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Natural Flicker Noise («1/*f* Noise») in Music.

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Abstract. – A large class of musical selections exhibits a spectral density of audio power fluctuations characterized by a low-frequency behaviour typical of $1/f$ noise. We show that this $1/f$ behaviour follows from natural flicker noise theory.

Flicker noise—also known as «excess noise» or «$1/f$ noise»—was first observed more than 60 years ago by JOHNSON, while studying fluctuation processes in electron tubes. Subsequent investigations revealed the existence of spectra of the form $1/f^{\nu}$ (with $f = \omega/2\pi$, and $\nu = 1$ in the low-frequency range) in a wide variety of fluctuation processes in nature and in the laboratory. More recently it was shown [1-3] that the spectral density of audio power fluctuations in music also behaves like $1/f$ up to some frequency $\omega_{\max}$. This result was obtained by investigating the long time dynamics of music recordings, that is by considering the time series generated by an audio variable on a time scale that characterizes the overall dynamics of the full piece, *i.e.* beyond the characteristic time scales commonly identified in music composition, like the reciprocal frequency of the notes, the time signature, or the tempo. VOOS and CLARKE [1] chose to measure the audio power as the characteristic slow variable of the music (instead of the audio signal itself); then, after low-pass filtering, they constructed the audio power spectral density and observed that for all selections (from BACH to STOCKHAUSEN) the spectral density of audio power fluctuations showed a $1/f$-type behaviour below ~ 1 Hz.

When it was discovered as a general feature of a large class of music pieces, the $1/f$ nature of music was recognized as an important step towards an objective characterization of music *a*) as intermediate between randomness and predictability [3, 4], and *b*) as having «time scaling» property [2, 4], *i.e.* the property of self-similarity typical of fractal objects (1).

(1) We have investigated this aspect of the problem for a variety of musical selections and found that they could be classified according to their fractal dimension as obtained from phase-space analysis; these results will be presented in a forthcoming paper [5].

Therefore, it was also suggested (and tested) [1, 3] that $1/f$ noise could provide an interesting basis for stochastic composition ([2]). On the other hand, it was clearly stated [1, 4] that there was no satisfactory theory that could explain this $1/f$ behaviour.

In the present paper we show that «$1/f$ noise» in music can be explained on the basis of natural flicker noise theory as developed in [7-9]. We first review the essential features of the theory; then we present its application to music.

1) Natural flicker noise arises as a consequence of the intrinsic structure of the system (the molecular structure in physical systems) in the frequency range $\omega_{\min} < \omega < \omega_{\max}$. The upper limit of the flicker noise domain is defined by

$$\omega_{\max} \sim D_{\text{eff}} L_{\text{eff}}^{-2}, \tag{1}$$

where D_{eff} is the effective diffusion coefficient and L_{eff} is the shortest effective dimension of the system. The lower limit $\omega_{\min}$ is set by the duration of observation: $\omega_{\min} \sim \tau_{\text{obs}}^{-1}$. (Flicker noise spectra have been measured down to frequencies $\sim (10^{-5} \div 10^{-6})$ Hz.)

2) The flicker noise intensity at a given frequency ($\omega_{\min} < \omega < \omega_{\max}$) is inversely proportional to N, the number of particles in the sample (*i.e.* in the finite system considered) [9]. So flicker noise may be termed «natural».

3) Flicker noise can be viewed as a feature of the spectral density of Brownian-type systems in the low-frequency domain: $\omega \ll D_{\text{eff}} L_{\text{eff}}^{-2} \equiv \tau_{\text{D}}^{-1}$, or equivalently for large correlation times: $\tau \gg \tau_{\text{D}}$. Under such conditions, the dimensions of the system play no significant role and the system can be treated as a point (*i.e.* as a system with zero dimension).

4) In the flicker noise domain, the spectral density of any fluctuating quantity δn_V (averaged over the volume V of the system) can be expressed in the following form [7-9]:

$$(\delta n_V \, \delta n_V)_\omega = \frac{\pi}{|\omega|} \, \frac{\langle (\delta n_V)^2 \rangle}{\ln(\tau_{\text{obs}}/\tau_{\text{D}})}; \qquad \tau_{\text{obs}}^{-1} \ll \omega \ll \tau_{\text{D}}^{-1}. \tag{2}$$

The mean-square fluctuations $\langle (\delta n_V)^2 \rangle$ are given by [9]

$$\langle (\delta n_V)^2 \rangle = \int_{\tau_{\text{obs}}^{-1}}^{\tau_{\text{D}}^{-1}} (\delta n_V \, \delta n_V)_\omega \, \frac{\mathrm{d}\omega}{\pi} \propto \frac{1}{N} \tag{3}$$

and so increase with the duration of observation. $\langle (\delta n_V)^2 \rangle$ is a characteristic thermodynamic quantity of the system and is temperature dependent.

5) The corresponding time correlation function reads [8, 9]

$$\langle (\delta n_V)^2 \rangle_\tau = \left[C - \frac{\ln(\tau/\tau_{\text{D}})}{\ln(\tau_{\text{obs}}/\tau_{\text{D}})} \right]; \qquad \tau_{\text{obs}} \gg \omega \gg \tau_{\text{D}}, \tag{4}$$

where C denotes the residual time correlations. Note that, since $\tau \gg \tau_{\text{D}}$, the time dependence in (4) is extremely weak and $\langle (\delta n_V)^2 \rangle_\tau$ differs very little from C [7, 8].

([2]) Note that various methods based on mathematical formulations have recently been proposed (and some of them used) as new tools for composition and for producing previously unheard sounds [6].

6) Flicker noise phenomena can be represented by a Langevin-type equation

$$(i\omega + \gamma_\omega)\,\delta n_V(\omega) = y(\omega)\ , \tag{5}$$

where $y(\omega)$ denotes the random source term and γ_ω is the dissipative coefficient, with

$$\gamma_\omega = \omega\ , \qquad \text{for } \tau_{\rm obs}^{-1} \ll \omega \ll \tau_{\rm D}^{-1}. \tag{6}$$

The intensity of the random source is given by

$$(yy)_\omega = 2\gamma_\omega \pi \ \frac{\langle(\delta n_V)^2\rangle}{\ln(\tau_{\rm obs}/\tau_{\rm D})}; \qquad \tau_{\rm obs}^{-1} \ll \omega \ll \tau_{\rm D}^{-1}. \tag{7}$$

This relation expresses the fluctuation-dissipation theorem for the flicker noise region. Note that for large frequencies $(\omega \gg \tau_{\rm D}^{-1})$, $\pi/\ln(\tau/\tau_{\rm D}) \to 1$ and (7) reduces to the classical fluctuation-dissipation theorem. Equation (5) yields straightforwardly the usual expression for the spectral density

$$(\delta n_V\,\delta n_V)_\omega = \frac{(yy)_\omega}{\omega^2 + \gamma_\omega^2}\ . \tag{8}$$

So flicker noise theory follows from the description of the diffusion type stage of fluctuation processes in finite-size systems.

The theory can be logically applied to flicker noise in music by considering the Langevin equation for a «wave function» $u_k(t)$ describing the audio signal, *i.e.*

$$\ddot{u}_k + 2\gamma_k \dot{u}_k + \omega_k^2 u_k = y_k\ , \tag{9}$$

where γ_k and ω_k denote the dissipative coefficient and eigenfrequency, respectively, for the space Fourier component with wave number k. For any finite system there exists a minimum value $k_{\min} \sim L_{\rm eff}^{-1}$; correspondingly $\gamma_m \equiv \gamma_{k_{\min}}$ and $\omega_m \equiv \omega_{k_{\min}}$ are the damping coefficient and the eigenfrequency respectively for the mode with the smallest wave number. The numerical value of ω_m can be evaluated from spectral analyses of the audio signal of musical selections (see, *e.g.*, fig. 2*a*) in ref. [1]: $\omega_m \leqslant 100$ Hz). However, in order to investigate experimentally more slowly varying quantities, the audio power $u^2(t)$ is taken instead of the audio signal $u(t)$ and its spectral density $(u^2u^2)_\omega$ is measured after low-pass filtering [1]. It is found that, whereas the audio signal spectral density

$$(uu)_\omega = 2\,{\rm Re}\int_0^{+\infty} {\rm d}\tau \langle u(t)\,u(t+\tau)\rangle \exp[i\omega\tau] \tag{10}$$

is distributed over the audio frequency range, the spectral density of the audio power fluctuations

$$(u^2u^2)_\omega = 2\,{\rm Re}\int_0^{+\infty} {\rm d}\tau \langle u^2(t)\,u^2(t+\tau)\rangle \exp[i\omega\tau] \tag{11}$$

exhibits the characteristic $1/f$ behaviour in the very low-frequency range, typically from 10^{-3} Hz to 1 Hz (see, *e.g.*, fig. 3*b*) in ref. [1]). Thus the quantity that was measured experimentally [1] is the monotonically varying audio power of the music which is proportional to the square of the audio signal. So, instead of $u_k(t)$, we should consider $u_k^2(t)$

whose time correlation function can be computed analytically from eq. (9) with the usual assumption $\langle u_k(t)\, y_k(t+\tau)\rangle = 0$. That the signal be decorrelated from the random source term is certainly valid here, since we consider long-time correlations (as we are interested in the low-frequency domain). One finds

$$\langle (u_k^2)^2\rangle_\tau \equiv \langle u_k^2(t)\, u_k^2(t+\tau)\rangle =$$

$$= \langle (u_k^2)^2\rangle \left[\frac{1}{2}\left(1+\frac{\gamma_m^2}{\beta^2}\right)+\frac{\gamma_m}{\beta}\sin(2\beta\tau)+\frac{1}{2}\left(1-\frac{\gamma_m^2}{\beta^2}\right)\cos(2\beta\tau)\right]\exp[-2\gamma_m\tau] \qquad (12)$$

with $\beta^2 = \omega_m^2 - \gamma_m^2$. According to flicker noise theory [9] as outlined above, we must investigate the corresponding power spectrum for the diffusion stage defined by $\omega \ll \gamma_m$,

$$(u^2 u^2)_\omega = \left(\frac{L_\omega^2}{2\pi}\right)^{3/2}\int \mathrm{d}\boldsymbol{k}\, S(\omega)\exp[-\tfrac{1}{2}L_\omega^2 k^2]\,\langle (u^2)^2\rangle\ , \qquad (13)$$

where $\langle (u^2)^2\rangle$ denotes the audio power mean-square fluctuations, that is the characteristic fluctuating thermodynamic quantity of the audio system. $S(\omega)$ is obtained by time Fourier transformation of (12), and is given by

$$S(\omega) = \frac{1}{2\beta^2}\frac{\omega_m^2}{\gamma_m} + \left(2-\frac{\omega_m^2}{2\beta^2}\right)\ \frac{\omega_m^2/\gamma_m}{\omega^2+(\omega_m^2/\gamma_m)^2}\, , \qquad (14)$$

where the «diffusion stage» condition $\omega \ll \gamma_m$ has been used. Assuming $\omega_m \propto |\boldsymbol{k}|$, the dispersion L_ω^{-2} is given by

$$L_\omega^2 = L^2 + c^2(\gamma_m\,\omega)^{-1}, \qquad (15)$$

where c denotes the sound velocity and L the size of the system. Then, for low frequencies, $c^2(\gamma_m\,\omega)^{-1} \gg L$ and $L_\omega^2 \simeq c^2(\gamma_m\,\omega)^{-1}$.

Now the flicker noise domain is defined by the frequency range

$$\tau_{\rm obs}^{-1} \ll \omega \ll \frac{\omega_m^2}{\gamma_m} = \frac{c^2 k^2}{\gamma_m} = \frac{D_{\rm eff}}{L_{\rm eff}^2} \equiv \tau_{\rm D}^{-1}\, . \qquad (16)$$

Since $\omega \ll \gamma_m$, it also follows from (16) that $\gamma_m^2/\omega_m^2 \ll 1$, which permits to simplify somewhat expression (14). Setting $x^2 = c^2 k^2(\gamma_m\,\omega)$, we find

$$(u^2 u^2)_\omega \simeq \frac{3}{\sqrt{2\pi}}\int_0^\infty \mathrm{d}x\,\frac{x^4}{1+x^4}\exp[-x^2/2]\,\frac{\langle (u^2)^2\rangle}{\omega}\, , \qquad (17)$$

where again we used the fact that $\omega \ll \gamma_m$.

The linear dependence $\omega_m = c|\boldsymbol{k}|$ used here is most logical; when a more general (ω, k) dependence of the type $\omega_m^2/\gamma_m \propto k^\delta\,\omega^{1-\alpha}$ is taken, it is easy to show that the main result, eq. (17), remains essentially unchanged, *i.e.* one finds

$$(u^2 u^2)_\omega \propto \frac{\langle (u^2)^2\rangle}{\omega}\, . \qquad (18)$$

So we have shown that under condition (16), audio power fluctuations in music selections exhibit a low-frequency spectral density with $1/\omega$ behaviour («$1/f$ noise»). Such a behaviour is subject to the existence of three characteristic quantities: a «thermodynamic function» given by the mean-square fluctuations in the audio system; an «effective diffusion» time τ_D, *i.e.* the time related to L_{eff} which here is the characteristic instrumental length; and τ_{obs}, the duration of observation time, here the length of the music piece. In conclusion flicker noise should exist for any audio system (*i.e.* for any music) for which inequalities (16) hold.

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