Near Optimal Linear Precoder for Multiuser MIMO for Discrete Alphabets

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***Abstract*—In this paper we look at the effect of discrete constellation alphabets on linear precoding for the downlink of multiuser (MU) MIMO in the context of LTE. We underline the fundamental difference in the approach of precoding if the alphabets are assumed to be discrete constellations rather than the idealized Gaussian assumption. We show that the problem of finding global optimal linear precoder taking into account discrete inputs is non-convex and we propose a method of finding a near optimal linear precoder. Underlining the viability of MU MIMO for future wireless communications as LTE, we further propose in this paper a precoding strategy based on low resolution LTE precoders which necessitate 2 bits feedback from the users. The proposed strategy encompasses geometrical interference alignment at eNodeB and the use of low complexity MU detectors at the users. On one hand, this strategy relegates the interference seen by each user by a geometric scheduling algorithm while on the other hand, users exploit the structure of this interference in the detection process. Simulation results validate improved performance of the proposed strategy over single user schemes.**

I. Introduction

Spatial dimension surfacing from the usage of multiple antennas promises improved reliability, higher spectral efficiency and spatial separation of users [1]. This spatial dimension (MIMO) is particularly beneficial for precoding in the downlink of multiuser (MU) cellular system, where spatial resources can be used to transmit data to multiple users simultaneously. For transmission in MU MIMO Gaussian broadcast channel, optimal precoding involves a theoretical pre-interference subtraction technique known as dirty paper coding (DPC) [2]. Due to highly nonlinear nature of signal processing involved in DPC, its practical implementation is far from realizable. Moreover its optimality is constrained to idealistic Gaussian alphabets. Although Gaussian inputs are optimum from a mutual information standpoint, they are too idealistic to be implemented in practical communication systems. The reason for this assumption is the convenience in mathematics to derive the elegant capacity formula [1].

Linear precoding, also based on the Gaussian assumption for alphabets, provides an alternative approach for the transmission in MU MIMO Gaussian downlink channel, trading off a reduction in precoder complexity for suboptimal performance. Interference cancellation based schemes use channel inversion (CI) and block diagonalization (BD) to transform the MU downlink into parallel single user systems [3]. Interference attenuation based schemes use regularized channel inversion (RCI) precoding as proposed in [4] which adds a multiple of the identity matrix before channel inversion. Optimum linear precoders [5] and optimum unitary linear precoders [6] for MU MIMO Gaussian broadcast channel have been derived in the literature but they are also based on the Gaussian assumption of alphabets.

Gaussian is the worst case interference and is devoid of any structure [7], so the recommended precoding strategies for such inputs are perceptibly pre-interference subtraction (DPC), interference cancellation (CI) and interference attenuation (RCI). These strategies strive to transform the cross-coupled channels into parallel noninteracting channels and consequently lead to simplified single user receivers. However they are void of exploiting the interference structure in mitigating its effect which is evident as Gaussian interference encompasses no structure to be manipulated.

In real world, inputs must be drawn from discrete constellations (often with very limited peak-to-average ratios) which may significantly depart from Gaussian idealization. The ensuing interferences (discrete alphabets) unlike Gaussian case have structures that can be exploited in the detection process. Under such a scenario, what should be the structure of a linear precoder to maximize the sum rate? In this paper we show that the problem of finding an optimal linear precoder for finite alphabets in the case of MU MIMO is non-convex and intractable even for the simple case of two single antenna users. However we propose a method of finding near optimal linear precoder and show that this precoder neither diagonalizes the channel nor attenuates or pre-subtracts the interference which validates the notion of exploitation of interference structure in the detection process rather than its suppression or cancellation. We then further argue that the precoders may be designed to manage the interference in a way that this interference can be exploited in the detection process at the receivers.

We then look at the low resolution precoders of LTE [8] and investigate that whether MU MIMO is a viable option with these precoders which necessitate barely two bits of feedback from the users. Using the idea of interference exploitation, we propose an algorithm basing on the geometrical alignment of interference at eNodeB which relegates the effective interference seen by each user. It is followed by the detection at the user terminals exploiting the structure of this interference. To this end, we recommend the use of recently proposed low complexity detectors [9] which not only reduce one complex dimension of the system but also exploit the interference structure in the detection process.

Regarding notations, we will use lowercase boldface letters for vectors and uppercase boldface letters for matrices as $\mathbf{I}_n$ indicates an $n \times n$ identity matrix. $|.|$ and $\|.\|$ designate norm of scalar and vector respectively while $\mathrm{Tr}(.)$ points to the trace of a matrix. $(.)^T$, $(.)^*$ and $(.)^\dagger$ represent transpose, conjugate and conjugate transpose operations respectively. The paper is divided into five sections. In section II we define the system model while section III derives the near optimal linear precoder in information theoretic perspective. Section IV is dedicated to the proposed transmission strategy for LTE MU MIMO which is followed by simulation results and conclusions.

II. System Model

Coherent with the next generation wireless systems as LTE [10] and IEEE 802.16m [11], we consider the downlink of a wireless system using bit interleaved coded modulation (BICM) [12] OFDM system for downlink transmission. We assume n_t transmit antennas and 2 single antenna users. After encoding and interleaving, the output bits are mapped onto the tone $x_{k,n}$ using the signal map $\chi_k \subseteq \mathcal{C}$ where $k = 1, 2$ (user) and n indicates the subcarrier. It is assumed that an appropriate length of cyclic prefix (CP) is used for each OFDM symbol. By doing so, OFDM converts downlink frequency selective channels into parallel flat fading channels denoted as $\mathbf{h}_{k,n}^\dagger \in \mathbb{C}^{n_t \times 1}$ where $\mathbf{h}_{k,n}^\dagger$ symbolizes the MISO channel from eNodeB to k-th user and $\mathbb{C}^{n_t \times 1}$ denotes the n_t-dimensional complex space. We assume a spatially uncorrelated flat Rayleigh fading channel model so that the elements of $\mathbf{h}_{k,n}^\dagger$ can be modeled as independent identically distributed (i.i.d) zero mean circularly symmetric complex Gaussian (ZMCSCG) random variables with variance of 0.5 per dimension. Symbol for each tone is then multiplied by the corresponding precoding vector $\mathbf{p}_{k,n}$. Cascading IFFT at eNodeB and FFT at the user with CP extension, transmission at n-th frequency tone for first user can be expressed as::-

$$y_{1,n} = \mathbf{h}_{1,n}^\dagger \mathbf{p}_{1,n} x_{1,n} + \mathbf{h}_{1,n}^\dagger \mathbf{p}_{2,n} x_{2,n} + z_{1,n}, \quad n = 1, 2, \cdots, N$$

where $y_{1,n}$ is the received symbol at user-1 and $z_{1,n}$ is ZMCSCG white noise of variance N_0. Complex symbols $x_{1,n}$ and $x_{2,n}$ are assumed to be independent and of variances σ_1^2 and σ_2^2 respectively. Transmitter is subjected to an average power constraint $\mathbb{E}\left\|\mathbf{p}_{1,n}x_{1,n} + \mathbf{p}_{2,n}x_{2,n}\right\|^2 \le P_t$. We assume that eNodeB has perfect channel state information of all users (perfect CSIT), and each user knows its own effective channel (scalar coefficient) and that of the other user perfectly. This implies that user-1 has perfect knowledge of the coefficients $\mathbf{h}_{1,n}^\dagger \mathbf{p}_{1,n}$ and $\mathbf{h}_{1,n}^\dagger \mathbf{p}_{2,n}$. For channel estimation by the users, eNodeB needs to transmit pilot symbols for the symbol intervals equal to the number of co-scheduled users (two). It would enable both the users not only to estimate their own coefficients but also the the coefficient of the other co-scheduled user. For notational convenience, we drop the frequency index for subsequent sections and rewrite the system equation as:-

$$y_1 = \alpha_1 x_1 + \beta_2 x_2 + z_1$$
$$y_2 = \beta_1 x_1 + \alpha_2 x_2 + z_2$$

where α is the effective channel of the desired signal and β is the effective channel of the interferer.

III. Information Theoretic Perspective

Sum rate of the downlink channel for a given precoder $\mathbf{P}$ is given as

$$I(\mathbf{P}) = \mu I_1(\mathbf{P}) + (1-\mu) I_2(\mathbf{P}) \qquad 0 \le \mu \le 1 \tag{1}$$

where $\mathbf{P} = [\mathbf{p}_1 \ \mathbf{p}_2]$ is the precoder matrix, $I_1(\mathbf{P})$ and $I_2(\mathbf{P})$ are the mutual information of the first and second user respectively and μ is the parameter that defines the rate distribution between two users. The rate distribution factor can be absorbed in the precoder i.e. μ in $\mathbf{p}_1$ and $(1-\mu)$ in $\mathbf{p}_2$ and the equation can be rewritten as

$$I(\mathbf{P}) = I_1(\mathbf{P}) + I_2(\mathbf{P}) \tag{2}$$

The mutual information for first user for finite size QAM constellation with $|\chi_1| = M_1$ takes the form as

$$\begin{aligned} I(Y_1; X_1|\alpha_1, \beta_2) &= \mathcal{H}(X_1|\alpha_1, \beta_2) - \mathcal{H}(X_1|Y_1, \alpha_1, \beta_2) \\ &= \log M_1 - \mathcal{H}(X_1|Y_1, \alpha_1, \beta_2) \end{aligned} \tag{3}$$

where $\mathcal{H}(.) = -E\log p(.)$ is the entropy function. The second term of eq. (3) is given as:-

$$\begin{aligned} &\mathcal{H}(X_1|Y_1, \alpha_1, \beta_2) \\ &= \sum_{x_1}\int_{y_1}\int_{\alpha_1}\int_{\beta_2} p(x_1, y_1, \alpha_1, \beta_2) \log \frac{1}{p(x_1|y_1, \alpha_1, \beta_2)} dy_1 d\alpha_1 d\beta_2 \\ &= \sum_{x_1}\sum_{x_2}\int_{y_1}\int_{\alpha_1}\int_{\beta_2} p(x_1, x_2, y_1, \alpha_1, \beta_2) \\ &\quad \times \log \frac{\sum_{x_1'}\sum_{x_2'} p\left(y_1|x_1', x_2', \alpha_1, \beta_2\right)}{\sum_{x_2'} p\left(y_1|x_1, x_2', \alpha_1, \beta_2\right)} dy_1 d\alpha_1 d\beta_2 \end{aligned} \tag{4}$$

Conditioned on the channel and the precoder, there is one source of randomness i.e. noise. So (4) can be extended as

$$\begin{aligned} &\mathcal{H}(X_1|Y_1, \alpha_1, \beta_2) \\ &= \frac{1}{M_1 M_2}\sum_{\mathbf{x}} E_{z_1} \log \frac{\sum_{\mathbf{x}'} \exp\left[-\frac{1}{N_0}\left|\alpha_1 x_1 + \beta_2 x_2 + z_1 - \alpha_1 x_1' - \beta_2 x_2'\right|^2\right]}{\sum_{x_2'} \exp\left[-\frac{1}{N_0}\left|\beta_2 x_2 + z_1 - \beta_2 x_2'\right|^2\right]} \\ &= \frac{1}{M_1 M_2}\sum_{\mathbf{x}} E_{z_1} \log \frac{\sum_{\mathbf{x}'} \exp\left[-\frac{1}{N_0}\left|\mathbf{h}_1^\dagger \mathbf{P}\left(\mathbf{x} - \mathbf{x}'\right) + z_1\right|^2\right]}{\sum_{\mathbf{x}_2'} \exp\left[-\frac{1}{N_0}\left|\mathbf{h}_1^\dagger \mathbf{P}\left(\mathbf{x} - \mathbf{x}_2'\right) + z_1\right|^2\right]} \end{aligned} \tag{5}$$

where $M_2 = |\chi_2|$, $\mathbf{x} = [x_1 \ x_2]^T$, $\mathbf{x}' = \left[x_1' \ x_2'\right]^T$ and $\mathbf{x}_2' = [x_1 \ x_2^\circ|\mathbf{x}]^T$. Note that the summation $\sum_{\mathbf{x}_2'}$ is only over x_2° as the value of x_1 in $\mathbf{x}_2'$ is decided by the outer summation $\sum_{\mathbf{x}}$. The mutual information for the first user can be rewritten as

$$I\left(Y_1; X_1|\mathbf{h}_1^\dagger, \mathbf{P}\right) = \log M_1 - \frac{1}{M_1 M_2}\sum_{\mathbf{x}} E_{z_1} \log \frac{\sum_{\mathbf{x}'} p\left(y_1|\mathbf{x}', \mathbf{h}_1^\dagger, \mathbf{P}\right)}{\sum_{\mathbf{x}_2'} p\left(y_1|\mathbf{x}_2', \mathbf{h}_1^\dagger, \mathbf{P}\right)}$$

Similarly the mutual information for user-2 is given as

$$I\left(Y_2; X_2|\mathbf{h}_2^{\dagger}, \mathbf{P}\right) = \log M_2 - \frac{1}{M_1 M_2}\sum_{\mathbf{x}} E_{z_2} \log \frac{\sum_{\mathbf{x}'} p\left(y_2|\mathbf{x}', \mathbf{h}_2^{\dagger}, \mathbf{P}\right)}{\sum_{\mathbf{x}_1'} p\left(y_2|\mathbf{x}_1', \mathbf{h}_2^{\dagger}, \mathbf{P}\right)}$$

where $\mathbf{x}_1' = [x_1^{\circ}\ x_2|\mathbf{x}]^T$. Again note that the summation $\sum_{\mathbf{x}_1'}$ is only over x_1° as the value of x_2 in $\mathbf{x}_1'$ is decided by the outer summation $\sum_{\mathbf{x}}$. The optimal linear precoder $\mathbf{P}$ can be solved by maximizing the cost function (6) given on the top of next page. In the equation, λ is a Lagrange multiplier, which is chosen to satisfy the power constraint. Note that the optimal linear precoder maximizing (6) would ensure the rate distribution between two streams. Using the facts that norm of a vector is convex and the log of sum of exponentials is also convex, we can write (6) as the sum of a convex and a concave function. Thus the cost function is non-convex and appears to be intractable. However we now propose a method based on local optimals to find near optimal linear precoder.

The local optimal linear precoders $\mathbf{P}$ which maximize the mutual information are the solution to the following equation:-

$$-\frac{\partial}{\partial \mathbf{P}^{\dagger}}\left(\frac{1}{M_1 M_2}\sum_{\mathbf{x}} E_{z_1} \log \frac{\sum_{\mathbf{x}'} p\left(y_1|\mathbf{x}', \mathbf{h}_1^{\dagger}, \mathbf{P}\right)}{\sum_{\mathbf{x}_2'} p\left(y_1|\mathbf{x}_2', \mathbf{h}_1^{\dagger}, \mathbf{P}\right)}\right) - \frac{\partial}{\partial \mathbf{P}^{\dagger}}\left(\frac{1}{M_1 M_2}\sum_{\mathbf{x}} E_{z_2} \log \frac{\sum_{\mathbf{x}'} p\left(y_2|\mathbf{x}', \mathbf{h}_2^{\dagger}, \mathbf{P}\right)}{\sum_{\mathbf{x}_1'} p\left(y_2|\mathbf{x}_1', \mathbf{h}_2^{\dagger}, \mathbf{P}\right)}\right) + \lambda \mathbf{P} = 0 \quad (8)$$

where we have used the identity $\frac{\partial}{\partial \mathbf{P}^{\dagger}} \mathrm{Tr}\left(\mathbf{P}^{\dagger}\mathbf{R}\right) = \mathbf{R}$ and $\mathrm{Tr}\left(\mathbf{AB}\right) = \mathrm{Tr}\left(\mathbf{BA}\right)$. We can interchange the order of derivative inline with the proof in Appendix A of [13]. Due to similarity, we solve only one derivative in (8) i.e.

$$\frac{\partial}{\partial \mathbf{P}^{\dagger}} \log \sum_{\mathbf{x}'} \exp\left(-\left|y_1 - \mathbf{h}_1^{\dagger}\mathbf{P}\mathbf{x}'\right|^2\right) = \frac{\sum_{\mathbf{x}'} \exp\left(-\left|y_1 - \mathbf{h}_1^{\dagger}\mathbf{P}\mathbf{x}'\right|^2\right)\left[\mathbf{h}_1\left(y_1 - \mathbf{h}_1^{\dagger}\mathbf{P}\mathbf{x}'\right)\mathbf{x}'^{\dagger}\right]}{\ln 2 \sum_{\mathbf{x}'} \exp\left(-\left|y_1 - \mathbf{h}_1^{\dagger}\mathbf{P}\mathbf{x}'\right|^2\right)}$$

where we have used the relation $\frac{\partial}{\partial \mathbf{H}^{\dagger}}\left(\mathbf{x}^{\dagger}\mathbf{H}^{\dagger}\mathbf{y}\right) = \mathbf{y}\mathbf{x}^{\dagger}$. Solving (8) leads to (9) at the top of next page.

Taking partial derivative of (6) w.r.t λ we get

$$\mathrm{Tr}\left(\mathbf{P}\mathbf{P}^{\dagger}\right) - P_t = 0 \quad (10)$$

So the precoders $\mathbf{P}$ being a function of λ which satisfy (6) and (10) are the local optimal precoders satisfying the power constraint. From (6), it is very likely that the local optimal precoders $\mathbf{P}$ will be general (non-diagonal) matrices. It is noted that it is very difficult to find a closed-form solution (if any) to (9) and (10) for $\mathbf{P}$. Nevertheless as we are considering only the two user case, solving the local optimal $\mathbf{P}$ is still feasible. However when the number of users is large, the computational complexity of solving for local optimal $\mathbf{P}$ is prohibitive. We developed an iterative algorithm employing gradient descent method to solve (9) via utilizing the partial derivative $\frac{\partial}{\partial \mathbf{P}^{\dagger}}\left[\mu I_1\left(\mathbf{P}\right) + \left(1-\mu\right) I_2\left(\mathbf{P}\right)\right]$ which is equal to the left hand side of (9) except $\lambda\mathbf{P}$. The algorithm is as under:-

- $\mathbf{P}_0 =$ initial guess as MF , CI and RCI precoder s.t it satisfies the power constraint
- $\mathbf{S}_0 = \frac{\partial}{\partial \mathbf{P}_0^{\dagger}} I(\mathbf{P}_0), \quad k = 0$

  while $I(\mathbf{P}_k)$ is increasing
  - Compute α_k i.e. step size which is a real number
  - $\mathbf{P}_{k+1} = \lambda_k\left(\mathbf{P}_k + \alpha_k \mathbf{S}_k\right)$,we induce λ_k once $\mathbf{P}_{k+1}$ does not satisfy power constraint. It is given as

    $$\mathrm{Tr}\left(\lambda_k\left(\mathbf{P}_k + \alpha_k\mathbf{S}_k\right)\lambda_k\left(\mathbf{P}_k + \alpha_k\mathbf{S}_k\right)^{\dagger}\right) = P_t \Longrightarrow$$
    $$\lambda_k = \sqrt{\frac{P_t}{\mathrm{Tr}\left(\left(\mathbf{P}_k + \alpha_k\mathbf{S}_k\right)\left(\mathbf{P}_k + \alpha_k\mathbf{S}_k\right)^{\dagger}\right)}}$$
  - $\mathbf{S}_{k+1} = \frac{\partial}{\partial \mathbf{P}_{k+1}^{\dagger}} I(\mathbf{P}_{k+1})$
  - $k = k + 1$

  end

Numerical optimization furnished some interesting insights into linear precoding. For gradient descent method, we used three precoders i.e. MF, CI and RCI precoder as the initial guess. Interestingly both CI and RCI converged to the same local optimal linear precoder however MF precoder converged to a different local optimal precoder which has higher mutual information as compared to the local optimal precoder in case of CI and RCI precoders. Local optimals were found with different initializations, but the local optimal with MF as the initial guess has the highest mutual information amongst all. Therefore we call this local optimal as near optimal linear precoder. It was also observed that the near optimal linear precoders does not attempt to cancel or attenuate the MU interference. Rather it enhances the strength of desired signal $|\alpha|$. It verifies our argument that in case of discrete constellations, strategy of exploiting interference structure shall lead to optimality compared to the strategy of interference cancellation or attenuation.

Fig. 1 shows the sum rate of a broadcast channel with 2 transmit antennas and 2 single antenna users for QPSK

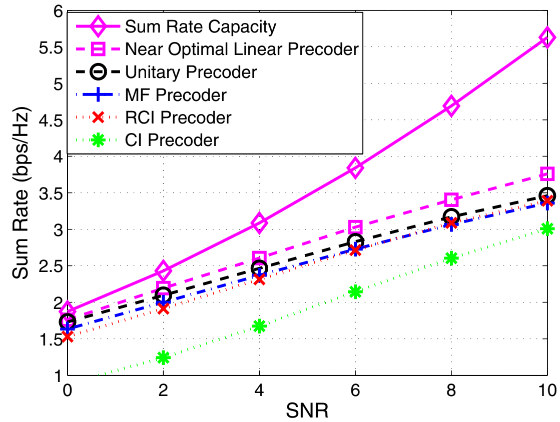

Fig. 1. Both users belong to QPSK constellations.

$$J = I\left(Y_1; X_1|\mathbf{h}_1, \mathbf{P}\right) + I\left(Y_2; X_2|\mathbf{h}_2, \mathbf{P}\right) + \lambda\left[\mathrm{Tr}\left(\mathbf{P}\mathbf{P}^\dagger\right) - P_t\right] \tag{6}$$

$$= \log M_1 M_2 - \frac{1}{M_1 M_2}\left(\sum_{\mathbf{x}} E_{z_1} \log \frac{\sum_{\mathbf{x}'} \exp\left[-\frac{1}{N_0}\left|\mathbf{h}_1^\dagger \mathbf{P}\left(\mathbf{x}-\mathbf{x}'\right)+z_1\right|^2\right]}{\sum_{\mathbf{x}_2'} \exp\left[-\frac{1}{N_0}\left|\mathbf{h}_1^\dagger \mathbf{P}\left(\mathbf{x}-\mathbf{x}_2'\right)+z_1\right|^2\right]} + \sum_{\mathbf{x}} E_{z_2} \log \frac{\sum_{\mathbf{x}'} \exp\left[-\frac{1}{N_0}\left|\mathbf{h}_2^\dagger \mathbf{P}\left(\mathbf{x}-\mathbf{x}'\right)+z_2\right|^2\right]}{\sum_{\mathbf{x}_1'} \exp\left[-\frac{1}{N_0}\left|\mathbf{h}_2^\dagger \mathbf{P}\left(\mathbf{x}-\mathbf{x}_1'\right)+z_2\right|^2\right]}\right) + \lambda\left[\mathrm{Tr}\left(\mathbf{P}\mathbf{P}^\dagger\right) - P_t\right] \tag{7}$$

$$\frac{-1}{M_1 M_2}\sum_{\mathbf{x}} E_{z_1}\left\{\frac{\sum_{\mathbf{x}'} \exp\left(-\left|y_1 - \mathbf{h}_1^\dagger \mathbf{P}\mathbf{x}'\right|^2\right)\left[\mathbf{h}_1\left(y_1 - \mathbf{h}_1^\dagger \mathbf{P}\mathbf{x}'\right)\mathbf{x}'^\dagger\right]}{\ln 2 \sum_{\mathbf{x}'} \exp\left(-\left|y_1 - \mathbf{h}_1^\dagger \mathbf{P}\mathbf{x}'\right|^2\right)} - \frac{\sum_{\mathbf{x}_2'} \exp\left(-\left|y_1 - \mathbf{h}_1^\dagger \mathbf{P}\mathbf{x}_2'\right|^2\right)\left[\mathbf{h}_1\left(y_1 - \mathbf{h}_1^\dagger \mathbf{P}\mathbf{x}_2'\right)\mathbf{x}_2'^\dagger\right]}{\ln 2 \sum_{\mathbf{x}_2'} \exp\left(-\left|y_1 - \mathbf{h}_1^\dagger \mathbf{P}\mathbf{x}_2'\right|^2\right)}\right\}$$
$$-\frac{1}{M_1 M_2}\sum_{\mathbf{x}} E_{z_2}\left\{\frac{\sum_{\mathbf{x}'} \exp\left(-\left|y_2 - \mathbf{h}_2^\dagger \mathbf{P}\mathbf{x}'\right|^2\right)\left[\mathbf{h}_2\left(y_2 - \mathbf{h}_2^\dagger \mathbf{P}\mathbf{x}'\right)\mathbf{x}'^\dagger\right]}{\ln 2 \sum_{\mathbf{x}'} \exp\left(-\left|y_2 - \mathbf{h}_2^\dagger \mathbf{P}\mathbf{x}'\right|^2\right)} - \frac{\sum_{\mathbf{x}_1'} \exp\left(-\left|y_2 - \mathbf{h}_2^\dagger \mathbf{P}\mathbf{x}_1'\right|^2\right)\left[\mathbf{h}_2\left(y_2 - \mathbf{h}_2^\dagger \mathbf{P}\mathbf{x}_1'\right)\mathbf{x}_1'^\dagger\right]}{\ln 2 \sum_{\mathbf{x}_1'} \exp\left(-\left|y_2 - \mathbf{h}_2^\dagger \mathbf{P}\mathbf{x}_1'\right|^2\right)}\right\} + \lambda\mathbf{P} = 0 \tag{9}$$

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alphabets. SNR is the transmit SNR i.e. $\frac{\sigma_1^2\|\mathbf{p}_1\|^2+\sigma_2^2\|\mathbf{p}_2\|^2}{N_0}$. Sum rate capacity (Gaussian broadcast channel) [2] and the sum rate of near optimal linear precoder along with unitary (based on QR-QL decomposition of the channel), MF, RCI and CI precoders are shown. In case of MF and unitary precoders, rate distribution between the two streams is optimized to maximize the sum rate. In the interesting low SNR regime, sum rates with the near optimal linear precoder, unitary precoder and MF precoder dominate those of CI and RCI precoders. It substantiates the argument that the precoding strategy of interference nulling or attenuation being devoid of exploiting interference structure will lead to degraded performance relative to other precoding strategies which allow interference to be propagated to the users. Detection of the desired signal is facilitated by the structure of the interference thereby leading to higher sum rate in the presence of interference. Therefore the degrees of freedom available at eNodeB should be used in improving the desired signal strength instead of exhausting them in nulling or attenuating the interference.

IV. MU MIMO IN LTE

We now look at the precoders for LTE and address the effectiveness of low resolutions of these precoders for MU MIMO (Transmission mode 5 [10]). For the case of eNodeB with 2 antennas, LTE proposes the use of following 4 precoders basing on 2 bit feedback from the users.

$$\mathbf{p} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} 1 \\ -j \end{bmatrix} \tag{11}$$

The number of precoders increases to 16 in the case of 4 transmit antennas however in this paper we restrict to the case of 2 transmit antennas. We propose the use of quantized MF precoders. i.e. the user measures its channel $\mathbf{h}_1^\dagger = [h_{11} \quad h_{12}]$ from eNodeB. Basing on MF, user then normalizes first channel coefficient i.e.

$$\mathbf{p}_{MF} = \frac{h_{11}^*}{|h_{11}|^2}\begin{bmatrix} h_{11} \\ h_{12} \end{bmatrix} = \begin{bmatrix} 1 \\ h_{11}^* h_{12}/|h_{11}|^2 \end{bmatrix} \tag{12}$$

Then basing on the miminum distance between $\mathbf{p}_{MF}$ and $\mathbf{p}$, one of the four precoders is selected by the user and the index of that precoder is fed back to eNodeB. From the geometrical perspective, this precoder would align h_{11} and h_{12} in the complex plane so as to maximize the received signal power i.e. $|h_{11}+h_{12}|^2$.

Assuming a densely populated cell, eNodeB selects the second user in each group of allocatable resource blocks whose requested precoder $\mathbf{p}_2$ is 180° out of phase from the precoder $\mathbf{p}_1$ of first user in the same resource blocks i.e. if $\mathbf{p}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ then $\mathbf{p}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ or if $\mathbf{p}_1 = \begin{bmatrix} 1 \\ j \end{bmatrix}$ then $\mathbf{p}_2 = \begin{bmatrix} 1 \\ -j \end{bmatrix}$ and vice versa.

Selection of the precoder for each user would ensure maximization of its desired signal strength i.e. $\left|\mathbf{h}_1^\dagger \mathbf{p}_1\right|^2$ for first user and $\left|\mathbf{h}_2^\dagger \mathbf{p}_2\right|^2$ for second user while selection of the user pairs with out of phase precoders would ensure minimization of interference strength seen by each user i.e. $\left|\mathbf{h}_1^\dagger \mathbf{p}_2\right|^2$ for first user and $\left|\mathbf{h}_2^\dagger \mathbf{p}_1\right|^2$ for second user. As an example, consider both h_{11} and h_{12} to have positive real and imaginary parts. Then the precoder selection would ensure desired signal strength to be $|h_{11}+h_{12}|^2$ and interference signal strength to be $|h_{11}-h_{12}|^2$.

Though this precoding and scheduling would ensure minimization of interference under the constraint of low resolution LTE precoders, the residual interference would still be significant. This interference is from finite alphabets and its structure can be exploited in the detection process. Here we propose to use the recently proposed low complexity detectors [9] by the users which on one hand reduce one complex dimension of the system while on the other exploit interference structure in the detection of desired stream. As the user already knows its own channel and the requested precoder, it can determine the effective channel of interference basing on the above discussed scheduling algorithm. Consequently there is

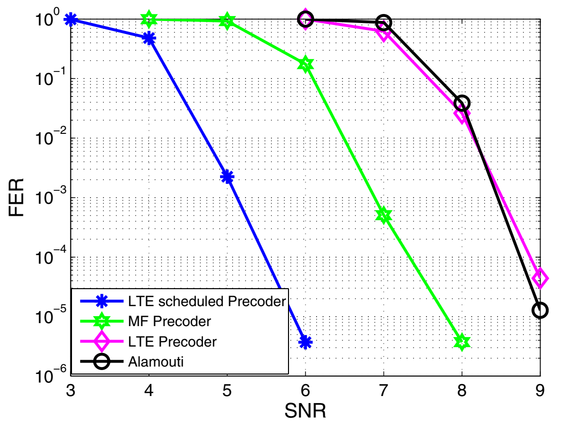

Fig. 2. Downlink channel with $n_t = 2$ and 2 single antenna users. 3GPP LTE punctured rate $1/2$ turbo code is used with maximum of 5 decoding iterations. Spectral efficiency is 2 bps/Hz.

no additional complexity in utilizing this detector as compared to using single user detectors.

We now simulate the downlink of LTE system by using different transmission schemes. As a reference we consider Alamouti scheme (fallback transmit diversity scheme in LTE) and compare it with the precoding schemes using LTE low resolution precoders. For these precoders we further consider two cases i.e no user scheduling and user scheduling with the precoders of two users to be $180°$ out of phase (opposite quadrants). As a reference, we also consider MF precoding based on full CSIT but without any scheduling. We consider BICM OFDM based transmission from eNodeB equipped with two antennas to two single antenna users using rate-$1/3$ LTE turbo code [14] with rate matching to rate $1/2$. We consider ideal OFDM system (no ISI) and analyze the system in the frequency domain where the channel has iid Gaussian matrix entries with unit variance and is independently generated for each channel use. We assume no power control so two users have equal power distribution. Perfect CSIT is assumed for the case of MF precoding while error free feedback of 2 bits is assumed for LTE precoders. Furthermore, all mappings of coded bits to QAM symbols use Gray encoding. We focus on frame error rates (FER) while the frame length is fixed to 1056 information bits. The users employ low complexity detectors [9] which have the inherent ability of exploiting interference structure in the detection of desired stream. Fig. 2 shows that the proposed transmission scheme with LTE precoders has more than 3 dB gain over Alamouti transmit diversity scheme and 2 dB gain over MF precoders with perfect CSIT at the same spectral efficiency. These results amply manifest the possible gains of MU MIMO in LTE even with the low resolution precoders. Gains may be further enhanced by improving the precoder resolutions.

V. Conclusion

In this paper we have questioned the idealized assumption of Gaussianity for alphabets and the subsequent linear precoding schemes concluding from this postulation. We have shown that the near optimal linear precoder based on realistic discrete alphabets neither cancels nor attenuates the interference. Rather it allows interference to be propagated to the users for its subsequent exploitation. We have demonstrated that with the low resolution precoders of LTE, significant performance improvement can be obtained by managing interference at eNodeB by their geometric alignment and then subsequently exploiting the structure of residual interference at the users.

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