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Nearest-Neighbour Predictions in Foreign Exchange Markets

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Abstract

The purpose of this paper is to contribute to the debate on the relevance of non-linear predictors of high-frequency data in foreign exchange markets. To that end, we apply nearest-neighbour (NN) predictors, inspired by the literature on forecasting in non-linear dynamical systems, to exchange-rate series. The forecasting performance of univariate and multivariate versions of such NN predictors is first evaluated from the statistical point of view, using a battery of statistical tests. Secondly, we assess if NN predictors are capable of producing valuable economic signals in foreign exchange markets. The results show the potential usefulness of NN predictors not only as a helpful tool when forecast daily exchange data but also as a technical trading rules.

JEL classification numbers: C53, F31, G12, G15

KEY WORDS: Nearest-neighbour prediction methods, Exchange rates, Technical trading rules.

Resumen

Con este trabajo pretendemos contribuir al debate sobre la relevancia del uso de predictores no lineales en datos de alta frecuencia relativos a mercados cambiarios. Para ello, aplicamos a tipos de cambio predictores por analogía, inspirados en la literatura sobre predición en sistemas dinámicos no lineales. Inicialmente se realiza una evaluación estadística de la capacidad predictiva de las versiones univariante y multivariante de estos predictores, mediante el uso de una amplia batería de contrastes. Posteriormente, se procede a examinar el valor económico de tales predictores, utilizándolos como señales para generar estrategias de contratación en los mercados. Los resultados obtenidos indican una potencial utilidad de estos predictores por analogía, no sólo como herramienta para la predicción del tipo de cambio diario, sino también como reglas técnicas de compra-venta.

JEL classification numbers: C53, F31, G12, G15

PALABRAS CLAVES: Predictores por analogía, Tipos de cambio, Reglas técnicas de negociación.

1. Introduction

Exchange rates have proven extremely difficult to forecast, despite a vast literature strewn with attempts. The structural exchange rate models have little success forecasting exchange rate compared with the forecast from a simple random walk model, as first noted by Messe and Rogoff (1983) and confirmed subsequently by many others [see, e. g., Frankel (1984) and Flood and Rose (1995)]. Therefore, beating the random walk still remains the standard metric by which to judge empirical exchange rate models.

Several explanations for the failure of structural models have been suggested, including misspecification of the models and poor modelling of expectations [Frankel and Rose (1995) for a survey]. Recently, interest has been shown in the possibility that non-linearities account for the apparent unpredictability of exchange rates, and some papers have highlighted the importance of non-linear adjustment of the exchange rate to the value implied by fundamentals, including Taylor and Peel (2000) and Kilian and Taylor (2001).

Dynamical systems theory suggests that non-linear behaviour may explain fluctuations in financial asset prices that appear to be random. One strand of the financial economics literature attempts to provide theoretical justification of the presence of non-linearities in assets prices. Another strand empirically tests for the presence of non-linearities in such prices. Regarding foreign exchange markets, economic theory highlights a number of potential sources for the presence of non-linearities in exchange rates. The main explanations include the concept of time deformation (Clark, 1973), diversity in agents' beliefs (Frankel and Froot, 1986), fads or noise trading (De Long *et al.*, 1990), technical traders (Bilson, 1990), hysteresis model with transaction cost bands (Baldwin, 1990), bubbles with self-fulfilling expectations (Froot and Obstfeld, 1991), target zone models (Krugman, 1991), non-linear risk premium model (Peel, 1993) and heterogeneity in investors' objectives arising from varying investment horizons and risk profiles (Peters, 1994). On the other hand, many empirical studies have uncovered significant non-linearities in exchange rates [see, for example, Hsieh (1989), Kugler and Lenz (1993) and Cecen and Erkal (1996)].

Recent breakthroughs pertaining to non-linear dynamics have helped empirical investigation of non-linear models and have fuelled the research agenda in this area, dramatically increasing the number of approaches to forecasting exchange rates in an attempt to guard against the failure to benefit from non-linearities due to the incorrect choice of functional form: chaotic dynamics (De Grauwe and Vansanten, 1990), turbulent motions (Ghahghaie et al., 1996) neural networks (Refenes, 1993), GARCH models (Bolleslev, Chou and Kroner, 1992), self-exciting threshold autoregressive models (Kräger and Kugler, 1993), etc.

One of these approaches to forecasting is the non-parametric, nearest neighbour (NN hereafter) forecasting technique. There are several ways of computing such forecasts. The basic idea behind these predictors, inspired in the literature on forecasting in non-linear dynamical systems, is that pieces of time series sometime in the past might have a resemblance to pieces in the future. In order to generate predictions, similar patterns of behaviour are located in terms of nearest neighbours. The time evolution of these nearest neighbours is exploited to yield the desired prediction. Therefore, the procedure only uses information local to the points to be predicted and does not try to fit a function to the whole time series at once [see, e. g., Bajo-Rubio Fernández-Rodríguez and Sosvilla-Rivero (1992a,b) and Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1997)].

The NN approach has been used in forecasting several financial time series, but the results are rather inconclusive. For exchange rates, while Diebold and Nason (1990), Hsieh (1991), Meese and Rose (1991) or Mizrach (1992) concluded that there was little gain in predictive accuracy over a simple random walk, more promising results were reported in Bajo-Rubio *et al.* (1992a,b), Lisi and Medio (1997), Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999), or Cao and Soofi (1999). Applications of NN to other financial variables include stock markets [Fernández-Rodríguez, Sosvilla-Rivero and García-Artiles (1997, 1999) and Andrada-Félix *et al.* (2001)] and interest rates (Bajo-Rubio *et al.,* 2001).

In this paper we will survey our contribution to this programme of research on predictability in financial markets, addressing the question of whether NN prediction methods can improve out-of-sample forecasting in foreign exchange markets, with special reference to the European Monetary System (EMS). The paper is organised as follows. The NN predictors are presented in Section 2. In Section 3 we discuss the EMS. In Section 4 the forecast accuracy of the NN predictors is assessed from a statistical point of view, while in Section 5 we use the NN predictions in a simple trading strategy to assess the economic significance of predictable patterns in the foreign exchange market. Some concluding remarks are provided in Section 6.

2. The NN approach to non-linear forecasting

2.1. Literature review

NN forecasting of time series has a dual and recent history. On one hand, in the statistical literature there were some attempts to develop a wide class of non-parametric regression estimates (see Stone, 1977 for a review) that culminated with the work of Cleveland (1979) on robust locally weighed regression, perhaps the first form of a NN predictor. On the other hand, independently and about ten years later than the statistics community, in the dynamical system literature some authors began to use NN predictors for nonlinear time series, associated with studies on deterministic chaos. Seminal papers by Farmer and Sidorowich (1987), Casdagli (1989) and Sugihara and May (1990) gave an important impulse to NN predictions.

2.1.1 The non-parametric regression approach

Accordingly to the non-parametric regression approach, if y_t (for $t = 1,...,T$) are observations of a dependent variable and $x_t = (x_{1t},...,x_{pt})$ (for $t = 1,...,T$) is a vector of explanatory independent variables, the model of local regression is given by

$$y_t = g(x_t) + \varepsilon_t, \tag{1}$$

where $g(x)$ is an unknown smooth function to be estimated from the data, and ε_t are i.i.d. normal random variables with mean 0 and variance σ^2. Note the sharp difference of the former assumption with respect to the standard paradigm of parametric regression: instead of a function from a specific parametric class, we just have an assumption about smoothness.

A smoother $\hat{g}(x_t)$ is an estimate of the conditional mean of the dependent variable, which is obtained by local averaging of the dependent variable at any given neighbourhood of the explanatory variables.

The estimator is constructed through the following steps:

1. Let W be a weighted function with the following properties:

   a. $W(x) > 0$ for $|x| < 1$;

   b. $W(-x) = W(x)$;

   c. $W(x)$ is a non-increasing function for $x \geq 0$; and

d. $W(x)=0$ for $|x| \geq 1$.

An example of function verifying the properties is

$W(x)=(1-|x|^3)^3$, for $|x|<1$, and, $W(x)=0$, for $|x| \geq 1$.

2. Let x be a point where we are going to compute $\hat{g}(x)$. Let f be a smoothing constant such that $0<f \leq 1$ and let be $q=int(fT)$, where *int(.)* rounds down to the nearest integer. Then rank the $x_t's$ by Euclidean distance from x. Call these $x_{t_1}, x_{t_2}, \ldots, x_{t_k}$, so that x_{t_1} is closest to x, x_{t_2} is second closest to x and so on.

3. Let $d(x,x_{t_k})=\left[\sum_{j=1}^{p}(x_j - x_{t_{k_j}})\right]^{1/2}$ be the Euclidean distance from x to its *k*-th closest neighbour.

4. Now a set of weights for the points (x_t, y_t) is defined by

$$w_t(x)=W\left(\frac{d(x_t,x)}{d(x,x_k)}\right) \tag{2}$$

5. The value of the regression surface at x is then computed using ordinary least squares as:

$$\hat{y}=\hat{g}(x)=\hat{\alpha}_0+\hat{\alpha}_1 x_{t_1}+\ldots+\hat{\alpha}_k x_{t_k}, \tag{3}$$

where $\hat{\alpha}_i$ minimise the expression

$$\sum_{t=1}^{T} w_t(x)\left(y_t-\alpha_0-\alpha_1 x_{t_1}-\ldots\ldots-\alpha_k x_{t_k}\right)^2 \tag{4}$$

2.1.2 The dynamical system approach

A second approach to NN predictors is related to the forecast in chaotic time series. A key result in this approach is given by Taken’s (1981) theorem.

Following Takens (1981), we say that the time series of real numbers x_t (for $t=0,\ldots,\infty$) has a smoothly deterministic explanation if there exists a system (h,F,a_0) such that $h:R^n \to R$ and $F:R^n \to R^n$ are smooth (i.e., twice differentiable almost everywhere) and

$$x_t = h(a_t), \; a_t = F(a_{t-1}), \; t = 1,2,... \; (a_0 \text{ is given}). \tag{5}$$

Note that the relationship $a_t = F(a_{t-1})$ is an unknown law of motion involving the unobserved *n* vector of state variables. Nevertheless, Takens suggests that we can learn about the underlying state variable a_t from "embeddings" of the observed x_t's in R^m for m sufficiently large. To that end, we compute a *m-history* as a vector of *m* observations sampled from the original time series[1]:

$$x_t^m = \left(x_t, x_{t-1}, x_{t-2}, ..., x_{t-(m-1)}\right), \quad t = m, m+1, ..., T, \tag{6}$$

with *m* referred to as the *embedding dimension* and the *m*-dimensional space R^m is referred to as the *reconstructed phase space* of time series. The Takens embedding theorem states that, if *m* is sufficiently large ($m \geq 2n+1$) the dynamics of the unknown law $a_t = F(a_{t-1})$ is equivalent to a dynamics of x_t^m. This means that the sequence of *m*-histories in this case can mimic the true data generation process. Therefore, the dynamics of the *m*-histories x_t^m provide a correct topological picture of the unknown chaotic dynamics.

More formally, if the time series has a smoothly deterministic explanations Takens showed that, for a embedding dimension *m* sufficiently large, there exists a function

$$\tilde{F} : R^m \to R^m \tag{7}$$

such that $x_{t+1}^m = \tilde{F}\left(x_t^m\right)$ and this map has the same dynamic behaviour as that of the original unknown system in the sense of topological equivalence.

The Takens embedding theorem provides a new geometrical interpretation for the NN predictors. In this case, the NN method can be seen as selecting some geometric segments in the past of the time series, similar to the last segment available before the observation we want to forecast [see Farmer and Sidorowich (1987)]. So *m*-histories with a similar dynamic behaviour are detected in the series and employed afterward in the prediction of the next term at the end of the series. This term is computed as some adequate average of the actually observed terms next to the *m*-history involved.

2.2. The approach in this paper

In our empirical implementations, we have followed the dynamical system approach. In what follows we shall succinctly described this approach for both

[1] Note that we are assuming daily data, and therefore the *m* observations are sampled from the original time series at intervals of one unit.

the univariate and multivariate cases follows [see Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999) for a more detailed account].

2.2.1 Univariate NN predictors

A forecast of a variable using information from its own time series can be produced through the following steps:

1) We first transform the scalar series x_t (t=1,...,T) into a series of m-dimensional vectors, x_t^m, t=m,...,T:

$$x_t^m = \left(x_t, x_{t-1}, x_{t-2}, \ldots, x_{t-(m-1)}\right)$$

with m referred to as the *embedding dimension*. These m-dimensional vectors are often called *m-histories*.

2) As a second step, we select the k m-histories

$$x_{t_1}^m, x_{t_2}^m, \ldots, x_{t_k}^m \tag{8}$$

most similar to the last available vector

$$x_T^m = \left(x_T, x_{T-1}, x_{T-2}, \ldots, x_{T-(m-1)}\right)$$

where k=$int(\lambda T)$ (0<λ<1), with $int(\cdot)$ standing for the integer value of the argument in brackets, and where the subscript t_r (r=1,2,...,k) is used to denote each of the k chosen m-histories.

To that end, we look for the closest k vectors in the phase space R^m, in the sense that they maximise the function:

$$\rho\left(x_t^m, x_T^m\right) \tag{9}$$

(i.e., looking for the highest serial correlation of all m-histories, x_t^m, with the last one, x_T^m)[2].

3) Finally, to obtain a predictor for x_{T+1}, we consider the following local regression model:

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[2] Alternatively, we could have established nearest neighbours to x_T^m by looking for the closest k points x_t^m that minimise the following functions $\left\| x_t^m - x_T^m \right\|$ or $1 - cos(x_t^m, x_T^m)$ (i.e., looking, respectively for the minimum distance or the lowset angle) (see Fernández-Rodríguez, 1992, for the relationship between the nearest neighbours obtained by using these three functions).

$$\hat{x}_{T+1} = \hat{\alpha}_0 x_T + \hat{\alpha}_1 x_{T-1} + ... + \hat{\alpha}_{m-1} x_{T-(m-1)} + \hat{\alpha}_m \quad (10)$$

whose coefficients have been fitted by a linear regression of x_{t_r+1} on $x_{t_r}^m = \left(x_{t_r}, x_{t_r-1}, x_{t_r-2}, ..., x_{t_r-(m-1)}\right), \; (r = 1, ..., k)$. Therefore, the $\hat{\alpha}_i$ are the values of α_i that minimise

$$\sum_{r=1}^{k} \left(x_{t_{r+1}} - \alpha_0 x_{t_r} - \alpha_1 x_{t_r-1} - ..... - \alpha_{m-1} x_{t_r-(m-1)} - \alpha_m\right)^2 \quad (11)$$

Therefore, in order to obtain a predictor for x_{T+1}, we look for similar behavioural patterns (or occurring analogues) in the time series x_t by means of choosing *k m*-histories whose resemblance to the last available *m*-history could help us to infer the likely future evolution of x_t from instant time *T* onward.

Alternatively, one can consider the weighted least squares algorithm in order to estimate the regression coefficients[3], that is minimising the expression

$$\sum_{r=1}^{k} w\left(x_{t_r}\right)\left(x_{t_{r+1}} - \alpha_0 x_{t_r} - \alpha_1 x_{t_r-1} - ..... - \alpha_{m-1} x_{t_r-(m-1)} - \alpha_m\right)^2 \quad (12)$$

where the weights $w\left(x_{t_r}\right)$ are assigned using the so-called "tri-cube" weight function:

$$w(x_{t_r}) = W\left(\frac{\left\| x_{t_r}^m - x_T^m \right\|}{\sum_{r=1}^{k} \left\| x_{t_r}^m - x_T^m \right\|} \right)$$

where $\|.\|$ is the Euclidean distance of the last *m*-history x_T^m to the *r*-th nearest neighbour $x_{t_r}^m$ and, for any *u*, $W(u) = (1 - u^3)^3$ for $0 \le u \le 1$ and $W(u) = 0$ otherwise.

Although this last weighting scheme has the theoretical attraction of allowing us to use an arbitrary amount of NN, weighted algorithms may be numerically less stable than unweighted algorithms. Wayland *et al.* (1994) showed that, in presence of noise, unweighted algorithms tend to yield superior results, because when calculating the parameters of the local regression, it is necessary to invert an X'X matrix whose rows are highly collinear since they are formed by the selected nearest neighbours that are all very similar to x_T^m.

[3] The fit of equation (10) may be also done by using more sophisticated tools as singular-valued decomposition of Broomhead and King (1986) [see Lisi and Medio (1997) for an application of this method predicting exchange rates].

When the X'X matrix is close to singular, the parameter estimates are numerically highly unstable. Numerical experiments reveal that this problem is specially acute for the weighted X'X matrix, because it is much more closer to singular than the unweighted matrix. We therefore focus our attention only on unweighted algorithms (see also Jaditz and Sayers, 1998).

2.2.2 Multivariate NN predictors

Hitherto we have consider a univariate version of the NN predictor. However, when we have a set of simultaneous time series, the NN prediction can be extended to a multivariate case using the *simultaneous nearest neighbour* (SNN hereafter) predictors in an attempt to also consider the information content in other related time series. To simplify, let us consider a set of two time series:

$$x_t \ (t=1,...,T),\ y_t \ (t=1,...,T)$$

We are interested in making predictions of an observation of one of these series (e.g., x_{T+1}), by simultaneously considering nearest neighbours in both series. To this end, we embed each of these series in the vectorial space R^{2m}, paying attention to the following vector:

$$\left(x_t^m, y_t^m\right) \in R^m x R^m$$

which gives us the last available *m*-history for each time series.

In order to establish nearest neighbours to the last *m*-histories $\left(x_t^m, y_t^m\right)$ we can look for the closest *k* points that maximise the function:

$$\rho\left(x_t^m, x_T^m\right) + \rho\left(y_t^m, y_T^m\right) \tag{13}$$

The predictor for x_{T+1} and y_{T+1} can be obtained from a linear autoregressive predictor with varying coefficients estimated by ordinary least squares:

$$\hat{x}_{T+1} = \hat{\alpha}_0 x_T + \hat{\alpha}_1 x_{T-1} + ... + \hat{\alpha}_{m-1} x_{T-(m-1)} + \hat{\alpha}_m \tag{14}$$

$$\hat{y}_{T+1} = \hat{\beta}_0 y_T + \hat{\beta}_1 y_{T-1} + ... + \hat{\beta}_{m-1} y_{T-(m-1)} + \hat{\beta}_m$$

The difference between this SNN predictor (14) and the NN predictor (10) is that now the nearest neighbours are established using criteria in which information on both series is used. Therefore, to obtain a predictor for x_{T+1}, we look now for similar behavioural patterns not only in the time series x_t, but also in the related time series y_t, whose resemblance to the last available *m*-history of x_t could be used to help us when projecting the likely future evolution of x_t from instant time T onward.

As in the NN case, the weighted least squares algorithm could also be used to estimate the regression coefficients, but the same arguments on numerical instability apply.

2.3 Selecting the key parameters in (S)NN predictions

Note that the NN predictors (both the univariate and the multivariate cases) depend on the values of two key parameters: the embedding dimension m and the number of closest k points in the phase space R^m.

The Takens embedding theorem provides no information upon these key parameters. The only limitation is to take $m \geq 2n+1$ where n is the dimension of compact manifold that have to be embedding in the R^m Euclidean space (Takens, 1981).

Even though these parameters are crucial in the determination of predicting map $\tilde{F}(.)$ in (7) since they are essential for the accuracy of the predictions, in empirical applications they are usually selected "based on a combination of judgement and of trial and error" (Cleveland, 1993, p.96). Nevertheless, some methods have been proposed in the literature to choose these key parameters.

2.3.1 Selection of m

A capital issue in (S)NN prediction is the appropriated choice of the embedding dimension for the time series. Among the basic methods proposed in the literature, we can name the following:

- Methods based in dynamical properties of the strange attractor, which rely upon computing some invariant of the chaotic motion as correlation dimension or Lyapunov exponents (e.g. Grassberger and Procaccia, 1983). However, one could argued that these methods are not relevant in financial time series where the chaotic nature of the series it is not so clear.
- Singular value decomposition methods (Broomhead and King ,1986).
- False neighbourhoods methods (Kennel *et al.*, 1992) and averaged false neighbours methods (Cao, 1997), and
- The zero order approximation method developed by Cao *et al.* (1998). This method consist on finding the embedding dimension by minimising

the average absolute one-step ahead prediction errors, using a zero-order approximation predictive model.

2.3.2 Selection of k

In chaotic time series, the ideal number of nearest neighbours depends on the complexity of the attractor and the number of observations in the time series. For experimental and financial data, there are a great diversity of approaches for selecting the number of nearest neighbours.

To fit the equation (10), k must be at least as big as $m+1$. When $k=m+1$, this method is equivalent to linear interpolation and the least-square problem has a unique solution. In practice, in order to ensure stability of the solution and to reduce the forecasting error, it would be an advantage to take $k>m+1$. Casdagli (1992) recommend to take $k = 2(m+1)$.

In general there is not a uniform guideline for selecting the number of nearest neighbours when forecasting a time series. It is usually claimed that standard forecasting errors diminish with the number of neighbours [see Casdagli (1992) and Jaditz and Sayers (1998)].

Casdagli (1992) have proposed a forecasting algorithm based on study the behaviour of the normalised root mean square error (RMSE) of forecasts when *m* and *k* vary. Although Casdagli uses this algorithm in order to distinguish low dimensional chaotic behaviour from linear stochastic behaviour by comparing the accuracy of short-term forecasts, this procedure may be used for selecting the embedding dimension and number of nearest neighbours. The procedure implies the selection of a "fitting set" $F_t=\{x_t : 1<t\le T\}$ and a “prediction set” $P_t=\{x_t : T<t\le N-1\}$, for some $T<N$. We choose the parameters *k* and *m* in such a way that they minimise the normalised RMSE of forecasts in the testing set:

$$RMSE_m(k)=\frac{\sqrt{\sum_{t=1}^{T}\left(\hat{x}_t(k)-x_t\right)^2}}{\sigma} \quad (15)$$

where σ is the standard deviation of the time series in the fitting set[4]. In order to prevent the over-fitting problem, we can assign a cost to the introduction of each additional unity of embedding dimension using the well-known Akaike (1973) information criterion.

In the empirical applications reviewed in Sections 4 and 5, m and k are chosen according to Casdagli’s (1991, 1992) algorithm.

[4] See Härdle (1990) for a discussion of this "cross-validation" method.

3. The European Monetary System

The European Monetary System (EMS) was initially planned as an agreement to reduce exchange rate volatility for a Europe in transition to a closer economic integration. Following its inception in March 1979, a group of European countries linked their exchange rates through formal participation in the Exchange Rate Mechanism (ERM). The ERM was an adjustable peg system in which each currency had a central rate expressed in the European Currency Unit (ECU), predecessor of the Euro. These central rates determined a grid of bilateral central rates *vis-à-vis* all other participating currencies, and defined a band around these central rates within which the exchange rates could fluctuate freely. In order to keep these bilateral rates within the margins, the participating countries were obliged to intervene in the foreign exchange market if a currency approached the limits of its band, for which some special credit facilities were established[5]. If the participating countries decided by mutual agreement that a particular parity could not be defended, realignments of the central rates were permitted.

The ERM was the most prominent example of a target zone exchange-rate system. In the 1990s an extensive literature appeared, building on the seminal paper by Krugman (1991), which studied the behaviour of exchange rates in target zones. The main result of the simple target zone model was that, with perfect credibility, the zone would exert a stabilising effect (the so-called "honeymoon" effect), reducing the exchange rate sensitivity to a given change in "fundamentals". However, in a target zone with credibility problems, expectations of future interventions would tend to destabilise the exchange rate, making it less stable than the underlying fundamentals (Bertola and Caballero, 1992). Therefore, credibility (i.e., the degree of confidence that the economic agents assign to the announcements made by policymakers) becomes a key variable. In the context of an exchange-rate target zone, credibility refers to the perception of economic agents with respect to the commitment to maintain the exchange rate around a central parity. Therefore, the possibility for the official authorities to change the central parity could be anticipated by the economic agents, triggering expectations of future changes in the exchange rate that can act as a destabilising element of the system. Nevertheless, in both cases (with perfect credibility or with credibility problems), the target zone model implies that the exchange rate is a non-linear function of the fundamentals, therefore providing a further theoretical foundation for the exchange rate to exhibit a non-linear data generating process.

[5] Following Basel/Nyborg agreement of September 1987, central banks were also empowered to intervene within margins before limits were reached.

The fact that the currencies participating in the ERM are institutionally related makes them a natural candidate among financial time series for the application of the SNN methodology in order to examine if forecast accuracy can be obtained by considering the information content of other related exchange rates. In other words, the use of SNN predictions in this context can be seen as an attempt to incorporate structural information into the non-parametric analysis.

4. Assessing the forecast accuracy of the NN predictors

A first contribution applying the NN methods is Bajo-Rubio, Fernández-Rodríguez and Sosvilla-Rivero (1992a), where daily data for the Spanish peseta-US dollar, spot and one- and three-month forward exchange rates, during the period January 1985-May 1991, were used in the empirical application; a deeper discussion of the methods used in the paper can be found in Bajo-Rubio, Fernández-Rodríguez and Sosvilla-Rivero (1992b). In that paper, several predictors based on the NN methodology were computed, and their performance was compared with that of a simple random walk, by calculating their respective forecasting errors, as measured by the root mean square error (RMSE). In general, the non-linear predictors outperformed the random walk in all cases for the forward rates, whereas for the spot rate this only occurred in four over nine cases.

The objective of Sosvilla-Rivero, Fernández-Rodríguez and Bajo-Rubio (1999) was to compute an indicator of volatility, defined as the (absolute value of the) forecast error, derived from the NN predictors, weighted by the standard deviation of the original series. This indicator was applied to six EMS currencies experiencing different evolutions after the crisis that affected the system after the summer of 1992, that lead to the broadening of the fluctuation bands in August 1993: two of them "temporarily" leaving the ERM (Italian lira and Pound sterling), two others forced to devalue (Spanish peseta and Portuguese escudo), and the remaining two not devaluing (French franc and Dutch guilder), with the sample period running from January 1974 to April 1995. The volatility indicators showed an initial low degree of exchange rate volatility, with a sudden increase from September 1992 onwards. Then, volatility remained high for the currencies that abandoned the ERM; however, for the rest of the currencies, the broadening of the bands after August 1993 would have led to a decrease in volatility to levels comparable with those prevailing before the crisis. These results were interpreted, rather than in terms of an unexpected loss of credibility, as being a consequence of the fragility of the EMS in a world of very high international capital mobility, which became evident with the problems associated with German reunification at the end of 1989 and the effects of self-fulfilling speculative attacks (Eichengreen and Wyplosz, 1993).

The above papers compute predictions for each variable using information from the own series [i.e., in a univariate (NN) context]. In a later contribution, Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999) applied the SNN predictors, therefore using the information content of a wider set of time series (nine currencies participating in the ERM) in an attempt to incorporate structural information into the non-parametric analysis. The data set includes daily observations of nine exchange rates (Belgian franc, Danish crown, Portuguese escudo, French franc, Dutch guilder, Irish pound, Italian lira, Spanish peseta, and Pound sterling) covering the period January 1978-December 1994. Given the central role of Germany in the European Union [see Bajo-Rubio *et al.* (2001)] they are expressed *vis-à-vis* the Deutschemark

As mentioned above, the predictors depend on the values of the embedding dimension m and the number of closest k points in the phase space R^m. Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999) chose them according to Casdagli´s (1991) algorithm, obtaining an embedding dimension $m=6$ and a number of nearest neighbourhood points k around a 2% of the sample. On the other hand, it is necessary to chose related exchange-rate series in order to establish occurring analogues in the SNN predictor. These authors consider three groups of currencies according to the credibility with respect to the commitment to maintain the exchange rate around the central parity (see Ledesma-Rodríguez *et al*., 2001). It is interesting to note that these groups roughly correspond to those found in Jacquemin and Sapir (1996), applying principal component and cluster analyses to a wide set of structural and macroeconomic indicators, to form a homogeneous group of countries. Moreover, these groups are basically the same that those found in Sosvilla-Rivero and Maroto (2001) when examining the duration of the central parities in the ERM.

After founding evidence of non-linear dependence in the series using the well-known BDS test statistic (see Brock *et al*., 1996), hence supporting their approach to forecasting, Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999) evaluated the forecasting performance for the whole sample using Theil's U statistic, a summary statistic that is based on standard symmetric loss function:

$$U = \frac{\sqrt{\sum_{t=T+1}^{T+N} (x_t - \hat{x}_t)^2}}{\sqrt{\sum_{t=T+1}^{T+N} (x_t - x_{t-1})^2}} \tag{16}$$

where x_t is the actual value and $\hat{x}_t$ the forecast value. Note that we have defined the U statistic is defined as the ratio of the RMSE of forecasts from a particular predictor to the RMSE of the naive random walk forecast. Therefore, a value of U less than one indicates better performance than the random walk specification.

Table 1 shows the forecasting performance, relative to the random walk, from both the SNN predictors and the traditional (linear) ARIMA(1,1,0) model. As can be seen, the U statistics were, for the SNN predictors, above one only in three of the nine cases, suggesting that the non-linear predictors marginally outperformed the random walk, despite the forecasting period being very long and heterogeneous, with the best SNN predictor presenting an improvement of 18.9%. We can also see that the predictors from an ARIMA(1,1,0) model always show U statistics below one, the best one showing an improvement of 12.2% out of sample. Nevertheless, in six out of nine cases, the SNN predictors show lower U statistics than the ARIMA(1,1,0) model.

Table 1: Forecast accuracy (U statistic)[a]

	SNN predictor	ARIMA (1,1,0) predictor
BFR[b]	0.984	0.995
DKR[b]	0.939	0.954
ESC[c]	1.016	0.997
FF[b]	0.908	0.952
HFL[b]	0.811	0.878
IRL[d]	1.014	0.997
LIT[c]	0.973	0.981
PTA[c]	0.995	0.999
UKL[d]	1.022	0.999

Notes: [a] BFR, DKR, ESC, FF, HFL, IRL, LIT, PTA and UKL denote, respectively, the Belgian franc, the Danish crown, the Portuguese escudo, the French franc, the Dutch guilder, the Irish pound, the Italian lira, the Spanish peseta and the British sterling.
[b] Time series used in establishing occurring analogues in the SNN predictor: BFR, DKR, FF, and HFL.
[c] Time series used in establishing occurring analogues in the SNN predictor: ESC, LIT and PTA.
[d] Time series used in establishing occurring analogues in the SNN predictor: IRL and UKL.
Source: Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999).

As Boothe and Glassman (1987) observed, a relevant test of forecasting performance relative to a random walk is the accuracy in predicting the direction of exchange rate movements. This is because obtaining the sign right in the prediction matters in markets with low transaction costs, like foreign exchange markets. To explore this possibility, Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix. (1999) also computed the percentage of correct predictions as a further test of forecasting accuracy. Table 2 reports the results, which are rather promising. In eight of nine cases, the SNN predictors show a value higher than 50%, clearly outperforming the random walk directional forecast[6]. Note also that in all the cases, the predictors from an ARIMA(1,1,0) model show a value greater than 50%. Finally, in seven out of the nine cases, the SNN predictors offer higher values than the ARIMA model.

Table 2: Directional forecast[a]

	SNN predictor	ARIMA (1,1,0) predictor
BFR[b]	63.23	52.19
DKR[b]	67.35	63.73
ESC[c]	55.95	53.86
FF[b]	67.43	62.02
HFL[b]	69.19	65.34
IRL[d]	57.73	51.77
LIT[c]	57.25	54.83
PTA[c]	50.64	55.33
UKL[d]	47.58	51.21

Notes: [a] Percentage of correct forecast direction.
[b] Time series used in establishing occurring analogues in the SNN predictor: BFR, DKR, FF, and HFL.
[c] Time series used in establishing occurring analogues in the SNN predictor: ESC, LIT and PTA.
[d] Time series used in establishing occurring analogues in the SNN predictor: IRL and UKL.

Source: Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999).

To formally assess the forecast accuracy of the local predictors, Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix. (1999) used the test proposed by Diebold and Mariano (1995) on the (corrected) value of the differential between two forecasting errors. Let $\hat{x}_{1_t}$ and $\hat{x}_{2_t}$ denote alternative forecasts of the variable $x_t, t = T+1,....,T+N$, where N is the number of forecast. Let e_{1_t} and e_{2_t} denote the corresponding forecast errors ($x_t - \hat{x}_{1_t}$ and $x_t - \hat{x}_{2_t}$, respectively). The Diebold-Mariano test is given by:

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi \hat{f}_d(0)}{N}}} \tag{17}$$

where $\bar{d}$ is an average (over N observations) of a general loss differential function and $\hat{f}_d(0)$ is a consistent estimate of the spectral density of the loss

[6] The value of 50% is the ususl benchmark. However, the results must be treated with caution, since numbers of positive changes do not necessarily coincide with the number of negative changes.

differential at frequency zero. Diebold and Mariano show that the DM statistic is asymptotically distributed as standard normal under the null of equal forecast accuracy. Therefore, a significant and positive (negative) value for *DM* would indicate a significant difference between the two forecasting errors, which would mean a better accuracy of the $\hat{x}_{2_t}$ ($\hat{x}_{1_t}$) predictor. In line with a large literature [see, e. g., Mark (1995) and Killian (1999)], the loss differential function considered was the difference between the absolute forecast error.

As can be seen in Table 3, the null hypothesis of no difference in the loss function can be rejected in eight out of the nine cases when assessing forecast accuracy of the random walk *versus* the SNN predictor. The results suggest that the SNN predictor outperforms the random walk at the 1% significance level for the Belgian franc, Danish crown, French franc and Dutch guilder, while the random walk presents superior performance relative to the NN local predictor for the Portuguese escudo and Pound sterling at the 1% significance level.

When comparing the predictors from an ARIMA model with the SNN predictors, the null hypothesis of equal absolute error is rejected in four of the nine cases. The results indicate that the SNN predictor outperforms the ARIMA model at the 1% significance level for the Belgian franc, French franc and Dutch guilder, while the ARIMA model presents superior performance relative to the SNN predictor for the Pound sterling at the 1% significance level.

Table 3: The Diebold Mariano test statistic[a]

	SNN predictor	ARIMA (1,1,0) predictor
BFR[b]	5.11*	4.16*
DKR[b]	12.12*	0.95
ESC[c]	-2.17**	-0.54
FF[b]	14.35*	11.22*
HFL[b]	12.83*	7.72*
IRL[d]	2.97*	-0.21
LIT[c]	3.19*	1.53
PTA[c]	1.14	0.38
UKL[d]	-3.14*	-3.16*

Notes: [a] * and ** denote significance at the 1% and 5% levels, respectively.
[b] Time series used in establishing occurring analogues in the SNN predictor: BFR, DKR, FF, and HFL.
[c] Time series used in stablishing occurring analogues in the SNN predictor: ESC, LIT and PTA.
[d] Time series used in establishing occurring analogues in the SNN predictor: IRL and UKL.
Source: Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999).

Next, Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999) computed the Pesaran-Timmerman non-parametric test proportion of correctly predicted signs. Let $z_{t+l} = 1$ if $(x_{t+l}\hat{x}_{t+l} > 0)$ and $z_{t+l} = 0$ otherwise. Let $P_x = Pr(x_{t+l} > 0)$ $P_{\hat{x}} = Pr(\hat{x}_{t+l} > 0)$ and

$$\hat{P} = \frac{1}{N}\sum_{t=T+1}^{T+N} z_{t+l} \tag{18}$$

be the percentage of correct sign predictions. Denoting the *ex ante* probability that the sing will correctly be predicted as $P*$, then

$$P^* = Pr(z_{t+l} = 1) = Pr(x_{t+l}.\hat{x}_{t+l} > 0) = P_x \cdot P_{\hat{x}} + (1 - P_x)\cdot(1 - P_{\hat{x}})$$

Pesaran and Timmerman (1992) show that

$$PT_l = \frac{\hat{P} - P^*}{\sqrt{v\hat{a}r(\hat{P}) - v\hat{a}r(\hat{P}^*)}} \tag{19}$$

is asymptotically distributed as a standard normal variate under the null hypothesis of independence between corresponding actual and forecast values, where $\hat{P}*$ denotes estimated values,

$$v\hat{a}r(\hat{P}) = \frac{1}{T}\hat{P}^* \cdot (1 - \hat{P}^*)$$

and

$$v\hat{a}r(\hat{P}^*) = \left[\frac{1}{T}(2\cdot\hat{P}_x - 1)^2 \cdot \hat{P}_{\hat{x}} \cdot (1 - \hat{P}_{\hat{x}}) + \frac{1}{T}\cdot(2\cdot\hat{P}_{\hat{x}} - 1)^2 \cdot \hat{P}_x \cdot (1 - \hat{P}_x) + \frac{4}{T^2}\hat{P}_x \cdot \hat{P}_{\hat{x}}(1 - \hat{P}_x)\cdot(1 - \hat{P}_{\hat{x}})\right]$$

Notice that l denotes the number of periods ahead in the prediction, being $l=1$ in this case of daily forecasts.

As shown in Table 4, the results indicated that the probability of correctly predicting the sign of change is higher for the SNN predictors than the ARIMA case.

Table 4: The Pesaran Timmerman Test statistic[a]

	SNN predictor	ARIMA (1,1,0) predictor
BFR[b]	11.33*	3.45*
DKR[b]	13.10*	10.83*
ESC[c]	2.80*	1.54
FF[b]	13.01*	9.08*
HFL[b]	14.97*	11.89*
IRL[d]	6.91*	3.01*
LIT[c]	6.31*	3.79*
PTA[c]	0.53	1.15
UKL[d]	0.18	-0.37

Notes: [a] * denotes significance at the 1% level.
[b] Time series used in establishing occurring analogues in the SNN predictor: BFR, DKR, FF, and HFL.
[c] Time series used in stablishing occurring analogues in the SNN predictor: ESC, LIT and PTA.
[d] Time series used in establishing occurring analogues in the SNN predictor: IRL and UKL.

Source: Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999).

Overall, the evidence presented in Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999) suggested that, when predicting exchange-rate time series, some forecast accuracy can be gained by considering the information content of other related exchange rates through SNN predictors.

5. Assessing the economic value of the NN predictors

As pointed out by Satchell and Timmermann (1995), standard forecasting criteria are not necessarily particularly suited for assessing the economic value of predictions in financial time series. To assess the economic significance of predictable patterns detected in the EMS series, it is necessary to explicitly consider how investors may exploit the computed (S)NN predictions as a trading rule.

In fact, an important line of research has evaluated the relevance of technical analysis in foreign exchange market. As is well known, technical analysis involves the use of charts of financial price movements to infer the likely course of future prices and therefore construct forecasts and determine trading decisions. Technical analysis is used by the vast majority of active foreign exchange participants, who are mostly interested in the short term movements of the currencies [see, e. g., Allen and Taylor (1990), Taylor and Allen (1992), Lui and Mole (1998) and Oberlechner (2001)]. A considerable amount of work has provided support for the view that technical trading rules are capable of producing valuable economic signals in foreign exchange markets. Dooley and Shafer (1983) presented some of the earliest evidence suggesting that technical trading rules might be detecting changes in conditional mean returns in foreign exchange rate series, generating profits in excess of the buy-and-hold strategy. Later, Sweeney (1986) also found results supportive of the profitability of similar rules, whereas Taylor (1992) documented similar evidence for even more extensive sets of rules and data series. Moreover, LeBaron (1992) and Levich and Thomas (1993) followed the methodology of Brock *et al.* (1992) and used bootstrap simulations to demonstrate the statistical significance of the technical trading rules against several parametric null models of exchange rates, while Lee and Mathur (1996) showed that only in two of the six cases examined the trading rules were marginally profitable. More recently, Szakmary and Mathur (1997), LeBaron (1999), Sosvilla-Rivero, Andrada-Félix and Fernández-Rodríguez. (1999) and Neely and Weller (2001) discovered that excess returns from extrapolative technical trading rules in foreign exchange markets are high during periods of central bank intervention. Finally, Neely and Weller (1998), using genetic programming methodology, found that *ex ante* trading rules generate significant excess returns in three of the four cases considered, whereas and Gençay (1999), using both feed-forward networks and NN regressions, found statistically significant forecast improvements for the current returns over the random walk model of foreign exchange returns.

Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (2002) transformed the forecast from NN and SNN predictors into a simple technical trading strategy in which positive returns are executed as long positions and negative returns are executed as short positions. As shown in Clyde and Osler (1997), the non-linear NN forecasting technique can be viewed as a generalisation of graphical methods ("heads and shoulders", "rounded tops and bottoms", "flags, pennants and wedges", etcetera) widely used by practitioners. The profitability of this NN strategy is evaluated against the traditional moving average trading rules. Furthermore, unlike previous empirical evidence, when evaluating trading performance, they considered both interest rates and transaction cost, as well as a wider set of profitability indicators than those usually examined.

Regarding the MA trading rules, if E_t is the daily exchange rate, the moving average $m_t(n)$ when it is defined as:

$$m_t(n) = \frac{1}{n}\sum_{i=0}^{n} E_{t-i}$$

where n is the length of the moving average. Very simple technical trading rules consider the signal $s_t(n_1, n_2)$ defined by

$$s_t(n_1, n_2) = m_t(n_1) - m_t(n_2)$$

where $n_1 < n_2$, and where n_1 and n_2 are the short and the long moving averages, respectively. When $s_t(n_1, n_2)$ exceeds zero, the short term moving average exceeds the long term moving average to a certain extent, and a "buy" signal is generated. Conversely, when $s_t(n_1, n_2)$ is negative, and a "sell" signal is given. As can be seen, the moving average rule is essentially a trend following system because when prices are rising (falling), the short-period average tends to have larger (lower) values than the long-period average, signalling a long (short) position. In particular, the following popular moving average rules were evaluated: [1,50], [1,150], [1, 200], [5, 50], [5,150] and [5,200], where the first number in each pair indicates the days in the short period (n_1) and the second number shows the days in the long period (n_2) (see, e. g., Brock *et al.*, 1992).

In order to assess the economic significance of a trading strategy, we can consider the estimated total return of such a strategy:

$$R_t = \sum_{t=1}^{N} z_t \cdot r_t \tag{20}$$

where r_t is the return from a foreign currency position over the period *(t, t+1)*, z_t is a variable interpreted as the recommended position which takes either a value of -1 (for a short position) or +1 (for a long position), and *N* is the number of observations.

Given that trading in the spot foreign exchange market requires consideration of interest rates when evaluating trading performance, we can use overnight interest rates to compute r_t as follows:

$$r_t = ln(E_{t+1}) - ln(E_t) - \left(ln(1 + i_t) - ln\left(1 + i_t^*\right)\right)$$

where *E* represents the spot exchange rate expressed vis-à-vis the Deustche mark, *i* is the domestic daily interest rate and *i** is the German daily interest rate.

On the other hand, assuming that transaction costs of *c*% are paid each time a new position (i. e., from short to long or from long to short) is established, the net return of the technical trading strategy is given by:

$$R_n = \sum_{t=1}^{N} z_t \cdot r_t + nrt \cdot \{ln(1 - c) - ln(1 + c)\} \tag{21}$$

where *nrt* is the number of round-trip trades. The last term in the equation reflects the transaction costs that are assumed to be paid whenever a new position is established. Following Levich and Thomas (1993) and Osler and Chang (1995), transaction costs of 0.05% were considered.

Tables 5 and 6 report the estimated mean annual total and net return, respectively, as well as its associated *t*-statistics [with corrections for serial correlation and heteroskedasticity, see Hamilton (1994)] as a measure of the statistical significance of the results. As can be seen in Table 5, only in 1 of the 9 cases considered (Pound sterling) the MA trading rules outperformed the trading strategy based on a non-linear (NN or SNN) predictor. Nevertheless, the *t*-statistic suggests that in this case the null hypothesis that the mean annual return is equal to zero cannot be rejected. From Table 5, we can also see that in 6 out of 9 cases (Belgian franc, Danish crown, Portuguese escudo, Dutch guilder, Italian lira and Spanish peseta), the mean annual total returns when using the SNN predictors are the highest, while for the cases of the French franc and Irish pound, it is the trading system based on the NN predictor that yields the highest mean annual total returns. It should be noted that for the later cases all *t*-statistics reject the null hypothesis that the mean annual total returns are equal to zero. On the other hand, and as can be seen from Table 6, the mean annual net return from the non-linear trading rule dominates that from the MA trading rules in all the cases, except for the Pound sterling case. In 6 such cases (Belgian franc, Danish crown, Portuguese escudo, Dutch guilder, Italian lira and Spanish peseta), the trading system based on the SNN predictor give the highest mean annual net returns, whereas in the cases of the French franc and Irish pound, the highest mean annual net return is obtained when using the NN predictors. Finally, note that in 7 of the 9 exchange rate examined (Belgian franc, Danish crown, French franc, Dutch guilder, Irish pound, Italian lira and Pound sterling) the results are statistically significant (at least at the 5% level) as indicated by the t-statistics.

Besides the total and net returns, Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (2002) also took into account other two profitability measures: the ideal profit and the Sharpe ratio. We consider a version of the ideal profit that measures the net returns of the trading system against a perfect predictor and is calculated by:

$$R_I = \frac{\sum_{t=1}^{N} \hat{z}_t \cdot r_t + nrt \cdot \{ln(1-c) - ln(1+c)\}}{\sum_{t=1}^{N} |r_t| + nrt \cdot \{ln(1-c) - ln(1+c)\}} \qquad (22)$$

TABLE.5: Mean annual total return[ab]

Exchange rates	Non-linear trading rules		Linear trading rules					
	NN predictor	SNN predictor	[1,50]	[1,150]	[1,200]	[5,50]	[5,150]	[5,200]
BFR[c]	0.0631 (5.9861^{x})	0.0670 (7.8334^{x})	-0.0437 (-5.0054^{x})	-0.0282 (-3.5077^{x})	-0.0298 (-3.6200^{x})	-0.0035 (-0.4854)	-0.0071 (-0.9706)	-0.0057 (-0.7981)
DKR[c]	0.2614 (11.7277^{x})	0.2822 (11.5596^{x})	-0.1734 (-6.1999^{x})	-0.1312 (-5.0620^{x})	-0.1367 (-5.4208^{x})	-0.0516 (-2.7768^{x})	-0.0323 (-2.0516^{xx})	-0.0406 (-2.5656^{xx})
ESC[d]	0.0323 (0.6578)	0.0891 (2.2230^{x})	0.0317 (0.7455)	0.0385 (1.1145)	0.0381 (1.0782)	0.0673 (1.7961^{xxx})	0.0329 (0.9258)	0.0391 (1.1333)
FF[c]	0.2448 (13.9101^{x})	0.2427 (13.6570^{x})	-0.1115 (-5.5922^{x})	-0.0745 (-4.2358^{x})	-0.0657 (-4.4343^{x})	-0.0085 (-0.6926)	-0.0204 (-1.8477)	-0.0211 (-1.8279^{xxx})
HFL[c]	0.1403 (17.6453^{x})	0.1471 (18.1531^{x})	-0.1131 (-11.6381^{x})	-0.0984 (-8.8918^{x})	-0.0845 (-8.3545)	-0.0324 (-5.9658^{x})	-0.0237 (-4.7103^{x})	-0.0222 (-4.6854^{x})
IRL[e]	0.0527 (3.1079^{x})	0.0305 (1.8536^{xxx})	-0.0378 (-2.3394^{xx})	-0.0242 (-1.6387)	-0.0308 (-1.9767^{xx})	-0.0065 (-0.4002)	-0.0094 (-0.6427)	-0.0099 (-0.7225)
LIT[d]	0.2681 (6.2970^{x})	0.3348 (9.1434^{x})	-0.1664 (-3.5986^{x})	-0.0449 (-1.2011)	-0.0425 (-1.1516)	-0.0332 (-0.8943)	-0.0032 (-0.0922)	-0.0008 (-0.0244)
PTA[d]	0.0405 (0.9921)	0.0677 (2.0879^{xx})	0.0626 (1.7485^{xxx})	0.0193 (0.4924)	0.0473 (1.2748)	0.0483 (1.2894)	0.0368 (0.9768)	0.0349 (0.9290)
UKL[e]	-0.0663 (-1.4951)	-0.0079 (-0.2688)	0.0561 (1.4503)	0.0076 (0.1952)	0.0139 (0.3612)	0.0352 (0.9124)	0.0111 (0.2851)	0.0076 (0.1943)

Notes: [a] Returns generated by each forecasting method over the forecast sample, before transaction fees are taken into account [see equation (20) in the text].

[b] t- statistics (corrected for serial correlation and heteroskedasticity) in parenthesis: ^{x}, ^{xx}, and ^{xxx} denote significance at the 1%, 5% and 10% levels, respectively.

[c] Time series used in establishing occurring analogues in the SNN predictor: BFR, DKR, FF and HFL.

[d] Time series used in establishing occurring analogues in the SNN predictor: ESC, LIT and PTA.

[e] Time series used in establishing occurring analogues in the SNN predictor: IRL and UKL.

Source: Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (2002).

TABLE 6: Mean annual net return[ab]

Exchange rates	Non-linear trading rules		Linear trading rules					
	NN predictor	SNN predictor	[1,50]	[1,150]	[1,200]	[5,50]	[5,150]	[5,200]
BFR[c]	-0.0619 (-5.8806[x])	-0.0580 (-6.7737[x])	-0.1687 (-19.3104)	-0.1532 (-19.0678[x])	-0.1548 (-18.8113[x])	-0.1285 (-17.8063[x])	-0.1321 (-17.9502[x])	-0.1307 (-18.3446[x])
DKR[c]	0.1364 (6.1189[x])	0.1572 (6.4393[x])	-0.2984 (-10.6697[x])	-0.2562 (-9.8857[x])	-0.2617 (-10.3778[x])	-0.1766 (-9.5070[x])	-0.1573 (-9.9967[x])	-0.1656 (-104596[x])
ESC[d]	-0.0927 (-1.8860[xxx])	-0.0359 (-0.8958)	-0.0934 (-2.1990[xx])	-0.0865 (-2.5006[xx])	-0.0869 (-2.4568[xx])	-0.0577 (-1.5379)	-0.0921 (-2.5919[x])	-0.0859 (-2.4909[xx])
FF[c]	0.1198 (6.8069[x])	0.1177 (6.6234[x])	-0.2365 (-11.8608[x])	-0.1995 (-11.3436[x])	-0.1907 (-12.8718[x])	-0.1335 (-10.8287[x])	-0.1454 (-13.1503[x])	-0.1461 (-12.6432[x)]
HFL[c]	0.0153 (1.9241[xxx])	0.0221 (2.7314[x])	-0.2381 (-24.4976[x])	-0.2234 (-20.1831)	-0.2095 (-20.7122[x])	-0.1574 (-28.9952[x])	-0.1487 (-29.5279[x])	-0.1472 (-31.0157[x])
IRL[e]	-0.0723 (-4.2672[x])	-0.0945 (-5.7488[x])	-0.1628 (-10.0772[x])	-0.1492 (-10.0892[x])	-0.1558 (-9.9999[x])	-0.1315 (-8.1011[x])	-0.1344 (-9.2135[x])	-0.1349 (-9.8889[x])
LIT[d]	0.1431 (3.3608[x])	0.2098 (5.7296[x])	-0.2914 (-6.3014[x])	-0.1699 (-4.5449[x])	-0.1675 (-4.5404[x])	-0.1582 (-4.2609[x])	-0.1282 (-3.7280[x])	-0.1258 (-3.6508[x])
PTA[d]	-0.0845 (-2.0689[xx])	-0.0573 (-1.7653[xxx])	-0.0624 (-1.7425[xxx])	-0.1057 (-2.6970[x])	-0.0777 (-2.0937[xx])	-0.0767 (-2.0490[xx])	-0.0883 (-2.3459[xx])	-0.0901 (-2.3998[xx])
UKL[e]	-0.1913 (-4.3125[x])	-0.1329 (-4.4966[x])	-0.0690 (-1.7841[xxx])	-0.1174 (-3.0194[x]	-0.1111 (-2.8958[x])	-0.0898 (-2.3284[xx])	-0.1139 (-2.9346[x])	-0.1174 (-2.9940[x])

Notes:
- [a] Returns generated by each forecasting method over the forecast sample, after transaction fees are taken into account [see equation (12) in the text].
- [b] t- statistics (corrected for serial correlation and heteroskedasticity) in parenthesis: [x], [xx], and [xxx] denote significance at the 1%, 5% and 10% levels, respectively.
- [c] Time series used in establishing occurring analogues in the SNN predictor: BFR, DKR, FF and HFL.
- [d] Time series used in establishing occurring analogues in the SNN predictor: ESC, LIT and PTA.
- [e] Time series used in establishing occurring analogues in the SNN predictor: IRL and UKL.

Source: Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (2002).

According to this equation, $R_I = 1$ if the indicator variable takes the correct trading position for all observations in the sample. If all trade positions are wrong, then the value of this measure is $R_I = -1$. An $R_I = 0$ value is considered as a benchmark to evaluate the performance of an investment strategy. Regarding the Sharpe ratio (Sharpe, 1966), it is simply the annual mean net return of the trading strategy divided by its standard deviation:

$$S_R = \frac{\mu_{R_n}}{\sigma_{R_n}} \tag{23}$$

As can be seen, the higher the Sharpe ratio, the higher the mean annual net return and the lower the volatility. The results for these additional profitability measures are reported in Tables 7 and 8.

As shown in Table 7, the MA trading rules always render negative values for the ideal profit ratio. In contrast, the trading strategy based on the non-linear (NN or SNN) predictors renders positive values in 4 out of the 9 cases considered (Danish crown, French franc, Dutch guilder and Italian lira). In all cases, except for the Pound sterling, the use of non-linear predictors to generate sell/buy signals produces higher values of this profitability measure than those from the MA trading rules. As for the Sharpe ratio, a similar pattern emerges from Table 8: the trading strategy based on the non-linear predictors yields the highest Sharpe ratios in 8 out of the 9 cases (Belgian franc, Danish crown, Portuguese escudo, French franc, Dutch guilder, Irish pound, Italian lira and Spanish peseta), while for the Pound sterling the highest (less negative) value is obtained from the MA trading rules.

TABLE 7: Ideal profit ratio[a]

Exchange rates	Non-linear trading rules		Linear trading rules					
	NN predictor	SNN predictor	[1,50]	[1,150]	[1,200]	[5,50]	[5,150]	[5,200]
BFR[b]	-0.5479	-0.5127	-1.4925	-1.3548	-1.3691	-1.1366	-1.1688	-1.1559
DKR[b]	0.2388	0.2753	-0.5225	-0.4486	-0.4582	-0.3092	-0.2754	-0.2900
ESC[c]	-0.1793	-0.0695	-0.1807	-0.1673	-0.1681	-0.1116	-0.1782	-0.1663
FF[b]	0.2599	0.2554	-0.5132	-0.4328	-0.4137	-0.2897	-0.3155	-0.3170
HFL[b]	0.1356	0.1963	-2.1112	-1.9809	-1.8574	-1.3953	-1.3186	-1.3054
IRL[d]	-0.2839	-0.3710	-0.6389	-0.5858	-0.6115	-0.5161	-0.5274	-0.5293
LIT[c]	0.1538	0.2255	-0.3133	-0.1827	-0.1801	-0.1701	-0.1378	-0.1353
PTA[c]	-0.1359	-0.0921	-0.1004	-0.1701	-0.125	-0.1234	-0.142	-0.1450
UKL[d]	-0.2932	-0.2037	-0.1057	-0.1799	-0.1703	-0.1376	-0.1746	-0.1799

Notes: [a] The ideal profit measures the returns of the trading system against a perfect predictor [see equation (22) in the text].
[b] Time series used in establishing occurring analogues in the SNN predictor: BFR, DKR, FF and HFL.
[c] Time series used in establishing occurring analogues in the SNN predictor: ESC, LIT and PTA.
[d] Time series used in establishing occurring analogues in the SNN predictor: IRL and UKL.

Source: Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (2002).

TABLE 8: Sharpe ratio[a]

Exchange rates	Non-linear trading rules		Linear trading rules					
	NN predictor	SNN predictor	[1,50]	[1,150]	[1,200]	[5,50]	[5,150]	[5,200]
BFR[b]	-0.1583	-0.1484	-0.4285	-0.3875	-0.3917	-0.3243	-0.3335	-0.3298
DKR[b]	0.1635	0.1900	-0.3483	-0.2964	-0.3031	-0.2023	-0.1800	-0.1897
ESC[c]	-0.0880	-0.0342	-0.0886	-0.0821	-0.0825	-0.0548	-0.0874	-0.0816
FF[b]	0.1835	0.1801	-0.3437	-0.2878	-0.2748	-0.1916	-0.2087	-0.2097
HFL[b]	0.0555	0.0814	-0.8277	-0.7685	-0.7046	-0.5119	-0.4825	-0.4775
IRL[d]	-0.1045	-0.1363	-0.2349	-0.2152	-0.2247	-0.1895	-0.1936	-0.1943
LIT[c]	0.0941	0.1392	-0.1898	-0.1101	-0.1085	-0.1025	-0.0830	-0.0815
PTA[c]	-0.0732	-0.0497	-0.0541	-0.0916	-0.0673	-0.0665	-0.0765	-0.0781
UKL[d]	-0.1646	-0.1142	-0.0593	-0.1008	-0.0955	-0.0772	-0.0979	-0.1008

Notes: [a] The Sharpe ratio is obtained dividing the mean return of the trading system by its standard deviation [see equation (23) in the text].
[b] Time series used in establishing occurring analogues in the SNN predictor: BFR, DKR, FF and HFL.
[c] Time series used in establishing occurring analogues in the SNN predictor: ESC, LIT and PTA.
[d] Time series used in establishing occurring analogues in the SNN predictor: IRL and UKL.
Source: Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (2002).

6. Concluding remarks

There is a growing consensus in the literature on the fact that foreign exchange markets have become increasingly complex and therefore less amenable to forecasting over time when using standard linear models. As an alternative approach, recent advances in both analytic and computational methods have suggested new lines of research exploring the possible non-linear dynamics in foreign exchange rates. This approach has been supported by a burgeoning literature providing a theoretical justification for the presence of non-linearities in foreign exchange rates, while many empirically papers have found non-linearities in foreign exchange markets.

The purpose of this paper has been to contribute to the debate on the relevance of these non-linear forecasting methods for high-frequency data presenting the results of applying the nearest neighbour (NN) approach to forecasting to nine currencies participating in the exchange rate mechanism of the EMS.

The NN forecasting approach relies on the premise that short-term predictions can be made based on past patterns of the time series, therefore circumventing the need to specify an explicit econometric model to represent the time series. Therefore, it is philosophically very different from the Box-Jenkins methodology. In contrast to the traditional Box-Jenkins (linear) models (see Box and Jenkins, 1976), where extrapolation of past values into the immediate future is based on correlation among lagged observations and error terms, NN methods select relevant prior observations based on their levels and geometric trajectories, not their location in time.

The research reviewed in the paper illustrate the relevance of the approach, both from the point of view of the statistical forecasting accuracy and from the point of view of the economic value as derived from it use as a trading rule. The results have showed the potential usefulness of NN predictors not only as useful tool when forecast daily exchange data but also as a technical trading rule capable of producing valuable economic signals in foreign exchange markets. Therefore, further consideration of NN predictors could be a fruitful enterprise.

There are a number of directions that extensions from the present research might take. Two avenues that seem worthy of further research are:

i) the combination of NN methods with other non-linear forecasting methods such as artificial neural networks.

ii) the application of NN methods to other currencies, such as the Dollar/Euro or the Dollar/Yen exchange rates.

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