INSTITUT NATIONAL DE LA STATISTIQUE ET DES ETUDES ECONOMIQUES
Série des Documents de Travail du CREST
(Centre de Recherche en Economie et Statistique)

n° 2006-16

Negishi-Solow Efficiency Wages, Unemployment Insurance and Dynamic Deterministic Indeterminacy

J.-M. GRANDMONT[1]

[1] ICEF, Department of Economics, University Ca Foscari di Venezia at San Giobbe, Italy and CREST (GREQSTA, UMR 2773), 15 Boulevard Gabriel Péri, 92245 Malakoff Cedex, France.

Abstract

This paper introduces efficiency wages designed to provide workers with incentives to make appropriate effort levels, and involuntary unemployment, along the pioneering lines of Negishi (1979), Solow (1979), Shapiro and Stiglitz (1984), in a dynamic model involving heterogeneous agents and financial constraints as in Woodford (1986) and Grandmont, Pintus and de Vilder (GPV, 1998). Effort varies continuously while there is unemployment insurance funded out of taxation of labour incomes. Increasing unemployment insurance is beneficial to employment along the deterministic stationary state, and can even in some cases lead to a Pareto welfare improvement for all agents, through general equilibrium effects, by generating higher individual real labour incomes, hence larger consumptions of employed and unemployed workers, and thus a higher production. On the other hand, the local (in)determinacy properties of the stationary state are opposite to those obtained in the competitive specification of the model (GPV, 1998) : local determinacy (indeterminacy) occurs for elasticities of capital-efficient labour substitution lower (larger) than a quite small bound. Increasing unemployment insurance is more likely to lead to local indeterminacy and thus to generate dynamic inefficiencies due to the corresponding expectations coordination failures.

JEL Classification Numbers : E24, E32, C62.

Keywords : Efficiency wages, involuntary unemployment, unemployment insurance, effort incentives, local indeterminacy, capital-labour substitution, local bifurcations.

Résumé

Ce papier introduit des salaires d'efficience destinés à inciter les travailleurs à fournir un niveau d'effort approprié, et du chômage involontaire, selon les avancées pionnières de Negishi (1979), Solow (1979), Shapiro et Stiglitz (1984), dans un modèle dynamique avec agents hétérogènes et contraintes financières comme dans Woodford (1986) et Grandmont, Pintus et de Vilder (GPV, 1998). L'effort des travailleurs varie continûment, tandis qu'une assurance chômage est mise en oeuvre par taxation des revenus du travail. Un accroissement de l'assurance chômage est bénéfique à l'emploi le long de l'état stationnaire déterministe, et peut dans certains cas conduire à une amélioration Parétienne du bien être de tous les agents, par des mécanismes d'équilibre général, en augmentant les revenus réels individuels du travail, donc les consommations des travailleurs employés et au chômage et par conséquent la production. D'autre part, les propriétés d'(in)détermination locale de l'état stationnaire sont opposées à celles obtenues dans la spécification concurrentielle du modèle (GPV, 1998) : on observe la détermination (indétermination) locale pour des élasticités de substitution capital-travail inférieures (supérieures) à une borne qui est petite. Accroître l'assurance chômage rend plus probable l'indétermination locale et donc les inefficacités dynamiques dues aux difficultés correspondantes de coordination des anticipations.

Numéros de classification JEL : E24, E32, C62.

Mots clés : Salaires d'efficience, chômage involontaire, assurance chômage, incitations à l'effort, indétermination locale, substitution capital-travail, bifurcations locales.

1 Introduction

In his seminal contribution on *The Microeconomic Foundations of Keynesian Macroeconomics* (1979), Professor Takashi Negishi made the fundamental and insightful observation that in order to show a Keynesian equilibrium involving involuntary unemployment to exist, it was not enough to show that workers would resist wage cuts : its was necessary that firms on the short side of the labor market be also unwilling to cut down wages, or to hire unemployed workers at a lower wage, despite the existence of notional labour excess supply.

The particular argument he was the first to introduce along this line and to explore theoretically in a general equilibrium framework, was that labour productivity was likely to depend positively on nominal or real wages, so that firms' profit maximization would lead them to choose high real "*efficiency wages*" that maximize in a sense productivity and that may generate excess notional supply of labour (Negishi, 1979, Ch. 9). This efficiency wage argument, also independently discovered by Solow (1979), has been since a cornerstone of modern theorizing of unemployment.

The aim of this paper is, in the footsteps of Professor Negishi, to introduce efficiency wage arguments and involuntary unemployment in an otherwise standard finance constrained economy, as in Woodford (1986) and Grandmont, Pintus and de Vilder (GPV, 1998), and to study the consequences of unemployment insurance (financed by taxation of labour income) on the local dynamics near a stationary state. We use a specification that is directly inspired from the "shirking" formulation of Coimbra (1999), Alexopoulos (2004), Nakajima (2006), Aloi and Lloyd-Braga (2006), following the early contribution of Shapiro and Stiglitz (1984), see also Danthine and Donaldson (1990). The efficiency wage contract offered by firms specifies the wage of an employed worker as well as the required effort level. We focus here on a specification involving effort level as a continuous variable, in the spirit of Uhlig and Xu (1996). An employed worker who "shirks" faces a positive probability of getting caught, in which case he is fired and gets the unemployment insurance paid to unemployed workers. Firms will choose accordingly an efficiency wage contract that minimizes the real cost of labour per unit of effort, subject to a "non-shirking condition" that states that for any given effort level, the wage paid has to be high enough to induce employed workers not to shirk. We shall focus on the case where the resulting real wage per unit of effort generates notional excess supply on the labour market. To simplify matters, we shall assume that the chances for a worker to get a job offer in any period do not depend on his past employment status. We abstract accordingly from any complication, in particular unemployment persistence, arising for instance from job search, and from the possible influence of unemployment insurance on the duration of unemployment.

The economy is otherwise identical to the one studied in GPV (1998) with a competitive market for output, two assets (capital and money) and two classes of households, "capitalists" and "workers". The latter face a finance constraint : they cannot borrow against the money receipts they will get at the end of any period, as labour income (wages) or unemployment insurance. So if workers are significantly more impatient than capitalists, if the rate of unemployment insurance is high and if workers display low risk aversion, the households' behaviors display the same properties as in Woodford (1986) or GPV (1998) along equilibria near a stationary state : capitalists hold capital and no money, while workers hold no capital and finance consumption in a any period from the cash receipts they got one period earlier as their wage income or unemployment insurance.

We focus in this paper on deterministic perfect foresight intertemporal equilibria and on the properties of, and the local dynamics near, a stationary state, leaving aside for further research the existence and properties of endogenous stochastic fluctuations (sunspots) that are bound to occur as well in such a context in the case of indeterminacy. It turns out that increasing the rate of unemployment insurance has here beneficial effects on employment, and in some cases can generate a Pareto welfare improvement for all agents, along a stationary state. This outcome arises, despite the fact that increasing unemployment insurance leads to efficiency wage contracts involving higher real wages per unit of effort and thus in principle lower labour demand, because it leads also to higher real incomes, hence consumptions, of employed and unemployed workers, and therefore in equilibrium, to higher production and employment. On the other hand, the local dynamics near a stationary state is in the efficiency wage context considered here, in fact governed by the same equations as in the competitive case studied by GPV (1998), with the difference that the workers' competitive offer curve $c = \gamma(l)$ in GPV is replaced here by a function $c = g(l)$ describing the (employed and unemployed) workers' equilibrium aggregate consumption as a function of equilibrium (efficient) employment. The essential difference between the two frameworks, as far as local (in)determinacy is concerned, is that the elasticity of the competitive offer curve in GPV was bound to exceed 1 $(\varepsilon_\gamma > 1)$ as a result of the fact that competitive labour supply should depend positively on the real wage, while the elasticity of the consumption function g in the efficiency wage case is bound to be between 0 and 1 $(0 < \varepsilon_g < 1)$, with ε_g

close to 1 when there is little unemployment insurance and close to 0 when unemployment insurance is large. The consequence is that, in contrast with the competitive case of GPV where local indeterminacy could occur only for very low and implausible elasticities of substitution between capital and labor, local indeterminacy can be obtained in the efficiency wage specification considered here for elasticities of capital-labor substitution that exceed a small lower bound. The range of such elasticities leading to local indeterminacy is thus quite large and empirically plausible. It includes in particular Cobb-Douglas production functions, a conclusion that seems to be agreement with similar findings obtained after introducing unemployment in that sort of model, e.g. through unions, and imperfect unemployment insurance financed through labour income taxation as here (Dufourt, Lloyd-Braga and Modesto (2004)). Our analysis shows in particular that local indeterminacy is more likely to occur for low elasticities ε_g, i.e. when unemployment insurance is significant. So increasing unemployment insurance can have a positive impact on employment and the agents welfare when focusing on the stationary state, but can also generate dynamic inefficiencies due to expectations coordination failures in the case of local indeterminacy.

The paper is organized as follows. The agents behavior is specified in section 2, and analyzed further in more detail in Appendix A. Efficiency wage contracts are studied in section 3 and in appendix B. In section 4, the equations governing the local dynamics near a stationary state are presented. We consider there the existence and comparative statics of such a stationary state, in particular when unemployment insurance increases. Deterministic local indeterminacy and local bifurcations are analyzed in section 5. Conclusions and a few hints for future research are given in Section 6.

2 Agents Behavior

I describe in this section how efficiency wages can be formally introduced in a finance constrained economy of the Woodford type (1986), with capital-labour substitution as generalized in Grandmont, Pintus and De Vilder (GPV, 1998). I focus here on the overlapping generations structure arising from the infinite horizon version of the model, near the monetary steady state. I make precise in Appendix A how the complete specification of the infinite horizon model with eficiency wages, can be reduced to the overlapping generations specification considered directly here.

The economy involves three types of agents (firms, "capitalists" and "workers"), three commodities (capital, labor and a consumption good) and two assets (physical capital and fiat money).

There is a continuum of identical small firms, the total size of which is normalized to 1. Firms in period t combine physical capital $k_{t-1} \geqq 0$ with *efficient labour* $l_t = n_t\ x_t \geqq 0$, where $n_t \geqq 0$ is the number of labour units employed and $x_t \geqq 0$ is the effort level (assumed to be the same across identical workers), to produce output $y_t \geqq 0$ available in the same period according to a constant return technology given by the production function $y_t = AF(k_{t-1}, l_t)$. When $l > 0$, I shall represent this technology by $y = lAf(a)$, where $a = k/l$ is the capital-efficient labour ratio, $Af(a)$ is the reduced production function $AF(a, 1)$ and $A > 0$ is a scaling productivity parameter. I shall assume throughout

(2.a) *The reduced production function* $y/l = Af(a)$ *is a continuous function of the capital-efficient labour ratio* $a = k/l \geqq 0$, *with* $f(0) = 0$, *and has continuous derivatives of all required orders for* $a > 0$, *with* $f'(a) > 0$, $f''(a) < 0$. *In particular, the marginal productivity of capital,* $A\rho(a) = Af'(a)$, *decreases from* $+\infty$ *to* 0, *while the marginal productivity of efficient labour* $A\omega(a) = A(f(a) - af'(a))$ *increases from* 0 *to* $+\infty$, *when the capital-efficient labour ratio* a *goes up from* 0 *to* $+\infty$.

Firms' profit maximizing behavior, as specified precisely shortly below, will imply that production takes place in period t so that the capital-efficient labour ratio $a_t = k_{t-1}/l_t$ equates 1) the real rental rate of capital services $\rho_t = r_t/p_t$, where $r_t > 0$ is the nominal rental rate and $p_t > 0$ the price of output, with the marginal productivity of capital $A\rho(a_t)$, and 2) the real wage per unit of effort $\omega_t/x_t = w_t/(p_t x_t)$, where $w_t > 0$ is the nominal wage and $x_t > 0$ the effort level, with the marginal productivity of efficient labour $A\omega(a_t)$. As capital is assumed to depreciate at the constant rate $0 < \delta < 1$, the real gross rate of return on capital $R_t = \rho_t + 1 - \delta$, will then be equal to $R(A, a_t) = A\rho(a_t) + 1 - \delta$.

There is a second continuum of agents, called "capitalists", the size of which is also normalized to 1. They do not work and to simplify matters, are assumed to have Cobb-Douglas instantaneous utility for consumption, Log c_{tc}, and to have an infinite horizon, so that at date t they maximize the discounted intertemporal utility $\sum_{j=0}^{\infty} (\beta_c)^j\ Log\ c_{t+j,c}$, where the discount factor satisfies $0 < \beta_c < 1$, given current and expected (non random) prices of consumption, (p_{t+j}), and real gross rates of return on capital $(R_{t+j}), j \geqq 0$. Capitalists can save in the form of cash balances as well as of physical capital. We assume perfect foresight and shall focus on equilibria such that

$R_{t+1} = \overline{R} = 1/\beta_c > 1 = p_t/p_{t+1}$ at the steady state and thus $R_{t+1} > p_t/p_{t+1}$ nearby for all t. Physical capital dominates then money as an asset, so that capitalists choose to hold capital only, implying budget constraints of the form $c_{tc} + k_{tc} = R_t k_{t-1,c}$ and an optimal consumption, capital investment policy given by

$$c_{tc} = (1-\beta_c) R_t k_{t-1,c} \quad , \quad k_{tc} = \beta_c R_t k_{t-1,c}. \tag{2.1}$$

There is finally a third continuum of identical small agents, called "workers", the size of which is also normalized to 1. Each individual worker supplies one unit of labour. Firms' profit maximizing behavior will result, as described precisely shortly below, in an "*efficiency wage contract*" (w_t, x_t), that specifies the nominal wage $w_t > 0$ to be paid to an employed worker provided that he produces the required effort level $x_t > 0$. Firm's behavior will generate also a demand $n_t > 0$ of labour units to be employed. We shall focus here on unemployment, i.e. on configurations where $n_t < 1$. Then it is assumed that at date t, n_t individual workers are drawn independently at random and offered employment with the efficiency wage contract (w_t, x_t). As such a contract is designed to induce workers to make indeed the required effort level x_t, no worker will turn down the job offer : unemployment will be involuntary.

This formulation implies that there are $1-n_t > 0$ workers who are unemployed at date t. It is assumed that there is unemployment insurance, so that unemployed workers get the income $\nu w_t \geqq 0$, where the rate of compensation $0 \leqq \nu < 1$ is fully anticipated by all agents. Unemployment compensation is funded by taxing wages and unemployed workers income at the uniform rate $0 \leqq (1-d_t) \leqq 1$, so that the rate d_t is obtained in equilibrium from the employment and wage rates (n_t, w_t) by

$$(1-n_t) d_t \nu w_t = (1-d_t) n_t w_t. \tag{2.2}$$

Workers are assumed to have instantaneous utility for consumption $U(c_{tw})$, and disutility for effort, $V(x_{tw})$.They are assumed also to have an infinite horizon, so that their objective is to maximize at date t the expectation of the discounted intertemporal utility, $\sum_{j=0}^{\infty} (\beta_w)^j \left(U\left(c_{t+j,w}\right) - \beta_w V\left(x_{t+j,w}\right)\right)$, where $0 < \beta_w < 1$ is their discount factor, subject to the appropriate budget constraints, given current and expected (again, non random) prices for consumption, (p_{t+j}), real gross rates of return on capital (R_{t+j}), employment rates (n_{t+j}), efficiency wage contracts (w_{t+j}, x_{t+j}), and wage income taxation rates $(1-d_{t+j})$. In this framework, the wage contracts do specify the required effort levels, so that the (random) sequence $x_{t+j,w}$ is given, since it is equal to x_{t+j} with probability n_{t+j} (when the worker is employed) and to 0 with probability $1-n_{t+j}$ (when he is unemployed). On the other hand, a worker's (random) consumption $c_{t+j,w}$ will depend on the history of his employment status. The specific feature of the workers' decision making problem assumed here as well as in Woodford (1986) or GPV (1998), is that they are subject to a finance constraint stating that their money demand $m_{t+j,w} \geqq 0$ in period $t+j$ cannot be less than their disposable money wage income $d_{t+j}w_{t+j}$ (if employed) or their disposable money unemployment compensation $d_{t+j}\nu w_{t+j}$ (if unemployed) at that date (for instance, they will receive this income in cash at the end of the period, but cannot borrow against it). Such a finance constraint is necessarily binding at a monetary steady state (there is no point in keeping a constant positive amount of idle cash in excess of the stationary disposable wage income or unemployment compensation) and therefore nearby. On the other hand, we shall focus on intertemporal equilibria with perfect foresight that satisfy at a steady state and thus nearby (assuming continuously differentiable utility and positive consumption) :

$$U'(c_{tw}) > \beta_w E_t[R_{t+1}U'(c_{t+1,w})]. \tag{2.3}$$

This inequality states that the marginal cost, in terms of the utility of foregone consumption, of investing in one additional unit of physical capital, exceeds the corresponding expected utility gain. It implies that thanks to the finance constraint, workers will keep their wage income or unemployment compensation in cash, but will choose not to make any additional saving in physical capital even though this asset dominates money.

A worker's behavior exhibits therefore a simple two-periods overlapping generations structure, since he will keep at date t his disposable wage income $d_t w_t$ (when employed, with probability n_t) or his after-tax unemployment compensation $d_t\nu w_t$ (when unemployed, with probability $1-n_t$) in the form of a money balance m_{tw} or m^u_{tw} when "young", and will spend it one period later, when "old", to finance consumption $c_{t+1,w}$ or $c^u_{t+1,w}$ at the price p_{t+1} :

$$\begin{array}{rcll} d_t w_t = & m_{tw} = & p_{t+1}c_{t+1,w} & \text{if employed at } t, \text{with probability } n_t, \\ d_t\nu w_t = & m^u_{tw} = & p_{t+1}c^u_{t+1,w} & \text{if unemployed at } t, \text{with probability } 1-n_t. \end{array} \tag{2.4}$$

We see accordingly that (2.3) will obtain at a deterministic stationary state and nearby, since one will have $R_{t+1} = \overline{R} = 1/\beta_c > 1$ at such a steady state, when the workers' discount factor β_w is significantly lower than β_c, i.e. when workers are significantly more impatient than capitalists (this condition is similar to that of Woodford (1986) or GPV (1998) in the case of a competitive labor market), when the workers' consumption does not vary much with their employment status (when unemployment insurance is significant, ν is close to 1), and when the workers' utility for consumption $U(c)$ displays low risk aversion (marginal utility $U'(c)$ does not decrease too fast).

It remains to specify how firms determine at date t their production plans $(y_t, k_{t-1}, l_t = n_t x_t)$ and the efficiency wage contract (w_t, x_t) that they offer to workers, given the price $p_t > 0$ of output, the rental rate $r_t > 0$ of capital services, the rate of unemployment insurance ν and the rate of taxation $(1 - d_t)$ of wage income. Firms seek to maximize their real profit

$$AF(k_{t-1,}l_t) - \frac{\omega_t}{x_t} l_t - \rho_t k_{t-1}, \tag{2.5}$$

where $\omega_t = w_t/p_t$ is the real wage and $\rho_t = r_t/p_t$ is the real rental rate of capital services. Given ρ_t and the real wage per unit of effort ω_t/x_t, profit maximization in (2.5) with respect to (k_{t-1}, l_t) leads to a capital - efficient labour ratio $a_t = k_{t-1}/l_t$ that equates, as announced earlier, ρ_t with the marginal productivity of capital, and ω_t/x_t with the marginal productivity of efficient labour

$$\rho_t = A\rho\left(a_t\right) \text{ and } \omega_t/x_t = A\omega\left(a_t\right). \tag{2.6}$$

Profit maximization makes on the other hand firms seek to minimize the real cost of labor per unit of effort ω_t/x_t. I follow here the fundamental insight provided by Negishi (1979, Ch. 9), Solow (1979), who noted that firms are constrained in their choice of a wage contract by the fact that workers' productivity (here their incentives to provide a required effort level) depends positively on the wage offered in the contract. I adopt here a specification that is directly inspired from the "shirking" formulation of Coimbra (1999), Alexopoulos (2004), Nakajima (2006), Aloi and Lloyd-Braga (2006), following the early contribution of Shapiro and Stiglitz (1984), see also Danthine and Donaldson (1990), Uhlig and Xu (1996). At date t, under our maintained assumption (2.3), an employed worker under the wage contract (w_t, x_t) who provides the required effort level $x_{tw} = x_t$, gets the money wage w_t, keeps the corresponding disposable wage income in cash, $m_{tw} = d_t w_t$, in order to finance his consumption one period later, $c_{t+1,w} = d_t \omega_t p_t/p_{t+1}$, as in (2.4), the corresponding utility level achieved being $U(d_t \omega_t p_t/p_{t+1}) - V(x_t)$. A worker employed under the same contract (w_t, x_t) who "shirks", i.e. who provides no effort, $x_{tw} = 0$, faces a probability $0 < \theta < 1$ of getting caught. In that case, he is fired, gets the unemployment compensation νw_t, keeps the after-tax income $d_t \nu w_t = m^u_{tw}$ in cash in order to finance his consumption one period later $c^u_{t+1,w} = d_t \nu \omega_t p_t / p_{t+1} = \nu c_{t+1,w}$. The "*Non-Shirking Condition" (NSC)* stipulates that the wage rate w_t, the effort level x_t and penalties for shirking should be such that

(NSC) The disutility of effort is outweighted by the expected utility gain of not shirking

$$\begin{array}{c} V(x_t) - V(0) \leqq \theta \left[U(c_{t+1,w}) - U(\nu c_{t+1,w}) \right] \\ \textit{where} \;\; c_{t+1,w} = d_t \omega_t p_t / p_{t+1} \;. \end{array} \tag{2.7}$$

Firms' profit maximizing behavior will make them accordingly to choose a production plan $(y_t, k_{t-1}, l_t = n_t x_t)$ satisfiying the standard marginal productivity conditions (2.6), and offer a wage contract (w_t, x_t) that minimizes the real cost of labour per unit of effort, ω_t / x_t, subject to the non-shirking conditions (NSC) (2.7). We note that this condition ensures that an employed non-shirking worker is at least as well off as when employed but shirking, and thus strictly better off than if he were unemployed. Hence unemployment is involuntary. We analyze in the next section the characteristics of the resulting optimum efficiency wage contract.

3 Efficiency wage contracts

Minimizing the real cost of labour per unit of effort ω_t / x_t, given $d_t, p_t / p_{t+1}$, amounts to minimizing consumption $c_{t+1,w} = d_t \omega_t p_t / p_{t+1}$ per unit of effort. It follows that *efficiency wage contracting will generate an effort level* x^* *and a non-shirking employed worker consumption* c^* *that is constant over time and that maximizes effort per unit of consumption* x/c *subject to the non-shirking schedule*

(NSS) $$V(x) - V(0) = \theta \left[U(c) - U(\nu c) \right] \overset{def}{\equiv} \varphi(c, \nu) \,. \tag{3.1}$$

We shall assume throughout :

$(3.a)$ *Disutility of effort satisfies* $V(0) = 0, V(x) > 0$ *for* $0 < x < \overline{x}$ *where* $\overline{x}$ *is the maximum effort level (possibly infinite). There is a fixed cost of providing effort,* $0 < x_o = \lim_{x \to 0} V(x)$. *Disutility of effort* $V(x)$ *is continuously differentiable of every required order for* $0 < x < \overline{x}$, *increasing* $(V'(x) > 0)$, *strictly convex* $(V''(x) > 0)$ *with* $\lim_{x \to \overline{x}} V'(x) = +\infty$.

(3.b) *Utility of consumption* $U(c)$ *is continuous for* $c \geqq 0$, *continuously differentiable of every required order for* $c > 0$, *increasing* ($U'(c) > 0$) *and strictly concave* ($U''(c) < 0$). *Moreover,* $cU'(c)$ *is increasing for* $c > 0$ ($-cU''(c)/U'(c) < 1$, *intertemporal gross substitutability) and* $\lim_{c\to 0} U'(c) = +\infty$, $\lim_{c\to\infty} U'(c) = 0$.

We note that when $cU'(c)$ is increasing, the function $\varphi(c,\nu)$ appearing in the NSS (3.1) is increasing in c, since $c\varphi_c'(c,\nu) = \theta\,[cU'(c) - \nu cU'(\nu c)] > 0$. If we assume in addition

$$(3.c) \qquad 0 = \varphi(0,\nu) < x_o = \lim_{x\to 0} V(x) < \lim_{c\to\infty} \varphi(c,\nu) \leqq \lim_{x\to\overline{x}} V(x),$$

we may conclude since $V(0) = 0$

Lemma 3.1. *Under assumptions (3.a), (3.b), (3.c), the Non-Shirking Schedule (NSS) (3.1) determines uniquely effort* x *as a function of consumption* $x = h(c,\nu)$ *by* $h(0,\nu) = 0$ *when* $c = 0$, *and by* $h(c,\nu) \overset{def}{\equiv} V^{-1}(\varphi(c,\nu))$ *for* $c > c_o$ *where* $c_o > 0$ *is given by* $x_o = \varphi(c_o,\nu)$.

Under the above assumptions, an optimum efficiency wage contract is characterized by an employed worker consumption $c^* > c_o$ and effort level $x^* = h(c^*,\nu)$ that maximizes effort per unit of consumption $x/c = h(c,\nu)/c$. We may expect existence and unicity of such a maximum when $h(c,\nu)$ is strictly concave in consumption for $c > c_o$ (Fig. 1. Note the importance of a fixed disutility cost of effort $x_o > 0$ to ensure $c^* > c_o > 0$ and $x^* > 0$). Since $V(x)$ is strictly convex, this property will be guaranteed if the function $\varphi(c,\nu)$ appearing in the NSS (3.1) is itself strictly concave with respect to c. As

$$c^2\varphi''_{c^2}(c,\nu) \equiv \theta\left[c^2U''(c) - \nu^2c^2U''(\nu c)\right],$$

this will be the case if we assume on top of (3.b)

$$(3.d) \qquad -c^2U''(c) \textit{ is increasing } (-cU'''(c)/U''(c) < 2).$$

In the case of a constant elasticity specification, $U(c) = c^\eta$, assumptions (3.b) and (3.d) are satisfied whenever $\eta < 1$.

Fig. 1

Proposition 3.2. *Under assumptions (3.a), (3.b), (3.c), (3.d), there is a unique worker's effort level* $x^* > 0$ *and consumption* $c^* > c_o$ *that maximizes effort per unit of consumption* x/c *under the Non-Shirking Schedule (NSS)*

$$V(x) = \theta\left[U(c) - U(\nu c)\right] \equiv \varphi(c, \nu).$$

The optimum is defined uniquely by the first order condition

$$c^* \varphi_c'(c^*, \nu) = x^* V'(x^*).$$

The (simple) proof of this fact is given in appendix B. Intuitively, one should expect that the maximum effort per unit of consumption x^*/c^* involved in the optimum efficiency wage contract, goes down when the unemployment compensation rate $0 \leqq \nu < 1$ goes up, since firms have to pay higher wages, other things being equal, in order to compensate the greater incentive for workers to shirk. Formally, the function $\varphi(c, \nu)$ appearing in the NSS decreases, so that the graph of $x = h(c, \nu)$ in Fig. 1 shifts to the right, making the optimum ratio x^*/c^* to go down. It can be verified that the workers consumption c^* then actually increases, while the consequence on the effort level x^* is ambiguous.

Corollary 3.3. *Under assumptions* (3.*a*), (3.*b*), (3.*c*), *(3.d), increasing the rate* $0 \leqq \nu < 1$ *of unemployment insurance makes the effort per unit of consumption* x^*/c^* *of the optimum efficiency wage contract to go down, the corresponding consumption* c^* *to increase, whereas the variation of the effort level* x^* *is ambiguous.*

The analytical proof of this fact may be found in Appendix B. Explicit calculations that can be verified by direct inspection, show that in the case of a constant elasticity specification, the effort level x^* involved in the optimum efficiency wage contract is actually constant, while the corresponding consumption c^* increases and goes up to infinity when $0 \leqq \nu < 1$ increases and tends to 1.

Corollary 3.4 *Let* $V(x) = x_o + x^{\xi}$ *for* $0 < x < +\infty$, *and* $U(c) = c^{\eta}$ *for* $c > 0$, *with* $0 < \eta < 1 < \xi$ *(Assumptions (3.a), (3.b), (3.c), (3.d)).*

The Non-Shirking Schedule (NSS) is given by

$$x = h(c, \nu) = (\theta(1 - \nu^{\eta})c^{\eta} - x_o)^{1/\xi} \quad \textit{for} \quad c > c_o = \left[x_o/(\theta(1 - \nu^{\eta}))\right]^{1/\eta}.$$

The maximum of effort per unit of consumption x/c *in the optimum efficiency wage contract is obtained for* $c^* = c_o(\xi/(\xi - \eta))^{1/\eta} > c_o$ *and stipulates the effort level* $x^* = (x_o\eta/(\xi - \eta))^{1/\xi}$.

When the rate of unemployment insurance $0 \leqq \nu < 1$ *goes up and tends to 1, the optimum effort level* x^* *stays constant while the worker's consumption* c^* *increases and goes to* $+\infty$.

4 Perfect foresight intertemporal unemployment equilibria

We analyze in this section deterministic intertemporal equilibria with perfect foresight, under the maintained hypothesis :

$(4.a)$ *Assumptions* $(2.a)$ *on firms, and* $(3.a)$, $(3.b)$,$(3.c)$, $(3.d)$ *on workers, hold.*

We keep on focusing on equilibria statisfying $R_t > p_t/p_{t+1}$ and (2.3) for all $t \geqq 0$, so that capitalists save only in the form of capital, $k_{tc} > 0$, while workers hold cash only. Equilibrium at date t on the market for capital services k_{t-1} and for the amount of newly invested capital stock k_t, together with capitalists' saving behavior, $k_t = \beta_c R_t k_{t-1}$ with $R_t = \rho_t + 1 - \delta$, and firms' profit maximization (2.6), yields

$$k_t = \beta_c R(A, a_t) k_{t-1}, \text{ with } R(A, a_t) = a\rho(a_t) + 1 - \delta. \tag{4.1}$$

Under the maintained assumption of involuntary unemployment, i.e. $n_t < 1$ for all $t \geqq 0$, equilibrium on the labour maket is achieved through profit maximizing efficiency wage contracting and rationing of the labour supply down to the firms' labour demand n_t. The efficiency wage offered by firms determines $x_t = x^* > 0$ and $c_{t+1,w} = c^* > 0$ for a worker employed at t, for all $t \geqq 0$. Given the initial capital stock $k_{t-1} > 0$, determination of the capital-efficient labour ratio $a_t = k_{t-1}/(n_t x^*)$ at date t is then equivalent to the determination of the real rental rate of capital services $\rho_t = A\rho\left(a_t\right)$, or of the real wage per unit of effort $\omega_t/x^* = A\omega\left(a_t\right)$, as well as of the current employment rate n_t.

We consider finally the equilibrium of the money market at date $t+1$ (which will imply, by Walras law, equilibrium of the remaining market for output). We assume that the money supply is constant and equal to $M > 0$ for all t. At the beginning of period $t+1$, the whole money stock M is held by workers, who got it at date t either as wage income if they were employed, or as unemployment compensation otherwise. Thanks to the binding finance constraint, workers spend their money balances in period $t+1$ to finance their consumption, either c^* if previously employed at t, or νc^* if not. This implies

$M/p_{t+1} = n_t c^* + (1 - n_t)\nu c^*$. On the other hand, the whole money stock will end up at $t+1$ in the pocket of workers as well, in the form of wage income for workers who are employed then, or of unemployment compensation for those who are not. Since unemployment insurance is entirely financed out of wages (see (2.2)), this means that $M/p_{t+1} = n_{t+1}\omega_{t+1}$, where $n_{t+1} < 1$ is employment and ω_{t+1} is the real wage at date $t+1$.

As the real wage per unit of effort at $t+1$ is also given by $\omega_{t+1}/x^* = A\omega\left(a_{t+1}\right)$ and $n_{t+1}x^* = l_{t+1} = k_t/a_{t+1}$, we get

$$k_t A\omega\left(a_{t+1}\right)/a_{t+1} = g(k_{t-1}/a_t), \text{ where } g(l) = l\frac{c^*}{x^*} + (1 - \frac{l}{x^*})\nu c^* \qquad (4.2)$$

which determines implicitly, given $k_{t-1} > 0$ and $a_t > 0$, and thus the new equilibrium capital stock $k_t > 0$ through (4.1), the equilibrium capital-efficient labour ratio $a_{t+1} = k_t/l_{t+1} > 0$ at $t+1$.

Definition 4.1. *A perfect foresight intertemporal unemployment equilibrium is a sequence of capital stocks $k_{t-1} > 0$ and of capital-efficient labour ratios $a_t = k_{t-1}/l_t > 0$ with $n_t = k_{t-1}/\left(a_t x^*\right) < 1$ for all $t \geqq 0$, such that*

$$\begin{array}{rclll} k_t & = & \beta_c R(A, a_t) k_{t-1}, \textit{ where} & R(A, a_t) = & A\rho(a_t) + 1 - \delta, \\ k_t A\omega(a_{t+1})/a_{t+1} & = & g\left(k_{t-1}/a_t\right), \quad \textit{where} & g(l) \quad = & l\frac{c^*}{x^*} + (1 - \frac{l}{x^*})\nu c^* \end{array}$$

A distinctive feature of the above two-dimensional deterministic dynamics is that its structure is in fact identical to the dynamics that would arise in the competitive specification with elastic labour supply $l > 0$ analyzed in Woodford (1986) or GPV (1998). The only but essential difference is that the function $g(l)$ appearing here, that stands for aggregate consumption of employed and unemployed workers as a function of employment of efficient labour $l = nx^* < x^*$, would be replaced there by the workers' offer curve $\gamma(l)$, i.e. their aggregate consumption as a function of their labour supply $l > 0$ resulting from their utility maximizing consumption-leisure choice under competitive conditions. We shall exploit this feature systematically in the next section, to show that it is much likely to generate deterministic local indeterminacy and bifurcations, in particular when the unemployment insurance rate $\nu < 1$ is large. We focus here on existence and unicity of a deterministic stationary solution $k_t = \overline{k} > 0$, $a_t = \overline{a} > 0$ for all t, of the dynamical system in Definition 4.1, and how it varies, as well as the agents welfare, when unemployment insurance increases.

Such a stationary state is characterized by $\beta_c R(A, \overline{a}) = 1$, which determines uniquely $\overline{a} > 0$, given the technology, thus A, since under assumption $(2.a)$, $\beta_c R(A, a)$ decreases from $+\infty$ to $\beta_c(1 - \delta) < 1$ when a goes up from 0 to $+\infty$. Given this $\overline{a}$, the stationary capital stock $\overline{k} > 0$ or equivalently employment $\overline{n} = \overline{k}/(\overline{a}x^*)$, is determined by $\overline{n}A\omega(\overline{a}) = g(\overline{n}x^*)/x^* = \overline{n}(c^*/x^*) + (1-\overline{n})\nu(c^*/x^*)$. The left hand side can be interpreted as the workers' stationary real demand for money, since it is equal to the real wage bill, per unit of effort. Given $\overline{a} > 0$, it increases linearly, when stationary employment $\overline{n}$ varies between 0 and 1, from 0 to $A\omega(\overline{a})$. The right hand side stands for the workers' stationary real supply of money when buying consumption, again per unit of effort. It increases also linearly, from $\nu c^*/x^*$ to c^*/x^*, when $\overline{n}$ varies from 0 to full employment. The intersection of these "demand and supply" schedules determines a unique stationary employment $0 < \overline{n} < 1$ if and only if there is unemployment insurance, $\nu > 0$, and the technology is productive enough, i.e. when the marginal productivity of one additional unit of labor who provides the effort level x^*, $Ax^*\omega(\overline{a})$, covers the extra consumption c^* needed to induce workers not to shirk (Fig. 2).

<u>Fig. 2</u>

We look next at the consequence on stationary employment $\overline{n}$ of increasing unemployment insurance $0 < \nu < 1$. One might expect such a policy to be detrimental to employment since by reducing workers' incentives not to shirk, it should lead, other things being equal, to a higher real cost of labour per unit of effort ω_t/x_t to be paid by firms through efficiency wage contracts. It turns out that the conclusion has to be in fact reversed when general equilibrium effects, as a consequence of increasing workers' consumption, are taken into account. The stationary capital-efficient labour ratio, solving $\beta_c R(A, \overline{a}) = 1$, and thus the real "demand" schedule $\overline{n}A\omega(\overline{a})$ in Fig 2, is independent of ν. On the other hand, we know from Corollary 3.3 that increasing the rate of unemployment insurance makes consumption per unit of effort c^*/x^* to go up, so that the real "supply" schedule $\overline{n}(c^*/x^*) + (1-\overline{n})\nu(c^*/x^*)$ is pushed up on two counts in Fig. 2 : directly through the increase of ν and indirectly through the induced increase of c^*/x^*. All this makes stationary employment to go up.

The ultimate consequences of increasing unemployment insurance on the agents' welfare are nevertheless ambiguous since we know that in the general case, the corresponding variation of the effort level x^* is ambiguous (Corollary 3.3). Sharp conclusions can be obtained in the case of the constant elasticity specification considered in Corollary 3.4, since the effort level x^* is then unchanged. Increasing stationary employment $\overline{n}$ as a result of greater unemployment insurance raises the stationary capital stock $\overline{k} = \overline{a}\overline{n}x^*$ and generates actually a Pareto welfare improvement along the stationary state for all agents.

Proposition 4.2. *Under the maintained assumption (4.a) :*

1. A deterministic stationary state, i.e. a stationary solution $k = \overline{k} > 0, a_t = \overline{a} > 0$ *of the dynamical system in Definition 4.1, is characterized by a unique capital-efficient labour ratio that solves* $\beta_c R(A, \overline{a}) = 1$. *It involves a rate of employment* $\overline{n} = \overline{k}/(\overline{a}x^*)$ *given by*

$$\overline{n} = \frac{\nu c^*/x^*}{A\omega(\overline{a}) + \nu(c^*/x^*) - (c^*/x^*)}$$

that is positive whenever there is unemployment insurance, $\nu > 0$, *and corresponds to unemployment,* $\overline{n} < 1$, *if and only if the technology is productive enough,* $Ax^*\omega(\overline{a}) > c^*$.

2. Increasing the rate of unemployment insurance $0 < \nu < 1$, *other things being equal, makes the stationary rate of employment* $\overline{n}$ *to go up. In the constant elasticity specification considered in Corollary 3.4, it leads to a Pareto welfare improvement, as it increases the discounted intertemporal utility of capitalists along the stationary state (*$\overline{k}$ *goes up), as well as the expectation of the workers' discounted intertemporal utility, whether they are initially employed or unemployed (*c^* *goes up,* x^* *is unchanged,* ν *increases and the probability of employment* $\overline{n}$ *is larger) as long as* $c^* < Ax^*\omega(\overline{a})$.

Our whole analysis rests upon the assumption that there is excess supply on the labour market, $n_t < 1$ for all $t \geqq 0$. If we fix the technology, in particular the productivity parameter A and thus $\overline{a}$, and increase the rate of unemployment insurance toward 1, it may be that the employed workers stationary consumption per unit of effort c^*/x^* increases above $A\omega(\overline{a})$ (as in the constant elasticity specification) so that the stationary value of $\overline{n}$ found in 1) of Proposition 4.2 exceeds 1. In such a case, there would be excess demand for labor, the equilibrium on that market being achieved through rationing of the firms' labor demand, in the spirit of traditional disequilibrium analysis (Barro and Grossman (1976), Benassy (1982, 2002), Drèze (1991), Malinvaud (1977)). We focus here on unemployment configurations only.

Remark 4.3. One verifies easily that along a stationary state, the real gross rate of return on capital is $\overline{R} = 1/\beta_c > 1 = p_t/p_{t+1}$, so indeed $R_t > p_t/p_{t+1}$ for all intertemporal equilibria in a small enough neighborhood of the stationary state. Capitalists hold only capital and no money along such equilibria, as indeed postulated in the text. On the other hand, along a stationary state, inequalities (2.3) will read in any period t :

1) $U'(c^*) > (\beta_w/\beta_c)U'(c^*)$ (when employed at dates $t-1$ and t),

2) $U'(\nu c^*) > (\beta_w/\beta_c)U'(c^*)$ (when unemployed at $t-1$ and employed at t),

3) $U'(\nu c^*) > (\beta_w/\beta_c)U'(\nu c^*)$ (when unemployed in both periods) and

4) $U'(c^*) > (\beta_w/\beta_c)U'(\nu c^*)$ (when employed at $t-1$ and unemployed at t).

The first three inequalities are satisfied when $\beta_c > \beta_w$. The fourth one requires that β_c is significantly larger than β_w, that the rate of unemployment insurance ν is close enough to 1, and that the workers' utility for consumption $U(c)$ displays low risk aversion (marginal utility $U'(c)$ does not decrease too fast). In that case, inequalities (2.3) will also hold along intertemporal equilibria within a small enough neighborhood of the stationary state, so that workers hold cash only and no capital, as indeed postulated in the text. In the constant elasticity specification, $U(c) = c^\eta$, inequalities 1) to 4) hold whenever $\nu > (\beta_w/\beta_c)^{1/(1-\eta)}$.

5 Deterministic local indeterminacy and bifurcations

We study in this section the deterministic local dynamics $(k_{t-1}, a_t) \to (k_t, a_{t+1})$ near the stationary state, as defined implicitly in Definition 4.1, in relation with the elasticities of the various functions involved there, in particular with the share $s = \overline{a}\rho(\overline{a})/f(\overline{a})$ of capital in total income, the elasticity of substitution σ between capital and efficient labour, and the elasticity of $g(l) = c^*[(l/x^*) + \nu(1 - (l/x^*))]$ giving the workers aggregate consumption as a function of efficient employment $l = nx^*$, all evaluated at the stationary state $(\overline{k}, \overline{a})$. As mentioned earlier, our analysis will be greatly simplified by the fact that the dynamics in the case of a competitive labour market and an elastic labour supply, as studied in GPV (1998), is actually the same as in Definition 4.1, with the function $g(l)$ being replaced there by the workers' competitive offer curve $c = \gamma(l)$.

In a nutshell, the elasticity $\varepsilon_\gamma(\overline{l}) = \overline{l}\gamma'(\overline{l})/\gamma(\overline{l})$ of the competitive offer curve in GPV (1998) is replaced in our context by

$$\varepsilon_g(\overline{l}) = \frac{\overline{l}g'(\overline{l})}{g(\overline{l})} = \frac{(1-\nu)\overline{n}}{\nu + (1-\nu)\overline{n}} \quad . \tag{5.1}$$

The essential characteristic of the competitive specification in GPV (1998) is that $\varepsilon_\gamma > 1$, because the workers' competitive labour supply, defined implicitly as a function of the real wage ω by $\omega l = \gamma(l)$, has an elasticity equal to

$1/[\varepsilon_\gamma(l)-1]$ that is assumed to be positive. The consequence of this fact in the competitive case is that local indeterminacy and bifurcations can occur only for very low capital-labour elasticities of substitution, $\sigma < s$, too low to be empirically relevant. By contrast, the elasticity $\varepsilon_g(\bar{l})$ in (5.1) is clearly less than 1 and can be actually quite small when unemployment insurance ν is large (close to 1). As will be made precise shortly, this feature will reverse drastically the qualitative conclusions, in the sense that in the efficiency wage case, local determinacy will always occur for $\sigma < s$, while local indeterminacy will be observed for larger and quite plausible values of σ and ν close enough to 1.

The approach just outlined rests on the possibility to make the rate ν of unemployment insurance vary close enough to 1 so as to make the elasticity $\varepsilon_g(\bar{l})$ small. This may lead to problematic self-contradictory configurations if we fix completely the technology while doing so, since in such cases the stationary value of $\bar{n}$ in Proposition 4.2 may involve "overemployment", $\bar{n} > 1$. In order to avoid such configurations here, we shall employ a simple trick, namely to fix all fundamental characteristics of the economy, including firms' technology, except the productivity parameter $A > 0$, that will be in fact systematically "scaled" so as to guarantee that stationary employment will be "normalized" to a pre-specified value $0 < \bar{n} < 1$. That such a scaling procedure is indeed feasible is easily seen by remarking that the first equation characterizing a stationary state $(\bar{k}, \bar{a})$, i.e. $\beta_c R(A, \bar{a}) = 1$, defines implicitly $\bar{a}$ as a function $\alpha(A)$ that is increasing from 0 to $+\infty$ as the productivity parameter A varies from 0 to $+\infty$. So given the pre-specified "normalized" value of employment $\bar{n}$, the second equation characterizing the stationary state, i.e. $\bar{n}A\omega(\alpha(A)) = \bar{n}(c^*/x^*) + (1-\bar{n})\nu(c^*/x^*)$, leads to a unique solution for the scaling productivity parameter A, thanks to assumption $(2.a)$ on firms, however close to 1 is the rate ν of unemployment insurance, and correlatively, however large is the employed workers' consumption per unit of effort c^*/x^* involved in the induced efficiency wage contract. We shall assume accordingly form now on :

$(5.a)$ *Assumption $(4.a)$ is supposed to hold. Employment along the deterministic stationary state $(\bar{k}, \bar{a})$, solution of $\beta_c R(A, \bar{a}) = 1$ and $\bar{n}A\omega(a) = \bar{n}(c^*/x^*) + (1-\bar{n})\nu(c^*/x^*)$, with $\bar{k} = \bar{n}x^*\bar{a}$, is assumed to be normalized to a pre-specified value $0 < \bar{n} < 1$, through an appropriate choice of the productivity parameter A.*

The local dynamics near such a normalized stationary state is well defined, thanks to the implicit function theorem, as long as we assume $\varepsilon_\omega = \bar{a}\omega'(\bar{a})/\omega(\bar{a}) \neq 1$.

Proposition 5.1. *Let assumption (5.a) hold. Let* $\varepsilon_\omega = \overline{a}\omega'(\overline{a})/\omega(\overline{a}) = s/\sigma$ *be the elasticity of the marginal productivity of efficient labour,* $|\varepsilon_R| = |\overline{a}R'(\overline{a})/R(\overline{a})| = \delta^*(1-s)/\sigma$ *be the elasticity of the real gross rate of return on capital, and* $\varepsilon_g = \overline{l}g'(\overline{l})/g(\overline{l}) = (1-\nu)\overline{n}/[\nu+(1-\nu)\overline{n}]$ *be the elasticity of the workers' aggregate consumption as a function of employment* $l = nx^*$*, all evaluated at the normalized stationary state, where* $s = \overline{a}\rho(\overline{a})/f(\overline{a})$ *is the share of capital in total income,* σ *is the elasticity of substitution between capital and efficient labour, and* $\delta^* = 1-\beta_c(1-\delta)$ *is assumed to be positive.*

Assume $\varepsilon_\omega \neq 1$ *or* $\sigma \neq s$*. Then Definition 4.1 defines uniquely a local dynamics* $(k_{t-1}, a_t) \to (k_{t-1}, a_{t+1})$ *near the normalized stationary state* $(\overline{k}, \overline{a})$*. The linearized dynamics for the deviations* $dk = k - \overline{k}, da = a - \overline{a}$ *are determined by*

$$dk_t = dk_{t-1} - \overline{l}\mid\varepsilon_R\mid da_t, \quad da_{t+1} = \frac{1}{\overline{l}}\frac{\varepsilon_g - 1}{\varepsilon_\omega - 1}dk_{t-1} - \frac{\varepsilon_g - \mid\varepsilon_R\mid}{\varepsilon_\omega - 1}da_t\ .$$

The sum T *and product* D *of the two eigenvalues solutions of the corresponding characteristic polynomial* $Q(z) \equiv z^2 - Tz + D = 0$ *are given by*

$$T = T_1 - (\varepsilon_g - 1)\sigma/(s-\sigma) \quad , \quad D = \varepsilon_g D_1$$

where $T_1 = 1 + D_1$ *and* $D_1 = (\delta^*(1-s) - \sigma)/(s-\sigma)$.

Proof : This is Proposition 4.1 together with (2.9) and (2.10) in GPV (1998), with ε_g replacing here ε_γ there, and δ^* here instead of the rate δ of capital depreciation there, on account of the fact that the capitalists' discount factor β_c was assumed to be equal to 1 in GPV (1998) whereas $\beta_c < 1$ here. Q.E.D.

We employ the same geometrical method as in GPV (1998), that allows to evaluate local stability and bifurcations of the two eigenvalues of the local linearized dynamics without actually computing them explicitly, by locating their sum and product in the (T, D) plane and by looking at how T and D move as a function of a few key economic parameters.

In the (T, D) plane, the line (AC) of equation $D = T - 1$ is the locus where one eigenvalue of the local dynamics is equal to $+1$. The line (AB) of equation $D = -T - 1$ corresponds to one eigenvalue equal to -1. On the segment [BC] of equation $D = 1$, $\mid T\mid \leqq 2$, the two local eigenvalues are complex conjugates with modulus 1 (Fig. 3). Since the two eigenvalues are stable when $T = 0, D = 0$, one gets by continuity two stable eigenvalues, hence local indeterminacy in the framework considered here with one single predetermined variable k_{t-1}, in the interior of the triangle ABC. Saddle point determinacy obtains in the two regions between the two lines (AB) and (AC) determined by $\mid T \mid > \mid 1 + D \mid$. The stationary state is a source, hence locally determinate in the regions between the two lines (AB) and (AC) above the segment [BC], and below the point A. When considering a one dimensional family of economies with varying characteristics so that (T, D) moves along a curve in the plane, crossing the line (AC) leads generically to a so-called transcritical bifurcation (since we have here a persistent stationary state). Crossing the line (AB) leads generically to a Flip bifurcation involving a cycle of period 2 near the stationary state, whereas crossing the interior of the segment [BC] leads generically to a Hopf bifurcation involving an invariant closed curve near the stationary state.

In what follows, we shall fix the capitalists and workers characteristics, the "normalized" stationary employment rate $0 < \overline{n} < 1$ and all characteristics of the technology except the productivity parameter A, that is assumed to adjust as in assumption $(5.a)$. We shall conduct the analysis as if the three parameters $\sigma, s = \overline{a}\rho(\overline{a})/f(\overline{a})$ and ε_g appearing in Proposition 5.1 can be made to vary independently. One should keep in mind nevertheless when applying our results that, strictly speaking, when ε_g is made to move through for instance a variation of the rate of unemployment insurance ν, "normalized" stationary employment $\overline{n}$ being fixed as in assumption $(5.a)$, the stationary capital-efficient labour ratio $\overline{a}$, hence the share s of capital in total income, is bound to move as well (except in the specific case of a Cobb-Douglas production function).

The key observation is here as in GPV (1998, Section 2), that T and D appearing in Proposition 5.1 vary linearly with ε_g, when s, σ (and δ^*) are kept fixed. The corresponding point (T, D) in the plane moves accordingly then along a straight line $\Delta(\sigma)$ going through the point $M_1(\sigma) = (T_1(\sigma), D_1(\sigma))$ for $\varepsilon_g = 1$ that belongs to the line (AC) since $T_1(\sigma) = 1 + D_1(\sigma)$ with $D_1(\sigma) = (\delta^*(1-s) - \sigma)/(s-\sigma)$, and having a slope equal to $1 - (\delta^*(1-s)/\sigma)$. The part of $\Delta(\sigma)$ that was relevant in the competitive case studied in GPV (1998) was the half $-$subline corresponding to $\varepsilon_g > 1$ (see the dashed half-lines in Fig. 3, 4, 5). In the efficiency wage specification considered here, on has $0 < \varepsilon_g < 1$ (see (5.1)), so the relevant portion is the segment (drawn in plain in Fig. 3, 4, 5) generated by varying ε_g or ν between 0 and 1 and joining the point $M_1(\sigma)$ on (AC) (for $\varepsilon_g = 1$ or $\nu = 0$) to the point $M_0(\sigma) = (T_0(\sigma), 0)$ on the horizontal axis $D = 0$ (corresponding to $\varepsilon_g = 0$ or $\nu = 1$) with $T_0(\sigma) = 1 + \delta^*(1-s)/(s-\sigma)$. The qualitative picture one gets then (Fig.3) makes clear right at the begining that local indeterminacy (i.e. intersection with the stability triangle ABC) cannot occur for the same range of elasticities σ in the competitive case of GPV (1998) and the efficiency wage model considered here, since for any σ, the half $-$subline of $\Delta(\sigma)$ corresponding to $\varepsilon_g > 1$ and the segment $[M_0(\sigma), M_1(\sigma)]$ stand on opposite sides of (AC).

Fig. 3

How the segment $[M_0(\sigma), M_1(\sigma)]$ varies with σ becomes in fact a simple consequence of the analysis made in GPV (1998, Section 2). We focus here as well on the case where $\delta^*(1-s)/s < 1$ ($\delta^* = 1-\beta_c(1-\delta)$ is bound to be quite small since the period is short). When $\sigma = 0$, the line $\Delta(\sigma)$ crosses (AC) at a point below C with a positive ordinate ($0 < D_1(0) = \delta^*(1-s)/s < 1$), and is vertical (its slope is $-\infty$). When σ increases toward s, the point $M_1(\sigma)$ move down (AC) toward infinity, with $D_1(\sigma)$ going through 0 for $\sigma = \delta^*(1-s)$. The slope of $\Delta(\sigma)$ increases when σ goes up from 0 to s, from $-\infty$ to $1-\delta^*(1-s)/s < 1$ (it goes through 0 also for $\sigma = \delta^*(1-s)$). The outcome in the competitive case is that when $\sigma < s$, since the half $-$subline of $\Delta(\sigma)$ for $\varepsilon_g > 1$ stands above (AC) and intersects the stability triangle ABC only when $\sigma < \sigma_I$ where $\sigma_I = [\delta^*(1-s)+s]/2 < s$ solves $D_1(\sigma_I) = -1$, local indeterminacy and bifurcations occur then only for very low elasticities of substitution σ between capital and efficient labour (see the dashed lines in Fig. 4). Still when $\sigma < s$, the diagnosis is reversed as expected in the efficiency wage specification considered here, since for $0 < \varepsilon_g < 1$, the relevant segment $[M_0(\sigma), M_1(\sigma)]$ stands below (AC) and never intersects the stability triangle (Fig. 4) : the stationary state is then a saddlepoint, hence locally determinate, and no bifurcations occur.

Fig. 4 : The case $\sigma < s$

When $\sigma > s$ increases toward $+\infty$, the point $M_1(\sigma)$ goes down along (AC) from infinity to the point C, while the slope of $\Delta(\sigma)$ goes up from $1-\delta^*(1-s)/s$ to 1. The half $-$subline corresponding to $\varepsilon_g > 1$, for the competitive case, stands now below (AC) (see the dashed lines in Fig. 5). So when $\sigma > s$, in the competitive case, the stationary state is always a saddle, hence locally determinate for any $\varepsilon_g > 1$. Again, the diagnosis is quite the opposite in the efficiency wage specification considered here. When $\sigma > s$ and $0 < \varepsilon_g < 1$, the relevant segment $[M_0(\sigma), M_1(\sigma)]$ stands above (AC), with $M_1(\sigma)$ going along (AC) down to C, while $M_0(\sigma)$ is moving form $-\infty$ on the horizontal axis toward the point (1, 0) on (AC), when σ goes up from s to $+\infty$. So when $s < \sigma < \sigma_H$, where σ_H is determined by the property that $\Delta(\sigma_H)$ goes through $B = (-2, 1)$, the segment $[M_0(\sigma), M_1(\sigma)]$ never intersects the stability triangle ABC, so that the stationary state is always determinate (a source or a saddle) with a possible Flip bifurcation involving a cycle of period 2 as ε_g goes through ε_{gF} when the segment intersects (AB) (see Fig. 5). When $\sigma_H < \sigma < \sigma_{-1}$, where σ_{-1} is defined by the property that $T_0(\sigma_{-1}) = -1$, the segment $[M_0(\sigma), M_1(\sigma)]$ does intersect the stability triangle ABC, so that the stationary state is locally indeterminate for the intermediate range of elasticities $\varepsilon_{gF} < \varepsilon_g < \varepsilon_{gH}$, with the stationary state becoming a source through a Hopf bifurcation when ε_g goes up through ε_{gH} (when the segment crosses [BC]), or a saddle through a Flip bifurcation when ε_g goes down ε_{gF} (when the segment intersects (AB)). For higher values $\sigma_{-1} < \sigma$, the bifurcation value ε_{gF} becomes negative, so that local indeterminacy occurs for the whole range $0 < \varepsilon_g < \varepsilon_{gH}$. For a graphical illustration of these facts, see Fig. 5.

Fig. 5 : The case $s < \sigma$

Proposition 5.2. *Under the assumptions and notations of Proposition 5.1, let us assume* $\delta^*(1-s)/s < 1$.

1) $0 < \sigma < s$. *The stationary state is a saddle (locally determinate) for any* $0 < \varepsilon_g < 1$.

2) $s < \sigma < \sigma_H = \dfrac{s}{2}\left[1 + \dfrac{\delta^*(1-s)}{s} + \left(1 - \dfrac{\delta^*(1-s)}{s}\right)^{1/2}\right]$. *The stationary state is a saddle (locally determinate) for* $0 < \varepsilon_g < \varepsilon_{gF} = 1 - 2[2\sigma - s - \delta^*(1-s)]/[2\sigma - \delta^*(1-s)] < 1$, *a source when* $\varepsilon_{gF} < \varepsilon_g$. *A Flip bifurcation occurs generically when* ε_g *goes up through* ε_{gF}.

3) $\sigma_H < \sigma < \sigma_{-1} = s + \delta^*(1-s)/2$. *The stationary state is a saddle (locally determinate) for* $0 < \varepsilon_g < \varepsilon_{gF}$, *a sink (locally indeterminate) for* $\varepsilon_{gF} < \varepsilon_g < \varepsilon_{gH} = (\sigma - s)/(\sigma - \delta^*(1-s))$, *a source (locally determinate) for* $\varepsilon_{gH} < \varepsilon_g < 1$. *Generically, a Flip bifurcation occurs when* ε_g *goes down through* ε_{gF}, *a Hopf bifurcation when* ε_g *goes up through* ε_{gH}.

4) $\sigma_{-1} < \sigma$. *The stationary state is a sink (locally indeterminate) for* $0 < \varepsilon_g < \varepsilon_{gH}$, *a source (locally determinate) for* $\varepsilon_{gH} < \varepsilon_g < 1$. *Generically, a Hopf bifurcation occurs when* ε_g *goes up through* ε_{gH}.

Proof : This formal statement is adapted from Proposition 2.2 in GPV (1998) in view of the preceding discussion and Fig. 4, 5, with ε_g and δ^* replacing ε_γ and δ there. The expressions of ε_{gF} and ε_{gH} above are identical to the critical values γ_F and γ_H given in GPV, again up to $(\varepsilon_\gamma, \delta) \rightarrow (\varepsilon_g, \delta^*)$. The expression of σ_H above is obtained from the condition that $\Delta(\sigma_H)$ goes through $B = (-2, 1)$, which leads to a second degree equation, and by picking up the solution $\sigma_H > s$ (whereas GPV focused on the other solution $\sigma_H < s$). Q.E.D.

One may remark finally that the above critical values σ_{-1} and ε_{gH} determining the range of local indeterminacy, are compatible with quite plausible estimations of the corresponding elasticities. In particular, if we focus on the case of a Cobb-Douglas production function with a share of capital in total income of about $s = 1/3$, then $\sigma = 1$ is bound to exceed undoubtedly $\sigma_{-1} = s + \delta^*(1-s)/2$ (or even $\sigma_0 = s + \delta^*(1-s)$ for which $M_0(\sigma_0)$ coincides with the origin $(0,0)$, see Fig. 5), because $\delta^* = 1 - \beta_c(1-\delta)$ must be very small since the period is quite short, to be compatible with the liquidity constraint that workers face. Then for $\sigma = 1$, the critical value ε_{gH} is bound to be slightly larger but quite close to $1 - s \backsimeq 2/3$. If we focus on a "normalized" rate of employment $\overline{n} = 0.9$ to fix ideas, the corresponding threshold value for the rate of unemployment insurance derived from (5.1), $\nu_H = \overline{n}(1-\varepsilon_{gH}) / [\varepsilon_{gH} + \overline{n}(1-\varepsilon_{gH})]$ is slightly below but quite close to 0.31. In such a case, when the technology is Cobb-Douglas, $\sigma = 1$, local indeterminacy is occurring in the efficiency wage model considered here for a wide and quite plausible range of unemployment insurance rates $\nu > \nu_H \backsimeq 0.31$. Our analysis shows that such a result is robust to moderate changes of the economic parameters $\sigma, \delta^*, \overline{n}$, etc.... Such a conclusion is to be contrasted with the outcome obtained in the competitive case, as in GPV (1998), where local indeterminacy could occur only for implausibly low elasticities $\sigma < \sigma_I < s$.

6 Conclusion

We showed in this paper that introducing unemployment through efficiency wage contracts *à la* Negishi-Solow, and unemployment insurance financed by taxation of labour income, could generate deterministic local indeterminacy for a wide and plausible range of elasticities of substitution between capital and (efficient) labour (for elasticities greater than a small lower bound), that includes in particular Cobb-Douglas production functions. While raising unemployment insurance has a positive impact on the rate of employment and can in some cases generates a Pareto welfare improvement for all agents along the deterministic stationary state, it increases the likelihood to get local indeterminacy and thus possible inefficiencies due to expectations coordination failures. These results are to be contrasted with the findings obtained in the competitive case, where local indeterminacy could occur only for low capital-labour elasticities of substitution (as in GPV (1998)). One way out that has been much explored in the literature has been to consider increasing returns through productive externalities with the unfortunate drawback however, that their size had to be often too large to be empirically plausible in order to get local indeterminacy (Barinci and Cheron (2001), Benhabib and Farmer (1994), Cazzavillan, Lloyd-Braga and Pintus (1998), Hintermaier (2003)). The results obtained in this paper suggest that an alternative promising avenue in this respect would be to introduce unemployment, confirming in that other similar findings by Dufourt, Lloyd-Braga and Modesto (2004).

These results suggest quite a few directions for future research. First, we focused here on deterministic perfect foresight equilibria, with the consequence that efficiency wage contracts led to a consumption and an effort levels for workers that were constant over time. This somewhat unrealistic feature goes away as soon as we consider stationary stochastic endogenous fluctuations generated by sunspots, since then workers' consumption and effort should be random. Studying the existence and properties of such stationary rational expectations sunspots equilibria should be of interest both from an analytical and an economic viewpoints. Analytically, existence of these sunspot equilibria in relation with deterministic local indeterminacy is non-standard in the sense that it does not fit the general framework studied in GPV (1998, Section 3) : its study should raise interesting theoretical issues. Economically, analyzing the time series properties of such stochastic sunspot equilibria, and their possible connections with so-called "New Keynesian Economics" (Mankiw and Romer (1991)) should also raise interesting issues. In particular, it should be of interest to analyze the dynamic properties of such unemployment business cycles depending upon whether firms offer efficiency wage contracts that are fully indexed on sunspots ("complete" contracts leading to "real wage" rigidities as in Grandmont (1989)), or that are only partially indexed or even completely predetermined (predetermined nominal wages in the spirit of traditional disequilibrium theory as in Barro and Grossman (1976), Benassy (1982, 2002), Dreze (1991), Grandmont and Laroque (1976), Malinvaud (1977), Negishi (1979)). There is in fact a whole array of interesting possibilities along such lines, e.g. staggered efficiency wage setting, whether exogenous or modeled as the rational response of firms under the gradual diffusion of “sticky information” about sunspots (see Taylor (1999), Mankiw and Reiss (2003)).

The analysis of this paper has been also made by relying on oversimplified assumptions that it would be interesting to relax. One set of assumptions, namely that workers were significantly more impatient than capitalists, that unemployment insurance was high, and that workers had low risk aversion, was here to guarantee that both types of workers, whether employed or unemployed, did not save more than the cash receipts (wage income or unemployment insurance) they got. While this feature led to great simplifications of the analysis, it is somewhat extreme. By relaxing slightly these conditions, one should get a picture where capitalists would hold capital only, unemployed workers would hold money only, while employed workers would hold both assets (money as a result of the finance constraint and capital as additional saving). Even though the analysis would be more complex technically since one would lose the two-periods overlapping generations structure of the workers' behavior, that would generate a sort of heterogeneity that appears not to be completely irrelevant and might be interesting to explore further, by contrast with standard "representative agent" models. Another simplifying assumption we made here, namely that the probability for a worker to get a job offer in any period t was equal to the firms' labor demand $n_t < 1$, and thus independent of the worker's past employment status, should be relaxed as it neglects important channels generating unemployment persistence, such as for instance job search. Relaxing such an assumption would allow in particular to take into account one possible negative consequence of raising unemployment insurance that we abstracted from, namely that increased unemployment compensation might lower the intensity of job search and thus hurt employment (Baily (1978), Nicholson and Needels (2006)).

Appendix A

We specify precisely in this appendix the infinite horizon decision problem of the agents (capitalists and workers) and the first order Euler conditions characterizing their choices.

At date t, capitalists have (perfect foresight) expectations of (non random) future prices of the good $p_{t+j} > 0$ and real gross rates of return on capital $R_{t+j} > 0$ for $j \geqq 1$, and observe these quantities in the current period $j = 0$. They have to choose current and future consumptions $c_{t+j,c} \geqq 0$, capital and money holdings, $k_{t+j,c} \geqq 0$ and $m_{t+j,c} \geqq 0$, for $j \geqq 0$, given their initial capital and money stocks $k_{t-1,c} > 0$ and $m_{t-1,c} \geqq 0$, so as to maximize their discounted intertemporal utility $\sum_{j=0}^{\infty} (\beta_c)^j \, Log c_{t+j,c}$ under the current and expected budget constraints, for $j \geqq 0$

$$p_{t+j}c_{t+j,c} + p_{t+j}k_{t+j,c} + m_{t+j,c} \leqq p_{t+j}R_{t+j}k_{t+j-1,c} + m_{t+j-1,c} \ .$$

In view of the logarithmic instantaneous utility function, optimum consumption is always positive, $c_{t+j,c} > 0$. The first order conditions (FOC) for capital and money holdings become then

$$(k_{t+j,c} \geqq 0) \qquad c_{t+j+1,c} \geqq \beta_c \ R_{t+j+1} \ c_{t+j,c}, \tag{A.1}$$

$$(m_{t+j,c} \geqq 0) \qquad p_{t+j+1} \ c_{t+j+1,c} \geqq \beta_c \ p_{t+j} \ c_{t+j,c}, \tag{A.2}$$

with complementary slackness in each set of inequalities. They express as usual that the marginal cost, in terms of the utility of foregone current consumption, of investing at the margin in capital or money, is at least as large as the marginal expected utility gain of increased future consumption. These FOC have to be supplemented with the transversality condition $\lim_{j\to\infty} (\beta_c)^j \left(k_{t+j,c} + (m_{t+j,c}/p_{t+j})\right)/c_{t+j,c} = 0$.

Since we focus on perfect foresight intertemporal equilibria near a stationary state with $p_t = \overline{p} > 0$ and $c_{t,c} = \overline{c}_c > 0$ for all $t \geqq 0$, the second inequality relative to money (A.2) will be always strict since $\beta_c < 1$, so capitalists will choose not to hold money, $m_{t+j,c} = 0$ for all $j \geqq 0$. The budget constraints reduce then to $c_{t+j,c} + k_{t+j,c} = R_{t+j} \ k_{t+j-1,c}$. The optimum capital stocks are positive (to finance positive consumption) so the corresponding FOC (A.1) is an equality that reads $c_{t+j+1,c} = \beta_c \, R_{t+j+1} \, c_{t+j,c}$ or :

$$\begin{aligned} \frac{k_{t+j,c}}{c_{t+j,c}} &= \beta_c \left(1 + \frac{k_{t+j+1,c}}{c_{t+j+1,c}}\right) \\ &= \beta_c + \beta_c^2 + \ldots + \beta_c^n \left(1 + \frac{k_{t+j+n,c}}{c_{t+j+n,c}}\right). \end{aligned}$$

This leads to the policy function $k_{t+j,c}/c_{t+j,c} = \beta_c/(1-\beta_c)$ as long as $\lim_{n\to\infty} \beta_c^n k_{t+j+n,c}/c_{t+j+n,c} = 0$, which is in fact the transversality condition. One gets then $c_{tc} = (1-\beta_c) R_t k_{t-1,c}$ and $k_{tc} = \beta_c R_t k_{t-1,c}$ as stated in (2.1) in the text.

At date t, workers have (perfect foresight) expectations of (non random) future prices of the good $p_{t+j} > 0$, real gross rates of return on capital $R_{t+j} > 0$, employment rates $0 \leqq n_{t+j} < 1$, efficiency wage contracts (w_{t+j}, x_{t+j}) and wage income tax rates $0 \leqq 1 - d_{t+j} \leqq 1$ for $j \geqq 1$, and observe these quantities in the current period $j = 0$. They determine in particular the wage or unemployment insurance incomes of a worker at $t, b_t = d_t w_t$ if he is employed or $b_t = d_t \nu w_t$ if he is not, as well as his (rational) expectations of the corresponding random incomes in the future for $j \geqq 1, b_{t+j} = d_{t+j} w_{t+j}$ if he is employed at $t + j$ (with probability n_{t+j}), and $b_{t+j} = d_{t+j} \nu w_{t+j}$ if he is not (with probability $(1 - n_{t+j})$). Workers have to choose current and future consumptions $c_{t+j,w} \geqq 0$, capital and money holdings $k_{t+j,w} \geqq 0$ and $m_{t+j,w} \geqq 0$ for $j \geqq 0$, given their initial capital and money stocks $k_{t-1,w} \geqq 0$ and $m_{t-1,w} > 0$, under the current and expected budget constraints, for $j \geqq 0$

$$p_{t+j}c_{t+j,w} + p_{t+j}k_{t+j,w} + m_{t+j,w} \leqq p_{t+j}R_{t+j}k_{t+j-1,w} + m_{t+j-1,w} + b_{t+j},$$

and the current and expected finance constraints $m_{t+j,w} \geqq b_{t+j}$ or

$$p_{t+j}c_{t+j,w} + p_{t+j}k_{t+j,w} \leqq p_{t+j}R_{t+j}k_{t+j-1,w} + m_{t+j-1,w}.$$

In this framework, the choices planned for any future period $t + j, j \geqq 1$, i.e. $(c_{t+j,w}, k_{t+j,w}, m_{t+1,w})$ are random since they may be affected by the random income b_{t+j}, and may in fact depend on the whole history of the worker's employment status. Workers seek accordingly to maximize at t the expectation of the discounted intertemporal utility $E_t[\sum_{j=0}^{\infty} (\beta_w)^j$ $(U(c_{t+j,w}) - \beta_w V(x_{t+j,w}))]$, in which the (random) sequence $x_{t+j,w}$ is actually given by the expected future efficiency wage contracts, since it is equal to the required effort level x_{t+j} if he is employed at $t + j$ (with probability n_{t+j}) and to 0 if he is not (with probability $1 - n_{t+j}$).

In view of the Inada conditions satisfied by $U(c)$ (assumption $(3.b)$), optimum consumption is always positive, $c_{t+j,w} > 0$. The first order conditions (FOC) for capital and money holdings and for consumptions become then for $j \geqq 0$:

$$(k_{t+j,w} \geqq 0) \quad U'(c_{t+j,w}) \geqq \beta_w E_{t+j}\left[R_{t+j+1} U'(c_{t+j+1,w})\right], \tag{A.3}$$

$$(m_{t+j,w} \geqq 0) \qquad \lambda_{t+j} \geqq (\beta_w)^{j+1} E_{t+j}\left[\frac{U'(c_{t+j+1,w})}{p_{t+j+1}}\right], \tag{A.4}$$

$$(c_{t+j,w} > 0) \qquad \lambda_{t+j} + \mu_{t+j} = (\beta_w)^j \frac{U'(c_{t+j,w})}{p_{t+j}}, \tag{A.5}$$

where $\lambda_{t+j} > 0$ and $\mu_{t+j} \geqq 0$ are the shadow multipliers of the budget constraint and the finance constraint at $t+j$, respectively, with complementary slackness in each set of inequalities (A.3) and (A.4), together with the relevant transversality condition, $\lim_{j\to\infty} (\beta_w)^j (k_{t+j,w} + (m_{t+j,w}/p_{t+j}))U'(c_{t+j,w}) = 0$. These FOC express as usual that the marginal utility cost of investing in capital or money is at least as large as the marginal expected utility gain.

As stated in the text (see (2.3)), we focus on intertemporal equilibria near a stationary state satisfying $R_t = 1/\beta_c > 1$ and such that $U'(c_{tw}) > \beta_w E_t\left[R_{t+1} U'(c_{t+1,w})\right]$. As made precise in the discussion following (2.4) and in Remark 4.3, this condition will be satisfied ex post if $\beta_c > \beta_w$ and if $U'(c^*) > (\beta_w/\beta_c) U'(\nu c^*)$, i.e. when unemployment insurance is high (ν is close to 1) and $U(c)$ displays low risk aversion. It implies that the inequality (A.3) is strict, so that $k_{t+j,w} = 0$, for all $j \geqq 0$. Money holdings have then to be positive in order to finance positive consumption, $m_{t+j,w} > 0$, so (A.4) is an equality for all $j \geqq 0$. The fact that the finance constraint is binding, or $\mu_{t+j} > 0$, is then expressed by the strict inequality

$$\frac{U'(c_{t+j,w})}{p_{t+j}} > \beta_w E_{t+j}\left[\frac{U'(c_{t+j+1,w})}{p_{t+j+1}}\right],$$

which is always verified near a stationary state as long as (A.3) holds, since the deterministic sequences R_{t+j+1} and p_{t+j}/p_{t+j+1} are then close to $1/\beta_c > 1$ and 1 respectively. All the statements about the workers choices, in particular the fact that they display a simple two-periods overlapping generations structure, follow then immediately.

Appendix B

The proofs of Proposition 3.2 and Corollary 4.3 are gathered in this appendix

Proof of Proposition 3.2 : The issue is to find the maximum for $c > c_0$ of $h(c,\nu)/c = V^{-1}(\varphi(c,\nu))/c$. From Lemma 3.1 and the ensuing discussion leading to assumption $(3.d)$, $h(c,\nu)$ is a positive, increasing, strictly concave function of c for $c > c_0$. It follows immediately that $h(c,\nu)/c$ is increasing from 0 when c increases from and near c_0, and globally single peaked on $(c_0, +\infty)$. Indeed the elasticity of $h(c,\nu)/c$ has the same sign as $h'_c(c,\nu)\,c - h(c,\nu)$, which is positive for $c > c_0$ close to c_0, and is decreasing since its derivative with respect to c is $h''_{c^2}(c,\nu)\,c < 0$.

The only case where $h(c,\nu)/c$ does not reach a maximum at finite distance $c^* > c_0$ would be when $h(c,\nu)/c$ is non-decreasing on $(c_{0,} + \infty)$. That would imply that $h(c,\nu)$ is unbounded above, so that $\overline{x} = +\infty$. That would imply further that the elasticity of $h(c,\nu)/c$ with respect to c for $c > c_0$

$$\frac{c\varphi'_c(c,\nu)}{h(c,\nu)V'(h(c,\nu))} - 1 \tag{B.1}$$

should tend to -1 when c goes to $+\infty$, since in that case $c/h(c,\nu)$ should be non-increasing, while $\varphi'_c(c,\nu)$ would be decreasing from assumption $(3.d)$ and $V'(h(c,\nu))$ would tend to $+\infty$ from assumption $(3.a)$. Such a contradiction shows that $h(c,\nu)/c$ reaches a maximum at finite distance $c^* > c_0$. It is obtained by setting the elasticity (B.1) equal to 0, or equivalently $c^*\varphi'_c(c^*,\nu) = x^*V'(x^*)$, with $x^* = h(c^*,\nu)$. Q.E.D.

Proof of Corollary 3.3. It is clear graphically from Fig. 1 that the maximum of $h(c,\nu)/c$ decreases when ν goes up. To verify this analytically, one must consider the equations defining implicitly c^* and x^* :

$$V(x) = \varphi(c,\nu) = \theta\left[U(c) - U(\nu c)\right], \tag{B.2}$$

$$xV'(x) = c\varphi'_c(c,\nu) = \theta\left[cU'(c) - \nu c U'(\nu c)\right]. \tag{B.3}$$

By total differenciation of (B.2), with all partial derivatives evaluated at the optimum x^* and (c^*,ν), one gets :

$$V'dx = \varphi'_c dc + \varphi'_\nu d\nu, \tag{B.4}$$

which on account of (B.3) leads to :

$$c\varphi'_c \left[\frac{dx}{x} - \frac{dc}{c} \right] = \varphi'_\nu d\nu < 0,$$

and proves indeed that the optimum x^*/c^* goes down when ν increases.

To prove that c^* goes up when ν increases, we differenciate totally (B.3) and substract (B.4) member to member from the outcome. This leads to :

$$\begin{aligned} xV''dx &= c\varphi''_{c^2}dc + c\varphi''_{c\nu}d\nu - \varphi'_\nu d\nu \\ &= c\varphi''_{c^2}dc - \theta\nu c^2 U''(\nu c)d\nu. \end{aligned} \tag{B.5}$$

Replacing in (B.5) dx by $\left[\varphi'_c dc + \varphi'_\nu d\nu\right]/V'$ as implied by (B.4) leads to, after rearranging terms :

$$\left(\frac{xV''}{V'}\varphi'_\nu + \theta\nu c^2 U''(\nu c) \right) d\nu = \left(c\varphi''_{c^2} - \frac{xV''}{V'}\varphi'_c \right) dc.$$

since both factors multiplying $d\nu$ and dc are negative under our assumptions, one gets indeed $dc/d\nu > 0$. Q.E.D.

Acknowledgements

This paper was prepared for a special issue of the *International Journal of Economic Theory (IJET)* in honor of Professor Takashi Negishi. The idea of this work originates from my attending a lecture by T. Nakajima on his paper (2006) in Kyoto, on the occasion of the Second International Conference on Economic Theory jointly organized on December 17-18, 2004 by the 21st century COE programs, Keio University and Kyoto University. I had useful exchanges with Nicolas Dromel, Stephane Gauthier and Stephane Gregoir while writing the paper. I am grateful to Stefano Bosi for his useful comments on an earlier draft. Efficient typing by Nadine Guedj is gratefully acknowledged. I am indebted to Nicolas Dromel, who translated very kindly my handwritten Figures into print. The usual caveat applies.

References

Alexopoulos, M. (2004), “Unemployment and the Business Cycle”, *Journal of Monetary Economics, 51*, 277-298.

Aloi, M. and T. Lloyd-Braga (2006), “National Labor Markets, International Factor Mobility and Macroeconomic Instability”, mimeo, FCEE, Universidade Catolica Portuguesa, Lisbon.

Baily, M.N. (1978), “Some Aspects of Optimal Unemployment Insurance”, *Journal of Public Economics, 10*, 379-402.

Barinci, J. P. and A. Cheron (2001), “Sunspots and the Business Cycle in a Finance Constrained Economy”, *Journal of Economic Theory, 97*, 30-49.

Barro, R. J. and H. I. Grossman (1976), *Money, Employment and Inflation,* Cambridge University Press.

Benassy, J. P. (1982), *The Economics of Market Disequilibrium*, Academic Press.

Benassy, J. P. (2002), *The Macroeconomics of Imperfect Competition and Non-clearing Markets : A Dynamic General Equilibrium Approach*, The MIT Press.

Benhabib, J. and R. E. A. Farmer (1994), “Indeterminacy and Increasing Returns”, *Journal of Economic Theory, 63*, 19-41.

Cazzavillan, G., Lloyd-Braga, T. and P.A. Pintus (1998), “Multiple Steady States and Endogenous Fluctuations with Increasing Returns to Scale in Production”, *Journal of Economic Theory, 80,* 60-107.

Coimbra, R. (1999), “Efficiency Wages, Increasing returns and Endogenous Fluctuations”, Working Paper 1999-06, University of York.

Danthine, J.-P. and J. B. Donaldson (1990, “Efficiency Wages and the Business Cycle Puzzle”, *European Economic Review*, *34*, 1275-1301.

Dreze, J. (1991), *Underemployment Equilibria,* Cambridge University Press.

Dufourt, F., Lloyd-Braga, T. and L. Modesto (2004), “Indeterminacy, Bifurcations and Unemployment”, mimeo FCEE, Universidade Catolica Portuguesa, Lisbon, forthcoming in *Macroeconomic Dynamics.*

Grandmont, J.-M. (1989), “Keynesian Issues and Economic Theory”, *Scandinavian Journal of Economics, 91*, 265-293.

Grandmont, J.-M. and G. Laroque (1976), “On Temporary Keynesian Equilibria”, *Review of Economic Studies*, *43*, 53-67.

Grandmont, J.-M., Pintus, P. A. and R. de Vilder (1998), “Capital-Labour Substitution and Competitive Nonlinear Endogenous Business Cycles”, *Journal of Economic Theory, 80*, 14-59.

Hintermaier, T. (2003), “On the Minimum Degree of Returns to Scale in Sunspot Models of the Business Cycle”, *Journal of Economic Theory, 110*, 400-409.

Malinvaud, E. (1977), *The Theory of Unemployment Reconsidered*, Basil Blackwell.

Mankiw, N. G. and D. Romer (eds.) (1991), *New Keynesian Economics*, vols I, II, The MIT Press.

Mankiw, N. G. and R. Reiss (2003), "Sticky Information : A Model of Monetary Nonneutrality and Structural Slumps" in *Knowledge, Information and Expectations in Modern Macroeconomics*, Aghion, P., Frydman, R., Stiglitz, J. E. and M. Woodford (Eds.), Princeton University Press.

Nakajima, T. (2006), "Unemployment and Indeterminacy", *Journal of Economic Theory, 126*, 314-327.

Negishi, T. (1979), *Microeconomic Foundations of Keynesian Macroeconomics*, North-Holland.

Nicholson, W. and K. Needels, "Unemployment Insurance : Strengthening the Relationship between Theory and Policy", *Journal of Economic Perspectives, 20*, 47-70.

Shapiro, G. and J. E. Stiglitz (1984), "Equilibrium Unemployment as a Worker Discipline Device", *American Economic Review, 74*, 433-444.

Solow, R. M. (1979), "Another Possible Source of Wage Stickiness", *Journal of Macroeconomics, 1*, 79-82.

Taylor, J. B. (1999), "Staggered Price and Wage Setting in Macroeconomics", in *Handbook of Macroeconomics*, vol. 1., Taylor, J.B. and M. Woodford (eds.), Elsevier Science B.V..

Uhlig, H. and Y. Xu (1996) "Effort and the Cycle : Cyclical Implications of Efficiency Wages", Working paper, CentER, Tilburg University.

Woodford, M. (1986), "Stationary Sunspot Equilibria in a Finance Constrained Economy", *Journal of Economic Theory, 40*, 128-137.

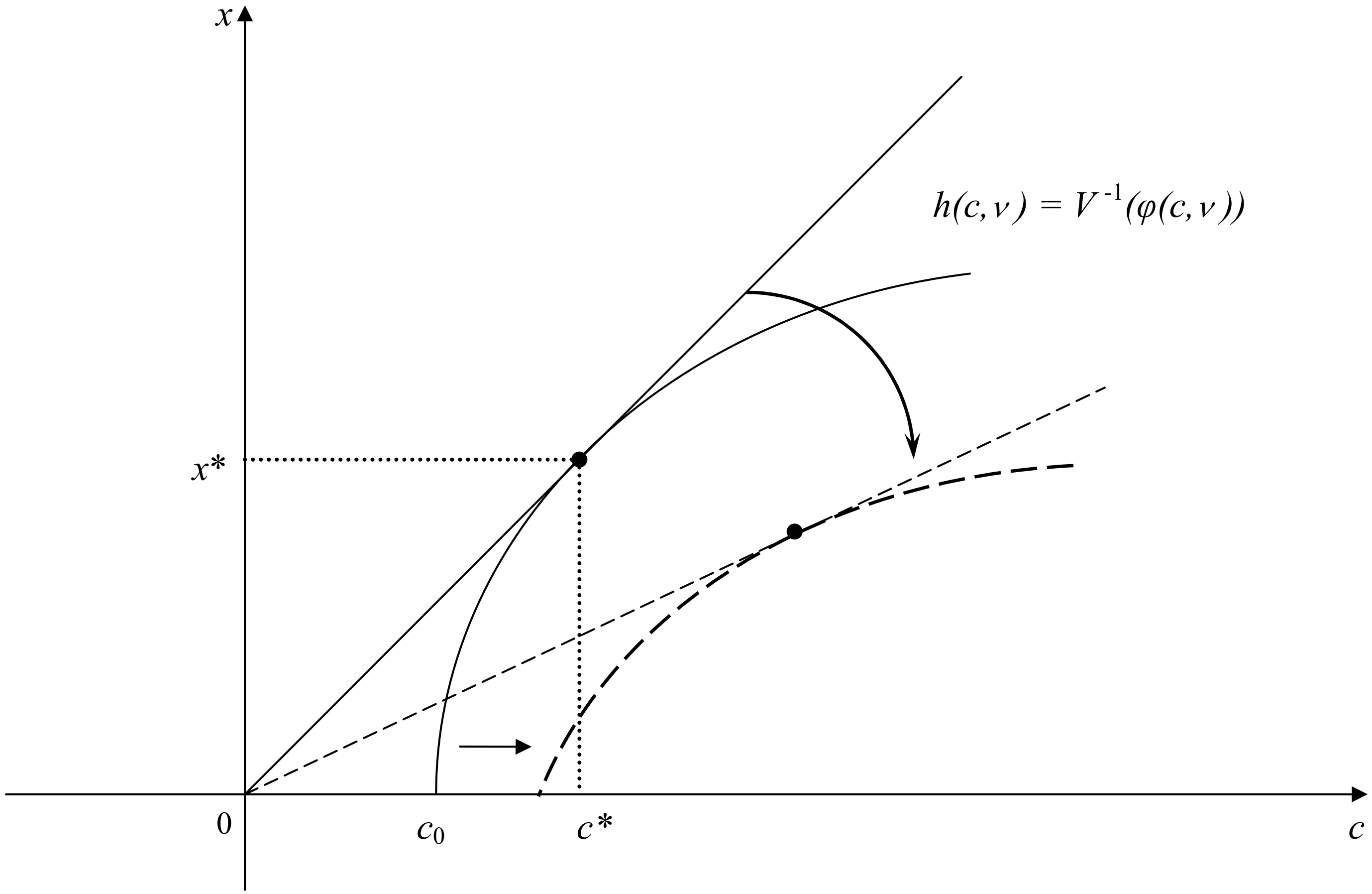


Fig. 1

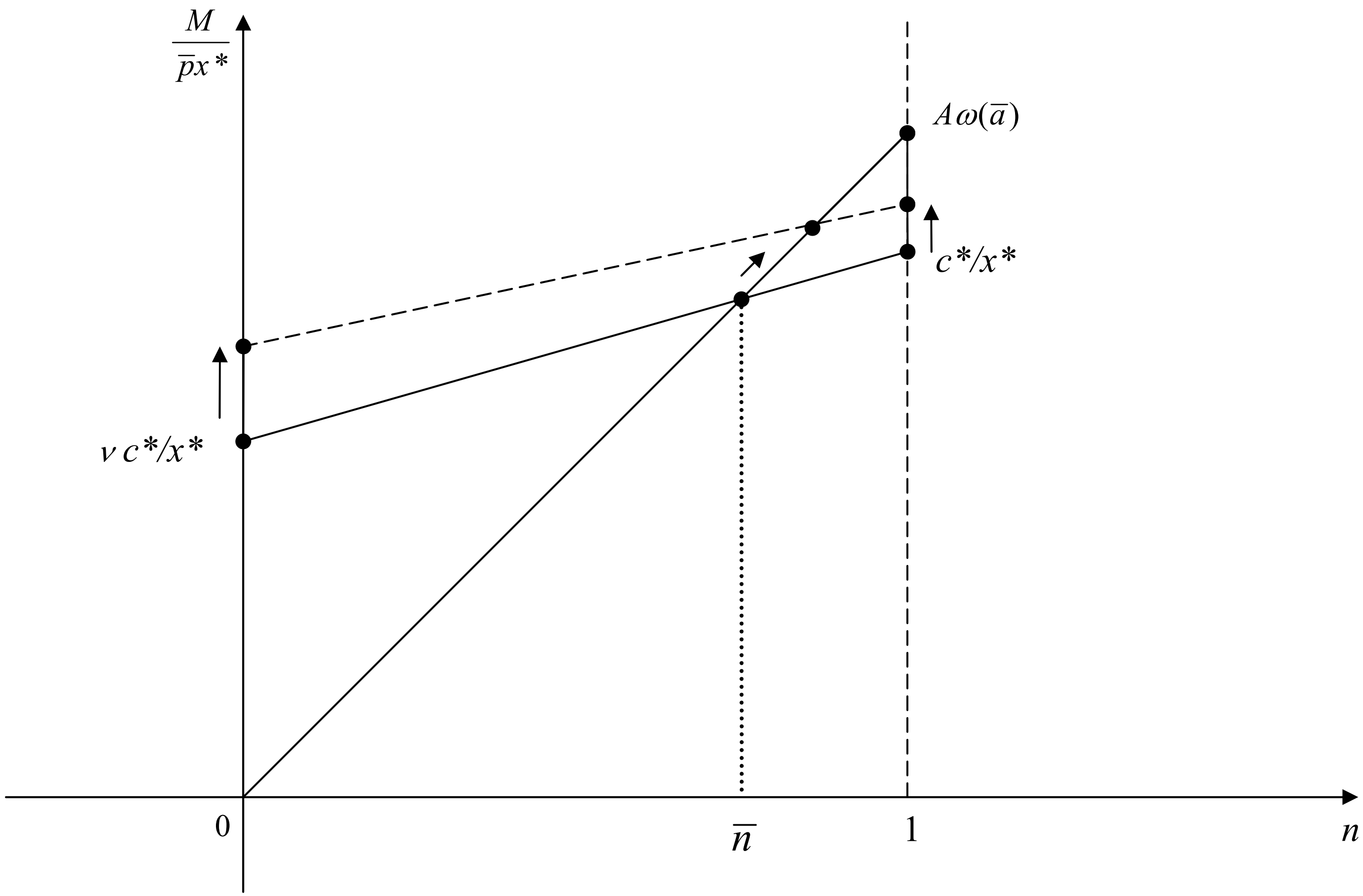
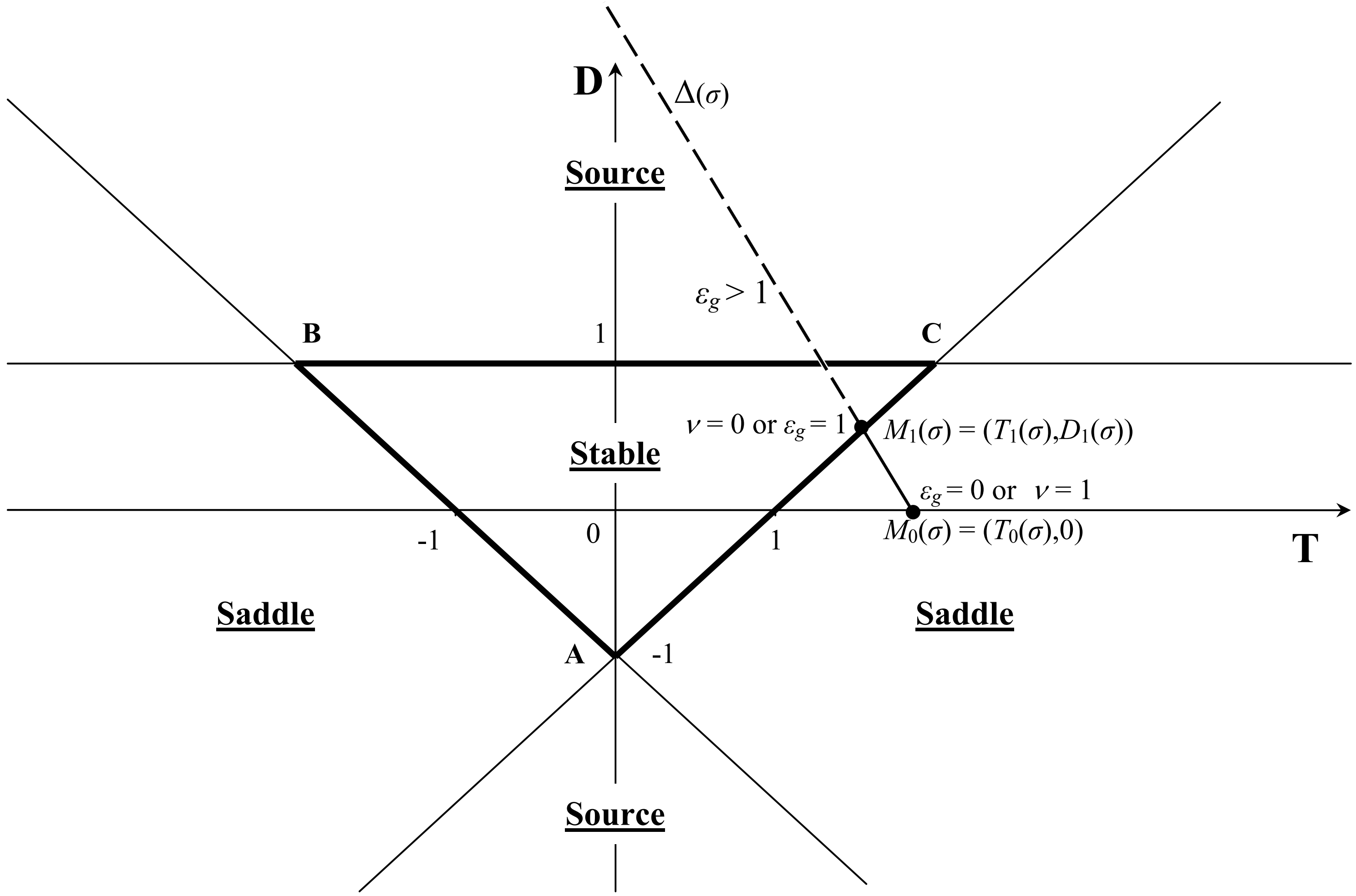


Fig. 2



D
Δ(σ)
Source
εg > 1
B
1
C
ν = 0 or εg = 1
M1(σ) = (T1(σ),D1(σ))
Stable
εg = 0 or ν = 1
-1
0
1
M0(σ) = (T0(σ),0)
T
Saddle
Saddle
A
-1
Source

Fig. 3

*

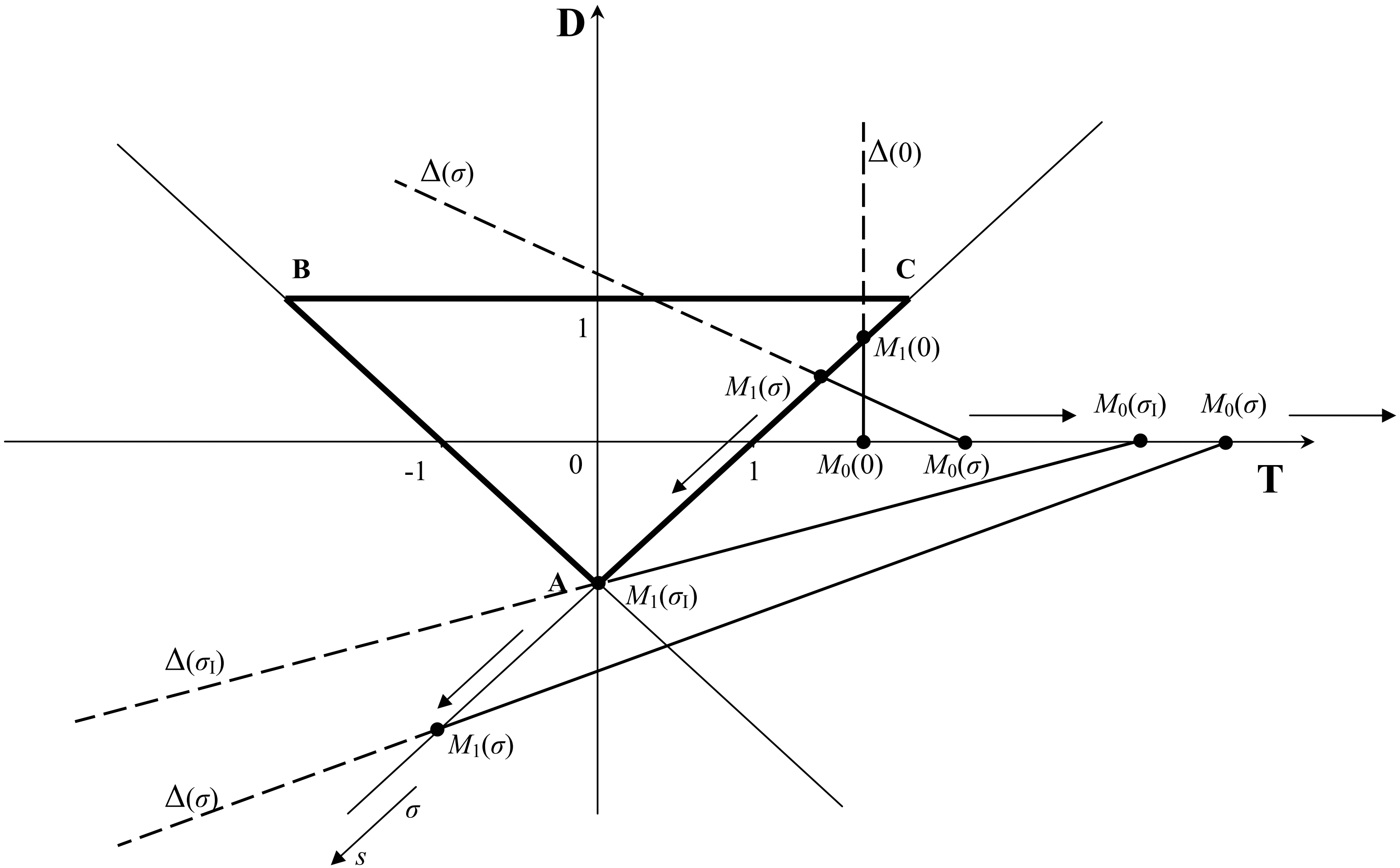


Fig. 4: The case $\sigma < s$

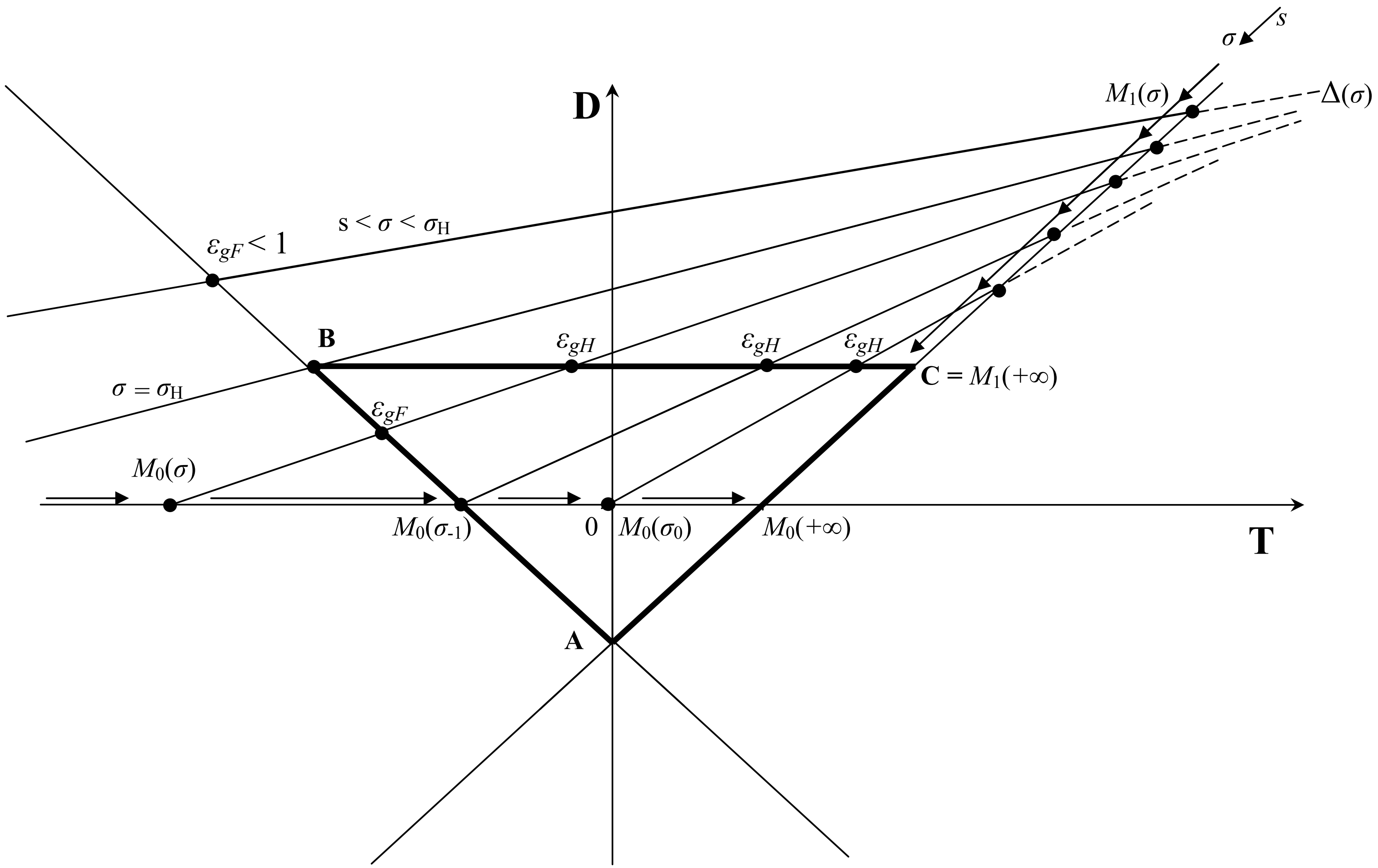


Fig. 5: The case $s < \sigma$