7$, $T_c \approx 269 \text{ K}$, $c_p \approx 1005 \text{ J kg}^{-1} \text{ K}^{-1}$, $\sigma_R \approx 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$, $F_{in} \approx F_L \approx 155 \text{ W m}^{-2}$, $\rho \approx 1 \text{ kg m}^{-3}$, and $\mu \sim 16$), we get $\text{TCAPE} \approx 2500 \text{ J kg}^{-1}$, $\sigma \approx 5.0 \times 10^{-4}$, and $w \approx 12 \text{ m s}^{-1}$. These numbers are of the order of magnitude of the values (for undiluted drafts) observed in deep tropical convection (Williams et al. 1992; Black et al. 1994). However, since irreversible processes are present in real convection our values for the convective velocities are only an upper bound to the observed values.

To “independently” check the equilibrium TCAPE value predicted by the heat engine theory, we use Eq. (21) to compute the mechanical energy available from observed convective systems. In a typical tropical storm system the updrafts are at least 2 K warmer than the downdrafts (Zipser 1977). Thus, assuming $\eta = 0.1$, $\Delta T > 2 \text{ K}$, $\Delta r > 0.002$ (from the assumption that the surface temperature is 299 K, and that both the updrafts and the downdrafts are saturated), we get $\text{TCAPE} > 700 \text{ J kg}^{-1}$. In a typical midlatitude storm system, $\Delta T > 4 \text{ K}$ (Newton 1966). In this case, we get $\text{TCAPE} > 1650 \text{ J kg}^{-1}$. Again, these results are of the same order of magnitude as the TCAPE values predicted by the heat engine theory (see section 8).

TABLE 1. The total heat flux into the surface air, F_{in} , predicted by the heat engine framework, for various values of the fractional area covered by updrafts, σ . TCAPE $\sim 200 \text{ J kg}^{-1}$ is computed based on the observations for the thermal plume whose properties are displayed in Fig. 1 (top). It is also assumed $T_h \approx 295 \text{ K}$ and $T_c \approx 275 \text{ K}$. These assumptions are based on the observations of Rennó and Williams (1995).

σ	F_{in} (W m^{-2})	Remarks
0.4	3000	—
0.1	1000	—
0.07	500	observed value
0.02	100	—
0.001	10	—

8. Discussion

Atmospheric convection is formulated as a heat engine. We show that the volume integral of the work produced by a convective heat engine is a fundamental global number qualifying the state of the planet's atmosphere in quasi-equilibrium conditions. This number is a measure of the quasi-equilibrium amount of total CAPE present on the planet's atmosphere. It follows from Eqs. (39) and (40) that increases in the surface temperature lead to an increase in the total CAPE value through increases in the heat engine efficiency. Increases on the earth's global temperature are likely to produce increases in the atmosphere's opacity to infrared radiation (mainly due to increases in its water vapor content). Increases in the opacity lead to increases in the surface temperature (T_h) and to decreases in the atmosphere's emission temperature (T_c). It also leads to increases in the heat input at the surface through increases in the downward longwave radiation flux (Rennó et al. 1994). Thus, it follows from Eqs. (39) and (40) that the total amount of CAPE present in a semiopaque convecting atmosphere should increase with increases in the global temperature (or the atmosphere's opacity to infrared radiation). Therefore, on earth, the average total CAPE value should be larger in a warmer and moister climate regime and smaller in a colder climate regime than the presently observed value. This argument is in agreement with observations (Williams et al. 1992; Williams 1994) and the results obtained by Rennó et al. (1994) in a climate study with a one-dimensional radiative-convective equilibrium model using various cumulus models. The results also support Williams (1992, 1994) hypothesis that the Shumann resonance is a global tropical thermometer. It follows from our results [the second term on the right-hand side of Eq. (21)] that only a fraction of the "latent heat released" by moist convection is available to be transformed into kinetic energy. Since decreases in the temperature of the heat sink, T_c , lead to increases in the heat engine efficiency, η , it also follows from Eq. (21) that increases in the height of the latent heat re-

lease—the heat output—lead to increases in the intensity of convective systems. These results are in agreement with observations of deep moist convection (Cotton and Anthes 1989).

Table 1 displays the value of the total heat flux at the surface, F_{in} , predicted by the heat engine model [Eq. (24)] for various values of the fractional area covered by updrafts (or downdrafts). Observations show that the fractional area covered by convective boundary layer plumes is about 5% (Wyngaard 1990). Since the midday net heat flux at the surface over the New Mexico desert in August is $\sim 500 \text{ W m}^{-2}$, the prediction of the heat engine model is in agreement with the observations of boundary layer convection. Table 2 displays the predicted TCAPE $\approx 2 \times$ CAPE values for the tropical atmosphere [Eq. (24)]. Our calculations are based on data from Peixoto and Oort (1991), and on various assumptions about the fractional area covered by updrafts (or downdrafts). Observations show that, at each instant, only about 3% of the Tropics is covered by active cumulus clouds. Furthermore, they also show that only about 1% of the area covered by active clouds is covered by undiluted convective drafts (Cotton and Anthes 1989; Black et al. 1994). Thus, for the present climate, the heat engine framework predicts a CAPE value of the order of 1000 J kg^{-1} for the tropical atmosphere. This value is in agreement with observations (Williams and Rennó 1993). Figure 5 shows the predicted TCAPE value, the magnitude of the convective velocity, and the fractional area covered by convective drafts predicted by our theory [Eqs. (40)–(42)] as a function of the heat input. The dotted line corresponds to $\eta = 0.01$, the solid line to $\eta = 0.1$, and the dotted-dashed line to $\eta = 1.0$. The solid dot corresponds to our estimate for the tropical atmosphere. The predicted TCAPE and w values are strongly dependent on both the heat engine efficiency and the heat input, which determine the energy available for mechanical work. However, the TCAPE value is insensitive to the

TABLE 2. TCAPE for the tropical atmosphere for various values of the fractional area covered by updrafts, σ . The total heat input ($F_{in} \approx 155 \text{ W m}^{-2}$) is computed based on the observations that the average precipitation in the Tropics is about 5.2 mm day^{-1} . The hot source temperature is assumed to be $T_h \approx 299 \text{ K}$, and the cold source temperature $T_c \approx 269 \text{ K}$ (i.e., $\eta \approx 0.1$). These assumptions are based on data from Peixoto and Oort (1991).

σ	TCAPE (J kg^{-1})	Remarks
0.5	20	—
0.2	40	—
0.1	70	—
0.01	310	—
0.001	1500	—
0.0005	2400	observed value
0.0002	4300	—
0.0001	6900	—
0.00001	32200	—

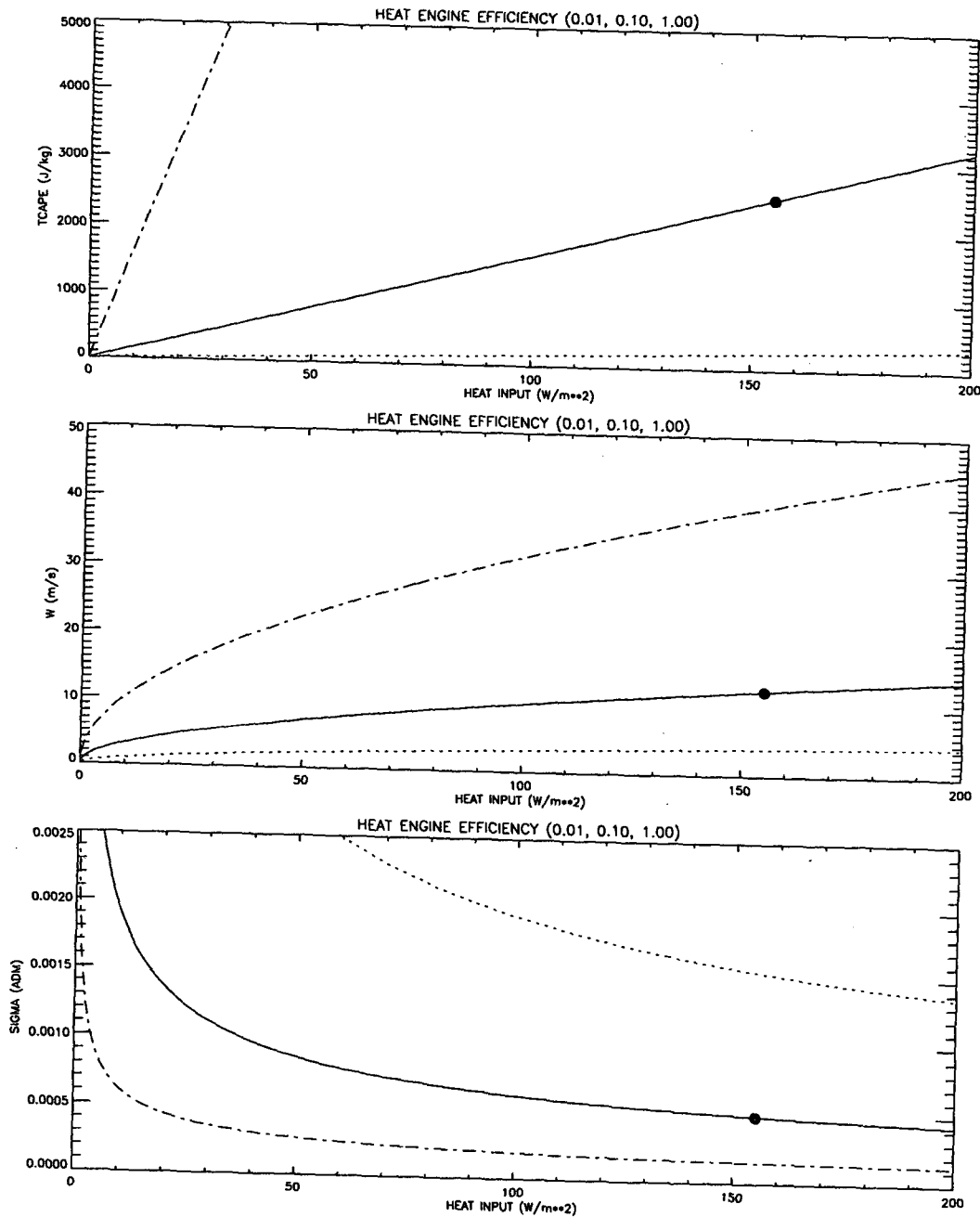


FIG. 5. The TCAPE value, the magnitude of the convective velocity, and the fractional area covered by convective drafts predicted by Eqs. (40)–(42) as a function of the heat input. The dotted line corresponds to $\eta = 0.01$, the solid line to $\eta = 0.1$, and the dotted-dashed line to $\eta = 1.0$. The solid dot corresponds to our estimate for the tropical atmosphere. The calculations are based on the parameter values presented at the end of section 7.

coefficient of dissipation of mechanical energy, μ [see Eq. (40)]. Finally, for $F_{in} \approx 150 \text{ W m}^{-2}$, the predicted σ value presents only a small sensitivity to both F_{in} and η .

The heat engine framework also provides a physical interpretation for the observations that convective plumes cover only a small fractional area, and that the

frequency of the convective plumes decreases upward (Wyngaard 1990). For the closed convective circulation to be possible, heat has to be lost at the top layers of the convective plumes, and ultimately, radiated to space. Since the atmosphere's radiative timescale is small compared to the convective timescale, the surface area of the atmosphere's "heat radiator" has to be large

enough to compensate for its inefficiency. Thus, convective plumes must occupy a small fractional area [Eq. (38)]. The frequency of convective plumes decreases upward because of the increase, with height, in the atmosphere's radiative timescale. However, the reader should be cautious when applying our model to boundary layer convection over land, which is a non-equilibrium phenomena.

9. Conclusions

We presented a simple theory for quasi-equilibrium convection based on the heat engine framework. Our theory applies to either dry or moist convection. It predicts the buoyancy, the vertical velocity, and the fractional area covered by natural convection in a state of quasi-equilibrium. With increases in the global temperature, we predict increases in the total CAPE value and in the convective velocity, and decreases in the fractional area covered by convective updrafts. Moreover, we show that only a fraction η of the "latent heat released" by moist convection is available to be transformed into kinetic energy.

Since convecting atmospheres are "adjusted" by up-downraft couplets (that to a first order approximation are both adiabatic), we should expect the equilibrium free troposphere to approach an intermediate adiabat (between the updraft and the downraft adiabats). To balance mechanical friction, the convective drafts must be buoyant. Therefore, they must originate in superadiabatic boundary layers.

We also showed that the volume integral of the work produced by the planetary heat engine provides a measure of the quasi-equilibrium amount of total CAPE present on the planet's atmosphere. This CAPE value is a fundamental global number qualifying the state of the planet in quasi-equilibrium conditions. Increases in the atmosphere's opacity to infrared radiation lead to increases in the heat engine efficiency, which, in turn, lead to increases in the total value of CAPE.

APPENDIX

The Thermodynamics of CAPE

Below we show that our definition of total CAPE is equivalent to the "standard" meteorological definition. We show that the area enclosed by the hot and the cold adiabats, and the bottom and the top of the convective layer represents the total amount of work done by the buoyancy forces in moving an air parcel around the convective cycle (Brunt 1941).

Neglecting the effect of water vapor on density, the amount of energy available for the displacement of an air parcel along the convective updraft is

$$\begin{aligned} \text{CAPE} &= \int_{up} B_u dz \\ &= \int_{up} g \left(\frac{T_u - T_a}{T_a} \right) dz, \end{aligned} \quad (\text{A1})$$

where *up* refers to integration along the upward branch of the convective circulation (including the heat intake branch), B_u is the updraft parcel buoyancy, z is height above a reference level, and T_u and T_a are, respectively, the updraft and ambient temperatures.

Similarly, the amount of energy available for the displacement of an air parcel along the convective downraft is

$$\begin{aligned} \text{DCAPE} &= - \int_{down} B_d dz \\ &= - \int_{down} g \left(\frac{T_d - T_a}{T_a} \right) dz, \end{aligned} \quad (\text{A2})$$

where *down* refers to integration along the downward branch of the convective circulation (including the heat rejection branch), B_d is the downraft parcel buoyancy, and T_d is the downraft temperature.

Thus, the total amount of work done in moving an air parcel around the convective cycle, $W = \text{CAPE} + \text{DCAPE}$, is given by

$$\begin{aligned} W &= \int_{bottom}^{top} g \left(\frac{T_u - T_d}{T_a} \right) dz \\ &\approx - \int_{up} RT_u d \ln p + \int_{down} RT_d d \ln p \\ &\approx - RT d \ln p \\ &\approx p d\alpha \\ &\approx \text{TCAPE}, \end{aligned} \quad (\text{A3})$$

where we have used the hydrostatic approximation. In the above equations, *bottom* and *top* refer to limits of integration at the bottom and top of the convective layer, respectively.

Equation (A3) shows that our definition of *total* CAPE is equivalent to the "standard" meteorological definition. Furthermore, it shows that buoyancy is not essential for convection to occur.

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