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NEARSHORE CIRCULATIONS UNDER SEA BREEZE CONDITIONS AND WAVE-CURRENT INTERACTIONS IN THE SURF ZONE

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13. ABSTRACT

Numerical models for nearshore circulation patterns in the surf zone have been developed and applied to an observed condition subjected to a sea breeze environment. Bottom topography and input waves were derived from observed data to predict surf zone circulation as a function of time of day. It was found that many features observed in the surf zone were modeled but wave-current interactions are known to be important.

Wave-current interactions were modeled for shallow water assuming a two-dimensional motion which included rip current and longshore current components. The refraction effects caused by even small currents produce major changes in the wave induced driving forces in the surf zone which leads to the prediction of entirely different rip-current patterns when wave-current interactions are considered. Numerical results are presented and a discussion of the numerical techniques is included.

A review of water wave theories to include mass transport, vorticity and current was made for a vertical section in shallow water of constant depth.

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NEARSHORE CIRCULATIONS UNDER SEA BREEZE CONDITIONS AND WAVE-CURRENT INTERACTIONS IN THE SURF ZONE

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PREFACE AND ACKNOWLEDGEMENTS

The research effort described herein was performed at Tetra Tech during the past twelve months. In particular Chapter 2, "Circulation Under Sea Breeze Condition," was the contribution of Dr. Choule J. Sonu, Chapter 3, "Wave-Current Interaction Over Variabe Topography," was contributed by Dr. Edward K. Noda and Chapter 4, "Wave and Current," was the research effort of Dr. Viviane C. Rupert.

This study was sponsored by the Office of Naval Research, Geography Programs under Contract No. N00014-69-C-0107. The authors wish to acknowledge the contribution by discussion and encouragement of Dr. Bernard LeMehaute, Vice President, and many other members of Tetra Tech's staff. In particular the authors would like to thank Mr. Wayne Wier for considerable effort in computer programming and computer runs and plots.

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ABSTRACT

Numerical models for nearshore circulation patterns in the surf zone have been developed and applied to an observed condition subjected to a sea breeze environment. Bottom topography and input waves were derived from observed data to predict surf zone circulation as a function of time of day. It was found that many features observed in the surf zone were modeled but wave-current interactions are known to be important.

Wave-current interactions were modeled for shallow water assuming a two-dimensional motion which included rip current and longshore current components. The refraction effects caused by even small currents produce major changes in the wave induced driving forces in the surf zone which leads to the prediction of entirely different rip-current patterns when wave-current interactions are considered. Numerical results are presented and a discussior of the numerical techniques is included.

A review of water wave theories to include mass transport, vorticity and current was made for a vertical section in shallow water of constant depth.

1. INTRODUCTION AND SUMMARY

1.1 INTRODUCTION

In the nearshore area waves arriving from offshore continuously bring in momentum, energy and mass. Since the shoreline provides a fixed boundary the momentum and energy fluxes are dissipated in the surf zone. Most of the energy is converted to turbulence in the breaker zone but enough is left to supply a nearshore current system and move loose bed material. The momentum brought in by the waves will drive the nearshore current system and cause a local set-up or set-down of the mean water level.

Over the past four and one half years a series of analytic developments have been attempted to model some of the more pertinent characteristics of the surf zone. The work is conveniently divided into two broad groups: statistical and deterministic. In the statistical approach (Collins, 1971 and Collins and Wier, 1969) a relatively simple beach topography was assumed and the effects on wave height statistics computed together with longshore currents and wave set-up. More recently, (Noda, 1972, 1973) a deterministic approach employing monochromatic waves and much more complex beach topographies has been explored.

A number of sub-tasks have been investigated during the past year. The three specific sub-tasks receiving intensive investigation include:

a) the application of wave-induced circulation computations on beaches having rythmic topography.

b) the development of a numerical model for wave induced circulation which includes wave-current interaction.

c) the analytical investigation of wave, current, and vorticity interaction.

The following sections of this report present details of the work performed. The subsections below pre ent a brief review of some earlier work and a summary of the work completed during the past year.

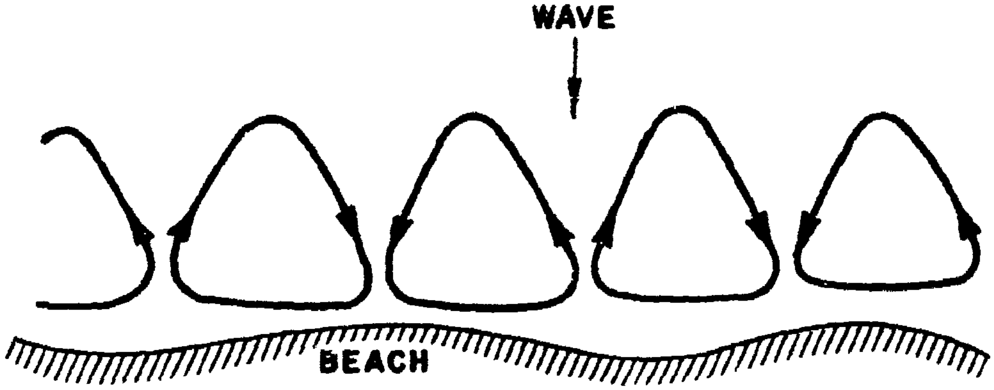
1.2 REVIEW OF EARLIER WORK

In a recent study by Noda (1972, 1973) the solution to wave-induced nearshore circulation due to the incoming wave-bottom topography interaction was studied. Results for both normal and oblique wave incidence were presented and while the results generally agreed with recent field data from Sonu (1972), Figures 1.1, 1.2 and 1.3, the numerically derived circulation velocities tended to be larger than measured in the field, Figures 1.4, 1.5 and 1.6.

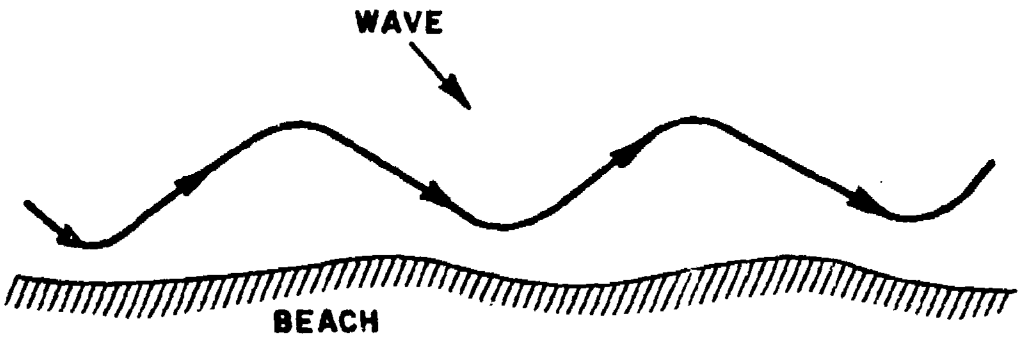
Several possible reasons for the apparent discrepancies can be postulated including;

a) neglect of wave-current interaction
) bottom fiction approximation
c) choice of wave breaking criteria
d) assumption of monochromatic waves which consequently all break at the same location
e) over-estimates of the incoming wave height or errors in direction
f) approximations made in the analytical developments.

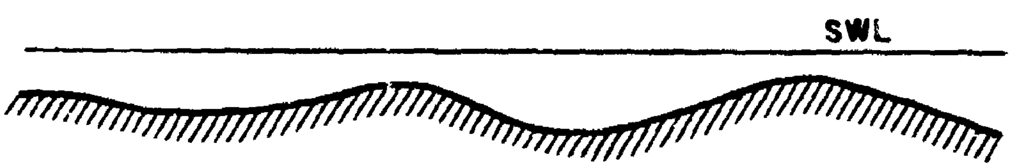
Of the possible reasons for differences the assumptions made to comply with (c), (d) and (e) produce similiar effects in that the nearshore circulation pattern is strongly influenced by the wave breaker location. The dominant driving forces are produced by the radiation stresses induced by breaking waves. Also, because of this it must be realized that even relatively weak currents change the breaker location and characteristics hence, the importance of wave-current interaction is a major one. Therefore reason (a) is of prime importance.



1. CIRCULATION UNDER NORMAL WAVE INCIDENCE



2. MEANDER UNDER OBLIQUE WAVE INCIDENCE



3. LONGSHORE SURF ZONE PROFILE

Figure 1.5 Dependence of Current Patterns on Wave Incidence Angles and Surf Zone Topography [From Sonu, 1972]

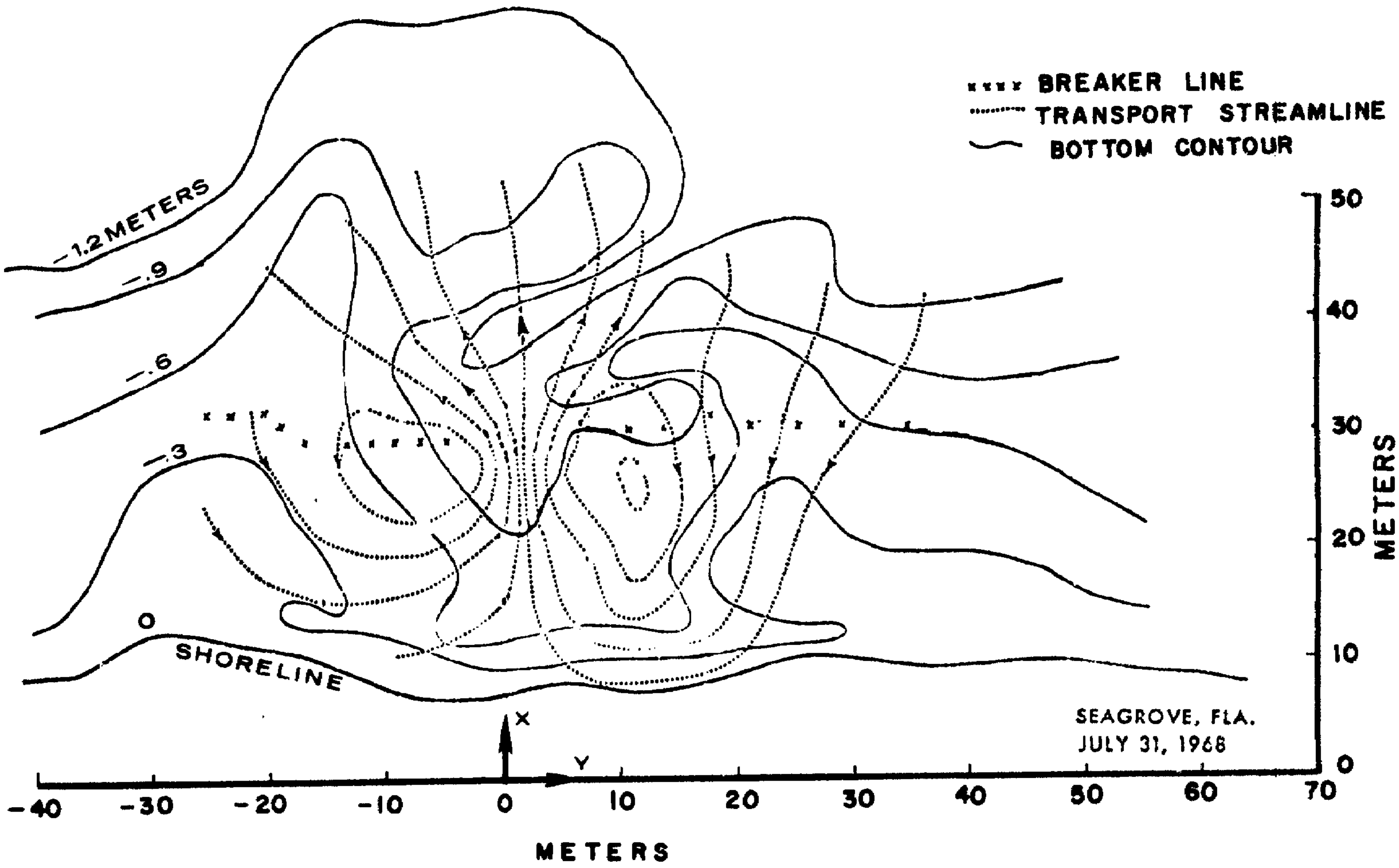


Figure 1.2: Distribution of Streamlines and Depth Contours in a Circulation Cell Under Normal Wave Incidence [From Sonu, 1972]

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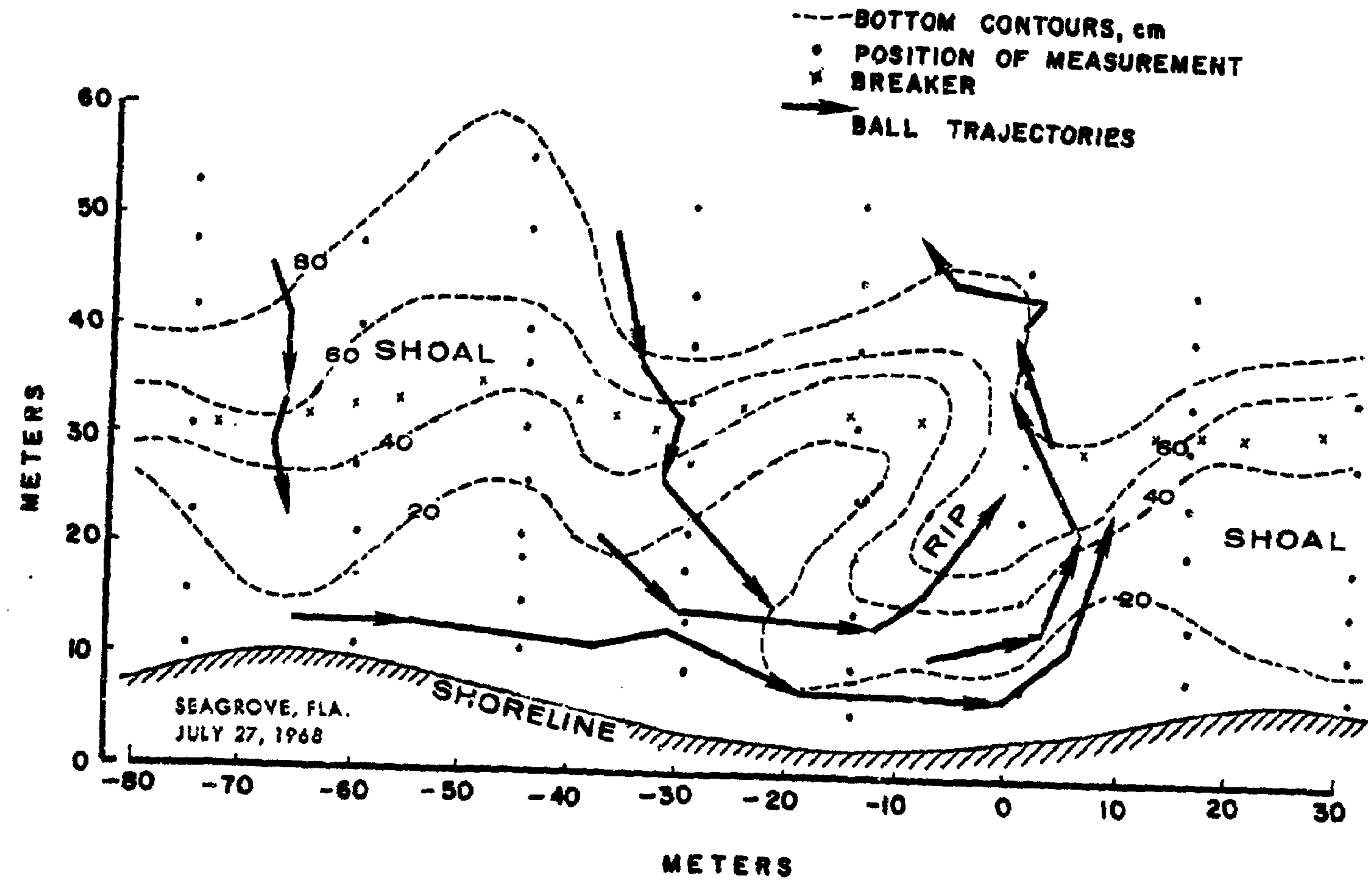

Figure 1.3: Ball Trajectories and Bottom Contours for Oblique Wave Incidence
[From Sonu, 1972]

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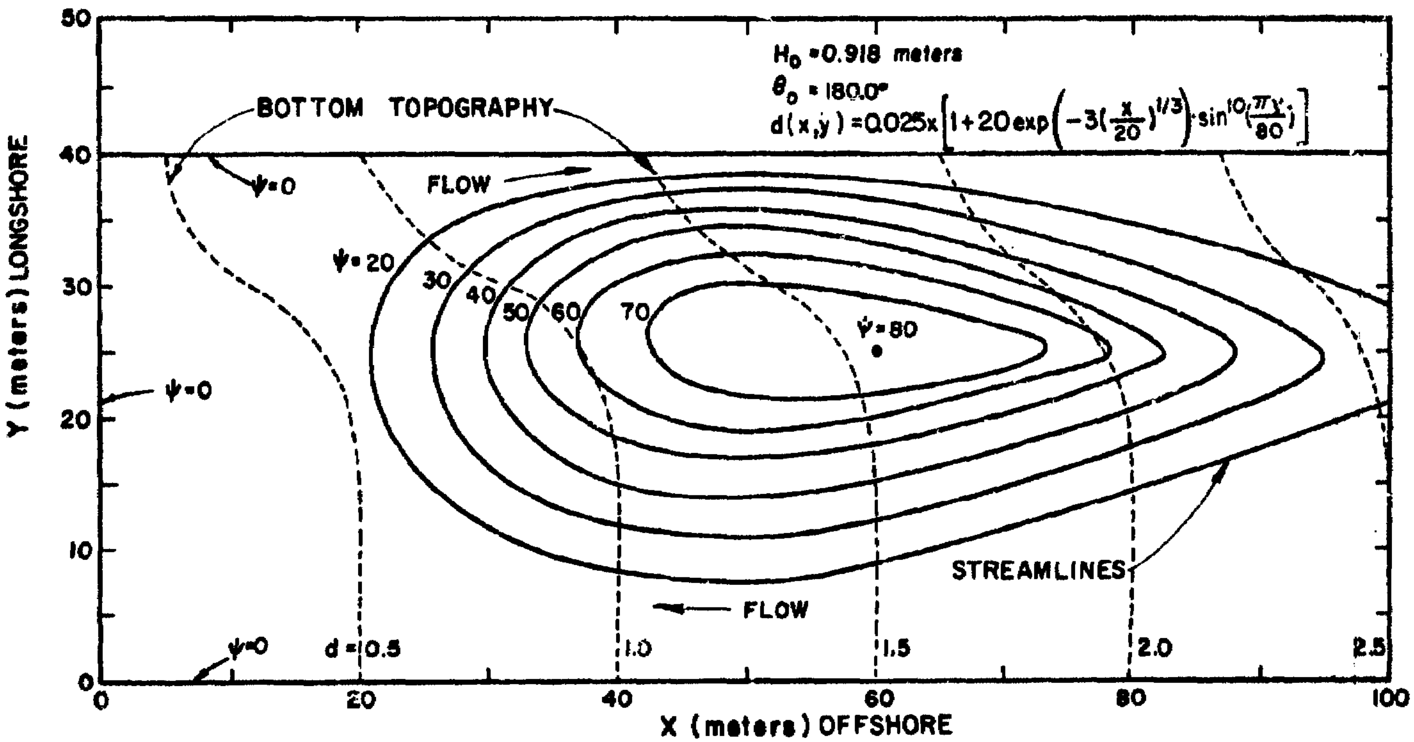

Figure 1.4: Streamline Flow Due to Normal Wave Incidence [Noda (1973)]

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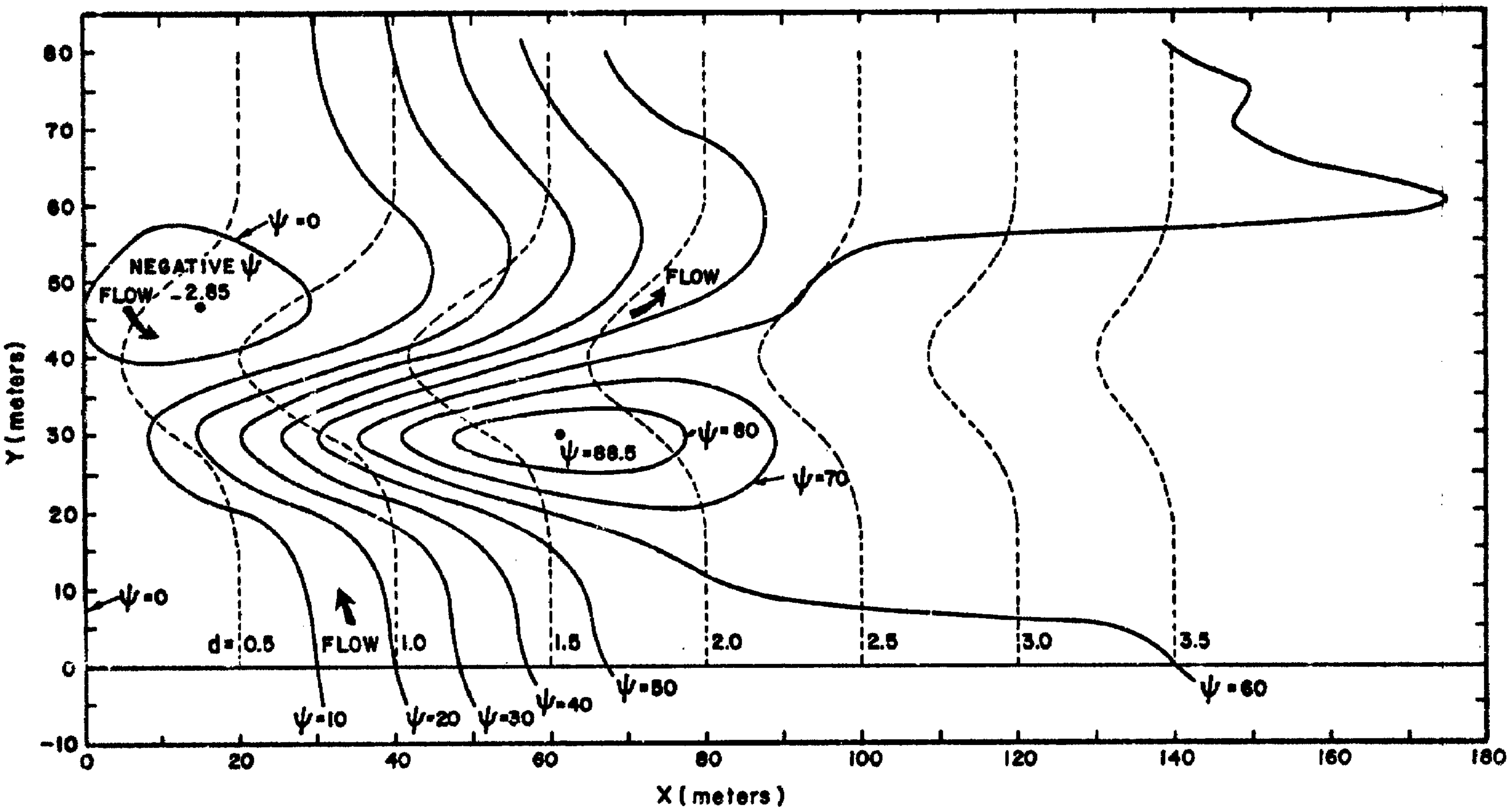

Figure 1.5 Stream Function Solution for Oblique Wave Incidence [Noda (1973)]

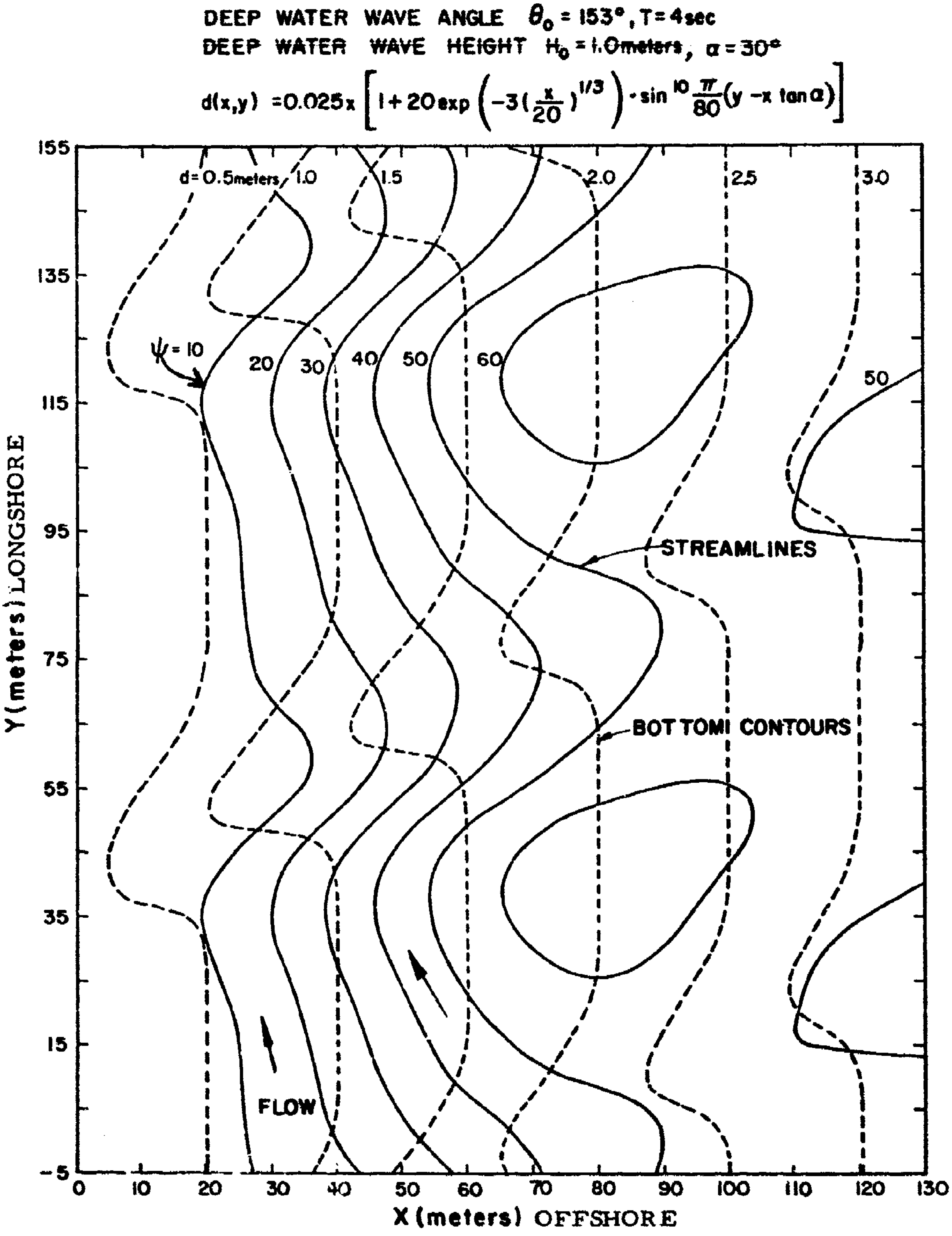

Figure 1.6: Meandering Rip-Current Flow Over a Skewed Channel [Noda (1973)]

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It is believed that bottom friction effects are important and hence are never ignored in these investigations. However, approximations are necessary in order to yield a tractible numerical model. It is apparent that considerable room for improvement exists in the approximations generally made.

The numerical techniques employed were found to influence the predicted currents and a certain effort was needed to refine the earlier more crude methods. New approximations include the choice of more realistic beach topographic model and the procedures to solve the governing differential equations.

The following subsection (1. 3) of this report presents a brief summary of the technical work which has been oriented towards improvements and refinements in the nearshore circulation models. More complete details are presented in Sections 2 and 3.

1.3 SUMMARY

1.3.1 Wave Induced Circulation Over a Rhythmic Topography

The data obtained by the Coastal Studies Institute (SALIS by Sonu et. al., 1973) has been investigated and attempts have been made to model the wave induced circulation using the procedures developed by Noda (1972) in an earlier phase of the work.

The steps required are:

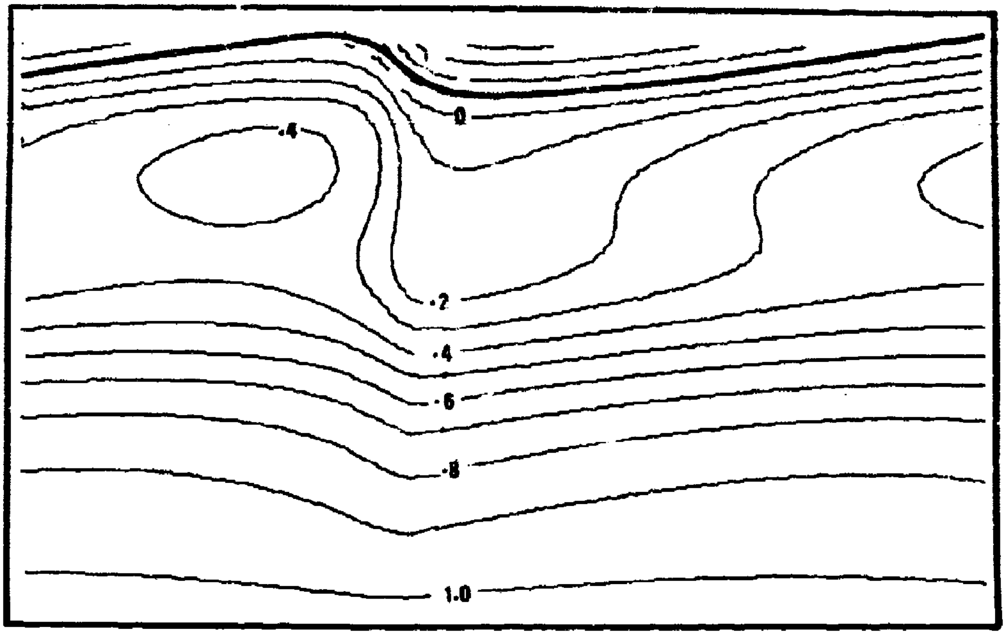
a) topographic model

b) wave height-wave direction field

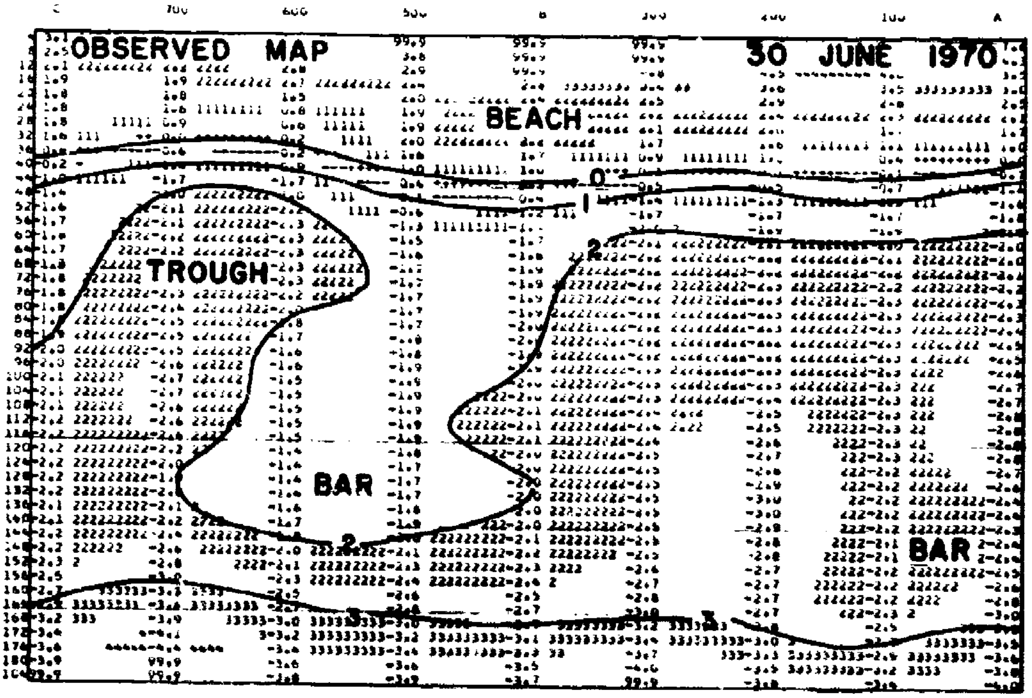
c) solution of the momentum equations

d) comparison with observed data

The field data was used to provide (a) and the offshore wave conditions. The topographic model was developed by choosing empirical functions and constants to closely simulate the observed topography. Fig. 1.7 presents a sample of the topographic simulation as used. More



(a) Analytical Simulation



(b) Observed Nearshore Topography (Davis & Fox, 1971)

Figure 1. 7: Comparison of Analytical Model and an Observed Nearshore Topography

details will be included in Section 2.

The wave height-direction field has been computed using the ray equations (see Noda, 1972) but some variations on the numerical techniques were found which yielded significant improvements in accuracy and speed of computation. The revised technique is based on a relaxation procedure rather than the previous method of marching along rays. Wave heights and directions are computed directly at the required grid points and yield considerable savings in computing time otherwise spent on interpolation subroutines. The techniques are fully detailed in Section 3.

The solution of the momentum equations for the wave induced circulation follows the procedures outlined in Technical Report No. 3 (Noda, 1972).

One apparent difference between the numerical results and typical observed data is that the numerical predictions show a too strong concentrating effect of the circulation near the transverse bar (or shoal) and a number of localized eddies in the nearshore area. Possible reasons for such effects have been indicated in Section 1.2 and these are also discussed in Section 2.4. However, in spite of some obvious shortcomings it is apparent that the approach and results outlined in Section 2 have yielded a reasonable modeling capability for many features of the nearshore circulation over a rhythmic topography.

1.3.2 Wave and Current Interaction Over a Rhythmic Topography

It is apparent from even a casual glance at a beach that incoming waves interact strongly with the local currents which are themselves induced by the waves. Section 3 presents a detailed analysis of this problem. The interaction produces two dominant effects:

the currents change the wave refraction and also change the breaker locations.

These wave-induced nearshore circulation patterns were derived assuming no wave-current interaction. Thus interest was developed to determine if the effects of wave-current interaction produced significant changes in the nearshore circulation patterns as observed in prototype. Section 3 deals with the theoretical development and numerical computations of this process as affecting the circulation within the nearshore zone.

The initial computational steps followed those given in Section 1.3.1, i.e. topography-wave height and direction field-solution of the momentum equations. Then the resulting circulation velocities were considered as an existing mean current system, and waves were again propagated into this system and a new wave height, direction and nearshore circulation pattern obtained. It was hoped that continual interaction would finally yield an "equilibrium" solution including wave-current interaction. However, attempts to directly impose this derived mean current system in an interaction process with the incoming waves lead to failures of the technique because the mean current system derived for no wave-current interaction was too large. Hence, conditions arise where the local waves were no longer able to propagate into some areas.

An attempt was made to take only a percentage of the initially derived current system and then include interaction with the waves. This was partly successful and indicated that considerations of wave-current interaction were extremely important. Some major changes in the computed nearshore circulation system were produced. Section 3.3.3 presents some of the results.

It has been demonstrated that wave-current interactions are of major importance in determining the nearshore circulation but a complete solution was not possible because of the occurrance of regions in the nearshore zone where waves could no longer propogate when opposed by a current. Two major conclusion are deduced,

a) the current-wave interaction theory needs further development to include the special case of "no wave propagation" in some regions;

b) the nearshore circulation system is basically a non-steady pulsating system in that the breaking waves initially produce a circulation system which shuts off the waves in some regions and decays until the waves are re-established and reproduce the initial circulation.

There seems to be a considerable amount of qualitative field data to support the second hypothesis (see, for instance, Sonu, 1972).

1.3.3 Wave-Current Interaction with Vorticity (Two-Dimensional)

As waves approach a shoreline they transport energy, momentum and mass from deep water towards the shore. Many aspects of the momentum balance have been evaluated in Sections 2 and 3 and summarized above. Mass transport by waves is closely related to the vorticity present in the water column. Section 4 of this report presents a detailed review of wave motion, currents, mass transport and vorticity and their interaction in two-dimensions. The early work of Dubreil-Jacotin (1934) and others is reviewed. It is shown that there are an infinite number of solutions for periodic waves in a perfect inviscid fluid associated with the presence of a more or less arbitrary vorticity distribution. A current having a velocity profile which varies over a vertical has an associated vorticity distribution and hence the form of periodic waves present do not necessarily follow the classical Stokes solution.

In Section 4 of this report the equations required to solve at least up to the first order the problem of small amplitude wave propagation in the presence of an arbitrary current for an arbitrary wave spectrum have been presented, and a review of the special solutions previously obtained for a single wave length has been made. The problem in general requires lengthy numerical computations.

However, for a current whose velocity distribution can be approximated by a linear depth dependence, it has been shown that, at most, a single numerical quadrature was required to obtain the average velocity components. This method may then be used to estimate the forces due to wave action in the presence of a current. An experimental knowledge of the current velocity at but a few depths (two minimum) will define the parameters necessary to completely solve this problem.

2. CIRCULATIONS UNDER THE SEA BREEZE CONDITION

2.1 INTRODUCTION

When the nearshore wave field is strongly influenced by a sea breeze, local wind waves undergo diurnal changes in height, period, and incidence angles. In the northern hemisphere, the wave direction rotates clockwise, while heights and periods both grow steadily toward late afternoon. Usually, a background swell is superimposed on these wind waves.

Nearshore circulations, which are sensitive to breakers and their incidence angles, will undergo rapid changes accordingly. Diurnal changes in nearshore and surf zone topography under this condition are probably more gentle. This situation is known to develop at a number of tropical and subtropical regions of the world.

In this chapter, a series of computations are performed to simulate successive stages of nearshore circulation under the influence of a day-time sea breeze condition. Some of the basic considerations included in the present computation are summarized as follows:

1) In reality, the change in the circulation velocity field occurs as a continuous process. However, a finite-difference solution of time-dependent equations involves technical difficulties as well as a considerable amount of computer time. Instead, the computation is performed for four discrete stages of circulation development (at three hourly intervals) using steady-state equations.

2) Quadratic inertia terms impose difficult, if not insurmountable, restrictions to the computation. Consequently, the equations of motion are linearized by neglecting the inertia terms.

3) Velocity variations over a vertical are neglected.

4) The formulation of the bottom friction term in the momentum equations was derived following the assumption that circulation velocity components are small as compared to wave orbital velocity, as in the previous report (Noda, 1972; Thornton, 1969).

5) The effect of interactions between wave and circulation, as discussed in detail in Section 3, is not included in the computations presented in this section.

6) When wind waves and swell coexist as separate wave trains, there will be an interaction not only between them but also between the currents they drive simultaneously. This situation is extremely complex and involves a number of mechanisms which are not well understood. As an alternative, the case of coexisting wind wave and swell is treated by vector addition of the velocity fields associated with each of the wave trains.

2.2 GOVERNING EQUATIONS

The method of computation is to solve by a finite difference approximation a set of steady-state linear equations of motion and a continuity equation. Basic mathematics of this method have been discussed in detail in the previous report (Noda, 1972). However, for the benefit of the reader, these will be briefly summarized:

Equations of motion (vertically integrated) are:

$$g \frac{\partial \eta}{\partial x} = M_x - F_x \qquad 2.1$$

$$g \frac{\partial \eta}{\partial y} = M_y - F_y \qquad 2.2$$

and a continuity equation is:

$$\frac{\partial}{\partial x}\left[u(\eta+d)\right]+\frac{\partial}{\partial y}\left[v(\eta+d)\right]=0\ . \qquad 2.3$$

where x and y are taken normal and parallel to the coast, respectively.

M_x and M_y denote radiation stress terms (Longuet-Higgins, 1964), given by

$$M_x = -\frac{1}{\rho(\eta+d)}\left(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}\right) \qquad 2.4$$

$$M_y = -\frac{1}{\rho(\eta+d)}\left(\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{yx}}{\partial x}\right) \qquad 2.5$$

where, in shallow water,

$$\sigma_{xx}=\frac{1}{16}\rho g H^2\left[3\cos^2\theta+\sin^2\theta\right] \qquad 2.6$$

$$\sigma_{yy}=\frac{1}{16}\rho g H^2\left[3\sin^2\theta+\cos^2\theta\right] \qquad 2.7$$

and

$$\tau_{xy}=\tau_{yx}=\frac{1}{16}\rho g H^2\sin^2\theta \qquad 2.8$$

The friction terms are simplified as:

$$F_x=\frac{2\bar{c}Hu}{(\eta+d)T\sinh kd}\equiv F.d.u \qquad 2.9$$

$$F_y=\frac{2\bar{c}Hv}{(\eta+d)T\sinh kd}\equiv F.d.v \qquad 2.10$$

where $\bar{c}$ is friction coefficient (0.01 in our computation); d is the water depth, and η is a set-up or set-down relative to the mean sea level.

Defining a stream function given by

$$\frac{\partial \psi}{\partial y} = -ud, \quad \frac{\partial \psi}{\partial x} = +vd \qquad 2.11$$

and assuming

$$\eta + d \simeq d \qquad 2.12$$

Equations 2.1-2.3 reduce to a single equation:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\frac{\partial F}{\partial y}}{F} \frac{\partial \psi}{\partial y} + \frac{\frac{\partial F}{\partial x}}{F} \frac{\partial \psi}{\partial x} =$$

$$\frac{1}{\rho F} \left\{ \frac{\partial}{\partial y} \left[\frac{1}{d} \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) \right] - \frac{\partial}{\partial x} \left[\frac{1}{d} \left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} \right) \right] \right\} \qquad 2.13$$

The boundary conditions are:

$$\frac{\partial \psi}{\partial x} = 0 \quad \text{at } x = 0 \text{ and } \infty, \qquad 2.14$$

and

$$\psi(y, z) = \psi(y + \lambda, x) \qquad 2.15$$

The latter condition implies that the circulation field is periodic along the shore at a spacing equal to the wavelength λ of the bo'tom topography.

The computation solves Eq. 2.13 using a relaxation (or Gauss-Seidell) method, as already discussed in the previous report. The ψ values at the inshore and offshore boundaries can be chosen arbitrarily. I.. this case, ψ is chosen to be zero at $x = 0$ and ∞. The iterative procedure was continued until a condition

$$|\psi_{j+1} - \psi_j| / |\psi_j| \leq 0.05 \qquad 2.14$$

was achieved between successive interation cycles ϕ_j and ϕ_{j+1}.

The radiation stress field to be entered into Equation 2.13 is provided from a wave ray equation which combines effects of shoaling and refraction

$$\frac{D^2\beta}{Ds^2} + p(s)\frac{D\beta}{Ds} + q(s)\beta = 0 \qquad 2.15$$

where

$$p(s) = -\cos\theta\left[\frac{1}{C}\frac{\partial C}{\partial x}\right] - \sin\theta\left[\frac{1}{C}\frac{\partial C}{\partial y}\right]$$

$$q(s) = \sin^2\theta\left[\frac{1}{C}\frac{\partial^2 C}{\partial x^2}\right] - 2\sin\theta\cos\theta\left[\frac{1}{C}\frac{\partial^2 C}{\partial x\partial y}\right] + \cos^2\theta\left[\frac{1}{C}\frac{\partial^2 C}{\partial y^2}\right] \qquad 2.16$$

where

s is the arc length along the ray

β is the wave intensity, and C is the celerity.

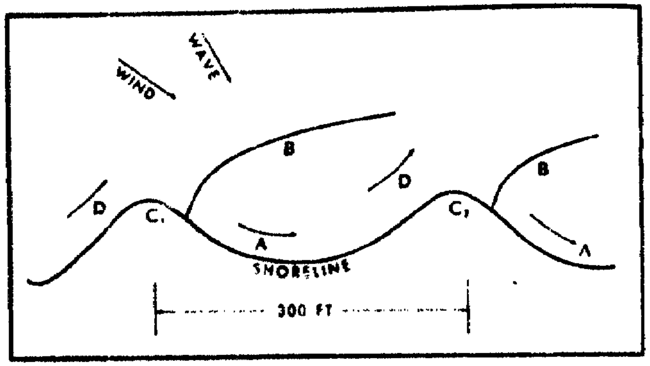
Previously, the ray equation was solved by a fourth order Runge-Kutta scheme. In the present report, this equation is solved by a relaxation technique, as described in detail in Section 3. This method computes incident wave heights and angles directly on the grid, whereas the previous method traced wave rays individually, which required additional visual inspection of wave ray density and interpolation steps to transfer the ray data onto the grid. The new method thus allows the entire wave field computation to be carried out in a single run of computer processing, resulting in a substantial improvement with respect to both speed and accuracy.

2.3 BOTTOM TOPOGRAPHY

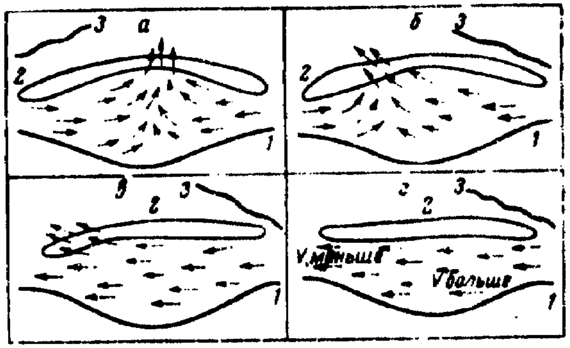
The input information for bottom topography and wave characteristics is derived from the observations carried out by the Coastal Studies Institute on Santa Rosa Island, Florida, in 1972 (SALIS Project, see Sonu et al., 1973). The CSI data are especially pertinent to our study because they contained detailed characteristics of the surf zone topography to which the nearshore circulation is known to be sensitive (Sonu, 1972, 1973). The CSI data also contained general information of circulation pattern and current velocities as revealed from repeated dye experiments.

Importance of bottom topography, particularly that of undulations in the surf zone bottom, to nearshore circulation has been pointed out by a number of field observers, among them Evans (1939), McKenzie (1958), Shadrin (1961), Davis and Fox (1971, 1972), and Sonu (1972, 1973). Surf zone topographies as reported by these investigators are summarized in Figure 2.1. Evans reported a meandering current consisting of an inflow across the bar and an outflow originating from the shoreline embayment. McKenzie reported an inflow across the bank (shoal) and an outflow along a conspicuous rip channel (or depression) between banks. It should be noted that although a schematic presented by McKenzie depicts the shoreline with a straight line, his photographs indicated a periodically curved shoreline. According to Shadrin, an outflow generally initiated in the embayment, but its orientation depended upon not only wave direction but also wave height. Davis and Fox reported meandering currents under wind wave conditions. These rhythmic topographies had wavelengths ranging between 70 and 200 meters.

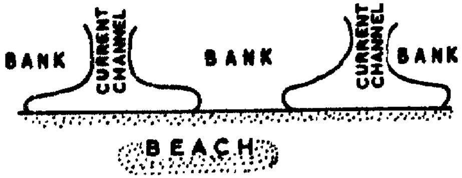
Figure 2.2 shows the surf zone topography at the site of the CSI project. Note that a cuspate portion of the rhythmic shoreline descends directly to a shoal in the surf zone. A line of longshore bar exists approximately 30 meters from the average shoreline position. This



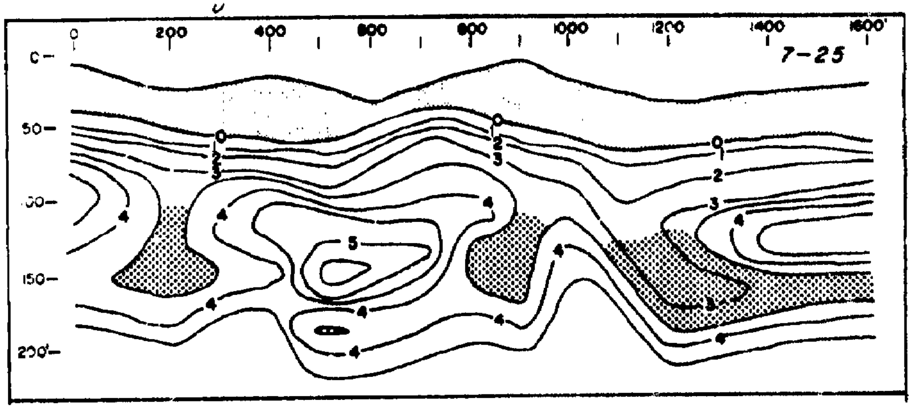
Evans (1939)



Shadrin (1961)



McKenzie (1958)



Davis and Fox (1971)

Figure 2.1: Rhythmic Surf Zone Structures as Reported by Various Investigators

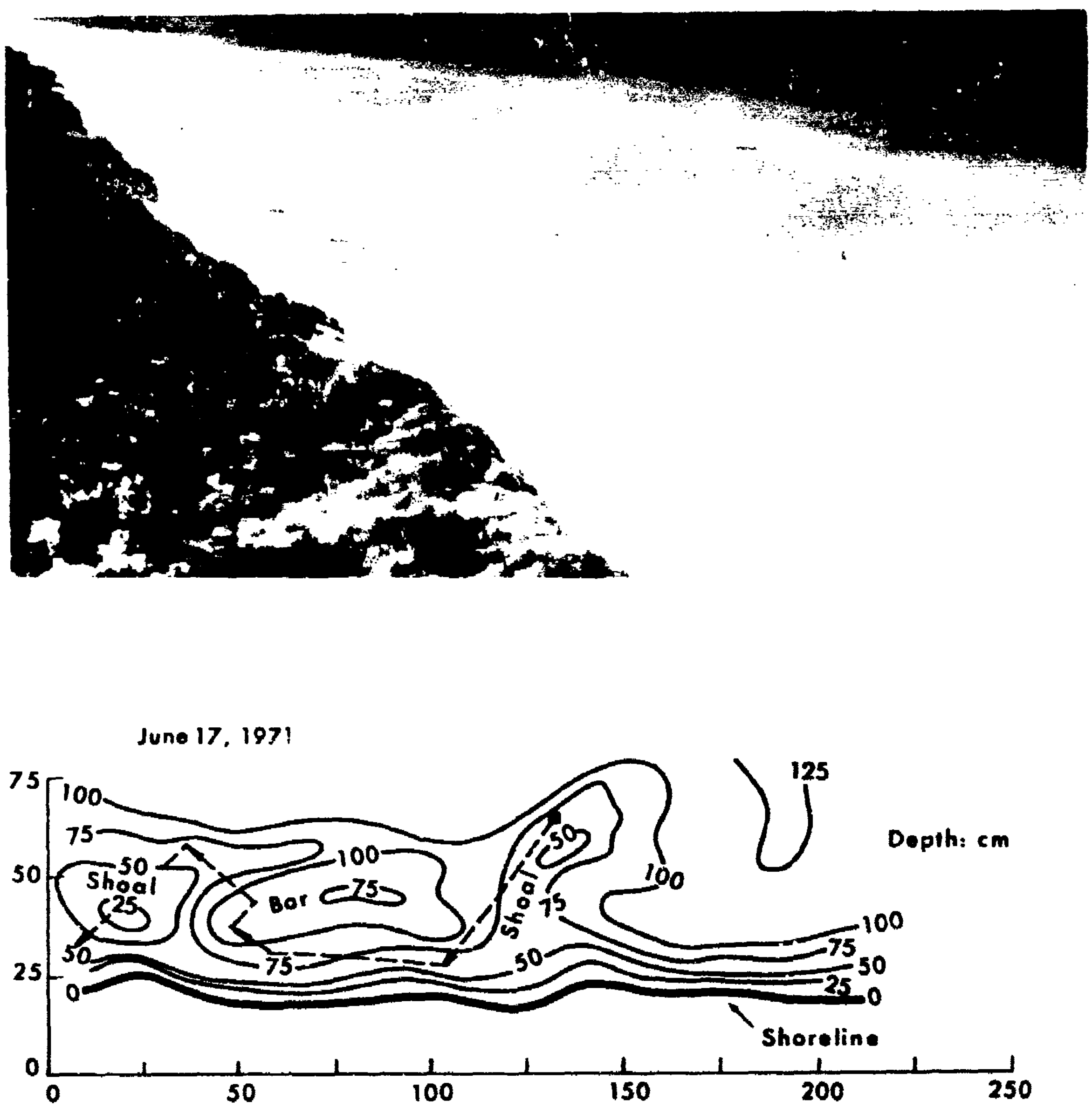


Figure 2-2: Rhythmic Topography at the CSI Study Site

rhythmic topography was formed at the time of a strong local storm and remained essentially unchanged for as long as 14 days while a local sea breeze dominated the area.

Figure 2.2 also shows a typical example of water movement as revealed from the movement of dye. The dye, initially injected at the break point on a shoal, streaked toward an embayment in approximately the same direction as the breaker. It then travelled parallel to the shoreline for some distance before making a seaward turn. The outflow across the surf zone usually occurred on the depression, which, upon reaching a break point, tended to turn alongshore and eventually returned shoreward across the downstream shoal. This type of meandering current pattern was typical of afternoon conditions when wind waves associated with the sea breeze arrived obliquely to the coast. Current speeds in the meandering currents generally amounted to 30 cm/sec in the inflow current across the shoal, 10 - 15 cm/sec in the parallel current near the shoreline, and about 20 cm/sec in the outflow or rip.

During the morning hours when the wave field was dominated by the background swell, the currents tended to form closed circulations of minor velocities, consisting of an inflow on the shoal and an outflow on the depression. Maximum speed under this condition was no more than 20 cm/sec.

For mathematical representation, a rhythmic topography may be broken up into three components, (1) mean profile, (2) longshore bar, and (3) longshore undulations.

The mean profile of a coast is generally concave upward and may be approximated by

$$d_1 = \mu x^{\gamma} \qquad 2.17$$

in which d_1 is the depth measured from the mean sea level, x is the distance seaward from the shoreline, and μ and γ are numerical coefficients; especially, $\gamma < 1$ to ensure the concavity of the profile. Bruun (1973) showed on the basis of a wide range of evidence, that γ varies between about 2/3 nearshore and about 1/2 offshore.

The bar can be defined, for the sake of simplicity, as a symmetrical hump superimposed on the mean profile. Assuming a bell-shaped configuration similar to an error function, the bar profile is given by,

$$d_2 = b \cdot \exp\left[-(x - x_b)^2 / (x_b/2)^2\right] \qquad 2.18$$

The longshore undulation is generally confined within the surf zone, and its amplitude attenuates quite rapidly outside the breaker line. Thus, we assume a longshore undulation whose amplitude decreases linearly toward zero at $x = l_b$, i.e.

$$d_3 = a\,(1 - x/l_b)\sin\frac{2\pi}{\lambda}(y - \delta) \qquad 2.19$$

in which a is the maximum amplitude and λ is the wave length of the undulation. The term δ in Equation 2.19 represents a degree of distortion to be introduced in the geometry of the undulation. Normally, this will consist of two parts:

$$\delta = \delta_1 + \delta_2 \,. \qquad 2.20$$

Where a longshore current is significant, the longshore cross-section of the sinusoidal undulation is skewed, yielding a steeper slope facing the downstream side. Furthermore, under this condition, the crest-line of the undulation will extend obliquely seaward from the shore-line.

The first of these effects, the skewness, can be incorporated in Equation 2.19 by considering δ_1 of the form

$$\delta_1 = \delta_{max} \sin \frac{2\pi}{\lambda} (y - \delta_1). \qquad 2.21$$

In other words, the symmetrical sinusoid of the original undulation, $\sin \frac{(2\pi y)}{\lambda}$, is distorted by displacing the coordinate y by a variable distance δ_1 in such a way as to achieve a steep downstream slope. The displacement is maximum (δ_{max}) along the crest of undulation, e.g. at $y = \frac{\pi}{4}(2n+1) + \delta_{max}$, decreasing in both directions away from this in proportion to $\sin \frac{2\pi}{\lambda}(y-\delta_1)$.

The oblique downstream orientation of the crest of the undulation can be represented by δ_2 of the form

$$\delta_2 = x \tan \alpha \qquad 2.22$$

in which α is the angle between the normal to the shoreline and the crest of undulation.

Thus, combining the mean profile, a bar, and skewed undulations, the general expression for the rhythmic topography is

$$d = d_1 - d_2 + d_3$$

$$= \mu x^{\gamma} - b.\ \exp\left[-(x-x_b)^2/(x_b/2)^2\right] + a\,(1-x/l_b) \sin \frac{2\pi}{\lambda} (y-\delta_1-\delta_2) \qquad 2.23$$

Figure 2.3 shows successive superimposition of d_1, d_2 and d_3, in which $\mu = 0.075$, $\gamma = 0.600$, $b = 0.300$ (meters), $x_b = 30.00$ (meters), $a = 0.200$ (meters), $l_b = 80$ (meters), $\lambda = 115$ (meters), and $\alpha = 20^\circ$.

2.4 WAVES

Figures 2.4(a), (b), and (c) show diurnal changes in wave characteristics. Typically, the waves during the morning were dominated by the background swell arriving normal to the shore. As the sea breeze began to increase between 1100-1200 hours, small wind waves became superimposed on swell. Wind waves subsequently grew both in height and period, while rotating its direction clockwise, until they dominated the sea state around 1500-1800 hours in the afternoon. In the evening hours after 1800 hours, wind waves steadily attenuated and were gradually replaced by the background swell until the next morning.

In Figure 2.4(a) and 2.4(b), it is seen that the wind waves (0.3-0.7 cps) were strongly coupled with sea breeze, so that the period increased rapidly from about 1 sec at 1000 hours to 3 sec at 1600 hours, the time of maximum sea breeze. The direction of wind waves also increased from about 20° to 40° against the normal to the shoreline (Fig. 2.4(c)). The swell spectrum underwent a slight change, its direction remaining essentially perpendicular to the shoreline.

From these data, the wave heights, periods, and directions to be input into the computation were determined, as shown in Table 2.1. The significant wave height was computed from the power spectrum according to

$$H_{1/3} = 4\left[\int_{f_1}^{f_2} S(f)\,df\right]^{\frac{1}{2}}$$

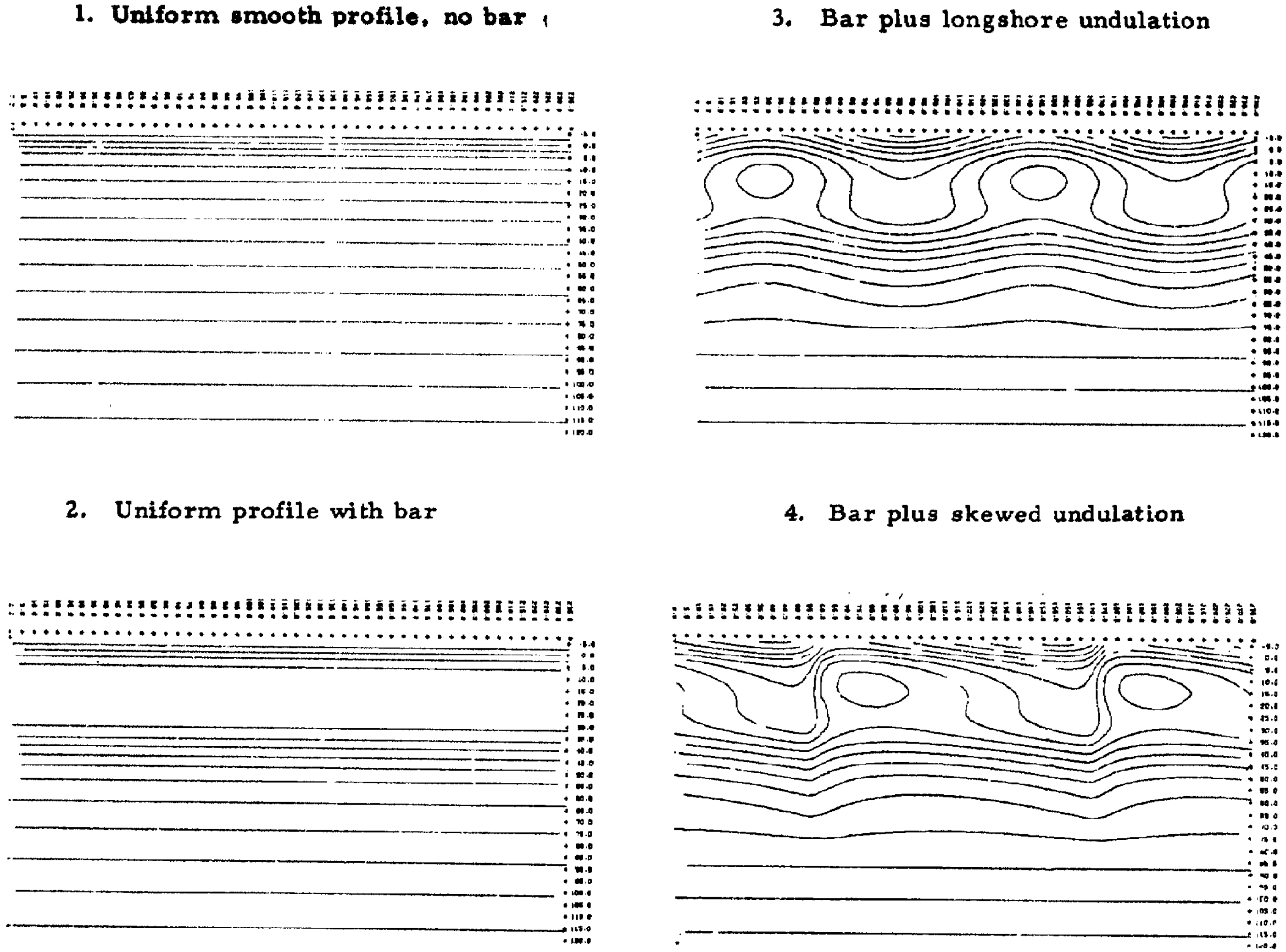
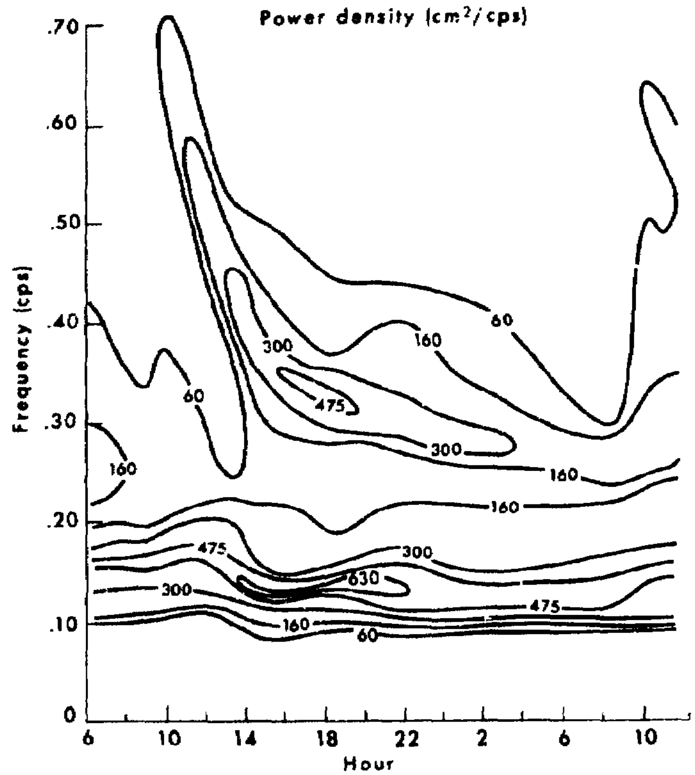


Figure 2-3: Mathematical Simulation of Bottom Topography



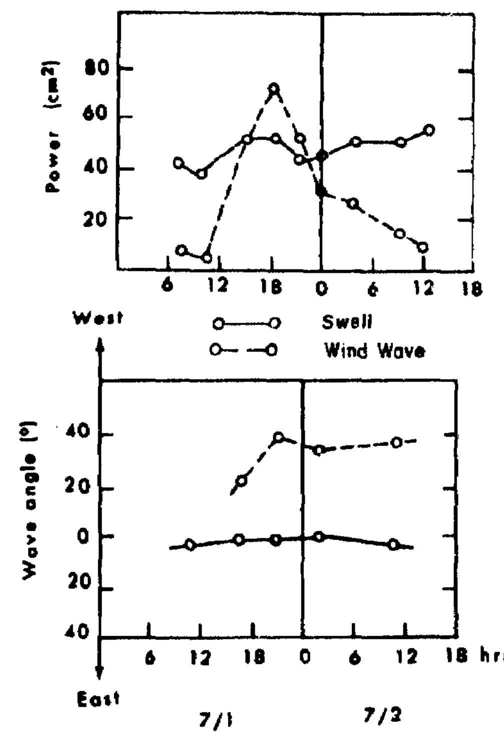


Figure 2-4: (a) Wave spectrum at the outer bar over a 28-hour period showing the presence of sea breeze wind waves (as curving ridge) at high frequency and swell (as straight ridge) as low frequency.
(b) Power content of wind wave and swell peaks over 28 hours at outer bar (see Fig. 2.4(a) tor spectrum); change in relative height of wind waves and swell is shown.
(c) Direction of approach of swell and wind waves during 28-hour period.

(data processed and organized by Suhayda)

TABLE 2.1

INPUT WAVE CHARACTERISTICS

Time of Day hours	Wind Wave $H_{1/3}$ cm	Wind Wave T sec	Wind Wave θ °	Swell $H_{1/3}$ cm	Swell T sec	Swell θ °
1200	17.9	1.62	0.0	26.0	7.00	-2.0
1500	28.8	2.41	15.0	28.8	7.70	-1.0
1800	33.9	2.75	25.0	29.0	7.90	-0.5
2100	29.0	2.96	40.0	25.5	7.90	0

The term inside the parentheses denotes either wind-wave or swell portions of the power spectrum, as plotted in Figure 2.4(b). Wave periods were obtained directly from the spectral density peaks for wind-wave and swell.

2.5 RESULTS

2.5.1 Circulations Under Wind Waves

Figures 2.5 and 2.6 show streamlines caused by wind waves only at 1200, 1500, 1800 and 2100 hours. Note that the streamline separation represents .2 m^3/sec.

According to Figure 2.5 (case of normal wave incidence), an inflow dominates the area of shoals (y = 30-60, 140-170 meters). An inflow also occurs at part of the depression immediately to the right of the shoal. However, most of the depression area (y = 90-120 meters) is dominated by outflow. Thus, there is a general indication that an inflow is strong on the shoal and an outflow is strong on the depression.

However, a detailed streamline distribution is more complex and includes some departures from the general rule. There is a small but well-defined eddy immediately to the right of the shoal (y = 70-90, 185-205), which surrounds an area marked by a contour 0.4 meters. Another eddy of much smaller velocity is located almost directly offshore. These eddies have not been noticed during the field observation. It must be noted that, although the congested streamlines give the impression of a strong current, they only involve velocities on the order of a few cm/sec. Normal velocity components are clearly larger than the longshore components. The inflow velocity on the shoal is on the order of 1.5 cm/sec; the outflow velocity in the middle of the depression is on the order of 1.8 cm/sec. The maximum inflow velocity reaches about 8.6 cm/sec at y = 70; the maximum outflow velocity reaches about 7.6 cm/sec at y = 85. Maximum velocity outside of the surf zone is only about 4 cm/sec.

These low velocities are typical of weak breaker activities prior to the arrival of the sea breeze wave front in the surf zone. It is noticed in Figure 2.5 that waves are breaking only in the immediate vicinity of a shoal.

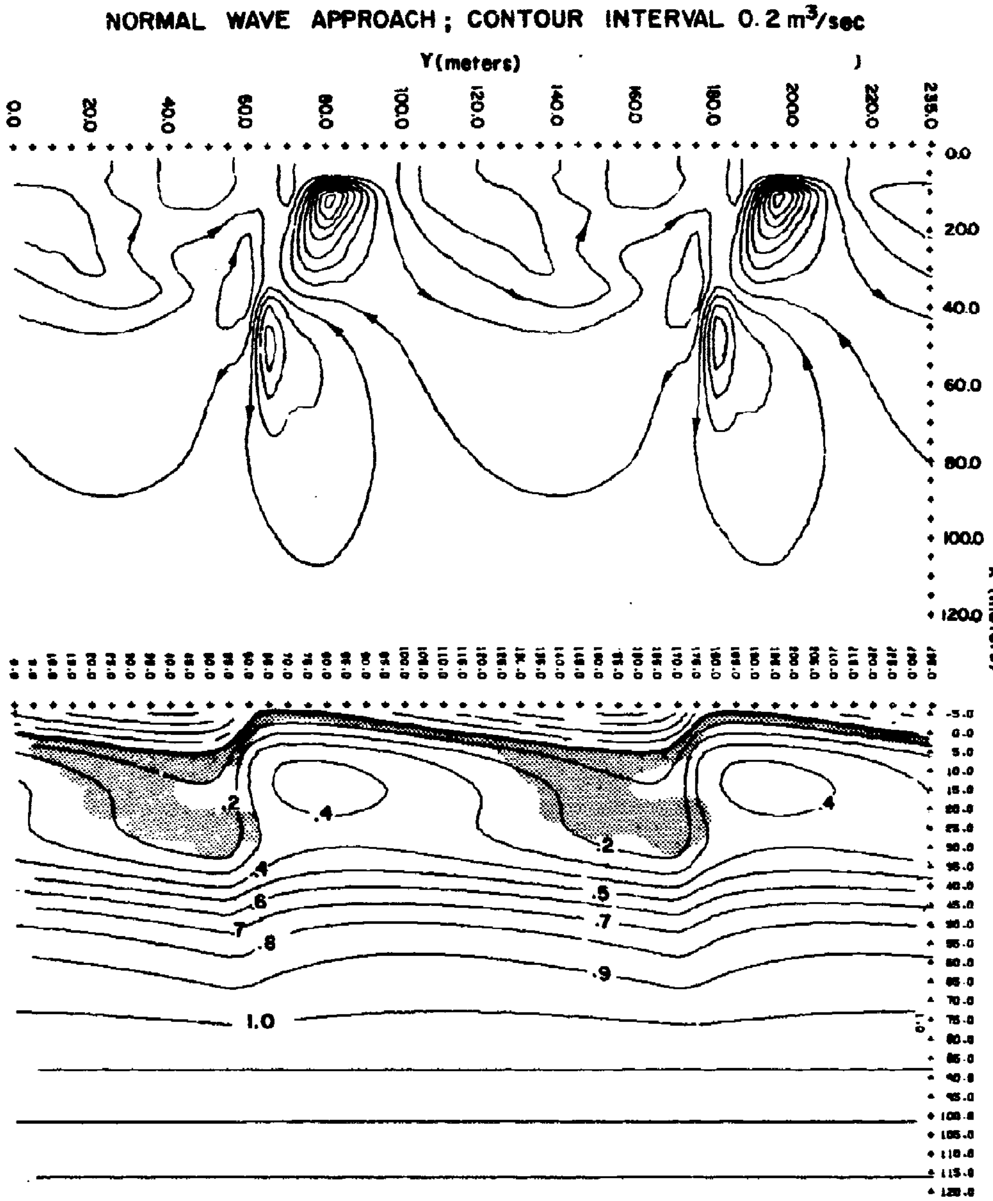


Figure 2-5: Streamlines and Bottom Topography for Wind Waves at 1200 hours Normal Incidence

Figure 2.6 shows cases of oblique wind waves in the afternoon. These streamlines now exhibit a stronger tendency for meander than in the case of normal incidence of Fig. 2.5. The outflow portion of the meander is located in the depression. However, an inflow also occurs at the depression nearer an upstream shoal. Small eddies tend to persist throughout the period of computation.

One of the conspicuous features of the afternoon situations is the tendency for the longshore current along the bar crest to intensify in proportion to the breaking activity. Fig. 2.7 shows the distribution of breakers at times corresponding to Fig. 2.6. Breaking is the most intensive at 1800 hours, e.g. at the peak of sea breeze activity, generating a strongest current along the bar crest (maximum 23 cm/sec). Both before and after this event (e.g., at 1500 and 2100 hours) when the breaker zone was narrower, current speeds along the bar reached a maximum of only 15 cm/sec. Concentration of longshore current velocity in the breaker zone arises from the longshore wave thrust generated directly by a breaking phenomenon, in proportion to the rate of shoreward decrease in the flux of longshore momentum across a plane parallel to the shore.

2.5.2. Circulations Under Swell

Figure 2.8 show streamlines associated with swell. Only two cases are shown inasmuch as the swell characteristics changed little under the sea breeze condition. Streamline separation is .6 m^3/sec.

A salient feature of these streamlines is the occurrence of a local circulation immediately to the right of the shoal (y = 60-80, 175-185), which contains velocities as high as 120 cm/sec seawards and 80 cm/sec onshore. These circulation are located somewhat offshore of the eddies as noticed in the case of wind waves (compare with Figures 2.5 and 2.6).

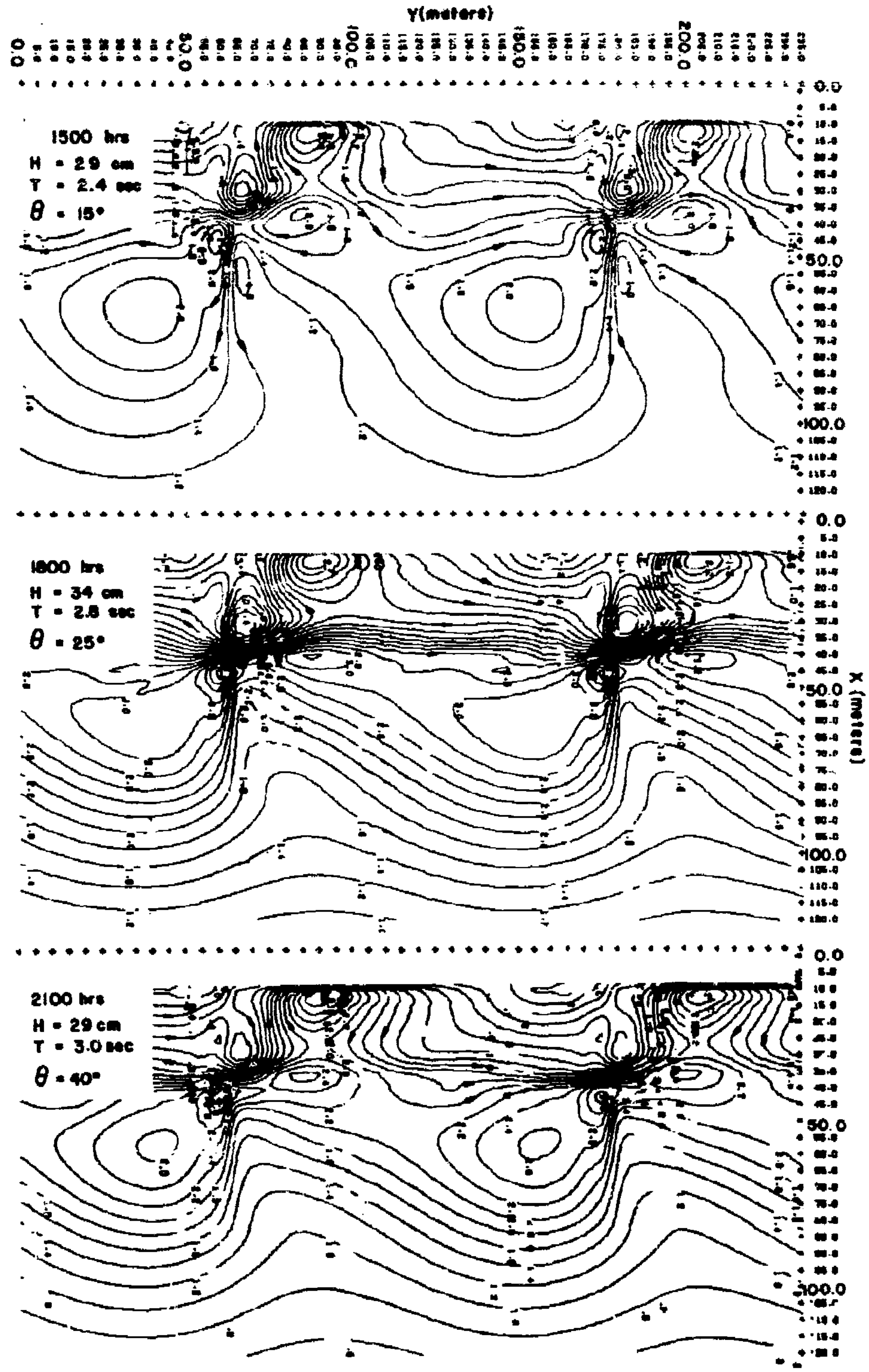


Figure 2-6: Streamlines During Oblique Wave Incidences in the Afternoon

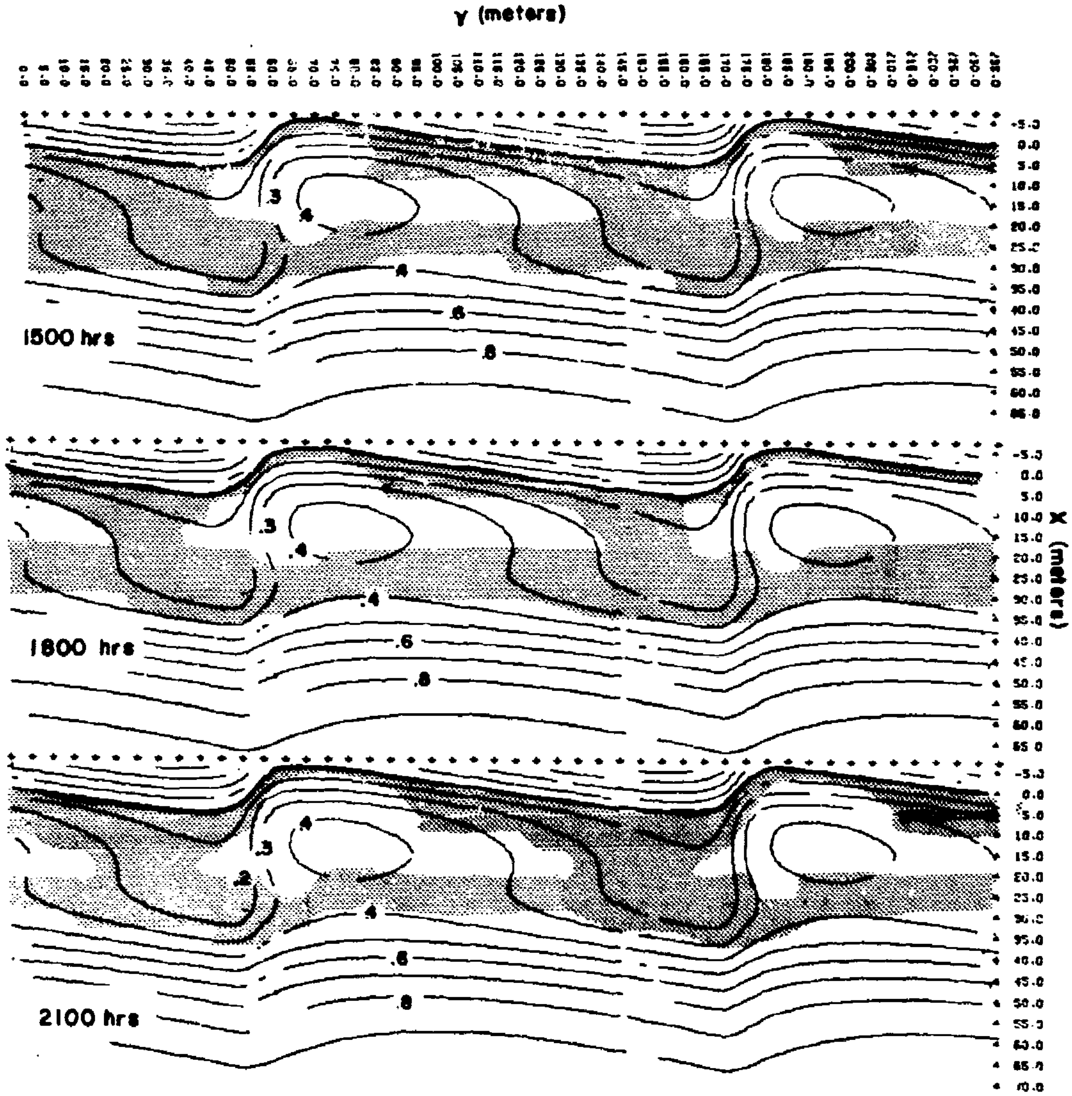


Figure 2-7: Breaker Distribution

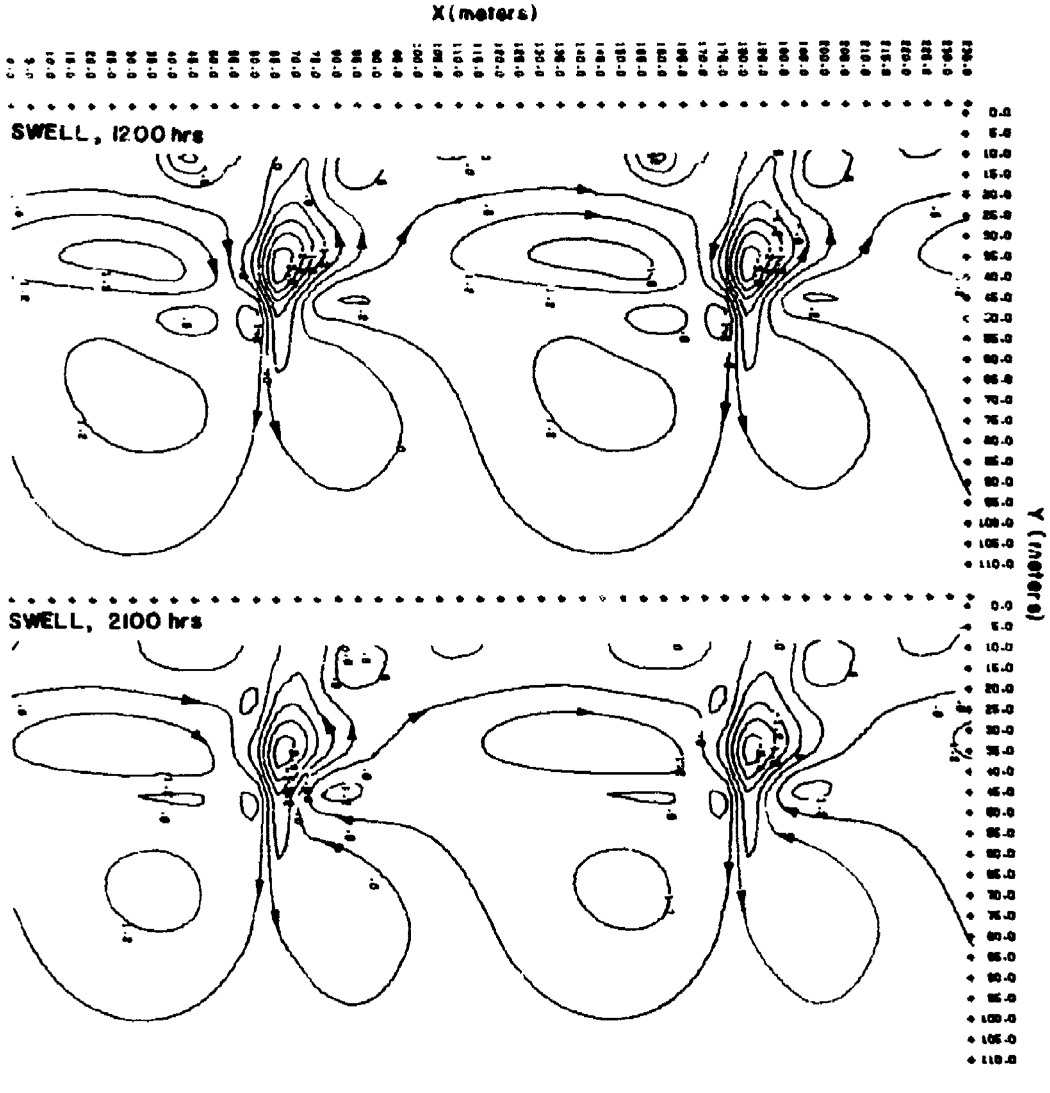


Figure 2.8: Typical Streamlines Under Swell

These velocity patterns contrast strongly with the case of closed circulations previously observed by Sonu off the Seagrove beach (Sonu, 1972; See Figure 1.2 in this report). In the latter case, the bottom undulation was symmetrical, containing a broad shoal and narrow depression. In the present case, a depression occupies a larger area than a shoal, and the shoal is non-symmetrical, causing a more complex distribution of radiation stresses than in the case of a symmetrical broad shoal.

2.5.3. Circulations Under Coexisting Wind Waves And Swell

Figure 2.9 shows superimposition of streamlines associated with wind waves and swell. Again, streamline separation is 0.6 m/sec.

As expected, the results generally indicated both features of wind-wave and swell cases. At 1200 hours, when wind waves produce weak breakers, the current field is dominated by swell. Effects of wind waves steadily increase through 1500 hours toward 1800 hours, the tendency for current meander becoming gradually more evident. At 1800 hours, a current arriving at a shoal partly escapes seaward and partly meanders back shoreward. A local circulation near the tip of a shoal persists, reflecting a complicated radiation stress distribution over the sharply skewed bottom topography. It is also noted that a strong longshore current along the bar crest remains in force during the time of maximum sea breeze at 1800 hours. In general, current activities are concentrated around the steep fall of this shoal where the breaker height variation is most pronounced.

2.6 DISCUSSIONS

The simulated streamlines indicate both similarities and differences as compared with field observations. In general, the feature of inflow dominance over the shoal and outflow dominance over the depression is revealed in the computed streamlines, but it is also disrupted to various degrees by the occurrence of localized eddies and small

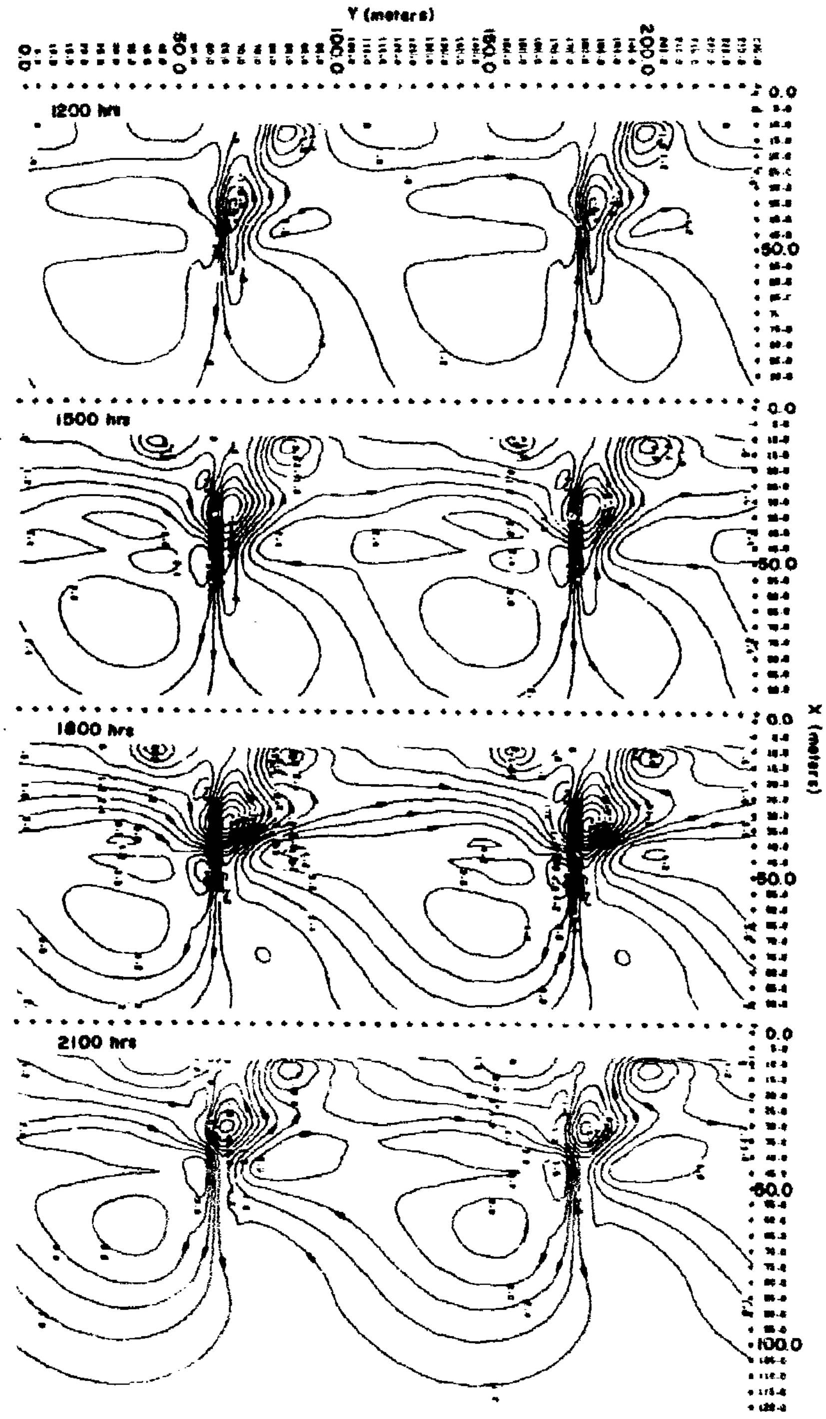


Figure 2-9: Streamlines Under Combined Effects of Wind Waves and Swell

circulations persisting near the steep face of the skewed bottom undulation. Especially in the case of swell, these localized flows tend to dominate the overall streamline distribution. Also, the computed velocities tend to be higher than observations by a substantial margin especially in the case of swell.

Several approaches seem possible to improve the degree of reliability of numerical simulation for nearshore circulations.

First, the criterion for breaking inception and the estimation of wave heights during breaking should be improved. The present computation uses the Miche criterion,

$$\left(\frac{H}{L}\right)_b = 0.12 \tanh 2\pi \left(\frac{d}{L}\right)_b$$

for both breaking inception and post-breaking wave height. Since this criterion requires wave heights to diminish to zero at the shoreline, the rate of wave height reduction during breaking, hence the magnitude of radiation stress, may result in over-estimation. This could be one of the causes for overestimation of velocities.

There exists a critical deficiency of knowledge on the behavior of breaking waves. One way to overcome this difficulty may be to take into consideration a wave set-up in the water depth estimation in the surf zone. This problem has been handled numerically in a two-dimensional case (Hwang and Divoky, 1970). In the three-dimensional case, as in our study, this problem could be handled by stepwise approximation. First, the result of the computation which is based on the assumption (Eq. 2.12).

$$\eta + d \cong d$$

could be substituted into the starting equations 2.1-2.3 to determine η_1.

In the next iteration, η_1 will be added to the mean-sea-level water depth d and the new wave field and streamlines will be determined. This result will again be recycled to the starting equations to determine η_2 and initiate the second iteration, and so on. These procedures will result in a slower breaker height reduction on the shoal and hence smaller radiation stresses and weaker currents.

The second approach is to take into consideration the randomness in the incident waves. Since wave breaking will occur in a zone instead of at a point, the radiation stresses will be spread more broadly, resulting in a general lowering of peak current velocities. In the case of two-dimensional longshore currents, this approach has resulted in a velocity distribution comparable to a derivation using a turbulent momentum mixing or eddy viscosity assumption (Collins, 1972).

Third, a more rigorous formulation of the bottom friction term may be needed. In the present computation, the bottom friction is associated primarily with wave orbital motion. Retardation of circulation velocity, presumably of considerable magnitude, is not taken into consideration in full value. As already mentioned, this approach requires readjustment of numerical scheme to ensure a sufficient degree of computational stability.

Fourth, it must be noted that the present computation does not consider interactions between wave and circulation. Therefore, there is an implicit assumption as if the wave field had been abruptly removed after driving the current instantaneously. However, the current, once produced, will interact with waves at all phases of its development. It is possible that the effect of such interactions is to produce an equilibrium circulation with less current velocities than obtained in the present computation, or a pulsation of the circulation around a certain mean equilibrium state.

3. WAVE CURRENT INTERACTION OVER VARIABLE TOPOGRAPHY

3.1 INTRODUCTION AND REVIEW OF HISTORICAL WORK

The available literature on surface wave-current interaction is not extensive. Unna (1942) and Sverdrup (1944) considered the case of deep-water waves encountering a following or opposing current and applied their results to waves in tidal entrances. Johnson (1941) discussed the refraction of deep-water waves encountering a uniform current moving at an angle to the wave system. Arthur (1950) studied the problem of shallow-water waves being refracted by both changes in bottom bathymetry and a nonuniform current system. Application of refraction effects due to a current distribution similar to an intense rip current was solved by considering the analogous problem of determining the minimum flight path of an airplane flying in a variable wind field.

Taylor (1955) investigated the influence an outward flowing surface current would have in preventing the passage of waves coming in from the sea. This study was in association with the concept of utilizing a surface current produced by a curtain of air bubbles as a "pneumatic breakwater". Evans (1955) performed an experimental investigation of this concept.

Ursell (1960) and Whitham (1960) developed the general geometrical equation governing the interaction of a variable current and any type of wave motion. In a classic series of papers by Longuet-Higgins and Stewart (1960, 1961, 1962) and by Whitham (1962) the conservation equations of mass, momentum and energy per unit area for a wave system superimposed on a variable current system were derived. A very good summary of this work is given by Phillips (1966). Taylor (1962) studied the characteristics of free-standing waves on either a contracting or expanding current and provided experimental data. Hughes and Stewart (1961) also conducted experimental investigations to determine the characteristics of gravity waves on a shear flow.

Recently Jonsson, Skougaard and Wang (1970) concentrated attention on the "current-wave set-down" for two-dimensional wave current propagation over a gently sloping bed. Kenyon (1971) studied the kinematics of deep-water waves in conjunction with a variable current to show the possibility of either the trapping or total reflection of waves by the current.

To carry out the basic objective of this study as indicated at the very outset of this introduction, the important kinematic and dynamic relationships are first set forth. Numerical techniques are developed to solve these relationships so that the stream function and associated circulation pattern can be obtained. The basic philosophy is to first solve the nearshore wave-induced circulation problem with no wave-current interaction. Then the output of these circulation velocities are now considered the existing mean current system, and waves are again propagated into this system and a new wave height, direction and nearshore circulation pattern obtained. It is hoped that this continual interaction will finally yield an "equilibrium" solution.

3.2 WAVE CURRENT INTERACTION

3.2.1 Wave Kinematics

Inherent in the concept of three-dimensional waves is the motion of a "wave front". Crests and troughs of a wave often tend to maintain their identity as they propagate, which is represented by surfaces everywhere perpendicular to the direction of wave motion. These surfaces are called "surfaces of constant phase" or phase surfaces. The propagation of gravity water waves can be represented by a form

$$\zeta(\vec{x},t) = a(\vec{x},t)\, e^{i\varphi(\vec{x},t)} \tag{3.1}$$

where $a(\vec{x},t)$ is an amplitude function and the sinusoidal term provides for the motion of the wave, where the surfaces $\varphi(\vec{x},t) =$ constant are the surfaces of constant phase [Morse and Feshbach (1953), Phillips (1966)].

This physical interpretation of the phase surface function φ yields the definition of the wave-number vector field $\vec{k}$ and the scalar wave-frequency field $\bar{\imath}$ in terms of the phase function:

$$\vec{k} = \nabla\varphi \tag{3.2}$$

and

$$\bar{\imath} = -\frac{\partial\varphi}{\partial t} \tag{3.3}$$

In particular, the classic solution for the surface oscillation of a progressive water wave moving in the +x direction [Lamb (1945), Stoker (1957), Wiegel (1965)] is given by

$$\zeta_s(x,t) = a \sin 2\pi\left(\frac{x}{L} - \frac{t}{T}\right) \tag{3.4}$$

where

a is the wave amplitude

L is the wave length

and T is the wave period.

Thus application of (3. 2) and (3. 3) to (3. 4) where $\varphi = 2\pi(\frac{x}{L} - \frac{t}{T})$ yields the wave-number in the +x direction as

$$k_o \equiv \frac{2\pi}{L} \tag{3.5}$$

and the wave-frequency

$$\bar{\omega}_o \equiv \frac{2\pi}{T} \tag{3.6}$$

Note that in Equations (3. 5) and (3. 6) a subscript o has been utilized. In all following analyses this subscript refers to conditions of no wave-current interaction and not to deep-water conditions as is often denoted in the literature. For deep-water conditions the subscript d will be utilized.

Since the curl (grad φ) $\equiv 0$, then Equation (3. 2) becomes

$$\nabla \times \vec{k} = 0 \tag{3.7}$$

and consequently the wave-number vector field is irrotational. Moreover if $\varphi(\vec{x}, t)$ is a continuous function then the order of differentiation yields identical results and consequently

$$\frac{\partial}{\partial t}(\nabla\varphi) = \nabla\left(\frac{\partial\varphi}{\partial t}\right) \tag{3.8}$$

Thus substituting from Equations (3. 2) and (3. 3) yields

$$\frac{\partial\vec{k}}{\partial t} + \nabla\bar{\omega} = 0 \tag{3.9}$$

Equation (3. 9) is a kinematical relationship which describes the conservation of wave number. Consider a single wave train being viewed by an "Eulerian" observer at a stationary point. The time rate of change of waves viewed by the observer must be balanced by the convergence or divergence of the wave frequency $\bar{\omega}$, which describes the flux of the number of waves.

Consider now the case of surface waves interacting with a mean current $\vec{U}$. Kinematical requirements yield that the wave frequency is given by

$$\bar{\omega} = \omega + \vec{k} \cdot \vec{U} \tag{3.10}$$

where the first term on the RHS is the wave number with respect to the current system where

$$\omega = \omega(k, \vec{x}) \tag{3.11}$$

In the following analysis concerning surface gravity waves it is assumed that the depth of water d and mean current $\vec{U}$ vary slowly so that the classical solutions for no wave-current interaction are valid during interaction such that

$$\omega^2 = gk \tanh(kd), \tag{3.12}$$

where g is the gravitational constant,
the phase velocity c in the local wave direction is

$$c^2 = \frac{g}{k} \tanh(kd) \tag{3.13}$$

and the group velocity is

$$(c_g)_i = \frac{\partial \omega}{\partial k_i} = \tfrac{1}{2} c_i \left(1 + \frac{2kd}{\sinh(2kd)}\right) \tag{3.14}$$

Figure (3.1) schematically describes the basic wave-current interaction terminology. Furthermore all following analyses will assume that averaging over the water depth or vertical integration has taken place. From the condition of the irrotationality of the wave number vector $\vec{k}$ in horizontal space coordinates x and y due to vertical integration, Equation (3.7) becomes, in cartesian coordinates,

$$\nabla_h \times \vec{k} = \frac{\partial k_x}{\partial y} + \frac{\partial k_y}{\partial x} = 0 \tag{3.15}$$

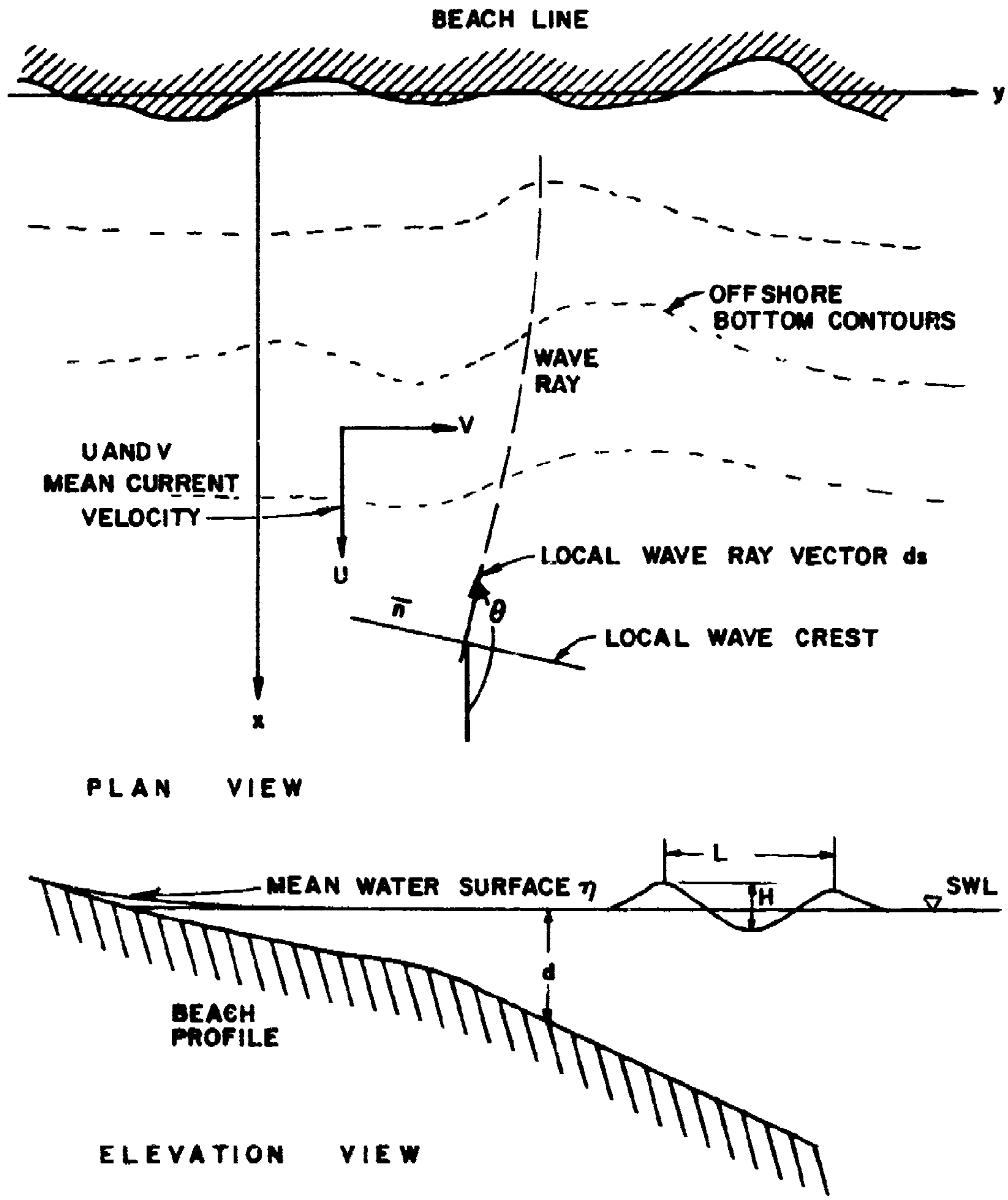


Figure 3.1: Schematic View of Nearshore Beach Terminology

where

$$k_x = k\cos\theta \tag{3.16}$$

and

$$k_y = k\sin\theta \tag{3.17}$$

Furthermore, assuming steady flow conditions exist, then $\frac{\partial}{\partial t} \equiv 0$ and Equation (3.9) becomes

$$\nabla_h \bar{\omega} = \nabla_h(\omega + \vec{k} \cdot U) = 0 \tag{3.18}$$

and for an arbitrary mean current system, the gradient of a scalar field can only be identically zero if

$$\omega + \vec{k} \cdot \vec{U} = \text{constant} \tag{3.19}$$

If $\vec{U} \equiv 0$ then Equation (3.19) becomes identically the invariant wave frequency ω_o and thus Equation (3.19) becomes in cartesian form

$$[gk\tanh(kd)]^{\frac{1}{2}} + U(x,y)k\cos\theta + V(x,y)k\sin\theta = \omega_o \tag{3.20}$$

where $\omega_o = 2\pi/T_o$ and after substitution of Equation (3.12).

Expanding Equation (3.15) yields

$$\cos\theta\frac{\partial\theta}{\partial x} + \sin\theta\frac{\partial\theta}{\partial y} = \cos\theta\frac{1}{k}\frac{\partial k}{\partial y} - \sin\theta\frac{1}{k}\frac{\partial k}{\partial x} \tag{3.21}$$

where the wave number k is defined by the transcendental relationship (3.20). Notice that if a local coordinate system s and $\bar{n}$ as shown in Figure (3.1) are utilized, the form of Equation (3.21) becomes

$$\frac{D\theta}{Ds} = \frac{1}{k}\frac{Dk}{D\bar{n}} \tag{3.22}$$

with

$$\frac{Dx}{Ds} = \cos\theta \tag{3.23}$$

and

$$\frac{Dy}{Ds} = \sin\theta \tag{3.24}$$

where the operators of s and $\bar{n}$ are

$$\frac{D}{Ds} = \cos\theta\frac{\partial}{\partial x} + \sin\theta\frac{\partial}{\partial y} \tag{3.25}$$

and

$$\frac{D}{D\bar{n}} = -\sin\theta\frac{\partial}{\partial x} + \cos\theta\frac{\partial}{\partial y} \tag{3.26}$$

Equations (3.22), (3.23) and (3.24) are very similar to the kinematical relationships obtained by Munk and Arthur (1951) starting from Fermat's principle of minimum travel time for a water wave ray or orthogonal except that Equation (3.22) is replaced instead by

$$\frac{D\theta}{Ds} = -\frac{1}{c}\frac{Dc}{D\bar{n}} \tag{3.27}$$

where c is the phase speed of the water wave as given by Equation (3.13). In fact, if the mean current is identically zero $U = V \equiv 0$, then it can be shown that Equation (3.22) reduces exactly to (3.27) and thus (3.22), (3.23) and (3.24) are the general relationships governing the ray path with wave-current interaction.

While the form of Equations (3.22) to (3.24) appear deceptively simple such that a standard numerical computational technique such as a Runge-Kutta or similar method could be utilized, an expansion of the RHS of Equation (3.22) yields a problem. Differentiating Equation (3.20) yields

$$\frac{\partial k}{\partial x} = \left\{k\frac{\partial\theta}{\partial x}\left(U\sin\theta - V\cos\theta\right) - k\left(\cos\theta\frac{\partial U}{\partial x} + \sin\theta\frac{\partial V}{\partial x}\right) - \frac{gk^2\operatorname{sech}^2(kd)}{2[gk\tanh(kd)]^{\frac{1}{2}}}\frac{\partial d}{\partial x}\right\} \div \left\{U\cos\theta + V\sin\theta + \frac{g[kd\operatorname{sech}^2(kd) + \tanh(kd)]}{2[gk\tanh(kd)]^{\frac{1}{2}}}\right\} \tag{3.28}$$

and

$$\frac{\partial k}{\partial y} = \left\{ k \frac{\partial \theta}{\partial y} (U\sin\theta - V\cos\theta) - k\left(\cos\theta \frac{\partial U}{\partial y} + \sin\theta \frac{\partial V}{\partial y}\right) - \frac{gk^2 \operatorname{sech}^2(kd)}{2[gk\tanh(kd)]^{\frac{1}{2}}} \frac{\partial d}{\partial y} \right\} \div \left\{ U\cos\theta + V\sin\theta + \frac{g[kd\operatorname{sech}^2(kd) + \tanh(kd)]}{2[gk\tanh(kd)]^{\frac{1}{2}}} \right\} \qquad (3.29)$$

and notice that both $\frac{\partial k}{\partial x}$ and $\frac{\partial k}{\partial y}$ each have a term $\frac{\partial \theta}{\partial x}$ and $\frac{\partial \theta}{\partial y}$, respectively. Thus Equation (3.22) does not explicitly yield an expression for the ray angle θ in terms of only changes along the ray path s . Hence the valuable technique of integrating along characteristic lines is no longer valid if Equation (3.21) is to be fully solved.

3.2.2 Wave Dynamics

As indicated in the introduction, Section 3.1, the objective of this current research effort is to determine the effects of wave-current interaction on the nearshore circulation characteristics. Thus of prime interest with respect to wave dynamics is the change in wave height characteristics as the wave interacts with the nearshore current distribution. The conservation for mass, momentum and energy per unit area due to the interaction of wave motion on a variable current have been given by Longuet-Higgins and Stewart (1960, 1961) and Whitham (1962). In this section the important relationship arises from the energy balance of the fluctuating motion of a wave train in which energy dissipation is negligible.

Vertically integrating the energy balance due to the fluctuating wave train superimposed on a variable current system and averaging over time during a wave period yields the energy relationship

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_i}\left\{ E[U_i + (c_g)_i \right\} + \sigma_{i,j}\frac{\partial U_j}{\partial x_i} = 0 \qquad (3.30)$$

where

$$E = \frac{1}{8}\rho g H^2 \text{ is the energy density per unit area} \qquad (3.31)$$

$\sigma_{i,j}$ is the "radiation stress" for surface waves defined by Longuet-Higgins and Stewart (1960, 1961)

and given by

$$\sigma_{xx} = E[(2n-\tfrac{1}{2})\cos^2\theta + (n-\tfrac{1}{2})\sin^2\theta] \qquad (3.32)$$

$$\sigma_{yy} = E[(2n-\tfrac{1}{2})\sin^2\theta + (n-\tfrac{1}{2})\cos^2\theta] \qquad (3.33)$$

$$\tau_{xy} = \tau_{yx} = \frac{E}{2} n \sin(2\theta) \qquad (3.34$$

where

$$n = \left(\frac{c_g}{c}\right)_i = \frac{1}{2}\left(1 + \frac{2kd}{\sin h(2kd)}\right) \qquad (3.35)$$

Since the region of primary concern is the nearshore coastal zone especially between the breaker zone and beachline, the tendency to consider $kd << 1$ as was assumed by Noda (1972, 1973) is very strong and outwardly very reasonable. But a more careful analysis of the physical processes involved in the breaker zone deems this unwise. In particular consider the degenerate case of surface waves propagating in the $+x$ direction on a variable current $U(x)$ in infinitely deep water. In this case since $\theta = 0$ everywhere the kinematic relationship Equation (3.21) is identically satisfied and Equation (3.20) yields a quadratic equation with solution

$$c = \frac{c_o}{2}\left[1 + \left(1 + \frac{4U}{c_o}\right)^{\frac{1}{2}}\right] \qquad (3.36)$$

where the positive sign in the square root term is taken so that

$$c = c_o = c_d = \left(\frac{g}{k_o}\right)^{\frac{1}{2}} \tag{3.37}$$

when U = 0. Notice the interesting effect that no solution to Equation (3.36) can exist if $U/c_o < -\frac{1}{4}$. At the critical velocity of $U = -\frac{c_o}{4}$ the square root term becomes zero and Equation (3.36) yields

$$c = \frac{c_o}{2} \tag{3.38}$$

and

$$\frac{U}{c} = -\frac{1}{2} \tag{3.39}$$

Since the local group velocity of the deep-water wave system is

$$c_g = \frac{1}{2}c \tag{3.40}$$

then Equation (3.39) physically implies that the wave system can no longer propagate when the mean current exactly opposes the energy propagating speed of the wave system.

For this special case the energy relationship Equation (3.30) becomes

$$\frac{1}{E}\frac{dE}{dx} + \frac{1}{(U+c_g)}\frac{d(U+c_g)}{dx} + \frac{1}{2(U+c_g)}\frac{dU}{dx} = 0 \tag{3.41}$$

and the solution to (3.41) is

$$\frac{H}{H_o} = \frac{c_o}{[c(c+2U)]^{\frac{1}{2}}} \tag{3.42}$$

These results were given by Longuet-Higgins and Stewart (1961).

Extension of these concepts to the nearshore coastal zone implies that the complicated vector direction of the mean current coupled with the wave direction could yield the equivalent situation where the mean current directly opposes the local energy propagation or group velocity of the wave. In the limit as this situation is approached the local wave length will approach zero with respect to a stationary observer. Thus while the local water depth d may be small, the local wave number $k = 2\pi/L$ may become very large such that the so called "shallow water" approximation may not be valid. Hence in all subsequent theoretical formulations with wave-current interaction, no approximations are made for the magnitude of the term kd.

Expanding Equation (3.30) in cartesian coordinates yields

$$(U + c_g \cos\theta)\frac{1}{E}\frac{\partial E}{\partial x} + (V + c_g \sin\theta)\frac{1}{E}\frac{\partial E}{\partial y}$$

$$+ \frac{\partial}{\partial x}(U + c_g \cos\theta) + \frac{\partial}{\partial y}(V + c_g \sin\theta)$$

$$+ \left[\bar{\sigma}_{xx}\frac{\partial U}{\partial x} + \bar{\tau}_{yx}\frac{\partial U}{\partial y} + \bar{\tau}_{xy}\frac{\partial V}{\partial x} + \bar{\sigma}_{yy}\frac{\partial V}{\partial y}\right] = 0 \qquad (3.43)$$

where

$$\bar{\sigma}_{xx} = (2n - \tfrac{1}{2})\cos^2\theta + (n - \tfrac{1}{2})\sin^2\theta \qquad (3.44)$$

$$\bar{\sigma}_{yy} = (2n - \tfrac{1}{2})\sin^2\theta + (n - \tfrac{1}{2})\cos^2\theta \qquad (3.45)$$

and $$\bar{\tau}_{xy} = \bar{\tau}_{yx} = \frac{n}{2}\sin(2\theta) \qquad (3.46)$$

Since E is defined by Equation (3.31), substituting for E in Equation (3.43) provides directly a relationship for the wave height

$$(U + c_g \cos\theta)\frac{2}{H}\frac{\partial H}{\partial x} + (V + c_g \sin\theta)\frac{2}{H}\frac{\partial H}{\partial y} + \frac{\partial}{\partial x}(U + c_g \cos\theta)$$

$$+ \frac{\partial}{\partial y}(V + c_g \sin\theta) + \bar{\sigma} = 0 \qquad (3.47)$$

where

$$\bar{\sigma} \equiv \left[\bar{\sigma}_{xx}\frac{\partial U}{\partial x} + \bar{\tau}_{yx}\frac{\partial U}{\partial y} + \bar{\tau}_{xy}\frac{\partial V}{\partial x} + \bar{\sigma}_{yy}\frac{\partial V}{\partial y}\right] \qquad (3.48)$$

Finally expanding Equation (3.47) fully yields

$$(U + c_g \cos\theta)\frac{2}{H}\frac{\partial H}{\partial x} + (V + c_g \sin\theta)\frac{2}{H}\frac{\partial H}{\partial y} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}$$

$$- c_g \sin\theta \frac{\partial \theta}{\partial x} + \cos\theta \frac{\partial c_g}{\partial x} + c_g \cos\theta \frac{\partial \theta}{\partial y} + \sin\theta \frac{\partial c_g}{\partial y}$$

$$+ \overline{\sigma} = 0 \qquad (3.49)$$

The group velocity and wave celerity functions are given by

$$c = \left[\frac{g}{k}\tanh(kd)\right]^{\frac{1}{2}} \qquad (3.50)$$

$$c_g = \frac{c}{2}\left[1 + \frac{2kd}{\sinh(2kd)}\right] \qquad (3.51)$$

$$\frac{\partial c_g}{\partial x} = \frac{c\left[k\frac{\partial d}{\partial x} + d\frac{\partial k}{\partial x}\right]\cdot\left[\sinh(2kd) - 2kd\cosh(2kd)\right]}{\sinh^2(2kd)}$$

$$+ \frac{1}{2}\left[1 + \frac{2kd}{\sinh(2kd)}\right]\frac{\partial c}{\partial x} \qquad (3.52)$$

$$\frac{\partial c_g}{\partial y} = \frac{c\left[k\frac{\partial d}{\partial y} + d\frac{\partial k}{\partial y}\right]\cdot\left[\sinh(2kd) - 2kd\cosh(2kd)\right]}{\sinh^2(2kd)}$$

$$+ \frac{1}{2}\left[1 + \frac{2kd}{\sinh(2kd)}\right]\frac{\partial c}{\partial y} \qquad (3.53)$$

where

$$\frac{\partial c}{\partial y} = \frac{g}{2k^2 c}\left[k\,\text{sech}^2(kd)\left(k\frac{\partial d}{\partial x} + d\frac{\partial k}{\partial x}\right) - \tanh(kd)\frac{\partial k}{\partial x}\right] \qquad (3.54)$$

$$\frac{\partial c}{\partial y} = \frac{g}{2k^2 c}\left[k\,\text{sech}^2(kd)\left(k\frac{\partial d}{\partial y} + d\frac{\partial k}{\partial y}\right) - \tanh(kd)\frac{\partial k}{\partial y}\right] \qquad (3.55)$$

and where $\frac{\partial k}{\partial x}$ and $\frac{\partial k}{\partial y}$ are given by Equations (3.28) and (3.29) respectively and k defined by the solution to Equation (3.20).

To understand some of the physical processes interacting within the energy equation (3.49), it is useful to transform (3.49) in terms of the local coordinate system s along the wave ray and $\bar{n}$ along the wave front. Utilizing operators defined by Equations (3.25) and (3.26), Equation (3.49) becomes

$$\frac{1}{H}\frac{DH}{Ds} + \frac{1}{2}\frac{D\theta}{D\bar{n}} + \frac{1}{2c_g}\frac{Dc_g}{Ds} + \frac{1}{2c_g}\left[\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right] + \frac{1}{c_g H}\left[U\frac{\partial H}{\partial x} + V\frac{\partial H}{\partial y}\right]$$
$$+ \frac{\bar{\sigma}}{2c_g} = 0 \qquad (3.56)$$

If the mean current velocity is now set equal to zero, $U = V = 0$, then Equation (3.56) becomes

$$\frac{1}{H}\frac{DH}{Ds} + \frac{1}{2}\frac{D\theta}{D\bar{n}} + \frac{1}{2c_g}\frac{Dc_g}{Ds} = 0 \qquad (3.57)$$

or in terms of the energy density

$$\frac{1}{E}\frac{DE}{Ds} + \frac{D\theta}{D\bar{n}} + \frac{1}{c_g}\frac{Dc_g}{Ds} = 0 \qquad (3.58)$$

Notice that both Equations (3.57) and (3.58) describe the changes in the wave height or energy as being related to the curvature of the wave front $\frac{D\theta}{D\bar{n}}$ and to the logarithmic change in group velocity along the ray path, $\frac{1}{c_g}\frac{Dc_g}{Ds}$. In point-of-fact $\frac{D\theta}{D\bar{n}}$ describes ray refraction and $\frac{1}{c_g}\frac{Dc_g}{Ds}$ wave shoaling.

The form of Equation (3.57) suggests a form of separation of variables where

$$H = H_r(\theta)H_{sh}(c_g) \qquad (3.59)$$

and substitution of (3.59) into (3.57) yields

$$\frac{1}{H_{sh}}\frac{DH_{sh}}{Ds} + \frac{1}{2c_g}\frac{Dc_g}{Ds} + \frac{1}{H_r}\frac{DH_r}{Ds} + \frac{1}{2}\frac{D\theta}{D\bar{n}} = 0 \qquad (3.60)$$

Since H_{sh} is only a function of c_g, and H_r only a function of θ, Equation (3.60) implies that

$$\frac{1}{H_{sh}} \frac{DH_{sh}}{Ds} + \frac{1}{2c_g} \frac{Dc_g}{Ds} = C \tag{3.61}$$

and

$$\frac{1}{H_r} \frac{DH_r}{Ds} + \frac{1}{2} \frac{D\theta}{D\bar{n}} = -C \tag{3.62}$$

where C is a constant. Equation (3.61) can easily be integrated and yields a solution

$$H_{sh} = \frac{C_1}{\sqrt{c_g}} \tag{3.63}$$

where C_1 is a new constant. The solution to Equation (3.62) is much more complicated. By considering the ray separation diagram shown in Figure 3.2, Munk and Arthur (1951) have shown that

$$\frac{1}{b} \frac{Db}{Ds} = \frac{D\theta}{D\bar{n}} \tag{3.64}$$

and defining $\beta = b/b_d$, the equation for ray separation (3.65) becomes

$$\frac{1}{\beta} \frac{D\beta}{Ds} = \frac{D\theta}{D\bar{n}} \tag{3.65}$$

Now substituting Equation (3.66) into (3.62) yields

$$\frac{1}{H_r} \frac{DH_r}{Ds} + \frac{1}{2\beta} \frac{D\beta}{Ds} = -C \tag{3.66}$$

and integrating directly yields

$$H_r = \frac{C_2}{\sqrt{\beta}} \tag{3.67}$$

where C_2 is a constant.

Thus finally substituting back into Equation (3.59) produces

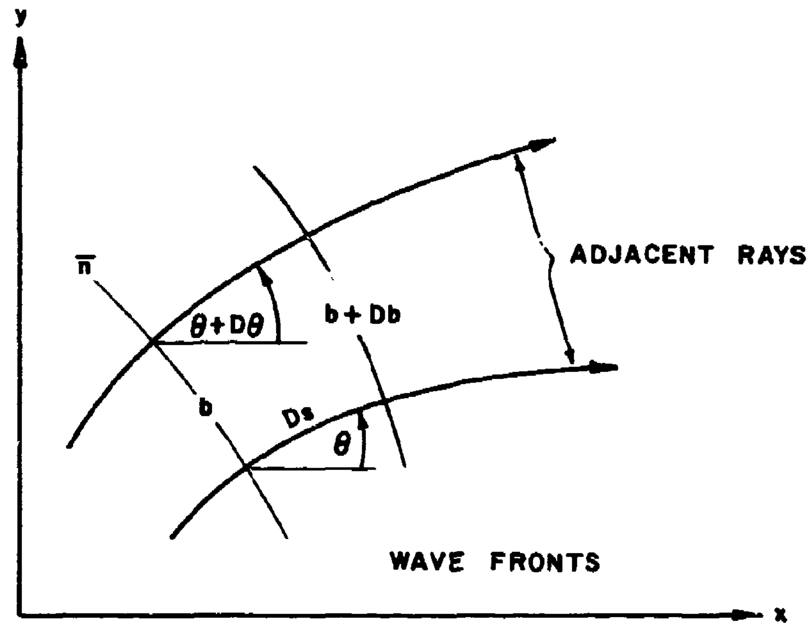


Figure 3.2: Ray and Wave Front Terminology

$$H = \frac{\bar{C}}{\sqrt{c_g \sqrt{\beta}}} \tag{3.68}$$

and in deep water $H = H_d$, $c_{g_d} = c_{g_d}$ and $\beta \to 1$ yields

$$\bar{C} = H_o \; c_{g_d} \tag{3.69}$$

and finally,

$$H = H_o \left(\frac{c_{g_d}}{c_g} \right)^{\frac{1}{2}} \frac{1}{\sqrt{\beta}} \tag{3.70}$$

where Equations (3.50) and (3.51) give

$$\left(\frac{c_{g_d}}{c_g} \right)^{\frac{1}{2}} = \left\{ \frac{1}{\tan h(kd) \left[1 + \frac{2\,kd}{\sin h(2kd)} \right]} \right\}^{\frac{1}{2}} \tag{3.71}$$

Equation (3.70) is the well known classic solution to waves undergoing transformation due to both shoaling and refraction.

The solution for H from Equation (3.70) is not fully complete since β is yet an unknown. Munk and Arthur (1951) have derived a differential equation for β , called the wave intensity

$$\frac{D^2\beta}{Ds^2} + p(s) \frac{D\beta}{Ds} + q(s)\beta = 0 \tag{3.72}$$

where

$$p(s) = -\cos\theta \left[\frac{1}{c} \frac{\partial c}{\partial x} \right] - \sin\theta \left[\frac{1}{c} \frac{\partial c}{\partial y} \right] \tag{3.73}$$

and

$$q(s) = \sin^2\theta \left[\frac{1}{c} \frac{\partial^2 c}{\partial x^2} \right] - 2 \sin\theta \cos\theta \left[\frac{1}{c} \frac{\partial^2 c}{\partial x^2} \right] + \cos^2\theta \left[\frac{1}{c} \frac{\partial^2 c}{\partial y^2} \right] \tag{3.74}$$

and solutions for β are shown in Noda (1972, 1973). In particular it can be shown that Equation (3.72) degenerates to the Snell's Law solution when $d = d(x)$ only

$$\beta = \left[\frac{\cos\theta}{\cos\theta_d}\right]^{\frac{1}{2}} \tag{3.75}$$

Returning back to the general equation (3.56), it is painfully evident that with wave-current interaction, the simple concept that the local wave height can be represented by a product of wave shoaling and wave refraction factors as described by Equation (3.59) is no longer valid. The combined dependency of refraction on shoaling and vice versa necessitates the solution of Equation (3.49) directly.

As the wave propagates from relatively deep water into shallow water or into an area where mean current conditions exist, Equation (3.49) will govern the local wave height until an instability occurs. This instability is usually wave breaking due to the effects of shoaling, refraction and wave-current interaction. In order to determine when breaking occurs it is assumed that spatial variation in U and V are sufficiently gradual so that an empirical breaking criteria is imposed, developed from the non wave-current interaction observation. The theoretical limiting wave steepness condition from Miche (1944) is

$$\frac{H_b}{L_b} = 0.142 \tanh\left(\frac{2\pi d_b}{L_b}\right) \tag{3.76}$$

where the subscript b indicates breaking conditions and the breaking wave length L_b is given by

$$L_b = \frac{2\pi}{k_b} \tag{3.77}$$

and k_b is derived for the transcendental Equation (3.20). An examination of experimental data of waves breaking over a horizontal bottom by Le Mehaute and Koh (1967) indicates a better limiting steepness criterion is

$$\frac{H_b}{L_b} = 0.12 \tanh\left(\frac{2\pi d_b}{L_b}\right) \tag{3.78}$$

Since the wave number k during wave-current interaction is obtained directly from Equation (3. 20), then Equation (3. 78) can be transformed to

$$H_b = \frac{(0.12)2\pi}{k_b} \tanh\left(\frac{d_b}{k_b}\right) \tag{3.79}$$

During computation if the solution for the local wave height H from Equation (3. 49) is less than H_b , then the wave height is H. If computation indicates that

$$H \geq H_b \tag{3.80}$$

then the local wave is considered to have broken and the empirical relationship Equation (3. 79) is imposed where $H = H_b$. Thus the effects of the mean current become critically important through the wave number k. In other words if the local mean current is in the same direction as the local wave direction then the local wave number k becomes smaller which requires a larger wave height for breaking to occur. On the other hand if the local mean current opposes the local wave direction then the local wave number k increases and the limiting local breaker height decreases. This phenomenon is easily seen at the entrances of river and estuaries when an outflowing current meets an incoming gravity wave system. The local limiting breaking wave height decreases so that even very small waves seem to "white cap" and break.

Application of the empirical breaking criterion Equation (3. 79) is indeed crude. Recent studies of breaking waves by Divoky, Le Mehaute and Lin (1970) indicate that wave breaking is dependent on a "characteristic" bottom slope and the research effort of Galvin (1969) centers on different types of breaker characteristics as a function of bottom slope. Thus the breaking criterion expressed by Equation (3. 79) hopefully indicates the major breaking process, sufficient to determine the merits of the concept of nearshore wave-induced circulation including wave-current interaction.

3.2.3 Nearshore Wave-Current Circulation Formulation

In the following theoretical formulation, the equations which describe the circulation pattern within the nearshore zone are derived. The formulation objective is to initially solve for the wave height and direction fields and subsequently for the wave-induced nearshore circulation pattern with no wave-current interaction [Noda (1972, 1973)]. The next step is to use this circulation pattern or more specifically the circulation velocity fields $U(x, y)$, $V(x, y)$ as the mean current to be input into a new calculation of the wave height and direction fields and again obtain the solution for the new wave-induced circulation pattern with wave-current interaction. In theory this iterative technique, which assumes a series of quasi-steady states, should hopefully in the limit, converge such that the circulation velocities are exactly equal to the previously imposed mean current.

The coordinate system is described in Figure 3.1. Vertically integrating the momentum and continuity equations and neglecting the nonlinear and time dependent terms yields

$$g \frac{\partial \eta}{\partial x} = - \frac{1}{\rho(\eta + d)} \left[\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right] - F_x \tag{3.81}$$

$$g \frac{\partial \eta}{\partial y} = - \frac{1}{\rho(\eta + d)} \left[\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} \right] - F_y \tag{3.82}$$

and

$$\frac{\partial}{\partial x} [u(\eta + d)] + \frac{\partial}{\partial y} [v(\eta + d)] = 0 \tag{3.83}$$

where

η is the mean water surface

ρ is the fluid density

F_x, F_y are bottom friction terms

and u, v are the new wave-induced circulation velocities.

The analysis of the bottom friction term is similar to that provided by Longuet-Higgins (1970) and a detailed formulation of this concept is given by Noda (1972) where

$$F_x = \frac{2\bar{c}Hu}{(\eta + d)T \sinh kd} \qquad (3.84)$$

and

$$F_y = \frac{2\bar{c}Hv}{(\eta + d)T \sinh kd} \qquad (3.85)$$

where

$\bar{c}$ is a constant bottom friction coefficient, usually $\bar{c} = 0.01$,

and T is the local wave period.

Assuming that

$$\eta + d \cong d \qquad (3.86)$$

and cross-differentiating Equations (3.81) and (3.82) and introducing a stream function ψ defined by

$$\frac{\partial \psi}{\partial y} \equiv -ud \qquad (3.87)$$

and

$$\frac{\partial \psi}{\partial x} \equiv +vd \qquad (3.88)$$

automatically satisfies the continuity equation (3.83) and yields the nearshore circulation equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{1}{F}\frac{\partial F}{\partial y}\frac{\partial \psi}{\partial y} + \frac{1}{F}\frac{\partial F}{\partial x}\frac{\partial \psi}{\partial x} = \frac{g}{F}\left\{ \frac{\partial}{\partial y}\left[\frac{1}{d}\left(\frac{\partial \sigma^*_{xx}}{\partial x} + \frac{\partial \sigma^*_{xy}}{\partial y}\right)\right] - \frac{\partial}{\partial x}\left[\frac{1}{d}\left(\frac{\partial \sigma^*_{yy}}{\partial y} + \frac{\partial \sigma^*_{yx}}{\partial x}\right)\right]\right\} \qquad (3.89)$$

where

$$F = \frac{2\bar{c}H}{d^2 T \sinh kd} \qquad (3.90)$$

$$\sigma^*_{xx} = \frac{H^2}{8}\left[(2n - \tfrac{1}{2}) \cos^2\theta + (n - \tfrac{1}{2})\sin^2\theta\right] \qquad (3.91)$$

$$\sigma^*_{yy} = \frac{H^2}{8}\left[(2n - \tfrac{1}{2}) \sin^2\theta + (n - \tfrac{1}{2})\cos^2\theta\right] \qquad (3.92)$$

$$\tau^*_{xy} = \tau^*_{yx} = \frac{H^2}{16}\, n \sin(2\theta) \qquad (3.93)$$

Typically since the nearshore coastal zone is of primary interest, the urge to utilize the so called "shallow water" approximation where the argument kd is considered small seems appropriate. Note the important concept distinction that vertically integrating the momentum and continuity equations is not identically synonymous to the shallow water approximation. As described in the previous section, while the water depth d may become small, the wave number k may become very large if the mean current opposes the wave ray direction and its magnitude approaches the energy propagating speed of the local wave; implying that the product kd may become very large. Thus the form of the circulation equation (3.84) should definitely consider this interactive concept. In particular the local wave period T in Equation (3.90) is not invariant to a stationary "Eulerian" observer and its variation must be considered.

Since the local wave period T is defined by

$$T = \frac{2\pi}{\omega} , \qquad (3.94)$$

then substituting Equation (3.94) into (3.90) after noting that ω is defined by Equation (3.12) yields

$$F = \frac{\bar{c}\sqrt{2g}}{\pi}\,\frac{H}{d^2}\left[\frac{k}{\sinh(2kd)}\right]^{\frac{1}{2}} \qquad (3.95)$$

and consequently its x and y derivatives are

$$\frac{\partial F}{\partial x} = \frac{\bar{c}\sqrt{2g}}{\pi}\left\{\frac{H}{2d^2}\left[\frac{\sinh(2kd)}{k}\right]^{\frac{1}{2}}\cdot\left[\frac{\sinh(2kd)\frac{\partial k}{\partial x} - 2k\left(k\frac{\partial d}{\partial x} + d\frac{\partial k}{\partial x}\right)\cosh(2kd)}{\sinh^2(2kd)}\right]\right.$$

$$\left. + \left[\frac{k}{\sinh(2kd)}\right]^{\frac{1}{2}}\left[\frac{d\frac{\partial H}{\partial x} - 2H\frac{\partial d}{\partial x}}{d^3}\right]\right\} \tag{3.96}$$

$$\frac{\partial F}{\partial y} = \frac{\bar{c}\sqrt{2g}}{\pi}\left\{\frac{H}{2d^2}\left[\frac{\sinh(2kd)}{k}\right]^{\frac{1}{2}}\cdot\left[\frac{\sinh(2kd)\frac{\partial k}{\partial y} - 2k\left(k\frac{\partial d}{\partial y} + d\frac{\partial k}{\partial y}\right)\cosh(2kd)}{\sinh^2(2kd)}\right]\right.$$

$$\left. + \left[\frac{k}{\sinh(2kd)}\right]^{\frac{1}{2}}\left[\frac{d\frac{\partial H}{\partial y} - 2H\frac{\partial d}{\partial y}}{d^3}\right]\right\} \tag{3.97}$$

and thus

$$\frac{1}{F}\frac{\partial F}{\partial x} = \frac{1}{2k}\left[\frac{\partial k}{\partial x} - \frac{2k\left(k\frac{\partial d}{\partial x} + d\frac{\partial k}{\partial x}\right)}{\tanh(2kd)}\right] + \frac{1}{Hd}\left(d\frac{\partial H}{\partial x} - 2H\frac{\partial d}{\partial x}\right) \tag{3.98}$$

and

$$\frac{1}{F}\frac{\partial F}{\partial y} = \frac{1}{2k}\left[\frac{\partial k}{\partial y} - \frac{2k\left(k\frac{\partial d}{\partial y} + d\frac{\partial k}{\partial y}\right)}{\tanh(2kd)}\right] + \frac{1}{Hd}\left(d\frac{\partial H}{\partial y} - 2H\frac{\partial d}{\partial y}\right) \tag{3.99}$$

where k is the solution to Equation (3.20).

Carrying through the derivatives of the RHS of the nearshore circulation equation (3.89) yields

$$\text{RHS [Eq. (3.84)]} = \frac{g}{F}\left\{\frac{1}{d}\left[\frac{\partial^2\sigma^*_{xx}}{\partial y\partial x} + \frac{\partial^2\tau^*_{xy}}{\partial y^2} - \frac{\partial^2\sigma^*_{yy}}{\partial x\partial y} - \frac{\partial^2\tau^*_{xy}}{\partial x^2}\right]\right.$$

$$\left. - \frac{\frac{\partial d}{\partial y}}{d^2}\left[\frac{\partial\sigma^*_{xx}}{\partial x} + \frac{\partial\tau^*_{xy}}{\partial y}\right] + \frac{\frac{\partial d}{\partial x}}{d^2}\left[\frac{\partial\sigma^*_{yy}}{\partial y} + \frac{\partial\tau^*_{xy}}{\partial x}\right]\right\} \tag{3.100}$$

and full computation of the derivatives in Equation (3.100) yields

$$\frac{\partial \sigma^*_{xx}}{\partial x} = \frac{H^2}{8}\left[-n\frac{\partial \theta}{\partial x}\sin(2\theta) + \frac{\partial n}{\partial x}(1+\cos^2\theta)\right] + \frac{2}{H}\frac{\partial H}{\partial x}\sigma^*_{xx} \tag{3.101}$$

$$\frac{\partial \sigma^*_{yy}}{\partial y} = \frac{H^2}{8}\left[n\frac{\partial \theta}{\partial y}\sin(2\theta) + \frac{\partial n}{\partial y}(1+\sin^2\theta)\right] + \frac{2}{H}\frac{\partial H}{\partial y}\sigma^*_{yy} \tag{3.102}$$

$$\frac{\partial \tau^*_{yx}}{\partial x} = \frac{H^2}{8}\left[n\frac{\partial \theta}{\partial x}\cos(2\theta) + \frac{n}{H}\frac{\partial H}{\partial x}\sin(2\theta)\right] + \frac{1}{n}\frac{\partial n}{\partial x}\tau^*_{yx} \tag{3.103}$$

$$\frac{\partial \tau^*_{xy}}{\partial y} = \frac{H^2}{8}\left[n\frac{\partial \theta}{\partial y}\cos(2\theta) + \frac{n}{H}\frac{\partial H}{\partial y}\sin(2\theta)\right] + \frac{1}{n}\frac{\partial n}{\partial y}\tau^*_{xy} \tag{3.104}$$

where

$$\frac{\partial n}{\partial x} = \frac{\left(k\frac{\partial d}{\partial x} + d\frac{\partial k}{\partial x}\right)}{\sinh(2kd)}\left[1 - \frac{2kd}{\tanh(2kd)}\right] \tag{3.105}$$

$$\frac{\partial n}{\partial y} = \frac{\left(k\frac{\partial d}{\partial y} + \frac{\partial k}{\partial y}\right)}{\sinh(2kd)}\left[1 - \frac{2kd}{\tanh(2kd)}\right] \tag{3.106}$$

and the second-order derivatives are

$$\begin{aligned}\frac{\partial^2 \sigma^*_{xx}}{\partial y \partial x} = \frac{H^2}{8}\Bigg\{ & -n\frac{\partial^2 \theta}{\partial y \partial x}\sin(2\theta) - 2n\frac{\partial \theta}{\partial x}\frac{\partial \theta}{\partial y}\cos(2\theta) - \frac{\partial \theta}{\partial x}\frac{\partial n}{\partial y}\sin(2\theta) \\ & - \frac{\partial \theta}{\partial y}\frac{\partial n}{\partial x}\sin(2\theta) + (1+\cos^2\theta)\frac{\partial^2 n}{\partial y \partial x}\Bigg\} \\ & + \frac{H}{4}\frac{\partial H}{\partial y}\left[-n\frac{\partial \theta}{\partial x}\sin(2\theta) + (1+\cos^2\theta)\frac{\partial^2 n}{\partial x}\right] \\ & + \frac{2}{H}\frac{\partial H}{\partial x}\frac{\partial \sigma^*_{xx}}{\partial y} + 2\sigma^*_{xx}\left[\frac{1}{H}\frac{\partial^2 H}{\partial y \partial x} - \frac{1}{H^2}\frac{\partial H}{\partial x}\frac{\partial H}{\partial y}\right]\end{aligned} \tag{3.107}$$

$$\frac{\partial^2 \tau^*_{xy}}{\partial y^2} = \frac{n}{8}\left\{ H^2\left[\frac{\partial^2\theta}{\partial y^2}\cos(2\theta) - 2\left(\frac{\partial\theta}{\partial y}\right)^2\sin(2\theta)\right] + 4H\,\frac{\partial\theta}{\partial y}\frac{\partial H}{\partial y}\cos(2\theta)\right.$$

$$\left. + \left[H\,\frac{\partial^2 H}{\partial y^2} + \left(\frac{\partial H}{\partial y}\right)^2\right]\sin(2\theta)\right\} + \frac{H}{8}\,\frac{\partial n}{\partial y}\left[H\,\frac{\partial\theta}{\partial y}\cos(2\theta) + \frac{\partial H}{\partial y}\sin(2\theta)\right]$$

$$+ \frac{1}{n}\,\frac{\partial n}{\partial y}\,\frac{\partial \tau^*_{xy}}{\partial y} + \tau^*_{xy}\left[\frac{1}{n}\,\frac{\partial^2 n}{\partial y^2} - \frac{1}{n^2}\left(\frac{\partial n}{\partial y}\right)^2\right] \qquad (3.108)$$

$$\frac{\partial^2 \sigma^*_{yy}}{\partial x\partial y} = \frac{H^2}{8}\left\{ n\,\frac{\partial\theta}{\partial y}\frac{\partial\theta}{\partial x}\cos(2\theta) + \frac{\partial^2\theta}{\partial x\partial y}\sin(2\theta)\right] + \frac{\partial\theta}{\partial y}\frac{\partial n}{\partial x}\sin(2\theta)$$

$$\left. + \frac{\partial n}{\partial y}\,\frac{\partial\theta}{\partial x}\sin(2\theta) + (1 + \sin^2\theta)\,\frac{\partial^2 n}{\partial x\partial y}\right\}$$

$$+ \frac{H}{4}\,\frac{\partial H}{\partial x}\left[n\,\frac{\partial H}{\partial y}\sin(2\theta) + \frac{\partial n}{\partial y}(1 + \sin^2\theta)\right]$$

$$+ \frac{2}{H}\,\frac{\partial H}{\partial y}\,\frac{\partial \sigma^*_{yy}}{\partial x} + 2\sigma^*_{yy}\left[\frac{1}{H}\,\frac{\partial^2 H}{\partial x\partial y} - \frac{1}{H^2}\,\frac{\partial H}{\partial y}\,\frac{\partial H}{\partial x}\right] \qquad (3.109)$$

$$\frac{\partial^2 \tau^*_{yx}}{\partial x^2} = \frac{n}{8}\left\{ H^2\left[\frac{\partial^2\theta}{\partial x^2}\cos(2\theta) - 2\left(\frac{\partial\theta}{\partial x}\right)^2\sin(2\theta)\right] + 4H\,\frac{\partial\theta}{\partial x}\frac{\partial H}{\partial x}\cos(2\theta)\right.$$

$$\left. + \left[H\,\frac{\partial^2 H}{\partial x^2} + \left(\frac{\partial H}{\partial x}\right)^2\right]\sin(2\theta)\right\} + \frac{H}{8}\,\frac{\partial n}{\partial x}\left[H\,\frac{\partial\theta}{\partial x}\cos(2\theta) + \frac{\partial H}{\partial x}\sin(2\theta)\right]$$

$$+ \frac{1}{n}\,\frac{\partial n}{\partial x}\,\frac{\partial \tau^*_{yx}}{\partial x} + \tau^*_{yx}\left[\frac{1}{n}\,\frac{\partial^2 n}{\partial x^2} - \frac{1}{n^2}\left(\frac{\partial n}{\partial x}\right)^2\right] \qquad (3.110)$$

where

$$\frac{\partial \sigma^*_{xx}}{\partial y} = \frac{H^2}{8}\left[-n\frac{\partial \theta}{\partial y}\sin(2\theta) + (1 + \cos^2\theta)\frac{\partial n}{\partial y}\right] + \frac{2}{H}\frac{\partial H}{\partial y}\sigma^*_{xx} \tag{3.111}$$

and

$$\frac{\partial \sigma^*_{yy}}{\partial x} = \frac{H^2}{8}\left[n\frac{\partial \theta}{\partial x}\sin(2\theta) + (1 + \sin^2\theta)\frac{\partial n}{\partial x}\right] + \frac{2}{H}\frac{\partial H}{\partial x}\sigma^*_{yy} \ . \tag{3.112}$$

Notice all derivatives and parameters have been specified except for second-order derivatives in n , i.e. $\frac{\partial^2 n}{\partial x^2}$, $\frac{\partial^2 n}{\partial y^2}$ and $\frac{\partial^2 n}{\partial x \partial y}$. While algebraically feasible, these derivatives would contain 2nd order derivatives of k with respect to x and y and considering the complicated form of Equation (3. 28) and (3. 29), it was decided to use central differences of the first-order derivatives to compute the second-order derivatives of n.

Finally the RHS of Equation (3. 89) and the variable coefficients of the first-order derivatives of ψ are completely specified such that providing sufficient and necessary boundary conditions, the stream function can be found by utilizing an iterative relaxation technique.

3.3 Numerical Analysis

This section deals with the numerical techniques developed to determine, first, the wave characteristics of direction and height due to wave-mean current interaction, and second, to utilize these results to determine the resulting wave-current induced circulation pattern in the nearshore coastal zone. Hence, essentially two separate programs exist--one to obtain the wave local height and direction, and the second to solve for the stream function ψ once the wave height and direction fields are known.

In a previous study [Noda (1972, 1973)], the solution to wave-induced nearshore circulation was found excluding the effects of wave-current interaction. In that study, the wave height and direction fields were obtained by integrating along characteristic lines by utilizing a fourth-order Runge-Kutta scheme. As discussed previously, with wave-current interaction, characteristic lines no longer exist and thus a completely new numerical technique was developed. This technique first solves the kinematics problem directly on numerical grid points i, j and yields the ray directional field θ at all nodal points. Then the energy equation (3.49) is solved directly on the same numerical grid points i, j to obtain the wave height field.

The numerical technique discussed above to determine the wave characteristics for wave-current interaction makes important use of the fact that prototype data indicates a longshore periodicity of both the wave-induced circulation pattern and the bottom bathymetry such that

$$d(x,y) = d(x, y + m\lambda) \quad \text{where} \quad |m| = 1, 2, 3 \tag{3.113}$$

This longshore periodicity leads to the key boundary condition which emits the development of a highly efficient numerical algorithm for the solution of wave characteristics including wave-current interaction in the nearshore zone.

While longshore periodicity proved to be a valuable tool for the work herein, nevertheless, the method of integrating along characteristic lines is such a generally powerful method that continuing research efforts should be extended to be able to utilize this technique for wave-current interaction, even as an approximate approach.

3.3.1 Numerical Solution For Wave Characteristics With Wave-Current Interaction on a Longshore Periodic Beach

Figure 3.3 schematically describes a longshore periodic beach with wavelength λ. Equation 3.21 describes the variation of the wave direction field θ as a function of x, y, U, V, and T where the wave number k is defined by Eq. 3.20. Rewrite Eq. 3.21 in the form

$$\cos\theta\left[\frac{\partial\theta}{\partial x} - \frac{1}{k}\frac{\partial k}{\partial y}\right] + \sin\theta\left[\frac{\partial\theta}{\partial y} + \frac{1}{k}\frac{\partial k}{\partial x}\right] = 0 \qquad (3.114)$$

and rewrite the derivatives of k from Eqs. 3.28 and 3.29 as:

$$\frac{1}{k}\frac{\partial k}{\partial x} = \frac{\partial\theta}{\partial x}\frac{[U\sin\theta - V\cos\theta]}{A} + \overline{\frac{1}{k}\frac{\partial k}{\partial x}} \qquad (3.115)$$

$$\frac{1}{k}\frac{\partial k}{\partial y} = \frac{\partial\theta}{\partial y}\frac{[U\sin\theta - V\cos\theta]}{A} + \overline{\frac{1}{k}\frac{\partial k}{\partial y}} \qquad (3.116)$$

where $$A = U\cos\theta + V\sin\theta + \frac{1}{2}\left[1 + \frac{2kd}{\sinh(2kd)}\right]\left[\frac{T_o}{k} - U\cos\theta - V\sin\theta\right] \qquad (3.117)$$

$$\overline{\frac{1}{k}\frac{\partial k}{\partial x}} = \frac{-\left[\frac{\partial U}{\partial x}\cos\theta + \frac{\partial V}{\partial x}\sin\theta\right] - \frac{\left[T_o - Uk\cos\theta - Vk\sin\theta\right]\frac{\partial d}{\partial x}}{\sinh(2kd)}}{A} \qquad (3.118)$$

$$\overline{\frac{1}{k}\frac{\partial k}{\partial y}} = \frac{-\left[\frac{\partial U}{\partial y}\cos\theta + \frac{\partial V}{\partial y}\sin\theta\right] - \frac{\left[T_o - Uk\cos\theta - Vk\sin\theta\right]\frac{\partial d}{\partial y}}{\sinh(2kd)}}{A} \qquad (3.119)$$

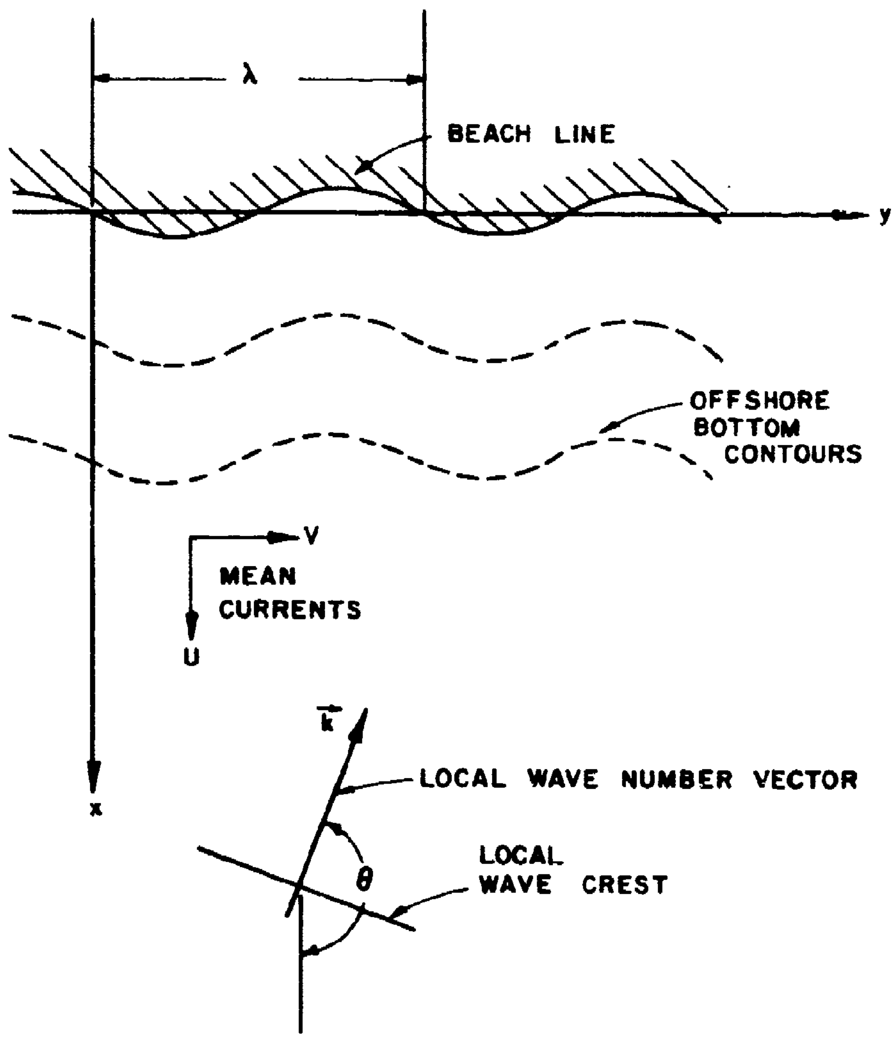


Figure 3.3: **Schematic Illustration of Periodic Beach Terminology**

Now Eq. 3.114 can be written:

$$\frac{\partial\theta}{\partial x}\left[\cos\theta + \frac{\sin\theta\,(U\sin\theta - V\cos\theta)}{A}\right] + \frac{\partial\theta}{\theta y}\left[\sin\theta - \frac{\cos\theta\,(U\sin\theta - V\cos\theta)}{A}\right] =$$

$$\overline{\frac{1}{k}\frac{\partial k}{\partial y}}\cos\theta - \overline{\frac{1}{k}\frac{\partial k}{\partial x}}\sin\theta \qquad (3.120)$$

Thus, at any grid point i, j utilize a forward difference derivative in x and a backward difference derivative in y and solving for $\theta_{i,j}$ yields

$$\theta_{i,j} = \frac{1}{B_{i,j}}\left\{\overline{\frac{1}{k}\frac{\partial k}{\partial y}}\cos\theta_{i,j} - \overline{\frac{1}{k}\frac{\partial k}{\partial x}}\sin\theta_{i,j} + \frac{\theta_{i,j-1}}{\Delta y}\left[\sin\theta_{i,j} - \frac{\cos\theta_{i,j}}{A_{i,j}}\right.\right.$$

$$\left.\left.(U\sin\theta_{i,j} - V\cos\theta_{i,j})\right] - \frac{\theta_{i+1,j}}{\Delta x}\left[\cos\theta_{i,j}(U\sin\theta_{i,j} - V\cos\theta_{i,j})\right]\right\}$$

$$(3.121)$$

where

$$B_{i,j} = \frac{\sin\theta_{i,j}}{\Delta y} - \frac{\cos\theta_{i,j}}{\Delta x} - \frac{(U\sin\theta_{i,j} - V\cos\theta_{i,j})}{A}\left[\frac{\cos\theta_{i,j}}{\Delta y} + \frac{\sin\theta_{i,j}}{\Delta x}\right]$$

$$(3.122)$$

and $\overline{\frac{1}{k}\frac{\partial k}{\partial y}}$ and $\overline{\frac{1}{k}\frac{\partial k}{\partial x}}$ are evaluated at i, j .

The RHS of Eq. 3.121 contains terms $\cos\theta_{i,j}$ and $\sin\theta_{i,j}$ and a variety of approximations for these terms can be utilized to update $\theta_{i,j}$. The simplest would be to use the previous value of $\theta_{i,j}$. A more sophisticated approach is to approximate these sinusoidal function at i, j in terms of the four surrounding grid point and going to 2nd order using Taylor series expansions yields

$$\sin\theta_{i,j} = \frac{1}{4}\left[\sin\theta_{i+1,j} + \sin\theta_{i-1,j} + \sin\theta_{i,j+1} + \sin\theta_{i,j-1}\right]$$

$$+ \frac{1}{8}\Big[(\theta_{i+1,j} - \theta_{i-1,j})(\cos\theta_{i-1,j} - \cos\theta_{i+1,j})$$

$$+ (\theta_{i,j+1} - \theta_{i,j-1})(\cos\theta_{i,j-1} - \cos\theta_{i,j+1})\Big] \qquad (3.123)$$

and

$$\cos\theta_{i,j} = \frac{1}{4}\left[\cos\theta_{i+1,j} + \cos\theta_{i-1,j} + \cos\theta_{i,j+1} + \cos\theta_{i,j-1}\right]$$

$$+ \frac{1}{8}\Big[(\theta_{i+1,j} - \theta_{i-1,j})(\sin\theta_{i+1,j} - \sin\theta_{i-1,j})$$

$$+ (\theta_{i,j+1} - \theta_{i-1,j})(\sin\theta_{i,j+1} - \sin\theta_{i,j-1})\Big] \qquad (3.124)$$

The need for the sophistication of Eqs. 3. 123 and 3. 124 is questionable, although utilizing this scheme to iterate for $\theta_{i,j}$ yields amazing results. For instance, if $d = d(x) = mx$ (a plane beach), then starting with a boundary condition offshore and setting the initial θ field equal to π, a solution using Eq. 3. 121 will converge to within 1% of the Snell's Law solution after only two iterations through the full field ! Note that it is not necessary to relax over the whole field, but the solution could have been obtained by iterating row by row or along i = constant , Figure 3. 4, working inward toward the beach. In this case, the approximation given in Eqs. 3. 123 and 3. 124 for $\sin\theta_{i,j}$ and $\cos\theta_{i,j}$ should not contain values of θ on row $i-1$. Nevertheless, the extremely rapid convagence of this technique is sufficient to justify the numerical algorithm. The row by row technique will be utilized to find the wave height field, shown later in this section.

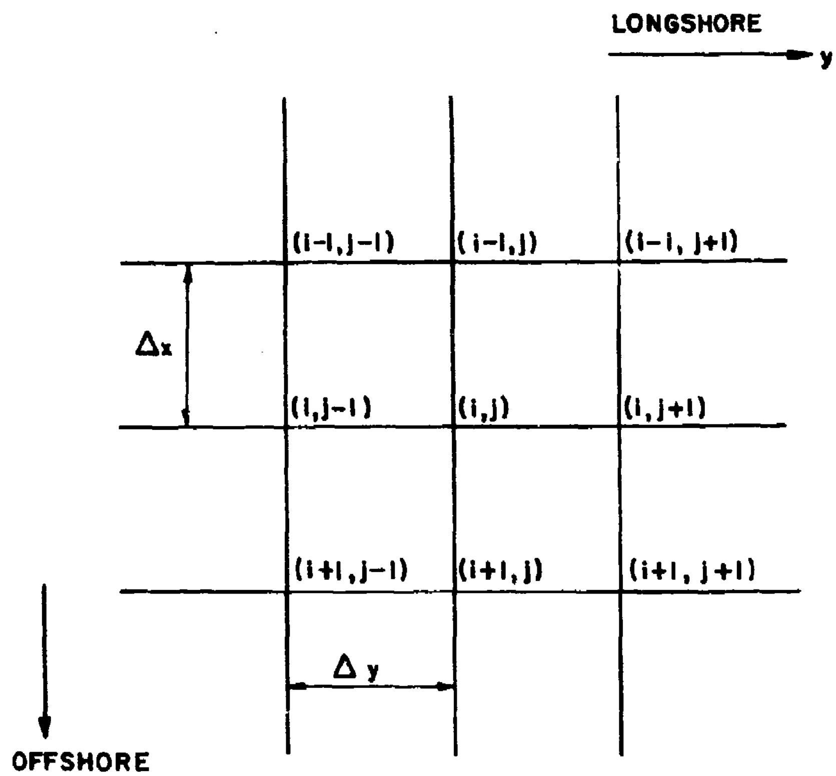


Figure 3.4: Local Grid Description

Before further exploring the numerical techniques, the important longshore boundary conditions should be examined. Figure 3.5 is a grid description of the entire field of calculation. Notice that while the beach is periodic in λ or from $j = 2$ to $j = r + 1$, a column has been added on either side. Thus, computations for either θ, H or ψ, are only performed inside of the boundary lines from $i = 2$ to $i = m-1$ and $j = 2$ to $j = r + 1$. To utilize the important longshore periodic boundary condition, for any computed variable f along each row i the following conditions are imposed as the computation proceeds toward the shore line in decreasing values of i:

$$f_{i,\, r+1} = f_{i,\, 2} \tag{3.125}$$

$$f_{i,\, r+2} = f_{i,\, 3} \tag{3.126}$$

$$f_{i,\, 1} = f_{i,\, r} \tag{3.127}$$

Imposing conditions 3.125 to 3.127 upon θ, H or ψ physically requires that the functions and moreover its derivatives are continuous and periodic in λ. Hence, iteration continues for $\theta_{i,j}$ until every updated value of $\theta_{i,j}$ is less than a specified percent of $\theta_{i,j}$ itself. A typical run will converge in between 3 to 7 iterations with a maximum relative error for each $\theta_{i,j}$ of 0.1% of itself.

At this point, it should be noted that for each updated value of $\theta_{i,j}$ a transcendental relationship for k must be solved defined by Eq. 3.20. To efficiently solve for k the Newton-Raphson method was utilized where

$$e(k) \equiv gk \tanh(kd_{i,j}) - \left[T_o - Uk\cos\theta_{i,j} - Vk\sin\theta_{i,j}\right]^2 \tag{3.128}$$

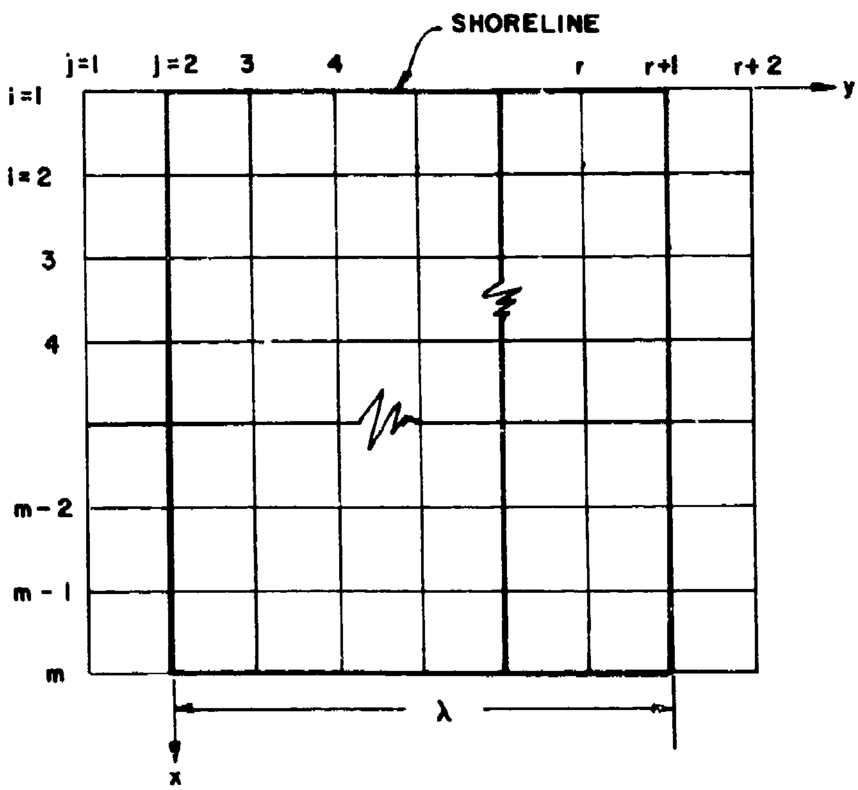


Figure 3.5: Full Grid Description

and

$$\frac{de}{dk} = e'(k) = g\left[kd_{i,j}\,\mathrm{sech}^2(kd_{i,j}) + \tanh(kd_{i,j})\right]$$

$$+ 2\left[U\cos\theta_{i,j} + V\sin\theta_{i,j}\right]\left[T_o - Uk\cos\theta_{i,j} - Vk\sin\theta_{i,j}\right] \quad (3.129)$$

Hence,

$$k_{new} = k_{old} - \frac{e\left(k_{old}\right)}{e'\left(k_{old}\right)} \quad (3.130)$$

and this iteration was performed until

$$\left|k_{new} - k_{old}\right| \le 0.0001\left|k_{new}\right| \quad (3.131)$$

Computation starts far offshore where the periodic beach $d(x, y)$ is defined to be a plane beach $d = d(x) = ax$ starting at $i = m$ and offshore from this row. On the row $i = m$, the local wave angle is specified in reference to a chosen deep water wave angle, using Snell's Law. From this point, computation immediately begins with Eq. 3.121 and the output is the direction $\theta_{i,j}$ for a given relative accuracy.

The next series of calculations solves for the wave height field. Rewrite Eq. 3.49 to yield

$$(U + c_g\cos\theta)\frac{\partial H}{\partial x} + (V + c_g\sin\theta)\frac{\partial H}{\partial y} = \frac{H}{2}\Big\{c_g\sin\theta\frac{\partial\theta}{\partial x} - c_g\cos\theta\frac{\partial\theta}{\partial y} - \left[\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right] - \cos\theta\frac{\partial c_g}{\partial x} - \sin\theta\frac{\partial c_g}{\partial y} - \bar{\sigma}\Big\} \quad (3.132)$$

Define

$$Q_{i,j} = \frac{1}{2}\left\{c_g \sin\theta \frac{\partial\theta}{\partial x} - c_g \cos\theta \frac{\partial\theta}{\partial y} - \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) - \cos\theta \frac{\partial c_g}{\partial x} - \sin\theta \frac{\partial c_g}{\partial y} - \sigma\right\}_{i,j} \quad (3.133)$$

where $\left(\frac{\partial\theta}{\partial x}\right)_{i,j}$ and $\left(\frac{\partial\theta}{\partial y}\right)_{i,j}$ are obtained by central differences,

and taking a forward difference derivative in x and a backward difference in y and solving for $H_{i,j}$ yields:

$$H_{i,j} = \frac{\dfrac{(V + c_g \sin\theta)\, H_{i,j-1}}{\Delta y} - \dfrac{(U + cg \cos\theta)\, H_{i+1,j}}{\Delta x}}{\dfrac{(V + c_g \sin\theta)}{\Delta y} - \dfrac{(U + c_g \cos\theta)}{\Delta x} - \dfrac{Q_{i,j}}{2}} \quad (3.134$$

In the computation for the wave height $H_{i,j}$, a row by row relaxation technique is utilized starting on row $i = m-1$, and proceeding inward to row $i = 2$. On each row, the boundary conditions 3.125 and 3.127 are utilized and the solution is reached when

$$|H_{new} - H_{old}| \le (0.001)\, |H_{new}| \quad (3.135)$$

The convergence of this scheme is amazingly rapid for even the most complicated bottom bathymetry and mean velocity distribution. Usually, only 2 to 3 row iterations are required to meet the criterion defined by Eq. 3.135.

Similar to the θ calculations, the wave height calculations start at row $i = m-1$ where values of the wave height on row $i = m$ are used, derived from the Snell's Law relationship. During each update calculation of $H_{i,j}$, the breaking height is also calculated, and if

$H_{i,j}$ exceeds this value, then the wave breaking height is automatically imposed and a flag is set to denote if the breaking height condition is being utilized at the point i,j . Computer outputs of this breaking index will be shown later.

Now that the H and θ field has been specified, the numerical algorithm proceeds to the computation of the stream function field ψ. The technique used to find the stream function is very similar to the technique used by [Noda (1972, 1973)]. Equation 3.89 is rewritten

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \left(\frac{1}{F}\frac{\partial F}{\partial x}\right)\frac{\partial\psi}{\partial y} + \left(\frac{1}{F}\frac{\partial F}{\partial y}\right)\frac{\partial\psi}{\partial y} = W \qquad (3.136)$$

where

$$W = \frac{g}{F}\left\{\frac{1}{d}\left[\frac{\partial^2\sigma^*_{xx}}{\partial y\partial x} + \frac{\partial^2\tau^*_{xy}}{\partial y^2} - \frac{\partial^2\sigma^*_{yy}}{\partial x\partial y} - \frac{\partial^2\tau^*_{xy}}{\partial x^2}\right] - \frac{\frac{\partial d}{\partial y}}{d^2}\left[\frac{\partial\sigma^*_{xx}}{\partial x} + \frac{\partial\tau^*_{xy}}{\partial y}\right] + \frac{\frac{\partial d}{\partial x}}{d^2}\left[\frac{\partial\sigma^*_{yy}}{\partial y} + \frac{\partial\tau^*_{xy}}{\partial x}\right]\right\} \qquad (3.137)$$

Utilizing a central difference form for the ψ derivatives, and gathering terms of $\psi_{i,j}$ yields:

$$\begin{aligned}\psi_{i,j} = \frac{1}{2\left[1+\left(\frac{\Delta x}{\Delta y}\right)^2\right]}\Bigg\{ & -W_{i,j}(\Delta x)^2 + \psi_{i-1,j}\left[1 - \frac{\left(\frac{1}{F}\frac{\partial F}{\partial x}\right)_{i,j}\Delta x}{2}\right] \\ & + \psi_{i+1,j}\left[1 + \frac{\left(\frac{1}{F}\frac{\partial F}{\partial x}\right)_{i,j}\Delta x}{2}\right] \\ & + (\Delta x)^2\,\psi_{i,j-1}\left[\frac{1}{(\Delta y)^2} - \frac{\left(\frac{1}{F}\frac{\partial F}{\partial y}\right)_{i,j}}{2\Delta y}\right] \\ & + (\Delta x)^2\,\psi_{i,j+1}\left[\frac{1}{(\Delta y)^2} + \frac{\left(\frac{1}{F}\frac{\partial F}{\partial y}\right)_{i,j}}{2\Delta y}\right]\Bigg\} \end{aligned} \qquad (3.138)$$

Note that before computation of ψ begins, the n field and its derivatives are first computed. Thus, the second-order derivatives for n can then be calculated from the first-order derivatives using central differences.

The longshore boundary conditions for ψ are given by Eqs. 3.125 and 3.127, and at the beachline ψ is defined to be $\psi = 0$. The final boundary condition is to move the final offshore grid row sufficiently far from the nearshore zone so that its influence on the nearshore circulation pattern is small. At this final offshore boundary $i = mm$, the stream function is again defined to be $\psi = 0$. With these boundary condition the iteration for ψ can begin using Eq. 3.138, and the criterion of convergence is assumed when

$$|\psi_{new} - \psi_{old}| \leq 0.001 |\psi_{new}| \quad (3.138)$$

In comparison to the numerical techniques for θ and H, the convergence of the ψ field requires more extensive iterating, usually 300 to 400 complete field relaxations before the solution converges as defined by Eq. 3.138. Note that condition 3.138 is not a completely sufficient condition to assure convergence since the solution may be asymptotically convergent or perhaps even divergent. Therefore, value of the relative error are varied to insure that Eq. 3.138 does indeed provide an acceptable convergence criterion.

In the zone between $i = m$ and $i = mm$, a plane beach profile is assumed so that calculation for θ and H are explicitly determined from the Snell's Law relationships once deep-water wave characteristics are defined.

3.3.2 Verification of the Wave-Current Interaction Algorithms

To test the numerical algorithm for the determination of the wave characteristics, the degenerate case of a wave propagating in the -x direction in deep water over a variable current system U(x) was utilized. The solution to this case was first given by Longuet-Higgins and Stewart (1961). For the case of a wave propagating in the -x direction, $\theta = 180^{o}$ and the analytic solution for the wave celerity is:

$$c = \frac{c_o}{2}\left[1 + (1 - \frac{4U}{c_o})\right] \quad (3.139)$$

where

$$c_o = \frac{T_o}{k_o} = \left(\frac{g}{k_o}\right)^{\frac{1}{2}} \quad (3.140)$$

but since

$$k_o = \frac{\omega_o^2}{g} \quad (3.141)$$

$$c_o = \frac{g}{T_o} = \frac{gT_o}{2\pi} \quad (3.142)$$

The wave height is given by:

$$H = \frac{H_o c_o}{[c(c - 2U)]^{1/2}} \quad (3.143)$$

A simple form for the mean current velocity was chosen to be

$$U(x) = 0.2x + \overline{b} \quad (3.144)$$

where $\frac{dU}{dx} = 0.2$.

Essentially, two verification tests were performed. First, the energy equation 3.49 was degenerated into an ordinary differential equation given by:

$$\frac{dH}{dx} = \frac{H}{2} \frac{\left[-\frac{3}{2}\frac{dU}{dx} + \frac{dc_g}{dx}\right]}{U - c_g} \tag{3.145}$$

and solved by a 4th order Runge-Kutta technique. Second, the finite difference Eq. 3.134 was utilized directly to obtain a solution for H. The final comparison was to check each solution directly with the analytic solution Eq. 3.143. Table 3.1 shows the analytic solutions for the wave celerity c and wave height H from Eqs. 3.139 and 3.143, respectively, for $T_o = 4$ seconds.

An arbitrary offshore point was chosen so that $U(x) = 0$ and $H_o = 1.0$ and as the solution proceeded shoreward both the Runge-Kutta technique and the finite difference scheme did not yield results comparable with the analytic solution. Table 3.2 describes the numerical solutions. While the difference between the Runge-Kutta solution with step size $dx = -0.1$ m and the finite difference scheme may possibly be acceptable, comparison to Table 3.1, the analytic solution was unacceptable. The problem is due to the unrealistic starting conditions imposed at $U(x) = 0$ where $\frac{dHo}{dx}$ is assumed to be zero. But there is an obvious discontinuity in the derivative of $\frac{dU}{dx}$ at the point where $U(x) = 0$ if $\frac{dHo}{dx}$ is set equal to zero. Thus, a second series of calculations were performed at a start point where $U(x) = -1.0$ m/sec. The Runge-Kutta program then computed its own starting derivative and the starting wave height was obtained from the analytic solution $H_{start} = 0.77475$ meters. Table 3.3 describes the results. Comparison of the Runge-Kutta solution with the analytic results of Table 3.1 show an exact correlation and the results of the finite difference solution compare very closely with the Runge-Kutta solutions. Appendix A contains the Runge-Kutta computer program for the above case.

TABLE 3.1

Analytic Solution For Wave-Current Interaction: Check Case

T_o = 4 seconds, g = 9.80621, k_o = 0.25161618, c_o = 6.2428272

H/H_o Eq. (3.143)	c (m/sec) Eq. (3.139)	U (m/sec)
1.615187	4.992348	1.0
3.761628	3.737028	1.5
6.417806	3.379949	1.55
13.424308	3.187839	1.56
1.0	6.242827	0.0
0.774750	7.119669	-1.0
0.648222	7.836164	-2.0
0.564575	8.457301	-3.0
0.504132	9.013317	-4.0
0.457911	9.521207	-5.0
0.421147	9.991652	-6.0

TABLE 3.2

Numerical Results for the Wave Height H
For the Test Case

Mean Current U (m/sec)	Runga-Kutta Solution $\frac{dHo}{dx} = 0.0$		Finite Difference Solution
	Step-Size $dx = -1.0m$	Step Size $dx = -0.1m$	Grid Size $dx = 0.01m$
0	1.0	1.0	1.0
-0.02		0.99471	0.99365
-0.04		0.98848	0.98742
-0.06		0.98232	0.98129
-1.0	0.78302	0.77558	
-2.0	0.65514	0.64891	
-3.0	0.57060	0.56518	
-4.0	0.50952	0.50467	

TABLE 3. 3

Numerical Results for the Wave Height H
For the Test Case

Mean Current U (m/sec)	Runga-Kutta Solution $\frac{dHo}{dx} = 0.0$		Finite Difference Solution
	Step-Size dx = -0. 10m	Step-Size dx = 0. 01 m	Grid Size dx = 0. 005m
-1. 0	0. 77475	0. 77475	0. 77475
-1. 02	0. 77156	0. 77156	0. 77156
-1. 04	0. 76840	0. 76840	
-1. 06	0. 76527	0. 76528	
-2	0. 64822		
-3	0. 56457		
-4	0. 50413		
-5	0. 45791		

3.3.3 Numerical Results for Wave-Current Interaction

The periodic bottom bathymetry used to test the influence of wave-current interaction is given by:

$$d(x,y) = mx\left[1 + ae^{\left(\frac{-x^{1/3}}{b}\right)}\sin^{10}\frac{\pi}{\lambda}(y - x\tan\alpha)\right] \quad (3.146)$$

where the first-order derivatives are given by:

$$\frac{\partial d}{\partial x} = m - \frac{10\pi max}{\lambda}\tan\alpha\, e^{\left(\frac{-x^{1/3}}{b}\right)}\sin^{9}\frac{\pi}{\lambda}(y - x\tan\alpha)\cos\frac{\pi}{\lambda}(y - x\tan\alpha)$$

$$+ mae^{\left(\frac{-x^{1/3}}{b}\right)}\sin^{10}\frac{\pi}{\lambda}(y - x\tan\alpha)\left[1 - \frac{x^{1/3}}{3b}\right] \quad (3.147)$$

and

$$\frac{\partial d}{\partial y} = \frac{10\pi max}{\lambda}e^{\left(\frac{-x^{1/3}}{b}\right)}\sin^{9}\frac{\pi}{\lambda}(y - x\tan\alpha)\cos\frac{\pi}{\lambda}(y - x\tan\alpha) \quad (3.148)$$

where the constants are given by:

$m = 0.025$, $a = 20$ meters, $\lambda = 80$ meters

$\alpha = 30°$, $b = \frac{(20)^{1/3}}{3}$ meters$^{1/3}$

The computation starts by first assuming no wave-current interaction exists such that initially $U = V = 0$. The wave height H and θ fields are obtained and the stream function ψ solved for. The algorithm then computes the circulation velocities defined by Eqs. 3.87 and 3.88, using central differences. These computed circulation velocities are now the mean current system which must now interact with the original incoming wave system.

Attempts to directly impose this derived mean current system in an interaction process with the incoming waves leads to failures of the technique because the mean current system derived for no wave-current interaction is too large. Hence, conditions arise where the local wave is no longer able to propagate into some areas and k passes through zero and becomes negative.

It is important to understand the specific processes involved in the numerical algorithm. In particular, under prototype conditions as a constant wave system begins to attack a coastal area, an instantaneous interplay of wave characteristics, bottom sediment movement and wave-generated current effects exist simultaneously until some type of "equilibrium" condition exists where the dynamic and kinematic requirements are all satisfied. The numerical model does not allow changes in bottom configuation. Moreover, the nearshore circulation system may be sensitive to large changes in the wave height and direction field due to the instantaneous application of the full mean current system derived from the noninteractive case. Thus, attempts were made to proceed much more slowly by multiplying the noninteractive current field by a constant less than one. This still preserved the continuity conditions of the mean flow, but yet allowed interaction to take place. This technique then envisioned some type of step by step series of quasi-steady circulation solutions until the full current system could be applied and the interactive results yields the input current system.

Unfortunately, while seemingly a logical procedure, the wave-current interactive system is sensitive to the rate at which the mean current is applied. Presently, the maximum interactive current applicable to wave-current interaction is about 50% of the noninteractive circulation system.

Figure 3. 6 graphically shows the stream function solution ψ for the no wave-current interaction case where $H_d = 1.0$ meters, $T_d = 4$ sec., $\theta_d = 150°$, and the depth is given by Eq. 3. 146. Tables 3. 4 through 3. 8 provide the wave direction θ , wave height H , breaking index IB, u velocity and v velocity field for this case. Notice the existence of the counter eddy field defined where $\psi < 0$, which is the degeneration of the normal incidence negative ψ field. Also, all above and following tables will only include the region of data where Eq. 3. 146 applies. Offshore from this row at x = 200 meters, the Snell's Law region is utilized and the H and θ field are easily derived. For all cases herein, the maximum distance offshore is x = 345 meters. The maximum meandering "rip-current" velocity is about 2. 7 meters/sec.

Figure 3. 7 is the solution for the stream function field ψ with wave-current interaction where only 20% of the noninteractive current velocities from the solution in Figure 3. 6 were utilized. These noninteractive velocities are shown in Tables 3. 7 and 3. 8. Notice that the counter eddy field has decreased a little but the general pattern of the circulation pattern remains much the same as the noninteractive case. Tables 3. 9 through 3. 13 provide the values of θ, H, IB, u and v for this case. For this case, an examination of Tables 3. 12 and 3. 13 shows that the maximum meandering rip-current velocity has been reduced slightly to a maximum value of about 2. 5 meters/sec.

Figure 3. 8 is the solution when for the stream function field ψ with wave-current interaction when 50% of the noninteractive current velocity field from Tables 3. 7 and 3. 8 are utilized directly. Tables 3. 14 through 3. 18 provide the resulting data for the spacial fields θ , H , IB , u and v .

The resulting stream function pattern from Figure 3. 8 is indeed startling and unexpected. The counter eddy field has now become stronger, and while the outgoing meandering rip current is similar to the previous case, a very strong inflowing meandering current now exists. The maximum magnitude of this in rip-current is about 4. 1 meter/sec. Presently, it is unclear what is causing this sudden change in the circulation pattern, since the 20% interaction case produces no significant effects.

One possible influence could be the sensitivity of the circulation pattern to the imposed bottom topography. In reality, the interaction of the ability of the bottom to change form in accord with the current intensity may be as critical a factor of consideration when the full wave-current interaction problem is considered. Another possibility is that the stream function solution is sensitive to the magnitude of the mean current and too intense a current produces spurious results.

Figure 3. 9 shows the stream function solution ψ when the wave height H_d has been reduced to $H_d = 0.5$ meters for no wave-current interaction. The circulation pattern appears reasonable with respect to the bottom contours with a maximum outflowing velocity of about 1. 6 meters/sec. Tables 3. 19 through 3. 23 describe the resulting spacial variables θ , H , IB , u , and v field respectively.

Figure 3. 10 shows the stream function solution ψ with a wave-current interaction of 50% of the noninteraction case described in Figure 3. 9. For this case, 50% of the velocity field shown in Tables 3. 22 and 3. 23 were interacted with the original incoming wave system and the results for the variables θ , H , IB , u , and v fields are given in Tables 3. 24 to 3. 28, respectively. Again as in Figure 3. 8, the results show extreme changes in the circulation pattern from the noninteractive case. The maximum in rip-current velocity is about 2. 6 meters/sec.

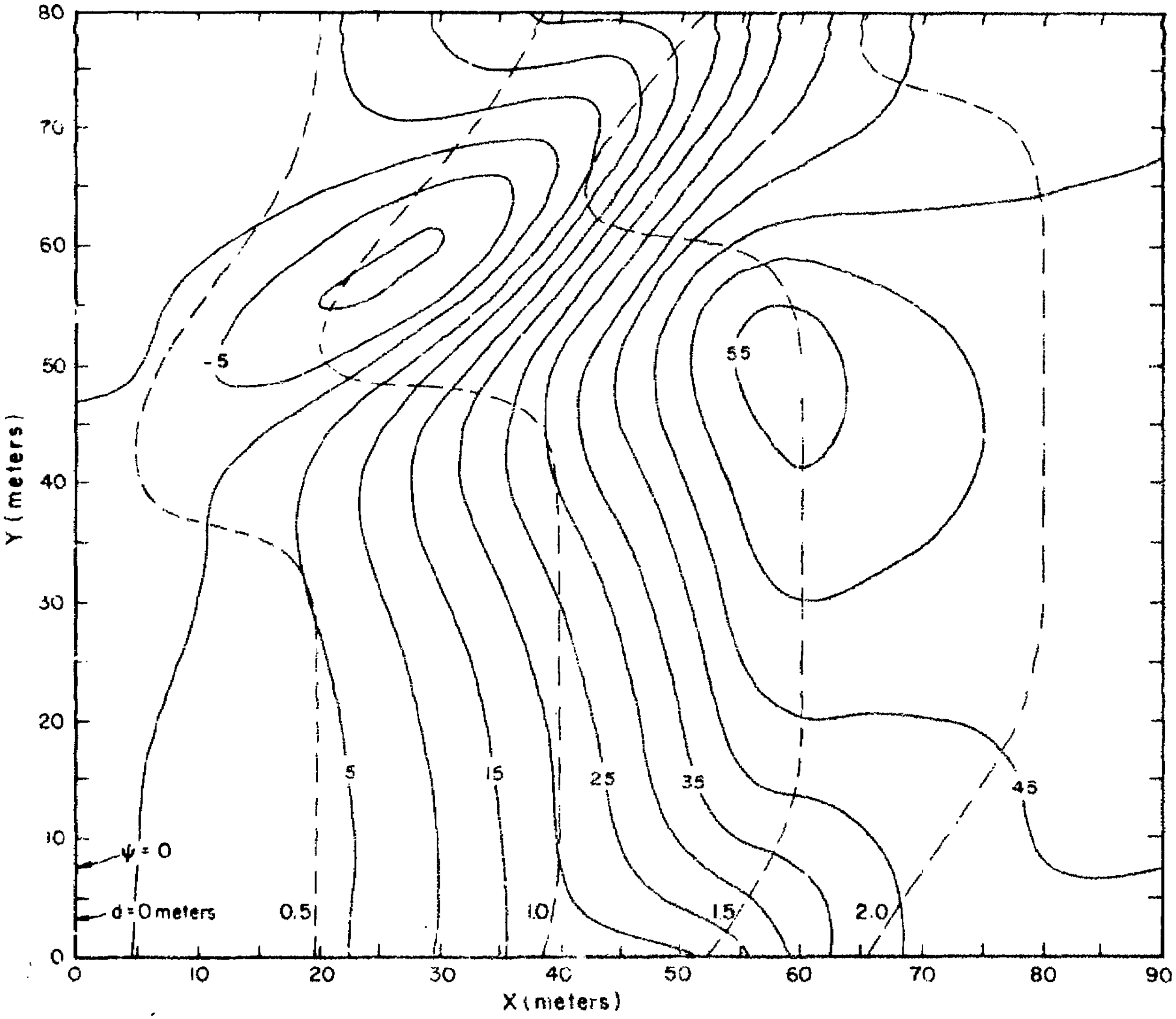


Figure 3.6: Stream Function Field ψ For No Wave-Current Interaction: ($H_d = 1.0m$, $\theta_d = 150°$, $T_o = 4$ sec.)

TABLE 3.4

NO WAVE-CURRENT INTERACTION:

H_d = 1.0 m, T_d = 4 sec., θ_d = 150° WAVE DIRECTION FIELD θ(degrees)

y (meters) ⟶

x (meters) ↓	-5	0	5	10	15	20	25	30	35	40
0	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
	171.46	172.62	173.34	173.81	174.15	175.28	181.49	194.27	203.51	196.73
	168.01	169.57	170.55	171.21	171.67	172.11	173.78	180.72	192.28	195.65
	165.60	167.28	168.40	169.16	169.71	170.11	170.72	173.28	180.46	188.39
20	163.96	165.58	166.74	167.55	168.13	168.57	168.94	169.84	173.16	179.79
	162.88	164.26	165.39	166.22	166.82	167.27	167.61	168.02	169.33	173.17
	162.35	163.23	164.28	165.09	165.69	166.15	166.50	166.78	167.31	169.11
	162.42	162.50	163.34	164.12	164.71	165.16	165.51	165.78	166.06	166.81
40	163.05	162.14	162.57	163.27	163.84	164.28	164.62	164.89	165.11	165.40
	164.08	162.20	162.01	162.52	163.06	163.49	163.82	164.08	164.30	164.50
	165.17	162.67	161.72	161.89	162.35	162.76	163.08	163.34	163.55	163.73
	165.97	163.41	161.76	161.42	161.72	162.10	162.41	162.66	162.86	163.03
60	166.21	164.14	162.11	161.18	161.18	161.49	161.78	162.02	162.22	162.38
	165.80	164.58	162.62	161.20	160.78	160.94	161.20	161.43	161.62	161.78
	164.83	164.56	163.08	161.44	160.56	160.46	160.66	160.88	161.06	161.22
	163.56	164.02	163.27	161.78	160.56	160.11	160.17	160.36	160.53	160.68
80	162.24	163.10	163.08	162.04	160.72	159.91	159.74	159.87	160.03	160.18
	161.10	161.99	162.49	162.06	160.93	159.88	159.43	159.43	159.57	159.71
	160.21	160.90	161.63	161.75	161.04	159.97	159.25	159.05	159.13	159.26
	159.54	159.97	160.66	161.15	160.93	160.08	159.20	158.77	158.73	158.84
100	159.02	159.25	159.75	160.35	160.55	160.08	159.24	158.60	158.39	158.44
	158.59	158.69	158.98	159.51	159.96	159.89	159.27	158.55	158.14	158.09
	158.20	158.24	158.38	158.74	159.24	159.48	159.19	158.55	158.00	157.79
	157.82	157.85	157.91	158.11	158.51	158.91	158.95	158.52	157.93	157.57
120	157.47	157.49	157.51	157.60	157.86	158.26	158.53	158.39	157.90	157.43
	157.13	157.14	157.16	157.19	157.32	157.64	158.00	158.11	157.83	157.36
	156.80	156.81	156.82	156.83	156.89	157.09	157.43	157.71	157.66	157.30
	156.49	156.49	156.50	156.50	156.53	156.64	156.90	157.22	157.38	157.20
140	156.18	156.19	156.19	156.19	156.21	156.27	156.44	156.72	156.59	157.00
	155.89	155.89	155.89	155.90	155.92	155.96	156.06	156.27	156.55	156.71
	155.61	155.60	155.60	155.61	155.64	155.69	155.75	155.88	156.11	156.34
	155.33	155.33	155.33	155.34	155.38	155.42	155.48	155.56	155.71	155.94
160	155.07	155.06	155.06	155.09	155.13	155.18	155.23	155.29	155.38	155.55
	154.82	154.80	154.81	154.85	154.90	154.95	155.00	155.04	155.09	155.20
	154.59	154.56	154.58	154.62	154.68	154.74	154.79	154.82	154.85	154.90
	154.38	154.33	154.35	154.41	154.48	154.50	154.58	154.60	154.62	154.64
180	154.20	154.12	154.15	154.22	154.29	154.34	154.37	154.39	154.40	154.41
	154.05	153.95	153.97	154.04	154.11	154.1[illegible]	154.18	154.19	154.20	154.20
	153.92	153.81	153.81	153.88	153.94	153.97	153.99	153.99	154.00	154.00
	153.77	153.70	153.70	153.74	153.78	153.80	153.80	153.80	153.80	153.80
200	153.62	153.62	153.62	153.62	153.62	153.62	153.62	153.62	153.62	153.62

TABLE 3.4

NO WAVE-CURRENT INTERACTION:

$H_d = 1.0\,m$, $T_d = 4\,sec.$, $\theta_d = 150^{\circ}$ WAVE DIRECTION FIELD θ(degrees)

(CONTINUED)

x (meters) ↓ \ y (meters) →	45	50	55	60	65	70	75	80	85
0-									
	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
	182.72	169.33	160.07	159.45	164.89	169.29	171.46	172.62	173.34
20-	187.93	176.59	166.52	161.32	162.18	165.46	168.01	169.57	170.55
	188.29	180.62	170.97	163.72	161.57	163.27	165.60	167.28	168.40
	184.40	182.05	174.75	167.04	162.56	162.29	163.96	165.58	166.70
	17[illegible].56	180.51	176.81	170.19	164.51	162.27	162.88	164.26	165.39
40-	17[illegible].07	176.91	176.79	172.44	166.83	163.09	162.35	163.23	164.28
	169.04	172.69	174.91	173.30	168.88	164.51	162.42	162.50	163.34
	166.49	169.00	171.95	172.67	170.16	166.11	163.05	162.14	162.57
	164.95	166.32	168.84	170.86	170.35	167.43	164.08	162.20	162.01
60-	163.95	164.59	166.23	168.46	169.46	168.07	165.17	162.67	161.72
	163.19	163.47	164.35	166.11	167.82	167.88	165.97	163.41	161.76
	162.53	162.69	163.09	164.19	165.87	166.91	166.21	164.14	162.11
	161.92	162.05	162.24	162.81	164.05	165.45	165.80	164.58	162.62
80-	161.35	161.47	161.59	161.86	162.60	163.86	164.83	164.56	163.08
	160.82	160.93	161.04	161.18	161.55	162.43	163.56	164.02	163.27
	160.31	160.43	160.53	160.62	160.80	161.30	162.24	163.10	163.08
	159.83	159.95	160.04	160.13	160.23	160.48	161.10	161.99	162.49
100-	159.38	159.49	159.59	159.67	159.75	159.87	160.21	160.90	161.63
	158.96	159.06	159.16	159.24	159.31	159.38	159.54	159.97	160.66
	158.55	158.65	158.74	158.82	158.89	158.94	159.02	159.25	159.75
	158.17	158.27	158.35	158.43	158.49	158.54	158.59	158.69	158.98
120-	157.81	157.90	157.98	158.05	158.11	158.16	158.20	158.24	158.38
	157.49	157.55	157.62	157.69	157.75	157.79	157.82	157.85	157.91
	157.22	157.22	157.28	157.35	157.40	157.44	157.47	157.49	157.51
	157.02	156.93	156.96	157.02	157.06	157.10	157.13	157.14	157.16
140-	156.90	156.69	156.66	156.70	156.75	156.78	156.80	156.81	156.82
	156.81	156.51	156.39	156.40	156.44	156.47	156.49	156.49	156.50
	156.73	156.38	156.17	156.13	156.15	156.17	156.18	156.19	156.19
	156.60	156.29	156.00	155.88	155.87	155.88	155.89	155.89	155.89
160-	156.39	156.18	155.88	155.67	155.61	155.60	155.61	155.60	155.60
	156.09	156.03	155.77	155.52	155.38	155.34	155.33	155.33	155.33
	155.74	155.80	155.65	155.39	155.19	155.10	155.07	155.06	155.06
	155.37	155.51	155.48	155.27	155.04	154.88	154.82	154.80	154.81
180-	155.02	155.18	155.25	155.14	154.91	154.70	154.59	154.56	154.58
	154.71	154.84	154.96	154.95	154.78	154.54	154.38	154.33	154.35
	154.44	154.53	154.66	154.72	154.62	154.40	154.20	154.12	154.15
	154.21	154.26	154.35	154.45	154.43	154.26	154.05	153.95	153.97
200-	154.00	154.02	154.08	154.16	154.18	154.08	153.92	153.81	153.81
	153.80	153.81	153.83	153.88	153.91	153.87	153.77	153.70	153.70
	153.62	153.62	153.62	153.62	153.62	153.62	153.62	153.62	153.62

TABLE 3.5

NO WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ WAVE HEIGHT FIELD H(meters)

y (meters) ⟶

x (meters) ↓	-5	0	5	10	15	20	25	30	35	40
0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	9.32637F-02	9.32618F-02	9.32618F-02	9.32625E-02	9.33749F-02	9.57007E-02	.11194	.16572	.26115	.34688
	.18461	.18456	.18456	.18456	.18457	.18514	.19218	.22711	.31517	.43440
	.27433	.27391	.27391	.27391	.27391	.27598	.27596	.29175	.34963	.46191
20	.36358	.36141	.36132	.36132	.36132	.36132	.36168	.36695	.39641	.47816
	.45511	.44747	.44682	.44681	.44681	.44681	.44684	.44810	.45975	.50726
	.55329	.53348	.53054	.53038	.53038	.53038	.53039	.53058	.53406	.55609
	.66240	.62236	.61303	.61209	.61207	.61207	.61207	.61208	.61282	.62081
40	.78315	.71822	.69588	.69211	.69187	.69187	.69187	.69187	.69197	.69414
	.90994	.82432	.78210	.77114	.76985	.76981	.76981	.76981	.76982	.77023
	.93297	.93904	.87540	.85093	.84626	.84591	.84590	.84590	.84590	.84595
	.92931	.92718	.93415	.93447	.92192	.92024	.92016	.92016	.92016	.92016
60	.92910	.92055	.92220	.92841	.93398	.93807	.94119	.94381	.94625	.94861
	.92948	.91802	.91415	.91740	.92234	.92641	.92966	.93248	.93513	.93766
	.92850	.91760	.90979	.90910	.91265	.91656	.91992	.92292	.92574	.92841
	.92560	.91728	.90816	.90370	.90490	.90829	.91169	.91485	.91783	.92061
80	.92116	.91587	.90774	.90097	.89927	.90150	.90480	.90807	.91118	.91403
	.91584	.91310	.90716	.90006	.89588	.89627	.89911	.90242	.90561	.90851
	.91034	.90932	.90569	.89981	.89444	.89276	.89461	.89777	.90100	.90388
	.90523	.90503	.90325	.89926	.89422	.89103	.89142	.89408	.89722	.90005
100	.90085	.90078	.90015	.89799	.89430	.89072	.88966	.89138	.89421	.89689
	.89728	.89698	.89682	.89604	.89404	.89120	.88928	.88976	.89194	.89434
	.89441	.89386	.89370	.89372	.89322	.89175	.88992	.88930	.89046	.89235
	.89211	.89140	.89107	.89140	.89195	.89194	.89098	.88986	.88986	.89093
	.89027	.88951	.88906	.88941	.89054	.89168	.89192	.89105	.89015	.89017
120	.88882	.88808	.88763	.88795	.88929	.89114	.89245	.89234	.89113	.89011
	.88774	.88704	.88669	.88707	.88845	.89058	.89258	.89333	.89240	.89071
	.88698	.88635	.88616	.88671	.88812	.89024	.89248	.89364	.89353	.89177
	.88651	.88599	.88600	.88679	.88828	.89026	.89237	.89395	.89425	.89292
140	.88633	.88593	.88619	.88724	.88885	.89067	.89242	.89385	.89449	.89382
	.88641	.88617	.88670	.88801	.88973	.89138	.89271	.89375	.89442	.89432
	.88675	.88670	.88752	.88906	.89083	.89230	.89323	.89380	.89425	.89448
	.88733	.88752	.88862	.89035	.89209	.89332	.89391	.89406	.89420	.89449
160	.88818	.88861	.88999	.89182	.89343	.89439	.89467	.89453	.89440	.89459
	.88933	.88997	.89157	.89342	.89481	.89545	.89548	.89518	.89490	.89493
	.89083	.89159	.89332	.89506	.89616	.89651	.89635	.89599	.89568	.89561
	.89275	.89347	.89516	.89669	.89746	.89756	.89730	.89697	.89671	.89661
180	.89506	.89561	.89705	.89824	.89871	.89864	.89838	.89813	.89796	.89789
	.89767	.89796	.89894	.89972	.89994	.89982	.89962	.89948	.89940	.89936
	.90033	.90039	.90082	.90116	.90123	.90115	.90107	.90102	.90099	.90098
	.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271
200										

TABLE 3.5

NO WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ WAVE HEIGHT FIELD H(meters)

(CONTINUED)

y (meters) ⟶ ; x (meters) ↓

x (meters)	45	50	55	60	65	70	75	80	85
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
	.35434	.27700	.17837	.11721	9.67929E-02	9.34615E-02	9.32637E-02	9.32618E-02	9.32618E-02
	.50392	.46536	.35178	.24826	.19855	.18601	.18461	.18456	.18456
	.57498	.60204	.51991	.39647	.31154	.28014	.27435	.27391	.27391
20	.60005	.68477	.66271	.55279	.44002	.38034	.36358	.36141	.36132
	.60853	.72258	.76515	.70006	.58182	.49181	.45511	.44747	.44682
	.62364	.73462	.82423	.82035	.72631	.61696	.55329	.53348	.53054
	.65725	.74272	.84983	.90297	.85736	.75160	.66240	.62236	.61303
40	.70986	.76322	.85976	.93675	.94124	.88452	.78315	.71822	.69588
	.77550	.80249	.87227	.95119	.95162	.93960	.90994	.82432	.78210
	.84726	.85810	.89917	.96070	.95873	.94114	.93297	.93904	.87540
	.92038	.92376	.94328	.96486	.95989	.94359	.92951	.92718	.93415
60	.95098	.95375	.95716	.95903	.95526	.94391	.92910	.92055	.92220
	.94003	.94231	.94495	.94756	.94732	.94122	.92948	.91802	.91415
	.93084	.93292	.93689	.93711	.93847	.93615	.92850	.91760	.90979
	.92307	.92507	.92665	.92819	.92975	.92966	.92560	.91728	.90816
80	.91649	.91843	.91979	.92081	.92191	.92267	.92116	.91587	.90774
	.91093	.91277	.91399	.91470	.91527	.91597	.91584	.91310	.90716
	.90624	.90796	.90904	.90957	.90978	.91007	.91034	.90932	.90569
	.90229	.90387	.90481	.90521	.90522	.90514	.90523	.90503	.90325
100	.89898	.90040	.90120	.90149	.90140	.90110	.90085	.90078	.90015
	.89623	.89747	.89813	.89832	.89818	.89778	.89728	.89698	.89682
	.89397	.89502	.89554	.89565	.89547	.89503	.89441	.89386	.89370
	.89215	.89298	.89338	.89343	.89322	.89277	.89211	.89140	.89107
120	.89077	.89133	.89150	.89162	.89140	.89095	.89027	.88951	.88906
	.88988	.89005	.89020	.89018	.88996	.88950	.88882	.88808	.88763
	.88957	.88919	.88915	.88910	.88888	.88842	.88774	.88704	.88669
	.88988	.88882	.88847	.88835	.88813	.88766	.88698	.88635	.88616
140	.89073	.88901	.88822	.88794	.88769	.88719	.88651	.88599	.88600
	.89190	.88979	.88846	.88790	.88753	.88699	.88633	.88593	.88619
	.89306	.89103	.88926	.88826	.88767	.88704	.88641	.88617	.88670
	.89398	.89249	.89058	.88903	.88812	.88734	.88675	.88670	.88752
160	.89459	.89387	.89225	.89041	.88895	.88790	.88733	.88752	.88862
	.89499	.89501	.89400	.89213	.89019	.88877	.88818	.88861	.88999
	.89539	.89588	.89559	.89406	.89184	.89002	.88933	.88997	.89157
	.89596	.89662	.89689	.89594	.89380	.89168	.89083	.89159	.89332
180	.89682	.89740	.89794	.89758	.89586	.89373	.89275	.89347	.89516
	.89798	.89836	.89890	.89893	.89783	.89607	.89506	.89561	.89705
	.89939	.89958	.89993	.90010	.89959	.89847	.89767	.89796	.89894
	.90099	.90105	.90118	.90130	.90117	.90073	.90033	.90039	.90082
200	.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271

TABLE 3.6

NO WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ BREAKING INDEX FIELD IB

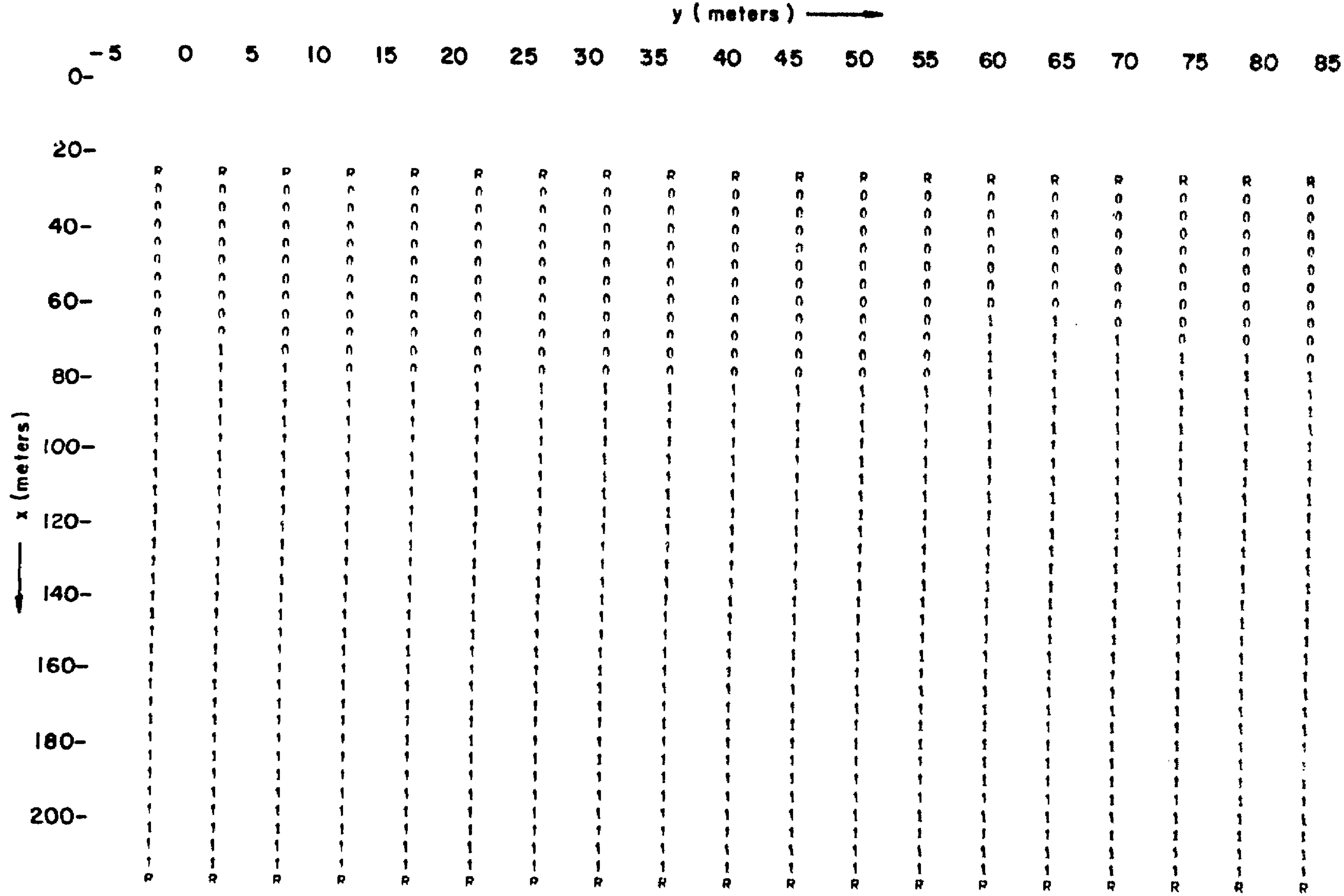


TABLE 3.7

NO WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ u VELOCITY FIELD (meters/sec)

y (meters) ⟶

x (meters) ↓	-5	0	5	10	15	20	25	30	35	40
0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	3.54019E-02	1.27970E-02	8.56794E-03	2.49352E-02	8.25483E-02	.20868	.39186	.08801	.52828	3.20339E-02
	9.42971E-02	5.17759E-02	1.91970E-02	7.79514E-05	-4.30651E-03	2.91599E-02	.15046	.25793	.13027	.45949
	8.46222E-02	7.74334E-02	2.81524E-02	-2.19476E-02	-7.56042E-02	-.13342	-.15669	-7.14560E-02	6.59319E-02	.50003
20	-3.75515E-02	7.16849E-02	3.37185E-02	-3.67473E-02	-.12090	-.22673	-.33542	-.34063	-8.22505E-02	.47988
	-.31190	6.59321E-03	3.00419E-02	-4.39359E-02	-.14318	-.26516	-.40611	-.48789	-.30518	.31695
	-.73308	-.15909	-2.07742E-03	-4.92055E-02	-.15139	-.27321	-.41430	-.53605	-.47850	1.22510E-02
	-1.1881	-.46505	-.10262	-6.82003E-02	-.15524	-.26917	-.39555	-.52313	-.56165	-.28465
40	-1.3403	-.91522	-.32763	-.13182	-.17106	-.26539	-.36990	-.48132	-.56408	-.47056
	-.76325	-1.1849	-.71652	-.28259	-.22233	-.27223	-.34661	-.42928	-.51176	-.52208
	5.24162E-02	-.78618	-.89192	-.53929	-.36581	-.30107	-.32923	-.37477	-.42966	-.46681
	.37860	-.22850	-.78441	-.76273	-.51186	-.35014	-.31864	-.31991	-.33525	-.34967
	.24983	-4.84784E-04	-.30232	-.50138	-.53181	-.43577	-.35151	-.30911	-.28538	-.25561
60	.10216	1.97624E-02	-.10469	-.24869	-.32658	-.31948	-.28119	-.24407	-.20746	-.15578
	6.07531E-03	-3.45280E-02	-4.30923E-02	-.13181	-.17889	-.21449	-.20758	-.18117	-.14497	-9.35289E-02
	-1.88089E-02	-8.97039E-02	-6.07840E-02	-3.95532E-02	-7.65025E-02	-.12480	-.14126	-.12806	-9.87836E-02	-5.65748E-02
	4.93770E-03	-.10480	-.10294	-4.05005E-02	-1.99231E-02	-5.21253E-02	-8.29488E-02	-8.52300E-02	-6.60933E-02	-3.49955E-02
80	4.00359E-02	-7.64025E-02	-.12671	-7.49902E-02	-9.22958E-03	-2.41154E-03	-3.20154E-02	-4.94313E-02	-4.30000E-02	-2.25561E-02
	6.06633E-02	-2.72870E-02	-.11341	-.10843	-3.35682E-02	1.67489E-02	8.39064E-03	-1.72484E-02	-2.51837E-02	-1.53591E-02
	6.07827E-02	1.56319E-02	-7.04414E-02	-.11551	-7.03857E-02	4.17424E-03	3.08179E-02	1.15433E-02	-8.62462E-03	-1.03841E-02
	4.73236E-02	3.79269E-02	-1.99796E-02	-9.01734E-02	-9.46428E-02	-2.93553E-02	2.85260E-02	3.22083E-02	8.71171E-03	-4.73868E-03
100	3.02532E-02	4.01961E-02	1.79059E-02	-4.50185E-02	-9.10823E-02	-6.40218E-02	2.70684E-03	3.70057E-02	2.49026E-02	3.84655E-03
	1.63966E-02	3.11406E-02	3.54454E-02	-1.81462E-04	-6.10777E-02	-8.07886E-02	-3.47303E-02	2.10571E-02	3.37791E-02	1.53928E-02
	7.90143E-03	1.98672E-02	3.63726E-02	2.96563E-02	-1.93758E-02	-7.14056E-02	-6.60561E-02	-1.18723E-02	2.81021E-02	2.61879E-02
	3.79126E-03	1.17248E-02	2.93141E-02	4.10659E-02	1.65896E-02	-4.19233E-02	-7.70987E-02	-4.90270E-02	5.30647E-03	2.94366E-02
	2.25620E-03	7.95317E-03	2.17568E-02	3.92215E-02	3.65289E-02	-7.43683E-03	-6.47817E-02	-7.47743E-02	-2.85202E-02	1.90905E-02
120	2.00628E-03	7.50153E-03	1.73532E-02	3.18725E-02	4.00238E-02	1.77604E-02	-3.78197E-02	-7.90711E-02	-6.03454E-02	-5.05251E-03
	2.48941E-03	8.93837E-03	1.63440E-02	2.47028E-02	3.29092E-02	2.72706E-02	-1.02854E-02	-6.26587E-02	-7.69220E-02	-3.52080E-02
	3.60891E-03	1.13319E-02	1.73825E-02	1.98039E-02	2.19785E-02	2.31139E-02	6.95935E-05	-3.55952E-02	-7.20606E-02	-5.88910E-02
	5.49119E-03	1.02433E-02	1.90332E-02	1.66284E-02	1.15707E-02	1.16635E-02	1.09243E-02	-1.04121E-02	-4.96844E-02	-6.57210E-02
140	8.47150E-03	1.74463E-02	2.03043E-02	1.37318E-02	2.98977E-03	-8.83126E-04	5.39333E-03	4.90765E-03	-2.07466E-02	-5.32453E-02
	1.31683E-02	2.07767E-02	2.05591E-02	9.95395E-03	-4.21288E-03	-1.09028E-02	-3.28693E-03	9.52612E-03	3.55209E-03	-2.79793E-02
	2.03299E-02	2.41787E-02	1.93487E-02	4.77161E-03	-1.06937E-02	-1.73136E-02	-1.00143E-02	7.58849E-03	1.72790E-02	-1.09952E-03
	3.01610E-02	2.77805E-02	1.64456E-02	-1.75644E-03	-1.65027E-02	-2.03078E-02	-1.25345E-02	4.21596E-03	2.08196E-02	1.79641E-02
160	4.12714E-02	3.17028E-02	1.20693E-02	-9.03211E-03	-2.10265E-02	-2.02613E-02	-1.10182E-02	2.56943E-03	1.82135E-02	2.55953E-02
	5.00535E-02	3.54481E-02	7.09378E-03	-1.58216E-02	-2.32830E-02	-1.74384E-02	-6.85449E-03	3.12959E-03	1.36753E-02	2.38005E-02
	5.15364E-02	3.72387E-02	2.94658E-03	-2.01617E-02	-2.22773E-02	-1.22845E-02	-1.75157E-03	4.65502E-03	9.58157E-03	1.72435E-02
	4.19679E-02	3.42400E-02	1.10887E-03	-1.96108E-02	-1.73885E-02	-5.83711E-03	2.61182E-03	5.58259E-03	6.44965E-03	1.01950E-02
180	2.18657E-02	2.45727E-02	2.60020E-03	-1.22563E-02	-8.95362E-03	7.97419E-05	4.75689E-03	5.00679E-03	4.03420E-03	5.06556E-03
	-2.39126E-03	1.10853E-02	8.14629E-03	1.28122E-03	6.71650E-04	3.96179E-03	3.84676E-03	3.11197E-03	2.31929E-03	2.54882E-03
	-1.87371E-02	5.96361E-03	1.92838E-02	1.47437E-02	5.33785E-03	9.23176E-04	6.85049E-04	1.27899E-03	1.71182E-03	2.32946E-03
200										

TABLE 3.7

NO WAVE-CURRENT INTERACTION:

$H_d = 1.0\,m$, $T_d = 4$ sec., $\theta_d = 150^\circ$ u VELOCITY FIELD (meters/sec)

(CONTINUED)

y (meters) ⟶ ; x (meters) ↓ (axis ticks 0–200)

45	50	55	60	65	70	75	80	85
0.	0.	0.	0.	0.	0.	0.	0.	0.
-.55481	-.50416	-3.58394E-02	.15994	.13199	7.92388E-02	3.54019E-02	1.27970E-02	8.56794E-03
.56080	-.28618	-1.0262	-.63813	-8.26782E-02	9.59063E-02	9.42971E-02	5.17759E-02	1.91970E-02
1.1356	.60012	-.86401	-1.3541	-.65595	-9.24337E-02	8.46222E-02	7.74334E-02	2.81524E-02
1.3041	1.3760	-1.76165E-02	-1.4607	-1.3406	-.50925	-3.75515E-02	7.16849E-02	3.37185E-02
1.2254	1.7800	.94949	-.84269	-1.6981	-1.0768	-.31190	6.59321E-03	3.00419E-02
.94665	1.8027	1.6520	.14922	-1.4109	-1.5312	-.73308	-.15909	-2.07742E-03
.49594	1.4735	1.8765	1.0868	-.49589	-1.4928	-1.1881	-.46505	-.10262
3.80466E-02	.93591	1.6027	1.5160	.61786	-.68247	-1.3403	-.91522	-.32763
-.26431	.42519	1.1680	1.4162	1.1904	.37414	-.76325	-1.1849	-.71652
-.36532	8.57336E-02	.81866	1.1770	1.1311	.81584	5.24162E-02	-.78618	-.89192
-.29830	2.35972E-02	.68193	1.0637	.93186	.67483	.37460	-.22850	-.78441
-.16710	.11901	.59505	.84162	.67412	.42700	.24983	-4.88784E-04	-.30232
-5.72740E-02	.13633	.39717	.54352	.44619	.23622	.10216	1.97624E-02	-.10469
-1.41085E-02	.10779	.25931	.36381	.32686	.15720	6.07531E-03	-3.45280E-02	-4.30923E-02
1.39850E-03	7.68136E-02	.16481	.23977	.24578	.13855	-1.88089E-02	-8.97439E-02	-6.07840E-02
5.37103E-03	5.25661E-02	.10314	.15229	.17750	.13165	4.93770E-03	-.10480	-.10294
4.66934E-03	3.48699E-02	6.44795E-02	9.35355E-02	.11944	.11465	4.00359E-02	-7.64025E-02	-.12671
2.28289E-03	2.20630E-02	4.04166E-02	5.66742E-02	7.47420E-02	8.74969E-02	6.06633E-02	-2.72874E-02	-.11341
-4.19236E-04	1.28299E-02	2.51322E-02	3.46432E-02	4.44587E-02	5.84811E-02	6.07827E-02	1.56319E-02	-7.04414E-02
-2.47029E-03	6.29562E-03	1.52151E-02	2.15843E-02	2.61559E-02	3.48189E-02	4.73236E-02	3.79269E-02	-1.99796E-02
-2.57553E-03	1.99228E-03	8.79504E-03	1.36698E-02	1.58239E-02	1.91384E-02	3.02532E-02	4.01961E-02	1.79059E-02
8.97580E-04	2.89606E-05	4.82937E-03	8.77325E-03	9.97636E-03	1.02461E-02	1.63966E-02	3.11406E-02	3.54454E-02
8.87412E-03	1.20839E-03	2.85201E-03	5.75846E-03	6.43867E-03	5.56681E-03	7.90143E-03	1.98672E-02	3.63726E-02
1.98528E-02	6.56438E-03	3.03565E-03	4.02319E-03	4.09273E-03	2.97727E-03	3.79126E-03	1.17248E-02	2.93141E-02
2.89475E-02	1.60551E-02	6.13373E-03	3.43187E-03	2.40284E-03	1.31553E-03	2.25620E-03	7.95314E-03	2.17568E-02
2.93942E-02	2.70308E-02	1.28016E-02	4.41657E-03	1.19578E-03	1.20092E-04	2.00628E-03	7.50153E-03	1.73532E-02
1.67044E-02	3.39671E-02	2.22484E-02	7.80546E-03	7.23883E-04	-6.92951E-04	2.48941E-03	8.93837E-03	1.63440E-02
-7.17229E-03	3.07307E-02	3.10723E-02	1.39962E-02	1.74713E-03	-9.08995E-04	3.60891E-03	1.13319E-02	1.73825E-02
-3.34090E-02	1.47288E-02	3.37144E-02	2.16687E-02	5.21543E-03	9.03869E-05	5.49119E-03	1.42433E-02	1.90332E-02
-5.06540E-02	-1.00687E-02	2.52593E-02	2.70615E-02	1.13194E-02	3.31781E-03	8.47150E-03	1.74463E-02	2.03043E-02
-5.14157E-02	-3.41797E-02	5.23748E-03	2.50670E-02	1.83346E-02	9.68225E-03	1.31683E-02	2.07767E-02	2.05591E-02
-3.62530E-02	-4.75632E-02	-2.06075E-02	1.23073E-02	2.23688E-02	1.89429E-02	2.03299E-02	2.41787E-02	1.93487E-02
-1.30397E-02	-4.52427E-02	-4.25889E-02	-9.66934E-03	1.89927E-02	2.87402E-02	3.01610E-02	2.77805E-02	1.64456E-02
8.03382E-03	-2.99007E-02	-5.22242E-02	-3.38469E-02	6.34660E-03	3.48131E-02	4.12714E-02	3.17028E-02	1.20693E-02
2.00482E-02	-9.80499E-03	-4.68825E-02	-5.09079E-02	-1.24603E-02	3.30224E-02	5.00535E-02	3.54481E-02	7.09378E-03
2.18857E-02	6.31563E-03	-3.09077E-02	-5.42484E-02	-3.00088E-02	2.22448E-02	5.15364E-02	3.72387E-02	2.94658E-03
1.69612E-02	1.37865E-02	-1.25891E-02	-4.36110E-02	-3.85116E-02	5.82282E-03	4.19679E-02	3.42400E-02	1.10887E-03

TABLE 3.8

NO WAVE-CURRENT INTERACTION:

H_d = 1.0 m, T_d = 4 sec., θ_d = 150° v VELOCITY FIELD (meters/sec)

y (meters) ⟶

x (meters) ↓ (axis marks: 0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200)

-5	0	5	10	15	20	25	30	35	40
8.69577E-02	4.96657E-02	3.57713E-02	1.53938E-02	-6.38921E-02	-.32709	-.76551	-1.2281	-1.5472	-2.1484
.46406	.39210	.36052	.35370	.35992	.35299	.25096	2.71749E-02	-.11770	-.10520
.74490	.67366	.63533	.63572	.68068	.78792	.94796	1.0212	.88759	.77879
.90303	.87714	.84338	.84500	.89243	1.0030	1.2049	1.4573	1.4658	1.0580
.93341	1.0058	.98934	.98963	1.0278	1.1119	1.2578	1.4771	1.6459	1.4578
.81219	1.0487	1.0776	1.0798	1.1073	1.1648	1.2536	1.3890	1.5721	1.6599
.52064	.97219	1.1024	1.1232	1.1457	1.1858	1.2388	1.3083	1.4307	1.6186
8.45130E-02	.73321	1.0429	1.1216	1.1532	1.1874	1.2224	1.2549	1.3110	1.4639
-.12795	.32043	.87215	1.0639	1.1365	1.1780	1.2073	1.2218	1.2323	1.3008
.39558	.16319	.60348	.91141	1.1001	1.1673	1.1988	1.2043	1.1874	1.1795
[illegible]	.47333	[illegible]	.79707	1.0648	1.1666	1.2055	1.2051	1.1731	1.1133
[illegible]	.81132	.48532	.46942	.59331	.72966	.79657	.81389	.79309	.73861
.65618	.73802	.50053	.24431	.13198	.14344	.15891	.15610	.13007	7.35389E-02
.34471	.44534	.40481	.27111	.10483	-5.28203E-03	-6.29853E-02	-.10605	-.15322	-.21336
.15801	.23082	.27133	.22191	.11682	2.27477E-02	-4.00478E-02	-8.64252E-02	-.12988	-.17323
7.84580E-02	8.99771E-02	.14689	.16463	.11208	3.60032E-02	-2.48723E-02	-6.79857E-02	-.10305	-.13280
6.11231E-02	2.27567E-02	4.61629E-02	9.63872E-02	9.52717E-02	4.42078E-02	-1.22407E-02	-5.22878E-02	-7.97270E-02	-9.92475E-02
6.33555E-02	1.02546E-02	-1.37651E-02	2.60500E-02	6.34779E-02	4.69732E-02	8.69802E-04	-3.80057E-02	-6.09621E-02	-7.36437E-02
6.16868E-02	2.38652E-02	-2.95269E-02	-2.83[illegible]85E-02	1.84904E-02	3.86069E-02	1.31678E-02	-2.30973E-02	-4.55721E-02	-5.47855E-02
5.05703E-02	3.72336E-02	-1.55495E-02	-5.24634E-02	-2.77595E-02	1.50762E-02	1.91952E-02	-7.34190E-03	-3.11817E-02	-4.06172E-02
3.40063E-02	3.97749E-02	6.67789E-03	-[illegible].64565E-02	-5.87201E-02	-1.94117E-02	1.26285E-02	6.00641E-03	-1.60134E-02	-2.86401E-02
1.80844E-02	3.18799E-02	2.15532E-02	-2.39462E-02	-6.46758E-02	-5.19582E-02	-8.19606E-03	1.10706E-02	-8.40912E-04	-1.65025E-02
6.52432E-03	1.92688E-02	2.40914E-02	-1.43332E-03	-4.91774E-02	-6.86274E-02	-3.64375E-02	3.08134E-03	1.03062E-02	-3.31252E-03
-2.77505E-04	7.57877E-03	1.72655E-02	1.09600E-02	-2.50629E-02	-6.39541E-02	-5.91595E-02	-1.69013E-02	1.19489E-02	9.02902E-03
-3.77864E-03	-4.90950E-04	6.97576E-03	1.17168E-02	-5.13146E-03	-4.38221E-02	-6.54482E-02	-4.02891E-02	1.14372E-03	1.59750E-02
-5.56520E-03	-5.09846E-03	-2.11075E-03	5.00099E-03	4.26867E-03	-2.00419E-02	-5.35301E-02	-5.51754E-02	-1.88342E-02	1.27968E-02
-6.61687E-03	-7.62328E-03	-8.25173E-03	-3.67405E-03	3.83593E-03	-2.48052E-03	-3.09006E-02	-5.39698E-02	-3.86776E-02	-1.59134E-03
-7.39224E-03	-9.29894E-03	-1.19201E-02	-1.06144E-02	-1.75829E-03	5.19536E-03	-8.31556E-03	-3.80842E-02	-4.82678E-02	-2.21952E-02
-8.10855E-03	-1.07957E-02	-1.42414E-02	-1.47484E-02	-7.6[illegible]155E-03	5.01287E-03	6.63573E-03	-1.60183E-02	-4.32127E-02	-3.96812E-02
-8.88565E-03	-1.23747E-02	-1.59924E-02	-1.65960E-02	-1.10[illegible]66E-02	1.53350E-03	1.21489E-02	2.68846E-03	-2.72558E-02	-4.59327E-02
-9.72161E-03	-1.40939E-02	-1.74225E-02	-1.69396E-02	-1.16342E-02	-1.40108E-03	1.09009E-02	1.25173E-02	-8.97691E-03	-3.91650E-02
-1.03657E-02	-1.58671E-02	-1.84425E-02	-1.61907E-02	-1.00792E-02	-2.25870E-03	6.93680E-03	1.32763E-02	3.84031E-03	-2.44329E-02
-1.02637E-02	-1.73828E-02	-[illegible].87944E-02	-1.44262E-02	-7.34017E-03	-1.48903E-03	3.04004E-03	8.24782E-03	7.90783E-03	-9.63268E-03
-8.85140E-03	-1.79742E-02	-1.81030E-02	-1.16501E-02	-4.17024E-03	-3.56035E-04	-8.38804E-05	1.28485E-03	4.62534E-03	-6.60979E-04
-6.23871E-03	-1.66531E-02	-1.58565E-02	-8.00546E-03	-1.27483E-03	-5.49462E-05	-3.03402E-03	-5.16469E-03	-2.16251E-03	9.88001E-04
-3.75562E-03	-1.25583E-02	-1.14468E-02	-3.89050E-03	5.15687E-04	-1.40255E-03	-6.53129E-03	-1.03043E-02	-8.82272E-03	-2.45856E-03
-3.50186E-03	-5.74650E-03	-4.52909E-03	-1.68794E-05	2.46315E-04	-4.78141E-03	-1.07085E-02	-1.41299E-02	-1.34034E-02	-7.46097E-03
-6.55215E-03	2.26537E-03	4.07909E-03	2.47030E-03	-2.93001E-03	-1.00078E-02	-1.49644E-02	-1.65975E-02	-1.55773E-02	-1.14177E-02
-1.05754E-02	8.83676E-03	1.10915E-02	1.60941E-03	-9.26984E-03	-1.60097E-02	-1.80965E-02	-1.74147E-02	-1.58547E-02	-1.35150E-02
-8.28203E-03	1.11989E-02	9.39013E-03	-6.07859E-03	-1.76732E-02	-2.02997E-02	-1.85489E-02	-1.63678E-02	-1.49822E-02	-1.41928E-02
-1.77380E-02	-2.54489E-03	3.11658E-03	-3.26671E-03	-1.13797E-02	-1.48137E-02	-1.50243E-02	-1.46349E-02	-1.45614E-02	-1.48484E-02

TABLE 3.8

NO WAVE-CURRENT INTERACTION:

H_d = 1.0 m, T_d = 4 sec., θ_d = 150^o v VELOCITY FIELD (meters/sec)

(CONTINUED)

y (meters) ⟶

x (meters) ↓	45	50	55	60	65	70	75	80	85
0-									
	-1.2527	.15240	.62690	.58673	.38968	.19086	8.69577E-02	4.96657E-02	3.57713E-02
20-	-.65476	-1.1825	-.55521	.31942	.63319	.57949	.46406	.39210	.36052
	.10781	-1.1159	-1.2968	-.30226	.52290	.77685	.74450	.67366	.63533
	.81267	-3.90040E-02	-1.2022	-.93578	.10815	.74639	.90303	.87714	.84338
	1.1722	.85181	-.36402	-1.0881	-.38998	.51649	.93341	1.0058	.98934
40-	1.5242	1.3906	.68033	-.55663	-.68224	.14460	.81219	1.0487	1.0776
	1.7076	1.7542	1.5109	.47770	-.45683	-.22706	.52064	.97219	1.1024
	1.7248	1.9487	2.0002	1.5397	.42141	-.28142	8.45130E-02	.73321	1.0429
	1.5237	1.8614	2.0889	2.0510	1.4235	.32750	-.12795	.32043	.87215
60-	1.2758	1.5483	1.8401	1.8972	1.7901	1.1998	.39558	.16319	.60348
	1.0836	1.1824	1.3629	1.3848	1.4018	1.4115	1.0537	.47333	.44855
	.04736	.58161	.58011	.66091	.74673	.95844	1.0389	.81132	.48532
	-2.17034E-02	-.13780	-.14630	7.06024E-02	.28814	.48337	.65618	.73802	.50053
80-	-.28844	-.35237	-.29385	-7.59714E-02	.11618	.22709	.34471	.44534	.40481
	-.21352	-.22791	-.16913	-3.46023E-02	8.46889E-02	.12933	.15801	.23082	.27133
	-.15301	-.15170	-.10847	-1.95550E-02	6.97201E-02	.10008	7.84580E-02	8.99771E-02	.14689
	-.10880	-.10290	-7.38387E-02	-1.68893E-02	5.16934E-02	8.73734E-02	6.11231E-02	2.27567E-02	4.61629E-02
100-	-7.76507E-02	-7.10061E-02	-5.15375E-02	-1.67243E-02	3.17967E-02	7.12469E-02	6.33555E-02	1.07546E-02	-1.37651E-02
	-5.59628E-02	-4.99358E-02	-3.65728E-02	-1.55669E-02	1.51990E-02	5.02928E-02	6.16868E-02	2.38652E-02	-2.95269E-02
	-4.08563E-02	-3.58674E-02	-2.65818E-02	-1.35939E-02	4.38930E-03	2.96780E-02	5.05703E-02	3.72336E-02	-1.55495E-02
	-3.00912E-02	-2.63729E-02	-1.99971E-02	-1.17321E-02	-1.43935E-03	1.38882E-02	3.40063E-02	3.97749E-02	6.67789E-03
120-	-2.16619E-02	-1.98682E-02	-1.57058E-02	-1.04372E-02	-4.34037E-03	4.02173E-03	1.80844E-02	3.18799E-02	2.15532E-02
	-1.35674E-02	-1.51276E-02	-1.29525E-02	-9.71571E-03	-5.92424E-03	-1.34239E-03	6.52432E-03	1.92688E-02	2.40914E-02
	-4.30833E-03	-1.08655E-02	-1.11649E-02	-9.42035E-03	-6.97144E-03	-4.14842E-03	-2.77505E-04	7.57877E-03	1.72655E-02
	5.90387E-03	-5.70830E-03	-9.68519E-03	-9.38037E-03	-7.74316E-03	-5.71297E-03	-3.77864E-03	-4.90950E-04	6.97576E-03
140-	1.42634E-02	1.08767E-03	-7.56112E-03	-9.33662E-03	-8.29848E-03	-6.65154E-03	-5.56520E-03	-5.09846E-03	-2.11075E-03
	1.61743E-02	8.47493E-03	-3.77952E-03	-8.76432E-03	-8.60136E-03	-7.20753E-03	-6.61687E-03	-7.62328E-03	-8.25173E-03
	8.06066E-03	1.31601E-02	1.83897E-03	-6.82137E-03	-8.45012E-03	-7.48833E-03	-7.39224E-03	-9.29894E-03	-1.19201E-02
	-9.29391E-03	1.09339E-02	7.61636E-03	-2.79160E-03	-7.34460E-03	-7.50883E-03	-8.10855E-03	-1.07957E-02	-1.42414E-02
160-	-2.96103E-02	-4.18105E-04	1.00198E-02	2.94466E-03	-4.56943E-03	-7.09483E-03	-8.88565E-03	-1.23747E-02	-1.59924E-02
	-4.40221E-02	-1.84680E-02	5.41952E-03	8.13590E-03	2.08643E-04	-5.79113E-03	-9.72161E-03	-1.40939E-02	-1.74225E-02
	-4.64665E-02	-3.62485E-02	-7.08801E-03	9.14818E-03	6.00242E-03	-3.06536E-03	-1.03657E-02	-1.58671E-02	-1.84425E-02
	-3.73129E-02	-4.59320E-02	-2.38473E-02	3.04320E-03	1.00723E-02	1.00857E-03	-1.02637E-02	-1.73828E-02	-1.87944E-02
180-	-2.24386E-02	-4.36841E-02	-3.78318E-02	-9.86914E-03	8.88241E-03	4.95669E-03	-8.85140E-03	-1.79742E-02	-1.81030E-02
	-9.03102E-03	-3.18316E-02	-4.27651E-02	-2.50873E-02	3.50246E-04	5.89332E-03	-6.23871E-03	-1.66531E-02	-1.58565E-02
	1.62395E-03	-1.68597E-02	-3.68369E-02	-3.59888E-02	-1.40250E-02	9.27355E-04	-3.75562E-03	-1.25583E-02	-1.14468E-02
	-7.21811E-04	-5.27456E-03	-2.41215E-02	-3.79091E-02	-2.90523E-02	-1.04700E-02	-3.50186E-03	-5.74650E-03	-4.52909E-03
200-	-3.99682E-03	-4.82462E-04	-1.12933E-02	-3.11952E-02	-3.84408E-02	-2.46689E-02	-6.55215E-03	2.26537E-03	4.07909E-03
	9.54256E-03	-2.26503E-03	-4.27324E-03	-2.10428E-02	-3.84236E-02	-3.41274E-02	-1.05750E-02	8.83676E-03	1.10915E-02
	-1.24355E-02	-8.30342E-03	-6.33918E-03	-1.50367E-02	-2.95651E-02	-2.93599E-02	-8.28203E-03	1.11989E-02	9.39013E-03
	-1.50275E-02	-1.40217E-02	-1.25344E-02	-1.50667E-02	-2.28660E-02	-2.68035E-02	-1.77388E-02	-2.54489E-03	3.11658E-03

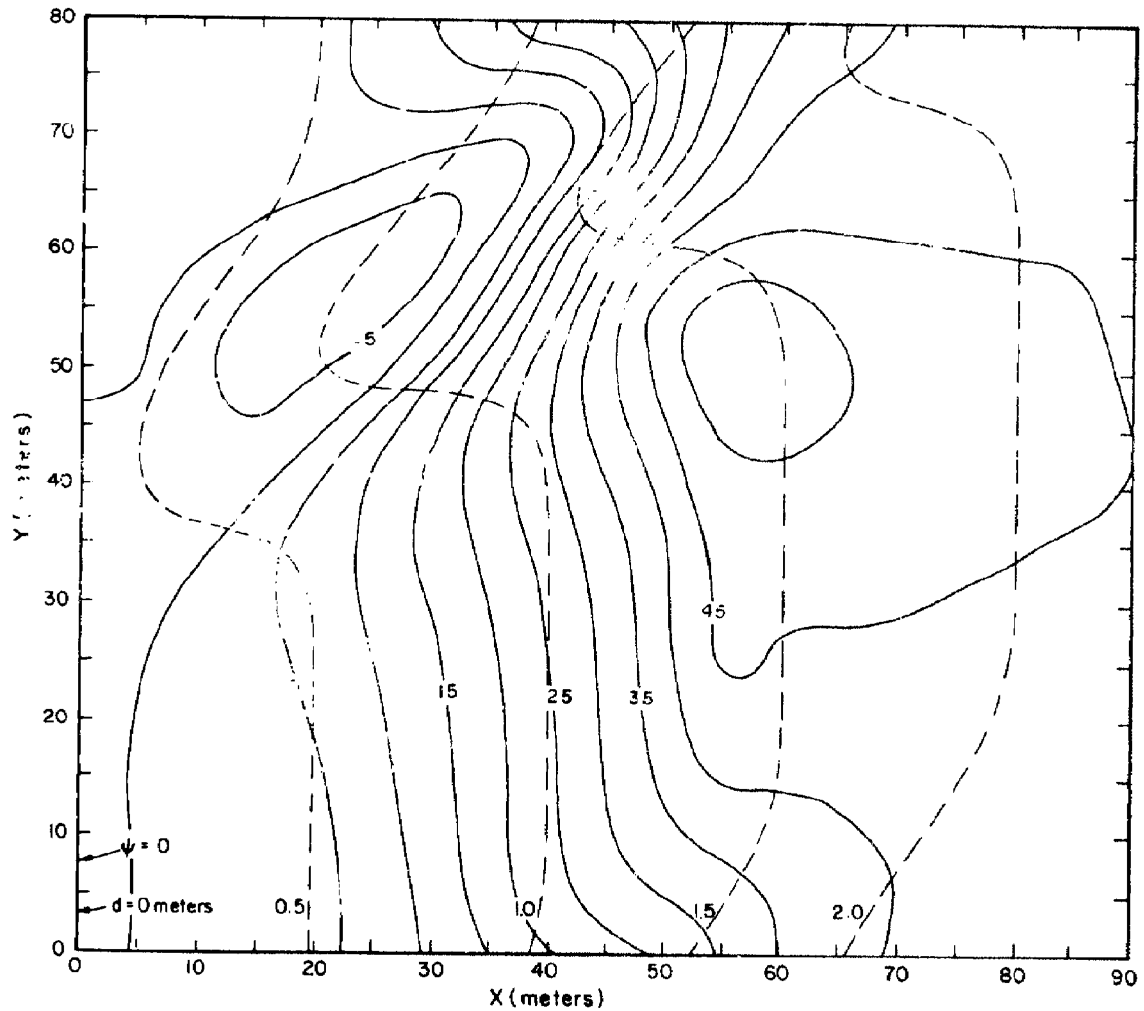


Figure 3. 7: Stream Function Field ψ For 20% Wave-Current Interaction: (H_d = 1.0m, θ_d = 150°, T_d = 4 sec.)

TABLE 3.9

20% WAVE-CURRENT INTERACTION:

H_d = 1.0 m, T_d = 4 sec., $\theta_d = 150^o$ WAVE DIRECTION FIELD θ(degrees)

y (meters) ⟶

x (meters) ↓ : 0– 20– 40– 60– 80– 100– 120– 140– 160– 180– 200–

-5	0	5	10	15	20	25	30	35	40
180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
171.43	172.00	172.60	173.53	174.57	175.90	180.32	188.43	192.51	191.73
167.63	168.50	169.55	171.16	173.02	174.42	175.05	175.85	178.06	184.09
164.85	165.63	166.98	169.01	171.19	172.71	172.69	170.95	170.03	174.51
162.95	163.37	164.95	167.29	169.53	170.97	170.94	169.02	166.78	168.26
161.87	161.73	163.48	166.02	168.17	169.41	169.47	168.07	165.77	165.19
161.65	160.88	162.66	165.19	167.08	168.06	168.11	167.17	165.38	163.95
162.28	161.02	162.55	164.69	166.15	166.84	166.79	166.10	164.86	163.47
163.61	162.18	163.01	164.32	165.24	165.67	165.49	164.88	164.05	163.07
165.36	164.03	163.68	163.85	164.25	164.47	164.19	163.63	163.06	162.51
167.08	165.25	163.82	163.31	163.31	163.25	162.92	162.47	162.11	161.87
167.93	165.61	163.67	162.68	162.28	162.11	161.92	161.66	161.49	161.48
167.91	165.65	163.27	161.95	161.62	161.60	161.47	161.29	161.23	161.34
167.02	165.51	163.33	161.77	161.22	161.14	161.04	160.92	160.92	161.00
165.72	165.14	163.47	161.82	160.93	160.67	160.56	160.49	160.51	160.62
164.17	164.43	163.52	162.05	160.84	160.27	160.09	160.05	160.11	160.24
162.61	163.39	163.29	162.26	160.93	160.01	159.65	159.61	159.71	159.87
161.25	162.16	162.71	162.29	161.10	159.92	159.31	159.21	159.34	159.50
160.18	160.94	161.84	162.04	161.21	159.96	159.10	158.86	158.97	159.13
159.39	159.90	160.84	161.48	161.13	160.05	159.04	158.60	158.62	158.76
158.79	159.10	159.88	160.69	160.80	160.07	159.08	158.45	158.31	158.40
158.31	158.50	159.07	159.82	160.21	159.92	159.14	158.42	158.08	158.05
157.90	158.05	158.45	159.00	159.47	159.55	159.12	158.45	157.93	157.74
157.52	157.69	157.98	158.31	158.70	159.00	158.94	158.46	157.86	157.50
157.18	157.38	157.60	157.76	157.99	158.35	158.58	158.38	157.84	157.36
156.87	157.09	157.28	157.32	157.40	157.69	158.07	158.15	157.80	157.30
156.59	156.82	156.97	156.96	156.93	157.11	157.49	157.77	157.67	157.27
156.33	156.56	156.67	156.62	156.54	156.62	156.93	157.29	157.42	157.21
156.08	156.29	156.37	156.30	156.21	156.24	156.44	156.78	157.06	157.07
155.85	156.03	156.06	155.98	155.91	155.92	156.04	156.30	156.63	156.81
155.62	155.76	155.76	155.68	155.62	155.64	155.72	155.90	156.18	156.45
155.38	155.48	155.46	155.38	155.36	155.40	155.47	155.57	155.76	156.02
155.13	155.20	155.16	155.11	155.11	155.18	155.24	155.30	155.39	155.58
154.88	154.92	154.88	154.85	154.89	154.97	155.03	155.05	155.07	155.18
154.64	[illegible]	154.62	154.63	154.69	154.77	154.81	154.80	154.79	154.85
154.41	154.38	154.38	154.42	154.50	154.56	154.58	154.57	154.55	154.58
154.21	154.16	154.17	154.24	154.31	154.35	154.36	154.34	154.34	154.37
154.05	153.97	153.99	154.06	154.12	154.14	154.14	154.15	154.16	154.18
153.92	153.82	153.83	153.89	153.93	153.95	153.96	153.97	153.99	154.00
153.78	153.70	153.69	153.73	153.77	153.78	153.79	153.80	153.81	153.81
153.62	153.62	153.62	153.62	153.62	153.62	153.62	153.62	153.62	153.62

TABLE 3.9

20% WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ WAVE DIRECTION FIELD θ(degrees)

(CONTINUED)

y (meters) →

x (meters) ↓	45	50	55	60	65	70	75	80	85
0									
	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
20	189.82	172.32	161.79	162.29	166.88	170.08	171.43	172.00	172.60
	192.64	186.01	172.09	164.65	164.20	166.20	167.63	168.50	169.55
	185.29	188.69	178.99	169.37	164.87	164.28	164.85	165.63	166.98
	176.17	184.45	182.67	174.85	167.50	163.91	162.95	163.37	164.95
40	169.44	177.31	181.50	178.34	170.81	164.72	161.87	161.73	163.48
	165.52	171.00	177.25	178.61	173.31	166.06	161.65	160.88	162.66
	163.52	166.75	172.50	176.28	173.74	167.26	162.28	161.02	162.55
	162.59	164.27	168.78	172.86	172.14	167.75	163.61	162.18	163.01
60	162.11	162.93	166.09	169.48	169.68	167.58	165.36	164.03	163.68
	161.77	162.18	163.93	166.36	167.77	168.00	167.08	165.25	163.82
	161.65	161.84	162.35	163.97	166.66	168.57	167.93	165.61	163.67
	161.49	161.35	161.29	162.66	165.73	168.17	167.91	165.65	163.27
80	161.04	160.93	161.05	162.15	164.43	166.63	167.02	165.51	16[illegible].33
	160.70	160.74	160.91	161.56	162.96	164.74	165.72	165.14	163.47
	160.36	160.47	160.66	161.04	161.81	163.01	164.17	164.43	163.52
	160.02	160.15	160.34	160.59	160.98	161.64	162.61	163.39	163.29
100	159.65	159.79	159.97	160.18	160.36	160.63	161.25	162.16	162.71
	159.27	159.41	159.59	159.77	159.86	159.90	160.18	160.94	161.84
	158.89	159.03	159.21	159.38	159.41	159.34	159.39	159.90	160.84
	158.51	158.65	158.84	158.99	158.98	158.85	158.79	159.10	159.88
120	158.14	158.29	158.48	158.60	158.56	158.40	158.31	158.50	159.07
	157.79	157.95	158.13	158.22	158.13	157.97	157.90	158.05	158.45
	157.47	157.63	157.80	157.84	157.72	157.55	157.52	157.69	157.98
	157.21	157.32	157.46	157.46	157.31	157.16	157.18	157.38	157.60
140	157.03	157.04	157.13	157.08	156.91	156.80	156.87	157.09	157.28
	156.92	156.80	156.79	156.70	156.54	156.46	156.59	156.82	156.97
	156.86	156.61	156.48	156.34	156.19	156.17	156.33	156.56	156.67
	156.80	156.46	156.20	156.00	155.88	155.90	156.08	156.29	156.37
160	156.69	156.34	155.97	155.71	155.60	155.66	155.85	156.03	156.06
	156.49	156.22	155.81	155.48	155.36	155.43	155.62	155.76	155.76
	156.18	156.05	155.69	155.32	155.16	155.22	155.38	155.48	155.46
	155.79	155.83	155.58	155.22	155.00	155.01	155.13	155.20	155.16
180	155.39	155.54	155.45	155.15	154.89	154.82	154.88	154.92	154.88
	155.02	155.22	155.26	155.06	154.80	154.65	154.64	154.64	154.62
	154.70	154.89	155.00	154.93	154.70	154.50	154.41	154.38	154.38
	154.44	154.58	154.70	154.72	154.58	154.37	154.21	154.16	154.17
200	154.22	154.29	154.39	154.46	154.40	154.23	154.05	153.97	153.99
	154.01	154.04	154.09	154.17	154.18	154.07	153.92	153.82	153.83
	153.81	153.81	153.84	153.88	153.91	153.87	153.78	153.70	153.69
	153.62	153.62	153.62	153.62	153.62	153.62	153.62	153.62	153.62

TABLE 3.10

20% WAVE-CURRENT INTERACTION:

$H_d = 1.0\,m$, $T_d = 4\,sec.$, $\theta_d = 150^o$ WAVE HEIGHT FIELD H (meters)

y (meters) →

-5	0	5	10	15	20	25	30	35	40
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
.09328	.09328	.09328	.09328	.09337	.09566	.11177	.16500	.25976	.34789
.18467	.18465	.18468	.18469	.18473	.18533	.19237	.22699	.31331	.42955
.27465	.27421	.27424	.27430	.27441	.27467	.27684	.29254	.34856	.45457
.36465	.36214	.36200	.36207	.36225	.36260	.36334	.36861	.39651	.47218
.45783	.44899	.44792	.44791	.44816	.44861	.44917	.45060	.46112	.50410
.55889	.53627	.53205	.53177	.53207	.53263	.53327	.53385	.53673	.55576
.67168	.62692	.61499	.61356	.61400	.61472	.61548	.61608	.61673	.62298
.79490	.72467	.69854	.69391	.69016	.69507	.69598	.69666	.69698	.69830
.91694	.83054	.78651	.77442	.77299	.77396	.77498	.77559	.77578	.77583
.95265	.92183	.88110	.85598	.85119	.85186	.85265	.85283	.85256	.85212
.95020	.91586	.90456	.90811	.91516	.91823	.91859	.92122	.92505	.92395
.94126	.91303	.90350	.90580	.91124	.91396	.91513	.91905	.92692	.93588
.93340	.91173	.90440	.90694	.91046	.91112	.91158	.91539	.92254	.93008
.92661	.91145	.90522	.90565	.90686	.90632	.90663	.91039	.91695	.92379
.92049	.91099	.90643	.90457	.90313	.90157	.90195	.90582	.91197	.91810
.91484	.91001	.90754	.90400	.89981	.89719	.89776	.90182	.90758	.91293
.90949	.90848	.90810	.90391	.89739	.89352	.89419	.89833	.90359	.90817
.90447	.90659	.90790	.90380	.89595	.89088	.89132	.89531	.89997	.90386
.90004	.90445	.90699	.90325	.89517	.88939	.88928	.89279	.89677	.90010
.89648	.90222	.90547	.90209	.89457	.88887	.88815	.89084	.89407	.89696
.89393	.90011	.90341	.90037	.89382	.88894	.88789	.88957	.89197	.89448
.89234	.89827	.90095	.89816	.89281	.88916	.88829	.88906	.89058	.89267
.89153	.89673	.89833	.89562	.89158	.88931	.88901	.88928	.88996	.89152
.89123	.89541	.89575	.89300	.89023	.88934	.88977	.89000	.89009	.89099
.89120	.89420	.89337	.89058	.88893	.88931	.89045	.89095	.89081	.89099
.89123	.89300	.89126	.88860	.88789	.88930	.89106	.89189	.89177	.89136
.89116	.89179	.88950	.88720	.88730	.88942	.89163	.89270	.89267	.89190
.89093	.89062	.88818	.88647	.88726	.88978	.89216	.89330	.89328	.89237
.89056	.88959	.88734	.88639	.88778	.89042	.89267	.89368	.89357	.89268
.89013	.88882	.88706	.88694	.88879	.89130	.89310	.89383	.89359	.89285
.88978	.88844	.88735	.88802	.89016	.89233	.89356	.89382	.89351	.89304
.88967	.88854	.88820	.88952	.89171	.89337	.89394	.89379	.89352	.89342
.88993	.88917	.88954	.89129	.89327	.89433	.89432	.89392	.89380	.89406
.89068	.89030	.89125	.89314	.89471	.89518	.89480	.89437	.89485	.89497
.89195	.89190	.89318	.89493	.89597	.89600	.89551	.89523	.89548	[illegible]
.89374	.89387	.89519	.89655	.89710	.89691	.89654	.89646	.89676	.89721
.89593	.89607	.89715	.89800	.89823	.89605	.89789	.89794	.89815	.89839
.89833	.89836	.89894	.89939	.89951	.89945	.89944	.89950	.89958	.89964
.90062	.90058	.90077	.90097	.90105	.90106	.90107	.90108	.90107	.90106
.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271

x (meters) ↓ : 0– 20– 40– 60– 80– 100– 120– 140– 160– 180– 200–

TABLE 3.10

20% WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^o$ WAVE HEIGHT FIELD H (meters)

(CONTINUED)

y (meters) ⟶ ; x (meters) ↓

x (meters)	45	50	55	60	65	70	75	80	85
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	.35648	.27771	.17824	.11718	.09679	.09347	.09328	.09328	.09326
20	.50274	.46874	.35416	.24906	.19879	.18610	.18467	.18465	.18468
	.56608	.60299	.52508	.39977	.31298	.28075	.27465	.27421	.27424
	.58612	.67768	.66698	.55928	.44429	.38244	.36465	.36214	.36200
	.59586	.70785	.76238	.70704	.58982	.49700	.45783	.44899	.44792
40	.61570	.71862	.81245	.82201	.73644	.62647	.55889	.53627	.53205
	.65423	.73114	.83405	.89587	.86525	.76476	.67168	.62692	.61499
	.71085	.75765	.84719	.93793	.95346	.89710	.79490	.72467	.69854
	.77931	.80191	.86620	.96327	.98566	.96777	.91694	.83058	.78651
60	.85220	.86028	.89784	.97763	1.00275	.99103	.95265	.92183	.88110
	.92284	.92522	.94342	.98673	1.00682	.99275	.95020	.91586	.90456
	.94248	.94909	.96504	.98752	.99830	.97929	.94126	.91303	.90350
80	.93619	.94366	.95835	.97559	.97940	.96177	.93340	.91173	.90440
	.93004	.93772	.94873	.95898	.95939	.94607	.92661	.91145	.90522
	.92364	.92996	.93759	.94354	.94281	.93343	.92049	.91099	.90643
	.91766	.92282	.92820	.93124	.92938	.92273	.91484	.91001	.90754
100	.91220	.91656	.92049	.92154	.91854	.91345	.90949	.90848	.90810
	.90743	.91129	.91419	.91376	.90980	.90554	.90447	.90659	.90790
	.90340	.90691	.90894	.90733	.90274	.89913	.90004	.90445	.90699
	.90010	.90325	.90443	.90189	.89705	.89419	.89648	.90222	.90587
120	.89744	.90015	.90046	.89720	.89250	.89061	.89393	.90011	.90341
	.89535	.89745	.89689	.89318	.88899	.88825	.89234	.89827	.90095
	.89371	.89504	.89369	.88982	.88644	.88690	.89153	.89673	.89833
	.89244	.89288	.89088	.88717	.88480	.88639	.89123	.89541	.89575
140	.89149	.89099	.88854	.88529	.88401	.88650	.89120	.89420	.89337
	.89086	.88947	.88681	.88419	.88305	.88702	.89123	.89300	.89126
	.89056	.88848	.88580	.88386	.88446	.88772	.89116	.89179	.88950
	.89061	.88816	.88559	.88424	.88535	.88842	.89093	.89062	.88818
160	.89097	.88854	.88618	.88519	.88643	.88901	.89056	.88959	.88734
	.89157	.88956	.88746	.88658	.88757	.88944	.89013	.88882	.88706
	.89235	.89098	.88921	.88823	.88872	.88977	.88978	.88848	.88735
	.89327	.89252	.89113	.89001	.88989	.89014	.88967	.88850	.88820
180	.89430	.89397	.89294	.89180	.89115	.89071	.88993	.88917	.88954
	.89538	.89521	.89448	.89351	.89257	.89163	.89068	.89030	.89125
	.89643	.89627	.89577	.89511	.89416	.89295	.89195	.89190	.89318
	.89744	.89727	.89695	.89659	.89586	.89468	.89374	.89387	.89519
200	.89847	.89834	.89816	.89802	.89758	.89668	.89593	.89607	.89715
	.89964	.89957	.89951	.89947	.89928	.89880	.89833	.89836	.89894
	.90105	.90103	.90102	.90101	.90097	.90083	.90062	.90058	.90077
	.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271

TABLE 3.11

20% WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ BREAKING INDEX IB

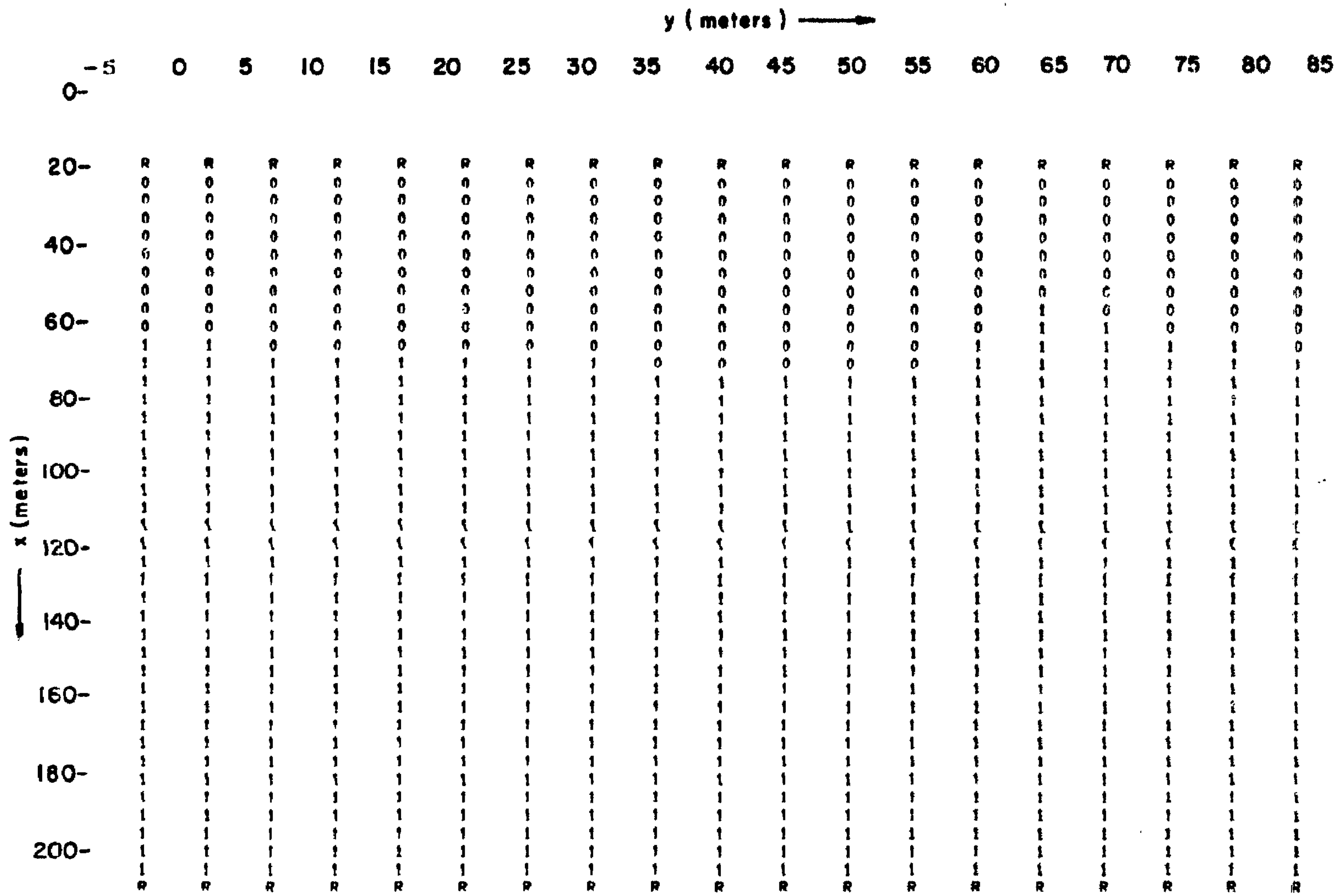


TABLE 3.12

20% WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ u VELOCITY FIELD (meters/sec)

y (meters) →

x (meters) ↓	-5	0	5	10	15	20	25	30	35	40
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	.05415	.02953	-.00322	-.01268	.01283	.10760	.41161	.95128	.88111	-.12420
	.13843	.07408	-.00741	-.08232	-.14052	-.17080	-.06807	.52332	1.29329	.53210
	.13569	.10287	-.02212	-.14387	-.24205	-.32574	-.31582	.09122	1.08677	1.21236
20	-.03200	.06444	-.05813	-.19046	-.28131	-.35887	-.38212	-.16757	.53580	1.31348
	-.41581	-.08872	-.11971	-.20886	-.26066	-.30960	-.34651	-.28352	.07627	.84398
	-.96245	-.38533	-.20475	-.18748	-.19647	-.22691	-.26814	-.29829	-.19530	.28460
	-1.40399	-.78798	-.30861	-.13645	-.12005	-.15162	-.19038	-.25898	-.30967	-.12433
40	-1.47319	-1.14481	-.43768	-.11928	-.08339	-.11390	-.13928	-.20923	-.32575	-.34676
	-.78365	-1.07613	-.67098	-.24628	-.14110	-.13112	-.12252	-.17117	-.29615	-.42189
	-.01878	-.52030	-.60260	-.41973	-.31054	-.22687	-.13416	-.13039	-.24403	-.40107
	.33621	.02955	-.42323	-.61373	-.53713	-.34695	-.16160	-.09150	-.16824	-.31470
60	.29785	.12517	-.14970	-.32179	-.35755	-.28467	-.17057	-.12271	-.20160	-.33259
	.17841	.12606	-.02913	-.18958	-.25606	-.21406	-.14038	-.11900	-.16470	-.21260
	.08080	.05593	-.00742	-.10218	-.16069	-.14462	-.10142	-.09185	-.11803	-.13115
	.05180	-.00483	-.04255	-.07222	-.08853	-.07780	-.05886	-.06347	-.08606	-.08787
80	.07110	-.03341	-.09679	-.08480	-.00296	-.01788	-.01857	-.04064	-.06440	-.06000
	.10738	-.02498	-.13945	-.12308	-.02878	.02798	.01768	-.02197	-.04898	-.04028
	.13359	.00327	-.15205	-.16126	-.00168	.05046	.04609	-.00463	-.03545	-.02475
	.13862	.02800	-.13541	-.17677	-.06655	.04603	.06035	.01220	-.02038	-.01142
100	.12442	.03411	-.10458	-.15992	-.08339	.02109	.05465	.02529	-.00236	.00114
	.09839	.01997	-.07651	-.11720	-.07750	-.00938	.02912	.02834	.01634	.01370
	.06759	-.00679	-.05978	-.06493	-.04702	-.02896	-.00759	.01630	.02948	.02550
	.03646	-.03600	-.05309	-.01857	-.00275	-.02824	-.04129	-.00975	.02955	.03339
120	.00732	-.05972	-.04992	.01442	.03945	-.00937	-.05920	-.04133	.01276	.03248
	-.01823	-.07358	-.04408	.03447	.06776	.01639	-.05670	-.06644	-.01706	.01906
	-.03845	-.07660	-.03268	.04578	.07863	.03620	-.03957	-.07589	-.04902	-.00566
	-.05166	-.07006	-.01640	.05224	.07508	.04240	-.01933	-.06809	-.07014	-.03417
140	-.05685	-.05621	.00201	.05563	.06258	.03463	-.00629	-.04887	-.07176	-.05532
	-.05417	-.03806	.01933	.05575	.04568	.01754	-.00414	-.02692	-.05366	-.06004
	-.04498	-.01868	.03291	.05167	.02700	-.00256	-.00967	-.00852	-.02364	-.04661
	-.03120	-.00066	.04099	.04273	.00805	-.02013	-.01627	.00453	.00686	-.02201
160	-.01486	.01419	.04267	.02926	-.00951	-.03119	-.01821	.01324	.02821	.00212
	.00244	.02497	.03807	.01296	-.02318	-.03372	-.01356	.01853	.03589	.01631
	.01865	.03162	.02856	-.00316	-.03028	-.02800	-.00470	.01981	.03105	.01828
	.03065	.03424	.01699	-.01506	-.02924	-.01720	.00317	.01620	.01920	.01260
180	.03457	.03309	.00767	-.01942	-.02176	-.00681	.00605	.00926	.00762	.00640
	.02797	.02815	.00422	-.01568	-.01255	-.00114	.00422	.00304	.00151	.00357
	.01375	.02030	.00563	-.00798	-.00574	.00039	.00193	.00079	.00068	.00278
	-.00381	.00746	.00714	.00219	.00174	.00235	.00144	.00052	.00071	.00168
	-.01411	.00422	.01556	.01262	.00492	.00026	-.00094	-.00066	.00012	.00104
200										

TABLE 3.12

20% WAVE-CURRENT INTERACTION:

H_d = 1.0 m, T_d = 4 sec., θ_d = 150° u VELOCITY FIELD (meters/sec)

(CONTINUED)

x (meters) ↓ \ y (meters) →	45	50	55	60	65	70	75	80	85
0-	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	-.82796	-.56224	.00163	.20349	.16059	.10298	.05415	.02053	-.00322
	-.02545	-.30072	-1.00945	-.81079	-.11822	.12397	.13843	.07408	-.00741
20-	.49052	.18477	-.57375	-1.46140	-.78781	-.09716	.13569	.10287	-.02212
	1.11587	.74042	.00697	-1.24250	-1.43168	-.59229	-.03200	.06444	-.05813
	1.41458	1.25171	.57840	-.53620	-1.45605	-1.17919	-.01581	-.08872	-.11971
	1.11681	1.47559	1.05662	.26337	-.82964	-1.46548	-.96245	-.38533	-.20475
40-	.52512	1.20095	1.21930	.94035	.12744	-1.16176	-1.44399	-.78798	-.30861
	-.01638	.60348	.96530	1.20969	1.01540	-.31319	-1.47319	-1.14481	-.43768
	-.35821	.04719	.64048	1.15048	1.26798	.43898	-.78365	-1.07613	-.62098
	-.47131	-.23149	.50395	1.04679	.97493	.57111	-.01878	-.52030	-.60264
60-	-.38466	-.16572	.50988	.89708	.62291	.40535	.33621	.02955	-.42323
	-.33473	-.06639	.41748	.61331	.38673	.28405	.29785	.12517	-.10974
	-.15907	.04368	.28932	.35884	.23450	.16104	.17841	.12606	-.02913
	-.08216	.03490	.17535	.24333	.20130	.12126	.08080	.05593	-.00742
80-	-.04405	.03265	.10999	.15997	.16440	.11988	.05180	-.00483	-.04255
	-.02120	.02643	.06144	.09244	.12700	.13310	.07110	-.03341	-.09679
	-.00623	.02016	.07668	.04092	.09100	.13968	.10738	-.02498	-.13945
	.00317	.01205	-.00056	.00628	.06377	.13602	.13359	.00327	-.15205
100-	.00774	.00177	-.02065	-.01306	.04904	.12656	.13862	.02800	-.13541
	.00816	-.01012	-.03498	-.01983	.04637	.11650	.12442	.03411	-.10458
	.00573	-.02210	-.04331	-.01679	.05237	.10803	.09839	.01997	-.07651
	.00265	-.03187	-.04513	-.00661	.06251	.10005	.06759	-.00679	-.05978
120-	.00127	-.03659	-.04021	.00791	.07246	.09012	.03646	-.03604	-.05309
	.00269	-.03373	-.02898	.02379	.07880	.07631	.00732	-.05972	-.04992
	.00532	-.02253	-.01256	.03819	.07933	.05827	-.01823	-.07358	-.04408
	.00519	-.00560	.00700	.04903	.07347	.03741	-.03845	-.07664	-.03268
140-	-.00156	.01044	.02629	.05549	.06245	.01643	-.05146	-.07006	-.01640
	-.01540	.01766	.04045	.05795	.04919	-.00145	-.05685	-.05621	.00201
	-.03214	.01066	.04424	.05710	.03755	-.01328	-.05417	-.03806	.01933
	-.04514	-.00942	.03458	.05308	.03068	-.01725	-.04498	-.01868	.03291
160-	-.04931	-.03471	.01323	.04489	.02910	-.01322	-.03124	-.00066	.04099
	-.04385	-.05429	-.01267	.03141	.02972	-.00317	-.01486	.01419	.04267
	-.03191	-.05958	-.03339	.01314	.02720	.00893	.00244	.02097	.03807
	-.01791	-.04868	-.04154	-.00667	.01748	.01806	.01865	.03162	.02856
180-	-.00523	-.02735	-.03595	-.02275	.00125	.02013	.03065	.03424	.01699
	.00400	-.00570	-.02189	-.03023	-.01572	.01391	.03457	.03309	.00767
	.00822	.00758	-.00727	-.02722	-.02593	.00207	.02797	.02815	.00422
	.00719	.01012	.00217	-.01651	-.02528	-.00959	.01575	.02030	.00563
200-	.00360	.00619	[illegible]	-.00427	-.01548	-.01575	-.00381	.00746	.00714
	.00204	.00348	.00426	-.00003	-.01150	-.02060	-.01411	.00422	.01556

TABLE 3.13

20% WAVE-CURRENT INTERACTION:

$H_d = 1.0\,m$, $T_d = 4$ sec., $\theta_d = 150^\circ$ u VELOCITY FIELD (meters/sec)

y (meters) ⟶

x (meters) ↓ (axis marks: 0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200)

-5	0	5	10	15	20	25	30	35	40
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
.45398	.33982	.30582	.35463	.46989	.61931	.67611	.43394	-.16849	-.96762
.71979	.60932	.57592	.64089	.78533	1.00712	1.27547	1.40312	1.17350	.36811
.84695	.81564	.81176	.88824	1.01083	1.16929	1.36452	1.52921	1.57847	1.35589
.82581	.96393	1.01985	1.09724	1.17263	1.24150	1.31392	1.41296	1.58248	1.77782
.66714	1.05470	1.19707	1.25563	1.26972	1.26635	1.25480	1.27819	1.43281	1.73967
.43009	1.06867	1.31165	1.33625	1.29720	1.25909	1.21605	1.19194	1.27173	1.53957
.22924	.95534	1.27850	1.30054	1.25634	1.22749	1.19206	1.15679	1.17384	1.33977
.35074	.66319	1.00046	1.13109	1.16501	1.18523	1.17986	1.15648	1.14567	1.21396
.89569	.52822	.66039	.87294	1.03549	1.14562	1.18635	1.17096	1.14523	1.15181
1.19964	.60185	.40533	.60467	.88593	1.07384	1.15001	1.14138	1.09661	1.05983
.86096	.60998	.37279	.32350	.37916	.43962	.48448	.50377	.49980	.50186
.41919	.46368	.35452	.17535	.04082	-.02902	-.04448	-.02510	.00057	-.00085
.18850	.27076	.25000	.17528	.09912	.03403	-.00508	-.01404	-.01943	-.07303
.05332	.13229	.17954	.16605	.10745	.04143	-.00290	-.02537	-.04539	-.08614
.01007	.03381	.09662	.13432	.10692	.04594	-.00256	-.02796	-.04491	-.06942
.00599	-.01650	.02505	.08892	.09512	.04674	-.00187	-.02603	-.03797	-.05468
.01526	-.02563	-.02052	.03671	.07052	.04540	.00337	-.01999	-.02989	-.04309
.02034	-.01034	-.03200	-.00739	.03307	.03681	.01134	-.01028	-.02218	-.03505
.01631	.00993	-.01565	-.03090	-.00841	.01690	.01670	.00180	-.01428	-.02922
.00621	.02373	.01197	-.02877	-.04059	-.01255	.01309	.01272	-.00539	-.02373
-.00412	.02879	.03524	-.01049	-.05369	-.04258	-.00254	.01675	.00377	-.01651
-.01008	.02829	.04646	.00958	-.04780	-.06187	-.02634	.00932	.01027	-.00652
-.01004	.02639	.04570	.02080	-.03181	-.05404	-.04845	-.00919	.01013	.00492
-.00499	.02523	.03710	.02012	-.01653	-.05135	-.05880	-.03180	.00114	.01309
.00272	.02443	.02510	.01046	-.00857	-.03221	-.05326	-.04786	-.01437	.01552
.01044	.02243	.01245	-.00294	-.00861	-.01518	-.03554	-.04939	-.02963	.00708
.01581	.01795	.00030	-.01561	-.01329	-.00460	-.01375	-.03590	-.03715	-.00939
.01733	.01974	-.01081	-.02508	-.01817	.00005	.00469	-.01426	-.03378	-.02745
.01467	.00161	-.02028	-.03050	-.02002	.00199	.01607	.00580	-.02266	-.03976
.00876	-.00805	-.02736	-.03172	-.01762	.00412	.02012	.01683	-.01071	-.04213
.00141	-.01664	-.03131	-.02883	-.01151	.00712	.01821	.01651	-.00372	-.03509
-.00515	-.02277	-.03155	-.02232	-.00352	.00938	.01176	.00760	-.00294	-.02242
-.00912	-.02539	-.02798	-.01345	.00365	.00854	.00225	-.00424	-.00532	-.00873
-.00989	-.02374	-.02100	-.00430	.00709	.00331	-.00794	-.01312	-.00667	.00199
-.00839	-.01761	-.01190	.00221	.00501	-.00483	-.01519	-.01566	-.00546	.00668
-.00687	-.00841	-.00281	.00372	-.00133	-.01182	-.01688	-.01308	-.00422	.00404
-.00798	-.00088	.00301	.00077	-.00767	-.01450	-.01470	-.01060	-.00650	-.00388
-.01264	.00224	.00681	-.00100	-.01069	-.01501	-.01471	-.01238	-.01179	-.01164
-.01088	.00332	.00438	-.00728	-.01638	-.01794	-.01628	-.01510	-.01482	-.01456
-.01701	-.00469	.00128	-.00384	-.01088	-.01410	-.01479	-.01503	-.01523	-.01531

TABLE 3.13

20% WAVE-CURRENT INTERACTION:

H_d = 1.0 m, T_d = 4 sec., θ_d = 150° u VELOCITY FIELD (meters/sec)

(CONTINUED)

y (meters) ⟶

x (meters)	45	50	55	60	65	70	75	80	85
0-	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00008	0.00000
	-.78168	-1.17449	-.77345	.28721	.67796	.61542	.45398	.33982	.30582
	-.07241	-.89425	-1.45523	-.47774	.48183	.77983	.71979	.60932	.57592
	.24861	-.11165	-.98748	-1.07689	-.05973	.66478	.84696	.81564	.81176
20-	1.22976	.36925	-.10576	-.89600	-.59035	.31492	.82581	.96393	1.01985
	1.81762	1.14644	.65897	-.07275	-.67091	-.11413	.66714	1.05470	1.19707
	1.87570	1.82396	1.39144	.87983	-.11314	-.34596	.43009	1.06867	1.31165
	1.69355	2.03899	1.99345	1.67819	.86696	-.07562	.22924	.95534	1.27850
40-	1.46831	1.90348	2.16257	1.98341	1.62049	.76368	.35074	.66319	1.00086
	1.27156	1.58693	1.86147	1.66108	1.65213	1.51940	.89569	.52822	.66039
	1.06098	1.15952	1.20439	1.05113	1.18819	1.49604	1.19964	.60185	.40533
	.49749	.43584	.34628	.02668	.64733	.88937	.86096	.60998	.37279
60-	-.05762	-.18106	-.25511	-.01395	.22236	.33105	.41919	.46368	.35052
	-.18520	-.29349	-.28558	-.09181	.03963	.07434	.18050	.27076	.25000
	-.14866	-.18664	-.14301	-.03499	.02371	.01741	.05332	.13229	.17954
	-.10331	-.12349	-.09797	-.02526	.03231	.03087	.01007	.03381	.09662
80-	-.07755	-.08936	-.06836	-.01395	.03820	.04026	.00599	-.01650	.02545
	-.06016	-.06639	-.04805	-.00798	.03230	.04410	.01526	-.02563	-.02052
	-.04820	-.04910	-.03102	-.00266	.02141	.03152	.02034	-.01034	-.03200
	-.03898	-.03482	-.01691	.00147	.00992	.01308	.01631	.00993	-.01565
100-	-.03073	-.02244	-.00585	.00342	-.00063	-.00499	.00621	.02373	.01197
	-.02255	-.01159	.00169	.00257	-.01027	-.01895	-.00412	.02870	.03524
	-.01184	-.00245	.00532	-.00175	-.01898	-.02751	-.01008	.02829	.04646
	.00025	.00472	.00503	-.00819	-.02612	-.03062	-.01004	.02639	.04570
120-	.01290	.00992	.00154	-.01551	-.03066	-.02870	-.00499	.02523	.03710
	.02257	.01323	-.00346	-.02182	-.03162	-.02259	.00272	.02443	.02510
	.02444	.01426	-.00757	-.02512	-.02854	-.01370	.01044	.02243	.01245
	.01501	.01177	-.00865	-.02368	-.02172	-.00398	.01581	.01795	.00030
140-	-.00478	.00430	-.00621	-.01679	-.01227	.00445	.01733	.01074	-.01081
	-.02840	-.00798	-.00218	-.00567	-.00177	.00996	.01467	.00161	-.02028
	-.04620	-.02232	-.00023	.00591	.00790	.01203	.00876	-.00805	-.02736
	-.05060	-.03360	-.00362	.01259	.01472	.01153	.00141	-.01664	-.03131
160-	-.04045	-.03733	-.01281	.00998	.01672	.00998	-.00515	-.02277	-.03155
	-.02164	-.03279	-.02461	-.00225	.01255	.00825	-.00912	-.02539	-.02798
	-.00368	-.02358	-.03375	-.01914	.00244	.00561	-.00989	-.02378	-.02100
	.00564	-.01480	-.03583	-.03270	-.01112	.00024	-.00839	-.01761	-.01190
180-	.00437	-.00944	-.02982	-.03672	-.02385	-.00887	-.00687	-.00841	-.00281
	-.00308	-.00730	-.01805	-.03056	-.03116	-.01983	-.00798	-.00088	.00301
	-.01004	-.00752	-.00893	-.01935	-.03085	-.02860	-.01264	.00224	.00681
	-.01319	-.00965	-.00686	-.01180	-.02272	-.02454	-.01088	.00332	.00838
200-	-.01522	-.01446	-.01346	-.01546	-.02140	-.02424	-.01701	-.00469	.00125

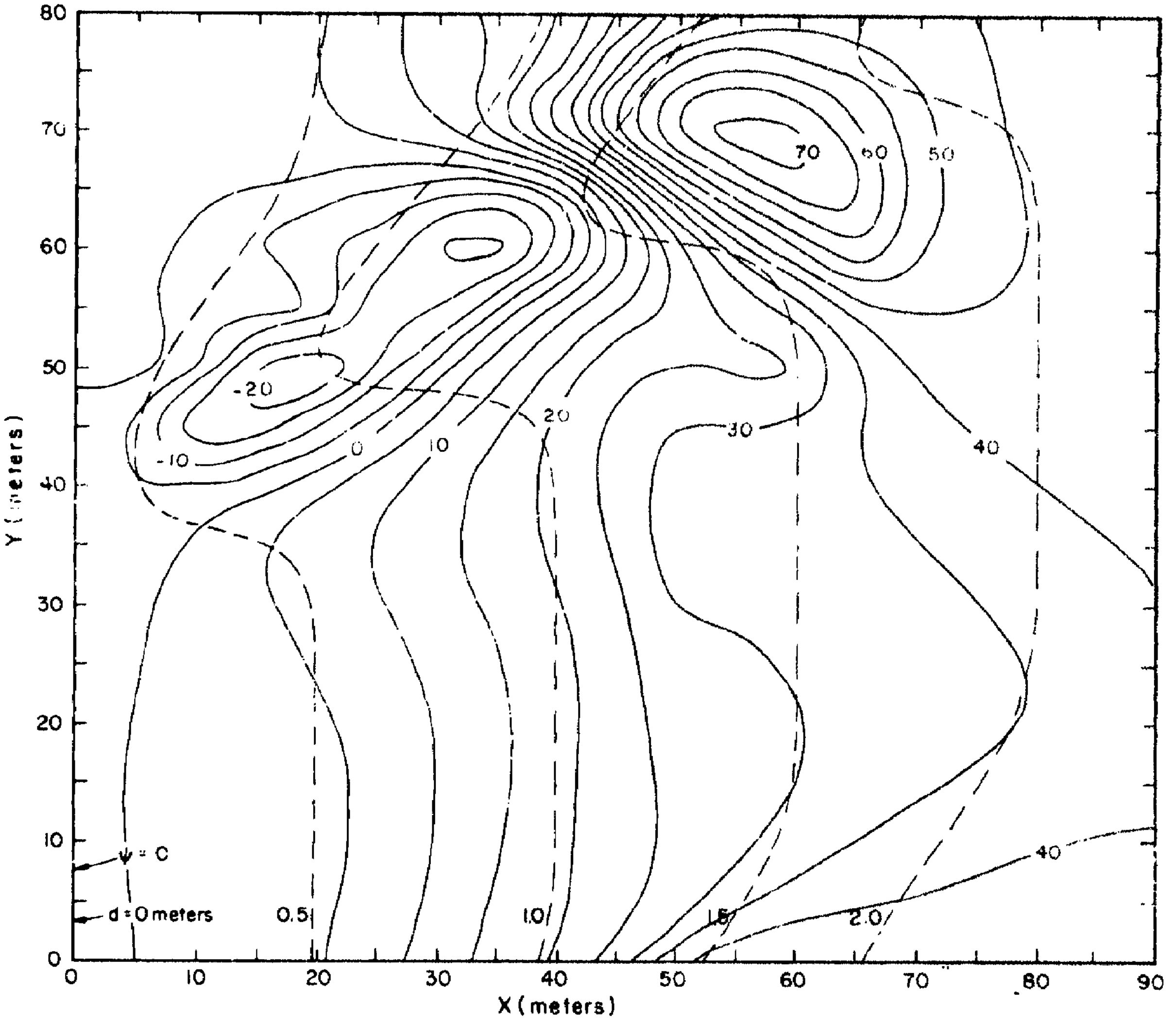


Figure 3.8: Stream Function Field ψ For 50% Wave-Current Interaction: (H_d = 1.0m, θ_d = 150°, T_d = 4 sec.)

TABLE 3.14

50% WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d \doteq 150^{\circ}$ WAVE DIRECTION FIELD θ(degrees)

y (meters) →

x (meters) ↓	-5	0	5	10	15	20	25	30	35	40
0-	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000
	172.56757	173.98263	174.89679	175.28747	174.96485	174.37475	180.25457	196.31770	198.48777	185.73320
	169.40856	171.50689	173.09429	174.34038	175.28230	175.58147	175.94449	180.83711	181.43589	173.56536
	167.69987	169.60769	171.41334	172.93481	174.25884	175.16107	175.07544	174.47912	172.57750	166.87601
20-	167.32558	168.30846	169.91710	171.39577	172.66332	173.58069	173.52654	171.63952	168.02472	163.11364
	168.37068	167.64053	168.59749	169.79267	170.80826	171.46308	171.43504	169.81334	166.06359	161.35104
	171.10195	167.76891	167.50986	168.19001	168.89817	169.21976	169.10251	168.01939	165.12987	160.85316
	175.81147	168.89009	166.73729	166.63553	167.01546	167.02655	166.76777	166.06836	164.22102	160.83384
40-	181.38268	171.03263	166.39578	165.21656	165.20019	164.92917	164.56298	164.06906	162.99943	160.64068
	183.30184	173.73141	166.70158	164.12461	163.49297	162.93355	162.55902	162.19550	161.57622	160.02247
	179.93807	174.39970	167.50658	163.61431	161.84739	161.08057	160.83226	160.60278	160.22044	159.15518
	175.62843	172.40572	167.07175	162.74092	160.58396	159.90695	159.72937	159.61535	159.37081	158.56917
60-	172.56941	169.43723	165.46033	162.41021	160.56579	159.63583	159.36563	159.35509	159.22986	158.65837
	170.12820	167.86170	164.72441	162.04144	160.39315	159.69908	159.52943	159.53377	159.49564	159.24458
	167.95745	166.82313	164.32983	161.86924	160.32209	159.69892	159.56103	159.59869	159.66068	159.65959
	165.78395	165.78548	164.10694	161.88079	160.22167	159.50125	159.39520	159.50961	159.65877	159.77684
80-	163.70569	164.50318	163.76686	161.97678	160.25827	159.34537	159.19720	159.37780	159.59216	159.75250
	161.93653	163.01542	163.12330	161.99498	160.42256	159.32607	159.02738	159.20457	159.45107	159.61015
	160.59487	161.52872	162.14993	161.77321	160.60606	159.46632	158.95273	159.01740	159.24376	159.37725
	159.65465	160.24583	160.98020	161.22878	160.66351	159.70531	159.01617	158.86290	158.98887	159.07532
100-	159.00451	159.25683	159.81754	160.40523	160.47739	159.92650	159.20505	158.78968	158.71801	158.72208
	158.52253	158.53631	158.82155	159.45173	160.01354	160.00124	159.44342	158.81939	158.47411	158.33921
	158.11847	158.00082	158.05112	158.54256	159.34247	159.84092	159.61576	158.92265	158.29635	157.95938
	157.74131	157.56575	157.48033	157.79662	158.60459	159.43756	159.60978	159.02000	158.19599	157.62403
120-	157.36904	157.17380	157.04762	157.24819	157.93863	158.86815	159.36188	159.00938	158.14072	157.36794
	156.99556	156.79796	156.69655	156.86871	157.42340	158.25262	158.88545	158.80662	158.06211	157.19848
	156.62163	156.43209	156.39313	156.60600	157.06502	157.69243	158.26214	158.38234	157.88332	157.08556
	156.25042	156.08117	156.12444	156.41285	156.82113	157.23114	157.59796	157.77697	157.55348	156.97164
140-	155.88629	155.75414	155.88975	156.25697	156.63433	156.85549	156.97421	157.08215	157.07371	156.79686
	155.53477	155.46004	155.69153	156.11812	156.45504	156.52288	156.42479	156.39894	156.50034	156.52469
	155.20252	155.20607	155.52932	155.98122	156.24967	156.19112	155.94567	155.80020	155.92060	156.16287
	154.89675	154.99590	155.39638	155.83135	155.99985	155.83594	155.51936	155.31741	155.41480	155.75679
160-	154.62488	154.82774	155.27875	155.65247	155.69954	155.45513	155.13495	154.94984	155.02744	155.36632
	154.39501	154.69259	155.15599	155.42980	155.35362	155.06483	154.79507	154.68100	154.76027	155.03570
	154.21576	154.57421	155.00410	155.15469	154.97832	154.69271	154.51134	154.48966	154.58368	154.77639
	154.09307	154.45212	154.80120	154.83031	154.60143	154.37013	154.29410	154.35259	154.45469	154.57036
180-	154.02222	154.30717	154.53702	154.47671	154.25982	154.12254	154.14180	154.24319	154.33257	154.38719
	153.97866	154.12818	154.22503	154.13307	153.99099	153.95824	154.03454	154.13216	154.18906	154.20218
	153.91707	153.91999	153.91283	153.85331	153.81706	153.85747	153.93542	153.99386	154.01386	154.00639
	153.78961	153.71728	153.68306	153.68756	153.72209	153.76838	153.80340	153.81751	153.81583	153.80686
200-	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857

TABLE 3.14

50% WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^o$ WAVE DIRECTION FIELD θ(degrees)

(CONTINUED)

x (meters) ↓ \ y (meters) →	45	50	55	60	65	70	75	80	85
0	180.00000	180.00000	180.00000	180.00000	180.00000	180.00300	180.00000	180.00000	180.00000
20	166.24577	194.24399	168.02119	162.62909	166.97997	170.37858	172.56757	173.98263	174.89679
	169.59865	199.32770	197.79204	162.49436	163.63195	166.69357	169.40856	171.50689	173.09429
	164.13148	184.73822	205.73808	172.68998	166.39036	166.33668	167.69987	169.60769	171.41338
	160.22191	169.19121	195.16269	185.67172	171.92111	168.74314	167.32558	168.30846	169.91710
40	158.25212	161.06556	178.24829	187.09287	179.35859	174.20237	168.37068	167.64053	168.59749
	157.42271	157.75865	166.[illegible]26	178.51993	183.35785	181.62347	171.10195	167.76891	167.50986
	157.31565	156.22766	159.55927	168.01853	180.29090	187.68478	175.81147	168.89009	166.73729
	157.28587	155.11174	155.90893	160.65639	173.41590	188.04069	181.38268	171.03263	166.39578
60	157.01664	153.94131	153.30548	157.57749	169.11940	183.12029	183.30184	173.73141	166.70158
	156.58286	153.01651	151.96115	157.95466	169.87125	178.57368	179.93807	174.39970	167.50658
	156.41848	153.29361	153.35231	160.91643	171.78572	176.25026	175.62843	172.40572	167.07175
	157.23014	155.72833	157.34241	163.53578	170.51652	173.50006	172.56941	169.43723	165.46033
80	158.73784	158.59992	160.01090	163.16706	167.16159	170.02519	170.12820	167.86170	164.72441
	159.62136	159.79212	160.51062	161.97529	164.29179	166.83187	167.95745	166.82313	164.32983
	159.88195	160.07488	160.46369	161.14689	162.36094	164.19488	165.78395	165.78548	164.10694
	159.87258	160.01454	160.22832	160.54724	161.12182	162.22829	163.70569	164.50318	163.76686
100	159.69400	159.76578	159.87602	160.03829	160.30487	160.87981	161.93653	163.01542	163.12330
	159.40919	159.41505	159.45963	159.56067	159.71201	159.98799	160.59487	161.52872	162.14993
	159.05674	159.01153	159.01689	159.09906	159.22726	159.37641	159.65465	160.24583	160.98020
	158.66075	158.58420	158.57319	158.65708	158.79640	158.91155	159.00451	159.25683	159.81754
120	158.23727	158.15106	158.14442	158.24261	158.40006	158.51463	158.52253	158.53631	158.82155
	157.80123	157.72384	157.74016	157.86126	158.03341	158.14808	158.11847	158.00082	158.05112
	157.37415	157.31128	157.36555	157.51477	157.69470	157.79706	157.74131	157.56575	157.48033
	156.98845	156.92369	157.02217	157.20133	157.38062	157.45593	157.36904	157.17384	157.04762
140	156.68183	156.57820	156.70984	156.91602	157.08546	157.12109	156.99556	156.79796	156.69655
	156.48086	156.29997	156.43036	156.65148	156.80181	156.78876	156.62163	156.43209	156.39313
	156.38287	156.11429	156.19189	156.40035	156.52136	156.45520	156.25042	156.08117	156.12444
	156.34987	156.03070	156.00904	156.15976	156.23629	156.11754	155.88629	155.75414	155.88975
160	156.32011	156.02892	155.89458	155.93549	155.94225	155.77457	155.53477	155.46004	155.69153
	156.23180	156.05804	155.84577	155.74079	155.64272	155.42788	155.20252	155.20607	155.52932
	156.04786	156.05186	155.83489	155.58739	155.35151	155.08446	154.89675	154.99590	155.39638
	155.76999	155.95356	155.81297	155.47319	155.08935	154.75945	154.62488	154.82774	155.27875
180	155.43337	155.73813	155.72652	155.37581	154.87403	154.47606	154.39501	154.69259	155.15599
	155.08510	155.42135	155.53939	155.25851	154.70931	154.25940	154.21576	154.57421	155.00410
	154.76113	155.04924	155.24882	155.08597	154.58074	154.12447	154.09307	154.45212	154.80120
	154.47580	154.67480	154.88668	154.84195	154.46134	154.06399	154.02222	154.30717	154.53702
200	154.22622	154.33656	154.50421	154.53904	154.32336	154.04286	153.97866	154.12818	154.22503
	154.00336	154.05012	154.14995	154.21342	154.14758	154.00251	153.91707	153.91999	153.91283
	153.80088	153.81337	153.85374	153.90441	153.92073	153.87389	153.78961	153.71728	153.68306
	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857

TABLE 3.15

50% WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ WAVE HEIGHT FIELD H (meters)

y (meters) →

x (meters) ↓: 0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200

-5	0	5	10	15	20	25	30	35	40
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
.09329	.09329	.09328	.09327	.09333	.09552	.11133	.16438	.25869	.34674
.18471	.18468	.18470	.18471	.18472	.18522	.19195	.22603	.31423	.39931
.27482	.27428	.27435	.27446	.27463	.27491	.27706	.29275	.35044	.44150
.36519	.36232	.36227	.36256	.36297	.36355	.36460	.37034	.39947	.47776
.45932	.44952	.44856	.44899	.44970	.45063	.45180	.45402	.46558	.50991
.56250	.53783	.53350	.53377	.53477	.53603	.53749	.53907	.54294	.56252
.67948	.63113	.61820	.61706	.61820	.61974	.62140	.62308	.62478	.63138
.79287	.73478	.70534	.69946	.70015	.70184	.70357	.70529	.70677	.70870
.85227	.77113	.78146	.78236	.78114	.78253	.78410	.78561	.78702	.78798
.92468	.80405	.78306	.80128	.81996	.83917	.85705	.86407	.86502	.86570
.96276	.85081	.80096	.80450	.82478	.84743	.86708	.88307	.89193	.89207
.95650	.88139	.83234	.82313	.83750	.86025	.88259	.90045	.91099	.91493
.94412	.89589	.86169	.84864	.85441	.87311	.89507	.91351	.92548	.93263
.93526	.90248	.87675	.86491	.86847	.88307	.90130	.91830	.92898	.93485
.92905	.90469	.88435	.87461	.87727	.88939	.90525	.91871	.92610	.92832
.92432	.70496	.88780	.88003	.88335	.89456	.90800	.91799	.92156	.92009
.91992	.90439	.88939	.88326	.88768	.89856	.90967	.91597	.91578	.91136
.91500	.90307	.89028	.88560	.89103	.90156	.91028	.91288	.90931	.90286
.90940	.90068	.89085	.88780	.89396	.90373	.90988	.90899	.90262	.89505
.90356	.89716	.89099	.89020	.89679	.90524	.90854	.90453	.89612	.88824
.89799	.89297	.89053	.89276	.89969	.90621	.90637	.89969	.89015	.88272
.89303	.88884	.88960	.89521	.90262	.90674	.90357	.89473	.88497	.87868
.88880	.88542	.88869	.89724	.90524	.90685	.90044	.88994	.88077	.87623
.88532	.88306	.88832	.89879	.90703	.90642	.89724	.88571	.87772	.87535
.88261	.88189	.88880	.90000	.90755	.90512	.89416	.88237	.87595	.87586
.88070	.88188	.89017	.90098	.90672	.90269	.89125	.88020	.87555	.87753
.87968	.88291	.89220	.90171	.90473	.89915	.88849	.87930	.87656	.88007
.87960	.88480	.89456	.90203	.90191	.89497	.88596	.87961	.87888	.88322
.88048	.88732	.89682	.90175	.89864	.89084	.88395	.88095	.88225	.88672
.88226	.89019	.89862	.90074	.89527	.88745	.88289	.88307	.88620	.89029
.88477	.89305	.89968	.89903	.89221	.88531	.88312	.88576	.89015	.89359
.88779	.89558	.89986	.89690	.88983	.88470	.88470	.88879	.89353	.89621
.89103	.89748	.89924	.89475	.88852	.88563	.88740	.89185	.89601	.89788
.89415	.89865	.89809	.89310	.88853	.88787	.89073	.89462	.89753	.89860
.89686	.89914	.89691	.89240	.88988	.89098	.89406	.89682	.89829	.89869
.89891	.89925	.89623	.89293	.89231	.89433	.89688	.89833	.89866	.89864
.90026	.89939	.89650	.89466	.89533	.89735	.89893	.89930	.89906	.89890
.90110	.89992	.89789	.89725	.89830	.89968	.90031	.90019	.89983	.89970
.90172	.90102	.90017	.90017	.90083	.90136	.90143	.90126	.90108	.90103
.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271	.90271

TABLE 3.16

50% WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^\circ$ BREAKING INDEX IB

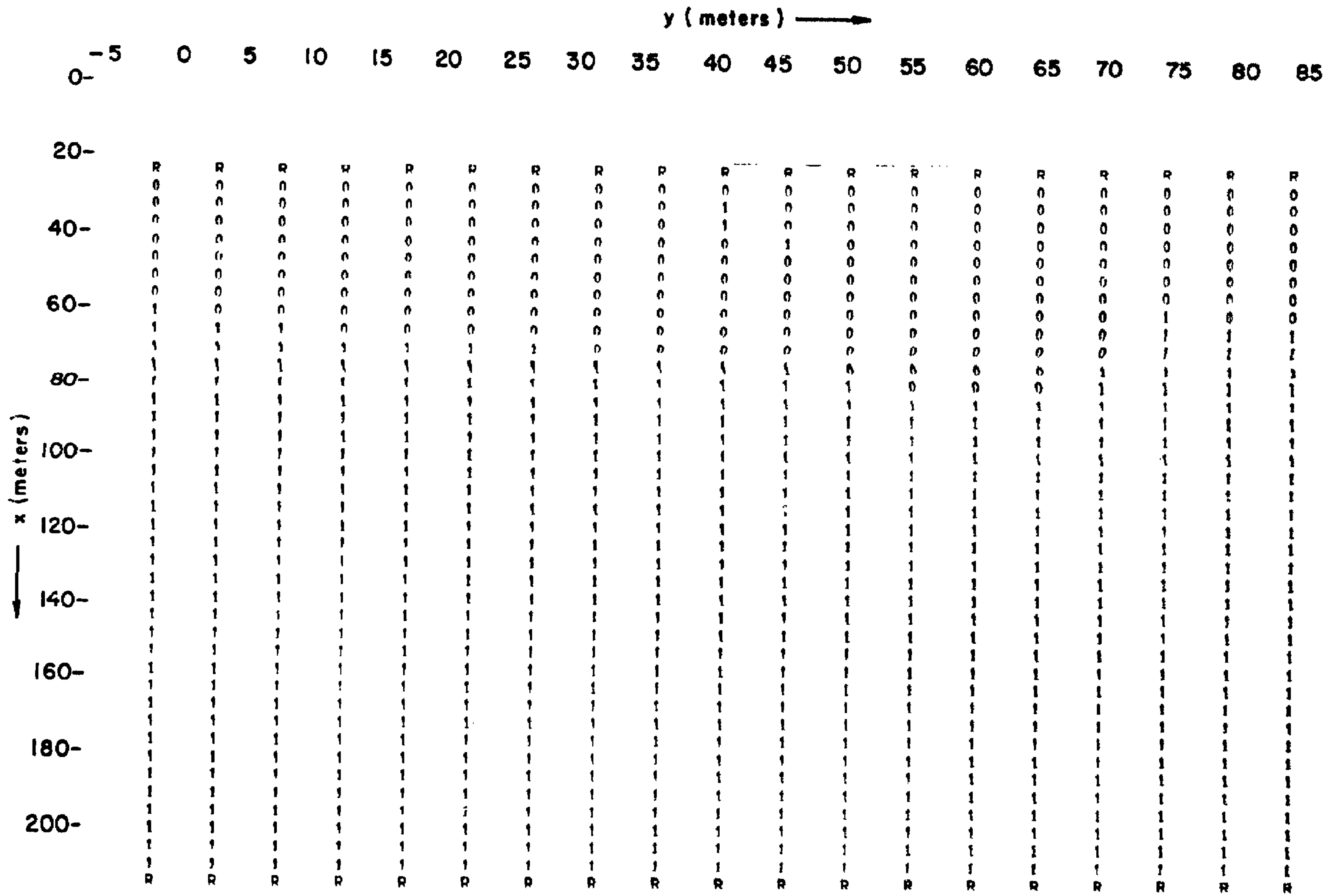


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TABLE 3.17

50% WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ u VELOCITY FIELD (meters/sec)

y (meters) ⟶

x (meters) ↓

x (meters)	-5	0	5	10	15	20	25	30	35	40
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	.02522	.00241	-.00890	-.00969	.02094	.10670	.27400	.81979	1.20273	.98439
	.05536	.05748	.02253	-.03081	-.10290	-.18341	-.25636	-.04689	1.42284	3.17752
	.02578	.12787	.08546	-.00875	-.14936	-.33240	-.50209	-.26573	1.17275	3.33640
20	-.11153	.18144	.16735	.04524	-.13274	-.35200	-.53208	-.35413	.71901	2.51748
	-.34284	.16272	.24537	.11584	-.07395	-.28254	-.44869	-.35839	.33166	1.59000
	-.41310	.05408	.28384	.17884	-.00289	-.17827	-.32263	-.30849	.05068	.84155
	.13656	.01731	.23037	.18840	.04893	-.08410	-.21128	-.24842	-.10048	.32556
40	1.13014	.53652	.14151	.07176	.04808	-.02554	-.15259	-.21767	-.16860	.04676
	1.75024	1.53090	.35647	-.00516	.03879	-.00222	-.15832	-.24130	-.19351	-.01087
	1.76560	1.99835	.93884	.14540	-.11364	-.17239	-.23970	-.26661	-.17091	.13087
	1.56744	1.64788	1.03629	.39667	.04350	-.07963	-.15517	-.21408	-.14097	.18023
60	1.23407	1.10606	.85339	.47822	.16480	-.01950	-.11125	-.14096	-.09671	.10207
	.80986	.72155	.55900	.38565	.22997	.07249	-.05274	-.11[illegible]	-.13175	-.07879
	.34323	.40770	.39857	.32650	.21596	.09115	-.02553	-.11829	-.17457	-.19784
	.10580	.19247	.27235	.27515	.20514	.09287	-.02918	-.13065	-.18932	-.20784
80	.00697	.05441	.16878	.22953	.19086	.08128	-.04612	-.14118	-.17928	-.17049
	-.01338	-.00918	.08844	.17723	.16599	.06413	-.06183	-.14286	-.15418	-.11873
	-.00348	-.01033	.04560	.12140	.12378	.03761	-.07316	-.13330	-.11948	-.06528
	.00821	.02450	.04701	.07614	.06458	-.00423	-.08286	-.11224	-.07737	-.01455
100	.01343	.06766	.08154	.05443	-.00149	-.06286	-.09616	-.08238	-.02929	.03191
	.01563	.10272	.12602	.05651	-.05930	-.13102	-.11651	-.04957	.02190	.07324
	.02119	.12569	.15925	.06839	-.09814	-.19324	-.14159	-.02076	.07080	.10819
	.03439	.13944	.17056	.07115	-.11812	-.23218	-.16176	-.00041	.10998	.13423
	.05571	.14787	.15926	.05237	-.12823	-.23722	-.16305	.01247	.13249	.14713
120	.08258	.15261	.12992	.01093	-.13784	-.20937	-.13471	.02435	.13502	.14200
	.11080	.15230	.08790	-.04525	-.14937	-.15884	-.07654	.04292	.11989	.11596
	.13580	.14370	.03747	-.10414	-.15751	-.09784	-.00054	.07086	.09418	.07105
	.15338	.12349	-.01788	-.15382	-.15366	-.03531	.07466	.10217	.06589	.01550
140	.16002	.09002	-.07384	-.18477	-.13123	.02351	.13210	.12456	.03990	-.03797
	.15320	.04453	-.12417	-.19095	-.08913	.07469	.16159	.12627	.01670	-.07651
	.13190	-.00828	-.16089	-.17042	-.03275	.11346	.16042	.10276	-.00510	-.09194
	.09756	-.06044	-.17607	-.12605	.02726	.13405	.13197	.05921	-.02495	-.08359
160	.05484	-.10186	-.16479	-.06607	.07766	.13150	.08427	.00860	-.03888	-.05782
	.01171	-.12265	-.12814	-.00350	.10605	.10487	.02934	-.03317	-.04169	-.02503
	-.02217	-.11675	-.07485	.04606	.10505	.06042	-.01809	-.05353	-.03181	.00483
	-.03885	-.08620	-.02057	.06906	.07660	.01256	-.04435	-.04896	-.01391	.02127
180	-.03645	-.04383	.01657	.06079	.03462	-.02002	-.04345	-.02764	.00240	.02368
	-.02231	-.01052	.02502	.03229	.00194	-.02516	-.02341	-.00596	.00866	.01481
	-.01230	-.00446	.0106[illegible]	.00977	-.00349	-.00978	-.00454	.00241	.00496	.00376
	-.00672	-.00263	.00809	.00818	-.00003	-.00475	-.00317	.00004	.00158	.00418
200										

TABLE 3.17

WAVE-CURRENT INTERACTION:

4 sec., $\theta_d = 150^\circ$ u VELOCITY FIELD (meters/sec)

(CONTINUED)

y (meters) ⟶

45	50	55	60	65	70	75	80	85
[illegible]	[illegible]	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
[illegible]	[illegible]	.19574	.48247	.18179	.07229	.02522	.00241	-.00890
[illegible]	[illegible]	-.00067	-.92229	-.52639	-.07444	.05636	.05748	.02253
[illegible]	[illegible]	.17557	-.37286	-1.68299	-.51630	.02578	.12787	.08546
[illegible]	[illegible]	[illegible]	-.68070	-2.22740	-1.27979	-.11153	.18144	.16733
[illegible]	[illegible]	-.19125	-1.21415	-2.17369	-1.92478	-.34284	.16272	.24537
[illegible]	[illegible]	.42526	-.66730	-2.76747	-2.10988	-.41310	.05408	.28384
[illegible]	[illegible]	.23390	.27520	-3.03666	-2.21873	.13656	.01731	.23037
[illegible]	[illegible]	.96950	.51876	-2.91147	-1.98044	1.13014	.53652	.14151
[illegible]	[illegible]	1.25655	-.41946	-2.84466	-1.35715	1.75024	1.53090	.35647
[illegible]	[illegible]	.24967	-1.49086	-2.65009	-.72997	1.76560	1.99835	.93888
[illegible]	[illegible]	-.90479	-2.08269	-2.09208	-.08243	1.56744	1.64788	1.03629
[illegible]	[illegible]	.60109	-2.21810	-1.19586	.54159	1.23407	1.10606	.85339
[illegible]	[illegible]	.48441	-1.33278	-.20382	.61947	.80986	.72155	.55900
[illegible]	[illegible]	.59453	-.45150	-.07020	.20735	.34323	.40770	.39857
[illegible]	[illegible]	-.24210	-.13260	.01629	.07708	.10580	.19247	.27235
[illegible]	[illegible]	.09216	-.01999	.05006	.04641	.00697	.05441	.16878
[illegible]	[illegible]	.02178	.00977	.04125	.03372	-.01338	-.00918	.08844
[illegible]	[illegible]	.01284	.00792	.01053	.01293	-.00348	-.01033	.04560
[illegible]	[illegible]	.02757	-.00529	-.02586	-.01600	.00821	.02450	.04701
[illegible]	[illegible]	.02850	-.02285	-.05837	-.04449	.01343	.06766	.08154
[illegible]	[illegible]	.01885	-.04253	-.08304	-.06444	.01563	.10272	.12602
[illegible]	[illegible]	.00148	-.06290	-.09907	-.07165	.02119	.12569	.15925
[illegible]	[illegible]	.02044	-.08194	-.10661	-.06571	.03439	.13944	.17056
[illegible]	[illegible]	-.04339	-.09704	-.10576	-.04851	.05571	.14787	.15926
[illegible]	[illegible]	.06357	-.10569	-.09651	-.02282	.08258	.15261	.12992
[illegible]	[illegible]	-.07732	-.10611	-.07919	.00849	.11080	.15230	.08790
[illegible]	[illegible]	.08201	-.09758	-.05492	.04256	.13580	.14370	.03747
[illegible]	[illegible]	.07733	-.08053	-.02555	.07646	.15338	.12349	-.01788
[illegible]	[illegible]	.06618	-.05687	.00674	.10712	.16002	.09002	-.07334
[illegible]	[illegible]	-.05408	-.03017	.03966	.13157	.15320	.04453	-.12417
[illegible]	[illegible]	.04568	-.00547	.07054	.14719	.13190	-.00828	-.16089
[illegible]	[illegible]	.04676	.01213	.09579	.15187	.09756	-.06044	-.17607
[illegible]	[illegible]	.05248	.01958	.11092	.14409	.05484	-.10186	-.16479
[illegible]	[illegible]	.05819	.01754	.11180	.12333	.01171	-.12265	-.12814
[illegible]	[illegible]	.05763	.01019	.09695	.09137	-.02217	-.11675	-.07485
[illegible]	[illegible]	.04776	.00297	.06962	.05373	-.03885	-.08620	-.02057
[illegible]	[illegible]	.03107	-.00033	.03847	.02002	-.03645	-.04383	.01657
[illegible]	[illegible]	.01449	.00095	.01550	.00055	-.02231	-.01052	.02502
[illegible]	[illegible]	.00546	.00594	.01121	-.00055	-.01230	-.00436	.01069
[illegible]	[illegible]	-.00753	.00533	.01275	.00415	-.00672	-.00263	.00809

TABLE 3.18

50% WAVE-CURRENT INTERACTION:

H_d = 1.0 m, T_d = 4 sec., θ_d = 150° v VELOCITY FIELD (meters/sec)

y (meters) ⟶

x (meters) ⟵ (0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200)

-5	0	5	10	15	20	25	30	35	40
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
.54608	.47508	.43114	.43001	.49215	.61945	.71562	.65306	.34201	-.98244
.89125	.79328	.70088	.66065	.70912	.89228	1.21148	1.47060	1.39662	.52328
1.12800	1.05307	.92614	.84501	.84527	.95312	1.18280	1.39159	1.26764	1.20165
1.19813	1.23007	1.09815	.98776	.94678	.96817	1.04948	1.13124	1.16812	1.38153
1.08462	1.27457	1.18580	1.06974	1.00743	.96701	.93967	.92522	.99173	1.31151
.98075	1.12821	1.14113	1.07051	1.01820	.95180	.88086	.82403	.86470	1.16593
1.37696	.87720	.90671	.97375	.97950	.91636	.85588	.81418	.83901	1.03465
2.24586	1.01814	.72724	.85427	.90254	.85377	.83146	.88700	.88541	.97316
2.30295	1.44398	.76013	.59968	.67476	.76989	.84365	.90060	.94639	.94004
1.18795	1.21318	.81468	.46822	.39787	.45831	.48366	.48652	.50198	.46740
.08290	.53647	.64483	.47330	.28246	.19727	.14765	.10073	.05905	.05087
-.35354	.07125	.33397	.39257	.29371	.19367	.14408	.10868	.07741	.12218
-.42393	-.07209	.16211	.26462	.25293	.19398	.14055	.11929	.13784	.21802
-.38287	-.05935	.11127	.16362	.17286	.16272	.14528	.14635	.17434	.27685
-.27904	-.07300	.07197	.12557	.13055	.12650	.13431	.15458	.17503	.18291
-.17942	-.08582	.02706	.08595	.09760	.10266	.12209	.14576	.15289	.13227
-.08834	-.07787	-.01931	.04209	.07237	.09111	.11479	.13260	.12635	.09096
-.02001	-.04832	-.05110	-.00718	.04252	.08307	.11319	.12200	.10254	.05942
.01997	-.01399	-.05849	-.05010	.00642	.07144	.11315	.11407	.08166	.03410
.03570	.00948	-.04712	-.07201	-.02742	.05340	.10903	.10631	.06312	.01281
.03551	.01653	-.03081	-.06801	-.04504	.03252	.09632	.09508	.04650	-.00467
.02725	.00969	-.02012	-.04582	-.03693	.01766	.07413	.07664	.03093	-.01726
.01585	-.00502	-.01812	-.01794	-.00588	.01749	.04631	.04861	.01437	-.02418
.00342	-.02166	-.02190	.00669	.03499	.03292	.01998	.01196	-.00588	-.02682
-.00939	-.03598	-.02583	.02553	.07121	.05481	.00110	-.02766	-.03088	-.02474
-.02215	-.04544	-.02463	.04057	.09382	.07000	-.00978	-.06132	-.05733	-.02791
-.03391	-.04857	-.01531	.05461	.10056	.06951	-.01713	-.08098	-.07725	-.03008
-.04310	-.04449	.00223	.06812	.09317	.05198	-.02575	-.08360	-.08143	-.02982
-.04780	-.03281	.02538	.07838	.07440	.02219	-.03690	-.07150	-.06507	-.02208
-.04619	-.01402	.04939	.08086	.04688	-.01215	-.04759	-.04959	-.03144	-.00452
-.03710	.00994	.06837	.07152	.01370	-.04287	-.05238	-.02254	.00926	.01924
-.02050	.03508	.07652	.04916	-.02044	-.06266	-.04613	.00588	.04457	.04841
.00189	.05549	.06973	.01678	-.04889	-.06604	-.02696	.03170	.06457	.05004
.02588	.06466	.04753	-.01818	-.06402	-.05094	.00142	.04947	.06485	.04435
.04454	.05798	.01476	-.04521	-.05968	-.02098	.02927	.05315	.00741	.02692
.04976	.03596	-.01777	-.05372	-.03550	.01283	.04382	.03989	.02019	.00650
.03682	.00697	-.03558	-.03881	-.00166	.03371	.03640	.01460	-.00511	-.00783
.00993	-.01372	-.02803	-.00882	.02098	.02845	.01152	-.00919	-.01786	-.01195
-.00504	-.01433	-.01271	.00144	.01022	.00336	-.00941	-.01620	-.01527	-.00937
-.00767	-.00375	.00236	.00542	.00109	-.00693	-.01184	-.01189	-.00981	-.00787

TABLE 3.18

50% WAVE-CURRENT INTERACTION:

$H_d = 1.0$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ v VELOCITY FIELD (meters/sec)

(CONTINUED)

y (meters) →

45	50	55	60	65	70	75	80	85
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
-3.68555	-1.33444	-1.08934	-.43370	.38142	.58656	.54608	.47508	.03114
-1.52408	-3.23515	-.54671	-1.59329	.05809	.81287	.89125	.79328	.70088
1.03867	-2.15969	-.61824	-1.65916	-.54317	.84595	1.12800	1.05307	.92614
1.94582	.66987	-1.63015	-.88327	-.86299	.73361	1.19813	1.23007	1.09815
1.91541	2.00477	-.68725	-1.00660	-.59803	.76566	1.08462	1.27457	1.18580
1.68107	1.98745	1.04686	-.70632	-.31218	1.35981	.98075	1.12821	1.14113
1.43028	1.75058	1.68409	.82090	.45034	2.19211	1.37696	.87720	.90671
1.15408	1.37550	1.54600	1.90568	2.04347	2.51536	2.24586	1.01814	.72724
.83583	.79627	1.20940	2.13213	2.88472	2.12992	2.30295	1.44398	.76013
.29341	.11337	.92339	2.06141	2.31451	1.24528	1.18795	1.21318	.81468
.06666	-.01673	.76791	1.81040	1.41911	.24665	.08290	.53647	.64483
.32797	.65160	1.04136	1.34926	.37551	-.54990	-.35354	.07125	.33397
.44512	.92069	.76309	.06820	-.83198	-.95280	-.42393	-.07209	.16211
.32386	.47561	.09457	-.64315	-.99313	-.79540	-.38287	-.05935	.11127
.17223	.12611	-.06762	-.33030	-.86490	-.43224	-.27904	-.07300	.07197
.08420	.01079	-.09532	-.19301	-.22943	-.21724	-.17942	-.08582	.02706
.03505	-.03069	-.09468	-.13101	-.12180	-.00424	-.08834	-.07787	-.01931
.00588	-.04530	-.08403	-.09505	-.06921	-.02871	-.02001	-.04832	-.05110
-.01351	-.05057	-.07060	-.06921	-.03904	.00320	.01997	-.01399	-.05849
-.02731	-.05088	-.05634	-.04409	-.01872	.01814	.03570	.00948	-.04712
-.03660	-.04783	-.04130	-.02223	.00283	.02597	.03551	.01653	-.03081
-.04094	-.04083	-.02500	-.00147	.02007	.03073	.02725	.00969	-.02012
-.03894	-.02945	-.00733	.01724	.03376	.03283	.01585	-.00502	-.01812
-.02985	-.01360	.01105	.03305	.04262	.03140	.00342	-.02166	-.02190
-.01476	.00507	.02874	.04495	.04601	.02577	-.00939	-.03598	-.02583
.00289	.02711	.04409	.05193	.04399	.01618	-.02215	-.04544	-.02463
.01831	.04290	.05533	.05349	.03720	.00379	-.03391	-.04857	-.01531
.02755	.05195	.06092	.05011	.02680	-.00971	-.04310	-.04449	.00223
.02929	.05466	.05959	.04313	.01452	-.02242	-.04780	-.03281	.02538
.02534	.04720	.05048	.03406	.00255	-.03238	-.04619	-.01402	.04939
.01901	.02628	.03394	.02365	-.00709	-.03751	-.03710	.00994	.06837
.01297	.00235	.01229	.01163	-.01335	-.03595	-.02050	.03508	.07652
.00812	-.01680	-.00985	-.00242	-.01666	-.02683	.00189	.05549	.06973
.00418	-.02540	-.02660	-.01745	-.01829	-.01146	.02588	.06466	.04753
.00115	-.02238	-.03330	-.03017	-.01903	.00600	.04454	.05798	.01476
-.00020	-.01161	-.02907	-.03592	-.01831	.01903	.04976	.03596	-.01777
.00037	.00032	-.01739	-.03134	-.01506	.02111	.03682	.00697	-.03558
.00134	.00725	-.00402	-.01745	-.01022	.00901	.00993	-.01372	-.02803
-.00102	.00479	.00315	-.00167	-.00136	.00058	-.00504	-.01433	-.01271
-.00540	-.00533	-.00847	-.01205	-.01220	-.00989	-.00767	-.00375	.00236

x (meters) ↓ 0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200

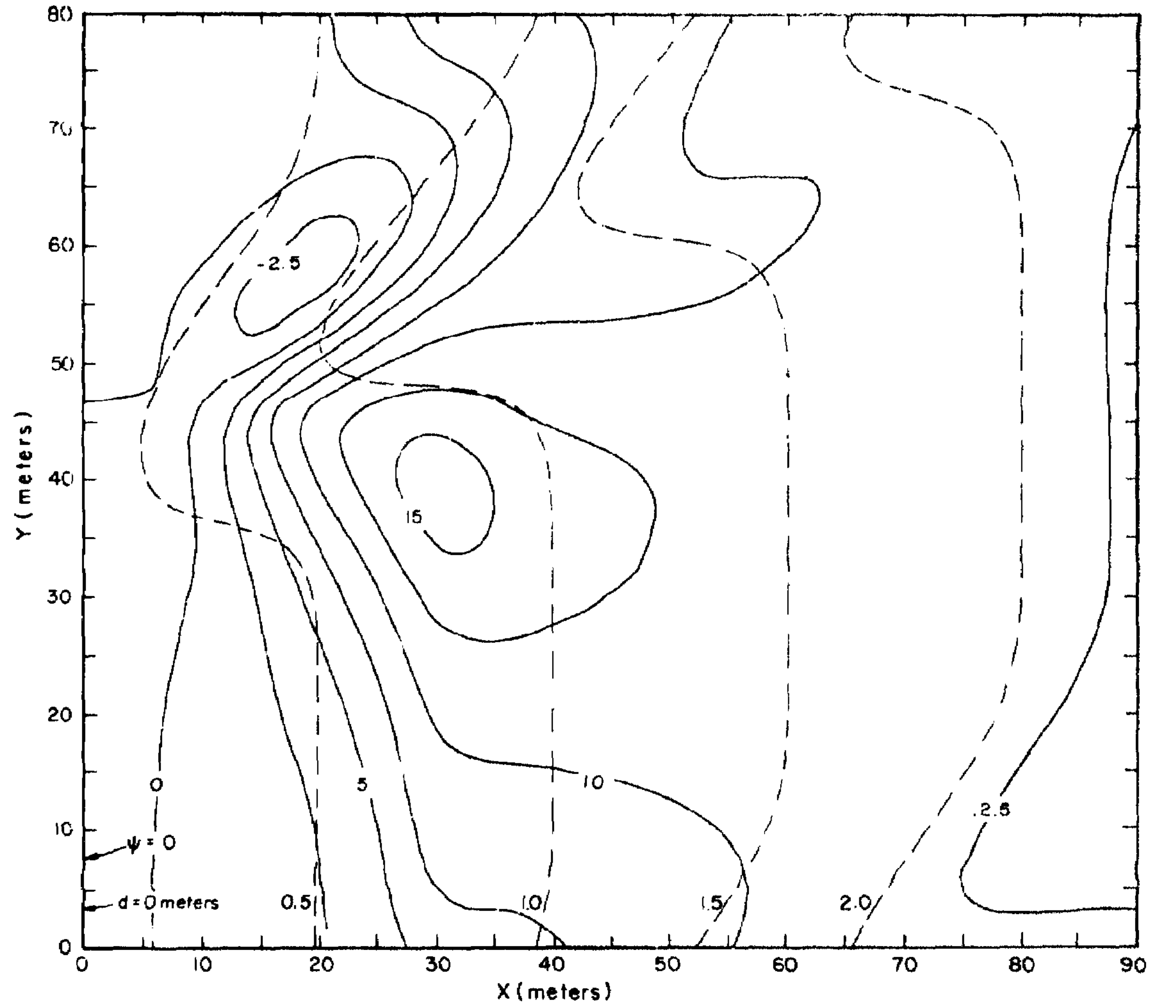


Figure 3.9: Stream Function Field ψ For No Wave-Current Interaction: (H_d = 0.5m, θ_d = 150°, T_d = 4 sec.)

TABLE 3.19

NO WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150°$ WAVE DIRECTION FIELD θ(degrees)

x (meters) ↓ \ y (meters) →	-5	0	5	10	15	20	25	30	35	40
0	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000
	171.46003	172.62020	173.33556	173.80893	174.14907	175.28262	181.48973	194.26828	203.50741	196.73183
	168.00991	169.56758	170.55199	171.20970	171.66696	172.10530	173.77977	180.71921	192.28419	195.68216
	165.60405	167.28434	168.40062	169.16479	169.70548	170.10917	170.72314	173.27719	180.46447	186.38643
20	163.96121	165.58145	166.73737	167.54985	168.13466	168.56502	168.93666	169.84031	173.16398	179.74841
	162.88286	164.25866	165.39377	166.21747	166.82038	167.26938	167.61336	168.01587	169.33472	173.17350
	162.35356	163.23255	164.27907	165.08969	165.69312	166.14789	166.49539	166.78498	167.30842	169.10831
	162.41701	162.50435	163.34251	164.11657	164.70871	165.15999	165.50786	165.78091	166.06006	166.80509
40	163.05127	162.13760	162.57381	163.26522	163.83697	164.27829	164.62139	164.89124	165.11489	165.44353
	164.07748	162.19950	162.00955	162.51990	163.05622	163.48108	163.81768	164.08276	164.29620	164.49972
	165.17142	162.67474	161.72024	161.80116	162.35198	162.75959	163.08368	163.34121	163.55007	163.72584
	165.97207	163.40899	161.76197	161.42273	161.72139	162.09663	162.40709	162.65674	162.86026	163.02925
	166.21120	164.13683	162.10961	161.17773	161.18257	161.48752	161.78140	162.02157	162.21889	162.38393
60	165.79797	164.58073	162.62269	161.19555	160.77867	160.93548	161.19968	161.42947	161.62008	161.78087
	164.83322	164.55525	163.08168	161.44015	160.56216	160.46157	160.65935	160.87537	161.05901	161.21529
	163.55569	164.02480	163.27451	161.78262	160.55648	160.10569	160.16700	160.35560	160.53173	160.68343
	162.24137	163.10088	163.07721	162.09160	160.71705	159.90942	159.74066	159.87011	160.03499	160.18211
80	161.09771	161.98753	162.49071	162.05690	160.92647	159.88250	159.42791	159.42761	159.56677	159.70867
	160.2[illegible]629	160.89654	161.62758	161.75019	161.03597	159.97482	159.24812	159.04998	159.12875	159.26093
	159.53981	159.96945	160.65959	161.14606	160.92677	160.08092	159.20392	158.76804	158.73097	158.83794
	159.02366	159.24635	159.74739	160.35236	160.55433	160.08020	159.20248	158.60393	158.39437	158.44261
100	158.58942	158.69142	158.98461	159.51152	159.95785	159.88693	159.27053	158.54754	158.14482	158.08568
	158.19568	158.24335	158.38499	158.74339	159.23701	159.48136	159.19173	158.54562	157.99618	157.78651
	157.82423	157.85206	157.91083	158.10709	158.50810	158.91079	158.94659	158.51686	157.93164	157.56583
	157.46901	157.48955	157.51294	157.60036	157.85867	158.26487	158.53309	158.38560	157.89916	157.43034
	157.12814	157.14434	157.15512	157.18750	157.32345	157.63689	158.00190	158.11282	157.82886	157.35820
120	156.80059	156.81278	156.81872	156.83006	156.89130	157.08915	157.43156	157.70861	157.66242	157.29991
	156.48548	156.49361	156.49668	156.50245	156.53061	156.63944	156.89537	157.22344	157.37656	157.19634
	156.18200	156.18604	156.18698	156.19264	156.21209	156.27724	156.43576	156.72433	156.98915	157.00385
	155.88934	155.88942	155.88903	155.89676	155.91806	155.96055	156.05876	156.26716	156.50811	156.71125
140	155.60676	155.60317	155.60261	155.61408	155.64106	155.68213	155.74692	155.87890	156.10920	156.34127
	155.33374	155.32687	155.32770	155.34479	155.37907	155.42454	155.47723	155.55757	155.71376	155.93803
	155.07072	155.06030	155.06456	155.08933	155.13187	155.18263	155.23244	155.28539	155.37732	155.54715
	154.82033	154.80374	154.81370	154.84829	154.89951	154.95457	155.00314	155.04315	155.09332	155.1[illegible]846
160	154.58822	154.55883	154.57616	154.62235	154.68187	154.73925	154.78511	154.81763	154.84544	154.89974
	154.38151	154.32975	154.35343	154.41216	154.47848	154.53534	154.57618	154.60210	154.61867	154.64255
	154.20438	154.12352	154.14854	154.21819	154.28832	154.34113	154.37472	154.39373	154.40385	154.41313
	154.05326	153.94816	153.96669	154.04089	154.10970	154.15455	154.17920	154.19181	154.19705	154.20038
180	153.91563	153.80839	153.81476	153.88102	153.94023	153.97329	153.98838	153.99468	153.99719	153.99825
	153.77480	153.70178	153.69862	153.73994	153.77722	153.79515	153.80150	153.80359	153.80427	153.80449
	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857
200										

TABLE 3.19

NO WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^{o}$ WAVE DIRECTION FIELD θ(degrees)

(CONTINUED)

x (meters) ↓ \ y (meters) →	45	50	55	60	65	70	75	80	85
0-	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000
	182.72258	169.32804	160.06917	159.45411	164.89265	169.28553	171.46003	172.62020	173.33556
	187.92863	176.58631	166.52056	161.31513	162.18330	165.46346	168.00991	169.56758	170.55199
	188.28856	180.62291	170.97068	163.72416	161.56933	163.27312	165.60405	167.28434	168.40062
20-	184.40406	182.05246	174.74565	167.04188	162.56435	162.28632	163.96121	165.58145	166.73737
	178.55614	180.51458	176.81026	170.18748	164.51276	162.26895	162.88286	164.25866	165.39377
	173.0714	176.91379	176.78657	172.43931	166.82656	163.09477	162.35356	163.23255	164.27907
	169.047	172.69238	174.90881	173.30446	168.88456	164.51363	162.41701	162.50435	163.34251
40-	166.[illegible]	168.99993	171.95406	172.67138	170.15784	166.11448	163.05127	162.13760	162.57381
	164.94982	166.32356	168.84101	170.85735	170.34636	167.42750	164.07748	162.19950	162.00955
	163.94739	164.58787	166.22762	168.46358	169.46429	168.07294	165.17142	162.67474	161.72024
	163.18559	163.47454	164.34692	166.10742	167.81834	167.87677	165.97207	163.40899	161.76197
60-	162.525[illegible]	162.68687	163.09473	164.18857	165.87111	166.91236	166.21120	164.13683	162.10961
	161.91890	162.04515	162.24174	162.80926	164.05023	165.45430	165.79797	164.58073	162.62269
	161.35046	161.46919	161.59305	161.86154	162.59993	163.85994	164.83322	164.55525	163.08168
	160.81562	160.93179	161.03682	161.17667	161.55131	162.42802	163.55569	164.02480	163.27451
80-	160.31124	160.42517	160.52570	160.62350	160.80388	161.30410	162.24137	163.10088	163.07721
	159.83471	159.94623	160.04443	160.13128	160.23252	160.48063	161.09771	161.98753	162.49071
	159.38378	159.49272	159.58809	159.67176	159.74801	159.86935	160.20629	160.89654	161.62758
	158.95651	159.06268	159.15574	159.23621	159.30527	159.37749	159.53981	159.96945	160.65959
100-	158.55134	158.65437	158.74442	158.82185	158.88742	158.94422	159.02366	159.24635	159.74739
	158.16804	158.26622	158.35295	158.42706	158.48935	158.54082	158.58942	158.69142	158.98461
	157.81036	157.89705	157.97990	158.05042	158.10926	158.15718	158.19568	158.24335	158.38499
	157.48918	157.54719	157.62398	157.69066	157.74587	157.79028	157.82423	157.85206	157.91083
120-	157.22192	157.22106	157.28436	157.34662	157.39808	157.43885	157.46901	157.48955	157.51294
	157.02437	156.92941	156.96195	157.01731	157.06488	157.10187	157.12814	157.14434	157.15512
	156.89688	156.68747	156.66176	156.70230	156.74535	156.77843	156.80059	156.81278	156.81872
	156.81461	156.50647	156.39426	156.40312	156.43875	156.46767	156.48508	156.49361	156.49668
140-	156.73170	156.38185	156.17247	156.12525	156.14504	156.16880	156.18200	156.18608	156.18698
	156.59933	156.28733	156.00356	155.87867	155.86621	155.88120	155.88934	155.88942	155.88903
	156.38640	156.18174	155.87930	155.67411	155.60795	155.60498	155.60676	155.60317	155.60261
	156.09112	156.02598	155.77331	155.51506	155.37933	155.34241	155.33374	155.32687	155.32770
160-	155.73897	155.79955	155.64959	155.39048	155.18849	155.09898	155.07072	155.06030	155.06456
	155.36969	155.50764	155.47779	155.27473	155.03547	154.88247	154.82033	154.80374	154.81374
	155.01995	155.17662	155.24585	155.13686	154.90698	154.69839	154.58822	154.55883	154.57616
	154.71056	154.84124	154.96359	154.95387	154.77850	154.54381	154.38151	154.32975	154.35343
180-	154.44404	154.53009	154.65654	154.71994	154.62366	154.40426	154.20438	154.12352	154.14854
	154.21139	154.25666	154.35395	154.44722	154.42535	154.25740	154.05326	153.94816	153.96669
	154.00117	154.01972	154.07701	154.15880	154.18201	154.08215	153.91563	153.80839	153.81476
	153.80499	153.81022	153.83322	153.87794	153.90646	153.86789	153.77480	153.70178	153.69862
200-	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857

TABLE 3.20

NO WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ WAVE HEIGHT FIELD H(meters)

y (meters) →

x (meters) ↓	-5	0	5	10	15	20	25	30	35	40
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	.09326	.09326	.09326	.09326	.09337	.09570	.11194	.16572	.26115	.34688
	.18461	.18456	.18456	.18456	.18457	.18514	.19218	.22711	.31517	.43440
	.27033	.27391	.27391	.27391	.27391	.27398	.27596	.29175	.34963	.46191
20	.36358	.36141	.36132	.36132	.36132	.36132	.36168	.36695	.39641	.47816
	.45511	.44747	.44682	.44681	.44681	.44681	.44684	.44810	.45975	.50726
	.51175	.52295	.53030	.53038	.53038	.53038	.53039	.53058	.53406	.50572
	.49386	.50429	.51146	.51609	.51893	.52059	.52156	.52229	.52368	.52663
40	.48065	.48971	.49660	.50119	.50412	.50593	.50705	.50786	.50879	.51070
	.47166	.47825	.48463	.48913	.49209	.49400	.49527	.49621	.49710	.49839
	.46659	.46955	.47490	.47923	.48218	.48416	.48554	.48662	.48760	.48870
	.46468	.46359	.46707	.47101	.47392	.47594	.47742	.47862	.47973	.48083
60	.46055	.46028	.46110	.46421	.46699	.46903	.47059	.47191	.47512	.47431
	.46474	.45901	.45707	.45870	.46117	.46320	.46483	.46624	.46756	.46883
	.46425	.45880	.45489	.45455	.45632	.45828	.45996	.46146	.46287	.46421
	.46280	.45864	.45408	.45185	.45245	.45415	.45585	.45742	.45892	.46031
80	.46058	.45793	.45387	.45048	.44963	.45075	.45240	.45403	.45559	.45702
	.45792	.45655	.45358	.45003	.44794	.44813	.44955	.45121	.45281	.45425
	.45517	.45466	.45284	.44990	.44722	.44638	.44730	.44889	.45050	.45198
	.45262	.45251	.45162	.44963	.44711	.44551	.44571	.44704	.44861	.45002
100	.45043	.45039	.45007	.44900	.44715	.44536	.44483	.44569	.44710	.44844
	.44864	.44849	.44841	.44802	.44702	.44560	.44464	.44488	.44597	.44717
	.44721	.44693	.44685	.44686	.44661	.44587	.44496	.44465	.44523	.44617
	.44605	.44570	.44554	.44570	.44598	.44597	.44549	.44493	.44493	.44547
120	.44513	.44476	.44453	.44471	.44527	.44584	.44596	.44552	.44508	.44508
	.44441	.44404	.44382	.44398	.44464	.44557	.44623	.44617	.44556	.44505
	.44387	.44352	.44335	.44354	.44422	.44529	.44629	.44666	.44620	.44536
	.44349	.44318	.44308	.44336	.44406	.44512	.44624	.44692	.44677	.44588
140	.44326	.44299	.44300	.44339	.44410	.44513	.44618	.44697	.44712	.44646
	.44317	.44296	.44309	.44362	.44443	.44533	.44621	.44692	.44725	.44691
	.44321	.44308	.44335	.44400	.44486	.44569	.44636	.44688	.44721	.44716
	.44337	.44335	.44376	.44453	.44542	.44615	.44662	.44690	.44713	.44724
160	.44366	.44376	.44431	.44517	.44604	.44666	.44695	.44703	.44710	.44725
	.44409	.44430	.44500	.44591	.44672	.44719	.44733	.44727	.44720	.44729
	.44466	.44498	.44579	.44671	.44740	.44773	.44779	.44759	.44745	.44747
	.44542	.44572	.44666	.44753	.44808	.44825	.44817	.44800	.44784	.44781
180	.44637	.44674	.44758	.44834	.44873	.44878	.44865	.44849	.44836	.44831
	.44753	.44780	.44852	.44912	.44935	.44932	.44919	.44907	.44898	.44898
	.44884	.44898	.44947	.44986	.44997	.44991	.44981	.44974	.44970	.44968
	.45017	.45020	.45041	.45058	.45062	.45058	.45053	.45051	.45050	.45049
200	.45135	.45135	.45135	.45135	.45135	.45135	.45135	.45135	.45135	.45135

TABLE 3.20

NO WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^{o}$ WAVE HEIGHT FIELD H(meters)

(CONTINUED)

x (meters) ↓ \ y (meters) →	45	50	55	60	65	70	75	80	85
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	.35434	.27700	.17837	.11721	.09679	.09346	.09326	.09326	.09326
	.50392	.46536	.35178	.24826	.19855	.18601	.18461	.18456	.18456
	.57498	.49823	.49209	.39647	.33154	.28014	.27433	.27391	.27391
20	.58433	.51785	.48813	.49061	.44002	.38030	.36358	.36141	.36132
	.56398	.53293	.49630	.48387	.49658	.49181	.45511	.44747	.44682
	.54693	.53390	.50602	.48400	.48259	.49637	.51175	.52295	.53030
	.52903	.52546	.51021	.48873	.47740	.48197	.49386	.50429	.51146
40	.51351	.51106	.50774	.49272	.47775	.47390	.48065	.48971	.49660
	.50065	.50277	.50132	.49318	.48002	.47090	.47166	.47825	.48463
	.49034	.49259	.49354	.49021	.48133	.47098	.46659	.46955	.47490
	.48212	.48394	.48572	.48525	.48045	.47188	.46468	.46359	.46707
60	.47549	.47687	.47858	.47951	.47763	.47195	.46455	.46028	.46110
	.47001	.47115	.47248	.47378	.47366	.47061	.46474	.45901	.45707
	.46542	.46646	.46745	.46856	.46923	.46807	.46425	.45880	.45489
	.46153	.46254	.46333	.46410	.46488	.46483	.46280	.45864	.45408
80	.45825	.45921	.45990	.46041	.46096	.46134	.46058	.45793	.45387
	.45547	.45639	.45700	.45735	.45764	.45799	.45792	.45655	.45358
	.45312	.45398	.45452	.45479	.45489	.45503	.45517	.45466	.45284
	.45115	.45193	.45241	.45261	.45261	.45257	.45262	.45251	.45162
100	.44949	.45020	.45060	.45070	.45070	.45055	.45043	.45039	.45007
	.44812	.44870	.44906	.44916	.44909	.44889	.44864	.44849	.44841
	.44698	.44751	.44777	.44783	.44773	.44752	.44721	.44693	.44685
	.44607	.44649	.44669	.44671	.44661	.44639	.44605	.44570	.44558
120	.44538	.44566	.44580	.44581	.44570	.44547	.44513	.44476	.44453
	.44494	.44503	.44510	.44509	.44498	.44475	.44441	.44404	.44382
	.44478	.44460	.44457	.44455	.44444	.44421	.44387	.44352	.44335
	.44494	.44441	.44423	.44418	.44407	.44383	.44349	.44318	.44308
140	.44537	.44450	.44411	.44397	.44384	.44359	.44326	.44299	.44300
	.44595	.44489	.44423	.44395	.44376	.44350	.44317	.44296	.44309
	.44653	.44551	.44463	.44413	.44383	.44352	.44321	.44308	.44335
	.44699	.44624	.44529	.44455	.44406	.44367	.44337	.44335	.44376
160	.44730	.44694	.44612	.44521	.44447	.44395	.44366	.44376	.44431
	.44750	.44750	.44700	.44607	.44510	.44439	.44409	.44430	.44500
	.44770	.44794	.44779	.44703	.44592	.44501	.44466	.44498	.44579
	.44798	.44831	.44844	.44797	.44690	.44584	.44542	.44579	.44666
180	.44841	.44870	.44897	.44879	.44793	.44687	.44637	.44674	.44758
	.44899	.44918	.44945	.44946	.44892	.44803	.44753	.44780	.44852
	.44970	.44979	.44997	.45005	.44979	.44923	.44884	.44898	.44987
	.45050	.45052	.45059	.45065	.45058	.45036	.45017	.45020	.45041
200	.45135	.45135	.45135	.45135	.45135	.45135	.45135	.45135	.45135

TABLE 3.21

NO WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^o$ BREAKING INDEX IB

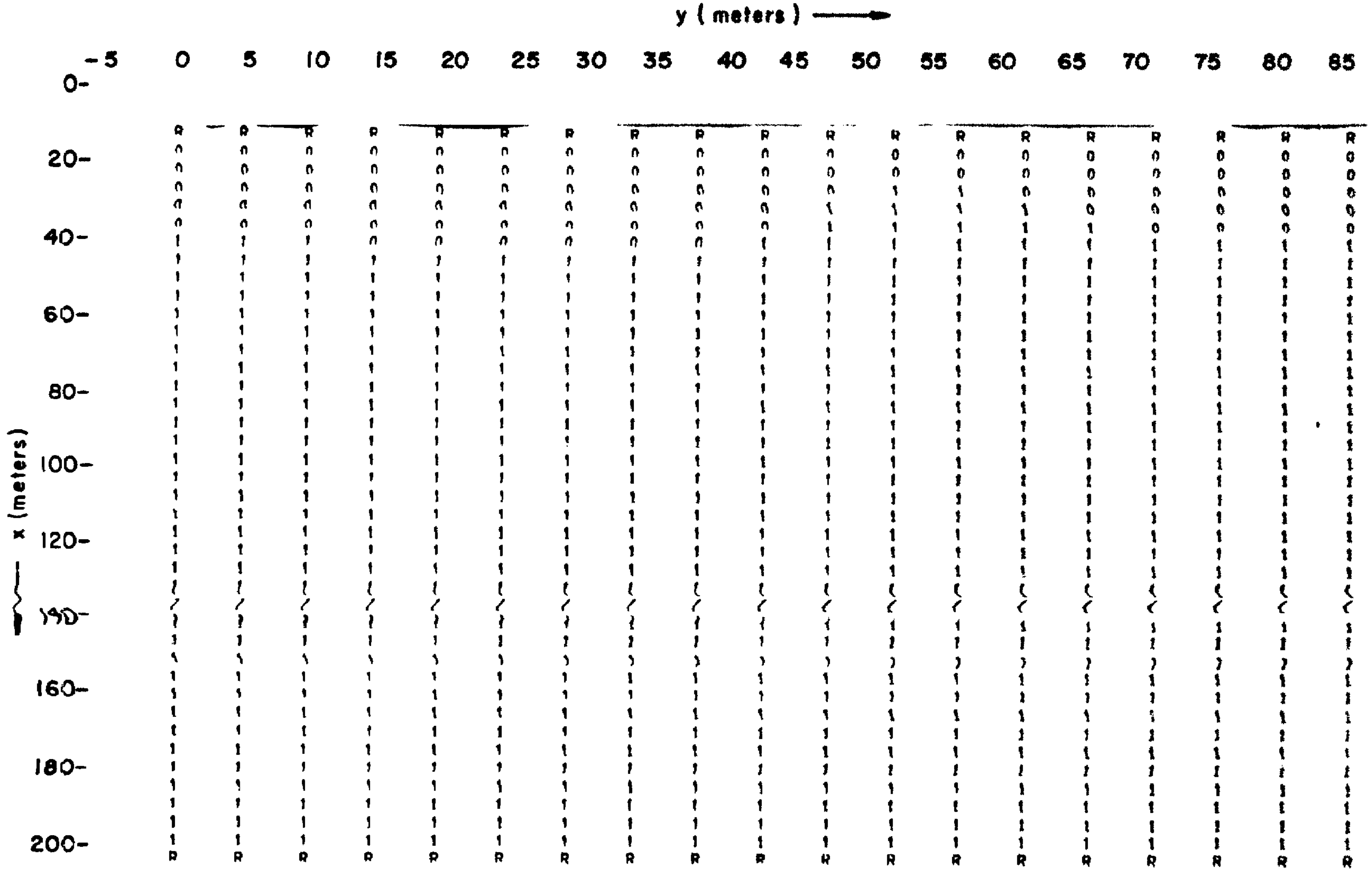


TABLE 3.22

NO WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ u VELOCITY FIELD (meters/sec)

y (meters) →

x (meters) ↓	-5	0	5	10	15	20	25	30	35	40
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	.03165	.00435	-.00250	.01051	.05924	.15997	.29377	.34844	.38355	-.06892
	.06647	.01219	-.02546	-.04884	-.06441	-.06073	-.00297	.00235	-.24625	-.03083
	.00722	-.00939	-.06133	-.10973	-.16829	-.25313	-.34541	-.40319	-.53341	-.46308
20	-.19993	-.07811	-.11420	-.16644	-.23520	-.35062	-.51331	-.65300	-.69277	-.45701
	-.40401	-.20662	-.20133	-.22120	-.26320	-.36075	-.51475	-.66963	-.65489	-.18020
	-.52272	-.40647	-.28706	-.24152	-.25072	-.31156	-.40740	-.48114	-.33876	.15505
	-.25032	-.25164	-.23215	-.23278	-.24441	-.26808	-.29836	-.28705	-.12206	.19840
40	-.06465	-.14421	-.17070	-.18123	-.19034	-.19962	-.20022	-.16515	-.05106	.13364
	.02550	-.05962	-.11235	-.13392	-.14184	-.14306	-.13276	-.09895	-.02604	.08478
	.05441	.00251	-.05905	-.09209	-.10202	-.10015	-.08783	-.06084	-.01510	.04997
	.03461	.03551	-.01245	-.05412	-.07006	-.06891	-.05798	-.03796	-.00893	.02847
60	-.01095	.03467	.02112	-.01883	-.04295	-.04619	-.03805	-.02376	-.00506	.01648
	-.05348	.00628	.03319	.01132	-.01767	-.02839	-.02455	-.01474	-.00267	.00997
	-.07152	-.03203	.02086	.02972	.00631	-.01208	-.01463	-.00891	-.00127	.00617
	-.06076	-.05968	-.00829	.02994	.02547	.00475	-.00565	-.00487	-.00054	.00366
80	-.03285	-.06430	-.03875	.01200	.03355	.02108	.00454	-.00118	-.00023	.00178
	-.00473	-.04740	-.05539	-.01531	.02625	.03242	.01635	.00373	.00017	.00025
	.01258	-.02098	-.05172	-.03849	.00592	.03319	.02738	.01088	.00155	-.00092
	.01739	.00171	-.03247	-.04675	-.01822	.02108	.03277	.01953	.00495	-.00135
100	.01397	.01378	-.00892	-.03781	-.03484	.00015	.02798	.02646	.01069	-.00024
	.00777	.01567	.00890	-.01815	-.03680	-.02056	.01260	.02701	.01727	.00320
	.00249	.01179	.01703	.00216	-.02489	-.03192	-.00813	.01801	.02116	.00875
	-.00060	.00666	.01720	.01570	-.00644	-.02989	-.02571	.00078	.01831	.01441
120	-.00183	.00299	.01357	.02059	.00987	-.01739	-.03315	-.01867	.00705	.01659
	-.00201	.00141	.00980	.01924	.01879	-.00184	-.02880	-.03265	-.00991	.01206
	-.00170	.00142	.00764	.01536	.01995	.00963	-.01685	-.03608	-.02621	.00088
	-.00114	.00232	.00720	.01169	.01610	.01372	-.[illegible]422	-.02913	-.03522	-.01444
140	-.00035	.00367	.00780	.00922	.01050	.01130	.00369	-.01666	-.03372	-.02652
	.00078	.00525	.00870	.00764	.00525	.00543	.00528	-.00483	-.02346	-.03051
	.00242	.00696	.00941	.00623	.00096	-.00089	.00234	.00238	-.00976	-.02501
	.00489	.00871	.00961	.00439	-.00262	-.00590	-.00205	.00447	.00187	-.01310
160	.00860	.01050	.00908	.00185	-.00583	-.00909	-.00542	.00342	.00844	-.00023
	.01366	.01238	.00770	-.00136	-.00869	-.01056	-.00666	.00173	.01009	.00894
	.01942	.01444	.00557	-.00495	-.01091	-.01051	-.00587	.00092	.00876	.01258
	.02408	.01645	.00315	-.00830	-.01200	-.00906	-.00376	.00123	.00651	.01162
180	.02514	.01753	.00115	-.01042	-.01147	-.00646	-.00118	.00203	.00450	.00835
	.02066	.01625	.00033	-.01011	-.00900	-.00321	.00103	.00253	.00297	.00486
	.01083	.01166	.00121	-.00638	-.00475	-.00022	.00214	.00228	.00181	.00234
	-.00121	.00512	.00412	.00045	.00009	.00130	.00172	.00137	.00100	.00113
200	-.00948	.00263	.00980	.00728	.00249	.00029	.00019	.00051	.00074	.00107

TABLE 3.22

NO WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ u VELOCITY FIELD (meters/sec)

(CONTINUED)

x (meters) ↓ \ y (meters) →	45	50	55	60	65	70	75	80	85
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	-.50585	-.30357	.14519	.23500	.15893	.08578	.03165	.00435	-.00250
	.49187	.25346	-.46998	-.43443	-.02730	.09599	.06647	.01219	-.02546
	.74709	1.26786	.13338	-.78968	-.55388	-.14008	.00722	-.00939	-.06133
20	.64105	1.37116	.89200	-.17400	-.79035	-.53644	-.19993	-.07811	-.11420
	.59653	.98122	.93147	.44044	-.32368	-.62667	-.40401	-.20662	-.20133
	.58083	.63628	.61451	.48513	.10466	-.34594	-.52272	-.40647	-.28706
	.39513	.35357	.31202	.31533	.18860	-.05677	-.23032	-.25164	-.23215
40	.27342	.24542	.10630	.14065	.15240	.06728	-.06465	-.14421	-.17070
	.18604	.19525	.09609	.02957	.06198	.08382	.02550	-.05962	-.11235
	.12010	.15166	.09501	.00341	-.01840	.03798	.05441	.00251	-.05905
	.07285	.10834	.09439	.01262	-.05263	-.02395	.03461	.03551	-.01245
60	.04190	.07022	.08065	.03768	-.04136	-.06428	-.01095	.03467	.02112
	.02344	.04120	.05870	.04909	-.00930	-.06755	-.05348	.00628	.03319
	.01304	.02212	.03661	.04479	.01807	-.04247	-.07152	-.03203	.02086
	.00706	.01088	.01937	.03199	.02965	-.00996	-.06076	-.05968	-.00829
80	.00330	.00463	.00834	.01820	.02748	.01310	-.03285	-.06430	-.03875
	.00068	.00105	.00229	.00778	.01864	.02158	-.00473	-.04740	-.05539
	-.00126	-.00119	-.00074	.00167	.00937	.01911	.01258	-.02098	-.05172
	-.00271	-.00267	-.00223	-.00117	.00286	.01202	.01739	.00171	-.03247
100	-.00356	-.00363	-.00296	-.00221	-.00050	.00522	.01397	.01378	-.00892
	-.00329	-.00407	-.00323	-.00242	-.00174	.00083	.00777	.01567	.00890
	-.00115	-.00379	-.00315	-.00229	-.00197	-.00124	.00249	.01179	.01703
	.00334	-.00224	-.00267	-.00200	-.00187	-.00192	-.00060	.00666	.01720
120	.00947	.00123	-.00152	-.00160	-.00173	-.00205	-.00183	.00299	.01357
	.01479	.00071	.00083	-.00099	-.00564	-.00365	-.00281	.00141	.00980
	.01584	.01295	.00082	.00017	-.00358	-.00294	-.00170	.00142	.00764
	.01024	.01721	.01017	.00239	-.00132	-.00201	-.00114	.00232	.00720
140	-.00120	.01630	.01523	.00595	-.00043	-.00179	-.00035	.00367	.00780
	-.01418	.00903	.01774	.01027	.00162	-.00103	.00078	.00525	.00870
	-.02304	-.00299	.01367	.01350	.00500	.00079	.00242	.00696	.00941
	-.02393	-.01499	.00416	.01307	.00889	.00419	.00489	.00871	.00961
160	-.01696	-.02194	-.00850	.00721	.01134	.00906	.00860	.01050	.00908
	-.00593	-.02124	-.01952	-.00340	.01011	.01427	.01366	.01238	.00770
	.00419	-.01409	-.02461	-.01534	.00420	.01767	.01942	.01444	.00557
	.00995	-.00448	-.02236	-.02397	-.00495	.01715	.02408	.01645	.00315
180	.01076	.00327	-.01480	-.02592	-.01366	.01206	.02514	.01753	.00115
	.00829	.00685	-.00598	-.02096	-.01804	.00402	.02066	.01625	.00033
	.00490	.00653	.00050	-.01204	-.01646	-.00400	.01083	.01166	.00121
	.00245	.00441	.00329	-.00373	-.01123	-.00983	-.00121	.00512	.00412
200	.00187	.00330	.00375	-.00033	-.00911	-.01449	-.00948	.00263	.00980

TABLE 3.23

NO WAVE-CURRENT INTERACTION:

H_d = 0.5 m, T_d = 4 sec., θ_d = 150° v VELOCITY FIELD (meters/sec)

y (meters) ⟶

x (meters) ↓	-5	0	5	10	15	20	25	30	35	40
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	.24890	.21303	.22452	.26394	.32181	.38266	.36918	.22265	.15345	.32694
	.42048	.41005	.43685	.50081	.60667	.78037	1.02529	1.19805	1.16817	1.26105
	.48155	.53991	.59462	.67523	.78397	.94560	1.20156	1.52601	1.62817	1.36824
20	.44957	[illegible]	.70415	.80495	.89835	1.00771	1.15817	1.36786	1.52112	1.27946
	.39386	.67067	.82989	.92495	.98659	1.03766	1.07986	1.11206	1.09311	.79904
	.28682	.36582	.42646	.47163	.51385	.53755	.52599	.45636	[illegible]	.03484
	.21612	.09788	.05656	.04987	.05684	.05249	.01793	-.06789	[illegible]	-.29744
40	.20752	.12785	.08562	.05834	.03342	.00407	-.04022	-.10760	-.18005	-.18463
	.18719	.13145	.08762	.05726	.03103	.00159	-.03512	-.07879	-.11432	-.10734
	.15488	.12757	.08703	.05518	.03028	.00517	-.02265	-.05059	-.06995	-.06419
	.10747	.11697	.08751	.05506	.03081	.01044	-.00975	-.02789	-.03914	-.03610
60	.05315	.09434	.08727	.05840	.03329	.01573	.00120	-.01082	-.01763	-.01611
	.00992	.05950	.08047	.06575	.03865	.02102	.00979	.00171	-.00255	-.00160
	-.00756	.02252	.06224	.06651	.04652	.02735	.01660	.01083	.00815	.00894
	.00059	-.00182	.03526	.06101	.05416	.03548	.02287	.01759	.01588	.01665
80	.02144	-.00508	.00988	.04505	.05686	.04452	.02991	.02311	.02162	.02244
	.04085	.00893	-.00255	.02347	.05053	.05141	.03819	.02867	.02617	.02691
	.05170	.02883	.00193	.00617	[illegible]	.05202	.04630	.03530	.03043	.03053
	.05390	.04457	[illegible]	.00132	.01773	.04396	.05105	.04285	.03539	.03379
100	.05097	.05204	.03506	.00463	.00620	.02939	.04910	.04942	.04156	.03734
	.04679	.05252	.04700	.02486	.00616	.01501	.03962	.05190	.04823	.04188
	.04371	.04931	.05146	.03906	.01616	.00805	.02611	.04781	.05313	.04745
	.04220	.04562	.05028	.04736	.02993	.01143	.01514	.03767	.05339	.05284
	.04173	.04304	.04665	.04917	.04111	.02229	.01225	.02578	.04744	.65562
120	.04163	.04164	.04302	.04668	.04657	.03472	.04836	.01809	.03691	.05334
	.04148	.04084	.04042	.04277	.04670	.04373	.02970	.01841	.02644	.04544
	.04112	.04008	.03868	.03936	.04389	.04750	.04083	.02600	.02107	.03439
	.04048	.03909	.03729	.03703	.04064	.04706	.04792	.03657	.02297	.02486
140	.03955	.03778	.03590	.03558	.03836	.04471	.05005	.04529	.03024	.02088
	.03835	.03615	.03442	.03463	.03728	.04241	.04858	.04937	.03853	.02334
	.03703	.03428	.03293	.03401	.03705	.04096	.04556	.04872	.04593	.02967
	.03589	.03233	.03157	.03371	.03722	.04013	.04239	.04499	.04478	.03590
160	.03521	.03067	.03056	.03374	.03745	.03933	.03946	.04013	.04177	.03905
	.03497	.02981	.03017	.03405	.03739	.03797	.03647	.03539	.03686	.03839
	.03450	.03018	.03072	.03447	.03664	.03566	.03308	.03117	.03189	.03503
	.03278	.03178	.03240	.03464	.03475	.03222	.02924	.02750	.02784	.03077
180	.02929	.03385	.03482	.03403	.03132	.02776	.02526	.02441	.02489	.02693
	.02522	.03510	.03635	.03166	.02622	.02283	.02176	.02207	.02281	.02394
	[illegible]	.03417	.03344	.02581	.02003	.01869	.01953	.02058	.02123	.02158
	.0	.02513	.02822	.02518	.02114	.01938	.01922	.01937	.01936	.01917
200										

TABLE 3.23

NO WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^o$ v VELOCITY FIELD (meters/sec)

(CONTINUED)

x (meters) ↓ / y (meters) →	45	50	55	60	65	70	75	80	85
0-	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	.15000	-.50773	-.38063	.22786	.42612	.34526	.24890	.21303	.22452
	1.08905	-.14377	-.88754	-.46241	.15085	.40185	.42048	.41006	.43685
20-	1.31414	.79818	-.15738	-.71399	-.29483	.22206	.48155	.53991	.59462
	.77903	.96259	.76280	.01103	-.24979	.07478	.48957	.59911	.70815
	.31616	.57373	.85105	.69761	.27607	.15930	.39386	.67067	.82989
	-.08083	.10160	.47338	.68015	.56550	.35571	.28682	.36582	.42646
40-	-.20304	-.02486	.17050	.59601	.47266	.38636	.21612	.09788	.05656
	-.10337	-.02042	.03605	.17204	.29579	.29471	.20752	.12785	.08562
	-.05022	.00850	.01843	.04730	.14661	.21066	.18719	.13145	.08762
	-.02488	.02406	.03679	.01445	.04501	.12308	.15488	.12757	.08703
60-	-.01168	.02744	.05032	.02848	.00422	.04669	.10747	.11697	.08751
	-.00234	.02527	.05229	.04831	.01005	.00238	.05315	.09434	.08727
	.00601	.02304	.04695	.05772	.03275	-.00375	.00992	.05950	.08043
	.01348	.02314	.04011	.05675	.05070	.01445	-.00756	.02252	.06224
80-	.01975	.02530	.03545	.05052	.05744	.03688	.00059	-.00182	.03526
	.02479	.02855	.03347	.04404	.05545	.05134	.02144	-.00508	.00988
	.02883	.03155	.03415	.03999	.04978	.05564	.04085	.00893	-.00253
	.03214	.03404	.03595	.03857	.04453	.05306	.05170	.02883	-.00193
100-	.03490	.03634	.03758	.03876	.04149	.04814	.05390	.04457	.01767
	.03730	.03828	.03901	.03954	.04047	.04409	.05097	.05209	.03506
	.03969	.03988	.04023	.04036	.04053	.04195	.04679	.05252	.04700
	.04261	.04129	.04115	.04100	.04091	.04130	.04371	.04931	.05146
120-	.04647	.04281	.04177	.04141	.04125	.04134	.04220	.04562	.05028
	.05094	.04494	.04232	.04160	.04144	.04152	.04173	.04304	.04663
	.05447	.04787	.04315	.04165	.04147	.04163	.04163	.04164	.04302
	.05471	.05101	.04469	.04178	.04136	.04158	.04148	.04080	.04042
140-	.04985	.05264	.04697	.04239	.04123	.04136	.04112	.04008	.03868
	.04029	.05066	.04913	.04384	.04134	.04100	.04048	.03909	.03729
	.02917	.04390	.04941	.04592	.04205	.04061	.03955	.03778	.03590
	.02092	.03385	.04600	.04751	.04354	.04044	.03835	.03615	.03442
160-	.01858	.02574	.03849	.04680	.04533	.04076	.03703	.03428	.03293
	.02194	.01759	.02874	.04237	.04604	.04156	.03589	.03233	.03157
	.02804	.01753	.02027	.03439	.04395	.04210	.03521	.03067	.03056
	.03328	.02176	.01628	.02516	.03804	.04096	.03497	.02981	.03017
180-	.03538	.02765	.01757	.01800	.02909	.03672	.03450	.03018	.03072
	.03409	.03172	.02220	.01525	.01973	.02915	.03278	.03178	.03240
	.03060	.03228	.02679	.01675	.01312	.02009	.02929	.03385	.03482
	.02638	.02946	.02837	.01989	.01115	.01333	.02522	.03510	.03635
200-	.02241	.02442	.02563	.02088	.01352	.01360	.02424	.03417	.03344
	.01904	.01949	.02016	.01878	.01474	.01269	.01729	.02513	.02822

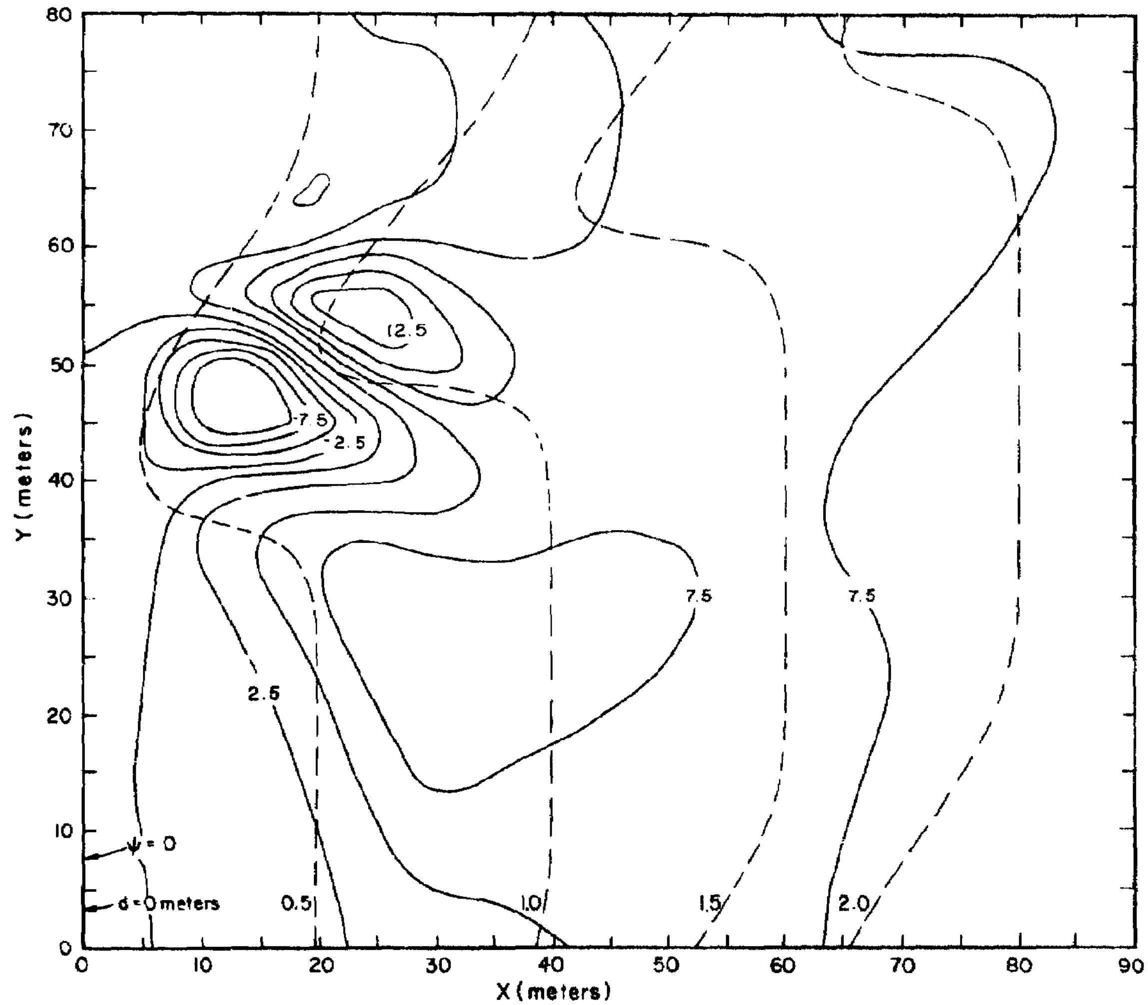


Figure 3.10: Stream Function Field ψ For 50% Wave-Current Interaction: (H_d = 0.5m, θ_d = 150°, T_d = 4 sec.)

TABLE 3.24

50% WAVE-CURRENT INTERACTION:

H_d = 0.5 m. T_d = 4 sec., θ_d = 150° WAVE DIRECTION FIELD θ (degrees)

y (meters) ⟶

x (meters) ↓	-5	0	5	10	15	20	25	30	35	40
0–	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000
	173.26660	173.97779	174.45422	174.53192	174.33556	174.59426	180.95820	195.30823	198.02439	183.92806
	169.62923	171.02209	172.09692	172.88018	173.76906	174.65041	176.06273	181.76025	182.72638	171.27229
	166.74618	168.21718	169.61572	170.56835	171.74343	173.10724	173.95736	174.59293	173.18575	163.13116
20–	165.02646	165.93028	167.27585	168.19122	169.26820	170.50434	170.96580	169.49044	164.96857	156.87337
	164.73149	164.43499	165.11582	165.91324	166.88110	167.75452	167.71764	165.27493	159.48695	154.66229
	164.47581	163.05985	163.41788	164.40612	165.25508	165.66261	165.12757	162.54739	158.13437	157.45086
	163.85233	162.85736	163.33436	163.97585	164.33278	164.34517	163.75466	162.13160	160.30174	161.34669
40–	164.26081	162.93306	162.98076	163.30814	163.50451	163.48329	163.15100	162.49082	161.98917	162.61424
	164.78936	162.83438	162.41862	162.59943	162.76632	162.79152	162.68245	162.50908	162.46993	162.85531
	165.46690	163.00998	162.00661	161.96110	162.11673	162.20710	162.24656	162.30604	162.46852	162.79661
	166.06330	163.46131	161.86038	161.43711	161.52805	161.67217	161.80590	161.98005	162.23084	162.54822
60–	166.31228	164.01951	162.01591	161.09683	161.00187	161.16091	161.35486	161.58826	161.87412	162.18371
	166.01988	164.44451	162.39014	160.99821	160.57480	160.67194	160.89997	161.16691	161.46351	161.75773
	165.15266	164.51779	162.80441	161.13801	160.30430	160.22639	160.45015	160.73753	161.03398	161.30618
	163.87256	164.11720	163.05124	161.42361	160.23274	159.86813	160.01918	160.31223	160.60329	160.84965
80–	162.46501	163.26913	162.96542	161.69595	160.34477	159.65012	159.63342	159.89880	160.17956	160.39810
	161.19461	162.14422	162.48087	161.78721	160.55084	159.60278	159.33424	159.50846	159.76549	159.95517
	160.19573	160.97678	161.65833	161.58015	160.71401	159.70124	159.16442	159.16382	159.36297	159.52100
	159.46600	159.95375	160.66028	161.05057	160.70159	159.85901	159.13999	158.89896	158.97983	159.09434
	158.92911	159.15130	159.67394	160.27766	160.43415	159.95701	159.22568	158.74464	158.63576	158.67631
100–	158.50055	158.54787	158.82966	159.40857	159.91570	159.88922	159.33607	158.70320	158.35952	158.27607
	158.12093	158.07793	158.16746	158.59068	159.23157	159.60269	159.36595	158.73156	158.17350	157.91449
	157.75948	157.67876	157.65918	157.91345	158.50900	159.11735	159.23033	158.74893	158.07247	157.61900
	157.40447	157.31062	157.25239	157.39404	157.85974	158.51526	158.89670	158.66668	158.01332	157.40809
120–	157.05330	156.95598	156.90284	157.00187	157.33990	157.90079	158.39604	158.42337	157.92699	157.27382
	156.70674	156.61106	156.58550	156.69311	156.94732	157.35370	157.80603	158.00877	157.74634	157.17667
	156.36652	156.27840	156.29124	156.43255	156.64608	156.90364	157.21350	157.46671	157.43447	157.05908
	156.03479	155.96254	156.01921	156.19901	156.39403	156.53624	156.67830	156.87441	157.00040	156.86975
140–	155.71422	155.66796	155.77076	155.98110	156.15842	156.21723	156.21962	156.30856	156.49511	156.58549
	155.40796	155.39818	155.54589	155.77143	155.91913	155.91882	155.82691	155.81823	155.98879	156.22053
	155.11946	155.15500	155.34158	155.56290	155.66588	155.61080	155.48053	155.41820	155.54120	155.81868
	154.85270	154.93755	155.15132	155.34776	155.39517	155.30149	155.16698	155.09969	155.18186	155.43159
160–	154.61298	154.74166	154.96543	155.11871	155.10904	154.99403	154.88335	154.84530	154.90836	155.09550
	154.40734	154.56021	154.77248	154.87140	154.81524	154.70184	154.63359	154.63814	154.69799	154.81976
	154.24222	154.38515	154.56221	154.60735	154.52742	154.44019	154.42195	154.46338	154.52206	154.59104
	154.11739	154.21041	154.33065	154.33658	154.26372	154.22112	154.24688	154.30644	154.35586	154.38760
180–	154.01899	154.03423	154.08675	154.07893	154.04228	154.04782	154.09769	154.15219	154.18337	154.19160
	153.91833	153.86124	153.85759	153.86190	153.87308	153.90926	153.95536	153.98757	153.99885	153.99570
	153.78326	153.70772	153.68759	153.71124	153.74676	153.77848	153.79900	153.80709	153.80662	153.80726
	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857
200–										

TABLE 3.24

50% WAVE-CURRENT INTERACTION:

H_d = 0.5 m. T_d = 4 sec., $\theta_d = 150^o$ WAVE DIRECTION FIELD θ(degrees)

(CONTINUED)

y (meters) ⟶

x (meters) ↓	45	50	55	60	65	70	75	80	85
0-	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000	180.00000
	163.55555	152.46635	181.31072	165.06790	169.22235	171.90980	173.26660	173.97779	174.45422
	159.33227	164.95616	196.05623	167.10288	165.88188	167.74609	169.62923	171.02209	172.09692
	155.36976	167.51434	195.74047	178.36453	168.05654	165.96734	166.74618	168.21718	169.61572
20-	156.94530	171.52242	187.63374	183.42879	172.53763	166.45745	165.02646	165.93028	167.27585
	162.54696	177.47848	182.53407	180.51334	174.15572	167.84692	164.73149	164.43499	165.11582
	167.14947	178.48033	180.61297	177.65555	173.00712	168.05549	164.47581	163.05985	163.41788
	167.18019	174.62974	177.97261	176.31017	172.13742	167.30646	163.85233	162.85736	163.33436
40-	165.30815	170.19594	174.51448	174.90775	171.89604	167.64531	164.26081	162.93306	162.98076
	164.10230	166.87289	170.69634	172.81862	171.57114	168.21375	164.78936	162.83438	162.41862
	163.41240	164.77935	167.36860	170.07877	170.61440	168.56899	165.46690	163.00998	162.00661
	162.91957	163.54121	164.95856	167.24712	168.90965	168.38433	166.06330	163.46131	161.86038
60-	162.46495	162.75723	163.39805	164.86671	166.75564	167.50482	166.31228	164.01951	162.01591
	161.99768	162.16337	162.40339	163.14413	164.63137	166.03637	166.01988	164.44451	162.39014
	161.51436	161.63125	161.70564	161.98991	162.88996	164.30604	165.15266	164.51779	162.80441
	161.02489	161.11482	161.13484	161.20287	161.62934	162.67862	163.87250	164.11720	163.05124
80-	160.53879	160.60477	160.61077	160.60659	160.76079	161.37687	162.46501	163.26913	162.96542
	160.06158	160.10368	160.10614	160.09465	160.13808	160.43541	161.19461	162.14422	162.48087
	159.59544	159.61577	159.61675	159.61959	159.64229	159.77055	160.19573	160.97678	161.65033
	159.14066	159.14402	159.14560	159.16703	159.20514	159.27210	159.46600	159.95375	160.66028
100-	158.69682	158.69019	158.69639	158.73610	158.79753	158.85660	158.92911	159.15130	159.67394
	158.26417	158.25542	158.27171	158.32929	158.41093	158.47805	158.50055	158.54787	158.82966
	157.84688	157.84064	157.87294	157.94842	158.04420	158.11715	158.12093	158.07793	158.16746
	157.45759	157.44761	157.50041	157.59362	157.69743	157.76770	157.75948	157.67876	157.65918
120-	157.11875	157.08180	157.15351	157.26340	157.36922	157.42793	157.40447	157.31062	157.25239
	156.85557	156.75580	156.83193	156.95486	157.05692	157.09664	157.05330	156.95598	156.90284
	156.68090	156.48883	156.53853	156.66415	156.75679	156.77215	156.70674	156.61106	156.58550
	156.58121	156.29864	156.28219	156.38830	156.46457	156.45260	156.36652	156.27840	156.29124
140-	156.51531	156.18777	156.07638	156.12850	156.17642	156.13642	156.03479	155.96254	156.01921
	156.42872	156.13347	155.93097	155.89241	155.89114	155.82284	155.71422	155.66796	155.77076
	156.27552	156.09051	155.84071	155.69175	155.61313	155.51278	155.40796	155.39818	155.54589
	156.03627	156.00698	155.77909	155.53411	155.35359	155.21085	155.11946	155.15500	155.34158
160-	155.72400	155.84459	155.70427	155.41312	155.12688	154.92730	154.85270	154.93755	155.15132
	155.37520	155.59329	155.57486	155.30528	154.94201	154.67758	154.61298	154.74166	154.96543
	155.03079	155.27348	155.36697	155.17731	154.79466	154.47701	154.40734	154.56021	154.77248
	154.71847	154.92462	155.08387	155.00047	154.66584	154.33180	154.24222	154.38515	154.56221
180-	154.44642	154.58668	154.75365	154.76383	154.52931	154.23132	154.11739	154.21041	154.33065
	154.20877	154.28525	154.41587	154.47953	154.36291	154.14717	154.01899	154.03423	154.08675
	153.99550	154.02751	154.10491	154.17680	154.15630	154.03986	153.91833	153.86124	153.85759
	153.79946	153.80809	153.83928	153.88647	153.90904	153.86958	153.78326	153.70772	153.68759
200-	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857	153.61857

TABLE 3.25

50% WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ WAVE HEIGHT FIELD H (meters)

y (meters) →

x (meters) ↓ (axis marks: 0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200)

-5	0	5	10	15	20	25	30	35	40
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
.09326	.09328	.09328	.09328	.09335	.09558	.11151	.16466	.25902	.34719
.18463	.18469	.18477	.18485	.18488	.18546	.19237	.22697	.31620	.38687
.27470	.27444	.27464	.27484	.27507	.27546	.27781	.29405	.35353	.42516
.36560	.36288	.36306	.36342	.36382	.36443	.36565	.37192	.40286	.46527
.45951	.45087	.45043	.45067	.45098	.45165	.45278	.45214	.45847	.51264
.45226	.47006	.48539	.49345	.49408	.48810	.47661	.46802	.48607	.55418
.45546	.46789	.47995	.48591	.48704	.48512	.48222	.48553	.51018	.56296
.46145	.46838	.47538	.47947	.48146	.48279	.48539	.49323	.51163	.53903
.46192	.46514	.46371	.47317	.47574	.47862	.48324	.49107	.50303	.51747
.46135	.46195	.46496	.46788	.47065	.47418	.47929	.48622	.49437	.50239
.46103	.45927	.46075	.46316	.46598	.46976	.47485	.48085	.48671	.49124
.46113	.45749	.45720	.45893	.46168	.46556	.47047	.47569	.48003	.48254
.46128	.45658	.45449	.45526	.45784	.46172	.46643	.47098	.47421	.47544
.46109	.45619	.45269	.45229	.45452	.45830	.46282	.46677	.46910	.46943
.46041	.45587	.45165	.45015	.45184	.45550	.45967	.46302	.46458	.46423
.45922	.45532	.45105	.44884	.44993	.45323	.45699	.45969	.46054	.45967
.45752	.45442	.45053	.44819	.44872	.45154	.45474	.45673	.45690	.45566
.45539	.45311	.44991	.44792	.44824	.45044	.45290	.45408	.45362	.45213
.45302	.45141	.44912	.44779	.44826	.44989	.45143	.45173	.45068	.44906
.45067	.44945	.44813	.44768	.44851	.44975	.45031	.44968	.44811	.44647
.44853	.44743	.44700	.44753	.44882	.44981	.44948	.44793	.44593	.44437
.44666	.44560	.44584	.44732	.44908	.44987	.44882	.44651	.44419	.44280
.44509	.44410	.44485	.44706	.44921	.44978	.44822	.44541	.44295	.44180
.44378	.44301	.44017	.44681	.44915	.44908	.44756	.44459	.44221	.44136
.44275	.44233	.44386	.44665	.44888	.44893	.44682	.44397	.44195	.44145
.44200	.44203	.44391	.44661	.44842	.44813	.44601	.44355	.44209	.44201
.44154	.44209	.44424	.44668	.44785	.44715	.44520	.44332	.44254	.44290
.44139	.44244	.44475	.44678	.44721	.44611	.44447	.44332	.44322	.44396
.44155	.44302	.44535	.44684	.44658	.44519	.44393	.44355	.44406	.44504
.44200	.44378	.44592	.44681	.44600	.44452	.44371	.44401	.44499	.44603
.44270	.44462	.44601	.44670	.44553	.44421	.44388	.44467	.44591	.44688
.44359	.44545	.44676	.44653	.44527	.44431	.44444	.44550	.44673	.44752
.44460	.44621	.44699	.44640	.44529	.44481	.44531	.44639	.44741	.44796
.44567	.44686	.44714	.44642	.44565	.44565	.44637	.44727	.44793	.44823
.44673	.44742	.44733	.44668	.44634	.44671	.44744	.44805	.44835	.44840
.44775	.44796	.44766	.44725	.44730	.44784	.44842	.44870	.44873	.44870
.44870	.44858	.44824	.44811	.44840	.44889	.44923	.44928	.44917	.44911
.44960	.44935	.44911	.44916	.44949	.44980	.44992	.44986	.44975	.44971
.45047	.45030	.45020	.45029	.45048	.45059	.45059	.45054	.45049	.45047
.45135	.45135	.45135	.45135	.45135	.45135	.45135	.45135	.45135	.45135

TABLE 3.25

50% WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^o$ WAVE HEIGHT FIELD H(meters)

(CONTINUED)

y (meters) ⟶

x (meters) ↓	45	50	55	60	65	70	75	80	85
0-	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	.35743	.27717	.17802	.11698	.09672	.09343	.09326	.09328	.09328
	.47496	.46206	.35561	.24939	.19873	.18599	.18463	.18469	.18477
20-	.54334	.57782	.51879	.40152	.31385	.28098	.27479	.27444	.27464
	.59508	.64961	.63766	.47084	.44189	.38386	.36560	.36288	.36306
	.59957	.69520	.65485	.52143	.45064	.44354	.45951	.45087	.45043
	.61177	.67243	.59847	.52624	.46994	.44638	.45226	.47006	.48539
40-	.60725	.60096	.56094	.51519	.47733	.45616	.45546	.46789	.47995
	.56007	.55868	.53727	.50645	.47884	.46295	.46145	.46838	.47538
	.52937	.53106	.51950	.49961	.47918	.46559	.46192	.46514	.46971
	.50863	.51064	.50510	.49286	.47843	.46676	.46135	.46195	.46496
60-	.49398	.49495	.49279	.48604	.47659	.46734	.46103	.45927	.46075
	.48321	.48294	.48211	.47926	.47384	.46727	.46113	.45749	.45720
	.47491	.47377	.47317	.47265	.47051	.46646	.46128	.45658	.45449
	.46823	.46667	.46603	.46658	.46681	.46502	.46109	.45619	.45269
80-	.46268	.46106	.46048	.46142	.46299	.46305	.46041	.45587	.45165
	.45798	.45655	.45622	.45735	.45941	.46066	.45922	.45532	.45105
	.45399	.45290	.45296	.45427	.45636	.45804	.45752	.45402	.45053
	.45063	.44997	.45049	.45200	.45395	.45547	.45539	.45311	.44991
100-	.44784	.44768	.44866	.45036	.45209	.45317	.45302	.45141	.44912
	.44562	.44596	.44734	.44917	.45065	.45120	.45067	.44945	.44813
	.44393	.44475	.44645	.44830	.44950	.44951	.44853	.44743	.44700
	.44278	.44401	.44590	.44766	.44850	.44803	.44666	.44560	.44584
120-	.44215	.44366	.44561	.44715	.44759	.44670	.44509	.44410	.44485
	.44201	.44364	.44549	.44671	.44672	.44549	.44378	.44301	.44417
	.44231	.44389	.44547	.44629	.44588	.44439	.44275	.44233	.44386
	.44297	.44434	.44551	.44587	.44506	.44342	.44200	.44203	.44391
140-	.44387	.44492	.44559	.44545	.44431	.44264	.44154	.44209	.44424
	.44488	.44554	.44568	.44506	.44366	.44207	.44139	.44244	.44475
	.44585	.44614	.44581	.44476	.44317	.44177	.44155	.44302	.44535
	.44668	.44667	.44598	.44461	.44292	.44175	.44200	.44378	.44592
160-	.44730	.44710	.44620	.44465	.44295	.44205	.44270	.44462	.44641
	.44772	.44741	.44647	.44490	.44330	.44267	.44359	.44545	.44676
	.44800	.44766	.44681	.44536	.44395	.44358	.44460	.44621	.44699
	.44821	.44791	.44720	.44600	.44488	.44472	.44567	.44686	.44714
180-	.44843	.44823	.44767	.44675	.44598	.44599	.44673	.44742	.44733
	.44873	.44865	.44823	.44759	.44716	.44729	.44775	.44796	.44766
	.44917	.44917	.44889	.44849	.44833	.44851	.44870	.44858	.44824
	.44976	.44979	.44964	.44942	.44942	.44959	.44960	.44935	.44911
200-	.45050	.45052	.45045	.45037	.45041	.45051	.45047	.45030	.45020
	.45135	.45135	.45135	.45135	.45135	.45135	.45135	.45135	.45135

TABLE 3.26

50% WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ BREAKING INDEX IB

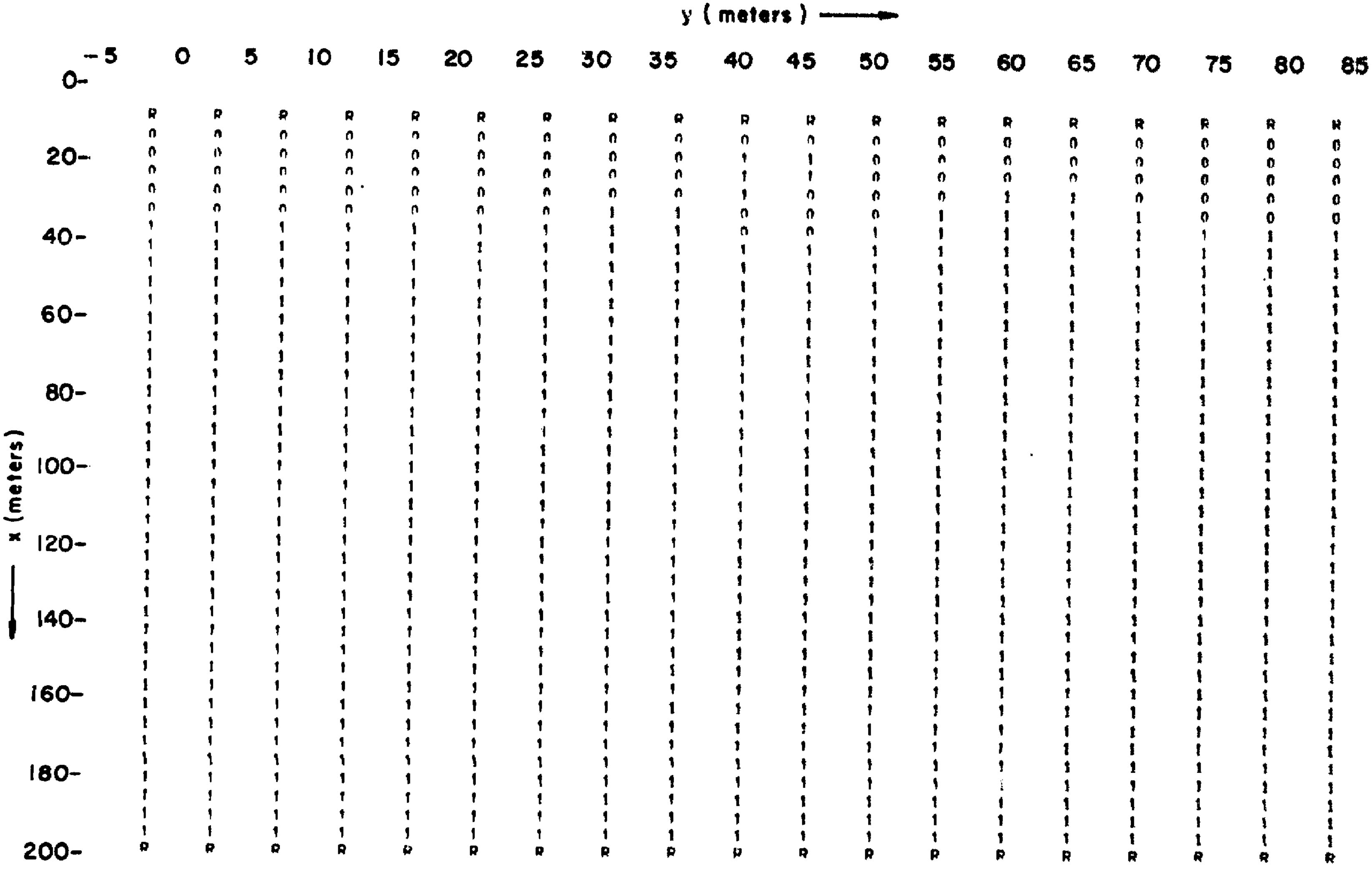


TABLE 3.27

50% WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^o$ u VELOCITY FIELD (meters/sec)

y (meters) ⟶ ; x (meters) ↓

x (meters)	-5	0	5	10	15	20	25	30	35	40
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	.07082	.01504	-.01087	-.01836	-.00157	.04001	.10239	.42145	.62586	.21932
	.17438	.05337	-.02432	-.08669	-.16912	-.27458	-.43058	-.52635	.57856	2.23844
	.13281	.03575	-.05749	-.14722	-.27549	-.44412	-.62366	-.51194	.81985	2.06651
20	-.13214	-.11208	-.13754	-.22682	-.35215	-.49234	-.56805	-.22352	1.01842	2.08282
	-.35326	-.30106	-.27396	-.33512	-.40717	-.43904	-.30564	.23336	1.13155	1.08920
	-.18189	-.37750	-.42084	-.37599	-.31701	-.21721	.01455	.41150	.76307	.21152
	-.09259	-.22345	-.28345	-.25671	-.19765	-.11794	.01612	.20232	.31570	.00286
40	-.08143	-.15747	-.18072	-.15907	-.11966	-.07100	-.00872	.06021	.11318	.09353
	-.04192	-.09141	-.11232	-.10331	-.07887	-.04878	-.01883	.01327	.05573	.10118
	-.01453	-.04414	-.06716	-.06702	-.05108	-.03193	-.01515	.00202	.02880	.07158
	-.01137	-.01375	-.03147	-.03890	-.03100	-.01868	-.00920	-.00093	.01330	.04171
60	-.03101	-.00350	-.00409	-.01353	-.01393	-.00855	-.00504	-.00311	.00323	.02046
	-.06030	-.01363	.01119	.00962	.00267	-.00055	-.00340	-.00629	-.00409	.00752
	-.08092	-.03716	.01077	.02691	.01926	.00677	-.00364	-.01037	-.00965	.00054
	-.08117	-.06031	-.00356	.03331	.03346	.01435	-.00483	-.01463	-.01360	-.00242
80	-.06271	-.06949	-.02346	.02664	.04065	.02146	-.00432	-.01795	-.01570	-.00248
	-.03709	-.05942	-.03700	.01033	.03666	.02508	-.00269	-.01892	-.01548	-.00025
	-.01602	-.03545	-.03545	-.00711	.02111	.02103	-.00062	-.01637	-.01229	.00391
	-.00445	-.00848	-.01851	-.01642	-.00119	.00678	-.00108	-.01027	-.00544	.00977
100	-.00079	.01274	.00620	-.01315	-.02212	-.01599	-.00736	-.00250	.00513	.01723
	-.00066	.02534	.02877	-.00026	-.03453	-.04113	-.02039	.00347	.01802	.02609
	-.00010	.03112	.04271	.01423	-.03630	-.06055	-.03703	.00452	.03010	.03550
	.00291	.03341	.04672	.02253	-.03103	-.06824	-.05075	.00021	.03736	.04344
120	.00852	.03472	.04301	.02107	-.02504	-.06323	-.05477	-.00628	.03703	.04681
	.01579	.03585	.03464	.01098	-.02298	-.04941	-.04576	-.00972	.02946	.04265
	.02344	.03627	.02390	-.00398	-.02547	-.03249	-.02580	-.00600	.01813	.03005
	.03023	.03491	.01191	-.01960	-.02983	-.01672	-.00115	.00504	.00780	.01138
140	.03513	.03081	-.00079	-.03261	-.03228	-.00350	.02105	.01927	.00177	-.00833
	.03736	.02357	-.01351	-.04090	-.03002	.00758	.03591	.03049	.00021	-.02344
	.03646	.01354	-.02503	-.04334	-.02231	.01707	.04173	.03394	.00071	-.03021
	.03241	.00186	-.03363	-.03962	-.01041	.02451	.03923	.02846	.00042	-.02821
160	.02584	-.00957	-.03755	-.03040	.00292	.02846	.03035	.01654	-.00176	-.01997
	.01800	-.01836	-.03561	-.01751	.01428	.02737	.01765	.00277	-.00469	-.00932
	.01053	-.02214	-.02791	-.00392	.02053	.02081	.00416	-.00813	-.00617	.00034
	.00485	-.01958	-.01624	.00673	.01988	.01034	-.00669	-.01300	-.00485	.00689
1:30	.00166	-.01158	-.00405	.01144	.01307	-.00037	-.01191	-.01137	-.00136	.00948
	.00041	-.00194	.00449	.00954	.00383	-.00684	-.01049	-.00573	.00203	.00837
	-.00063	.00366	.00672	.00434	-.00209	-.00652	-.00484	-.00036	.00311	.00496
	-.00398	.00137	.00440	.00233	-.00085	-.00179	-.00023	.00138	.00187	.00167
200	-.00698	.00025	.00676	.00567	.00141	-.00088	-.00055	.00041	.00093	.00078

TABLE 3.27

50% WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ u VELOCITY FIELD (meters/sec)

(CONTINUED)

x (meters) ↓ / y (meters) →	45	50	55	60	65	70	75	80	85
0-	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	-.05528	-.90317	-1.09261	.74171	.34491	.17238	.07082	.01504	-.01087
	1.19821	-2.12257	-2.13684	.49036	.20113	.27948	.17438	.05337	-.02432
	1.15665	-2.19371	-1.27138	1.02370	-.22876	.02208	.13281	.03575	-.05749
20-	-.09516	-2.05883	.00137	1.68392	.18658	-.29197	-.13214	-.11208	-.13754
	-.95905	-1.44475	.45430	1.35349	.53024	-.15695	-.35326	-.30106	-.27396
	-.89475	-.46734	.53184	.60365	.24841	.02757	-.18189	-.37750	-.42084
	-.33504	-.00439	.32866	.25744	.09668	.00579	-.09259	-.22345	-.28345
40-	.05085	.06847	.10897	.11245	.06882	.00282	-.08143	-.15747	-.18072
	.12886	.10500	.03893	.01538	.02286	.00483	-.04192	-.09141	-.11232
	.11584	.11629	.04309	-.02787	-.02818	-.00722	-.01453	-.04414	-.06716
	.08185	.10344	.06442	-.01955	-.06030	-.03774	-.01137	-.01375	-.03147
60-	.05068	.07981	.07401	.01246	-.05841	-.06819	-.03101	-.00350	-.00409
	.02949	.05644	.06895	.03934	-.03084	-.07886	-.06030	-.01363	.01114
	.01802	.03922	.05617	.04963	.00086	-.06500	-.08092	-.03716	.01077
80-	.01354	.02934	.04260	.04579	.02041	-.03799	-.08117	-.06031	-.00356
	.01335	.02518	.03205	.03477	.02474	-.01388	-.06271	-.06949	-.02346
	.01553	.02429	.02508	.02242	.01691	-.00174	-.03709	-.05942	-.03700
	.01881	.02459	.02044	.01'75	.00491	-.00143	-.01602	-.03545	-.03545
100-	.02235	.02470	.01656	.003[illegible]5	-.00688	-.00790	-.00445	-.00888	-.01851
	.02557	.02380	.01231	-.00374	-.01615	-.01563	-.00079	.01274	.00620
	.02812	.02153	.00719	-.00992	-.02247	-.02101	-.00066	.02534	.02877
	.02994	.01790	.00127	-.01557	-.02622	-.02267	-.00010	.03112	.04271
120-	.03106	.01335	-.00499	-.02048	-.02778	-.02078	.00291	.03341	.04672
	.03103	.00873	-.01079	-.02419	-.02733	-.01606	.00852	.03472	.04301
	.02860	.00488	-.01507	-.02611	-.02489	-.00931	.01579	.03585	.03464
	.02199	.00199	-.01681	-.02571	-.02052	-.00126	.02344	.03627	.02390
140-	.01008	-.00074	-.01554	-.02258	-.01434	.00745	.03023	.03491	.01191
	-.00607	-.00487	-.01201	-.01669	-.00655	.01618	.03513	.03081	-.00079
	-.02278	-.01141	-.00833	-.00878	.00257	.02432	.03736	.02357	-.01351
	-.03493	-.01971	-.00708	-.00062	.01251	.03137	.03646	.01354	-.02503
160-	-.03837	-.02720	-.00982	.00526	.02214	.03688	.03241	.00186	-.03363
	-.03214	-.03073	-.01587	.00676	.02958	.04029	.02584	-.00957	-.03755
	-.01916	-.02846	-.02239	.00347	.03261	.04071	.01800	-.01836	-.03561
	-.00477	-.02113	-.02595	-.00275	.02975	.03708	.01053	-.02214	-.02791
180-	.00592	-.01171	-.02440	-.00862	.02143	.02888	.00485	-.01958	-.01624
	.01035	-.00364	-.01811	-.01121	.01030	.01723	.00166	-.01158	-.00405
	.00910	.00097	-.00972	-.00958	.00042	.00529	.00041	-.00194	.00449
	.00498	.00205	-.00266	-.00505	-.00446	-.00297	-.00063	.00366	.00672
200-	.00120	.00080	.00071	-.00003	-.00297	-.00559	-.00398	.00137	.00440
	-.00005	-.00061	.00039	.00130	-.00177	-.00679	-.00698	.00025	.00676

TABLE 3.28

50% WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^o$ v VELOCITY FIELD (meters/sec)

y (meters) ⟶

x (meters) ↓	-5	0	5	10	15	20	25	30	35	40
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	.24320	.13990	.13647	.18853	.30947	.51322	.71563	.79488	.75728	-.15205
	.35528	.29511	.30926	.37592	.52088	.78582	1.15886	1.44948	1.38652	.45411
	.29291	.40797	.47843	.57515	.72306	.93166	1.18671	1.30308	.82648	.32652
20	.21428	.46985	.61942	.76932	.92791	1.07165	1.14232	.96662	.40078	.23583
	.13431	.28155	.50308	.67713	.77283	.77582	.63955	.30252	-.03505	.20364
	.17031	.13055	.19231	.23431	.21262	.12559	-.01565	-.14789	-.05623	.43257
	.18761	.14465	.06625	-.00570	-.07406	-.14068	-.17910	-.11824	.10481	.40501
40	.13556	.12198	.06693	.00726	-.04084	-.07740	-.08916	-.04211	.07293	.17122
	.11821	.09127	.05117	.01438	-.01441	-.03517	-.04204	-.02608	.00924	.04251
	.09747	.07852	.04931	.02259	.00164	-.01371	-.02171	-.02054	-.00874	.01498
	.07583	.07426	.05234	.02828	.01009	-.00237	-.00958	-.01065	-.00436	.01202
60	.04803	.06725	.05748	.03528	.01643	.00519	.00002	.00040	.00598	.01707
	.01967	.05226	.05954	.04321	.02327	.01163	.00845	.01093	.01676	.02430
	.00264	.03134	.05382	.04930	.03136	.01862	.01634	.02037	.02619	.03114
	.00483	.01297	.03960	.04971	.03934	.02709	.02441	.02888	.03396	.03658
80	.02225	.00657	.02184	.04175	.04416	.03661	.03340	.03665	.04016	.04039
	.04331	.01477	.00895	.02714	.04260	.04513	.04357	.04427	.04497	.04266
	.05870	.03132	.00722	.01208	.03392	.04964	.05365	.05201	.04854	.04361
	.06546	.04698	.01616	.00398	.02156	.04788	.06122	.05949	.05159	.04354
100	.06526	.05594	.02949	.00663	.01209	.04056	.06386	.06521	.05411	.04297
	.06144	.05740	.04047	.01787	.01139	.03196	.05907	.06685	.05594	.04260
	.05671	.05365	.04578	.03189	.02081	.02783	.04971	.06246	.05592	.04299
	.05233	.04776	.04581	.04351	.03654	.03168	.03977	.05196	.05243	.04398
120	.04840	.04204	.04306	.05062	.05258	.04234	.03383	.03806	.04450	.04446
	.04467	.03756	.04027	.05395	.06428	.05479	.03375	.02540	.03315	.04290
	.04100	.03463	.03922	.05557	.06999	.06360	.03767	.01811	.02179	.03866
	.03751	.03331	.04049	.05714	.07044	.06573	.04173	.01762	.01489	.03299
140	.03453	.03359	.04379	.05907	.06724	.06129	.04285	.02237	.01551	.02887
	.03253	.03548	.04827	.06061	.06161	.05250	.04045	.02941	.02356	.02925
	.03193	.03880	.05279	.06050	.05419	.04223	.03616	.03630	.03589	.03488
	.03304	.04309	.05603	.05763	.04547	.03302	.03260	.04203	.04810	.04351
160	.03596	.04746	.05671	.05160	.03638	.02679	.03190	.04657	.05645	.05111
	.04035	.05072	.05396	.04307	.02852	.02476	.03467	.04990	.05902	.05418
	.04515	.05161	.04771	.03381	.02379	.02720	.03971	.05122	.05576	.05152
	.04841	.04927	.03925	.02643	.02369	.03293	.04028	.04922	.04808	.04059
180	.04770	.04378	.03131	.02356	.02808	.03896	.04517	.04323	.03842	.03643
	.04165	.03679	.02736	.02623	.03409	.04110	.04042	.03448	.02972	.02996
	.03189	.03147	.02929	.03174	.03633	.03657	.03151	.02640	.02452	.02646
	.02699	.03032	.03090	.03044	.02957	.02703	.02413	.02288	.02336	.02495
	.02167	.02705	.03037	.02947	.02634	.02339	.02202	.02204	.02254	.02301
200										

TABLE 3.28

50% WAVE-CURRENT INTERACTION:

$H_d = 0.5$ m, $T_d = 4$ sec., $\theta_d = 150^{\circ}$ v VELOCITY FIELD (meters/sec)

(CONTINUED)

x (meters) ↓ \ y (meters) →	45	50	55	60	65	70	75	80	85
0-	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	-2.28200	-2.43481	1.10583	.73177	.77692	.48769	.24320	.13990	.13647
	-1.43618	-1.06910	1.07562	-.08257	.28108	.45557	.35528	.29511	.30926
	.47549	1.33069	1.41599	.18770	-.26710	.11173	.29291	.40797	.47843
20-	1.36363	1.96033	.83619	.63423	-.01501	-.07587	.21428	.46985	.61942
	1.39073	.85014	-.29283	.24413	.31864	.14452	.13431	.28155	.50308
	.82267	-.16463	-.63919	-.06721	.23566	.26668	.17031	.13055	.19231
	.17439	-.45449	-.32679	.01157	.12471	.16268	.18761	.14465	.06625
40-	-.03803	-.26121	-.13164	.05022	.10833	.12055	.13556	.12198	.06693
	.01303	-.03810	-.04379	.02476	.09256	.11978	.11821	.09127	.05117
	.03644	.04119	.01978	.00782	.04816	.08827	.09747	.07853	.04931
	.03838	.06195	.05705	.02065	.01198	.04729	.07563	.07426	.05239
60-	.03606	.06012	.06040	.04403	.00628	.01211	.04803	.06725	.05748
	.03567	.05167	.06641	.05908	.02331	-.00137	.01967	.05226	.05954
	.03577	.04401	.05673	.06220	.04351	.00909	.00264	.03134	.05342
	.03710	.03915	.04685	.05734	.05572	.03090	.00483	.01297	.03960
80-	.03816	.03659	.03986	.04987	.05883	.05027	.02225	.00657	.02184
	.03851	.03532	.03617	.04384	.05641	.06111	.04331	.01477	.00895
	.03815	.03469	.03502	.04089	.05276	.06401	.05870	.03132	.00722
	.03733	.03446	.03557	.04087	.05064	.06249	.06546	.04698	.01616
100-	.03641	.03472	.03727	.04291	.05078	.05996	.06526	.05594	.02989
	.03590	.03571	.03983	.04610	.05259	.05820	.06144	.05740	.04087
	.03646	.03766	.04310	.04975	.05505	.05735	.05671	.05365	.04578
	.03874	.04082	.04691	.05332	.05726	.05676	.05233	.04776	.04581
120-	.04287	.04538	.05104	.05639	.05861	.05570	.04840	.04204	.04306
	.04794	.05121	.05522	.05864	.05878	.05375	.04467	.03756	.04027
	.05207	.05747	.05919	.05987	.05770	.05083	.04100	.03463	.03922
	.05327	.06239	.06253	.06007	.05550	.04717	.03751	.03331	.04089
140-	.05072	.06378	.06437	.05947	.05252	.04315	.03453	.03359	.04379
	.04558	.06012	.06349	.05820	.04931	.03928	.03253	.03548	.04827
	.04038	.05173	.05877	.05600	.04643	.03617	.03193	.03880	.05279
	.03738	.04106	.05014	.05215	.04410	.03444	.03304	.04309	.05603
160-	.03711	.03166	.03919	.04597	.04194	.03453	.03596	.04746	.05671
	.03832	.02635	.02888	.03759	.03909	.03627	.04035	.05072	.05396
	.03914	.02588	.02224	.02849	.03483	.03864	.04515	.05161	.04771
	.03848	.02882	.02074	.02118	.02945	.03995	.04841	.04927	.03925
180-	.03646	.03260	.02347	.01790	.02439	.03853	.04770	.04378	.03131
	.03383	.03481	.02782	.01921	.02138	.03368	.04165	.03679	.02736
	.03095	.03405	.03089	.02340	.02093	.02640	.03189	.03147	.02929
	.02740	.02984	.02996	.02660	.02320	.02348	.02699	.03032	.03090
200-	.02345	.02371	.02333	.02184	.01986	.01909	.02167	.02705	.03037

The failure of the numerical program to compute wave-current interactions greater than about 50% of the noninteractive case is the inclusion in the algorithm of a condition which stops the calculation in the event that $k \leq 0$. This physically implies that the local wave is unable to penetrate beyond this point due to the magnitude of the opposing mean current. One possibility to extend computation is to assume that when $k \leq 0$, $H \equiv 0$. This will obviously yield large changes in the spacial derivatives of the H field. The effect of this change in the program is presently unknown, but if implemented wave diffraction effects should be considered due to possible sharp discontinuities in the wave height field.

3.4 CONCLUSIONS

1. When the process of wave-current interaction is important, the ability for the mean current to convect wave energy across orthogonal lines implies that the method of integrating along characteristic lines is no longer valid. Furthermore the wave height can no longer be simply represented by the product of independent refraction and shoaling coefficients.

2. While only limited interaction results have been obtained thus far, it is clear that wave-current interaction is an important physical process which can greatly affect incoming wave characteristics within the nearshore coastal zone. This influence would be especially important if a strong circulation pattern exists within the nearshore zone.

3. Since the effects of wave-current interaction on the nearshore circulation pattern is so startling, other parallel areas of effort should be also considered such as the simultaneous movement of bottom material as the current bottom shear stress becomes large. A conclusion of a previous study [Noda (1972)] showed that the circulation pattern and especially the magnitudes of the circulation velocities were very sensitive to bottom configurations. The interplay of all these factors must be seriously considered.

4. The preceeding results have opened up interesting areas for future research effort. For the present numerical model some nonlinearity could be included either from the inertia term or through the bottom function term. Also as the wave number becomes $k \leq 0$, the numerical algorithm should have $H \rightarrow 0$.

Future efforts should be expended on a perturbation expansion technique which would allow the use of integrating along characteristic lines to obtain approximate solutions for wave-current interaction.

Or on the practical side, these techniques could be directed toward an analytic quantification of such basic coastal engineering problems as groin spacing and design.

4. WAVE AND CURRENT

4.1 INTRODUCTION

The question of mass transport or current associated with wave propagation in the ocean as well as in channels or rivers has been investigated by numerous authors. Several types of problems arise in this connection but they can be roughly divided into two main classes. The first and most widely studied phenomenon is the mass transport due to gravity waves, the second is that of wave propagation in the presence of a current.

The mass transport due to gravity waves has been shown to be of second order with respect to wave height. M. L. Dubreil-Jacotin (1934) first proved the existence of waves associated with the rotational motion of a perfect inviscid fluid and determined that there are an infinity of such solutions associated with a more or less arbitrary distribution of vorticity. Since the motion is irrotational to the first order, Miche (1944) used the first order irrotational solution for a constant finite amplitude two dimensional motion and calculated the corresponding second order term; the results depend on an arbitrary function of depth which can be determined if the vorticity is specified. The effect of viscosity was first studied by Longuet-Higgins (1953 and 1960). He showed that the boundary layers near the surface and at the bottom were of very small extent and that the mass transport velocity above the bottom layer and its vertical gradient just below the free surface layer were independent of viscosity. His results confirmed the existence of vorticity in the mean flow and provided boundary conditions which may be used to define the unknown function suggested by Dubreil-Jacotin and used by Miche. Further computations of mass transport in cnoidal waves were presented by LeMéhauté (1968) who showed that the mass transport is uniform in a vertical plane to the second order of approximation. A further step in the theoretical study of mass transport in gravity wave was the study of a spectrum of random waves Ming-Shun Chang (1969 and 1970).

The problem of wave motion in the presence of a current has not been studied in the same systematic fashion. However, a number of particular solutions to the problem have been presented. For instance, Abdullah (1949) assumes an exponentional vorticity distribution, Biesel (1950), a constant vorticity and Eliasson and Engelund (1972) a hyperbolic distribution with depth.

Because of the importance of the latter type of motion, one obvious example being the interference of waves and rip current near shore, the assumptions and equations used in the last three references will be reviewed, then results summarized and an extension of Biesel's solution (1950) to a discrete wave spectrum will be presented.

4.2 BASIC EQUATIONS AND ASSUMPTIONS

The motion studied here is assumed two dimensional and the fluid is incompressible.

The basic coordinate system consists in a horizontal x axis laying along the mean water surface line, and a vertical y axis positive upwards. The following notation is used:

u, v	x and y components of a particle velocity
p	pressure
ν	kinematic viscosity
ρ	fluid density
ψ	stream function
g	gravity constant

The basic equations of motion are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x} = (\nu\nabla^2 u) \tag{4.2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial y} = -g + (\nu\nabla^2 v) \tag{4.3}$$

The use of these equations imply that any density gradient or Coriolis effects have been neglected. Moreover, the terms in parenthesis which will eventually be dropped imply a laminar flow, but could be replaced by other functions of u or v to approximate turbulent shear stresses.

Equation (4.1) is identically verified by the introduction of the stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \tag{4.4}$$

$$v = -\frac{\partial \psi}{\partial x} \tag{4.5}$$

Differentiating (4.2) with respect to y and (4.3) with respect of x and subtracting and replacing u and v by their values in terms of ψ yields the fundamental equation for the stream function

$$\frac{\partial}{\partial t}(\nabla^2\psi) + \frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\psi) - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\psi) = (\nu\nabla^2(\nabla^2\psi)) \tag{4.6}$$

or in another familiar form with $\zeta = -\frac{1}{2}\nabla^2\psi$

$$\frac{d\zeta}{dt} = (\nu\nabla^2\zeta) \quad . \tag{4.7}$$

For wave like solutions of wave speed c, ψ is a function of the variables (x - ct) and (y). Assuming a wave spectrum exists with wave numbers extending from k_1 to k_2, the stream function can be written as

$$\psi(x, y, t) = \int_{k_1}^{k_2} \psi_k \; (x - c_k t, \; y) \; dk \qquad (4.8)$$

where

c_k and ψ_k are functions of k.

Equation (4.6) is identically verified by the solution

$$\nabla^2 \psi = \int_{k_1}^{k_2} \nabla^2 \psi_k \; dk = \text{constant.}$$

The usual irrotational solution is obtained by setting the constant at zero.

When shear stresses are negligible, that is outside some bottom and free surface boundary layer, the right hand side of (4.6) can be neglected. In this region, the stream function must satisfy the oft used equation:

$$\frac{\partial}{\partial t} (\nabla^2 \psi) + \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} (\nabla^2 \psi) - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} (\nabla^2 \psi) = 0 . \qquad (4.9)$$

Assuming that the main effect of shear is to establish initially a non-zero vorticity throughout the fluid but that it can be neglected thereafter, solutions of (4.9) will be sought. The order of magnitude of the neglected term in (4.6) will then be evaluated to verify the validity of this approach.

A series expansion of the stream function and water height in terms of a small parameter ϵ will be used. Here ϵ is a measure of the wave height and it will be assumed that the surface equation $\eta(x, t)$ and the stream function $\Psi(x, y, t)$ can be written as

$$\eta = \sum_{m=1}^{N} \epsilon^m \eta_m \tag{4.10}$$

$$\Psi = \sum_{n=0}^{N} \epsilon^n \Psi_n \tag{4.11}$$

where the function Ψ_0 represents the main effect of the current, and will be assumed to depend on depth only. Note that here the functions η_m and Ψ_n are of the form (4.8) that is:

$$\eta_m = \int_{k_1}^{k_2} \eta_{km} (x - c_k t)\, dk \qquad m \geq 1 \tag{4.8a}$$

$$\Psi_n = \int_{k_1}^{k_2} \Psi_{kn} (x - c_k t, y)\, dk \qquad n \geq 1 \tag{4.8b}$$

Introducing expression (4.11) for Ψ into the defining equation (4.9), grouping terms in various powers of ϵ, and recalling that by hypothesis Ψ_0 is a function of y only (4.9) becomes

$$\epsilon \left[\frac{\partial}{\partial t} (\nabla^2 \Psi_1) + \frac{d\Psi_0}{dy} \frac{\partial}{\partial x} (\nabla^2 \Psi_1) - \frac{\partial \Psi_1}{\partial x} \frac{d^3 \Psi_0}{dy^3}\right] + 0(\epsilon^2) = 0.$$

Hence the zeroth and first order terms of Ψ are defined by the equations

$$\Psi_0 = \Psi_0(y) \tag{4.12}$$

$$\frac{\partial}{\partial t} (\nabla^2 \Psi_1) + \frac{d\Psi_0}{dy} \frac{\partial}{\partial x} (\nabla^2 \Psi_1) - \frac{\partial \Psi_1}{\partial x} \frac{d^3 \Psi_0}{\partial y^3} = 0 \quad . \tag{4.13}$$

But from (4. 8b)

$$\Psi_1 = \int_{k_1}^{k_2} \Psi_{k_1} \, (x - c_k t, \, y) \, dk \, . \tag{4.14}$$

An additional assumption is now made concerning the functional expression of Ψ_{k_1}, that is:

$$\Psi_{k_1} \, (x - c_k t, \, y) = \tilde{\Psi}_k \, (y) \cos \left[k \, (x - c_k t) \right] \, .$$

Hence,

$$\Psi_1 = \int_{k_1}^{k_2} \tilde{\Psi}_k \, (y) \cos \left[k \, (x - c_k t) \right] \, dk \tag{4.14a}$$

$$\nabla^2 \Psi_1 = \int_{k_1}^{k_2} \left[\frac{d^2 \tilde{\Psi}_k}{dy^2} - k^2 \tilde{\Psi}_k \right] \cos \left[k(x - c_k t) \right] \, dk \, . \tag{4.14b}$$

Introducing (4. 14a) and (4, 14b) in equation (4. 13), we get

$$\int_{k_1}^{k_2} \left[\left(\frac{d^2 \tilde{\Psi}_k}{dy^2} - k^2 \tilde{\Psi}_k \right) \left(c_k - \frac{d \Psi_0}{dy} \right) + \tilde{\Psi}_k \frac{d^3 \Psi_0}{dy^3} \right] dk = 0 \, .$$

This equation will be verified in particular if for all k's in the interval $k_1 \leq k \leq k_2$

$$\left(\frac{d \Psi_0}{dy} - c_k \right) \left(\frac{d^2 \tilde{\Psi}_k}{dy^2} - k^2 \tilde{\Psi}_k \right) = \frac{d^3 \Psi_0}{dy^3} \tilde{\Psi}_k \, . \tag{4.15}$$

Hence, given an arbitrary function $\Psi_0(y)$ defining the zeroth order stream function due to a depth dependent current, the components of the first order associated wave stream function are given by equation (4.15).

The assumptions leading to the equation are summarized below:

(1) There is no density gradient in the fluid.

(2) Coriolis forces are neglected.

(3) Shear stresses are neglected. The validity of this assumption will be tested for each case considered.

(4) Wave like solutions only are considered.

(5) The principal particle velocity component is due to the current and depends on depth only.

(6) The wave height is small compared to the characteristic length defining the depth variation of the current.

4.3 BOUNDARY CONDITIONS

Note that by definition (equations (4.4) and (4.5)) the stream function is known to within an arbitrary function of time. Hence, without loss of generality this arbitrary function can be set to zero.

1. The bottom is a line of current.

Assuming the bottom is defined by

$$y = -h(x)$$

$$-\frac{\partial \Psi}{\partial x}(x, t, -h) = -\frac{dh}{dx}\frac{\partial \Psi}{\partial y}(x, t, -h) .$$

For h constant this becomes

$$\Psi(x, t, -h) = \Psi_b$$

where Ψ_b is a constant.

Replacing Ψ by (4.11) yields the condition

$$\Psi_0(-h) = \Psi_b \tag{4.16}$$

$$\Psi_n(x, t, -h) = 0 \qquad n \geq 1 . \tag{4.17}$$

2. The free surface is a line of current

Since the free surface is defined by

$$y = \eta(x, t)$$

$$-\frac{\partial \Psi}{\partial x}(x, t, \eta) = \frac{\partial \eta}{\partial t} + \frac{\partial \eta}{\partial x}\frac{\partial \Psi}{\partial y}(x, t, \eta) .$$

Replacing y and Ψ by (4.10) and (4.11) and grouping terms in power of ϵ .

$$\epsilon\left[\frac{\partial \Psi_1}{\partial x}(x, t, 0) + \frac{\partial \eta_1}{\partial t}(x, t) + \frac{\partial \eta_1}{\partial x}(x, t)\frac{d\Psi_0}{dy}(0)\right] + 0\ (\epsilon^2) = 0.$$

Since Ψ_0 is a function of y only, $\Psi_0(0)$ is a constant. Since, moreover, Ψ is defined within an arbitrary constant, it is always possible to choose $\Psi_0(0) = 0$ in which case the value of Ψ_b is defined by other boundary conditions. Note that alternately Ψ_b could have been taken as zero and $\Psi_0(0) = \Psi_s$ determined subsequently.

The free surface condition hence becomes to first order:

$$\Psi_0(0) = 0 . \tag{4.18}$$

$$\frac{\partial \Psi_1}{\partial x} + \frac{\partial \eta_1}{\partial t} + \frac{\partial \eta_1}{\partial x} \frac{d \Psi_0}{dy} = 0 \text{ at } y = 0 . \qquad (4.19)$$

3. The pressure is known at the surface and is usually assumed constant

From equation (4.2) and (4.[illegible])

$$\frac{\partial^2 \Psi}{\partial t \, \partial y} + \frac{\partial \Psi}{\partial y} \frac{\partial^2 \Psi}{\partial x \, \partial y} - \frac{\partial \Psi}{\partial x} \frac{\partial^2 \Psi}{\partial y^2} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0$$

and

$$- \frac{\partial^2 \Psi}{\partial t \, \partial x} - \frac{\partial \Psi}{\partial y} \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial \Psi}{\partial x} \frac{\partial^2 \Psi}{\partial x \, \partial y} + \frac{1}{\rho} \frac{\partial p}{\partial y} = - g .$$

Hence,

$$\frac{\partial^2 \Psi}{\partial t \, \partial y} - \frac{\partial \eta}{\partial x} \frac{\partial^2 \Psi}{\partial t \, \partial x} + \frac{\partial \Psi}{\partial y} \left[\frac{\partial^2 \Psi}{\partial x \, \partial y} - \frac{\partial \eta}{\partial x} \frac{\partial^2 \Psi}{\partial x^2} \right]$$

$$- \frac{\partial \Psi}{\partial x} \left[\frac{\partial^2 \Psi}{\partial y^2} - \frac{\partial \eta}{\partial x} \cdot \frac{\partial^2 \Psi}{\partial x \, \partial y} \right] + g \frac{\partial \eta}{\partial x}$$

$$+ \frac{1}{\rho} \left[\frac{\partial p}{\partial x} + \frac{\partial \eta}{\partial x} \frac{\partial p}{\partial y} \right] = 0 .$$

Replacing η and Ψ by (4.10) and (4.11), at $y = 0$:

$$\epsilon \left[\frac{\partial^2 \Psi_1}{\partial t \, \partial y} + \frac{\partial \Psi_0}{\partial y} \frac{\partial^2 \Psi_1}{\partial x \, \partial y} - \frac{\partial \Psi_1}{\partial x} \frac{\partial^2 \Psi_0}{\partial y^2} + g \frac{\partial \eta_1}{\partial x} \right] + 0 (\epsilon^2) = 0$$

if $p(x, \eta) = p_0$ a constant.

Introducing expression (4.19)

$$\frac{\partial^2 \Psi_1}{\partial t \, \partial y} + \frac{d \Psi_0}{dy} \left[\frac{\partial^2 \Psi_1}{\partial x \, \partial y} + \frac{d^2 \Psi_0}{dy^2} \frac{\partial \eta_1}{\partial x} \right] + \frac{\partial \eta_1}{\partial t} \frac{d^2 \Psi_0}{dy^2} + g \frac{\partial \eta_1}{\partial x} = 0 \text{ at } y = 0 \qquad (4.20)$$

Now from (4.8a)

$$\eta_1 = \int_{k_1}^{k_2} \eta_{k_1} (x - c_k t) \; dk \; .$$

Assuming as for Ψ_1 (equation (4.14a) that

$$\eta_1 = \int_{k_1}^{k_2} \tilde{\eta}_k \cos \left[k(x - c_k t) \right] \; dk \; . \tag{4.21}$$

The above boundary conditions will be satisfied if:

from (4.17)

$$\tilde{\Psi}_k (-h) = 0 \tag{4.22}$$

from (4.19)

$$\tilde{\Psi}_k + \left[\frac{d \Psi_0}{dy} - c_k \right] \tilde{\eta}_k = 0 \text{ at } y = 0 \tag{4.23}$$

from (4.20)

$$\left[\frac{d \Psi_0}{dy} - c_k \right] \frac{d \tilde{\Psi}_k}{dy} + \left\{ \frac{d^2 \Psi_0}{dy^2} \left[\frac{d \Psi_0}{dy} - c_k \right] + g \right\} \tilde{\eta}_k = 0 \text{ at } y = 0. \tag{4.24}$$

In summary, given a function $\Psi_0 (y)$ which describes the effect of a current and such that $\Psi_0 (0) = 0$ the first order correction to Ψ_0 describing the effect of waves on the stream function is defined as $\Psi_1 (x, y, t)$ by equation (4.14a). This function must satisfy the differential equation (4.15) with boundary conditions described by equations (4.22) through (4.24).

For each function $\tilde{\Psi}_k$ a change in variables such that

$$\Psi_{rk} = \Psi_0 - c_k y \tag{4.25a}$$

$$u_k = u - c_k \tag{4.25b}$$

$$x_k = x - c_k t \tag{4.25c}$$

$$y_k = y \tag{4.25d}$$

$$v_k = v \tag{4.25e}$$

yields the following system of equations:

$$\frac{d\Psi_{rk}}{dy_k} \frac{d^2\tilde{\Psi}_k}{dy^2_k} - \left[k^2 \frac{d\Psi_{rk}}{dy_k} + \frac{d^3\Psi_{rk}}{dy_k^3} \right] \tilde{\Psi}_k = 0. \tag{4.26}$$

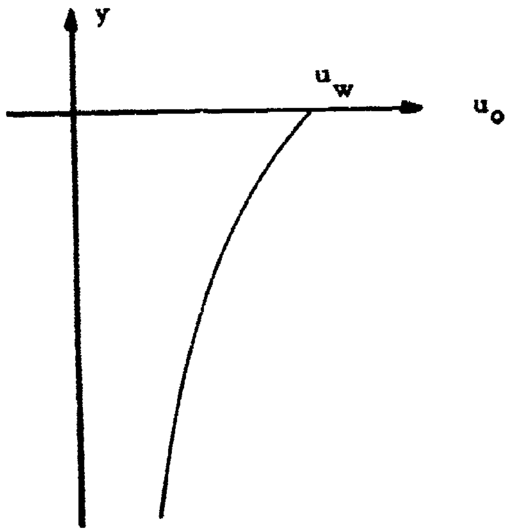
$$\tilde{\Psi}_k (-h) = 0. \tag{4.27}$$

$$\tilde{\Psi}_k + \frac{d\Psi_{rk}}{dy_k} \tilde{\eta}_k = 0 \text{ at } y = 0. \tag{4.28}$$

$$\frac{d\Psi_{rk}}{dy_k} \frac{d\tilde{\Psi}_k}{dy} + \left[\frac{d^2\Psi_{rk}}{dy_k^2} \frac{d\Psi_{rk}}{dy_k} + g \right] \tilde{\eta}_k = 0 \text{ at } y = 0 . \tag{4.29}$$

Special solutions of the set of equations (4.15, 4.22, 4.23 and 4.24) or (4.26 through 4.29) have been obtained for a single frequency component. These solutions are discussed in paragraphs 4.4 through 4.7 where the subscript k has been dropped. In paragraph 4.8, the extension of these cases to a finite wave spectrum will be discussed.

4.4 EXPONENTIAL DISTRIBUTION OF VORTICITY (Abdullah (1949))



Take

$$\zeta_0 = \nabla^2 \Psi_0 = \alpha u_w e^{\alpha y}$$

corresponding to

$$u_0 = u_w e^{\alpha y}$$

$$\Psi_0 = \frac{u_w}{\alpha} (e^{\alpha y} - 1)$$

or from equation (4, 25a)

$$\Psi_r = \frac{u_w}{\alpha} (e^{\alpha y} - 1) - cy \, .$$

Note that α is related to Ψ_b (equation (4. 16)) by

$$\Psi_b = \frac{u_w}{\alpha} (e^{-\alpha h} - 1) \, .$$

The first order solution is given by equation (4, 25)

$$\frac{d^2 \tilde{\Psi}_k}{dy^2} = \left[k^2 + \frac{\alpha^2 u_w e^{\alpha y}}{u_w^{\alpha y} - c} \right] \tilde{\Psi} = \left(k^2 + \frac{\alpha^2 u_o}{u_o - c} \right) \tilde{\Psi} \, .$$

This equation is Abdullah's equation 11. He gives a solution in terms of a power series in ku_o for an infinite fluid. The dispersion relation for this case is shown to be:

$$c = u_w - \tfrac{1}{2} u_w \frac{S_{01}}{S_{02}} \pm \sqrt{\left(\tfrac{1}{2} u_w \frac{S_{01}}{S_{02}} \right)^2 + \frac{g}{\alpha} \frac{S_{01}}{S_{02}}} \qquad (4.30)$$

where

$$S_{01} = \frac{(ku_w)^n}{kc(2n+1)} \left[1 + \frac{n}{2!(n+1)}\left(\frac{u_w}{c}\right) + \frac{n(4n+3)}{3!(n+1)(2n+3)}\left(\frac{u_w}{c}\right)^2 + ..\right]$$

$$S_{02} = \frac{(ku_w)^n}{kc(2n+1)} \left[n + \frac{n(n+1)}{2!(n+1)}\left(\frac{u_w}{c}\right) + \frac{n(n+2)(4n+3)}{3!(n+1)(2n+3)}\left(\frac{u_w}{c}\right)^2 + ..\right]$$

Equation (4.30) is an implicit relation b tween c and k. It was shown by Abdullah (1949), that the effect of the current is to lower the wave propagation speed for a fixed frequency.

Numerical solutions of equation (4.19) for small values of $\frac{u_w}{c}$ (smaller than .15) are presented in the cited reference.

Returning to equation (4.6), the solution given above verifies to order ϵ that the left hand side of the equation is zero. The right hand side is evaluated as

$$\nu\nabla^2(\nabla^2\Psi) \sim 0\,[\nu\nabla^2(\nabla^2\Psi_0)] \sim [\nu\alpha^3 u_w] .$$

Hence, neglecting the right hand side of equation (4.6) to order ϵ is only consistent if $\nu\alpha^3 u_w$ is of same order as the terms in ϵ^2, that is of order

$$\left[\epsilon^2\,\Psi_{1y}\,\frac{\partial}{\partial x}\,\nabla^2\Psi_1\right] .$$

But since $\Psi_1 = \tilde{\Psi}\cos k(x - ct) = \tilde{\Psi}\cos kX$ for a single frequency

$$\nabla^2\Psi_1 = \left[\frac{d^2\tilde{\Psi}}{dy^2} - k^2\tilde{\Psi}\right]\cos kX = \frac{\alpha^2 u_w e^{\alpha y}}{u_w e^{\alpha y} - c}\,\tilde{\Psi} .$$

From (4.28) and (4.29)

$$\tilde{\Psi} = 0\left(\frac{d\Psi_r}{dy}\right)$$

$$\frac{d\tilde{\Psi}}{dy} = 0 \left(\frac{d^2\Psi_r}{dy^2}\right)$$

$$\therefore \left(\epsilon^2 \Psi_{1y} \frac{\partial}{\partial x} \nabla^2 \Psi_1\right) = 0 \left[\epsilon^2 \upsilon_w \alpha\, k \frac{\alpha^2 u_w}{u_w - c} (u_w - c)\right]$$

$$= 0 \left[\epsilon^2 \alpha^3 u_w{}^2 k\right] .$$

To relate ϵ to known quantities, note that

$$u_k = \frac{d\Psi_r}{dy} + \epsilon \frac{d\Psi_1}{dy}$$

and the basic assumption is

$$\epsilon \frac{d\Psi_1}{dy} << \frac{d\Psi_0}{dy} .$$

In order of magnitude

$$\epsilon\, \alpha\, u_w << u_w .$$

Hence, ϵ is a small parameter for this problem provided $\epsilon\, \alpha << 1$.

From the data presented by Abdullah (1949), for $u_w = 15$ cm/s, $\alpha = 10^{-3}$ cm^{-1}

$$\epsilon << 10^3 \text{ cm} \qquad \text{say } \epsilon = 10 \text{ cm}$$

$$\nu \alpha^3 u_w = 0\ (10^{-2} \times 10^{-9} \times 10) = 0\ (10^{-10})\ \sec^{-2} .$$

By comparison

$$\epsilon^2 \alpha^3 u_w{}^2 k = 0\ (10^2 \times 10^{-9} \times 10^2\, k) \sec^{-2}$$

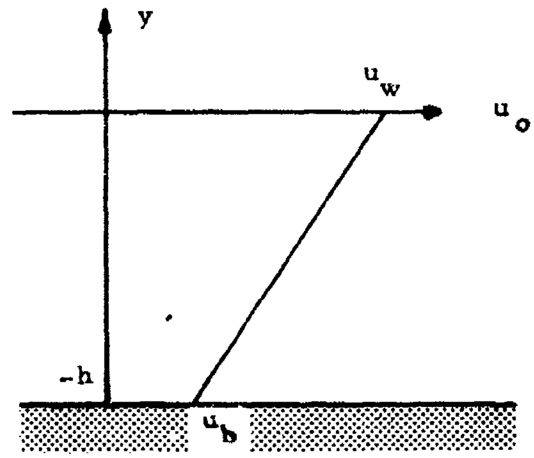
$$= 0\ (10^{-5}\, k) \sec^{-2} .$$

Hence, $0(\nu\alpha^3 u_w) \leq 0(\varepsilon^2\alpha^3 u_w{}^2 k)$ for wave lengths up to 600 m, and the assumption that the right hand side of (4.6) can be neglected is justified.

4.5 CONSTANT VORTICITY DISTRIBUTION (Biesel, 1950)

Take

$$\zeta_0 = \nabla^2 \Psi_0 = A \quad \text{a constant.}$$



Corresponding to

$$u_o = Ay + u_w$$

or

$$u_r = Ay + u_w - c = Ay + K$$

$$\Psi_r = \frac{A}{2}y^2 + Ky$$

so that Ψ_b and A are related by

$$\Psi_b = \frac{Ah^2}{2} - u_w h$$

or calling u_b the bottom velocity.

$$u_b = A(-h) + u_w \; .$$

$$A = \frac{u_w - u_b}{h} \; .$$

The first order solution is obtained from

$$\frac{d\tilde{\Psi}}{dy^2} = k^2\tilde{\Psi}$$

with the boundary conditions (4.27) and (4.28)

$$\tilde{\Psi} = - K\eta_1 \frac{\sinh k\,(y+h)}{\sinh\ kh} \quad .$$

Hence, for a single frequency component

$$\tilde{\Psi} = \frac{Ay^2}{2} + u_w y - \epsilon\,\tilde{\eta}_1\, K \frac{\sinh k\,(y+h)}{\sinh\ kh} \cos k\,(x - ct) + 0\,(\epsilon^2) \ .$$

The velocity components hence become

$$u = u_w + Ay + \epsilon\,\tilde{\eta}_1\, K\,k \frac{\cosh k\,(y+h)}{\sinh\ kh} \cos k\,(x - ct) + 0\,(\epsilon^2)$$

$$v = \epsilon\,\tilde{\eta}_1\ Kk \frac{\sinh k\,(y+h)}{\sinh\ kh} \sin\ k\,(x - ct) + 0\,(\epsilon^2) \ .$$

Introducing the expression for $\tilde{\Psi}$ in (4.25)

$$K^2 \left[k \frac{\cosh\ kh}{\sinh\ kh} - \frac{A}{K} \right] = g \tag{4.31}$$

which is the dispersion relation found by Biesel (1950).

Since $c = \frac{\omega}{k}$ where $\frac{2\pi}{\omega}$ is the wave period, Stokes dispersion relation $\omega^2 = g\,k \tanh kh$ is found for the special case where the vorticity vanishes ($A = 0$) and there is no current ($u_w = 0$).

For the case of no vorticity but a constant current, the relationship becomes

$$\omega'^2 = g\ k \tanh\ kh$$

where $\frac{2\pi}{\omega'}$ is an equivalent wave period defined by

$$\omega' = \omega - \frac{u_w}{k} \ .$$

In the general case where the vorticity is given as well as the surface currents, equation (4.31) gives the value of the wave propagation speed as a function of wave length. Equation (4.31) can be re-written:

$$(c-u_w)^2 \left[\frac{kh}{\tanh kh} - \frac{(c-u_w) - (c-u_b)}{(u_w-c)\ h}\right] = g$$

or

$$(c-u_w)^2 \left[\frac{kh}{\tanh kh} - \frac{(c-u_w) - (c-u_b)}{c-u_w}\right] = g\,h \quad .$$

Solutions only exist if

$$(c-u_w) \left[1 - \frac{(c-u_w) - (c-u_b)}{(c-u_w)}\right] \leq g\,h$$

that is

$$(c-u_w)\,(c-u_b) \leq g\,h$$

where u_b is the bottom (absolute) velocity.

For a uniform current of speed u_w (that is $A = 0$) equation (4.31) yields

$$(c_u - u_w)^2 \frac{kh}{\tanh kh} = g\,h \ .$$

Comparing with

$$(c - u_w)^2 \left[\frac{kh}{\tanh kh} + \frac{u_w - u_b}{c - u_w}\right] = g\,h$$

and assuming a realistic wind generated wave where $c > u_w$ (Wiegel, 1964) and $u_b < u_w$

$$c_u - u_w > c - u_w$$

$$c < c_u$$

so that when there is a vertical velocity gradient the wave speed is lower than for a constant current of same surface speed (but not necessarily lower than for a constant current at the average current speed). In this case

$\nabla^2(\nabla^2\Psi_0) \equiv 0$ and

$$\nabla^2(\nabla^2\Psi_1) = \nabla^2\left[\left(\frac{d^2\tilde{\Psi}_k}{dy^2} - k^2\tilde{\Psi}_k\right)\cos k(x - ct)\right] \equiv 0$$

so that the left hand side of (4.6) is irrelevant for zeroth and first order solutions.

4.6 HYPERBOLIC VORTICITY DISTRIBUTION (Eliasson and Engelund, 1972)

Here a single frequency component is considered and the following relationship is assumed

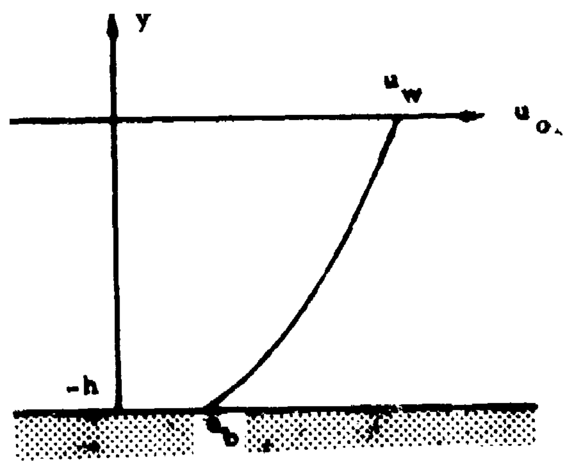
$$\zeta_0 = \nabla^2\Psi_0 = \nabla^2\Psi_r = \beta^2\Psi_r$$

and with boundary conditions (4.27) and (4.28)

$$\Psi_r = -\Psi_b\frac{\sinh\beta y}{\sinh\beta h}$$

or

$$\Psi_0 = -\Psi_b\frac{\sinh\beta y}{\sinh\beta h} + cy\ .$$



Hence,

$$u_0 = -\beta\Psi_b\frac{\cosh\beta y}{\sinh\beta h} + c$$

$$u_0(0) = u_w$$

or

$$K = u_w - c = -\beta\frac{\Psi_b}{\sinh\beta h}\ .$$

Hence,

$$\Psi_r = \frac{K}{\beta} \sinh \beta y$$

with

$$u_0 = K \sinh \beta y .$$

The first order solution is obtained by solving (4.6) which becomes

$$\frac{d^2\tilde{\Psi}}{dy^2} = (k^2 + \beta^2)\ \tilde{\Psi}$$

with boundary condition (4.27) and (4.28)

$$\tilde{\Psi} = - K\, \tilde{\eta}_1 \frac{\sinh\sqrt{k^2 + \beta^2}\,(y + h)}{\sinh\sqrt{k^2 + \beta^2}\ h} .$$

Hence,

$$\Psi = c\, y + \frac{K}{\beta} \sinh \beta y - \epsilon\, \tilde{\eta}_1\, K \frac{\sinh\sqrt{k^2 + \beta^2}\,(y + h)}{\sinh\sqrt{k^2 + \beta^2}\ h} \cos k\,(x - ct) + 0\,(\epsilon^2)$$

and

$$u = c + (u_w - c) \cosh \beta y - \epsilon\, \tilde{\eta}_1\, K\sqrt{k^2+\beta^2}\ \frac{\cosh\sqrt{k^2+\beta^2}\,(y+h)}{\sinh\sqrt{k^2+\beta^2}\,(h)} \cos k\,(x - ct) + 0\,(\epsilon^2)$$

$$v = - \epsilon\, \tilde{\eta}_1\, K\, k \frac{\sinh\sqrt{k^2 + \beta^2}\,(y+h)}{\sinh\sqrt{k^2 + \beta^2}\ h} \sin k\,(x - ct) + 0\,(\epsilon^2) .$$

Boundary condition (4.29) gives the following dispersion relation

$$K^2\sqrt{k^2 + \beta^2}\ \frac{\cosh\sqrt{k^2 + \beta^2}\ h}{\sinh\sqrt{k^2 + \beta^2}\ h} = g \quad \text{(Eliasson and Engelund, 1972)} .$$

Once again for zero vorticity ($\beta = 0$) and no current, this relation reduces to Stokes value where $K_s = c_s = \frac{\omega}{k}$

$$\frac{\omega^2}{k^2} k \frac{1}{\tanh kh} = g \quad \text{or} \quad \omega^2 = g\ k\ \tanh kh .$$

For non zero vorticity

$$(c - u_w)^2 = \frac{g}{\sqrt{k^2 + \beta^2}} \tanh\sqrt{k^2 + \beta^2}\ h \tag{4.32}$$

$$\left(\frac{c - u_w}{c_s}\right)^2 = \frac{\tanh\sqrt{k^2 + \beta^2}\ h}{\sqrt{k^2 + \beta^2}\ h} \frac{k\ h}{\tanh kd} < 1 .$$

Hence, the speed of the waves relative to the current is lower than their speed in the absence of current.

In order to evaluate the impact of the viscous terms, consider first the expression for u. The small parameter ϵ is defined by

$$\left| \epsilon(u_w - c) \sqrt{k^2 + \beta^2} \right| << |u_w|$$

or

$$\epsilon \sqrt{k^2 + \beta^2} << 1 .$$

Then

$$\nu \nabla^2 (\nabla^2 \Psi_0) = 0\ (\nu K \beta^3) = 0\ [\nu(u_w - c)\beta^3]$$

which needs to be compared to

$$[\epsilon^2 \Psi_1\ y\ \frac{\partial}{\partial x} \nabla^2 \Psi_1] = 0\ [\epsilon^2 K^2 \sqrt{k^2 + \beta^2}\ k\ \beta^2]$$

$$= 0\ [\epsilon^2 \sqrt{k^2 + \beta^2}\ k\ \beta^2 (u_w - c)^2] .$$

Hence, viscous term can be neglected if

$$\nu\beta \leq \epsilon^2 \sqrt{k^2+\beta^2}\ k\ |u_w - c| \quad .$$

Taking the dispersion relation into account

$$\nu\beta \leq \epsilon^2 \sqrt{k^2+\beta^2}\ k\sqrt{g}\ \sqrt{\frac{\tanh\sqrt{k^2+\beta^2}\ h}{\sqrt{k^2+\beta^2}}}$$

$$\nu\beta \leq \epsilon^2 (k^2+\beta^2)\sqrt{g\,h}\ \sqrt{\frac{k^2}{k^2+\beta^2}}\ \sqrt{\frac{\tanh\sqrt{k^2+\beta^2}\ h}{\sqrt{k^2+\beta^2}\ h}} \quad . \qquad (4.33)$$

In most real cases β will be relatively small, and for discussing equation (4.33), it will be assumed to be at most of order k. Then (4.33) can be approximated by

$$\nu\beta \leq \epsilon^2 k^2 \sqrt{g\,h}\ \sqrt{\frac{\tanh kh}{k\,h}} \quad .$$

For shallow water $k\,h < 1$ this becomes

$$\nu\beta \leq \epsilon^2 k^2 \sqrt{g\,h} \quad .$$

For example, assuming $\epsilon k = 10^{-2}$ $kd = 10^{-1}$

$$10^{-2}\ \beta \leq 10^{-4}\ \sqrt{10^3\ h} \qquad h \text{ and } \beta^{-1} \text{ in centimeters}$$

$$\beta \leq \sqrt{10^{-1}\ h} \quad \text{or} \quad \beta\sqrt{k} \leq 10^{-1} \quad .$$

Assuming β is of the same order as k, viscosity can be neglected for waves length larger than about 5 cm.

For deep water $k\,h >> 1$

$$\nu\beta \le \epsilon^2 k^2 \sqrt{\frac{g}{k}}$$

$$10^{-2}\ \beta \le 10^{-4}\sqrt{\frac{10^3}{k}} \quad \therefore \quad \beta\sqrt{k} \le 10^{-\frac{1}{2}}$$

so that the same order of magnitude as found above is still valid.

Hence, viscosity can reasonably be neglected for most of the expected wave spectra. It is interesting to compare this case with the case presented in paragraph 4.4.

In both cases the vorticity distribution is exponential in character. The main difference is that in the previous case, the solution which was obtained was such that the vorticity was a linear function of the stream function whereas here it is a linear function of the stream function relative to axes moving at the wave speed. The latter approach was used by Eliasson and Engelund (1972) for a single frequency as it yields a closed form solution for the velocity components and a fairly simple dispersion relation. However, if a wave spectrum is to be analyzed, individual solutions are obtained with respect to axes moving at different speeds and the combined solution may be difficult to obtain.

4.7 GENERAL SOLUTION

For any given absolute velocity distribution

$$u_0 = u\,(y)$$

$$\Psi_0 = \int_0^y u\,(t)\ dt$$

$$\Psi_r = \int_0^y u\,(t)\ dt - c\,y\ .$$

The first order solution is given by

$$\frac{d^2\tilde{\Psi}_r}{dy^2} = k^2 + \left(\frac{1}{u_0 - c} \quad \frac{d^2 u_0}{dy^2}\right) \quad \tilde{\Psi}_r \ . \tag{4.34}$$

Boundary conditions (4.27) and (4.28) define the two unknown constants of the second order differential equation.

The dispersion relation is then obtained from (4.29).

Equation (4.34) is singular for $u_0 = c$ or $\pm \infty$

and for

$$\frac{d^2 u_0}{dy^2} = - k^2 (u_0 - c) \ .$$

Since $|u_0| \rightarrow \infty$ is not a realistic assumption, the possible motions can be obtained either by direct integration of (4.34) as was done in paragraphs 4.5 and 4.6 or by power series approximation around the two other singular points as was done by Abdullah (1949). The dispersion relation in the latter case must be solved numerically by successive approximation.

It should be noted that in paragraphs 4.4, 4.5 and 4.6 the analyses have assumed a single wave length. Because of the non linearity of the basic equations, superposition of results cannot be readily performed. Some method of obtaining solutions for a wave spectrum may then be more valuable to the problem of estimating forces due to waves than determining a single component response in the presence of complex current distribution. The general discussion of paragraphs 4.2 and 4.3 will provide the means of obtaining this extension.

4.8 SOLUTIONS FOR A WAVE SPECTRUM

1. Consider first the exponential vorticity distribution. Since Ψ_0 is independent of k, as in paragraph 4.4

$$\Psi_0 = \frac{u_w}{\alpha} (e^{\alpha y} - 1)$$

and the equation for Ψ_k is from (4.15)

$$\frac{d^2\tilde{\Psi}_k}{dy^2} = k^2 + \left(\frac{\alpha^2 u_w e^{\alpha y}}{u_w e^{\alpha y} - c_k}\right) \Psi_k \qquad (4.35)$$

which is the equation solved by Abdullah (1949). The discussion of paragraph 4.4 applies for any wave number k.

This case corresponds to a wind induced current. Given the wind speed, the value u_w can be determined from Wiegel (1964). For each significant frequency in the wave spectrum, equations such as (4.30) can be numerically solved for or nomograms could be prepared to obtain c_k (k). Numerical integration of equation (4.35), then (4.14a) would yield the necessary information to compute forces due to the combined action of waves and current.

2. Constant vorticity

In this case

$$\Psi_0 = \frac{A y^2}{2} + u_w y .$$

Equation (4.26) gives

$$\frac{d^2\tilde{\Psi}_k}{dy^2} = k^2 \tilde{\Psi}_k .$$

Hence, as in paragraph 4.5

$$\tilde{\Psi}_k = - (c_k - u_w) \tilde{\eta}_k \frac{\sinh k (y + h)}{\sinh k h}$$

and

$$(u_w - c_k)^2 \left[k \frac{\cosh k h}{\sinh k h} - \frac{A}{u_w - c_k} \right] = g$$

which can readily be solved for.

Computing the velocity components from Ψ_1 (equation (4.14a)) and integrating over a fixed period, the average velocity components at a given fetch and depth are then obtained by a simple numerical quadrative in k.

3. Hyperbolic vorticity distribution

For a single frequency it was found that

$$\Psi_0 = \frac{u_w - c}{\beta} \sinh \beta y + c$$

where c depended on k according to equation (4.32). Hence, in this case Ψ_0 depends on k, and it is not possible to use the above expression for Ψ_0 where a wave spectrum is considered.

A possible extension of this solution can be obtained by first defining an average velocity $\bar{c}$ by

$$\bar{c} = \frac{1}{k_2 - k_1} \int_{k_1}^{k_2} c (k) \; d k$$

and taking

$$\Psi_0 = \frac{u_w - \bar{c}}{\beta} \sinh \beta y + \bar{c} y .$$

Then from (4.15)

$$[(u_w - \bar{c}) \cosh \beta y + \bar{c} - c_k] \left[\frac{d^2\tilde{\Psi}_k}{dy^2} - k^2 \tilde{\Psi}_k \right] = \beta^2 (u_w - \bar{c}) \cosh \beta y \, \tilde{\Psi}_k \; .$$

As mentioned in paragraph 4.6, the simple equation which is obtained when $\bar{c} = c_k$ occurs here at most for one wave number. In general the equation to be solved is

$$\frac{d^2\tilde{\Psi}_k}{dy^2} = k^2 + \left[\frac{\beta^2 (u_w - \bar{c}) \cosh \beta y}{(u_w - \bar{c}) \cosh \beta y + \bar{c} - c_k} \right] \tilde{\Psi}_k$$

which is of the form of the equation solved for numerically by Abdullah (1949).

4. General distribution

In order to obtain forces and moments on structures placed in a current in the presence of waves, it is necessary to compute the velocity components or stream function Ψ. For a first order solution, given the wave current effect Ψ_0, the stream function Ψ_1 is obtained by evaluating the integral (4.14a) when $\tilde{\Psi}_k(y)$ is a solution of equation (4.15) and when the functional relationship between c and k is usually given by boundary condition (4.24). The process is, therefore, usually a lengthy one even with the use of high speed digital computers.

Results are simplified if $\tilde{\Psi}_k(y)$ can be evaluated in closed form.

From (4.15) it can be seen that

$$\frac{d^2\tilde{\Psi}_k}{dy^2} - \left(k^2 + \frac{d^3\Psi_0/dy^3}{\frac{d\Psi_0}{dy} - c_k} \right) \tilde{\Psi}_k = 0$$

or

$$\frac{d^2\tilde{\Psi}_k}{dy^2} + f(y)\,\tilde{\Psi}_k = 0 \ .$$

Known solutions of such an equation can be obtained for

a) $f(y) = 0 \qquad \tilde{\Psi}_k = ay + b$

b) $f(y) = -\lambda^2 \qquad \tilde{\Psi}_k = ae^{\lambda y} + be^{-\lambda y}$

c) $f(y) = 2n+1-y^2 \quad \tilde{\Psi}_k = e^{-y^2/2} H_n(y)$ when n is an integer and H_n is a Hermite Polynomial

d) $f(y) = 1 + \frac{.25-\lambda^2}{y^2} \quad \tilde{\Psi}_k = \sqrt{y}\,[a\,J_\lambda(y) + bJ_{-\lambda}(y)]$

$J_\lambda(y)$ is a Bessel function and λ is not an integer

or $\tilde{\Psi}_k = \sqrt{y}\,[a\,J_n(y) + b\,Y_n(y)]$ if $\lambda = n$

is a integer.

Cases c) and d) already involve functions which must be evaluated numerically using a series representation.

The only fairly simple cases are cases a) and b).

For b)

$$k^2 + \frac{d^3\Psi_0/dy_3}{d\Psi_0/dy-c_k} = \lambda^2$$

$$\frac{d^3\Psi_0}{dy^3} = 0 \quad \Psi_0 = ay^2 + by \text{ since } \Psi_0(0) = 0$$

or

$$\frac{d^3\Psi_0}{dy^3} = (\lambda^2 - k^2)\left[\frac{d\Psi_0}{dy} - c_k\right]$$

$$\Psi_0 = C_k y + a\,[e^{\mu y} - e^{-\mu y}] \quad \mu^2 = \lambda^2 - k^2.$$

Both cases have been studied previously and it was shown that the first one only with Ψ_0 independent of k could be extended to a wave spectrum without undue complication of the computations.

4.9 CONCLUSION

Although the equations required to solve at least up to the first order the problem of small amplitude wave propagation in the presence of an arbitrary current for an arbitrary wave spectrum have been presented, and a review of the special solutions previously obtained for a single wave length has been made, the problem in general requires lengthy numerical computations.

However, for a current whose velocity distribution can be approximated by a linear depth dependence, it has been shown that, at most, a single numerical quadrature was required to obtain the average velocity components. This method may then be used to estimate the forces due to wave action in the presence of a current. An experimental knowledge of the current velocity at but a few depths (two minimum) will define the parameters necessary to completely solve this problem.

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APPENDIX A

NOTE ON THE BOTTOM FRICTION APPROXIMATION

APPENDIX A: NOTE ON THE BOTTOM FRICTION APPROXIMATION

A dissipative effect imposed by bottom friction comprises an important term in the momentum equations. The bottom friction is non-linear in nature, and the coexistence of wave orbital motion and circulation renders mathematical formulation of this effect even more complex in the surf zone.

The tangential bottom stress $\vec{B}$ of a quadratic form is given by

$$\vec{B} = \bar{c} \rho \mid \vec{V} \mid \vec{V} \qquad \text{A-1}$$

in which $\bar{c}$ is the friction coefficient, ρ the water density, $\vec{V}$ the resultant velocity vector combining wave orbital and circulation velocities, e.g.

$$\vec{V} = \vec{U}_o + (u, v) \qquad \text{A-2}$$

in which $\vec{U}_o$ is the wave orbital velocity vector, whose x and y components are, respectively;

$$\vec{U}_o \cos\theta \text{ and } - \vec{U}_o \sin\theta, \qquad \text{A-3}$$

and (u, v) is the circulation velocity vector whose x and y components are, respectively, u and v.

A full expression for $|\vec{V}|$ is then,

$$|\vec{V}| = \left[(U_o \cos\theta + u)^2 + (v - U_o \sin\theta)^2\right]^{1/2}$$

$$= \left[u^2 + v^2 + 2U_o u \cos\theta - 2U_o v \sin\theta + U_o^2\right]^{1/2} \qquad \text{A-4}$$

Several methods can be used to simplify Eq. A-4. In the previous investigation (Noda, 1972; also Thornton, 1969), two assumptions were made.

One was to consider the circulation velocity components u, v to be small as compared with the wave orbital motion, e.g.

$$|\vec{U}_o| >> u, v \qquad \text{A-5}$$

such that Eq. A-4 reduces to

$$|\vec{V}| \cong U_o \qquad \text{A-6}$$

Eq. A-1 is then rewritten

$$\vec{B} = \bar{c}\rho\, U_o\, [\vec{U}_o + (u, v)] \qquad \text{A-7}$$

The time average of the bottom stress is

$$< \vec{B} > = \bar{c}\, \rho < U_o > < [\vec{U}_o + (u, v)] > \qquad \text{A-8}$$

Using the linear theory, the term $< U_o >$ is written as

$$< U_o > = \frac{2H}{T \sinh kd} \qquad \text{A-9}$$

It was also assumed that the term $< [\vec{U}_o + (u, v)] >$ may be approximated by

$$< [\vec{U}_o + (u, v)] > \cong < \vec{U}_o > + < (u, v) > \qquad \text{A-10}$$

Since $< \vec{U}_o >$ will vanish over a wave cycle, Eq. A-10 now reduces to

$$< [\vec{U}_o + (u, v)] > = < (u, v) > \qquad \text{A-11}$$

Combining Eqs. A-8 and A-11, the time-averaged bottom stress takes the form

$$< \vec{B} > = \bar{c}\, \rho < U_o > < (u, v) > \qquad \text{A-12}$$

Noting that the friction terms in the momentum equations are defined by

$$\vec{F} = \frac{<\vec{B}>}{\rho(\eta + d)} \cong \frac{<\vec{B}>}{\rho d} \qquad \text{A-13}$$

then

$$F_x = \frac{2\bar{c}\,U_o}{\pi \cdot d} \cdot u \qquad \text{A-14}$$

$$F_y = \frac{2\bar{c}\,U_o}{\pi \cdot d} \cdot v \qquad \text{A-15}$$

In the present investigation, an attempt was made to carry out a more rigorous evaluation of these assumptions. Instead of Eq. A-5, we may assume

$$|\vec{U}_o|, \quad u >> v \qquad \text{A-16}$$

In other words, we assume that a nearshore circulation contains on- and offshore velocity component u which is much longer than the longshore component and is not negligible as compared with wave orbital motion. This assumption is obviously borne out in a circulation containing a strong outflow. Since, because of the refraction, the incidence wave angle in the surf zone is generally small, we may further assume

$$\theta \simeq 0 \qquad \text{A-17}$$

Using Eqs. A-16 and A-17, Eq. A-4 is simplified as

$$|\vec{V}| = \left[u^2 + 2uU_o + U_o^2\right]^{1/2} = u + U_o \qquad \text{A-18}$$

Using unit vectors $\vec{i}$ and $\vec{j}$ in the x and y directions, the bottom friction is now rewritten as

$$\vec{B} = \bar{c}\rho\,(u + U_o)\left[\vec{U}_o + (u, v)\right]$$
$$= \vec{i}\left[\bar{c}\rho\,(u + U_o)\,(u + U_o\cos\theta)\right]$$
$$+ \vec{j}\left[\bar{c}\rho\,(u + U_o)\,(v - U_o\sin\theta)\right] \qquad \text{A-19}$$

The time average of Eq. A-19 is evaluated separately for $\vec{i}$ and $\vec{j}$ components, e.g.

$$< \vec{B} > = \vec{i}\cdot\bar{c}\rho\left[u^2 + \frac{4uU_o}{\pi} + \frac{1}{2}U_o^2\right]$$
$$+ \vec{j}\cdot\bar{c}\rho\left[uv + \frac{2U_o}{\pi}v\right] \qquad \text{A-20}$$

Consequently, the friction terms in the momentum equations are rewritten as

$$F_x = \frac{\bar{c}}{d}\left[u^2 + \frac{4U_o}{\pi}u + \frac{1}{2}U_o^2\right] \qquad \text{A-21}$$

$$F_y = \frac{\bar{c}}{d}\left[u + \frac{2}{\pi}U_o\right]\cdot v \qquad \text{A-22}$$

where

$$U_o = \frac{\pi H}{T\sinh kd} \qquad \text{A-23}$$

The equation to solve to obtain stream function ψ is of the form [see Eq. (2.13) in the text]

$$\frac{\partial^2\psi}{\partial x^2} + f_1\frac{\partial\psi}{\partial x} + f_2\frac{\partial\psi}{\partial y} = f_3 \qquad \text{A-24}$$

The choice of the simple friction terms, Eqs. A-14 and A-15, give f_1, f_2 and f_3 as functions of x and y only, hence the problem reduces to solving a linear differential equation, [see Eq. (2.13) in the text]. The present computation has used this linear case.

On the other hand, the choice of the more complex friction terms, Eqs. A-21 and A-22, yields f_1, f_2 and f_3 as functions of x, y and ψ, thus the problem becomes non-linear. Numerical calculation of this non-linear case was attempted, but the result exhibited extreme instability, hence the lack of convergence. The form of our non-linear equation is more complex than any known equations for which the conditions for stable solution have been well explored. Further investigation of this problem is apparently outside the scope of the present investigation and should be reserved for a future study.

APPENDIX B

COMPUTER PROGRAMS FOR WAVE--CURRENT INTERACTION

```
      PROGRAM MAIN(INPUT=256,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7=512,
     *TAPE8=512,TAPE9=512,TAPE10=512)
C
C          PROGRAM COMPUTES WAVE KINEMATIC AND DYNAMICS BY FINITE
C          DIFFERENCE INSTEAD OF BY CHARACTERISTICS FOR THE
C          GENERAL CASE OF WAVE-CURRENT INTERACTION
C
      COMMON U(70,20),V(70,20),Z(70,20),SI(70,20),CO(70,20),H(70,20),
     1CG(41,19),S(41,19),HBREAK(41,19),TB(41,19),D(70,20),DDDX(70,20),
     2DDDY(70,20),PSI(70,20),FX(70,20),FY(70,20),W(70,20),UOLD(70,20),
     3VOLD(70,20),ULAST(70,20),VLAST(70,20)
      COMMON/CON/ G,PI,PI2,RAD,EPS,DX,DY,DX2,DY2,T,SIGMA,M,N,MM
C
C          READ AND WRITE INPUT
C
C         INTER=0---NO WAVE-CURRENT INTERACTION, U AND V = 0.0
C         INTER=1---WAVE-CURRENT INTERACTION, READ U AND V FROM PF CALLED NODAV
C         ISTORV=0--DO NOT STORE VELOCITY FIELDS U AND V ON PERMANENT FILES
C         ISTORV=1--STORE U AND V IN PERMANENT FILE CALLED NODAV
C         ISTORH=0--DO NOT STORE H AND THETA FIELDS ON PERMANENT FILES
C         ISTORH=1--STORE H AND THETA FIELDS IN PF CALLED NODAH
C         IREADH=0--CALCULATE THE H AND THETA FIELDS DIRECTLY
C         IREADH=1--READ THE H AND THETA FIELDS FROM PF CALLED NODAH
C         ISTORP=0--DO NOT STORE PSI FIELD ONTO PERMANENT FILE
C         ISTORP=1--STORE PSI FIELD ONTO PERMANENT FILE CALLED NODAPSI1
      READ(5,1) M,N,ITMAXZ,ITMAXH,ITMAXP,MM,INTER,ISTORV,ISTORH,IREADH,
     1ISTORP
    1 FORMAT(8I10)
      WRITE(6,10) M,N,ITMAXZ,ITMAXH,ITMAXP,MM,INTER,ISTORV,ISTORH,IREADH
     1,ISTORP
   10 FORMAT(10X,8I10/)
      READ(5,2) THETA0,HH,DX,DY,T,EPS,AM,FRICT,PSIMAX
    2 FORMAT(8F10.2)
      WRITE(6,11) THETA0,HH,DX,DY,T,EPS,AM,FRICT,PSIMAX
   11 FORMAT(10X,10G12.4/)
C
C         COMPUTE AND DEFINE CONSTANTS
C
      G=9.80621
      PI=3.1415926536
      PI2=PI*2.0
      RAD=180.0/PI
      DX2=DX*2.0
      DY2=DY*2.0
      SIGMA=PI2/T
      MM1=MM-1
      MMIN=M+1
      M1=M-1
      M2=M-2
      N1=N+1
      N2=N+2
C          CONVERT INITIAL VALUE OF THETA-DEEP WATER TO RADIANS
      THETA0=THETA0/RAD
C          WRITE CONSTANT
      WRITE(6,12) G,PI,PI2,RAD,DX2,DY2,SIGMA,THETA0,M1,M2,N1,N2
   12 FORMAT(1X,8G12.5,4I8/)
C
C          INITIALIZE VELOCITY,THETA AND WAVE HEIGHTVARIABLES
C
C         CONVERT DEEP WATER VALUES OF THETA0 AND HH
```

```
      XMAX=DX*FLOAT(M-1)
      DSTART=AM*XMAX
      CALL KVNUM(DSTART,0.0,0.0,0.0,0.0,RKSTART,A)
      ARG=DSTART*RKSTART
      TSTART=ASIN(SIN(THETAO)*TANH(ARG))
      TSTART=PI-TSTART
      ARG2=ARG*2.0
      HSHOAL=SQRT(1.0/(TANH(ARG)*(1.0+ARG2/SINH(ARG2))))
      HSTART=HH*SQRT(COS(THETAO)/COS(TSTART))
      HSTART=HSTART*HSHOAL
C        INITIALIZE FIELDS
      DO 50 I=1,MM
      ZZ=PI +(TSTART-PI)*FLOAT(I-1)/FLOAT(M1)
      HHH= HSTART*FLOAT(I-1)/FLOAT(M1)
      X= DX*FLOAT(I-1)
      SINZ=SIN(ZZ)
      COSZ=COS(ZZ)
      DO 50 J=1,N2
      U(I,J)= 0.0
      V(I,J)=0.0
      H(I,J)=HHH
      Z(I,J)=ZZ
      SI(I,J)=SINZ
      CO(I,J)=COSZ
   50 CONTINUE
      DO 55 I=2,M
      DO 55 J=1,N
      CALL DEPTH(I,J)
   55 CONTINUE
      DO 51 I=MMIN,MM
      DD= AM*DX*FLOAT(I-1)
      DO 51 J=1,N
      D(I,J)=DD
   51 CONTINUE
      READ(10) ((PSI(I,J),J=1,N2),I=1,MM)
      GO TO 300
      DO 30 I=1,MM
      PSI(I,1)= PSI(I,N)
      PSI(I,N2)= PSI(I,3)
   30 CONTINUE
  300 CONTINUE
      CALL SPEED
C        END INITIALIZING FIELDS
C
C        READ U AND V FROM PERMANENT FILES IF INTER=1
C
      IF (INTER .EQ. 0) GO TO 80
      READ(7) ((U(I,J),J=1,N2),I=1,MM)
      READ(7) ((V(I,J),J=1,N2),I=1,MM)
      REWIND 7
   80 CONTINUE
      GO TO 25
      WRITE(6,14)  ((U(I,J),J=1,10),I=1,M)
   14 FORMAT(//,(10F13.5))
      WRITE(6,15) ((U(I,J),J=11,N2),I=1,M)
   15 FORMAT(//,(9F13.5))
      WRITE(6,14) ((V(I,J),J=1,10),I=1,M)
      WRITE(6,15) ((V(I,J),J=11,N2),I=1,M)
      WRITE(6,14) ((H(I,J),J=1,10),I=1,M)
      WRITE(6,15) ((H(I,J),J=11,N2),I=1,M)
```

```
      WRITE(6,14) ((Z(I,J),J=1,10),I=1,M)
      WRITE(6,15) ((Z(I,J),J=11,N2),I=1,M)
      WRITE(6,14)((SI(I,J),J=1,10),I=1,M)
      WRITE(6,15)((SI(I,J),J=11,N2),I=1,M)
      WRITE(6,14)((CO(I,J),J=1,10),I=1,M)
      WRITE(6,15)((CO(I,J),J=11,N2),I=1,M)
   25 CONTINUE
      WRITE(6,13) XMAX,DSTART,RKSTART,ARG,TSTART,HSTART
   13 FORMAT(10X,8G13.5)
      KMAX=5
      DO 90 K=1,KMAX
      WRITE(6,91) K
   91 FORMAT(10X,*FEED BACK CYCLES=*,I3)
      DO 76 I=1,MM
      DO 76 J=1,N2
      IF (K .GT. 1) GO TO 77
      UOLD(I,J)= U(I,J)
      VOLD(I,J)= V(I,J)
      GO TO 78
   77 CONTINUE
      UOLD(I,J)= ULAST(I,J)
      VOLD(I,J)= VLAST(I,J)
   78 CONTINUE
      ULAST(I,J)= U(I,J)
      VLAST(I,J)= V(I,J)
      U(I,J)= 0.25*(UOLD(I,J)+ULAST(I,J))
      V(I,J)= 0.25*(VOLD(I,J)+VLAST(I,J))
   76 CONTINUE
      IFLAG=1
      IF (K .EQ. 1) GO TO 37
      EPSU= 0.05
      DO 33 I=1,MM
      DO 33 J=1,N2
      IF(ABS(UOLD(I,J)-ULAST(I,J)).GT.(EPSU*ABS(ULAST(I,J))))IFLAG=0
      IF(ABS(VOLD(I,J)-VLAST(I,J)).GT.(EPSU*ABS(VLAST(I,J))))IFLAG=0
   33 CONTINUE
      IF (IFLAG .EQ. 0) GO TO 35
      WRITE(6,34) EPSU
   34 FORMAT(10X,*ABOVE ITERATION ON U AND V CONVERGED FOR ERROR=*,
     1F7.4)
      CALL EXIT
   37 CONTINUE
   35 CONTINUE
      WRITE(6,14)  ((U(I,J),J=1,10),I=1,M)
      WRITE(6,15) ((U(I,J),J=11,N2),I=1,M)
      WRITE(6,14) ((V(I,J),J=1,10),I=1,M)
      WRITE(6,15) ((V(I,J),J=11,N2),I=1,M)
      IF (IREADH .EQ. 0) GO TO 85
      READ(8) ((H(I,J),J=1,N2),I=1,MM)
      READ(8) ((Z(I,J),J=1,N2),I=1,MM)
      REWIND 8
      WRITE(6,87)
   87 FORMAT(//,10X,*VALUES OF H AND THETA READ FROM PERMANENT FILE*,//)
      DO 88 I=1,M
      DO 88 J=1,N2
      ARGZ= Z(I,J)
      SI(I,J)= SIN(ARGZ)
      CO(I,J)= COS(ARGZ)
   88 CONTINUE
      GO TO 86
```

```
   85 CONTINUE
C
C        NOW SOLVE FOR THETA
C
      CALL ANGLE(ITMAXZ)
      CALL HEIGHT(ITMAXH)
   86 CONTINUE
      CALL SNELL(THETAO,HH,AM)
C
C        WRITE OUTPUT
C
      GO TO 79
      WRITE(6,60) ((D(I,J),J=1,10),I=1,M)
   60 FORMAT(10X,*WATER DEPTHS ARE*/,(10F13.5))
      WRITE(6,61) ((D(I,J),J=11,N2),I=1,M)
   61 FORMAT(///,(9F13.5))
   79 CONTINUE
      WRITE(6,62) ((H(I,J),J=1,10),I=1,M)
   62 FORMAT(//10X,*VALUES OF THE WAVE HEIGHT ARE*/,(10F13.5))
      WRITE(6,61) ((H(I,J),J=11,N2),I=1,M)
      WRITE(6,63) ((IB(I,J),J=1,N2),I=1,M)
   63 FORMAT(//10X,*VALUES OF THE BREAKING INDEX IB-- IB=1 NO BREAKING A
     1ND IB=0 BREAKING*/,(19I6))
      WRITE(6,64) ((HBREAK(I,J),J=1,10),I=1,M)
   64 FORMAT(//10X,*BREAKING WAVE HEIGHT*/,(10F13.5))
      WRITE(6,61) ((HBREAK(I,J),J=11,N2),I=1,M)
C
C        WRITE VALUES OF DEPTH,H AND THETA FOR SNELL↑S LAW REGION
C
      GO TO 26
      WRITE(6,70) ((D(I,J),J=1,10),I=MMIN,MM)
   70 FORMAT(//10X,*WATER DEPTHS FOR THE SNELL↑S LAW REGION*/,(10F13.5))
      WRITE(6,71) ((D(I,J),J=11,N2),I=MMIN,MM)
   71 FORMAT(///,(9F13.5))
      WRITE(6,72)  ((H(I,J),J=1,10),I=MMIN,MM)
   72 FORMAT(//10X,*WAVE HEIGHTS FOR THE SNELL↑S LAW REGION*/,(10F13.5))
      WRITE(6,71) ((H(I,J),J=11,N2),I=MMIN,MM)
      WRITE(6,73) ((Z(I,J),J=1,10),I=MMIN,MM)
   73 FORMAT(//10X,*THETA VALUES FOR THE SNELL↑S LAW REGION*/,(10F13.5))
      WRITE(6,71) ((Z(I,J),J=11,N2),I=MMIN,MM)
   26 CONTINUE
      IF (ISTORH .EQ. 0) GO TO 82
      WRITE(8) ((H(I,J),J=1,N2),I=1,MM)
      WRITE(8) ((Z(I,J),J=1,N2),I=1,MM)
      REWIND 8
   82 CONTINUE
C
C        INITIALIZE THE PSI FIELD
C
      IF (K .GE. 2) GO TO 98
      CALL PSIINT(PSIMAX)
   98 CONTINUE
C
C        SOLVE FOR PSI BY ITERATION
C
      CALL STREAM(ITMAXP,FRICT)
      IF (ISTORP .EQ. 0) GO TO 96
      WRITE(9) ((PSI(I,J),J=1,N2),I=1,MM)
      REWIND 9
   96 CONTINUE
```

```
      CALL SPEED
      IF (ISTORV .EQ. 0) GO TO 81
      WRITE(7) ((U(I,J),J=1,N2),I=1,MM)
      WRITE(7) ((V(I,J),J=1,N2),I=1,MM)
      REWIND 7
   81 CONTINUE
   90 CONTINUE
      STOP
```

```
      SUBROUTINE STREAM(ITMAXP,FRICT)
C
C        SUBROUTINE COMPUTES THE STREAMLINES GIVEN THE H AND THETA
C        FIELDS BY A GAUSS-SEIDEL RELAXATION TECHNIQUE BOTH FOR
C        WAVE AND NO WAVE-CURRENT INTERACTION
C
      DIMENSION DNDX(70,20),DNDY(70,20)
      COMMON U(70,20),V(70,20),Z(70,20),SI(70,20),CO(70,20),H(70,20),
     1CG(41,19),S(41,19),HBREAK(41,19),IB(41,19),D(70,20),DDDX(70,20),
     2DDDY(70,20),PSI(70,20),FX(70,20),FY(70,20),W(70,20)
      COMMON/CON/ G,PI,PI2,RAD,EPS,DX,DY,DX2,DY2,T,SIGMA,M,N,MM
      EQUIVALENCE (DNDX(1,1),CG(1,1)),(DNDY(1,1),HBREAK(1,1))
C
C        STATEMENT FUNCTION
C
      UPPSI(I,J)= (-DXSQ*W(I,J) + (1.0-FX(I,J)*DX/2.0)*PSI(I-1,J) +
     1(1.0 + FX(I,J)*DX/2.0)*PSI(I+1,J) + DXSQ*(1.0/DYSQ - FY(I,J)/DY2)*
     2PSI(I,J-1) + DXSQ*(1.0/DYSQ + FY(I,J)/DY2)*PSI(I,J+1))/CON
C        END STATEMENT FUNCTIONS
      MM1=MM-1
      N1=N+1
      N2=N+2
      DXSQ=DX**2
      DYSQ=DY**2
      RDXDY=DXSQ/DYSQ
      CON=2.0*(1.0 + RDXDY)
      EIGHT=1.0/8.0
      SIXTNT=1.0/16.0
      FCON= FRICT*SQRT(2.0*G)/PI
C        CALCULATE DNDX AND DNDY---NOTE THAT K,DKDX AND DKDY ARE
C        NOT STORED DUE TO CORE SPACE PROBLEMS
      DO 315 J=1,N2
      DNDX(1,J)= 0.0
      DNDY(1,J)= 0.0
  315 CONTINUE
      DO 316 I=2,MM
      DO 316 J=2,N1
      JJ=J-1
      DD=D(I,JJ)
      SS=SI(I,J)
      CC=CO(I,J)
      CALL WVNUM(DD,CC,SS,U(I,J),V(I,J),RK,A)
      ARG2= 2.0*RK*DD
      SINH2= SINH(ARG2)
      TT2= TANH(ARG2)
      DUDX= (U(I+1,J)-U(I-1,J))/DX2
      DUDY= (U(I,J+1)-U(I,J-1))/DY2
      DVDX= (V(I+1,J)-V(I-1,J))/DX2
      DVDY= (V(I,J+1)-V(I,J-1))/DY2
      DTDX= (Z(I+1,J)-Z(I-1,J))/DX2
      DTDY= (Z(I,J+1)-Z(I,J-1))/DY2
      FAC= U(I,J)*SS - V(I,J)*CC
      FF= U(I,J)*CC + V(I,J)*SS + 0.5*A*(1.0 + ARG2/SINH2)/RK
      DKDX= RK*(DTDX*FAC - CC*DUDX - SS*DVDX - A*DDDX(I,JJ)/SINH2)/FF
      DKDY= RK*(DTDY*FAC - CC*DUDY - SS*DVDY - A*DDDY(I,JJ)/SINH2)/FF
      FF= (1.0 - ARG2/TT2)/SINH2
      DNDX(I,J)= FF*(RK*DDDX(I,JJ) + DD*DKDX)
      DNDY(I,J)= FF*(RK*DDDY(I,JJ) + DD*DKDY)
  316 CONTINUE
      DO 317 I=2,MM
```

```
      DNDX(I,1)= DNDX(I,N)
      DNDY(I,1)= DNDY(I,N)
      DNDX(I,N2)= DNDX(I,3)
      DNDY(I,N2)= DNDY(I,3)
  317 CONTINUE
C      END CALCULATION FO DNDX AND DNDY
C
C         PRECOMPUTE DFDX/F,  DFDY/F AND W
C
      DO 300 I=2,MM1
      DO 300 J=2,N1
      JJ=J-1
      DD=D(I,JJ)
      DDX=DDDX(I,JJ)
      DDY=DDDY(I,JJ)
      DD2=DD**2
      H1=H(I,J)
      H2=H1**2
      DHDX=(H(I+1,J)-H(I-1,J))/DX2
      DHDY=(H(I,J+1)-H(I,J-1))/DY2
      DHDXY=(H(I+1,J+1)-H(I-1,J+1)-H(I+1,J-1)+H(I-1,J-1))/(DX2*DY2)
      DHDXX=(H(I-1,J)-2.0*H(I,J)+H(I+1,J))/DXSQ
      DHDYY=(H(I,J-1)-2.0*H(I,J)+H(I,J+1))/DYSQ
      DTDX=(Z(I+1,J)-Z(I-1,J))/DX2
      DTDY=(Z(I,J+1)-Z(I,J-1))/DY2
      DTDXY=(Z(I+1,J+1)-Z(I-1,J+1)-Z(I+1,J-1)+Z(I-1,J-1))/(DX2*DY2)
      DTDXX=(Z(I-1,J)-2.0*Z(I,J)+Z(I+1,J))/DYSQ
      DTDYY=(Z(I,J-1)-2.0*Z(I,J)+Z(I,J+1))/DYSQ
      SS=SI(I,J)
      CC=CO(I,J)
      SS2=SS**2
      CC2=CC**2
      SIN2=SIN(2.0*Z(I,J))
      COS2=COS(2.0*Z(I,J))
      CALL KVNUM(DD,CC,SS,U(I,J),V(I,J),RK,A)
      ARG2= 2.0*RK*DD
      SINH2=SINH(ARG2)
      ARG1= RK*DD
      TT=TANH(ARG1)
      TT2= TANH(ARG2)
      DUDX=(U(I+1,J)-U(I-1,J))/DX2
      DUDY=(U(I,J+1)-U(I,J-1))/DY2
      DVDX=(V(I+1,J)-V(I-1,J))/DX2
      DVDY=(V(I,J+1)-V(I,J-1))/DY2
      FAC=U(I,J)*SS - V(I,J)*CC
      FF=U(I,J)*CC + V(I,J)*SS + 0.5*A*(1.0 + ARG2/SINH2)/RK
      DKDX= RK*(DTDX*FAC - CC*DUDX - SS*DVDX - A*DDX/SINH2)/FF
      DKDY= RK*(DTDY*FAC - CC*DUDY - SS*DVDY - A*DDY/SINH2)/FF
C        NOTE-- FX(I,J)=DFDX/F AND FY(I,J)=DFDY/F
      HD=H1*DD
      RK2= RK*2.0
      DKDDX= RK*DDX + DD*DKDX
      DKDDY= RK*DDY + DD*DKDY
      FX(I,J)= (DKDX-RK2*DKDDX/TT2)/RK2 + (DD*DHDX-2.0*H1*DDX)/HD
      FY(I,J)= (DKDY-RK2*DKDDY/TT2)/RK2 + (DD*DHDY-2.0*H1*DDY)/HD
C        NOTE-- FN=N=CG/C
      FN= 0.5*(1.0 + ARG2/SINH2)
      COF1= 2.0*FN - 0.5
      COF2= FN - 0.5
      B= EIGHT*H2
```

```
      FS= FN*SIN2
      FC= FN*COS2
      CCC= 1.0 + CC2
      SSS= 1.0 + SS2
      SX= B*(COF1*CC2 + COF2*SS2)
      SY= B*(COF1*SS2 + COF2*CC2)
      TAU= SIXINT*H2*FN*SIN2
      DSXDX= B*(-FS*DTDX + DNDX(I,J)*CCC) + 2.0*DHDX*SX/H1
      DSXDY= B*(-FS*DTDY + DNDY(I,J)*CCC) + 2.0*DHDY*SX/H1
      DSYDY= B*(FS*DTDY + DNDY(I,J)*SSS) + 2.0*DHDY*SY/H1
      DSYDX= B*(FS*DTDX + DNDX(I,J)*SSS) + 2.0*DHDX*SY/H1
      DTAUX= B*(FC*DTDX + FS*DHDX/H1) + DNDX(I,J)*TAU/FN
      DTAUY= B*(FC*DTDY + FS*DHDY/H1) + DNDY(I,J)*TAU/FN
      DNDXX= (DNDX(I+1,J)-DNDX(I-1,J))/DX2
      DNDYY= (DNDY(I,J+1)-DNDY(I,J-1))/DY2
      DNDXY= (DNDY(I+1,J)-DNDY(I-1,J))/DX2
C        SECOND ORDER DERIVITIVES
      DSXDYX= B*(-FS*DTDXY-2.0*FC*DTDX*DTDY-SIN2*DTDX*DNDY(I,J)-SIN2*
     1DTDY*DNDX(I,J)+CCC*DNDXY) + 0.25*H1*DHDY*(-FS*DTDX+CCC*DNDX(I,J))
     2+ 2.0*DHDX*DSXDY/H1 + 2.0*SX*(DHDXY/H1-DHDX*DHDY/H2)
      DTAUYY= B*(FC*DTDYY-2.0*FS*DTDY**2) + 0.5*FC*H1*DTDY*DHDY + 0.125*
     1FS*(H1*DHDYY+DHDY**2) + B*DNDY(I,J)*(COS2*DTDY+SIN2*DHDY/H1) +
     2DNDY(I,J)*DTAUY/FN + TAU*(DNDYY/FN - (DNDY(I,J)/FN)**2)
      DSYDXY= B*((2.0*FC*DTDY*DTDX+FS*DTDXY) + SIN2*DTDY*DNDX(I,J)+SIN2*
     1DNDY(I,J)*DTDX+SSS*DNDXY) + 0.25*H1*DHDX*(FS*DTDY + SSS*DNDY(I,J))
     2+ 2.0*DHDY*DSYDX/H1 + 2.0*SY*(DHDXY/H1 - DHDY*DHDX/H2)
      DTAUXX= B*(FC*DTDXX-2.0*FS*DTDX**2) + 0.5*FC*H1*DTDX*DHDX + 0.125*
     1FS*(H1*DHDXX+DHDX**2) + B*DNDX(I,J)*(COS2*DTDX + DHDX*SIN2/H1) +
     2DNDX(I,J)*DTAUX/FN + TAU*(DNDXX/FN - (DNDX(I,J)/FN)**2)
      F=FCON*H1*SQRT(RK/SINH2)/DD2
      W(I,J)= G*((DSXDYX + DTAUYY - DSYDXY - DTAUXX)/DD - DDY*(DSXDX +
     1DTAUY)/DD2 + DDX*(DSYDY + DTAUX)/DD2)/F
  300 CONTINUE
      GO TO 360
      WRITE(6,390) ((FX(I,J),J=1,10),I=1,MM)
  390 FORMAT(//10X,*DFDX/F*/,(10G13.5))
      WRITE(6,391) ((FX(I,J),J=11,N2),I=1,MM)
  391 FORMAT(///,(9G13.5))
      WRITE(6,392) ((FY(I,J),J=1,10),I=1,MM)
  392 FORMAT(//10X,*DFDY/F*/,(10G13.5))
      WRITE(6,391) ((FY(I,J),J=11,N2),I=1,MM)
      WRITE(6,393) ((W(I,J),J=1,10),I=1,MM)
  393 FORMAT(//10X,*W(I,J)*/,(10G13.5))
      WRITE(6,391) ((W(I,J),J=11,N2),I=1,MM)
  360 CONTINUE
C
C        PERFORM ITERATION FOR PSI
C
      DO 310 IT=1,ITMAXP
      IFLAG=1
      DO 320 I=2,MM1
      PSINEW=UPPSI(I,2)
      IF (ABS(PSINEW-PSI(I,2)).GT. (EPS*ABS(PSINEW))) IFLAG=0
      PSI(I,2)=PSINEW
      PSI(I,N1)=PSI(I,2)
      PSINEW=UPPSI(I,3)
      IF (ABS(PSINEW-PSI(I,3)).GT. (EPS*ABS(PSINEW))) IFLAG=0
      PSI(I,3)=PSINEW
      PSI(I,N2)=PSI(I,3)
      DO 350 J=4,N
```

```
      PSINEW=UPPSI(I,J)
      IF (ABS(PSINEW-PSI(I,J)) .GT. (EPS*ABS(PSINEW))) IFLAG=0
      PSI(I,J)=PSINEW
  350 CONTINUE
      PSI(I,1)=PSI(I,N)
  320 CONTINUE
      IF (IFLAG .EQ. 1) GO TO 380
  310 CONTINUE
      WRITE(6,330) ITMAXP,EPS,K
  330 FORMAT(//10X,*RELAXATION FOR PSI FAILED AFTER*,I7,3X,*ITERATIONS W
     1ITH A REQUIRED ERROR OF*,F10.6,10X,2HK=,I4)
      WRITE(6,331) ((PSI(I,J),J=1,10),I=1,MM)
  331 FORMAT(/10X,*LAST VALUES OF PSI ARE*/,(10F13.5))
      WRITE(6,332) ((PSI(I,J),J=11,N2),I=1,MM)
  332 FORMAT(///,(9F13.5))
      GO TO 399
  380 WRITE(6,335) IT,EPS
  335 FORMAT(//10X,*RELAXATION FOR PSI CONVERGED AFTER*,I7,3X,*ITERATION
     1S WITH A MAXIMUM ERROR OF*,F10.6)
      WRITE(6,336) ((PSI(I,J),J=1,10),I=1,MM)
  336 FORMAT(/10X,*CONVERGED VALUES OF PSI ARE*/,(10F13.5))
      WRITE(6,332) ((PSI(I,J),J=11,N2),I=1,MM)
  399 RETURN
```

```
      SUBROUTINE ANGLE(ITMAX)
C
C        SUBROUTINE SOLVES FOR THETA BY RELAXATION INCLUDING
C        WAVE-CURRENT INTERACTION
C
      COMMON /CON/G,PI,PI2,RAD,EPS,DX,DY,DX2,DY2,T,SIGMA,M,N
      COMMON U(70,20),V(70,20),Z(70,20),SI(70,20),CO(70,20),H(70,20),
     1CG(41,19),S(41,19),HBREAK(41,19),IB(41,19),D(70,20),DDDX(70,20),
     2DDDY(70,20),PSI(70,20),FX(70,20),FY(70,20),W(70,20)
C
C     PERFORM ITERATION
C
      N1=N+1
      N2=N+2
      M1=M-1
      M2=M-2
      DO 200 IT=1,ITMAX
      IFLAG=1
      DO 210 II=1,M2
      I=M-II
      DO 210 J=2,N1
      CALL NEWANG(I,J,IFLAG)
  210 CONTINUE
      IF (IFLAG .EQ. 1) GO TO 250
  200 CONTINUE
      WRITE(6,220) ITMAX
  220 FORMAT(10X,33HRELAXATION FOR THETA FAILED AFTER,I6,3X,10HITERATION
     1S//)
      WRITE(6,221) ((Z(I,J),J=1,10),I=1,M)
  221 FORMAT(10X,24HLAST VALUES OF THETA ARE//,(10G13.5))
      WRITE(6,222) ((Z(I,J),J=11,N2),I=1,M)
  222 FORMAT(///,(9F13.5))
      CALL EXIT
  250 WRITE(6,251) IT,EPS
  251 FORMAT(1H1,//,10X,33HSOLUTION FOR THETA OBTAINED AFTER,I6,3X,43HIT
     1ERATIONS WITH A MAXIMUM RELATIVE ERROR OF,3X,F10.5)
      WRITE(6,252) ((Z(I,J),J=1,10),I=1,M)
  252 FORMAT(10X,23HSOLUTIONS FOR THETA ARE//,(10F13.5))
      WRITE(6,253) ((Z(I,J),J=11,N2),I=1,M)
  253 FORMAT(///,(9F13.5))
C
C        WRITE THETA IN DEGREES
C
      DO 260 I=1,M
      DO 260 J=1,N2
      Z(I,J)=Z(I,J)*RAD
  260 CONTINUE
      WRITE(6,251) IT,EPS
      WRITE(6,252) ((Z(I,J),J=1,10),I=1,M)
      WRITE(6,253) ((Z(I,J),J=11,N2),I=1,M)
      DO 270 I=1,M
      DO 270 J=1,N2
      Z(I,J)=Z(I,J)/RAD
  270 CONTINUE
      RETURN
```

```
      SUBROUTINE NEWANG(I,J,IFLAG)
C
C         SUBROUTINE COMPUTES THE UPDATED ANGLE THETA FOR THE RELAXATION
C         TECHNIQUE
C
      COMMON U(70,20),V(70,20),Z(70,20),SI(70,20),CO(70,20),H(70,20),
     1CG(41,19),S(41,19),HBREAK(41,19),IB(41,19),D(70,20),DDDX(70,20),
     2DDDY(70,20),PSI(70,20),FX(70,20),FY(70,20),W(70,20)
      COMMON /CON/ G,PI,PI2,RAD,EPS,DX,DY,DX2,DY2,T,SIGMA,M,N
C         STATEMENT FUNCTIONS
C
      C(I,J)=0.25*(CO(I+1,J)+CO(I-1,J)+CO(I,J+1)+CO(I,J-1))+ 0.125*((Z(I
     1+1,J)-Z(I-1,J))*(SI(I+1,J)-SI(I-1,J)) + (Z(I,J+1)-Z(I,J-1))*(SI(I,
     2J+1)-SI(I,J-1)))
      SS(I,J)=0.25*(SI(I+1,J)+SI(I-1,J)+SI(I,J+1)+SI(I,J-1))+0.125*((Z(I
     1+1,J)-Z(I-1,J))*(CO(I-1,J)-CO(I+1,J)) + (Z(I,J+1)-Z(I,J-1))*(CO(I,
     2J-1)-CO(I,J+1)))
      DUDX(I,J)=(U(I+1,J)-U(I-1,J))/DX2
      DUDY(I,J)=(U(I,J+1)-U(I,J-1))/DY2
      DVDX(I,J)=(V(I+1,J)-V(I-1,J))/DX2
      DVDY(I,J)=(V(I,J+1)-V(I,J-1))/DY2
C         NOTE IF CORE IS SUFFICIENT DVDX, DVDY,DUDX AND DUDY CAN BE
C         APRIORI CALCULATED AND ARRAY STORED
      F(I,J)=U(I,J)*COST + V(I,J)*SINT + 0.5*A*A*(1.0 + ARG2/SINH2)/RK
      DKDY(I,J)=(-(COST*DUDY(I,J) + SINT*DVDY(I,J)) - A*DDDY(I,J-1)/
     1SINH2)/FF
      DKDX(I,J)=(-(COST*DUDX(I,J) + SINT*DVDX(I,J)) - A*DDDX(I,J-1)/
     1SINH2)/FF
      FAC(I,J)= U(I,J)*SINT - V(I,J)*COST
C
C         BEGIN CALCULATION
C
      COST= C(I,J)
      SINT= SS(I,J)
      JJ=J-1
      CALL WVNUMB(D(I,JJ),COST,SINT,U(I,J),V(I,J),RK,A)
      ARG2=2.0*RK*D(I,JJ)
      SINH2=SINH(ARG2)
      FF=F(I,J)
      IF (FF .GT. 0.0) GO TO 450
      WRITE(6,451) I,J,D(I,JJ),COST,SINT,U(I,J),V(I,J),RK,A
  451 FORMAT(10X,*FF IS NEGATIVE--OUTPUT I,J,D,COST,SINT,U,V,RK,A*/,
     110X,2I5,7G13.5)
      CALL EXIT
  450 FACT=FAC(I,J)
      DEN1=(SINT-COST*FACT/FF)/DY
      DEN2=(COST+SINT*FACT/FF)/DX
      DEN= DEN1-DEN2
      ZNEW=(COST*DKDY(I,J) - SINT*DKDX(I,J) + Z(I,J-1)*DEN1 - Z(I+1,J)*
     1DEN2)/DEN
      IF (ABS(ZNEW-Z(I,J)) .GT. (EPS*ABS(ZNEW)))  IFLAG=0
      Z(I,J)=ZNEW
      CO(I,J)=COS(Z(I,J))
      SI(I,J)=SIN(Z(I,J))
      IF (J .NE. 2) GO TO 400
      M1=M+1
      Z(I,M1)=Z(I,2)
      CO(I,M1)=CO(I,J)
      SI(I,M1)=SI(I,J)
      GO TO 499
```

```
  400 IF (J .NE. 3) GO TO 401
      N2=N+2
      Z(I,N2)=Z(I,J)
      CO(I,N2)=CO(I,J)
      SI(I,N2)=SI(I,J)
      GO TO 499
  401 IF (J .NE. N) GO TO 402
      Z(I,1)=Z(I,N)
      CO(I,1)=CO(I,N)
      SI(I,1)=SI(I,N)
      GO TO 499
  402 N1=N+1
      IF (J .NE. N1) GO TO 499
      Z(I,2)=Z(I,J)
      CO(I,2)=CO(I,J)
      SI(I,2)=SI(I,J)
  499 RETURN
      END
vv
```

```
      SUBROUTINE WVNUM(D,COST,SINT,U,V,RK,A)
C
C     SUBROUTINE COMPUTES THE WAVE NUMBER K=2*PI/L INCLUDING WAVE-
C     CURRENT INTERACTION
C
      COMMON/CON/G,PI,PI2,RAD,EPS,DX,DY,DX2,DY2,T,SIGMA,M,N
      EPSK=0.001
      RK=PI2/(T*SQRT(G*D))
      DO 100 I=1,50
      A=SIGMA - U*RK*COST - V*RK*SINT
      A2=A**2
      ARG=RK*D
      E1=EXP(ARG)
      E2=1.0/E1
      SECH= 2.0/(E1+E2)
      SECH2=SECH**2
      TT=TANH(ARG)
      FK= G*RK*TT - A2
      FFK= G*(ARG*SECH2 + TT) + 2.0*(U*COST + V*SINT)*A
      RKNEW=RK - FK/FFK
      IF (ABS(RKNEW-RK) .LE. (ABS(EPSK*RKNEW))) GO TO 110
      RK=RKNEW
  100 CONTINUE
      WRITE(6,101)  I,RK,T,D,U,V
  101 FORMAT(///,10X,40HITERATION FOR K FAILED TO CONVERGE AFTER,I6,3X,
     15G13.5)
      CALL EXIT
  110 RK=RKNEW
      A=SIGMA - U*RK*COST - V*RK*SINT
      IF (RK .GT. 0.0) GO TO 120
      WRITE(6,130) D,COST,SINT,U,V,RK,A
  130 FORMAT(10X,*RK IS NEGATIVE--OUTPUT D,COST,SINT,U,V,RK,A*/,
     110X,7G13.5)
      CALL EXIT
  120 RETURN
      END
 vv
```

```
      SUBROUTINE GROUP(I,J,DCGDX,DCGDY,FF)
C
C
C         SUBROUTINE COMPUTES THE GROUP VELOCITY PARAMETERS CG,
C         DCGDX, DCGDY INCLUDING WAVE-CURRENT INTERACTION
C
      COMMON U(70,20),V(70,20),Z(70,20),SI(70,20),CO(70,20),H(70,20),
     1CG(41,19),S(41,19),HBREAK(41,19),IB(41,19),D(70,20),DDDX(70,20),
     2DDDY(70,20),PSI(70,20),FX(70,20),FY(70,20),W(70,20)
      COMMON /CON/ G,PI,PI2,RAD,EPS,DX,DY,DX2,DY2,T,SIGMA,M,N
C              STATEMENT FUNCTIONS
      DUDX(I,J)=(U(I+1,J)-U(I-1,J))/DX2
      DUDY(I,J)=(U(I,J+1)-U(I,J-1))/DY2
      DVDX(I,J)=(V(I+1,J)-V(I-1,J))/DX2
      DVDY(I,J)=(V(I,J+1)-V(I,J-1))/DY2
      DTDX(I,J)= (Z(I+1,J) - Z(I-1,J))/DX2
      DTDY(I,J)= (Z(I,J+1)-Z(I,J-1))/DY2
C         NOTE IF CORE IS SUFFICIENT DVDX, DVDY,DUDX AND DUDY CAN BE
C         APRIORI CALCULATED AND ARRAY STORED
      E(I,J)=U(I,J)*COST + V(I,J)*SINT + 0.5*A*(1.0 + ARG2/SINH2)/RK
      DKDX(I,J)= RK*((U(I,J)*SINT-V(I,J)*COST)*DTDX(I,J) -
     1 (COST*DUDX(I,J) + SINT*DVDX(I,J)) - A*DDDX(I,J-1)/SINH2)/EE
      DKDY(I,J)= RK*((U(I,J)*SINT - V(I,J)*COST)*DTDY(I,J) -
     1 (COST*DUDY(I,J) + SINT*DVDY(I,J)) - A*DDDY(I,J-1)/SINH2)/EE
C              END OF STATEMENT FUNCTIONS
C
C
      JJ=J-1
      DEP=D(I,JJ)
      COST=CO(I,J)
      SINT=SI(I,J)
      CALL WVNUM(DEP,COST,SINT,U(I,J),V(I,J),RK,A)
C         NEXT OPERATIONS COMPUTE THE WAVE BREAKING HEIGHT
      TA=TANH(RK*DEP)
      HBREAK(I,J)=0.12*PI2*TA/RK
C
      COSH1= COSH(RK*DEP)
      SECHSQ= 1.0/(COSH1**2)
      ARG2=2.0*RK*DEP
      SINH2=SINH(ARG2)
      COSH2=COSH(ARG2)
      SINHSQ=SINH2**2
      EE=E(I,J)
      C=SQRT(G*TA/RK)
      FF= 0.5*(1.0 + ARG2/SINH2)
      CG(I,J)= FF*C
      P= C*(SINH2 - ARG2*COSH2)/SINHSQ
      DKDDX= RK*DDDX(I,JJ) + DEP*DKDX(I,J)
      DKDDY= RK*DDDY(I,JJ) + DEP*DKDY(I,J)
      Q= 0.5*G/(C*RK**2)
      DCDX= Q*(RK*SECHSQ*DKDDX - TA*DKDX(I,J))
      DCDY= Q*(RK*SECHSQ*DKDDY - TA*DKDY(I,J))
      DCGDX= P*DKDDX + FF*DCDX
      DCGDY= P*DKDDY + FF*DCDY
      GO TO 1001
      WRITE(6,1000) I,J,RK,A,C,DCDX,DCDY,DKDDX,DKDDY
 1000 FORMAT(10X,2I5,7G15.5)
 1001 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DEPTH(I,J)
C
C        SUBROUTINE COMPUTES THE WATER DEPTH AND ITS SPACIAL DERIVITIVES
C
      COMMON U(70,20),V(70,20),Z(70,20),SI(70,20),CO(70,20),H(70,20),
     1CG(41,19),S(41,19),HBREAK(41,19),IB(41,19),D(70,20),DDDX(70,20),
     2DDDY(70,20),PSI(70,20),FX(70,20),FY(70,20),W(70,20)
      COMMON /CON/ G,PI,PI2,RAD,EPS,DX,DY,DX2,DY2,T,SIGMA,M,N
      AM=0.025
      THIRD=1.0/3.0
      B=(20.0**THIRD)/3.0
      FLAMDA=80.0
      A=20.0
      ALPHA=30.0
      ALPHA=ALPHA/RAD
      X=DX*FLOAT(I-1)
      Y=DY*FLOAT(J-1)
      TALPHA=TAN(ALPHA)
      ARG=(Y-X*TALPHA)*PI/FLAMDA
      S=SIN(ARG)
      S9=S**9
      S10=S9*S
      ARGE=-X**THIRD/B
      EE=EXP(ARGE)
      CON=10.0*AM*A*PI*X/FLAMDA
      C=COS(ARG)
      D(I,J)=B*X*(1.0 + A*EE*S10)
      DDDY(I,J)=CON*EE*S9*C
      DDDX(I,J)=AM - DDDY(I,J)*TALPHA + AM*A*EE*S10*(1.0 + ARGE/3.0)
```

```
      SUBROUTINE NEWHT(I,J,IFLAG)
C
C        SUBROUTINE COMPUTES THE UPDATED WAVE HEIGHT AND CHECKS FOR
C        BREAKING
      COMMON U(70,20),V(70,20),Z(70,20),SI(70,20),CO(70,20),H(70,20),
     1CG(41,19),S(41,19),HBREAK(41,19),IB(41,19),D(70,20),DDDX(70,20),
     2DDDY(70,20),PSI(70,20),FX(70,20),FY(70,20),W(70,20)
      COMMON /CON/ G,PI,PI2,RAD,EPS,DX,DY,DX2,DY2,T,SIGMA,M,N
C
C     COMPUTE NEW WAVE HEIGHT
C
      IB(I,J)=1
      N1=N+1
      N2=N+2
      CC1=(V(I,J) + CG(I,J)*SI(I,J))/DY
      CC2=(U(I,J) + CG(I,J)*CO(I,J))/DX
      HNEW=(CC1*H(I,J-1) - CC2*H(I+1,J))/(CC1 - CC2 - S(I,J)/2.0)
      IF (HNEW .LE. HBREAK(I,J)) GO TO 850
      HNEW= HBREAK(I,J)
      IB(I,J)=0
  850 CONTINUE
      IF (ABS(HNEW-H(I,J)) .GT. (EPS*ABS(HNEW)))  IFLAG=0
      H(I,J)=HNEW
      IF (J .NE. 2) GO TO 800
      H(I,N1)=H(I,J)
      IB(I,N1)=IB(I,J)
      GO TO 899
  800 IF (J .NE. 3) GO TO 801
      H(I,N2)=H(I,J)
      IB(I,N2)=IB(I,J)
      GO TO 899
  801 IF (J .NE. N) GO TO 802
      H(I,1)=H(I,J)
      IB(I,1)=IB(I,J)
      GO TO 899
  802 IF (J .NE. N1) GO TO 899
      H(I,2)=H(I,J)
      IB(I,2)=IB(I,J)
  899 RETURN
      END
vv
```

```
      SUBROUTINE SNELL(THETA0,HH,AM)
C
C        SUBROUTINE COMPUTES THE SNELL'S LAW WAVE HEIGHT, DEPTH AND
C        ORTHAGONAL ANGULAR DIRECTION OUTSIDE OF THE PERIODIC BEACH
C
      COMMON/CON/ G,PI,PI2,RAD,EPS,DX,DY,DX2,DY2,T,SIGMA,M,N,MM
      COMMON U(70,20),V(70,20),Z(70,20),SI(70,20),CO(70,20),H(70,20),
     1CG(41,19),S(41,19),HBREAK(41,19),IB(41,19),D(70,20),DDDX(70,20),
     2DDDY(70,20),PSI(70,20),FX(70,20),FY(70,20),W(70,20)
      MMIN=M+1
      N2=N+2
      DO 600 I=MMIN,MM
      DD=AM*DX*FLOAT(I-1)
      CALL WVNUM(DD,0.0,0.0,0.0,0.0,RK,A)
      AA=RK*DD
      ANG=ASIN(SIN(THETA0)*TANH(AA))
      ANG=PI - ANG
      ARG=2.0*AA
      SHOAL=SQRT(1.0/(TANH(AA)*(1.0+ARG/SINH(ARG))))
      REF=SQRT(COS(THETA0)/COS(ANG))
      WVHT=HH*SHOAL*REF
      SS=SIN(ANG)
      CC=COS(ANG)
      DO 600 J=1,N2
      D(I,J)=DD
      H(I,J)=WVHT
      Z(I,J)=ANG
      SI(I,J)=SS
      CO(I,J)=CC
      DDDX(I,J)=AM
      DDDY(I,J)=0.0
  600 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PSIINT(PSIMAX)
C
C        SUBROUTINE INITIALIZES PSI
C
      COMMON/CON/ G,PI,PI2,RAD,EPS,DX,DY,DX2,DY2,T,SIGMA,M,N,MM
      COMMON U(70,20),V(70,20),Z(70,20),SI(70,20),CO(70,20),H(70,20),
     1CG(41,19),S(41,19),HBREAK(41,19),IB(41,19),D(70,20),DDDX(70,20),
     2DDDY(70,20),PSI(70,20),FX(70,20),FY(70,20),W(70,20)
      N2=N+2
      MM1=MM-1
      FM=FLOAT(MM1)
      DO 650 J=1,N2
      PSI(1,J)=0.0
      PSI(MM,J)=0.0
  650 CONTINUE
      DO 660 I=2,MM1
      PSII=PSIMAX*SIN(PI*FLOAT(I-1)/FM)
      DO 660 J=1,N2
      PSI(I,J)=PSII
  660 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE HEIGHT(ITMAX)
C        SUBROUTINE COMPUTES THE WAVE HEIGHT BY RELAXATION INCLUDING
C        EFFECTS OF WAVE-CURRENT INTERACTION
C
      COMMON U(70,20),V(70,20),Z(70,20),SI(70,20),CO(70,20),H(70,20),
     1CG(41,19),S(41,19),HBREAK(41,19),IB(41,19),D(70,20),DDDX(70,20),
     2DDDY(70,20),PSI(70,20),FX(70,20),FY(70,20),W(70,20)
      COMMON /CON/ G,PI,PI2,RAD,EPS,DX,DY,DX2,DY2,T,SIGMA,M,N
      M1=M-1
      N1=N+1
C        COMPUTE VALUES OF S(I,J)
      DO 500 I=2,M1
      DO 500 J=2,N1
      CALL GROUP(I,J,DCGDX,DCGDY,FF)
      DUDX=(U(I+1,J)-U(I-1,J))/DX2
      DUDY=(U(I,J+1)-U(I,J-1))/DY2
      DVDX=(V(I+1,J)-V(I-1,J))/DX2
      DVDY=(V(I,J+1)-V(I,J-1))/DY2
      DZDX=(Z(I+1,J)-Z(I-1,J))/DX2
      DZDY=(Z(I,J+1)-Z(I,J-1))/DY2
      SS2= SI(I,J)**2
      CC2= CO(I,J)**2
      SIGXX= (2.0*FF - 0.5)*CC2 + (FF - 0.5)*SS2
      SIGYY= (2.0*FF - 0.5)*SS2 + (FF - 0.5)*CC2
      TAUXY= FF*SI(I,J)*CO(I,J)
      S(I,J)=  CG(I,J)*(SI(I,J)*DZDX - CO(I,J)*DZDY) - (DUDX + DVDY)  -
     1(CO(I,J)*DCGDX + SI(I,J)*DCGDY) - (SIGXX*DUDX + TAUXY*DUDY +
     2TAUXY*DVDX + SIGYY*DVDY)
      GO TO 595
      WRITE(6,590) I,J,CG(I,J),DCGDX,DCGDY,SIGXX,SIGYY,TAUXY,S(I,J)
  590 FORMAT(1X,2I5,7G15.5)
  595 CONTINUE
  500 CONTINUE
      N2=N+2
      M2=M-2
C
C        PERFORM ITERATION FOR THE WAVE HEIGHT FIELD
C
      DO 510 II=1,M2
      I=M-II
      DO 580 IT=1,ITMAX
      IFLAG=1
      DO 520 J=2,N1
      CALL NEWHT(I,J,IFLAG)
  520 CONTINUE
      IF (IFLAG .EQ. 1) GO TO 570
  580 CONTINUE
      WRITE(6,540) I,IT
  540 FORMAT(10X,49HRELAXATION FOR THE WAVE HEIGHT FAILED TO CONVERGE/,
     110X,9HON ROW I=,I5,5X,5HAFTER,I6,3X,10HITERATIONS)
      WRITE(6,541) (H(I,J),J=1,N2)
  541 FORMAT(10X,20HLAST VALUES OF H ARE/,(10G13.5))
      CALL EXIT
  570 WRITE(6,542) I,IT
  542 FORMAT(10X,43HRELAXATION FOR WAVE HEIGHT CONVERGED ON ROW,I6,3X,
     15HAFTER,I6,10HITERATIONS/)
  510 CONTINUE
      RETURN
      END
VV
```

```
      SUBROUTINE SPEED
C
C        SUBROUTINE COMPUTES THE U AND V  COMPONENTS OF THE CIRCULATION
C        VELOCITIES IN THE PSI FIELD
C
      COMMON U(70,20),V(70,20),Z(70,20),ST(70,20),CO(70,20),H(70,20),
     1CG(41,19),S(41,19),HBREAX(41,19),IB(41,19),D(70,20),DDDX(70,20),
     2DDDY(70,20),PSI(70,20),FX(70,20),FY(70,20),W(70,20)
      COMMON/CON/ G,PI,PI2,RAD,EPS,DX,DY,DX2,DY2,T,SIGMA,M,N,MM
      MM1=MM-1
      N2=N+2
      N1=N+1
      DO 750 J=2,N1
      U(1,J)=0.0
      V(I,J)= 0.0
      U(MM,J)=0.0
      V(MM,J)=-PSI(MM1,J)/(DX*D(MM,J-1))
      DO 750 I=2,MM1
      U(I,J)= -(PSI(I,J+1) - PSI(I,J-1))/(D(I,J-1)*DY2)
      V(I,J)= (PSI(I+1,J) - PSI(I-1,J))/(D(I,J-1)*DX2)
  750 CONTINUE
      DO 760 I=1,MM
      U(I,1)= U(I,N)
      U(I,N2)= U(I,3)
      V(I,1)= V(I,N)
      V(I,N2)=V(I,3)
  760 CONTINUE
      WRITE(6,751) ((U(I,J),J=1,10),I=1,MM)
  751 FORMAT(//10X,*U VELOCITIES*/,(10F13.5))
      WRITE(6,752) ((U(I,J),J=11,N2),I=1,MM)
  752 FORMAT(///,(9F13.5))
      WRITE(6,753) ((V(I,J),J=1,10),I=1,MM)
  753 FORMAT(//10X,*V VELOCITIES*/,(10F13.5))
      WRITE(6,752) ((V(I,J),J=11,N2),I=1,MM)
      RETURN
      END
```

```
      FUNCTION SINH(A)
      E=EXP(A)
      SINH= (E - 1.0/E)/2.0
      RETURN
      END
```

```
      FUNCTION COSH(A)
      E=EXP(A)
      COSH= (E + 1.0/E)/2.0
      RETURN
      END
```

41	17	300	50	400	70	0	1
1	0	1					
150.00	0.50	5.00	5.00	4.00	0.001	0.025	0.01
50.00							

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C        PROGRAM COMPUTES H(X) FROM THE DIFFERENTIAL EQUATION
C        FOR ENERGY FOR A WAVE SYSTEM PROPAGATING WITH A CURRENT
C        BY RUNGA-KUTTA
C
      DIMENSION Y(1),YP(1)
      COMMON CEL
      READ(5,1) NDX,T,DX
    1 FORMAT(I10,2F10.2)
      WRITE(6,10) NDX,T,DX
   10 FORMAT(10X,*NDX,T,DX*/,I10,2F10.2)
      G=9.80621
      PI= 3.1415926536
      PI2= PI*2.0
      CEL= G*T/PI2
C        INITIALIZE Y AND YP
      Y(1)= 0.774750
      X= 40.0
      INDX= 0
      CALL RUNGS(X,DX,1,Y,YP,INDX)
      WRITE(6,20) X,Y(1),YP(1)
   20 FORMAT(10X,*X=*,F5.1,5X,*H=*,G15.5,5X,*DHDX=*,G15.5)
      DO 100 I=1,NDX
      CALL RUNGS(X,DX,1,Y,YP,INDX)
      WRITE(6,21) X,Y(1),YP(1)
   21 FORMAT(12X,F5.1,7X,G15.5,10X,G15.5)
  100 CONTINUE
      STOP
      END
```

```
      SUBROUTINE  RUNGS (X,H,N,Y,YPRIME,INDEX)
      DIMENSION Y(1),YPRIME(1),Z(1),W1(1),W2(1),W3(1),W4(1)
CRUNGS - RUNGE-KUTTA SOLUTION OF SET OF FIRST ORDER O.D.E.  FORTRAN 99
C     DIMENSIONS MUST BE SET FOR EACH PROGRAM
C     X   INDEPENDENT VARIABLE
C     H   INCREMENT DELTA X, MAY BE CHANGED IN VALUE
C     N   NUMBER OF EQUATIONS
C     Y   DEPENDENT VARIABLE BLOCK     ONE DIMENSIONAL ARRAY
C     YPRIME  DERIVATIVE BLOCK  ONE DIMENSIONAL ARRAY
C     THE PROGRAMMER MUST SUPPLY INITIAL VALUES OF  Y(1) TO Y(N)
C     INDEX  IS A VARIABLE WHICH SHOULD BE SET TO ZERO BEFORE EACH
C     INITIAL ENTRY TO THE SUBROUTINE, I.E., TO SOLVE A DIFFERENT
C     SET OF EQUATIONS OR TO START WITH NEW INITIAL CONDITIONS.
C     THE PROGRAMMER MUST WRITE A SUBROUTINE CALLED DERIVE WHICH COM-
C     PUTES THE DERIVATIVES AND STORES THEM
C     THE ARGUMENT LIST IS   SUBROUTINE DERIVE(X,N,Y,YPRIME)
      IF (INDEX) 5,5,1
    1 DO 2 I=1,N
      W1(I)=H*YPRIME(I)
    2 Z(I)=Y(I)+(W1(I)*.5)
      A=X+H/2.
      CALL DERIVE(A,N,Z,YPRIME)
      DO 3 I=1,N
      W2(I)=H*YPRIME(I)
    3 Z(I)=Y(I)+.5*W2(I)
      A=X+H/2.
      CALL DERIVE(A,N,Z,YPRIME)
      DO 4 I=1,N
      W3(I)=H*YPRIME(I)
    4 Z(I)=Y(I)+W3(I)
      A=X+H
      CALL DERIVE (A,N,Z,YPRIME)
      DO 7 I=1,N
      W4(I)=H*YPRIME(I)
    7 Y(I)=Y(I)+(((2.*(W2(I)+W3(I)))+W1(I)+W4(I))/6.)
      X=X+H
      CALL DERIVE (X,N,Y,YPRIME)
      GO TO 6
    5 CALL DERIVE (X,N,Y,YPRIME)
      INDEX=1
    6 RETURN
      END
```

```
      SUBROUTINE DERIVE(X,N,Y,YP)
C
C        SUBROUTINE COMPUTES DERIVITIVES DYDX WHERE Y=FCT(X)
C
      DIMENSION Y(1),YP(1)
      COMMON CEL
      U= -9.0 + 0.2*X
      DUDX= 0.2
      RAD= SQRT(1.0 - 4.0*U/CEL)
      C= CEL*(0.5 + 0.5*RAD)
      CG= 0.5*C
      DCDX= -DUDX/RAD
      DCGDX= 0.5*DCDX
      YP(1)= Y(1)*(-1.5*DUDX + DCGDX)/(2.0*(U-CG))
      RETURN
      END

       200       4.00      -0.10

VV
```