Negative Domain Wall Resistance in Ferromagnets

R. P. van Gorkom,* Arne Brataas,† and Gerrit E. W. Bauer

Department of Applied Physics and Delft Institute of Microelectronics and Submicrontechnology, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands

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The electrical resistance of a diffusive ferromagnet with magnetic domain walls is studied theoretically, taking into account the spatial dependence of the magnetization. The semiclassical domain wall resistance is found to be either negative or positive depending on the difference between the spin-dependent scattering lifetimes. The predictions can be tested experimentally by transport studies in doped ferromagnets.

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A domain wall (DW) is the region between two ferromagnetic domains in which the direction of the magnetization rotates. A number of experiments have been conducted which show either an *increase* [1–3] or a *decrease* [4–8] of the resistance due to DWs compared to the resistance of a single-domain ferromagnet. These experiments have been done on thin films, structured thin films, and membranes in the diffusive transport regime, where the electron mean free path is shorter than the typical system size.

In the diffusive limit, Cabrera and Falicov [9] calculated an *increase* of the resistance caused by the backreflection of electrons by the domain wall. The reflection probability was found to be exponentially small in the ratio of the DW width to the Fermi wavelength. An *increase* of the semiclassical resistance has also been predicted by Tatara and Fukuyama [10] by linear response calculations assuming spin-independent relaxation times. Levy and Zhang [11] obtained the DW resistance from a Boltzmann equation. They showed that spin-dependent relaxation times can enhance this *positive* DW resistance, depending on the ratio of relaxation times of the majority and minority spin electrons. Brataas *et al.* [12] calculated the domain wall resistance generalizing the approach of Tatara and Fukuyama to include spin-dependent lifetimes with qualitatively similar results to Levy and Zhang.

The only intrinsic mechanism which explains a *decrease* of the resistance has been proposed by Tatara and Fukuyama [10], viz., the destruction of electron weak localization by the dephasing, caused by the domain wall, decreases the resistance. However, experimentally, the negative domain wall resistance persists up to relatively high temperatures [5,6], where localization does not play a role. Kent *et al.* [6] explain the negative DW resistance by an extrinsic effect: reduced surface scattering.

It is the purpose of this Letter to show that the semiclassical DW resistance of diffusive ferromagnets can be *negative* as well as *positive* when the electronic structure of the domain wall is taken into account semiclassically. The experimental results [1–8] may thus originate from the same intrinsic semiclassical effect.

Let us first describe the elementary physics of electron transport in a domain wall. The Drude resistivity of a single-domain ferromagnet reads, in the two-band Stoner model [13],

$$\rho = \frac{m}{e^2} \frac{1}{n_+\tau_+ + n_-\tau_-}, \tag{1}$$

where m is the mass of the electron, e is the charge of an electron, n_+ (n_-) is the density of spin-up (spin-down) electrons, and τ_+ (τ_-) is the scattering relaxation time for the spin-up (spin-down) electrons which at low temperatures depends on the (spin-dependent) impurity potential and (spin-dependent) density of states. A redistribution of the electrons between the spin-up and spin-down bands (i.e., a change in magnetization) modifies the resistivity when $\tau_+ \neq \tau_-$. With $n = n_+ + n_-$, $n_\pm = n_0^\pm + \delta n_\pm$, and $\delta n_+ = -\delta n_-$, the change in resistivity is found to be

$$\delta\rho \approx -\rho_0^2 \frac{e^2}{m} \delta n_+ (\tau_+ - \tau_-), \tag{2}$$

where ρ_0 is the resistivity of a single-domain ferromagnet. We see that a modified magnetization causes the resistivity to either increase or decrease, depending on the relaxation times. The relaxation times in a ferromagnet depend on the types of impurities that are present in the material [14]. The sign and magnitude of the resistivity change are therefore impurity specific. In the following we show that the magnetization is modified in a domain wall and contributes to the DW resistance on top of the DW scattering mechanisms discussed in literature [9–12].

We approximate the domain wall by a local constant rotation along the z direction [11] $\partial_z \phi(z) = \pi/\lambda_w = a_0$, where λ_w is the length of the domain wall, as indicated in Fig. 1. This is allowed when the domain wall is much wider than the Fermi wavelength. The DW resistance can in this limit be calculated by interpreting the DW as a finite slice of a so-called spin-spiral ferromagnet. The total resistance of a ferromagnet is determined by simply adding the resistivities of the DWs and the domains. The relative change in resistance due to the domain walls is $(R - R_0)/R_0 = [L\rho_0 + \lambda_w(\rho_0 + \delta\rho) - (L + \lambda_w)\rho_0]/[(L + \lambda_w)\rho_0] = \lambda_w \delta\rho/[(L + \lambda_w)\rho_0]$, where L is the length of the domain.

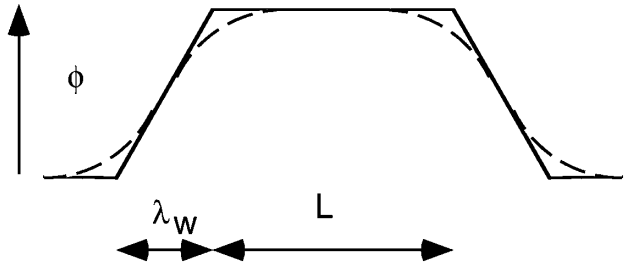

FIG. 1. A schematic picture of the magnetization angle as a function of position. The dashed line sketches a typical magnetization and the solid line sketches the piecewise spin-spiral approximation.

The magnetization is a result of the exchange interaction. We follow the common procedure to express the exchange energy by the continuum limit of the mean-field Heisenberg model,

$$E_{\rm ex} = \int d\mathbf{r} \int d\mathbf{r}' \, K(|\mathbf{r} - \mathbf{r}'|) \langle \mathbf{m}(\mathbf{r}') \rangle \cdot \mathbf{m}(\mathbf{r}) , \quad (3)$$

where $K(|\mathbf{r} - \mathbf{r}'|)$ is the exchange interaction between electrons in a volume element $\mathbf{r}$ and $\mathbf{r}'$, $\mathbf{m}(\mathbf{r}) = \mu_B \mathbf{s}(\mathbf{r})$ is the magnetization operator, and $\langle \mathbf{m}(\mathbf{r}) \rangle$ denotes the thermal ensemble average of the magnetization. The range $\lambda_{\rm ex}$ of the exchange interaction is of the order of the Fermi wavelength. In a DW, neighboring spins are canted. Hence the exchange energy is reduced. Using $\langle \mathbf{m}(\mathbf{r}') \rangle \cdot \mathbf{m}(\mathbf{r}) = \cos[\pi(z - z')/\lambda_w] \langle \mathbf{m}(\mathbf{r}) \rangle \cdot \mathbf{m}(\mathbf{r})$ and taking into account that $\lambda_w \gg \lambda_{\rm ex}$, the exchange energy becomes $E_{\rm ex} = \int d\mathbf{r} \, \mathbf{H}_{\rm ex}(\mathbf{r}) \cdot \mathbf{m}(\mathbf{r})$, where the exchange field is

$$\mathbf{H}_{\rm ex}(\mathbf{r}) \approx K \left[1 - \frac{1}{2} \left(\frac{\pi \lambda_{\rm ex}}{\lambda_w} \right)^2 \right] \langle \mathbf{m}(\mathbf{r}) \rangle , \quad (4)$$

and K is the total exchange integral. The exchange field, and therefore the splitting, decreases with decreasing λ_w and increasing $\lambda_{\rm ex}$. The effect of the reduced magnetization on the resistivity seems small, since the domain wall is much wider than the Fermi wavelength. However, we will show that the effect of the reduced magnetization on the DW resistance is of the same order as that of other mechanisms studied previously [10–12]. Expressing the exchange splitting as $2\Delta(\mathbf{r}) = 2Js(\mathbf{r})$, we have $J = \mu_B K[1 - (\pi \lambda_{\rm ex}/\lambda_w)^2/2]$ [the thermal average spin-density is $s(r) = |\langle s(\mathbf{r}) \rangle|$]. The relative change in the effective coupling constant J due to the domain wall can be written as $\delta J/J = -\kappa 2E_w/(\epsilon_F^+ + \epsilon_F^-)$, where $E_w = \hbar^2 a_0^2/(2m)$ is an energy parameter of the rotation of the magnetization, and $\epsilon_F^\pm = \hbar^2 (k_F^\pm)^2/(2m)$ is related to the (spin-dependent) Fermi wave vectors. Using an estimate for $\lambda_{\rm ex} \approx \lambda_F/2 = \pi/k_F$, we find $\kappa = \pi^2/2$. A longer range of the exchange interaction (3) will have even larger effects. Note that Δ decreases even faster than $\delta J/J$ because, in the self-consistent mean-field approximation, $|s|$ is also reduced in the DW, as will be shown below. Other effects that may change the magnetization and therefore also the resistivity are magnetostriction and the internal dipolar magnetic field. The former can change the exchange integral due to a change in lattice constant, caused by the domain wall. The latter directly affects the splitting. However, we expect these effects to be smaller than those discussed here.

The electronic structure of a two-band ferromagnet with noncollinear magnetization can be found from the Stoner Hamiltonian,

$$H = -\frac{\hbar^2}{2m} \nabla^2 + \mu_B \mathbf{H}_{\rm ex}(\mathbf{r}) \cdot \boldsymbol{\sigma} , \quad (5)$$

where the three components of $\boldsymbol{\sigma}$ are the Pauli spin matrices (σ_x, σ_y, and σ_z) and $\mathbf{H}_{\rm ex}(\mathbf{r})$ is the exchange field as described above. We disregard the spin-orbit interaction and the Lorentz force due to the internal magnetization, because the DW magnetoresistance can experimentally be separated from the anisotropic magnetoresistance (AMR) and the ordinary magnetoresistance (OMR) [6].

We solve the eigenvalue problem for the Hamiltonian equation (5) by a local gauge transformation, assuming translational symmetry in the x and y directions and introducing the Fourier transform of the gradient of the magnetization direction $\partial_z \phi(z) = \sum_q \exp(iqz) a_q$ [10,12], where $\phi(z)$ is the angle of magnetization in the rotation plane. After the gauge transformation, the Hamiltonian (5) becomes $\tilde{H} = H_0 + V$, where $H_0 = \sum_{\mathbf{k}\sigma} (\epsilon_{\mathbf{k}\sigma} - \mu) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$ ($\epsilon_{\mathbf{k}\sigma} = \hbar^2 k^2/2m - \sigma\Delta$) and the interaction V with the DW is specified in Refs. [10,12].

We proceed by calculating $\delta\rho$, the difference in resistivity of the spin spiral compared to the single-domain state. In the spin spiral, $\partial_z \phi$ is constant everywhere. The Hamiltonian (5) is diagonalized in spin space by $\mathbf{u}_\pm = \mathcal{N}_\pm [1, i(1 \mp \sqrt{1 + \alpha^2})/\alpha]^T$, where $\mathcal{N}_\pm$ is a normalization constant, $\alpha = k_z a_0/p^2$, and $p^2 = 2m\Delta/\hbar^2$ [12]. The eigenvalues are

$$E_k^\pm = \frac{\hbar^2}{2m} (k^2 + a_0^2 \mp \sqrt{k_z^2 a_0^2 + p^4}) . \quad (6)$$

Because of the rotation of the direction of the spin-quantization axis,

$$u_\pm^\dagger \sigma_z u_\pm = \pm \frac{1}{\sqrt{1 + \alpha^2}} , \quad (7)$$

the spin density in the direction of the local magnetization becomes

$$s_\pm = \pm \frac{1}{V} \sum_k \frac{1}{\sqrt{1 + \alpha^2}} f(E_k^\pm - \mu) , \quad (8)$$

whereas the electron densities of the spin-up and spin-down eigenstates remain as $n_\pm = (1/V) \sum_k f(E_k^\pm - \mu)$. The total spin density is $s = s_+ + s_-$.

At $T = 0$ K we find

$$n_\pm = \frac{1}{6\pi^2} (k_1^\pm)^3 \left(1 \pm \frac{E_w}{4\Delta} \right) \quad (9)$$

and

$$s_\pm = \pm\left(n_\pm - \frac{1}{30\pi^2}(k_1^\pm)^5\frac{\hbar^2 E_w}{\Delta^2 4m}\right), \qquad (10)$$

where $k_1^\pm = (2m/\hbar^2)^{1/2}(\mu \pm \Delta - \frac{1}{4}E_w)^{1/2}$. Without spin rotation ($a_0 \to 0$), n and s reduce to the familiar form $n_0^\pm = \pm s_0^\pm = (1/6\pi^2)(2m/\hbar^2)^{3/2}(\epsilon_F^\pm)^{3/2}$, where $\epsilon_F^\pm = \mu \pm \Delta$.

The numbers of spin-up and spin-down electrons have to be calculated self-consistently, since the effective spin splitting depends on the spin densities. A wide domain wall only weakly modifies the electronic structure. Therefore the reduced magnetization can be calculated by perturbation theory. Charge neutrality is taken into account as in Ref. [12] by introducing a shift in the chemical potential $\mu = \mu_0 + \delta\mu$. The spin densities are denoted as $s_\pm = \pm n_0^\pm + \delta s_\pm$, and the splitting of the bands becomes $\Delta = \Delta_0 + \delta\Delta$, where μ_0 is the single-domain chemical potential, $\delta\mu$ is the change in the chemical potential caused by the DW, δs_+ (δs_-) is the change in spin density in the spin-up (spin-down) band, and $\Delta_0 = (n_0^+ - n_0^-)J_0$. The exchange splitting is modified as

$$\delta\Delta = (|\delta s_+| - |\delta s_-|)J_0 + (n_0^+ - n_0^-)\delta J, \qquad (11)$$

where the first term is due to the reduced spin density in the spin spiral, and the second term reflects the reduced exchange interaction. We obtain

$$\delta n_\pm = N_\pm\left(\delta\mu \pm \delta\Delta - \frac{1}{4}E_w \pm \frac{E_w}{6\Delta}\epsilon_F^\pm\right) \qquad (12)$$

and

$$\delta s_\pm = \pm\left(\delta n_\pm - \frac{E_w}{15\Delta^2}N_\pm(\epsilon_F^\pm)^2\right), \qquad (13)$$

where N_+ (N_-) is the density of states of the spin-up (spin-down) band at the Fermi energy. With $n_\pm = 2N_\pm\epsilon_F^\pm/3$, which holds for parabolic bands, and Eqs. (11)–(13), we can find the electron densities, which substituted into Eq. (2) yield

$$\delta\rho = \frac{e^2\rho_0^2}{m}\frac{E_w}{J_0 D\Delta_0}(\tau_+ - \tau_-) \times\left(\frac{\kappa\Delta_0^2}{\epsilon_F^+ + \epsilon_F^-} - \frac{\epsilon_F^+ + \epsilon_F^-}{24} + \frac{J_0}{\Delta_0}\frac{N_+(\epsilon_F^+)^2 - N_-(\epsilon_F^-)^2}{30}\right), \qquad (14)$$

which is our main result. The first term is directly due to the reduced exchange interaction, the second term reflects the change in dispersion of the spin-up and spin-down bands [last term in Eq. (6)], and the third term is due to the reduced spin density [Eq. (7)]. The first term is almost always bigger than the other two terms. Equation (14) shows that the change in resistivity due to the reduced magnetization is of the same order as the effect of the spin-flip scattering [12], both scaling linearly with E_w. The dimensionless denominator $D = (N_+ + N_-)/(4N_+N_-J_0) - 1$ appears as a result of the self-consistency and is always positive (see below). D vanishes when the spontaneous magnetization disappears with decreasing J_0. In that case, the nondegenerate perturbation is not valid anymore, because $\delta\rho \to \infty$. When $\tau_+ < \tau_-$, the DW resistance is *negative.*

In order to estimate the importance of the effect, we introduce dimensionless variables: $\gamma \equiv k_+/k_-$ is the ratio of the Fermi wave vectors and $\beta \equiv \tau_+/\tau_-$ is the ratio of relaxation times. The dimensionless denominator then becomes $D = (\gamma + \gamma^{-1} - 2)/3 > 0$. Microscopic theory [12] reveals that additional spin-flip terms, which always increase the resistance, have to be added to the result derived here [15]. Figure 2 shows $\delta\rho/\rho_0$, as calculated from Eq. (14) (fine dashed line), due to the spin-flip scattering (taken from Ref. [12]) (coarse dashed line) and the sum of both effects (solid line) as a function of the ratio of relaxation times β, for two different ratios of the Fermi wavelengths γ. $E_w/\epsilon_F = 4.3 \times 10^{-4}$ for Co has been estimated from $\lambda_w = 15$ nm and $k_F = 1$ Å^{-1}. The exchange length is equal to $\lambda_{ex} = 2.8$ Å. In Fig. 2 we can see that the DW resistivity is between -5% and 10% for the parameters chosen, i.e., depending on the value of the exchange integral, the impurities, and the band structure of the material. Smaller spin splittings and larger asymmetries in the relaxation times increase the domain wall resistance.

Our results agree with the experimental finding that the DW resistance can be negative [4–8]. We have shown that the sign of the DW resistance depends on the difference of scattering relaxation times, which can be positive or negative. This difference is to a large extent determined by the kind of impurities present in the sample, e.g., theoretical calculations for Cr impurities

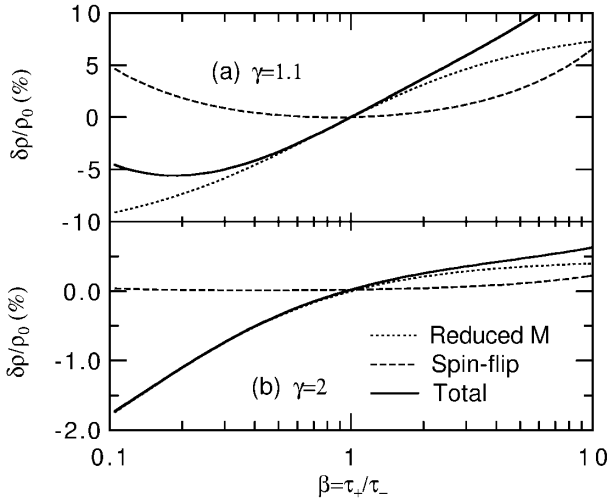

FIG. 2. The relative change in resistivity due to the reduced magnetization calculated here, the DW spin-flip scattering [12], and the sum of both effects [15] as a function of the ratio of relaxation times. (a) The ratio of Fermi wave vectors $\gamma = 1.1$ and (b) $\gamma = 2$.

in Fe give $\beta = 0.11$, i.e., $\tau_+ - \tau_- < 0$ whereas for Cu impurities $\beta = 3.68$, i.e., $\tau_+ - \tau_- > 0$ [16]. Also, for Ni, the DW resistance can have both signs, since $0.2 < \beta < 30$ [16]. Experimentally the parameter β is not available, but the ratio of the resistivities has been determined [14], yielding for Co $0.1 < \rho_+/\rho_- < 5$, Ni $0.05 < \rho_+/\rho_- < 8$, and Fe $0.1 < \rho_+/\rho_- < 10$, where $\rho_+/\rho_- = \tau_- n_- m_+/(n_+ \tau_+ m_-)$. (Only τ_+ and τ_- depend on the type of impurities [17].) Therefore, a reasonable agreement exists between experimental and theoretical values of the scattering relaxation times. This means that our predictions for the DW resistance can be experimentally tested by intentionally doping samples with different impurities and measuring the DW resistance as a function of type and concentration.

For realistic band structures the ballistic contribution to the domain wall resistivity from the spin-flip terms [10–12] is enhanced due to the near degeneracy of the different bands at the Fermi energy [18]. Similarly, we expect an enhancement of the present effect, because the exchange splitting is more sensitive to the gradient of the magnetization when the bands are nearly degenerate.

In conclusion, we have shown that a negative as well as a positive domain wall resistance is possible in the semiclassical regime, due to spin-dependent relaxation times and the spatial dependence of the magnetization.

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*Electronic address: gorkom@dimes.tudelft.nl

†Also at Philips Research Laboratories, Prof. Holstlaan 4, 5656 AA Eindhoven, The Netherlands.

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