Nature as a life support system. Renewable resources and environmental disruption

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Nature as a Life Support System. Renewable Resources and Environmental Disruption

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Natural resources and environmental disruption are analyzed as two distinct problems in the main stream of the economic literature[1]. In this paper, both problems are linked to each other. Nature is here conceived as providing a flow of goods such as the world's oxygen, the ozone layer as a protective shield for the earth, water supplies at given locations through the meteorological and ground water systems as well as the fish populations in the ocean. Natural resources "include all those natural endowments which constitute the life support system" (Krutilla, Smith and Kopp, 1977, p. 2).

One aspect of nature's production function is that regeneration is not only influenced by the stock of the resource but also by a vector of other variables one variable being the stock of pollutants. The acid rain in the Northern Hemisphere, the expected effects of freon on the ozone-layer and the possible impact of DDT on oxygen production in the world's oceans are examples where the stock of pollutants may have an impact on nature as a life support system. As the examples suggest, our frame of reference is a global regeneration function.

In this paper, we take into account the effect of the stock of pollution on the regeneration of a natural resource. In section 1,

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[1] An exception is Kneese (1976) who stresses a positive relationship between resource conservation and environmental quality management. Moreover, the literature on recycling (compare for instance Hoel 1978; Pethig 1979) represents a link between resource conservation and environmental economics.

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the regeneration function is discussed. In section 2, we specify the assumptions and describe the allocation problem. Then, the steady state solution is studied. In the last section some policy implications are discussed.

1. The Regeneration Function

Let R denote the stock of a resource and let S denote the stock of pollutants. Then the regeneration function may be written as

$$\dot{R} = \phi\,(R, S) = g\,(R) - aS \tag{1}$$

where $a > 0$ is constant, $g_R \gtreqless 0 \Leftrightarrow R \lesseqgtr R'$ and $g_{RR} < 0$. Also we assume, that $\dot{R} = 0$ for $R = \underline{R}$ or $\bar{R}$ and $S = 0$, so that there is a minimum and a maximum stock in the usual interpretation of the regeneration function (Plourde, 1970). In Fig. 1, curve 1 shows the regeneration of the resource for $S = 0$. If S increases, the curve shifts downward by the vertical distance aS (curve 2). Note that in Eq. (1), we have assumed seperability of the function in R and S so that the slope of the $\dot{R}$-curve remains the same for a given R and varying S. More specifically, at point R' we always have $g_R = 0$. Define a minimum stock $\underline{\underline{R}}$ and a maximum stock $\bar{\bar{R}}$ with $\dot{R} = 0$ for $S > 0$, then the minimum stock shifts to the right and the maximum stock shifts to the left with rising S.

The curves shown in Fig. 1a represent cuts through the three-dimensional regeneration space in Fig. 1b[2]. Note that for a given R, $d\dot{R}/dS = -a$ so that a cut through Fig. 1b for a given R is a straight line.

For our frame of reference of a global regeneration function, Fig. 1b can be interpreted as an illustration of the human living space indicating a trade-off between the withdrawal of goods from nature and the stock of pollutants. Potential withdrawals are reduced with an increasing stock of pollutants. Fig. 1b thus shows the transformation space (or the withdrawal and polluting possibilities of mankind). The level of pollution determines the amount of resources that we can withdraw from nature.

The reader should be aware that the negative effect of pollutants on the regeneration of natural systems is here interpreted as an a priori-hypothesis. No empirical estimate underlining the magnitude of this effect is given. Ecologists will argue that the negative pro-

[2] The diagram 1b was suggested to me by S. Toussaint.

ductivity effect of pollutants on the provision of nature's services is very important and that the transformation space shrinks quickly with an increasing level of pollution. Others will be less pessimistic.

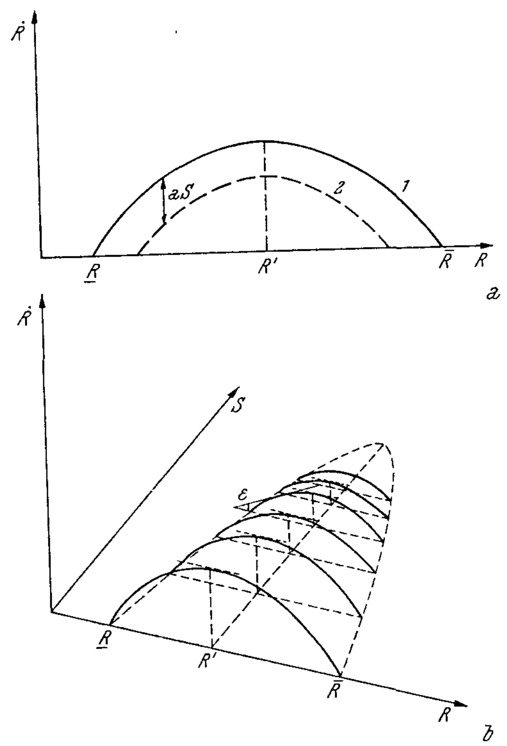

Fig. 1

They will assume that the regeneration of nature is not affected markedly, at least up to a large volume of pollution. It is quite apparent that the conclusion of our model will depend on the assumed magnitude of the negative productivity effect.

2. The Problem

Denote the quantity of natural resources being withdrawn from nature in each period with q. Thus, q may be interpreted as a flow of goods from nature. There are no extraction costs. Then the equation of motion for the resource stock is given by

$$\dot{R} = g(R) - aS - q. \qquad (1')$$

The resource is consumed. In the consumption process, pollutants are generated in a constant relation β per unit of resource consumed. The stock of pollutants is assimilated by the environment in a constant fraction π. Then the stock of pollutants changes according to[3]

$$\dot{S} = \beta q - \pi S. \tag{2}$$

Eq. (2) is a very rudimentary relationship expressing the phenomenon that pollutants are generated in economic activities. In a more complex framework, resources may be used in production processes where pollutants are generated, and the output of the production activity is used in consumption (Siebert, 1981).

Finally assume that welfare depends on the quantities of the resource consumed

$$W = W(q), \quad W_q > 0, \quad W_{qq} < 0. \tag{3}$$

This implies that pollutants do not enter the welfare function as an argument variable.

The problem of optimal use of the flow of the resource then consists of maximizing

$$\omega = \int_0^\infty e^{-\delta t} W(q_t)\, dt \tag{4}$$

$$\text{s. t. (1') and (2)}$$

R_t and S_t are the state variables of the system. q_t is the control variable.

From the Hamilton function

$$H = W(q) + \lambda [g(R) - aS - q] + \varrho [\beta q - \pi S] \tag{5a}$$

we have the implications

$$\frac{\partial H}{\partial q} = 0 \quad \Rightarrow \quad \lambda = W_q + \varrho\beta \tag{5b}$$

$$\dot{\lambda} = \varrho\lambda - \frac{\partial H}{\partial R} \quad \Rightarrow \quad \dot{\lambda} = \lambda(\delta - g_R) \tag{5c}$$

$$\dot{\varrho} = \delta\varrho - \frac{\partial H}{\partial S} \quad \Rightarrow \quad \dot{\varrho} = \varrho(\delta + \pi) + \lambda a \tag{5d}$$

The Eqs. (1') and (2) and the necessary conditions (5b) through *d* determine optimality conditions of the allocation problem. These conditions must be satisfied for each period of time.

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[3] For a similar function, compare Vogt (1981, S. 66).

3. Steady State

A steady state is defined by $\dot{R}=0$ and $\dot{S}=0$. From these conditions $\dot{R}=0$ and $\dot{S}=0$ we have

$$g(R)=\left(\frac{\pi}{\beta}+a\right)S \tag{6}$$

with

$$\left.\frac{dS}{dR}\right|_{\substack{\dot{R}=0\\ \dot{S}=0}}=\frac{g_R}{\frac{\pi}{\beta}+a}\gtreqless 0 \text{ for } R\lesseqgtr R' \tag{6a}$$

The $\dot{R}=\dot{S}=0$ curve has the shape shown in Fig. 2. Let $S=0$. Then we have $g(R)=0$ for $R=\underline{R}$ and $R=\bar{R}$. If the negative effect of a

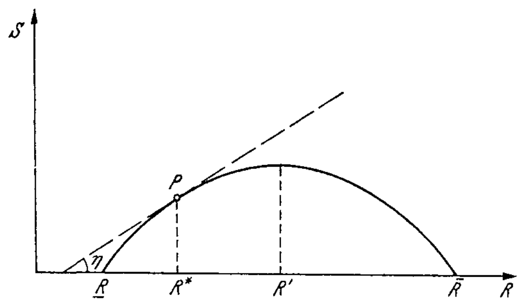

Fig. 2

unit of pollutant on the regeneration increases, i. e., the coefficient a rises, the curve is squeezed from above and the slope becomes smaller. Note that $dS/dR=0$ for $R=R'$.

The $\dot{R}=\dot{S}=0$ curve is constructed geometrically in Fig. 3. Quadrant I shows the regeneration curve $\dot{R}=0$ with curve 1 neglecting the negative effect of pollutants. Quadrant II contains the $\dot{S}=0$ curve with $\operatorname{tg}\gamma=\frac{\pi}{\beta}$. Assume quantity $\tilde{q}$ is to be withdrawn from nature. Then a stock of pollutants $\tilde{S}$ is allowed so that $\dot{S}=0$. With this stock of pollutants $\tilde{S}$ a negative productivity effect $a\tilde{S}$ occurs. Note that $\operatorname{tg}\varepsilon=a$. The negative productivity effect $a\tilde{S}$ shifts the $\dot{R}=0$ curve downward. The quantity $\tilde{q}$ to be withdrawn requires a resource stock $\tilde{R}$. The coordinates $\tilde{R}$, $\tilde{S}$ denote one point on the $\dot{R}=\dot{S}=0$ curve in quadrant IV. Note that a solution $\tilde{q}>0$ requires that $g(R)>aS$.

Fig. 3 also indicates the directions of motion in the $R-S$-diagram. The directions of motion for the $\dot{R}=0$ curve are well known. Let $\tilde{S}$ denote the stock of pollution in the steady state. Assume a given $\tilde{q}$ and let $S=S'<\tilde{S}$. Then a smaller amount of pollutants is assimilated by nature and the stock of pollutants rises. If $S=S''>\tilde{S}$, then a larger amount of pollutants is assimilated and S will decline.

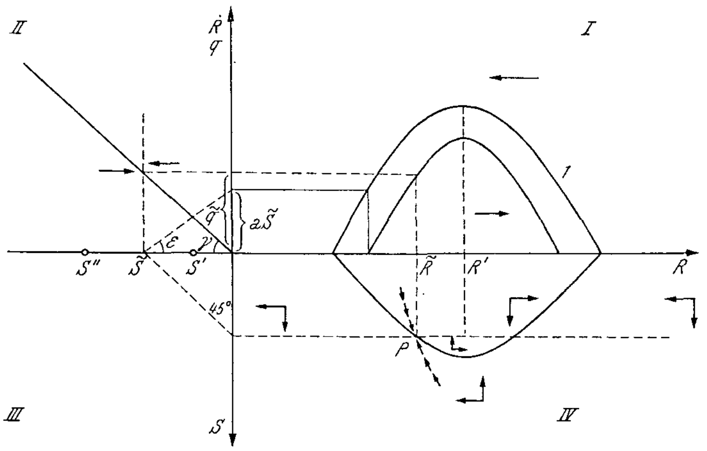

Fig. 3

The optimal stock of pollution $\tilde{S}$ is the deviding line for the regions $\dot{S}>0$ and $\dot{S}<0$. Note that quadrant IV of Fig. 3 shows the directions of motion with respect to $\tilde{S}$, the stock of pollutants of the stationary state[4].

The $\dot{R}=\dot{S}=0$ curve denotes the combinations of S and R that satisfy one condition of the stationary state. The price Eqs. (5 b) through d determine the optimal allocation S^*, R^*. Setting $\dot{\lambda}=0$ (in Eq. 5c) and $\dot{\varrho}=0$ (in Eq. 5d) and substituting Eq. (5b) we have

$$\frac{\delta}{\frac{\pi}{\beta}+a} = \frac{g_R(R)}{\frac{\pi}{\beta}+a} = \frac{\frac{1}{-\varrho}\left[(W_g+\varrho\beta)\,a+\varrho\pi\right]}{\frac{\pi}{\beta}+a} \quad (7)$$

[4] The stability of the steady state can be illustrated in quadrant IV of Fig. 3. Let P be the stationary point. Then the two Pontryagin paths shown are stable.

In Eq. (7), the second term represents the slope of the $\dot{R}=\dot{S}=0$ curve. For given π, β and a, the slope depends on g_R (R). The first expression of Eq. (7) is positive, so the solution requires $g_R>0$ and this implies $R<R'$. Once the properties of the $\dot{S}=\dot{R}=0$ curve, namely the parameters are given, the discount rate δ determines the optimal solution. Define $\operatorname{tg} \eta = \dfrac{\delta}{\dfrac{\pi}{\beta}+a}$ (Fig. 2). Then the tangent to the $\dot{R}=\dot{S}=0$ curve in point P determines the optimal solution.

In Eq. (7), the numeraters represent three different interest rates. δ denotes the time preference. g_R is the own rate of return of the resource when left in its natural environment. The expression on the far right side of the Eq. (7) is the own rate of return of a unit of pollutant, corrected for sign. The coefficient a denotes the negative productivity of one unit of pollution with respect to regeneration. $(W_q+\varrho\beta)$ represents the net marginal utility of one unit of the resource when consumed. Note that $\varrho\beta$ stands for the environmental costs per unit of resource used.

$\varrho<0$ is a negative shadow price per unit of pollutant and β the pollution arising per unit of resource consumed. Consequently, $(W_q+\varrho\beta)\, a$ is the marginal value product of a unit of pollution. In each period, π of each unit of pollution is assimilated, so that the net marginal value product of a unit of pollution is found by correcting with $\varrho\pi$. Finally, divide by ϱ, and you receive the value product in real terms. This value product is negative, since $\varrho<0$. Consequently, the expression is corrected in sign and can also be interpreted as the value product of a unit of pollutant not generated. Thus, the equality in the numerators in Eq. (7) requires the identity of three different interest rates in the steady state[5].

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[5] The movement of the system towards the steady state is controlled by the quantities withdrawn. The quantities withdrawn vary with the costate variables λ and ϱ. Differentiating Eq. (5b) with respect to time, we have

$$\dot{q}=\frac{1}{W_{qq}}\,(\dot{\lambda}-\beta\dot{\varrho})$$

and substituting Eqs. (5 c) and (5 d) we have

$$\dot{q}=\frac{1}{W_{qq}}\,[\lambda\,(\delta-g_R)-\beta\varrho\,(\delta+\pi)-\beta\lambda a]$$

$$\dot{q}\gtreqless 0 \Leftrightarrow \dot{\lambda}\gtreqless\beta\dot{\varrho}$$

Consider the case $R<R^*$ so that $\delta<g_R\,(R)$ and $\dot{\lambda}<0$. Then for $\dot{q}>0$, $\dot{\varrho}>0$

The condition $\delta = g_R(R)$ is sufficient to determine the optimum solution since $g(R)$ determines S according to Eq. (6). The optimal solution can also be illustrated in Fig. 1b (not drawn there). From the set of all resource stocks satisfying $\delta = g_R(R)$ (being determined by tg ε) only that resource stock R^* is optimal where $S = S^*$. For this optimal stock of pollutants S^* Eq. (7) must hold, i. e., the marginal value product of one unit of pollutants must be equal to the discount rate.

4. Some Policy Implications and Extensions

The steady state shifts parametrically with the discount rate. Assume that a society introduces a higher discount rate so that future generations receive a lower weight. Then a smaller stock of resources will be held in the long-run; according to Eq. (6) the stock of pollutants will be lower. Since for a steady state we have $\dot{S} = 0$ and thus (from Eq. 2) $q = (\pi/\beta)\, S$, a lower level of pollution implies a smaller withdrawal. In Eq. (7), the expression on the far right side must rise. This can be achieved by having a lower shadow price ϱ for a unit of pollutants in absolute terms. Thus, a higher time preference reduces the stock of resources; at the same time the stock of pollutants is lowered. This result is due to the assumption of separability; once R is determined by δ, S is also specified. Consequently, S and R must change in the same direction with a change in the time preference. If, on the other hand a lower discount rate is introduced, a larger resource stock must be held; withdrawals are larger and a higher stock of pollutants is optimal. The shadow price ϱ for a unit of pollutants must be higher in absolute terms.

An interesting policy problem is how the system behaves in different initial situations. Consider $\tilde{R}$, $\tilde{S}$ in Fig. 3 as the optimal

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is sufficient. $\dot{\varrho} > 0$ requires from Eq. (5 d)

$$\delta < \frac{1}{-\varrho} \; (\lambda a + \varrho \pi)$$

In this case, the initial scarce resource stock requires a high shadow price λ and low withdrawals. λ will fall over time, and withdrawals increase. The shadow price of pollutants is high in absolute terms in the initial situation. ϱ falls over time in absolute terms.

Note that in this case ($R < R^*$ and $\dot{\lambda} < 0$), $\dot{q} > 0$ is also consistent with $\dot{\varrho} < 0$; $\dot{\varrho}$, however, is restricted in magnitude.

solution and assume different initial positions of the economy. We distinguish the following cases:

i) If initially $R < \tilde{R}$ and $S < \tilde{S}$, the system can move towards P by a policy of small withdrawals, thereby slowly increasing the stock of resources and the stock of pollution.

ii) If in the initial situation $R > \tilde{R}$ and $S > \tilde{S}$, the depreciation of the large pollution stock will bring pollution down; the $\dot{R} = 0$ curve shifts outward and more resources can be used.

iii) If initially $R > \tilde{R}$ and $S < \tilde{S}$, the system will not reach point P. In this case, we do not have a pressing policy problem. The low level of pollution allows a generous reproduction of nature's services, and withdrawals are not large enough to both increase pollution and reduce the stock of resources.

iv) If initially $R < \tilde{R}$ and $S > \tilde{S}$, we are in a dooms day deadlock. The negative effect of pollution is so important that regeneration cannot occur fast enough. The resource stock will decline.

Another policy problem can be illustrated by introducing a minimum withdrawal. Assume that some minimum withdrawal $q_{\min}$ necessary for survival can be defined. Then the following two cases may be distinguished: If the optimal value of withdrawals in the steady state is $q^* \geq q_{\min}$, the solution is not affected. If the restraint is binding, the discount rate no longer determines the optimal solution. Apparently, the solution depends on the properties of the regeneration and accumulation function. In Fig. 4 a

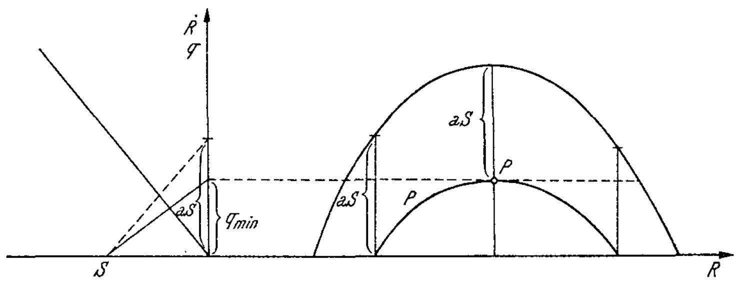

Fig. 4

special case is shown where the minimum withdrawal causes a "strong" negative effect of pollutants. Regeneration is just sufficient to withdraw the minimum amount. This case is shown by Point P in Fig. 4. A larger withdrawal than the minimum withdrawal shown in Fig. 4 cannot be satisfied by the system in the long run.

A potential extension of the analysis would be to include not only one natural resource but a set of different resources with interlinked regeneration functions. Then, some additional conditions on ecological equilibria may be developed[6].

Another extension is to uncouple the dependence of S or R in the optimal solution. One approach would be to introduce a "richer" description of the pollution behavior, for instance by explicitly considering production activities that generate pollution. Another approach would be to relax the separability assumption in the regeneration function.

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[6] The model may also be applied to extraction processes where extraction may have some negative feedback on the regeneration of resources.