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NEAR-SURFACE ELECTRON TEMPERATURE OF WEAKLY IONIZED PLASMA ACCORDING TO KINETIC THEORY

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ABSTRACT

Properties of electrons near absorbing and emitting surfaces have been studied for weakly ionized plasma by analyzing the Boltzmann equation governing the electrons. For simplicity, it was assumed that the electric field intensity is given a priori. It was shown that there exists a "nonequilibrium absorption layer", near the surface, wherein the kinetic distribution of electrons is completely out of equilibrium for all values of the mean free path, when the surface is highly absorbent with small or no electron emission. This layer is responsible for the large electron temperature jump at the surface, and it governs the electron temperature profile through the continuum as well as the rarefield plasmas. It was found from the analysis that the simple surface boundary condition for continuum electron energy equation previously employed by the present author is correct when there is no surface emission. Similar simple surface boundary condition is deduced for surfaces with given finite emission rates.

I. INTRODUCTION

Interaction of weakly ionized plasma with various electrically biased surfaces has been analyzed in recent years[1-15]. In all these analyses except those by Chung[7,8,9] and Burke[10] it was assumed that the electron temperature is either in equilibrium with the neutral gas temperature or a known constant. These assumptions in most cases are unrealistic ones, as was pointed out by Chung[7]. Nevertheless, these assumptions simplify the analyses greatly by a priori eliminating the consideration of the electron energy from the governing equations of the problem. With the assumptions, the interaction of the weakly ionized plasma with various solid bodies has been studied for the continuum range[1-10], intermediate range[11-14], and the collisionless range[15].

Rather complete theory has been developed and extensive solutions have been obtained for the continuum range*, so that all the basic features of the interaction, leading to such as characteristics of electrostatic probes, are now quite well understood, except for the behavior of the electron temperature. As it is to be expected, the analysis of the intermediate range lags behind the analyses of the other two ranges because of the difficulties associated with the solution of Boltzmann equation.

Now, focusing our attention on the continuum plasma, the elimination of the previously mentioned assumptions on the electron temperature, in order to make the analyses more realistic, does not simply imply the inclusion of the corresponding continuum relationship for the electron energy.

As it will be shown in this paper from Boltzmann equation, the transport of electrons becomes to be governed by a continuum equation as the suitably defined mean free path for collisions becomes sufficiently small, as compared with the characteristic length of the flow field, when the electron temperature is assumed to be known** a priori. If it is considered, however, that the electron energy, like the electron concentration, is governed by the Boltzmann equation, one finds that the continuum condition cannot be obtained near the surface, when the surface is a perfect absorber of electrons (perfect conductor) with a negligible emission rate, doesn't matter how small the mean free path becomes. As it will be seen later, this is because the kinetic distribution of electrons in the velocity space is always completely out of equilibrium near the surface, when the surface is a perfect absorber, for all non-zero values of the mean free path. This nonequilibrium distribution of electrons in velocity space, however, does not have any appreciable effect on the electron transport, since it is confined within few mean free paths from the surface, when the electron temperature is assumed to be known throughout the plasma. The nonequilibrium electron distribution near the surface, on the other hand, has a critical effect on the electron energy equation for it determines the wall boundary condition for this equation. The wall boundary condition, of course, affects the entire electron temperature profile across the plasma and, in turn, it affects the transport of electrons through its effect on the electric field intensity.

The problem of the wall boundary condition for the electron energy equation was first recognized by Chung[7]. Chung showed that a singularity exists for the continuum electron energy equation at the surface of a non-emitting conductor. Although the analysis was entirely based on continuum theory, the breakdown of the continuum theory was seen[7] to be the cause of the singularity. Mathematically, this singularity was found[7] to be removable and self-consistent solution of the electron energy equation, along with the other governing equations, was obtained.

Subsequently, Burke[10] faced the same problem of wall boundary condition for the electron energy equation. In the work of Burke[10], the various continuum governing equations, including the electron energy equation, were rederived rigorously by a near-equilibrium perturbation of the Boltzmann equation similar to the well-known Enskog-Chapman expansion[16,17]. This, however, did not help to resolve the problem of surface boundary condition, because, as it was stated before, the kinetic distribution of the electrons near the surface is so completely out of equilibrium that the near equilibrium perturbation is not applicable there. Consequently, an approximate boundary condition similar to that employed by Chung[7] was derived from the continuum equations.

In the present paper, the behavior of electrons in weakly ionized plasma will be studied by a nonequilibrium (not near-equilibrium) kinetic theory. A nonequilibrium kinetic theory analysis of the plasma consisting of neutral gas, ions, and electrons is a very formidable problem even with the assumptions of known electron temperatures (see references 11-14). The main purpose of the present analysis is to describe the electron energy near the surface and its effect on the electron temperature throughout the plasma. The only interaction of the electrons with the positive ions, in a weakly ionized plasma, is through the electric field governed by a Poisson equation (see references 1 and 2), which becomes a body force on the electrons. In order not to obscure the main purpose of the present study by the mathematical difficulties[11-14] associated with a complete kinetic theory description of the plasma, we shall consider that the electric field intensity acting on the electrons as a body force is given a priori. With the electric field intensity considered to be known, the governing equation for electrons is decoupled from that for ions and from the Poisson equation. The mathematical problem is then reduced to the solution of a Boltzmann equation for the behaviors of electrons and electron energy in the presence of collisons with the neutral gas particles and the given electric field intensity. Particular attention will be given to the properties of electrons near absorbing and thermionically emitting surfaces in the limit of small electron-neutral gas mean free paths wherein the electron and electron energy transports would be continuum processes if it were not for the surface phenomena described earlier.

It will be shown from the present analysis that the surface boundary condition employed by Chung[7] for the continuum electron energy equation is sufficiently accurate when the surface is a non-emitting conductor. The surface boundary condition for the thermionically emitting surfaces will be derived also from the analysis. Finally, the solution will show the effect of rarefaction (increasing mean free path) on the electron and electron energy profiles.

II. FORMULATION OF PROBLEM

The simplified problem to be analyzed in the present paper, in order to understand the behavior of electrons near surfaces affecting the electron temperature, was described in the preceding section along with the justification of the simplification.

Specifically, we consider that a weakly ionized plasma is confined between two infinite, parallel plates as shown in Fig.1. We consider that both plates absorb all the electrons colliding with the plates and, at the same time, may emit electrons which are in half-Maxwellian distribution, with all the particle velocities directed toward the plasma, at the temperature of the plates. This emission may be due to thermionic, surface ionization, or any other processes. The particular mechanism of electron emission is irrelevant to the present analysis. Also, the plates are sources and sinks for the ions; however, the ions are of no direct interest to the present problem.

Let us now consider the basic equation and boundary conditions which describe this problem.

Boltzmann Equation

The starting point of the present analysis is the following Boltzmann equation which governs the distribution function f_e of the electrons. For the plasma configuration given in Fig. 1, the Boltzmann equation is written as

$$v_e \frac{\partial f_e}{\partial y} + E \frac{\partial f_e}{\partial v_e} = \int\!\!\int\!\!\int\!\!\int\!\!\int_{-\infty}^{\infty} (\overline{f}_e \overline{f}_g - f_e f_g)\, V\, du_g\, dv_g\, dw_g p\, dp d\alpha \tag{1}$$

where u, v, and w are the x, y, and z components of the particle velocity respectively, and the subscripts e and g denote the electrons and neutral gas particles respectively. The symbol f represents the distribution function, and V is the relative velocity between an electron and a neutral gas particle before an encounter. Quantities p and α are the collision parameter and collision angle respectively. E(y) is the electric field intensity in the direction of y. The distribution functions with a bar denote those following a collision.

By definition, in weakly ionized plasma, the electron-electron and electron-ion collision frequencies are negligibly small as compared with the electron-neutral collision frequency. Hence, only the collisions between the electrons and the neutral gas particles are included in Eq.(1). Also, in weakly ionized plasma, the neutral gas is, for all practical purposes, unaffected by the presence of the ionized species[1-10]. Hence, the distribution function of the neutral gas particles is considered to be known from the existing analyses of neutral gases confined between two infinite parallel plates (see for instance references 18 and 19).

As it was mentioned toward the end of section I, the electric field intensity E is simply a body force acting on the electrons. It is immaterial what creates the electric field as far as the present study of the interaction of the electron energy with the surface is concerned. Therefore, as it was mentioned in section I, we consider that E is given. The only unknown function in Eq.(1) is, therefore, the electron distribution function f_e, and Eq.(1) is now determinate provided that the inter-molecular potential for the collisions is known. The intermolecular potential will be discussed later.

Boundary Conditions

In accordance with the physical problem described at the beginning of the present section, the boundary conditions are defined as follows.

At $Y = y/L = 0$

$$f_{e,w} = n'_{eb,w}\left(\frac{m_e}{2\pi kT'_{s,w}}\right)^{3/2} \exp\left[-\frac{m_e}{2kT'_{s,w}}\left(u_e^2+v_e^2+w_e^2\right)\right], \text{ for } v_e \geq 0 \tag{2}$$

and at $Y = y/L = 1$;

$$f_{e,\infty} = n'_{ea,\infty}\left(\frac{m_e}{2\pi kT'_{s,\infty}}\right)^{3/2} \exp\left[-\frac{m_e}{2kT'_{s,\infty}}\left(u_e^2+v_e^2+w_e^2\right)\right], \text{ for } v_e \leq 0 \tag{3}$$

In the above expressions, $n'_{eb,w}$ and $n'_{ea,\infty}$ are the number densities of the electrons being emitted at the lower and the upper plates respectively. Also, m and T' are the particle mass and the temperature respectively. The subscripts w and ∞ refer to the lower and to the upper plates respectively. L is the distance between the two plates. Note that when either surface is a perfect conductor without emission, $n'_{ea,\infty}$ or $n'_{eb,w}$ is zero. T'_s is the plate temperature.

Eqs.(1),(2)and(3)constitute a consistent boundary value problem. However, a direct solution of a Boltzmann equation, such as Eq.(1),for a completely nonequilibrium case is nearly impossible. We shall, in the present analysis, employ the moment method due to Mott-Smith[20] and Liu and Lees[18] to be discussed later.

As the necessary first step to a moment method, we shall show the generalized moment equation corresponding to Eq. (1).

Generalized Moment Equation

We let Q be a generalized function of the electron particle velocity. We then multiply Eq. (1) by Q and integrate each term with respect to the electron velocity space. After a rather standard manipulation (see references 16 and 17), the resulting equation is,

$$\frac{d}{dy}\iiint_{-\infty}^{\infty} f_e v_e Q du_e dv_e dw_e - \frac{E}{m_e}\iiint_{-\infty}^{\infty} f_e \frac{\partial Q}{\partial v_e} du_e dv_e dw_e = \Delta Q \tag{4}$$

where

$$\Delta Q = \iiiiiiiint_{-\infty}^{\infty} (\overline{Q}-Q)\ f_e f_g V\ p\ dp\ d\alpha\ du_e\ dv_e\ dw_e\ du_g\ dv_g\ dw_g \tag{5}$$

and $\overline{Q}$ denotes the value of Q following a collision.

The formulation of the problem is now complete. The solution of Eq. (1) to satisfy the boundary conditions, Eqs. (2) and (3), and the discussion of physical significance of the solution obtained will comprise the rest of the paper.

III. GOVERNING MOMENT EQUATIONS

We shall first make tractable the term ΔQ given by Eq. (5) by choosing the Maxwell's inverse fifth power intermolecular force[17]. We shall then develop from Eq. (4) a set of particular moment equations with the use of Liu and Lees[18] moment method. Solutions of the resulting equations will begin in the next section.

Intermolecular Force

It is known[11-14] that the Maxwell's intermolecular force law describes the collisions between electrons with low energies*** and neutral gas particles, especially the monatomic gas particles, quite well. As it was done in the previous analyses[11-14] of weakly ionized plasmas, we shall herein employ this force law which permits the integration of the collision integral. Thus, we consider that the repulsive force between a neutral gas particle and an electron is given[17] by $m_e m_g K_{eg} r^{-5}$, where K_{eg} is a constant and r is the distance between the colliding pair.

With the Maxwell's intermolecular force law, ΔQ becomes (see reference 17);

$$\Delta Q = \left[(m_g + m_e)K_{eg}\right]^{1/2} \iiiint\!\!\iint_{-\infty}^{\infty} J_p f_e du_e dv_e dw_e f_g du_g dv_g dw_g \tag{6}$$

where

$$J_p = \int_{\beta=0}^{\beta=\infty} \int_{\alpha=0}^{\alpha=2\pi} Q \, \beta d\beta d\alpha \tag{7}$$

Moment Equations

The particular moment method to be employed in the present study to solve the Boltzmann equation, Eq. (1), is that due to Mott-Smith [20], and Liu and Lees[18]. This method has been employed in various modified forms to describe the plasmas by kinetic theory when the electron temperature was assumed to be known[11-14].

The details of the general method are given in references 11 through 14, and will not be repeated here. One aspect of the method, however, as applied to a weakly ionized plasma should be discussed.

In the moment method due to Mott-Smith[20], and Liu and Lees[18], one first chooses a form of distribution function which will give the correct distribution functions in the two limits of the completely rarefied collisionless case and the collision dominated continuum case. This form of the distribution function is then substituted into the various particular moment equations derived from the general moment equation, Eq. (4). The resulting moment equations are then solved for the unknown functions comprising the chosen form of the distribution function.

Two half-Maxwellian distribution functions were employed by Wasserstrom et al[11] to analyze the plasma as it was done to analyze the Couette flow of a neutral gas by Liu and Lees[18]. As it was mentioned therein[11], this is not quite correct. It is because the distribution function of the electrons is not comprised of two half-Maxwellians in the collisionless limit, as it is with the neutral gas, since there exists a body force (electric field). The effect of using the two half-Maxwellian distribution functions on the analysis of plasmas was studied in detail by Chou et al[12]. This study[12] showed, as was expected, that the error caused by the use of the two half-Maxwellians increases with rarefaction, and that the error becomes negligible as the electron-neutral gas collision becomes frequent. Since it is the collisions which scatter the electrons toward two half-Maxwellian distributions, one for $v_e > 0$ and one for $v_e < 0$, it is seen[12] that the use of the two half-Maxwellians for a plasma will be as acceptable as for the neutral gas[18] when;

$$\frac{E\lambda}{kT'_e} \ll 1 \qquad (8)$$

where λ is the mean free path between electrons and neutral gas.

The present analysis is concerned with the cases wherein there is a sufficiently large number of collisions (see section I) such that the above relationship is satisfied. Hence, we assume that;

$$f_e = f_{ea} + f_{eb} \tag{9}$$

where

$$f_{ea} = n'_{ea}(y) \left[\frac{m_e}{2\pi kT'_{ea}(y)}\right]^{3/2} \exp\left[-\frac{m_e}{2kT'_{ea}(y)} (u_e^2 + v_e^2 + w_e^2)\right] \tag{10}$$

for $v_e \leqq 0$

$$f_{eb} = n'_{eb}(y) \left[\frac{m_e}{2\pi kT'_{eb}(y)}\right]^{3/2} \exp\left[-\frac{m_e}{2kT'_{eb}(y)} (u_e^2 + v_e^2 + w_e^2)\right] \tag{11}$$

for $v_e \geqq 0$

and, f_{ea} and f_{eb} are zero for $v_e > 0$ and $v_e < 0$ respectively.

There are four unknown functions, n'_{ea}, n'_{eb}, T'_{ea}, and T'_{eb}, in the present assumed form of the distribution function, Eqs. (10) and (11). We, therefore, need four moment equations. These equations are obtained from Eq. (4) by substituting the values, 1, v_e, $u_e^2 + v_e^2 + w_e^2$, and $v_e(u_e^2 + v_e^2 + w_e^2)$ sucessively for Q. In a one gas component system, such as those of references 18 and 20, the first three values of Q representing the mass, momentum, and the energy respectively were collisional invariants [16] which gave $\Delta Q = 0$. On the other hand, the mass and the energy were collisional invariants in the analyses[11-14] of weakly ionized plasma with given electron temperatures. In the present study, as it will be seen subsequently, only the mass, Q = 1, is a collisional invariant.

As we substitute $Q = 1$, v_e, $(u_e^2 + v_e^2 + w_e^2)$, and $v_e(u_e^2 + v_e^2 + w_e^2)$ sucessively in Eq.(4), and as we make use of Eqs. (6), (7), and (9) through (11), there result the following four moment equations after a considerable manipulation.

Continuity;

$$\frac{d}{dy}\left(n'_{eb}T'^{\frac{1}{2}}_{eb} - n'_{ea}T'^{\frac{1}{2}}_{ea}\right) = 0 \tag{12}$$

Momentum;

$$\frac{d}{dy}\left(n'_{eb}T'_{eb} + n'_{ea}T'_{ea}\right) - \frac{1}{k}\left(n'_{eb} + n'_{ea}\right) E$$

$$= -A\left(\frac{2K_{eg}}{\pi k}\right)^{\frac{1}{2}}\left(\frac{m_e}{m_g}\right)^{\frac{1}{2}} m_g n'_g \left(n'_{eb}T'^{\frac{1}{2}}_{eb} - n'_{ea}T'^{\frac{1}{2}}_{ea}\right) \tag{13}$$

Energy;

$$\frac{d}{dy}\left(n'_{eb}T'^{3/2}_{eb} - n'_{ea}T'^{3/2}_{ea}\right) - \frac{1}{2k} E\left(n'_{eb}T'^{\frac{1}{2}}_{eb} - n'_{ea}T'^{\frac{1}{2}}_{ea}\right)$$

$$= \frac{3}{2}\pi^{\frac{1}{2}} A\left(\frac{K_{eg}}{2k}\right)^{\frac{1}{2}}\left(\frac{m_e}{m_g}\right)^{3/2} m_g n'_g \left[\left(n'_{eb} + n'_{ea}\right)T'_g - \left(n'_{eb}T'_{eb} + n'_{ea}T'_{ea}\right)\right] \tag{14}$$

Energy Transport;

$$\frac{d}{dy}\left(n'_{eb}T'^{2}_{eb} + n'_{ea}T'^{2}_{ea}\right) - \frac{1}{k} E\left(n'_{eb}T'_{eb} + n'_{ea}T'_{ea}\right)$$

$$= -\frac{A}{5\pi^{\frac{1}{2}}}\left(\frac{2K_{eg}}{k}\right)^{\frac{1}{2}}\left(\frac{m_e}{m_g}\right)^{\frac{1}{2}} m_g n'_g\left(n'_{eb}T'^{3/2}_{eb} - n'_{ea}T'^{3/2}_{ea}\right) \tag{15}$$

where n'_g is the neutral gas number density.

In Eq. (13), the average diffusion velocity of neutral gas is considered to be negligibly small as compared with that of the electrons in accordance with the definition of the weakly ionized plasma. The general derivation of the right-hand side of Eq. (13), which comes from ΔQ, is given by Jeans.[17] The constant A which appears in the derivation of ΔQ was computed[17] to be 2.6595.

The ΔQ for $Q = v_e(u_e^2 + v_e^2 + w_e^2)$ was also derived by Jeans[17], but only for one gas component system wherein all the colliding particles have the same mass. This ΔQ and that for $Q = u_e^2 + v_e^2 + w_e^2$ are derived in the present analysis for $m_e/m_g << 1$ using the inverse fifth power law of collision, resulting in the right hand sides of Eqs. (15) and (14) respectively. These derivations are rather lengthy and tedious; however, they follow the standard method[17] and are not shown here.

Before we move on to the manipulations of Eqs. (12) through (15), it should be pointed out here that the present moment method of solution of the Boltzmann equation wherein the two half-Maxwellian distribution functions, Eqs. (10) and (11), are used automatically satifies the boundary conditions, Eqs. (2) and (3).

We shall now nondimensionalize the moment equations in the following.

Nondimensionalization of Moment Equations

There are two characteristic lengths in the present problem shown on Fig. 1. They are L and the mean free path λ which can be defined in terms of the present collision parameters as;

$$\lambda = \left(\frac{2\pi k T'_e}{m_e m_g K_{eg}}\right)^{1/2} \left(\frac{1}{A}\right)\left(\frac{1}{n'_g}\right) \tag{16}$$

Using L, λ_∞, $n_{ea,\infty}$ and $T'_{s,\infty}$ as the reference quantities, we transform Eqs. (12) through (15) into the following equations.

$$\frac{d}{dY}\left(n_b T_b^{1/2} - n_a T_a^{1/2}\right) = 0 \tag{17}$$

$$\varepsilon\left[\frac{d}{dY}\left(n_b T_b + n_a T_a\right) - F(n_b + n_a)\right] = -2N_g\left(n_b T_b^{1/2} - n_a T_a^{1/2}\right) \tag{18}$$

$$\varepsilon\left[\frac{d}{dY}\left(n_b T_b^{3/2} - n_a T_a^{3/2}\right) - (1/2)\,F\left(n_b T_b^{1/2} - n_a T_a^{1/2}\right)\right]$$
$$= \frac{3\pi}{2}\left(\frac{m_e}{m_g}\right) N_g\left[(n_b + n_a)\frac{1}{N_g} - (n_b T_b + n_a T_a)\right] \tag{19}$$

$$\varepsilon\left[\frac{d}{dY}\left(n_b T_b^2 + n_a T_a^2\right) - F\left(n_b T_b + n_a T_a\right)\right] = -\frac{8}{5}N_g\left(n_b T_b^{3/2} - n_a T_a^{3/2}\right) \tag{20}$$

where

$Y = y/L$, $T = T'_e/T'_{s,\infty}$, $n = n'_e/n'_{ea,\infty}$, $T_s = T'_s/T'_{s,\infty}$,

$N_g = n'_g/n'_{g,\infty}$, $F = EL/(kT'_{s,\infty})$

$$\varepsilon = \lambda_\infty/L = \left(\frac{2\pi k T'_{s,\infty}}{m_e m_g K_{eg}}\right)^{1/2}\left(\frac{1}{A}\right)\left(\frac{1}{L n'_{g,\infty}}\right) \tag{21}$$

We see in Eqs. (17) through (20) that m_e/m_g appears only in the right hand side of Eq. (19). The ratio m_e/m_g is of the order of 10^{-5}. Therefore, the collisional exchange of translational energies between the neutral gas particles and the electrons, which is represented by the right hand side of Eq. (19), is appreciable only as ε is reduced to the order of magnitude of $m_e/m_g \approx 10^{-5}$.

Since this elastic collisional energy exchange does not basically alter the behavior of the electrons near the surfaces, we confine our analysis to the values of $\varepsilon >> m_e/m_g$ and neglect the right-hand side of Eq. (19).

The body force E and the neutral gas density n_g' are known functions in the present problem as was explained earlier. Without loss of generality, we may let $N_g = 1$ and F to be constant for simplicity. Also we confine our analysis to $F\varepsilon << 1$ according to Eq. (8).

With the above mentioned conditions, we now seek the solution of Eqs. (17) through (20) for $\varepsilon << 1$. In these equations, the highest order derivative terms are multiplied by ε, and, hence, the equations could exhibit certain locally singular behavior. We shall first discuss the general behavior of Eqs. (17) through (20), and the physical implications of this behavior. Actual solution of the equations will then follow.

IV. GENERAL BEHAVIOR

In the original analysis[18] of the single component neutral gas by the method used herein, there was no singular region for $\varepsilon << 1$. One can show there[18] that in the limit of $\varepsilon \to 0$, the degenerated equations produce the continuum equations which are valid everywhere. Therefore, if one were to perturb the governing equations in the problem of reference 18, it would simply be a regular perturbation for small ε which is valid uniformly throughout the flow field.

It is noted here, however, that if the surface boundary condition in the problem of Liu and Lees[18] were that the gas particles are re-emitted specularly, instead of diffusely as it was, then there would have been a "Knudsen Layer" as $\varepsilon \to 0$ which is a singular region. That problem, however, could not be analyzed by the particular moment method.[18]

Now returning to the present problem described by Eqs. (17) through (20), we observe the following interesting phenomena. In the limit of $\varepsilon \to 0$, Eqs. (18) and (20) show that $n_a \to n_b$ and $T_a \to T_b$, which means, according to Eqs. (10) and (11), the distribution function approaches the full Maxwellian and, hence, the transport process becomes continuum, as was expected. There is, however, one fact, which is not immediately obvious, that breaks down the validity of the continuum limit near the surface, when the surface is a highly absorbent conductor with small or no emission.

Consider for the moment that there is no emission of electrons at the lower surface. Then $n_{bw} T_{bw}^{1/2} = 0$ according to our boundary condition. Now Eqs. (18) and (20) show that in the limit of $\varepsilon \to 0$,

$$(n_a T_a^{1/2})_w = 0$$

$$(n_a T_a^{3/2})_w = 0 \tag{22}$$

The above equations show that $n_{aw} = 0$, but T_{aw} can be any finite value, showing that $\varepsilon \to 0$ does not ensure the continuum near such a surface. With the preceding knowledges, we can now return to Eqs. (18) and (20) and examine these equations more carefully.

If $n_a \to n_b \approx 0$ near the lower surface, then there must exist a region near the surface wherein the right-hand sides of Eqs. (18) and (20) are of the order of the magnitude of ε or less for all values of $\varepsilon \ll 1$, including $\varepsilon = 0$. Therefore, in this region near the conductor, Eqs. (17) through (20) do not reduce to continuum relations even in the limit of $\varepsilon \to 0$. This region near a highly absorbent surface with no or small emission is a singualr region, and let us call it the "nonequilibrium absorption layer" for the lack of a better name.

Obviously the best suited method, in the light of the preceding discussion, of solution of Eqs. (17) through (20) is the method of "inner-and-outer" expansions described in references 21 through 23. We now proceed to solve Eqs. (17) through (20) by this method of singular perturbation.

V. SOLUTION OF GOVERNING EQUATIONS

In this section, we shall obtain the solution of Eqs. (17) through (20) for small values[ππ] of $\varepsilon = \lambda_\infty/L$ by the method of "inner-and-outer" expansions. We consider, henceforth, that $n_{eb,w} \ll n_{ea,\infty}$, and consider that it is the lower plate (w) which is the surface of our interest.

The general method of "inner-and-outer" expansions is now well known[21-23]. This method has been employed in the solution of many fluid mechanics problems (see for instance references 2, 5, 6, 24, and 25). We shall, therefore, apply this method to the present problem without much elaboration.

Outer Region; Continuum Region

We expand the dependent variables as;

$$n_i = n_{io} + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + O(\varepsilon^3)$$

$$T_i^{1/2} = t_i = t_{io} + \varepsilon t_{i1} + \varepsilon^2 t_{i2} + O(\varepsilon^3) \qquad (23)$$

where i may be either a or b.

A substitution of Eqs. (23) into Eqs. (17) through (20) results in sets of perturbation equations, out of which we write the first two sets as;

for O(1),

$$\frac{d}{dY}(n_{bo}t_{bo} - n_{ao}t_{ao}) = 0$$

$$n_{bo}t_{bo} - n_{ao}t_{ao} = 0 \qquad (24)$$

$$\frac{d}{dY}(n_{bo}t_{bo}^3 - n_{ao}t_{ao}^3) = \frac{F}{2}(n_{bo}t_{bo} - n_{ao}t_{ao})$$

$$n_{bo}t_{bo}^3 - n_{ao}t_{ao}^3 = 0$$

and for $O(\varepsilon)$,

$$\frac{d}{dY}(n_{b1}t_{bo} - n_{a1}t_{ao} + n_{bo}t_{b1} - n_{ao}t_{a1}) = 0$$

$$\frac{d}{dY}(n_{bo}t_{bo}^2 + n_{ao}t_{ao}^2) = -2(n_{b1}t_{bo} - n_{a1}t_{ao} + n_{bo}t_{b1} - n_{ao}t_{a1})$$

$$+ F(n_{bo} + n_{ao}) \qquad (25)$$

$$\frac{d}{dY}(3t_{bo}^2 t_{b1} n_{bo} - 3t_{ao}^2 t_{a1} n_{ao} + n_{b1}t_{bo}^3 - n_{a1}t_{ao}^3)$$

$$= (1/2)F(n_{b1}t_{bo} - n_{a1}t_{ao} + n_{bo}t_{b1} - n_{ao}t_{a1})$$

$$\frac{d}{dY}(n_{bo}t_{bo}^4 + n_{ao}t_{ao}^4) = -(8/5)(3t_{bo}^2 t_{b1}n_{bo} - 3t_{ao}^2 t_{a1}n_{ao} + n_{b1}t_{bo}^3$$

$$-n_{a1}t_{ao}^3) + F(n_{bo}t_{bo}^2 + n_{ao}t_{ao}^2)$$

Eqs. (24) readily give,

$$n_{bo} = n_{ao} = n_o$$

$$t_{bo} = t_{ao} = t_o \tag{26}$$

The first and the third equations of Eqs. (25) are readily integrated with the constants of integrations C_3 and C_4 respectively. The second and the fourth equations become with the use of the solutions of the other two equations as,

$$\frac{dn_o t_o^2}{dY} = - C_3 + Fn_o \tag{27}$$

$$\frac{dn_o t_o^4}{dY} = - \frac{4}{5}(\tfrac{1}{2} F C_3 Y + C_4) + Fn_o t_o^2 \tag{28}$$

Above equations can satisfy four boundary conditions. Two of the boundary conditions are given as,

$$n_o(1) = t_o^2(1) = 1 \tag{29}$$

The remaining two will be determined through the matching with the inner solution to be obtained in the following. Eqs. (27) and (28) are non-linear. After the remaining two boundary conditions are specified, Eqs. (27) and (28) are integrated numerically by a digital computer.

Inner Region; Nonequilibrium Absorption Layer

We first stretch this region by defining an independent variable

η as,

$$\eta = \frac{Y}{\varepsilon^q} \tag{30}$$

The dependent variables are then expanded as,

$$n_i = \varepsilon^s \hat{n}_{i1} + \varepsilon^{2s} \hat{n}_{i2} + O(\varepsilon^{3s})$$
$$T_i^{1/2} = t_i = \hat{t}_{io} + \varepsilon^h \hat{t}_{i1} + \varepsilon^{2h} \hat{t}_{i2} + O(\varepsilon^{3h}) \tag{31}$$

where i may be a or b and (^) denotes the inner region. As it was mentioned in section IV, it is the small values of n near the surface which is the cause of the singularity. Therefore, the first term in the expansion of n_i is considered to be of order ε^s in order that the correct singular behavior can become manifest.

Eqs. (30) and (31) are substituted into Eqs. (17) through (20). A study of the resulting equations showed that the various terms become of consistent orders of magnitude, and also the subsequent matching can be accomplished to the correct orders, if we let;

$$q = s = h = 1 \tag{32}$$

Sets of perturbation equations are generated from the above manipulations. The first two order perturbation equations are given below.

For O(1),

$$\frac{d}{d\eta}(\hat{n}_{b1}\hat{t}_{bo} - \hat{n}_{a1}\hat{t}_{ao}) = 0$$
$$\frac{d}{d\eta}(\hat{n}_{b1}\hat{t}^3_{bo} - \hat{n}_{a1}\hat{t}^3_{ao}) = 0 \tag{33}$$

For $O(\varepsilon)$,

$$\left.\begin{aligned}
&\frac{d}{d\eta}\left(\hat{n}_{b2}\hat{t}_{bo} - \hat{n}_{a2}\hat{t}_{ao} + \hat{n}_{b1}\hat{t}_{b1} - \hat{n}_{a1}\hat{t}_{a1}\right) = 0 \\
&\frac{d}{d\eta}\left(\hat{n}_{b1}\hat{t}^{2}_{bo} + \hat{n}_{a1}\hat{t}^{2}_{ao}\right) = -2\left(\hat{n}_{b1}\hat{t}_{bo} - \hat{n}_{a1}\hat{t}_{ao}\right) \\
&\frac{d}{d\eta}\left(3\hat{n}_{b1}\hat{t}^{2}_{bo}\hat{t}_{b1} - 3\hat{n}_{a1}\hat{t}^{2}_{ao}\hat{t}_{a1} + \hat{n}_{b2}\hat{t}^{3}_{bo} - \hat{n}_{a2}\hat{t}^{3}_{ao}\right) \\
&\qquad = (1/2)F\left(\hat{n}_{b1}\hat{t}_{bo} - \hat{n}_{a1}\hat{t}_{ao}\right) \\
&\frac{d}{d\eta}\left(\hat{n}_{b1}\hat{t}^{4}_{bo} + \hat{n}_{a1}\hat{t}^{4}_{ao}\right) = -\frac{8}{5}\left(\hat{n}_{b1}\hat{t}^{3}_{bo} - \hat{n}_{a1}\hat{t}^{3}_{ao}\right)
\end{aligned}\right] \qquad (34)$$

Eqs. (33), and the second and the third equations of Eqs. (34) are readily integrated to give,

$$\left.\begin{aligned}
&\hat{n}_{b1}\hat{t}_{bo} - \hat{n}_{a1}\hat{t}_{ao} = \hat{C}_1 \\
&\hat{n}_{b1}\hat{t}^{3}_{bo} - \hat{n}_{a1}\hat{t}^{3}_{ao} = \hat{C}_2 \\
&\hat{n}_{b1}\hat{t}^{2}_{bo} + \hat{n}_{a1}\hat{t}^{2}_{ao} = -2\,\hat{C}_1\eta + \hat{C}_3 \\
&\hat{n}_{b1}\hat{t}^{4}_{bo} + \hat{n}_{a1}\hat{t}^{4}_{ao} = -\frac{8}{5}\hat{C}_2\eta + \hat{C}_4
\end{aligned}\right] \qquad (35)$$

There are four constants of integrations in the above equations. We shall now apply the lower wall boundary conditions to Eqs. (35) and eliminate two of the four constants.

The emission rate of electrons at the lower plate, $\langle n'_e v_e \rangle_w$, whose distribution is given by Eq. (2) is,

$$\begin{aligned}
\langle n'_e v_e \rangle_w &= \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} v_e f_{e,w}\, du_e\, dv_e\, dw_e \\
&= \left(\frac{k}{2\pi m_e}\right)^{1/2} n'_{ea,\infty}\left(T'_{s,\infty}\right)^{1/2}\left(n_{b,w}t_{b,w}\right)
\end{aligned} \qquad (36)$$

As usual, the boundary condition must be satisfied by the lowest order term of the perturbation. Therefore, the boundary condition, Eq. (36), specifies $\hat{n}_{b1,w}\ \hat{t}_{bo,w}$. Since the electrons are emitted at the given plate temperature, we have $T_{b,w}^{1/2} = T_{s,w}^{1/2}$, and, hence Eq. (36) specifies $n_{b,w}$. The application of the boundary condition to Eqs. (35) that, at $\eta = 0$,

$$\hat{n}_{b1} = \hat{n}_{b1,w} = n_{b,w}/\varepsilon = \frac{n'_{eb,w}}{n'_{ea,\infty}} \Big/ \varepsilon \tag{37}$$

$$\hat{t}_{bo} = \hat{t}_{bo,w} = T_{s,w}^{1/2}$$

gives the relationships,

$$\begin{aligned} \hat{C}_3 &= (\hat{n}_{b1,w}\ \hat{t}^2_{bo,w}) + \left[\left(\hat{n}_{b1,w}\ \hat{t}_{bo,w} - \hat{C}_1\right)\left(\hat{n}_{b1,w}\ \hat{t}^3_{bo,w} - \hat{C}_2\right)\right]^{1/2} \\ \hat{C}_4 &= (\hat{n}_{b1,w}\ \hat{t}^4_{bo,w}) + \frac{(\hat{n}_{b1,w}\ \hat{t}^3_{bo,w} - \hat{C}_2)^{3/2}}{(\hat{n}_{b1,w}\ \hat{t}_{bo,w} - \hat{C}_1)^{1/2}} \end{aligned} \tag{38}$$

Eqs. (38) eliminates $\hat{C}_3$ and $\hat{C}_4$ as independent constants of integration. The remaining constants $\hat{C}_1$ and $\hat{C}_2$ will be determined through the matching to be described in the following.

<u>Matching</u>

The governing equations, Eqs. (17) through (20), describe the four moments $n_b T_b^{1/2} - n_a T_a^{1/2}$, $n_b T_b + n_a T_a$, $n_b T_b^{3/2} - n_a T_a^{3/2}$, and $n_b T_b^2 + n_a T_a^2$. These four moments become, with the aid of Eqs. (23) through (26), for the outer region as,

$$n_b T_b^{1/2} - n_a T_a^{1/2} = \varepsilon\ C_3 + O(\varepsilon^2) \tag{39}$$

$$n_bT_b + n_aT_a = 2\ n_o t_o^2 + \varepsilon \left[(n_{b1} + n_{a1})\ t_o^2 + 2\ n_o t_o\ (t_{b1} + t_{a1})\right] + O(\varepsilon^2) \quad (40)$$

$$n_bT_b^{3/2} - n_aT_a^{3/2} = \varepsilon(1/2\ F\ C_3\ Y + C_4) + O(\varepsilon^2) \quad (41)$$

$$n_bT_b^2 + n_aT_a^2 = 2\ n_o t_o^4 + \varepsilon \left[4\ t_o^3 n_o\ (t_{b1} + t_{a1}) + t_o^4\ (n_{b1} + n_{a1})\right] + O(\varepsilon^2) \quad (42)$$

The moments become, with the aid of Eqs. (30) through (35), for the inner region as,

$$n_bT_b^{1/2} - n_aT_a^{1/2} = \varepsilon\hat{C}_1 + O(\varepsilon^2) \quad (43)$$

$$n_bT_b + n_aT_a = \varepsilon(2\hat{C}_1\ \eta + \hat{C}_3) + O(\varepsilon^2) \quad (44)$$

$$n_bT_b^{3/2} - n_aT_a^{3/2} = \varepsilon\hat{C}_2 + O(\varepsilon^2) \quad (45)$$

$$n_bT_b + n_aT_a^2 = \varepsilon(-\frac{8}{5}\ \hat{C}_2\ \eta + \hat{C}_4) + O(\varepsilon^2) \quad (46)$$

Each moment is now matched between the two regions requiring that the outer solution as $Y \to 0$ should match, term by term with the inner solution as $\eta \to \infty$. The detailed explanation of a matching procedure is found elsewhere[21].

Equations (39) and (43), can be matched by observation to give,

$$C_3 = \hat{C}_1 \quad (47)$$

From Eq. (40) the two term inner expansion of the one term outer expansion (see reference 21) becomes as $Y \to 0$, in the inner variable,

$$n_bT_b + n_aT_a = 2\ (n_o t_o^2)_w + 2\ \varepsilon\left(\frac{dn_o t_o^2}{dY}\right)_w\ \eta \quad (48)$$

From Eq. (44), on the other hand, the one term outer expansion of the two term inner expansion becomes, in the outer variable, as $\eta \to \infty$,

$$n_b T_b + n_a T_a = - 2 \hat{C}_1 Y \tag{49}$$

Matching of Eqs. (48) and (49) by changing either η to Y or Y to η gives,

$$(n_o t_o^2)_w = 0 \tag{50}$$

$$\left(\frac{dn_o t_o^2}{dY} \right)_w = \hat{C}_1 \tag{51}$$

Equations (41) and (45), and Eqs. (42) and (46) are matched similarly, and there result the conditions,

$$C_4 = \hat{C}_2, \tag{52}$$

$$(n_o t_o^4)_w = 0 \tag{53}$$

$$\left(\frac{dn_o t_o^4}{dY} \right)_w = - \frac{4}{5} \hat{C}_2 \tag{54}$$

The matching is now completed for the lowest order. Construction of the complete solution is summarized below.

Complete Lowest Order Solution

Equations (50) and (53) obtained through the matching are the two of the four boundary conditions needed for the outer equations, Eqs. (27) and (28).

Equations (51) and (54) are not independent conditions. The outer equations, Eqs. (27) and (28), become Eqs. (51) and (54) respectively at Y = 0 when the conditions, Eqs. (50) and (53), are applied, thus showing the consistency of the matching.

Equations (27) and (28) are integrated numerically for the two functions, $(n_o t_o^2)$ and $(n_o t_o^4)$ to satisfy the four boundary conditions, Eqs. (29), (50) and (53). The solution gave $n_o(Y)$, t_o^2 (Y), C_3 and C_4, thus providing the lowest order values of n and T for the outer region from Eqs. (23).

With C_3 and C_4 known, the constants of integration for the inner region $\hat{C}_1$, $\hat{C}_2$, and $\hat{C}_3$, and $\hat{C}_4$ are readily obtained from Eqs. (47), (52), and (38), in terms of the surface conditions, $n_{b,w}$ and $T_{b,w} = T_{s,w}$, given by Eq. (36). Equations (35) are now determinate. The algebraic equations, Eqs. (35), though nonlinear, can be manipulated into explicit expressions for $\hat{n}_{a1}$, $\hat{n}_{b1}$, $\hat{t}_{ao}$ and $\hat{t}_{bo}$. Equations (31) then give the lowest order values of n_a, n_b, T_a, and T_b for the inner region. The lowest order n and T for the inner region are finally obtained as,

$$n = \frac{1}{n'_{ea,\infty}} \iiint_{-\infty}^{\infty} f_e \, du_e \, dv_e \, dw_e = (n_b + n_a)/\ 2$$

$$T = \frac{1}{3kT'_{s,\infty}} \frac{m_e}{n'_e} \iiint_{-\infty}^{\infty} \left[u_e^2 + (v_e - \langle v_e\rangle)^2 + w_e^2\right] f_e \, du_e \, dv_e \, dw_e$$

$$= \left(\frac{n_b T_b + n_a T_a}{n_b + n_a}\right) - \frac{2}{3\pi} \left(\frac{n_b T_b^{1/2} - n_a T_a^{1/2}}{n_b + n_a}\right)^2$$

where $\langle v_e \rangle$ is the average electron velocity in y direction.

VI DISCUSSION OF RESULTS

Typical electron temperature and electron number density profiles obtained from the solution are plotted in Figs. 2 through 5. The surface electron temperature jump is shown in Fig. 6.

It is clear from the preceding section[#] that the solution is determined, so far as the surface effect is concerned, when the surface temperature and $n_{b,w}/\varepsilon$, which is given by the surface emission rate,[##] is specified. The solutions are, therefore, plotted in Figs. 2 through 6 with $n_{b,w}/\varepsilon = (n'_{e,w}/n'_{ea,\infty})/\varepsilon$ as the parameter representing the surface emission rate.

The "nonequilibrium absorption layer" defined and discussed in section IV causes, among other things, the surface electron temperature jump. Figure 6 shows that the absorption layer disappears and the continuum process becomes valid all the way to the surface when $n_{b,w}/\varepsilon > O(10)$.

In this limit, the electrons at the surface is at the surface temperature. Also, the most nonequilibrium or "frozen" absorption layer exists when $n_{b,w}/\varepsilon \leqq O(10^{-2})$ when $T_{s,w}$ is of order one. In this "frozen" limit, the surface emission rate of electrons is so low that the electrons in the plasma are unaffected by the presence of the surface with a given surface temperature, and the surface electron temperature jump is maximum. Between these two limits of continuum (or equilibrium) and frozen absorption layers, the electrons near the surface feel a limited influence of the surface resulting in the electron temperature jump shown in Fig. 6.

We see in Figs. 2 through 5 that as $n_{b,w}/\varepsilon$ is increased and as the surface electron temperature jump is reduced, the effect of the surface temperature rapidly spreads deeply into the main body of the plasma. Therefore, for the small values of ε, for which the "continuum plasma theory"[1-10] has been developed, an extremely small value of $n_{b,w}$ or an extremely small surface emission rate can greatly alter the electron temperature profile from that corresponding to a non-emitting conductor.

Since the surface electron temperature jump increases with the decreasing $n_{b,w}/\varepsilon$, the jump increases as ε is increased for a given emission rate. This increasing surface electron temperature jump accompanying the increasing ε, however, is due to the rarefaction of the neutral gas, as seen in Eq. (21), which causes the kinetic distrubution of electrons to become nonequilibrium uniformly throughout the plasma. This effect is, therefore, basically different from that of decreasing $n_{b,w}$ for a given small value of ε where the large electron temperature jump occurs due to the nonequilibrium absorption layer near the wall. Of course, with the continuous increasing ε, the neutral gas distribution itself will become highly nonequilibrium and will create a neutral gas temperature jump at the surface in addition to the electron temperature jump.

Finally, we shall deduce, from the present analysis, the surface boundary condition for the electron energy equations appearing in the continuum plasma analyses such as those of references 7 through 10. We first recognize that the lowest order governing equations for the outer region, Eqs. (27) and (28), are precisely the continuum equations for the present problem of Fig. 1 governing the electrons. Equation (27) is the electron momentum equation and Eq. (28) is the electron energy equation. What we seek is, therefore, the suitable surface boundary conditions which may be applied to these equations directly, without going through the inner solutions, to produce n and T profiles which are acceptably close to the solutions shown on Figs. 2 through 5.

When the surface electron emission is sufficiently small such that $n_{b,w}/\varepsilon \leq O(10^{-1})$, Figs. 2 through 5 show that the outer solution describes the electron temperature correctly, practically all the way to the surface. The outer equations, Eqs. (27) and (28) can be combined as,

$$(n_o T_o) \frac{dT_o}{dY} + \frac{4}{5}\left(-\frac{1}{2} F \quad C_3 Y + C_4\right) - C_3 T_o = 0 \qquad (55)$$

where $T_o = t_o^2$. The surface boundary conditions, Eqs. (50) and (53) applied to Eqs. (27) and (28) can be shown to be equivalent to the boundary conditions,

$$n_{o,w} = 0 \qquad (56)$$

$$\left(\frac{dT}{dY}\right)_w = \frac{1}{5} F \qquad (57)$$

The electron energy equation employed by Chung[7-9] and others[10] is essentially Eq. (55) with the differences being only due to the different flow geometries, neutral gas compressibilities, and the different electron-neutral collision cross-sections. Furthermore, the surface boundary conditions employed by Chung[7-9] for the electron energy equation when the surface emission is zero is precisely

Eqs. (56) and (57) with the only algebraic differences being again due to the different flow geometries, etc. We have shown, therefore, that the surface boundary condition for the electron energy equation employed by Chung[7-9] for the continuum plasma is correct.

Now, let us consider the case for which the surface emission rate is such that $n_{b,w}/\varepsilon > 0(0\text{-}10^{-1})$. It is proposed, herein, that the continuum energy equation, Eq. (55), be solved with the wall boundary conditions,

$$\begin{aligned} n_o(Y = o) &= n_{b,w} \neq 0 \\ T(Y = o) &= T_w \end{aligned} \tag{58}$$

The solution of the continuum equations are obtained in the present study to satisfy Eqs. (58), and results for $n_{b,w}/\varepsilon = 4$ (shown in broken lines) are compared in Figs. 2 through 5 with the correct solutions. It is seen that the simple surface boundary conditions, Eqs. (58), when applied directly to continuum equations give the electron temperature profiles which are acceptably close, for "continuum plasma" studies, to the correct solutions. In the continuum plasma analyses, the accurate value of electron temperature within a few mean free paths of the surface itself is not a critical factor, as long as it does not affect the electron temperature in the main plasma.

VII CONCLUDING REMARKS

The interaction of electrons, in weakly ionized continuum plasma, with absorbing and emitting surfaces has been studied by analyzing the Boltzmann equation governing the electrons. In order not to obscure the interaction phenomenon by mathematical complications associated with a complete kinetic formulation of the plasma, it was assumed that the electric field intensity is given a priori. Solution was obtained by Mott-Smith[20], and Liu and Lees[18] moment method, and by using the method of "inner- and -outer" expansions.

It was shown that there exists a "nonequilibrium absorption layer", which corresponds to the "inner" region, wherein the kinetic distribution of electrons is completely out of equilibrium for all values of the mean free path, when the surface is highly absorbent with negligible emission. This layer is responsible for the surface electron temperature jump, and it governs the electron temperature profile through the continuum as well as the rarefied plasmas. The detailed mathematical and physical characteristics of the absorption layer are discussed in sections IV and VI.

It was found from the analysis that the simple surface boundary condition for the electron energy equation previously employed by Chung[7] in the continuum plasma analysis is correct in the limit of no surface emission. Similar simple surface boundary condition is deduced from the present analysis for the surfaces with given finite electron emission rates.

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LIST OF FOOTNOTES

* In order for the continuum equation to be applicable to plasma, the mean free path must be much smaller than the Debye shielding length in addition to the usual requirement of the collision dominence.

** As it was mentioned earlier, the usual assumption has been that the electron temperature is known to be either at the value of the neutral gas temperature or at another assigned constant value.

*** The electron energy in the weakly ionized plasmas of the present interest is less than 1 e.v.

ππ More precisely for $m_e/m_g \ll \varepsilon \ll 1$.

Particularly, see Eqs. (37).

See Eq. (36).

FIGURE CAPTIONS

FIG. 1 Plasma Configuration

FIG. 2 Electron Temperature and Density Profiles Near Surface for $F = 0$

FIG. 3 Electron Temperature and Density Profiles Near Surface for $F = 1.0$

FIG. 4 Electron Temperature and Density Profiles Near Surface for $F = -1$

FIG. 5 Electron Temperature Profile for $\varepsilon = 0.01$

FIG. 6 Electron Temperature Jump at the Surface

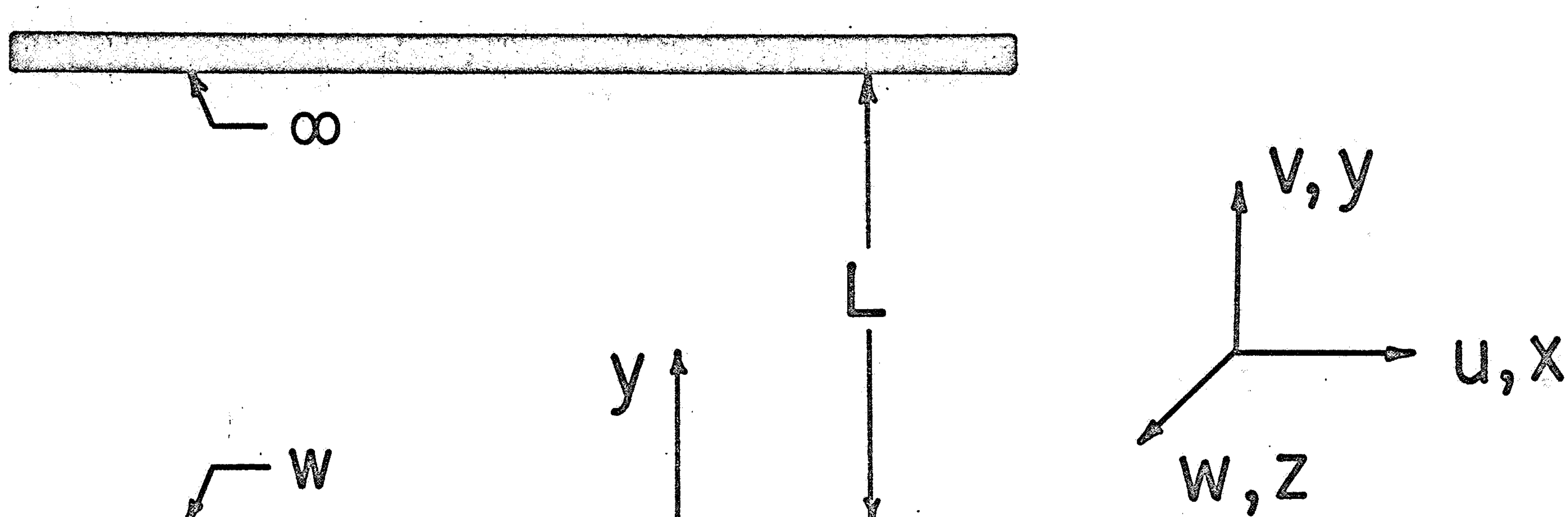

FIG. 1 PLASMA CONFIGURATION

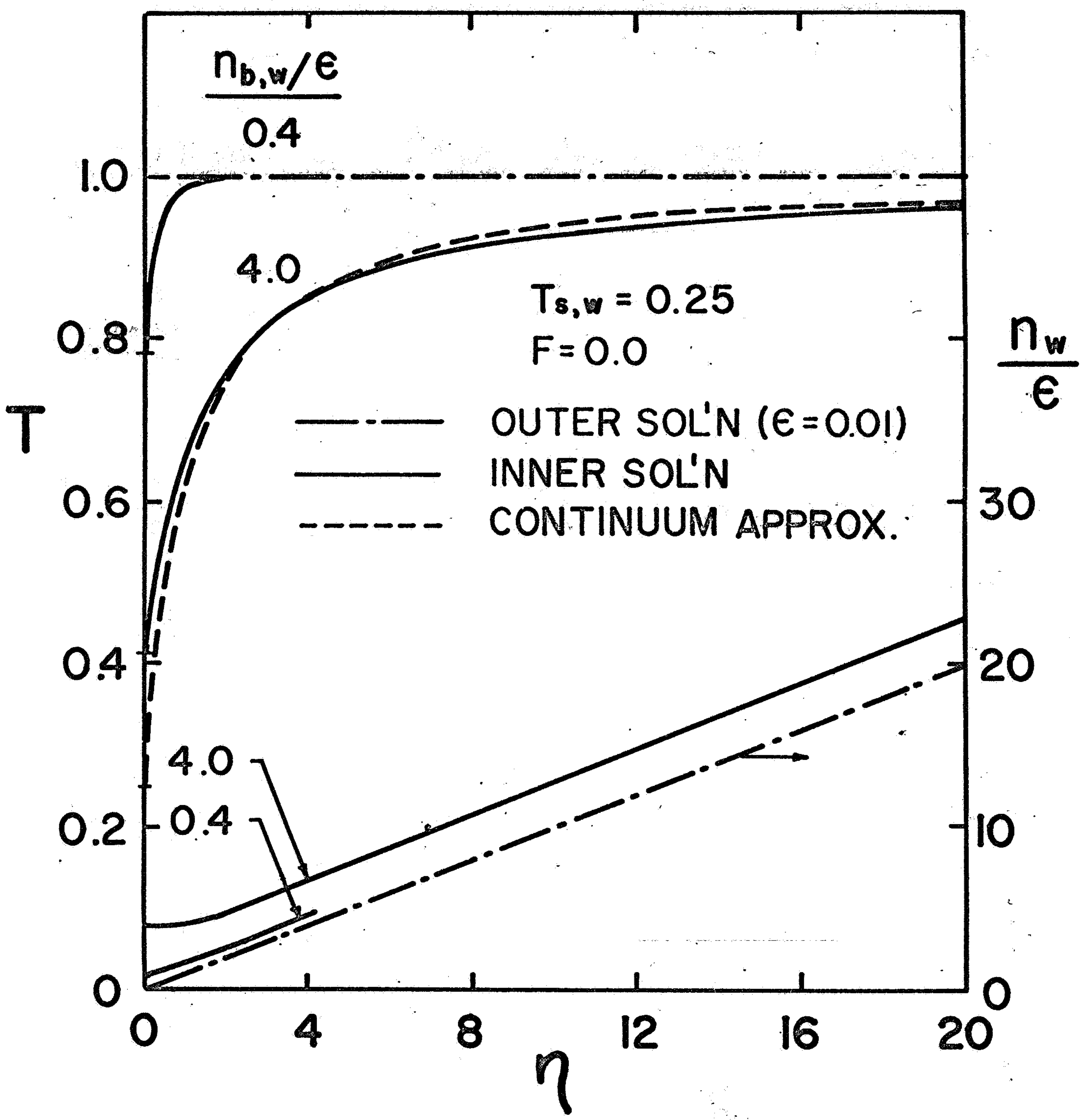

FIG. 2 ELECTRON TEMPERATURE AND DENSITY PROFILES NEAR SURFACE FOR F = 0

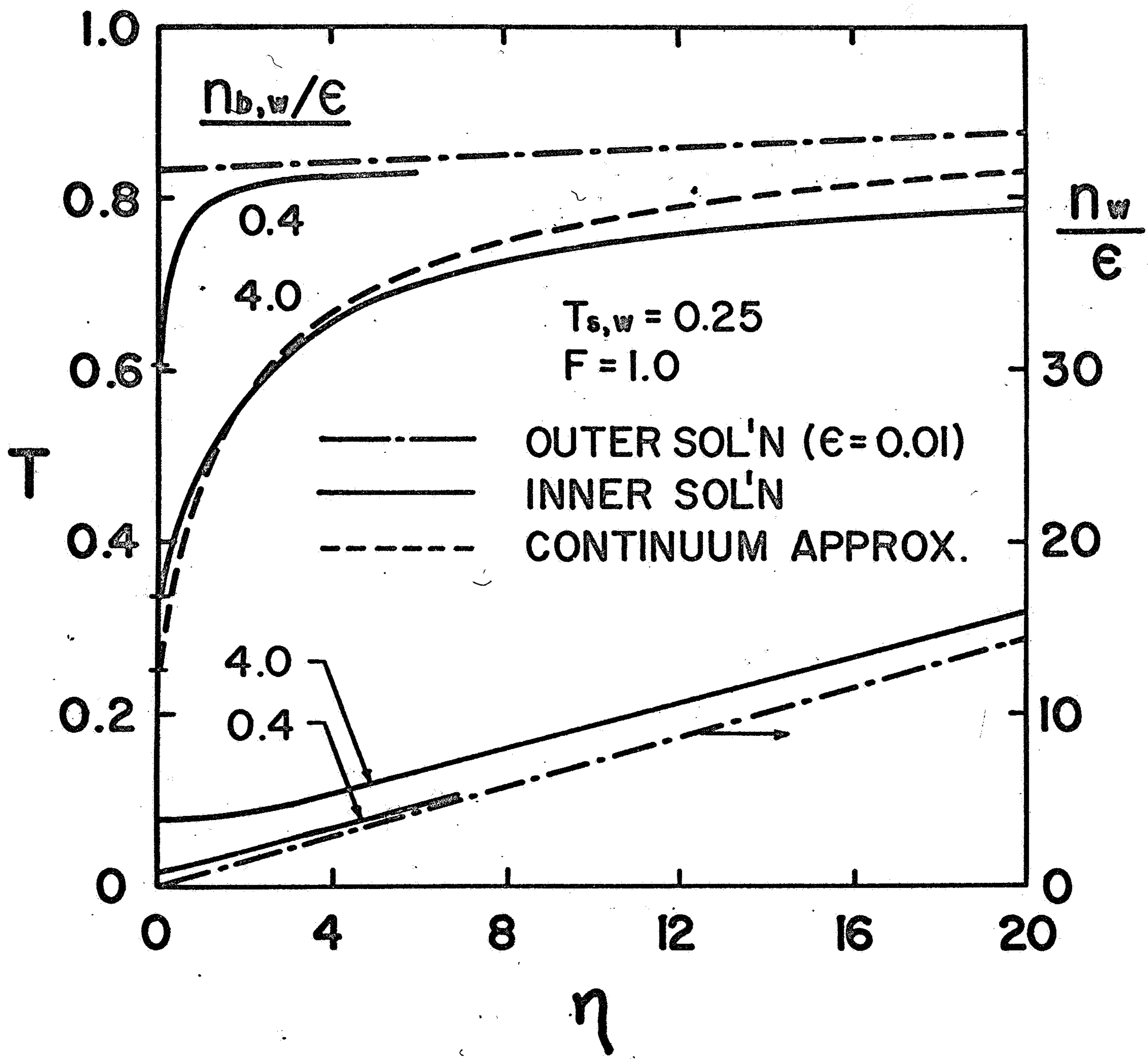

FIG. 3 ELECTRON TEMPERATURE AND DENSITY PROFILES NEAR SURFACE FOR F = 1.0

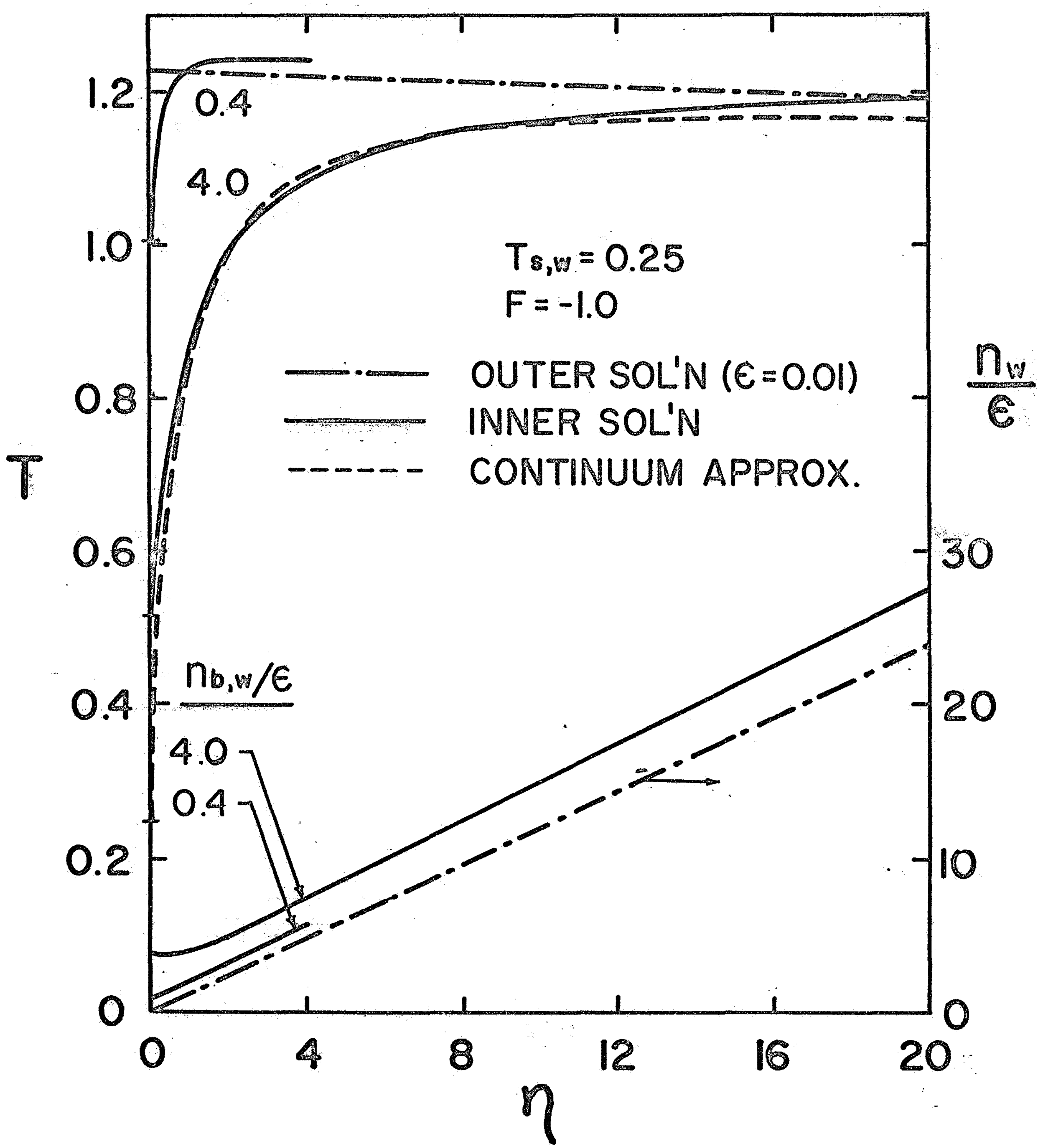

FIG. 4 ELECTRON TEMPERATURE AND DENSITY PROFILES NEAR SURFACE FOR F = - 1

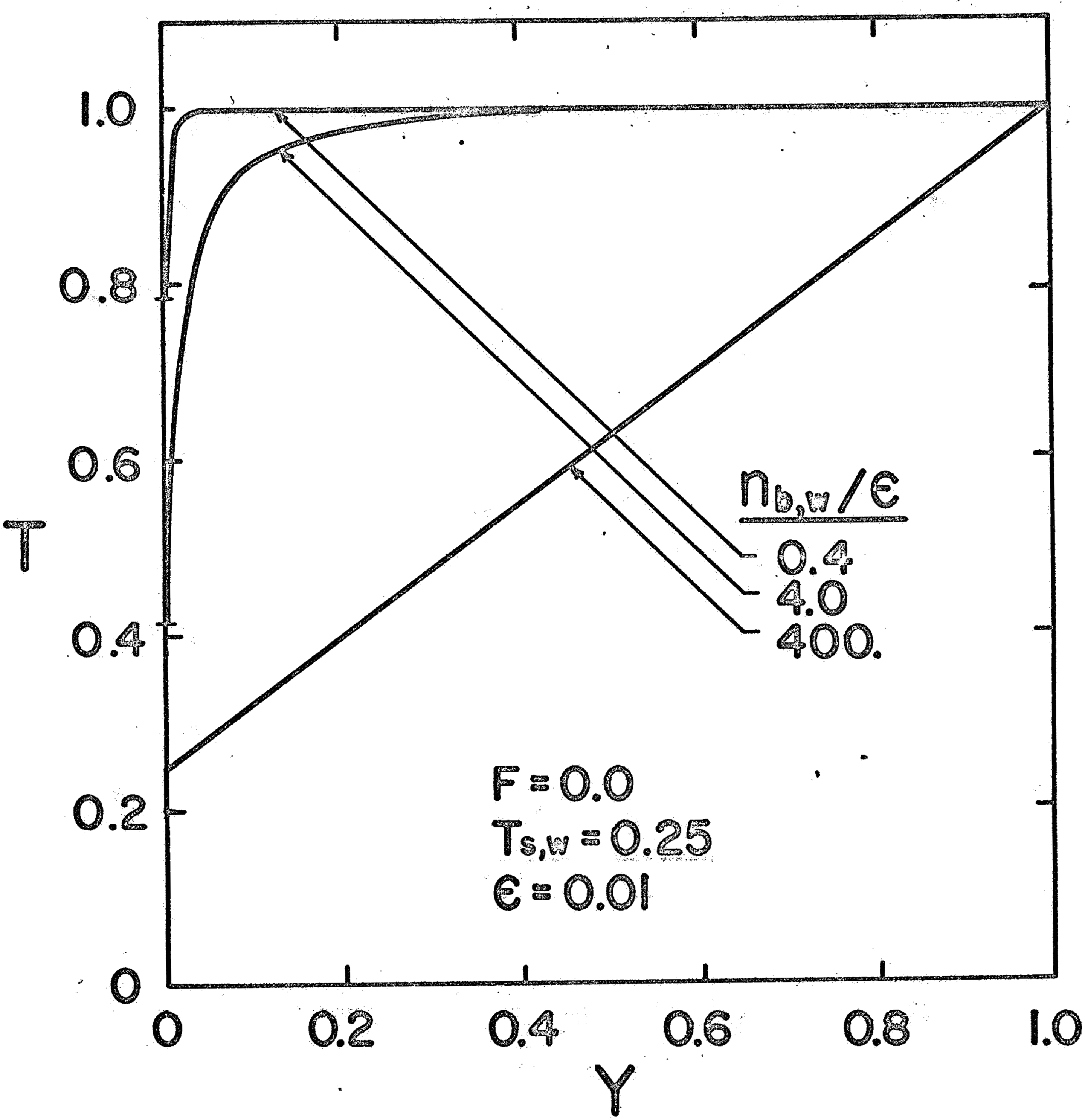

FIG. 5 ELECTRON TEMPERATURE PROFILE FOR $\varepsilon = 0.01$

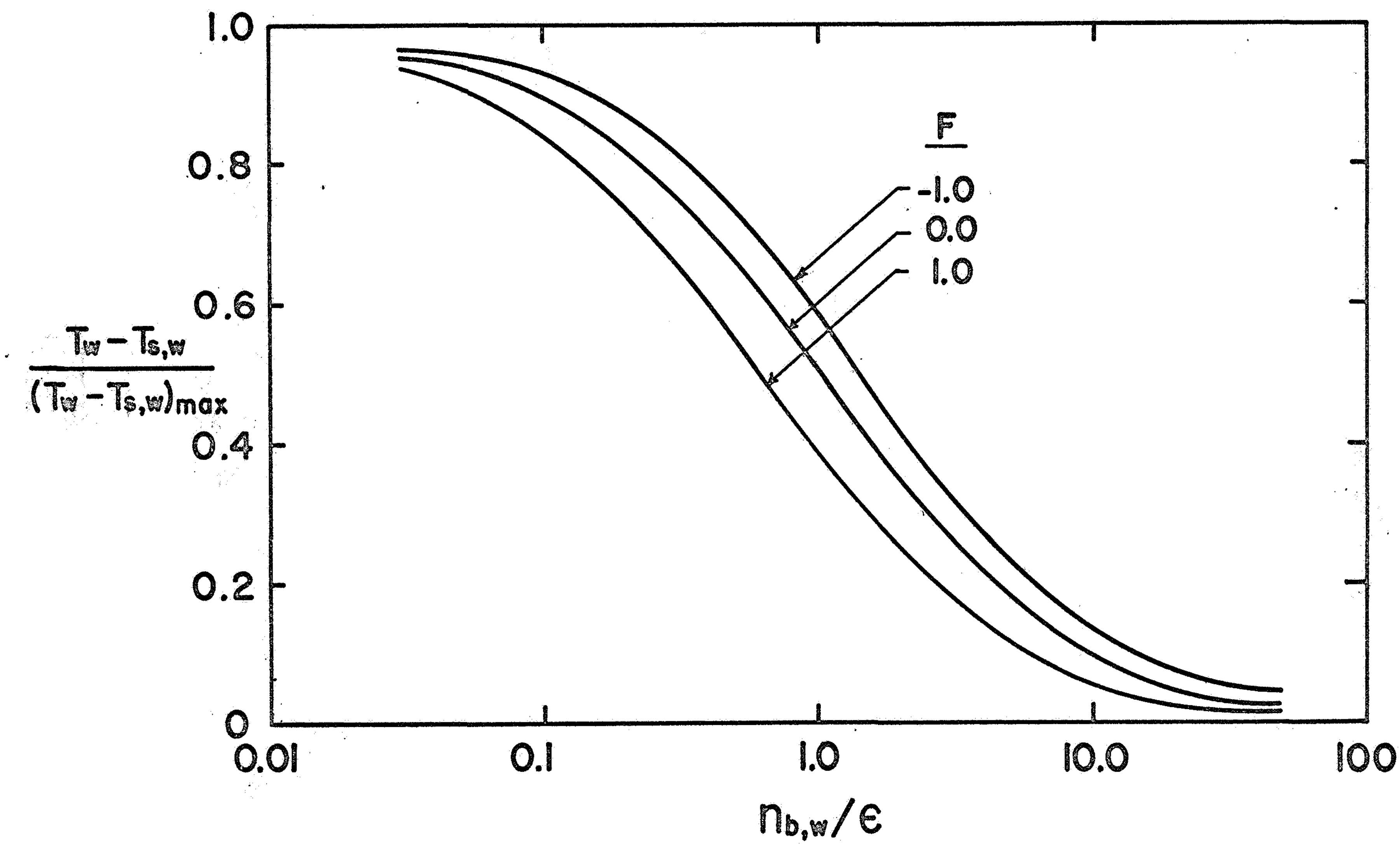

FIG. 6 ELECTRON TEMPERATURE JUMP AT THE SURFACE