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Negative Magnetoresistance of Ferromagnetic Metals due to Spin Fluctuations

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The magnetoresistance of ferromagnetic metals due to spin fluctuation through *s-d* interaction is calculated for the whole temperature region and for the weak magnetic field. The ladder approximation (RPA) is used in the calculation of the spin Green's functions and it is shown that the magnetoresistance has a logarithmic dependence on the temperature and the magnetic field both at low temperatures and near the transition temperature.

§ 1. Introduction

In a previous paper[1),*)] a simple theory of the magnetoresistance in magnetic metals due to electron scattering by localized spins has been developed in the frame-work of the molecular field approximation. It has been shown in I that the magnetoresistance is positive in the antiferromagnetic state, while it is negative in the ferro- and paramagnetic states. Several experimental tendencies of rare-earth metals have been qualitatively explained by this theory. However, the theory of I has severel unsatisfactory features characteristic of the molecular field approximation. In particular the low-temperature behaviour of the magnetoresistance proportional to $\exp(-\mathrm{const}/T)$ and the lack of the spacial fluctuation of the localized spins are important failings characteristic of the molecular field approximation. The latter point is particularly important for the temparature dependence of magnetoresistance near the critical point. It is the purpose of this paper to improve these failings of the results of I in the case of the ferromagnetic metals.

In the ferromagnetic metals a negative character of the magnetoresistance due to the electron-spin scattering arises from the following origin: The magnetic field increases the effective field acting on the localized spins and suppresses the fluctuation of spins in space and time, which leads to the decrease of the resistivity. The negative character of the magnetoresistance, therefore, will remain unchanged in the RPA calculation, as will be seen in this paper.

The negative magnetoresistance is also shown in dilute alloys with paramagnetic impurities. Yoshida[2)] first explained the negative magnetoresistance of the magnetic alloys. He showed in the molecular field approximation that when impurity spins formed antiferromagnetic ordering the negative magnetoresistance arises from the

*) Hereafter this paper will be referred to as I.

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cross term due to the magnetic and non-magnetic impurity scattering. Recently Williams[3] has also calculated the negative magnetoresistance of ferromagnetic dilute alloys in the spin wave approximation. Several works have been done on the magnetoresistance of dilute paramagnetic alloys concerning the Kondo problem.[4),5)]

In this paper we shall study the magnetoresistance of a ferromagnetic metal in which conduction electrons interact with the localized spins through the *s-d* interaction. The localized spins of this model, therefore, interact with each other through the conduction electrons, i.e., through the Ruderman-Kittel-Kasuya-Yoshida type interaction, and show the ferromagnetism below a transition temperature.

In § 2 we set up the model and study it in the Hartree-Fock approximation.

In § 3 we develop the perturbational treatment for the Green functions of the electron and the localized spin. We derive the explicit form of the Green functions in an R.P.A. approximation.

In § 4 using the Nakano-Kubo-Mori approximate formula we calculate the resistivity and magnetoresistivity due to electron scattering by the localized spins in the first Born approximation.

In § 5 we obtain the explicit form of the magnetic resistance for the whole temperature region. Section 6 will be devoted to the summary and discussion.

In the Appendix we calculate the magnetoresistance near T_c using the scaling hypothesis for the spin correlation function.

§ 2. Hamiltonian and Hartree-Fock approximation

We start with a simplified model of a ferromagnetic metal in an external field, in which conduction electrons interact with localized spins through the *s-d* interaction and there exists a uniform field acting upon the electron spins and the localized spins. In this model the magnetic field is assumed to be small and the cyclotron motion of the conduction electron is neglected. The Hamiltonian of the system is represented by

$$\mathcal{H}=\mathcal{H}_e+\mathcal{H}_{sd}+\mathcal{H}_{ex} \tag{2·1}$$

with

$$\mathcal{H}_e=\sum_{q\sigma}\xi_q C^*_{q\sigma}C_{q\sigma}\,, \tag{2·2}$$

$$\mathcal{H}_{sd}=-\frac{J}{N}\sum_q \boldsymbol{S}_\sigma(\boldsymbol{q})\cdot\boldsymbol{S}(-\boldsymbol{q}), \tag{2·3}$$

$$\mathcal{H}_{ex}=-H[\mu_e S_\sigma^z(0)+\mu_S S^z(0)] \tag{2·4}$$

and

$$\xi_q=\varepsilon_q-\mu\,, \tag{2·5}$$

$$S^\alpha(\boldsymbol{q})=\sum_l e^{-i\boldsymbol{q}\boldsymbol{R}_l}S_l^\alpha,\quad (\alpha=x,y,z) \tag{2·6}$$

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where $C_{q\sigma}^*(C_{q\sigma})$ is the creation (annihilation) operator of a conduction electron, ε_q and μ are the kinetic energy and the chemical potential of the conduction electron, S_l^α is the localized spin operator at the l-th lattice site and N is the total number of lattice sites. $\mu_e/2$ and μ_s are the magnetic moment of the conduction electron and the localized spin, respectively. $J(\boldsymbol{q})$ is the exchange interaction between the conduction electron and the localized spins. The direction of the external field is taken to the positive z direction. The components of $S_c(\boldsymbol{q})$ are explicitly given by

$$S_c^+(\boldsymbol{q}) = \sum_k C_{k+q\uparrow}^* C_{k\downarrow}, \tag{2·7}$$

$$S_c^-(\boldsymbol{q}) = \sum_k C_{k+q\downarrow}^* C_{k\uparrow}, \tag{2·8}$$

$$S_c^z(\boldsymbol{q}) = \frac{1}{2}\sum_k \sum_{\sigma=\pm} \sigma C_{k+q\sigma}^* C_{k\sigma}. \tag{2·9}$$

We introduce parameters m and M, which correspond to the magnetization of the conduction electron and the localized spin system, as follows:

$$\langle S^z(\boldsymbol{q})\rangle = N\delta_{q0}M, \quad \langle S_c^z(\boldsymbol{q})\rangle = N\delta_{q0}m \tag{2·10}$$

and rewrite $\mathcal{H}_{sd}$ in the following form:

$$\mathcal{H}_{sd} = JMmN - JMS_c^z(0) - JmS^z(0) - \frac{J}{N}\sum_q \boldsymbol{S}_c'(\boldsymbol{q})\cdot\boldsymbol{S}'(-\boldsymbol{q}), \tag{2·11}$$

where $a' = a - \langle a\rangle$. Using this transformation one can rewrite $\mathcal{H}$ as

$$\mathcal{H} = \mathcal{H}_{\rm HF} + \mathcal{H}', \tag{2·12}$$

$$\mathcal{H}_{\rm HF} = JmMN + \sum_q \sum_{\sigma=\pm} \xi_{q\sigma} C_{q\sigma}^* C_{q\sigma} - y\sum_l S_l^z, \tag{2·13}$$

$$\mathcal{H}' = -\frac{J}{N}\sum_q \boldsymbol{S}'_c(\boldsymbol{q})\cdot\boldsymbol{S}'(-\boldsymbol{q}) \tag{2·14}$$

with

$$y = \mu_s H + mJ, \qquad \xi_{q\sigma} = \xi_q - \frac{\sigma}{2}y_e,$$
$$y_e = \mu_e H + JM. \tag{2·15}$$

In the above equations $\mathcal{H}_{\rm HF}$ is the Hamiltonian in the Hartree-Fock approximation, and we take it as the zeroth-order Hamiltonian and the remainder part $\mathcal{H}'$ as the perturbation. In the Hartree-Fock approximation, the self-consistent equations are obtained from (2·13) in the form

$$M = b(\beta y), \tag{2·16}$$

$$m = \frac{1}{2N}\sum_k [f(\xi_{k\uparrow}) - f(\xi_{k\downarrow})], \tag{2·17}$$

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where

$$b(x)=\left(S+\frac{1}{2}\right)\coth\left[\left(S+\frac{1}{2}\right)x\right]-\frac{1}{2}\coth\frac{x}{2}, \tag{2·18}$$

$$f(x)=\frac{1}{e^{\beta x}+1}. \tag{2·19}$$

Equations (2·15), (2·16) and (2·17) are self-consistent equations to determine M and m. Since y_e is very small compared with the Fermi energy ε_F, we can expand Eq. (2·17) for m in terms of the small parameter

$$\frac{y_e}{\varepsilon_F}=\frac{\mu_e H+JM}{\varepsilon_F}\ll 1. \tag{2·20}$$

The self-consistent equation for M becomes

$$y=\mu_{\text{eff}}H+V_{\text{eff}}(0)M, \tag{2·21}$$

$$M=b(\beta y), \tag{2·22}$$

where

$$\mu_{\text{eff}}=\mu_S+\frac{V_{\text{eff}}(0)}{J}\mu_e, \tag{2·23}$$

$$V_{\text{eff}}(\boldsymbol{q}, i\zeta_n)=\frac{J^2}{2N}\chi(\boldsymbol{q}, i\zeta_n), \qquad V_{\text{eff}}(\boldsymbol{q})\equiv V_{\text{eff}}(\boldsymbol{q}, 0), \tag{2·24}$$

$$\chi(\boldsymbol{q}, i\zeta_n)=-\sum_{\boldsymbol{k}}\frac{f(\xi_{\boldsymbol{k}})-f(\xi_{\boldsymbol{k}+\boldsymbol{q}})}{i\zeta_n+\xi_{\boldsymbol{k}}-\xi_{\boldsymbol{k}+\boldsymbol{q}}} \tag{2·25}$$

and m is determined from M as

$$m=\frac{V_{\text{eff}}(0)}{J^2}\mu_{\text{eff}}H+\frac{V_{\text{eff}}(0)}{J}M. \tag{2·26}$$

In the above the effective interaction between localized spins $V_{\text{eff}}(\boldsymbol{q})$ is the usual R-K-K-Y interaction, and the self-consistent equations (2·21) and (2·22) are the same as the self-consistent equation of the spin system represented by a Heisenberg Hamiltonian with exchange interaction $V_{\text{eff}}(\boldsymbol{q})$.

The Curie temperature is given by

$$T_c=\frac{S(S+1)}{3}V_{\text{eff}}(0) \tag{2·27}$$

in the present approximation.

§ 3. Green's function and RPA approximation

3·1 *Green's function and perturbational expansion*

In this section we perform the formal expansion of Green's functions of the

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electrons and the localized spins in terms of $\mathcal{H}'$ following the method of the cumulant expansion extended by Brout et al.[5] and Larkin et al.[6]

The thermal Green function of the electrons and the localized spins are defined in the usual way:

$$\mathcal{G}_\sigma(\boldsymbol{q}, i\zeta_n) = \frac{1}{2}\int_{-\beta}^{\beta} d\tau e^{i\zeta_n\tau}\{-\langle T_\tau C_{q\sigma}^*(\tau) C_{q\sigma}(0)\rangle\}, \tag{3·1}$$

$$\mathcal{D}^{\alpha\gamma}(\boldsymbol{q}, i\zeta_n) = \frac{1}{2}\int_{-\beta}^{\beta} d\tau e^{i\zeta_n\tau}\langle T_\tau S'^\alpha(\boldsymbol{q}, \tau) S'^\gamma(-\boldsymbol{q}, 0)\rangle, \tag{3·2}$$

$$(\alpha, \gamma = +, -, z)$$

where

$$A(\tau) = e^{\mathcal{H}\tau} A e^{-\mathcal{H}\tau}, \tag{3·3}$$

$$\langle\cdots\rangle = (\mathrm{Sp} e^{-\beta\mathcal{H}})^{-1} \mathrm{Sp}(e^{\beta\mathcal{H}}\cdots), \tag{3·4}$$

$$S^\pm(\boldsymbol{q}) = S^x(\boldsymbol{q}) \pm iS^y(\boldsymbol{q}), \tag{3·5}$$

$$i\zeta_n = i\pi T n \tag{3·6}$$

and T_τ is the time ordering operator, n is an odd integer for $\mathcal{G}_\sigma$ and an even integer for $\mathcal{D}^{\alpha\gamma}$.

Using the Hamiltonian (2·12) and taking $\mathcal{H}_{\mathrm{HF}}$ as an unperturbed Hamiltonian the Green functions (3·1) and (3·2) can be expanded in terms of $\mathcal{H}'$. The Green function in the zeroth order is obtained as follows:

$$\mathcal{G}_\sigma^{(0)}(\boldsymbol{q}, i\zeta_n) = \frac{1}{i\zeta_n - \xi_{q\sigma}}, \tag{3·7}$$

$$\mathcal{D}^{-+(0)}(\boldsymbol{q}, i\zeta_n) = \frac{2NM}{i\zeta_n + y}, \tag{3·8}$$

$$\mathcal{D}^{zz(0)}(\boldsymbol{q}, i\zeta_n) = N\beta\delta_{n0} b'(\beta y), \tag{3·9}$$

where $b'(x) = db(x)/dx$. Each term in the expansion can be transformed into the product of the electron part and the spin part, both of which can be averaged independently. These terms are further expanded by using the Bloch-de Dominicis theorem for the electron part and the cumulant expansion method for the spin part.

The cumulant average $\langle\cdots\rangle_c$ of the spin operators are defined formally by the relation[5),7)]

$$\begin{aligned}&\langle T_\tau \exp[\xi_1 S^+(\boldsymbol{q}_1, \tau_1) + \xi_2 S^-(\boldsymbol{q}_2, \tau_2) + \xi_3 S'^z(\boldsymbol{q}_3, \tau_3)]\rangle_0 \\ &\quad = \exp\langle T_\tau \exp[\xi_1 S^+(\boldsymbol{q}_1, \tau_1) + \xi_2 S^-(\boldsymbol{q}_2, \tau_2) + \xi_3 S'^z(\boldsymbol{q}_3, \tau_3)]\rangle_c,\end{aligned} \tag{3·10}$$

where ξ_1, ξ_2 and ξ_3 are arbitrary parameters. The $\langle\cdots\rangle_0$ is defined as

$$\langle\cdots\rangle_0 = (\mathrm{Sp}\exp(-\beta y \sum_l S_l^z))^{-1} (\mathrm{Sp}\exp(-\beta y \sum_l S_l^z)\cdots). \tag{3·11}$$

As the localized spins of different sites are statistically independent in the Har-

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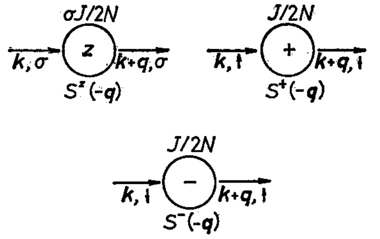
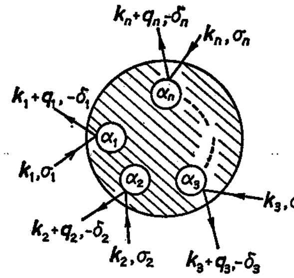


(a)

Fig. 1 (a). The bare vertex parts of the *s-d* interaction.



(b)

Fig 1 (b). A graph of the spin-cumulant function given by Eq. (3·13).

tree-Fock approximation as is seen from (2·13) and (3·11), the cumulant average which contains the spins of different sites are zero according to the property of of the cumulant average. This leads to the crystal momentum conservation at the spin vertex block as is seen in Eq. (3·13) below.

The resulting expansion of the Green function is expressed graphycally.[5),6)] Each graph is constructed from the free-electron line and the spin vertex blocks which are connected to each other at the bare vertex shown in Fig. 1(a). The spin vertex block is expressed as in Fig. 1(b) and is defined analytically as

$$\Gamma^{\alpha_1\alpha_2\cdots\alpha_n}(\boldsymbol{q}_1\boldsymbol{q}_2\cdots\boldsymbol{q}_n, i\zeta_1 i\zeta_2\cdots i\zeta_n)$$
$$=\frac{1}{2^n\beta^{n-1}}\int_{-\beta}^{\beta}d\tau_1\cdots d\tau_n\langle T_\tau S'^{\alpha_1}(\boldsymbol{q}_1,\tau_1)S'^{\alpha_2}(\boldsymbol{q}_2,\tau_2)\cdots S'^{\alpha_n}(\boldsymbol{q}_n,\tau_n)\rangle_c\,. \quad (3\cdot12)$$

Using the property of the cumulant average, (3·12) can be written as

$$\Gamma^{\alpha_1\alpha_2\cdots\alpha_n}(\boldsymbol{q}_1\boldsymbol{q}_2\cdots\boldsymbol{q}_n, i\zeta_1 i\zeta_2\cdots i\zeta_n)$$
$$=N\sum_{\boldsymbol{G}}\delta(\boldsymbol{q}_1+\boldsymbol{q}_2+\cdots+\boldsymbol{q}_n+\boldsymbol{G})\Gamma^{\alpha_1\alpha_2\cdots\alpha_n}(i\zeta_1 i\zeta_2\cdots i\zeta_n) \quad (3\cdot13)$$

with

$$\Gamma^{\alpha_1\alpha_2\cdots\alpha_n}(i\zeta_1 i\zeta_2\cdots i\zeta_n)=\frac{1}{2^n\beta^{n-1}}\int_{-\beta}^{\beta}d\tau_1 d\tau_2\cdots d\tau_n$$
$$\times e^{i\zeta_1\tau_1+i\zeta_2\tau_2+\cdots+i\zeta_n\tau_n}\langle T_\tau S'^{\alpha_1}(\tau_1)S'^{\alpha_2}(\tau_2)\cdots S'^{\alpha_n}(\tau_n)\rangle_c\,, \quad (3\cdot14)$$

where $\boldsymbol{G}$ is the reciprocal lattice vector. In Eq. (3·14) the cumulant average is taken at one site (i.e., by the Hamiltonian $\mathcal{H}=-yS^z$). In our treatment the umklapp process at the spin vertex block is not taken into account and we set $\boldsymbol{G}=0$ in (3·13).

Dyson's equations are written in the following form:

$$\mathcal{G}_\sigma(\boldsymbol{q}, i\zeta_n)=\frac{1}{\mathcal{G}_\sigma^{(0)}(\boldsymbol{q}, i\zeta_n)^{-1}-\Sigma_\sigma(\boldsymbol{q}, i\zeta_n)}\,, \quad (3\cdot15)$$

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$$\mathscr{D}^{\alpha\gamma}(\boldsymbol{q}, i\zeta_n) = \frac{1}{\Pi^{\alpha\gamma}(\boldsymbol{q}, i\zeta_n)^{-1} - (J/2N)^2 \chi^{\alpha\gamma}(\boldsymbol{q}, i\zeta_n)}, \tag{3·16}$$

where $\Sigma_\sigma(\boldsymbol{q}, i\zeta_n)$ is the irreducible electron selfenergy part, and $\chi^{\alpha\gamma}(\boldsymbol{q}, i\zeta_n)$ is the electron spin propagator which cannot be separated into two parts by cutting a part of each graph which starts from $S^\gamma(\boldsymbol{q})$ and ends at $S^\alpha(\boldsymbol{q})$. $\Pi^{\alpha\gamma}(\boldsymbol{q}, i\zeta_n)$ is defined by

$$\Pi^{\alpha\gamma}(\boldsymbol{q}_1, \boldsymbol{q}_2 : i\zeta_1, i\zeta_2) = \delta_{\boldsymbol{q}_1+\boldsymbol{q}_2,0}\delta_{\zeta_1+\zeta_2,0}\Pi^{\alpha\gamma}(\boldsymbol{q}, i\zeta_1) \tag{3·17}$$

and $\Pi^{\alpha\gamma}(\boldsymbol{q}_1\boldsymbol{q}_2, i\zeta_1 i\zeta_2)$ is the sum of diagrams starting from $S^\gamma(\boldsymbol{q}_2)$ and ending at $S^\alpha(\boldsymbol{q}_1)$ and cannot be separated into two parts by cutting a line which starts from $S_c^\gamma(\boldsymbol{q})$ and ends at $S_c^\alpha(\boldsymbol{q})$.

3·2 *R.P.A. approximation*

The simplest correction for the Green's functions to the Hartree-Fock approximation is obtained by using the lowest graph for $\Sigma_\sigma(\boldsymbol{q}, i\zeta_n)$, $\Pi^{\alpha\gamma}(\boldsymbol{q}, i\zeta_n)$ and $\chi^{\alpha\gamma}(\boldsymbol{q}, i\zeta_n)$. The lowest term $\Pi^{\alpha\gamma(0)}(\boldsymbol{q}, i\zeta_n)$ of $\Pi^{\alpha\gamma}(\boldsymbol{q}, i\zeta_n)$ is the two-particle spin vertex block $\Gamma^{\alpha\gamma}$. From Eqs. (3·2), (3·13) and (3·17) we have

$$\Pi^{\alpha\gamma(0)}(\boldsymbol{q}, i\zeta_n) = \mathscr{D}^{\alpha\gamma(0)}(\boldsymbol{q}, i\zeta_n). \tag{3·18}$$

For the lowest order of $\chi^{\alpha\gamma}(\boldsymbol{q}, i\zeta_n)$, we obtain

$$\chi^{-+(0)}(\boldsymbol{q}, i\zeta_n) = -\sum_{\boldsymbol{k}} \frac{f(\xi_{\boldsymbol{k}\uparrow}) - f(\xi_{\boldsymbol{k}+\boldsymbol{q}\downarrow})}{i\zeta_n + \xi_{\boldsymbol{k}\uparrow} - \xi_{\boldsymbol{k}+\boldsymbol{q}\downarrow}}, \tag{3·19}$$

$$\chi^{zz(0)}(\boldsymbol{q}, i\zeta_n) = -\sum_{\boldsymbol{k}} \sum_{\sigma=\pm} \frac{f(\xi_{\boldsymbol{k}\sigma}) - f(\xi_{\boldsymbol{k}+\boldsymbol{q}\sigma})}{i\zeta_n + \xi_{\boldsymbol{k}\sigma} - \xi_{\boldsymbol{k}+\boldsymbol{q}\sigma}}. \tag{3·20}$$

Inserting Eqs. (3·18), (3·19) and (3·20) into Eq. (3·16), we have

$$\mathscr{D}^{-+}(\boldsymbol{q}, i\zeta_n) = \frac{2NM}{i\zeta_n + y - (MJ^2/2N)\chi^{-+(0)}(\boldsymbol{q}, i\zeta_n)} \tag{3·21}$$

and

$$\mathscr{D}^{zz}(\boldsymbol{q}, i\zeta_n) = \frac{N\delta_{n0}\beta b'(\beta y)}{1 - (J^2/4N)\chi^{zz(0)}(\boldsymbol{q}, 0)\beta b'(\beta y)}. \tag{3·22}$$

One of the self-consistent ways to determine the parameter m and M is to minimize the free energy with respect to m and M, which is calculated from the R.P.A. diagram.[6] Here we use the iterative method to determine m and M, assuming that the correction to the zeroth term is small, and the solution of the Hartree-Fock approximation is used for m and M appeared in the Green function.

Inserting Eq. (2·21) for y into Eq. (3·21) we have

$$\mathscr{D}^{-+}(\boldsymbol{q}, i\zeta_n) = \frac{2NM}{i\zeta_n + \mu_s H + mJ - (MJ^2/2N)\chi^{-+(0)}(\boldsymbol{q}, i\zeta_n)}. \tag{3·23}$$

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If we neglect the terms higher than $(\mu_S H/\varepsilon_F)^3$ or $(\mu_e H/\varepsilon_F)^3$, Eqs. (3·23) and (3·24) reduce to

$$\mathcal{D}^{-+}(\boldsymbol{q}, i\zeta_n) = \frac{2NM}{i\zeta_n + \mu_{\text{eff}}H + M[V_{\text{eff}}(0,0) - V_{\text{eff}}(\boldsymbol{q}, i\zeta_n)]} \tag{3·24}$$

and

$$\mathcal{D}^{zz}(\boldsymbol{q}, i\zeta_n) = \frac{N\delta_{n0}\beta b'(\beta y)}{1 - \beta b'(\beta y) V_{\text{eff}}(\boldsymbol{q}, 0)}, \tag{3·25}$$

where $V_{\text{eff}}(\boldsymbol{q}, i\zeta_n)$ is defined by (2·24) and (2·25). The expression (3·24) shows that the localized spins interact indirectly through the interaction $V_{\text{eff}}(\boldsymbol{q}, i\zeta_n)$, i.e., through the Ruderman-Kittel-Kasuya-Yoshida type interaction. The expression (3·23) with $M=S$ has been obtained by several authors,[8)~11)] and the nature of the spin collective mode is studied in some details.[10)] The energy of this collective mode is determined by the pole of $D^{-+}(\boldsymbol{q}, i\zeta_n)$, i.e.,

$$-\omega + \mu_{\text{eff}}H + M[V_{\text{eff}}(0,0) - V_{\text{eff}}(\boldsymbol{q}, \omega + i0)] = 0\,. \tag{3·26}$$

If the ω dependence of $V_{\text{eff}}(\boldsymbol{q}, \omega)$ is taken into account, there are two branches in the spin-wave spectrum, i.e., the optical mode and the acoustical mode, and also there appears a dip in the acoustical spin-wave mode in the high-energy region near the continuum of the Stoner excitation.[10)]

In this paper we are concerned with the effect of spin fluctuation with the approximation $V_{\text{eff}}(\boldsymbol{q}, \omega) \simeq V_{\text{eff}}(\boldsymbol{q}, 0) \equiv V_{\text{eff}}(\boldsymbol{q})$. Inclusion of ω dependence of $V_{\text{eff}}(\boldsymbol{q}, \omega)$ will have only a small correction except in the temperature region much higher than the Curie temperature in which the contribution of high-frequency spin fluctuation becomes appreciable. In this approximation the spin-wave spectrum has the well-known form

$$\omega_{\boldsymbol{q}} = \mu_{\text{eff}}H + M[V_{\text{eff}}(0) - V_{\text{eff}}(\boldsymbol{q})]\,. \tag{3·27}$$

Performing the integration given in Eq. (2·25), one has

$$\omega_{\boldsymbol{q}} = \mu_{\text{eff}}H + \frac{3M}{16}\frac{N_e}{N}\frac{J^2}{\mu}\left\{1 - \frac{4p_{\text{F}}^2 - q^2}{4p_{\text{F}}q}\ln\left|\frac{2p_{\text{F}}+q}{2p_{\text{F}}-q}\right|\right\}. \tag{3·28}$$

For small q such as $q/(2p_{\text{F}}) \ll 1$, $\omega_{\boldsymbol{q}}$ becomes

$$\omega_{\boldsymbol{q}} = \mu_{\text{eff}}H + cq^2 \tag{3·29}$$

with

$$c = \frac{M}{8p_{\text{F}}^2}V_{\text{eff}}(0) \quad \text{and} \quad V_{\text{eff}}(0) = \frac{3}{8}\frac{J^2}{\mu}\frac{N_e}{N}\,. \tag{3·30}$$

§ 4. The resistivity due to spin fluctuation

In this section we calculate the resistivity resulting from the scattering of

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conduction electrons by the fluctuation of the localized spins. For this purpose we adopt the hydrodynamical approximation of Kubo's formula[7),12),13)]

$$\rho^{\alpha\gamma}=\mathrm{Re}\lim_{\delta\to+0}\frac{\int_0^\infty dte^{-t\delta}\langle\dot{\mathcal{J}}^\alpha(t),\dot{\mathcal{J}}^\gamma(0)\rangle}{\langle\mathcal{J}^\alpha,\mathcal{J}^\gamma\rangle^2},$$

$$(\alpha,\gamma=x,y,z) \qquad (4\cdot1)$$

where

$$\mathcal{J}^\alpha=\frac{e}{\sqrt{V}}\sum_q\sum_{\sigma=\pm}\frac{\partial\varepsilon_q}{\partial q^\alpha}C_{q\sigma}^*C_{q\sigma}, \qquad (4\cdot2)$$

$$\langle A(t),B\rangle=\int_0^\beta d\lambda\langle e^{i\mathcal{H}(t-i\lambda)}Ae^{-i\mathcal{H}(t-i\lambda)}B^*\rangle, \qquad (4\cdot3)$$

$$\langle C\rangle=(\mathrm{Sp}e^{-\beta\mathcal{H}})^{-1}(\mathrm{Sp}e^{-\beta\mathcal{H}}C), \qquad (4\cdot4)$$

$$\dot{\mathcal{J}}^\alpha=\frac{1}{i}[\mathcal{J}^\alpha,\mathcal{H}], \qquad (4\cdot5)$$

and A, B and C represent arbitrary operators and B^* is the hermite conjugate of B, and V is the volume of the system. There are various ways of deriving Eq. (4·1). One way is to regard the charge flow as a hydrodynamical mode, and to consider the damping of this flow to be caused by the ramdom force acting on the flow. The rate of this damping is determined by the correlation of this random force as in the case of Brownian motion of a particle suspended in a liquid.[13)] Expression (4·1) corresponds to the constant relaxation-time approximation of the exact Kubo formula at least in the case of the first Born approximation and leads to the well-known results in the case of the resistivity due to phonons and impurities.[12)] It should be noted that in Eq. (4·1) the difference of the Fermi surfaces or the numbers of the up-spin electrons and the down-spin electrons is not taken into account. As we are concerned with the resistivity in the first Born approximation, such effects are of higher order of (J/ε_F). The difference of Fermi surfaces of up and down spin electrons becomes important when the spin-independent scattering is present as in the alloys with paramagnetic impurities.[2)]

For the denominator of (4·1), we have

$$\langle\mathcal{J}^\alpha,\mathcal{J}^\gamma\rangle=\frac{1}{V}\sum_q\sum_{\sigma=\pm}\frac{\partial\varepsilon_q}{\partial q^\alpha}\frac{\partial\varepsilon_q}{\partial q^\gamma}\left(-\frac{\partial}{\partial\varepsilon_q}\langle C_{q\sigma}^*C_{q\sigma}\rangle\right) \qquad (4\cdot6)$$

from the current sum rule. Using the result of the Hartree-Fock approximation for $\langle C_{q\sigma}^*C_{q\sigma}\rangle$ and the effective mass approximation for ε_q, (4·6) reduces to

$$\langle\mathcal{J}^\alpha,\mathcal{J}^\gamma\rangle=\delta_{\alpha\gamma}\frac{n_e}{m^*}, \qquad (4\cdot7)$$

where $\varepsilon_q=q^2/2m^*$, and n_e is the number of conduction electrons per unit volume.

For the calculation of the numerator of Eq. (4·1), it is convenient to em-

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ploy the thermal Green's-function method used in § 3. The diagram chosen here is the simplest one corresponding to the Born approximation to the scattering process of conduction electrons. The numerator $P^{\alpha\gamma}$ of Eq. (4·1) is obtained from the thermal Green function $Q^{\alpha\gamma}(i\omega_n)$ by the following procedure:

$$P^{\alpha\gamma}=\mathrm{Re}\left[-i\frac{\partial}{\partial\omega}Q^{\alpha\gamma}(i\omega_n)_{i\omega_n\to\omega+i\delta}\right]_{\substack{\omega=0,\\ \delta\to+0}}, \tag{4·8}$$

where

$$P^{\alpha\gamma}\equiv\mathrm{Re}\lim_{\delta\to+0}\int_0^\infty dte^{-t\delta}\langle\mathcal{J}^\alpha(t),\mathcal{J}^\gamma\rangle \tag{4·9}$$

and

$$Q^{\alpha\gamma}(i\omega_n)\equiv\frac{1}{2}\int_{-\beta}^{\beta}d\tau e^{i\omega_n\tau}\langle T_\tau\mathcal{J}^\alpha(\tau)\mathcal{J}^{\gamma *}\rangle\equiv\langle\mathcal{J}^\alpha(\tau)\mathcal{J}^{\gamma *}\rangle_{i\omega_n} \tag{4·10}$$

with

$$\mathcal{J}^\alpha(\tau)=e^{\tau\mathcal{H}}\mathcal{J}^\alpha e^{-\tau\mathcal{H}}. \tag{4·11}$$

From Eqs. (4·5) and (2·1) we have

$$\mathcal{J}^\alpha=\frac{1}{i\sqrt{V}}\frac{e}{m^*}\frac{J}{N}\sum_q q^\alpha S_\sigma(\boldsymbol{q})\cdot\boldsymbol{S}(-\boldsymbol{q}). \tag{4·12}$$

From Eqs. (4·10) and (4·12), $Q^{\alpha\gamma}(i\omega_n)$ has the form

$$Q^{\alpha\gamma}(i\omega_n)=\frac{1}{V}\left(\frac{e}{m^*}\right)^2\left(\frac{J}{N}\right)^2\sum_q\sum_{q'}q^\alpha q'^\gamma$$

$$\times\langle[S_\sigma(\boldsymbol{q})\cdot\boldsymbol{S}(-\boldsymbol{q})](\tau)S_\sigma(\boldsymbol{q}')\cdot\boldsymbol{S}(-\boldsymbol{q}')\rangle_{i\omega_n}. \tag{4·13}$$

We can expand $Q^{\alpha\gamma}(i\omega_n)$ in the same way as in § 3 using the cumulant expansion method and taking $\mathcal{H}_{\mathrm{HF}}$ as the zeroth Hamiltonian and $\mathcal{H}'$ as the perturbation. The simplest diagram for $Q^{\alpha\gamma}(i\omega_n)$ is shown in Fig. 2. The real line represents the electron Green function given by Eqs. (3·24) and (3·25). The terms $Q^{\alpha\gamma}_{-+}(i\omega_n)$ and $Q^{\alpha\gamma}_{+-}(i\omega_n)$ represented by (a) and (b) in Fig. 3 correspond to the scattering of conduction electron due to transverse spin fluctuation and $Q^{\alpha\gamma}_{zz}(i\omega_n)$ represented by (c) in Fig. 3 corresponds to the scattering due to the longitudinal spin fluctuation. If we replace the spin line in Figs. 2 (a), (b) and (c) by a phonon line or an impurity line the resulting resistivity gives the Bloch-Grüneisen law for the phonon case or the well-known form of residual resis-

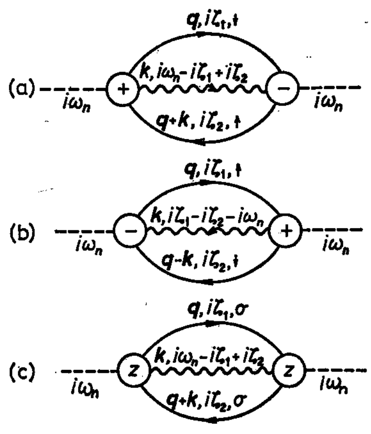

Fig. 2. The first-order diagrams of $Q^{\alpha\gamma}(i\omega_n)$ given by Eq. (4·10).

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tivity for the impurity case.[12])

In the present paper we study the resistivity due to the scattering of electrons by the acoustical spin-wave mode and neglect the dependence of effective inter-spin interaction. Then the wavy line in (a) and (b) in Fig. 2 represents the renormalized Green function

$$\mathscr{D}^{-+}(\boldsymbol{q}, i\zeta_n) = \frac{2NM}{i\zeta_n + \omega_{\boldsymbol{q}}} \tag{4·14}$$

with

$$\omega_{\boldsymbol{q}} = \mu_{\text{eff}} H + M[V_{\text{eff}}(0) - V_{\text{eff}}(\boldsymbol{q})]. \tag{4·15}$$

First let us consider the resistivity due to transverse spin fluctuation. From the diagrams (a), (b) given in Fig. 2 we have

$$\begin{aligned} &Q^{\alpha\tau}_{-+}(i\omega_n) + Q^{\alpha\tau}_{+-}(i\omega_n) \\ &= \frac{1}{V}\left(\frac{e}{m^*}\right)^2\left(\frac{J}{2N}\right)^2 NM \sum_{\boldsymbol{k}} \sum_{\boldsymbol{q}} q^\alpha q^\tau \\ &\times \left\{ \frac{[f(\xi_{\boldsymbol{k}+\boldsymbol{q}\uparrow}) - f(\omega_{\boldsymbol{q}} + \xi_{\boldsymbol{k}\downarrow})][f(\xi_{\boldsymbol{k}\downarrow}) + n(-\omega_{\boldsymbol{q}})]}{i\omega_n - \omega_{\boldsymbol{q}} - \xi_{\boldsymbol{k}\downarrow} + \xi_{\boldsymbol{k}+\boldsymbol{q}\uparrow}} \right. \\ &\quad \left. - \frac{[f(\xi_{\boldsymbol{k}+\boldsymbol{q}\downarrow}) - f(-\omega_{\boldsymbol{q}} + \xi_{\boldsymbol{k}\uparrow})][f(\xi_{\boldsymbol{k}\uparrow}) + n(\omega_{\boldsymbol{q}})]}{i\omega_n + \omega_{\boldsymbol{q}} - \xi_{\boldsymbol{k}\uparrow} + \xi_{\boldsymbol{k}+\boldsymbol{q}\downarrow}} \right\}. \end{aligned} \tag{4·16}$$

From the relation (4·8) we obtain

$$\begin{aligned} P^{\alpha\tau}_{-+} + P^{\alpha\tau}_{+-} = 4\pi\beta\left(\frac{e}{m^*}\right)^2\left(\frac{J}{2N}\right)^2 NM \sum_{\boldsymbol{k}} \sum_{\boldsymbol{q}} q^\alpha q^\tau \\ \times f(\xi_{\boldsymbol{k}\downarrow})[1 - f(\xi_{\boldsymbol{k}+\boldsymbol{q}\uparrow})] n(\omega_{\boldsymbol{q}}) \delta(\xi_{\boldsymbol{k}\downarrow} + \omega_{\boldsymbol{q}} - \xi_{\boldsymbol{k}+\boldsymbol{q}\uparrow}), \end{aligned} \tag{4·17}$$

where $n(x) = (e^{\beta x} - 1)^{-1}$. In the above, $P^{\alpha\tau}_{-+}$ and $P^{\alpha\tau}_{+-}$ are the part of $P^{\alpha\tau}$ which corresponds to (a) and (b) in Fig. 2, respectively. The resistivity due to transverse spin fluctuation ρ_t is obtained by Eqs. (4·1), (4·7) and (4·17) as

$$\rho_t = \frac{1}{6(2\pi)^5} \frac{J^2 M\beta}{e^2 n_e^2} \frac{V}{N} \int_0^{q_D} d^3q\, q^2 n(\omega_{\boldsymbol{q}}) I(\boldsymbol{q}) \tag{4·18}$$

with

$$I(\boldsymbol{q}) = \int d^3k\, f(\xi_{\boldsymbol{k}\downarrow})[1 - f(\xi_{\boldsymbol{k}+\boldsymbol{q}\uparrow})] \delta(\xi_{\boldsymbol{k}\downarrow} + \omega_{\boldsymbol{q}} - \xi_{\boldsymbol{k}+\boldsymbol{q}\uparrow}), \tag{4·19}$$

where q_D is some cutoff wave number. The summation over $\boldsymbol{q}$ and $\boldsymbol{k}$ was transformed into integration in the above expression. In the lowest-order approximation, neglecting the terms of the order of (y_e/ε_F), and noting $\omega_{\boldsymbol{q}} = \omega_q$ for isotropic system we have

$$I(q) = \frac{2\pi(m^*)^2}{|q|} \frac{\omega_q}{1 - e^{-\beta\omega_q}} \quad \text{for } 0 < q < 2p_F. \tag{4·20}$$

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Then ρ_t is given as

$$\rho_t=\frac{8\rho(\infty)}{S(S+1)}\frac{M}{q_c^4}\int_0^{q_c}dq\cdot q^3\frac{\beta\omega_q}{(e^{\beta\omega_q}-1)(1-e^{-\beta\omega_q})} \tag{4·21}$$

with

$$\rho(\infty)=\frac{S(S+1)}{48\pi^3}\left(\frac{m^*}{en_e}\right)^2 J^2\frac{V}{N}\frac{q_C^4}{4}, \tag{4·22}$$

where q_C is q_D or $2p_F$ according to $q_D<2p_F$ or $2p_F<q_D$, respectively.

The resistivity due to the longitudinal spin fluctration ρ_l corresponding to Fig. 2(c) can be obtained in a similar way. The result is

$$\rho_l=\frac{4\rho(\infty)}{S(S+1)}\frac{1}{q_C^4}\int_0^{q_c}dqq^3\frac{b'(\beta y)}{1-\beta b'(\beta y)V_{\mathrm{eff}}(q)}. \tag{4·23}$$

The total resistivity due to spin fluctuation ρ is given by

$$\rho=\rho_l+\rho_t. \tag{4·24}$$

The set of Eqs. (4·21), (4·23) and (4·24) together with the self-consistent equations (2·21) and (2·22) determine the temperature and magnetic field dependence of the resistivity due to the spin fluctuation over the whole temperature region and for the weak magnetic field.

§ 5. The temperature and magnetic field dependence of the resistivity

First let us briefly mention the temperature dependence of the resistivity in the absence of the external magnetic field. At low temperatures $T/T_C\ll 1$, $b'(\beta y)$ in Eq. (4·23) is exponentially small and ρ_l can be neglected. We can easily obtain ρ_t from Eq. (4·21) by scaling $\omega_q(\simeq cq^2)$ by temperature and extending the integration limit to infinity. The result is

$$\rho\left(\frac{T}{T_c}\ll 1\right)=\frac{4\pi^2}{3}\frac{S}{S(S+1)}\frac{\rho(\infty)}{C^2q_C^4}T^2. \tag{5·1}$$

This T^2-dependence of the resistivity which arises from the scattering of conduction electrons by spin waves agrees wih the result of the Boltzmann-equation approach, including the proportionality constant. In the paramagnetic region we put $b'(\beta y)=S(S+1)/3$ in Eq. (4·23), then ρ_l becomes

$$\rho_l=\frac{\rho(\infty)}{3}\frac{4}{q_C^4}\int_0^{q_c}dq\frac{q^3}{1-\{S(S+1)/3T\}V_{\mathrm{eff}}(q)}. \tag{5·2}$$

For ρ_t in the paramagnetic region, we use the relation

$$\lim_{H\to 0}\frac{M}{\beta\omega_q}=\frac{S(S+1)/3}{1-\{S(S+1)/3T\}V_{\mathrm{eff}}(q)}, \tag{5·3}$$

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which is obtained by solving the self-consistent equation for M in a finite field H. Then Eq. (4·21) becomes

$$\rho_t=\frac{2}{3}\rho(\infty)\frac{4}{q_C^4}\int_0^{q_C}dq\frac{q^3}{1-\{S(S+1)/3T\}V_{\rm eff}(q)} \tag{5·4}$$

and we have for the total resistivity

$$\rho(T>T_c, H=0)=\rho(\infty)\frac{4}{q_C^4}\int_0^{q_C}dq\frac{q^3}{1-\{S(S+1)/3T\}V_{\rm eff}(q)}\,. \tag{5·5}$$

When $q_C=2p_F$, the value of $\rho(\infty)$ of (4·22) becomes

$$\rho(\infty)=\frac{m^*}{e^2n_e}\left[\frac{\pi}{2}S(S+1)J^2\frac{V}{N}g(0)\right], \tag{5·6}$$

where $g(0)$ is the density of state per unit volume at the Fermi surface. If we set $V_{\rm eff}(q)=\{3/S(S+1)\}T_c(\sin\alpha q/\alpha q)$ with some parameter α characterizing the range of potential, then (5·5) reduces to the expression of de Gennes and Friedel.[16]

It is necessary to know the temperature and magnetic field dependence of M and $b'(\beta y)$, in order to know the behaviour of the resistivity from Eqs. (4·21) and (4·23). As mentioned earlier, the iterative way is used to determine M and y, assuming the correction to the Hartree-Fock approximation is small, and the solution of Eqs. (2·21) and (2·22) is used for Eqs. (4·21) and (4·23) in the treatment below.

According to the temperature dependence of the resistivity due to the spin fluctuation in the weak external magnetic field, we can divide the temperature region into three; i.e., for the high-temperature region ($T\gg T_c$), the one near the transition temperature ($T\sim T_c$) and for the spin-wave region ($T\ll T_c$). The second region can be devided into two, according to the relative magnitude of $\tau=(T-T_c)/T_c$ and $h=\mu_{\rm eff}H/T_c$. The temperature and the magnetic field dependence of M and $b'(\beta y)$ for various regions can be obtained from (2·24) and (2·25) and can be summarized as follows:[6]

(1) $a\tau^3\gg ch^2$, $\tau\gg 1$

$$M=a\frac{h}{\tau}-\frac{c}{3}\left(\frac{h}{\tau}\right)^3, \qquad b'(\beta y)=a-c\left(\frac{h}{\tau}\right)^2, \tag{5·7}$$

(2) $a\tau^3\gg ch^2$, $1\gg\tau\gtrsim 0$

$$M=a\frac{h}{\tau}-\frac{c}{3}\frac{h^3}{\tau^4}, \qquad b'(\beta y)=a-c\left(\frac{h}{\tau}\right)^2, \tag{5·8}$$

(3) $ch^2\gg a|\tau|^3$

$$M=\left(\frac{3a^4}{c}h\right)^{1/3}, \qquad b'(\beta y)=a-c\left(\frac{3a}{c}h\right)^{2/3}, \tag{5·9}$$

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(4) $a(-\tau)^3 \gg ch^2,\ 1 \gg (-\tau) \gtrsim 0$

$$M = \left[\frac{3a^3}{c}(-\tau)\right]^{1/2} + \frac{a}{2}\frac{h}{(-\tau)},$$

$$b'(\beta y) = a - 3a(-\tau) - \frac{h}{(-\tau)}[3ac(-\tau)]^{1/2}, \tag{5·10}$$

(5) $1 \gg \dfrac{T}{T_c}$

$$M \sim S, \qquad b'(\beta y) \sim 0, \tag{5·11}$$

where

$$\tau = \frac{T - T_c}{T_c}, \quad h = \frac{\mu H}{T_c}, \quad a = \frac{S(S+1)}{3}, \quad C = \frac{S(S+1)(S^2+S+\frac{1}{2})}{15}.$$

In the following we treat the case in which the effective inter-spin interaction satisfies

$$1 \gg \alpha q_c^2, \tag{5·12}$$

where α is given by

$$V_{\text{eff}}(q) = V_{\text{eff}}(0)(1 - \alpha q^2) + 0(q^4). \tag{5·13}$$

When $V_{\text{eff}}(q)$ is of the form of the R-K-K-Y type [Eq. (3·28)] α is $M/8p_F^2$ from (3·30) and the above case corresponds to the case when cut-off wave number q_D is much smaller than $2p_F$. First we consider the resistivity due to the longitudinal spin fluctuations. From Eq. (4·23) we have

$$\rho_l = \frac{2\rho(\infty)(a-\delta)(1+\tau)}{S(S+1)q_c^2\alpha}\left[1 - \frac{A}{q_c^2}\ln\left(1 + \frac{q_c^2}{A}\right)\right] \tag{5·14}$$

with the use of Eq. (5·13), where

$$A \equiv \frac{1}{\alpha}\left(\tau + \frac{\delta}{a}\right), \qquad b'(\beta y) \equiv a - \delta. \tag{5·15}$$

The value of δ is given by Eqs. (5·7)$\sim$(5·10) for various regions and we have

(1) $\tau \gg 1 \gg h \gtrsim 0$

$$\rho_l = \frac{\rho(\infty)}{3}\left[1 + \frac{1}{\tau} - \frac{c}{a}\left(\frac{h}{\tau}\right)^2\right], \tag{5·16}$$

(2) $a\tau^3 \gg ch^2,\ 1 \gg \tau \gtrsim 0$

$$\rho_l = \frac{2}{3}\frac{\rho(\infty)}{\alpha q_c^2}\left(1 - \frac{\tau}{\alpha q_c^2}\ln\left|\frac{\tau}{\alpha q_c^2}\right| - \frac{\tau}{\alpha q_c^2}\left|\ln\frac{\tau}{\alpha q_c^2}\right|\frac{c}{a}\frac{h^2}{\tau^3}\right) \tag{5·17}$$

(3) $ch^2 \gg a|\tau|^3,\ 1 \gg h, |\tau|$

$$\rho_l = \frac{2}{3}\frac{\rho(\infty)}{\alpha q_c^2}\left\{1 - \left[\frac{1}{\alpha q_c^2}3\left(\frac{c}{3a}\right)^{1/3}h^{2/3}\right]\left|\ln\left[\frac{1}{\alpha q_c^2}3\left(\frac{c}{3a}\right)^{1/3}h^{2/3}\right]\right|\right\}, \tag{5·18}$$

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(4) $a(-\tau)^3 \gg ch^2,\ 1 \gg (-\tau) \gtrsim 0$

$$\rho_l = \frac{2}{3}\frac{\rho(\infty)}{\alpha q_c^2}\left[1 - \frac{2(-\tau)}{\alpha q_c^2}\left|\ln\frac{2(-\tau)}{\alpha q_c^2}\right| - \frac{2(-\tau)}{\alpha q_c^2}\sqrt{\frac{3ch^2}{4a(-\tau)^3}}\left|\ln\frac{2(-\tau)}{\alpha q_c^2}\right|\right]. \tag{5·19}$$

In the spin wave region at low temperature, b' in (4·23) is exponentially small and ρ_l is also negligibly small compared with ρ_t.

Next we consider the resistivity due to transverse spin fluctuations. From (4·21) and (5·13) ρ_t is expressed as

$$\rho_t = \frac{4\rho(\infty)}{S(S+1)}\frac{1}{q_c^4 M}\left(\frac{a}{\alpha}\right)^2\left(\frac{T}{T_c}\right)^2\int_0^{x_c} dx \frac{x(\tilde{H}+x)}{(e^{x+\tilde{H}}-1)(1-e^{-x-\tilde{H}})}, \tag{5·20}$$

where

$$\tilde{H} = \frac{\mu_{\text{eff}}H}{T}, \qquad x_C = M\frac{\alpha}{a}\frac{T_c}{T}q_C^2. \tag{5·21}$$

For the high-temperature limit $\tau \gg 1$ or near T_c, when weak external field exists, the inequality $1 \gg x_c$ holds, and the integrand in (5·20) can be expanded by x and then ρ_t takes the form

$$\rho_t = \frac{4}{3}\frac{\rho(\infty)}{\alpha q_C^2}(1+\tau)\left[1 - u\ln\left(1+\frac{1}{u}\right)\right] \tag{5·22}$$

with

$$u = \frac{ah}{M\alpha q_c^2}. \tag{5·23}$$

Using (5·7)～(5·9) we have

(1) $\tau \gg 1 \gg h \gtrsim 0$

$$\rho_t = \frac{2}{3}\rho(\infty)\left[1 + \frac{1}{\tau} - \frac{c}{3a}\left(\frac{h}{\tau}\right)^2\right], \tag{5·24}$$

(2) $a\tau^3 \gg ch^2,\ 1 \gg \tau \gtrsim 0$

$$\rho_t = \frac{4}{3}\frac{\rho(\infty)}{\alpha q_c^2}\left[1 - \frac{\tau}{\alpha q_c^2}\left|\ln\frac{\tau}{\alpha q_c^2}\right| - \frac{c}{3a}\frac{h^2}{\tau^3}\frac{\tau}{\alpha q_c^2}\left|\ln\frac{\tau}{\alpha q_c^2}\right|\right], \tag{5·25}$$

(3) $ch^2 \gg a|\tau|^3,\ 1 \gg h,\ |\tau|$

$$\rho_t = \frac{4}{3}\frac{\rho(\infty)}{\alpha q_c^2}\left\{1 + \tau - \left[\frac{1}{\alpha q_c^2}\left(\frac{c}{3a}\right)^{1/3}h^{2/3}\right]\left|\ln\left[\frac{1}{\alpha q_c^2}\left(\frac{c}{3a}\right)^{1/3}h^{2/3}\right]\right|\right\}, \tag{5·26}$$

(4) $a(-\tau)^3 \gg ch^2,\ 1 \gg (-\tau) \gtrsim 0$

$$\rho_t = \frac{4}{3}\frac{\rho(\infty)}{\alpha q_c^2}\left\{1 - (-\tau) - \frac{1}{\alpha q_c^2}\sqrt{\frac{c}{3a}}\frac{h}{\sqrt{-\tau}}\left|\ln\left[\frac{1}{\alpha q_c^2}\sqrt{\frac{c}{3a}}\frac{h}{\sqrt{-\tau}}\right]\right|\right\}. \tag{5·27}$$

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For the spin-wave region at low temperature ($T/T_c \ll 1$), we have $x_c \gg 1$, and (5·20) becomes

$$\rho_t = \frac{4}{9}\frac{S(S+1)}{S}\frac{\rho(\infty)}{(\alpha q_c^2)^2}\left(\frac{T}{T_c}\right)^2 I \tag{5·28}$$

with

$$I = \int_0^\infty dx \frac{x(\tilde{H}+x)}{(e^{x+\tilde{H}}-1)(1-e^{-x-\tilde{H}})}. \tag{5·29}$$

The integral I can be devided into three parts:

$$I = I_1 + I_2 + I_3 \tag{5·30}$$

with

$$I_1 = 2\int_0^\infty dy \frac{y}{e^y-1}, \qquad I_2 = -2\int_0^{\tilde{H}} dy \frac{y}{e^y-1}, \qquad I_3 = -\tilde{H}\int_{\tilde{H}}^\infty dy \frac{1}{e^y-1}. \tag{5·31}$$

The value of I_1 and I_3 is obtained easily. The integrand in I_2 can be expanded in powers of y when the field is weak $1 \gg \tilde{H} > 0$. Then we have finally

$$\rho_t = \frac{4\pi^2}{27}\frac{S(S+1)}{S}\frac{\rho(\infty)}{(\alpha q_c^2)^2}\left(\frac{T}{T_c}\right)^2\left[1 - \frac{3}{\pi^2}\tilde{H}|\ln \tilde{H}| - \frac{6}{\pi^2}\tilde{H}\right]. \tag{5·32}$$

The total resistivity $\rho = \rho_t + \rho_l$ due to the fluctuation of localized spins is summarized from Eqs. (5·16)~(5·19), Eqs. (5·24)~(5·27) and (5·32) as follows:

(1) $\tau \gg 1 \gg h \gtrsim 0$ (the high-temperature limit)

$$\frac{\rho}{\rho(\infty)} = 1 + \frac{1}{\tau} - \frac{5}{9}\frac{c}{a}\left(\frac{h}{\tau}\right)^2, \tag{5·33}$$

(2) $a\tau^3 \gg ch^2, 1 \gg \tau \gtrsim 0$ (near and above the transition temperature and the weak magnetic-field region)

$$\frac{\rho}{\rho(\infty)} = \frac{2}{\alpha q_c^2}\left(1 - \frac{\tau}{\alpha q_c^2}\left|\ln\frac{\tau}{\alpha q_c^2}\right|\right) - \frac{10}{9}\frac{1}{(\alpha q_c^2)^2}\frac{c}{a}\left|\ln\frac{\tau}{\alpha q_c^2}\right|\left(\frac{h}{\tau}\right)^2, \tag{5·34}$$

(3) $ch^2 \gg a|\tau|^3$, $1 \gg h, |\tau|$ (just above or below the transition temperature)

$$\frac{\rho}{\rho(\infty)} = \frac{2}{\alpha q_c^2}\left(1 + \frac{2}{3}\tau\right) - \frac{10}{3}\left[\frac{1}{\alpha q_c^2}\left(\frac{c}{3a}\right)^{1/3} h^{2/3}\right]\left|\ln\left[\frac{1}{\alpha q_c^2}\left(\frac{c}{3a}\right)^{1/3} h^{2/3}\right]\right|, \tag{5·35}$$

(4) $a(-\tau)^3 \gg ch^2, 1 \gg (-\tau) \gtrsim 0$ (near and below the transition temperature and the weak magnetic field region)

$$\frac{\rho}{\rho(\infty)} = \frac{2}{\alpha q_c^2} - \frac{2}{3}\frac{2(-\tau)}{(\alpha q_c^2)^2}\left|\ln\frac{2(-\tau)}{\alpha q_c^2}\right| - \frac{4}{3}\frac{1}{\alpha q_c^2}\left[\frac{1}{\alpha q_c^2}\sqrt{\frac{c}{3a}}\frac{h}{\sqrt{-\tau}}\right]$$

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$$\times\left|\ln\left[\frac{1}{\alpha q_c^2}\sqrt{\frac{c}{3a}}\frac{h}{\sqrt{-\tau}}\right]\right|, \tag{5·36}$$

(5) $1 \gg \dfrac{T}{T_c}$, $1 \gg \dfrac{\mu_{\rm eff}H}{T}$ (spin-wave region)

$$\frac{\rho}{\rho(\infty)}=\frac{4\pi^2}{27}\frac{S(S+1)}{S}\frac{1}{(\alpha q_c^2)^2}\left(\frac{T}{T_c}\right)^2\left[1-\frac{3}{\pi^2}\frac{\mu_{\rm eff}H}{T}\left|\ln\frac{\mu_{\rm eff}H}{T}\right|-\frac{6}{\pi^2}\frac{\mu_{\rm eff}H}{T}\right]. \tag{5·37}$$

It is seen from Eqs. (5·16)~(5·18) and Eqs. (5·33)~(5·35) that the ratio of the negative magnetoresistivity $\Delta\rho_t(H)$ of transverse part to the longitudinal part $\Delta\rho_l$ is

$$\left|\frac{\Delta\rho_t(H)}{\Delta\rho_l(H)}\right|=\frac{2}{3} \quad \text{for regions (1), (2) and (3)} \tag{5·38}$$

and

$$|\Delta\rho_t(H)| \gg |\Delta\rho_l(H)| \quad \text{for regions (4) and (5).} \tag{5·39}$$

A very low temperatures satisfying the condition $\mu_{\rm eff}H/T \gg 1$, it is seen that the magnetoresistance is proportional to $\exp(-\mu_{\rm eff}H/T)$. This is the same situation as the decrease of resistivity when the spin-wave spectrum has an energy gap due to the internal anisotropy field as was first pointed out by Mackintosh.[19]

§ 6. Summary and discussion

The magnetic-field-dependent part $\Delta\rho$ of ρ defined as

$$\Delta\rho(T,H)=\frac{\rho(T,H)-\rho(T,0)}{\rho(\infty)}$$

is obtained from Eqs. (5·33)~(5·37), and is shown schematically in Fig. 3. When we compare the present results with that obtained by the simple molecular-field approximation in I [Fig. 3], we see that due to the spacial fluctuations of the localized spins there appears a logarithmic singularity in the magnetoresistance near the transition temperature T_c as seen from Eqs. (5·34), (5·35) and (5·36). In the spin wave region, on the other hand, the magnetoresistance is proportional to $T^2(\mu_{\rm eff}H/T)|\ln(\mu_{\rm eff}H/T)|$ in contrast to the $\exp(-{\rm const}/T)$-type.

The low-temperature behaviour of the magnetoresistance obtained in this paper is exact in the sense that the spin-wave state of a ferromagnet is an exact eigenstate at low-energy region. The magnetoresistance at the high-temperature limit coincides with the result of the molecular field approximation and is also exact concerning the spin fluctuation. On the other hand, however, our approximation becomes poor as the temperature comes near the Curie point. Because in this region the fluctuation of the magnetization gets very large in space and time and the interaction between fluctuations, which is neglected in the R. P. A. ap-

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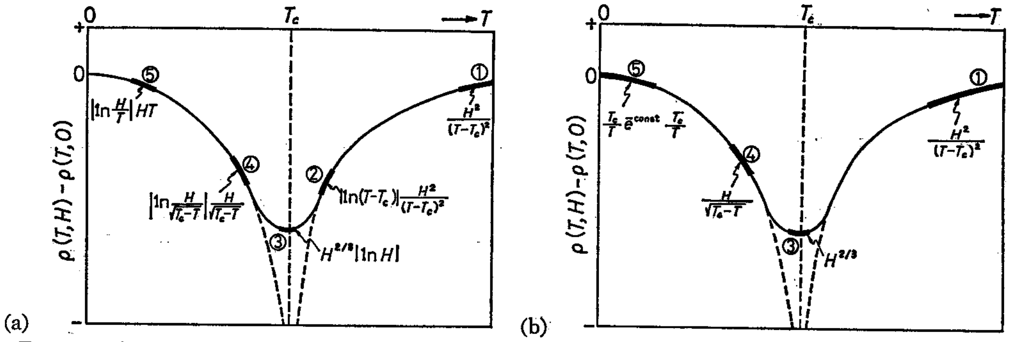

Fig. 3. The schematic graphs of the negative magnetoresistance. (a) Present results. (b) The results of the molecular-field approximation obtained in I. The number ①~⑤ in the graphs represents the following regions:

① $\frac{T-T_c}{T_c}\gg 1, \quad 1\gg\frac{\mu H}{T_c},$

② $\left(\frac{T-T_c}{T_c}\right)^3\gg\left(\frac{\mu H}{T_c}\right)^2, \quad 1\gg\frac{T-T_c}{T_c}\gtrsim 0,$

③ $\left(\frac{\mu H}{T_c}\right)^2\gg\left|\frac{T-T_c}{T_c}\right|^3, \quad 1\gg\frac{\mu H}{T_c}, \quad \left|\frac{T-T_c}{T_c}\right|,$

④ $\left(\frac{T_c-T}{T_c}\right)^3\gg\left(\frac{\mu H}{T_c}\right)^2, \quad 1\gg\frac{T_c-T}{T_c}\gtrsim 0,$

⑤ $1\gg\frac{T}{T_c}, \quad 1\gg\frac{\mu H}{T}.$

proximation, becomes essential. In this region the correct critical index of the magnetization, the susceptibility and the coherence length must be used, and a more careful treatment must be necessary in the calculation near T_c. In this case we could calculate the magnetoresistance with the use of the scaling hypothesis as was done by several authors in studying the resistive anomalies at the magnetic critical points.[16),17),18)] A simple treatment in this direction for the case $T>T_c$ is shown in the Appendix. The index of the magnetoresistivity $\Delta\rho\sim-|\ln(T-T_c)|\{H^2/(T-T_c)^2\}$, $-H^{2/3}|\ln H|$ in the regions (2) and (3) in Fig. 3 (a) changes into the form $-|\ln(T-T_c)|\{H^2(T-T_c)^a\}$, $-H^c$ or $-H^c|\ln H|$ and the indices a, b and c are given by the linear combination of the two critical index as shown in the Appendix.

Although we have started with the Hamiltonian representing rare earth metals, some other contributions characteristic of rare earth metals, for example the optical branch of spin waves, the appearance of a dip in the acoustical spin-wave branch or the effect of the Stoner excitation are not taken into account. Such effect may have the correction to the resistivity obtained in this paper especially in the high-temperature region.

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and critical reading of the manuscript.

Appendix

In this appendix we calculate the magnetoresistance near above T_c in the framework of the present calculation using the static scaling hypothesis for the spin Green's functions. Near T_c the most singular part of the resistivity due to magnetic scattering arises from $\mathscr{D}^{+-}(q,0)$ and $\mathscr{D}^{zz}(q,0)$, i.e., the part of $\zeta_n=0$, and then above the transition point

$$\rho(H)\propto\int_0^{q_c} q^3dq\mathscr{D}^{+-}(q,0) \tag{A·1}$$

similarly in Eqs. (5·3) and (5·4) in the text.

The static-scaling hypothesis postulates that $\mathscr{D}^{+-}(q,0)$ has the homogeneous property[20),21)]

$$\mathscr{D}^{+-}(q,0)=\frac{a}{\tau^\gamma f(q/\chi, H/\varepsilon)}, \tag{A·2}$$

where a is the constant, γ is the critical indices of the susceptibility, and

$$\chi=\chi_0\tau^\nu, \tag{A·3}$$

$$\varepsilon=\epsilon_0\tau^\Delta. \tag{A·4}$$

Here χ_0 and ϵ_0 are the some constants and the critical index satisfies the index relations

$$\Delta=\beta+\gamma, \qquad \nu=(2\beta+\gamma)/3 \tag{A·5}$$

using γ and the critical index of the magnetization denoted by β. The function $f(q/\chi, H/\epsilon)$ has the following properties for $q\ll\chi$, and $H\ll\epsilon$, i.e., for the hydrodynamic region,

$$f(q/\chi, H/\epsilon)\simeq 1+C_1(q/\chi)^2+C_2(H/\epsilon)^2, \tag{A·6}$$

while for $q\gg\chi$, $H\gg\epsilon$, i.e., for the critical region,

$$f(q/\chi, H/\epsilon)\simeq C_3(H/\epsilon)^{\gamma/\Delta}+C_4(q/\chi)^{\gamma/\nu}, \tag{A·7}$$

where C_1, C_2, C_3 and C_4 are the some dimensionless constants. Equation (A·7) tells us that $\mathscr{D}^{+-}(q,0)$ has a finite nonvanishing value at T_c for arbitrary q and $\mathscr{H}$ unless $q=H=0$.

From Eqs. (A·1), (A·2) and (A·6) we have

$$\frac{\rho(H)-\rho(0)}{\rho(0)}\propto\frac{\chi^2}{C_1\tau^\gamma}\int_0^{k_c}\left\{\frac{1}{1+C_1(q/\chi)^2}-\frac{[1+C_2(H/\epsilon)^2]q}{1+C_2(H/\epsilon)^2+C_1(q/\chi)^2}\right\}dq$$

$$=\frac{\chi^4}{2C_1^2\tau^\gamma}\Big\{\ln[1+C_1(k_c/\chi)^2]-[1+C_2(H/\epsilon)^2]$$

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$$\times \ln \frac{1+C_2(H/\epsilon)^2+C_1(k_c/\chi)^2}{1+C_2(H/\epsilon)^2}\Big\}, \tag{A·8}$$

where k_c is some cutoff momentum. It should be noted that although we use the expansion with respect to (q/χ) in the above calculation, the logarithmic singularity arises from the region $q\sim 0$ and does not concern with the behaviour of large q. From Eqs. (A·5) and (A·8) we finally get

$$\frac{\rho(H)-\rho(0)}{\rho(0)} \propto -\left(\tau^{4\nu-2\beta-3\gamma} \ln \frac{k_c}{\chi}\right) H^2 \tag{A·9}$$

for $H \ll \epsilon$.

Using Eq. (A·7), we have for $H \gg \epsilon$

$$\frac{\rho(H)-\rho(0)}{\rho(0)} \propto -H^{\gamma/\Delta} \int_0^{k_c} \frac{q^{3-\gamma/\nu} dq}{C_3(H/\epsilon_0)^{\gamma/\Delta}+C_4(q/\chi_0)^{\gamma/\nu}} \tag{A·10}$$

from Eqs. (A·1) and (A·2).

If we use the critical indices in the molecular-field approximation, i.e., $\nu=1/2$ $\gamma=1$ and $\beta=1/2$ in Eqs. (A·9) and (A·10), we get the results in the text.

According to the index relation $\gamma/\nu=2-\eta$, where η is the critical index for the spacial behaviour of the spin-correlation functions, we find $\gamma/\nu\leq 2$. Then we have

$$\frac{\rho(H)-\rho(0)}{\rho(0)} \propto -H^{\gamma/\Delta} = -H^{\gamma/(\beta+\gamma)} \tag{A·11}$$

for $\gamma/\nu<2$, and when $\gamma/\nu=2$, we get

$$\frac{\rho(H)-\rho(0)}{\rho(0)} \propto H^{\gamma/(\beta+\gamma)} \ln H. \tag{A·12}$$

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