Open access • Journal Article • DOI:10.1080/00207540600584821

Negotiation-Based Collaborative Planning in Divergent Two-Tier Supply Chains

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Published on: 18 Jan 2007 - International Journal of Production Research (Taylor & Francis)

Topics: Supply chain, Supply chain management and Purchasing

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To cite this version:

Gregor Dudek, Hartmut Stadtler. Negotiation-Based Collaborative Planning in Divergent Two-Tier Supply Chains. International Journal of Production Research, Taylor & Francis, 2007, 45 (02), pp.465-484. 10.1080/00207540600584821 . hal-00512887

HAL Id: hal-00512887

https://hal.archives-ouvertes.fr/hal-00512887

Submitted on 1 Sep 2010

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Journal:	*International Journal of Production Research*
Manuscript ID:	TPRS-2005-IJPR-0234.R1
Manuscript Type:	Original Manuscript
Date Submitted by the Author:	20-Nov-2005
Complete List of Authors:	Dudek, Gregor; Darmstadt University of Technology, Department of Production and Supply Chain Management Stadtler, Hartmut; University of Hamburg, Institute of Logistics and Transport
Keywords:	MATH PROGRAMMING, SUPPLY CHAIN MANAGEMENT
Keywords (user):	Collaborative Planning

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Total word count main text: ~ 6.400

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Abstract

Advanced Planning Systems are based on the principles of hierarchical planning, which – at least at the top level – grounds on centralized planning. However, central coordination requires access to all relevant information and the power to impose planning results on all organizational units. In consequence it can be realized only for parts of an inter-organizational supply chain, and the question arises whether there exist alternate ways to achieve coordination.

In this paper we describe a non-hierarchical, negotiation-based process, which can be used to synchronize plans between independent partners of a two-tier supply chain consisting of one supplier and several buyers. Assuming that all partners generate plans based upon mathematical programming – as in most Advanced Planning Systems at the master planning level – we show how modified versions of these models can be utilized to support the negotiation process by evaluating given purchasing orders or supplies and by generating counter-proposals. Resulting is an iterative, negotiation-like scheme, which establishes and subsequently improves a consistent overall plan based on a limited exchange of information between the supply chain partners.

Key Words: Collaborative Planning, Supply Chain Management, Mathematical Programming

1 Introduction

Coordinated planning and control of operations, i.e. production, storage, and distribution processes, is a central element of Supply Chain Management (SCM) (Stadtler (2005)).

One approach to coordinate operations is by centralized planning. Proponents of this approach usually suggest to implement hierarchical planning such that centralized coordination happens at a medium-term level, whereas it is left to the owners of the distinct operational processes to implement the results at the level of short-term planning and control (Shapiro (1999), Rohde / Meyr / Wagner (2000)).

However, centralized planning requires access to all relevant information. Moreover, it can fail simply because individual partners are involved in several SCs; for example suppliers typically serve more than a single customer. Therefore, alternate approaches are required which establish synchronized operations based on the exchange of few information and an acceptable coordination effort (e.g. Kilger / Reuter (2005)).

This paper provides such an alternate approach by laying out a negotiation-based process for aligning operations in a SC comprising several buyers and one common supplier as shown in Figure 1. Thereby, we focus on the medium-term task of Master Production Scheduling (MPS) (e.g. Silver / Pyke / Peterson (1998)) or Master Planning (Rohde / Meyr / Wagner (2000)).

We assume that several end products are sold by each buyer based on dynamic, but deterministic (or forecasted) demand by period. Each buyer's operations may comprise multiple stages and require a set of components that are purchased from the supplier. The supplier too may face a multi-stage production process. We suppose information is fully asymmetric, i.e. without additional communication each partner only possesses local information on his own operations and a local demand forecast. The supplier forecasts the demand of his end-products based on his best guess of the buyers' need of input materials. All partners are assumed to generate their local MPS with mathematical programming models.

Without any coordinating action, all parties, i.e. each buyer and the supplier, use their planning model with local information only (local optimization). Such isolated planning and operation typically results in poor performance with unnecessarily high costs, large inventory buffers, and amplified demand swings as described by the “bullwhip effect” (e.g. Lee / Padmanabhan / Whang (1997)).

In order to improve the SC’s performance, partner-specific MPS should be linked and synchronized with each other. A coordination scheme which synchronizes operations and improves total cost of a single buyer and supplier is described in Dudek / Stadtler (2004). In the following we expand its basic form in two directions. For one, a two-tier SC with one supplier and multiple buyers is covered. Second, the amount of information exchanged between the parties is farther reduced.

There is a large and growing amount of work on decentralized operation of SCs and associated contracting issues. Publications deal e.g. with the classical newsvendor problem (see e.g. Tsay (1999), Lariviere / Porteus (2001), Cachon (2003)), lot-sizing problems in a two-party setting (e.g. Monahan (1984), Wheng (1995)), or serial, multi-stage SCs (Lee / Whang (1999), Chen (1999)). In the following we limit our attention to the particular setting considered here, namely a SC consisting of one supplier and several buyers (for a more broad literature review refer to e.g. Tsay / Nahmias / Agrawal (1999), Cachon (2003), Thomas / Griffin (1996), Erengüc / Simpson / Vakharia (1999)).

<u>Existing work on SC coordination with a single supplier and multiple buyers focuses on replenishment policies of single items and associated contracting issues, while aspects of negotiations for aligning operations are not considered explicitly.</u> E.g. Cachon / Fisher (2000) study the value of information sharing when N identical buyers face stationary stochastic demand for a single product which they replenish from the supplier. They find that total SC costs decreases by 2.2% on average when the supplier has knowledge of the retailers’ downstream demand.

Chen / Federgruen / Zheng (2001) consider a scenario where the demand faced by the buyers depends on the price they charge. They show how a central planner’s solution can be established by the supplier under the assumption of perfect information by offering a discount scheme. Bilgic (2003) extends their model to the case of asymmetric information.

Fransoo / Wouters / de Kok (2001) assume stochastic demand at the buyers and study the effects of service level constraints imposed on the supplier. They show that if some buyers agree to jointly determine optimal service levels guaranteed by the supplier, notable reductions in inventory can be realized at both SC tiers.

Cachon (2003) deals with a setting where the buyers compete for the total market demand. He shows that, under full information, here too the supplier can induce the buyers to implement globally optimal policies by offering buy-back contracts.

Simpson / Erenguc (2001) deal with a SC comprising several retailers and one warehouse that is supplied by several manufacturers. Neglecting the production stage, the network represents a single supplier, multiple buyers setting. Replenishment and distribution plans are generated by solving mixed-integer programming (MIP) models. They compare central planning based on a single MIP model with a level-by-level or upstream planning scheme where plans are generated successively in upstream direction. While upstream planning is easy to implement and results in synchronized plans across the SC, it yields sub-optimization of the SC as a whole. In computational experiments Simpson / Erenguc observe an average gap in total cost of 14.1%.

A similar setting is studied by Ertogral/Wu (2000). They consider a central planning model (a multi-level capacitated lot-sizing problem) which is then decomposed into sub-models corresponding to individual SC partners. Drawing from well-known decomposition techniques, they develop a coordination mechanism where a central agent sets target values for supply quantities and penalty costs for deviations from these in order to achieve synchronized plans at acceptable total cost.

The remainder is organized as follows. In the next section we outline the planning model assumed to be used by all SC partners. Thereafter, section 3 describes the negotiation-based coordination scheme, followed by a computational evaluation in section 4. A summary and final remarks conclude the paper.

2 Planning model

Based on the SC setting described above and shown in figure 1, we assume that each SC partner uses a multi-level capacitated lot-sizing problem (MLCLSP, e.g. Stadtler (2003)) to generate his local MPS. Neglecting setup times and lead times, the MLCLSP can be formulated as follows:

Model 1 MLCLSP

Indices

t	*planning period*	$\in T$
j	*operation*	$\in J$
m	*resource*	$\in M$

Index sets

T	*set of planning periods*
J	*set of operations*
M	*set of constraint resources*
S_j	*set of direct successor operations of j*

Data

ch_j	*unit holding cost of operation j*
cf_j	*fixed setup cost of operation j*
co_m	*unit cost of overtime (capacity expansion) at resource m*
$D_{j,t}$	*(external) demand for operation j in period t*
$C_{m,t}$	*capacity at resource m in period t*
B	*large constant*
I_j	*starting inventory of operation j*
$a_{m,j}$	*unit requirement of resource m by operation j*
$r_{j,k}$	*unit requirement of operation j by successor operation k*

Variables

c	*total cost*
$x_{j,t}$	*output of operation j in period t*
$i_{j,t}$	*inventory level of operation j at the end of period t*
$y_{j,t}$	*setup variable of operation j in period t*

$o_{m,t}$ *overtime at resource m in period t*

Formulation

$$\min \quad c \tag{2.1}$$

s.t.

$$c = \sum_{t\in T}\sum_{j\in J}(ch_j i_{j,t} + cf_j y_{j,t}) + \sum_{t\in T}\sum_{m\in M} co_m o_{m,t} \tag{2.2}$$

$$i_{j,t-1} + x_{j,t} = D_{j,t} + \sum_{k\in S_j} r_{j,k} x_{k,t} + i_{j,t} \qquad \forall j\in J, t\in T \tag{2.3}$$

$$\sum_{j\in J} a_{m,j} x_{j,t} \le C_{m,t} + o_{m,t} \qquad \forall m\in M, t\in T \tag{2.4}$$

$$x_{j,t} \le B y_{j,t} \qquad \forall j\in J, t\in T \tag{2.5}$$

$$x_{j,t}, i_{j,t} \ge 0 \qquad \forall j\in J, t\in T \tag{2.6}$$

$$i_{j,0} = I_j \qquad \forall j\in J \tag{2.7}$$

$$o_{m,t} \ge 0 \qquad \forall m\in M, t\in T \tag{2.8}$$

$$y_{j,t} \in \{0,1\} \qquad \forall j\in J, t\in T \tag{2.9}$$

The model plans output ($x_{j,t}$) and inventory levels ($i_{j,t}$) for all operations considered as well as resource overtime ($o_{m,t}$) while minimizing total cost, which, due to (2.2), is captured by variable c. Constraints (2.3) ensure the flow balance between output, inventory and consumption by external demand or successor operations. Constraints (2.4) represent capacity restrictions, while lot-sizing relationships are expressed in (2.5).

Model 1 is used by each SC partner, i.e. each of the buyers and the supplier, to generate a locally cost-minimal MPS for all operations and resources under her/his control. Thereby, each partner uses her/his set of input parameters (e.g. J, M, $C_{m,t}$, $a_{m,j}$), including a local demand forecast $D_{j,t}$, and obtains local plan results (e.g. $x_{j,t}$, $i_{j,t}$). However, for the sake of simplicity we assume that all partners use an identical planning horizon of T periods.

Since such completely isolated planning yields sub-optimal SC performance as explained above, the question arises how the isolated planning models can be linked to achieve coordination and thus improved SC performance. One approach to achieve this linkage without fully centralized control is described in the next section.

3 Model-Based Negotiations

In this section we describe the model-based negotiation scheme. The following paragraph summarizes the decision situation faced by each of the buyers. Thereafter, we present the supplier's perspective in 3.2. The resulting negotiation process is discussed in 3.3.

3.1 The buyers' situation

Since each buyer negotiates with one (and the same) supplier only, associated planning steps resemble the case of a two-partner negotiation as described in Dudek / Stadtler (2004) and are only summarized in the following. During the negotiation, each buyer may face three distinct planning situations. First, he may determine his locally optimal plan without accounting forthe availability of supply material. Second, he may analyze the consequences of supply quantities proposed by the supplier. Finally, he may derive a compromise order proposal for supply items based on the given supplier proposal.

The locally optimal plan can be generated by applying Model 1 above. Resulting is the optimal cost outcome for any buyer k

$$C_{B,k}^{\min} = c^* \tag{3.1}$$

(c^* represents the value of c in the model's optimal solution).

The order pattern of input materials required from the supplier corresponding to the locally optimal plan can the derived for all supply items $j \in JS_k$ of buyer k from the output levels $x_{j,t}$

$$xo_{k,j,t} = \sum_{l \in S_j} r_{j,l} x_{l,t} \qquad \forall j \in JS_k, t \in T \tag{3.2}$$

In the second situation, a given set of supply quantities proposed by the supplier exists for all input materials $j \in JS_k$. Each buyer's best plan while obeying to the supply proposal can be obtained by solving Model 2.

Model 2 Buyer Proposal Analysis

Index sets

JS_k	*set of supplied items (operations)*

Data

$XS_{j,t}$ *proposed supply quantity of j in period t*

Variables

$xs_{j,t}$ *supply quantity of j in period t*

$is_{j,t}$ *supply inventory of j in period t*

Formulation

$$\min \quad c \tag{3.3}$$

s.t. (2.3) - (2.9)

$$c = \sum_{t\in T}\sum_{j\in J}(ch_j i_{j,t} + cf_j y_{j,t}) + \sum_{t\in T}\sum_{m\in M} co_m o_{m,t} + \sum_{t\in T}\sum_{j\in JS_k} ch_j is_{j,t} \tag{3.4}$$

$$is_{j,t-1} + xs_{j,t} = \sum_{l\in S_j} r_{j,l} x_{l,t} + is_{j,t} \qquad \forall j \in JS_k, t \in T \tag{3.5}$$

$$xs_{j,t} = XS_{j,t} \qquad \forall j \in JS_k, t \in T \tag{3.6}$$

$$xs_{j,t} \geq 0, is_{j,t} \geq 0 \qquad \forall j \in JS_k, t \in T \tag{3.7}$$

Proposed supply quantities $XS_{j,t}$ are input to the model by constraints (3.6). Balance equations (3.5) restrict internal operations by the availability of supplied items. Inventory holding of supplied quantities is however permitted, in order not to fully dictate internal operations by the supply proposal. Consequently, the cost function in (3.4) is enhanced by inventory holding costs of supply items.

Each buyer's k optimal solution to model 2 c^* is in the following referred to as

$$C_{B,k}^{prop,i} = c^* \tag{3.8}$$

where i represents an index to the underlying supply proposal. The difference $C_{B,k}^{prop,i} - C_{B,k}^{\min}$ resembles the cost increase accruing to buyer k for accepting the supply pattern proposed by the supplier.

The third planning situation, the generation of a compromise proposal, comprises two sub-steps. First, each buyer deduces a preferred solution from the given supplier proposal. This too can be achieved by solving model 2 after replacing constraints (3.6) by

$$xs_{j,t} + d_{j,t}^{+} + d_{j,t}^{-} = XS_{j,t} + d_{j,t-1}^{+} + d_{j,t+1}^{-} \qquad \forall j \in JS_k, t \in T \tag{3.9}$$

where new variables $d^{+}_{j,t} / d^{-}_{j,t}$ capture the modifications to the supply proposal.

Here, the allowable degree of modification can be limited by additional constraints

$$\sum_{s=1}^{t} xs_{j,s} \geq XS_{j,t}^{cum,\min} \qquad \forall j \in JS_k, t \in T \tag{3.10}$$

$$\sum_{s=1}^{t} xs_{j,s} \leq XS_{j,t}^{cum,\max} \qquad \forall j \in JS_k, t \in T \tag{3.11}$$

in order to account for the specific solution proposed by the supplier ($XS_{j,t}^{cum,\min}$ / $XS_{j,t}^{cum,\max}$ are cumulated minimum and maximum supply quantities over periods 1 to *t*. Based on a lot-sizing heuristic by Simpson / Erengüc (1998), the maximum modification can e.g. be defined as a shift of the entire period quantity to the next or previous period with a supply greater than zero as discussed in Dudek / Stadtler (2004)).

The optimal cost c* to the modified model is abbreviated by

$$C_{B,k}^{pref,i} = c^* \tag{3.12}$$

The sum of modifications (by item) made to a given proposal is referred to as

$$D_j^{\max,i} = \sum_{t \in T} (d_{j,t}^{+,*} + d_{j,t}^{-,*}) \qquad \forall j \in JS_k \tag{3.13}$$

The second sub-step finally generates the actual compromise proposal. The idea here is to balance between the solution $C_{B,k}^{prop,i}$ and the preferred outcome $C_{B,k}^{pref,i}$ such that only few, but highly cost effective modifications to the supply pattern remain. This can be realized by goal programming as shown in Model 3.

Model 3 Buyer Goal Programming

Variables

w_j *weight of item j in total deviation*

Variables

Δ *deviation from minimum cost*

d *total percentage distance in supply pattern*

Formulation

min	$\dfrac{\Delta}{C_{B,k}^{prop,i} - C_{B,k}^{pref,i}} + d$	(3.14)
s.t.	(2.3) - (2.9), (3.4), (3.5), (3.7), (3.9) - (3.11)	

$$c - \Delta = C_{B,k}^{pref,i} \tag{3.15}$$

$$d = \sum_{j \in JS_k} \frac{w_j}{D_j^{\max,i}} \sum_{t \in T} (d_{j,t}^{+} + d_{j,t}^{-}) \tag{3.16}$$

Constraints (3.15) and (3.16) measure the deviation from the goal target values. Since the first goal (3.15) is to come as close as possible to $C_{B,k}^{pref,i}$, variable Δ captures the difference between current and preferred cost. The other goal is to avoid modifications to the given supplier proposal. Therefore, (3.16) contains a measure of the total deviation as a summation across all supplied items and periods, normalized by the maximum deviation $D_j^{max,i}$ and averaged by item weights w_j. The objective function minimizes the sum of both goal deviation measures d and Δ. Since d takes values between 0 and 1, Δ is normalized over the interval $[C_{B,k}^{pref,i}, C_{B,k}^{prop,i}]$.

Solving model 3 produces a compromise between the $C_{B,k}^{prop,i}$ solution without modifications (model 2) and the $C_{B,k}^{pref,i}$ solution. As such it only contains modifications which contribute more strongly to cost savings below $C_{B,k}^{prop,i}$ than to the deviation measure.

The optimal cost c^* to model 3 is abbreviated by

$$C_{B,k}^{comp,i} = c^* \tag{3.17}$$

$C_{B,k}^{comp,i} - C_{B,k}^{\min}$ represents the cost increase above the buyer's local optimum resulting from complying with the compromise supply pattern.

3.2 The supplier's situation

The supplier principally faces similar planning situations during the course of negotiations. Here, we distinguish two cases: First, the evaluation of given order proposals requested by the buyers, and second the generation of a compromise counter-proposal. Compared to the buyer perspective, the situation is more complex, as the supplier collaborates with several buyers simultaneously.

The evaluation of buyer proposals can be accomplished by extending model 1 as given below.

Model 4 Supplier Proposal Analysis

Indices

k *buyer* $\in K$

Index sets

K *set of individual buyers*

JS_k *set of items (operations) ordered by buyer k*

Data

$XO_{k,j,t}$ *proposed order quantity by buyer k of item j in period t*

Variables

$xo_{k,j,t}$ *order quantity by buyer k of item j in period t*

Formulation

$$\min \quad c \tag{3.18}$$

s.t. (2.2), (2.4)-(2.9)

$$i_{j,t-1} + x_{j,t} = \sum_{k \in K} xo_{k,j,t} + \sum_{k \in S_j} r_{j,k} x_{k,t} + i_{j,t} \qquad \forall j \in J, t \in T \tag{3.19}$$

$$xo_{k,j,t} = XO_{k,j,t} \qquad \forall k \in K, j \in JS_k, t \in T \tag{3.20}$$

$$xo_{k,j,t} \geq 0 \qquad \forall k \in K, j \in JS_k, t \in T \tag{3.21}$$

The supplier cooperates with K buyers, referred to by index k. Assuming that each buyer k has announced order quantities $XO_{k,j,t}$, constraints (3.19) and (3.20) are used to incorporate the order proposals into the supplier's planning situation. Constraints (3.19) replace the original flow balance equations (2.3).

Thus, the supplier's local demand forecast $D_{j,t}$ is replaced by the order quantities requested by the buyers which are announced directly during the negotiation, avoiding local forecasting by the supplier. As there are several buyers, each ordering a specific set of items JS_k, constraints (3.21) are defined for each buyer and the items he orders.

The cost associated with the solution to model 4 is referred to as $c_S^{prop,i}$ in the following.

The compromise generation again comprises two sub-steps and starts by obtaining a preferred solution with maximal modifications to the buyers' proposals. This is realized as in the buyers' case by replacing constraints (3.20) by

$$xo_{k,j,t} + d^{+}_{k,j,t} + d^{-}_{k,j,t} = XO_{k,j,t} + d^{+}_{k,j,t-1} + d^{-}_{k,j,t+1} \qquad \forall k \in K, j \in JS_k, t \in T \qquad (3.22)$$

$$\sum_{s=1}^{t} xo_{k,j,s} \geq XO^{cum,\min}_{k,j,t} \qquad \forall k \in K, j \in JS_k, t \in T \qquad (3.23)$$

$$\sum_{s=1}^{t} xo_{k,j,s} \leq XO^{cum,\max}_{k,j,t} \qquad \forall k \in K, j \in JS_k, t \in T \qquad (3.24)$$

Proposal modifications are again introduced by deviation variables $d^{+}_{k,jt}$ / $d^{-}_{k,j,t}$. The extent of deviations is limited by cumulated minimum and maximum quantities, e.g. by shifts of a buyer's entire order quantity $XO_{k,j,t}$ to the previous or the next order period of item j.

The cost outcome to model 4 enhanced by constraints (3.22) to (3.24) is again abbreviated by $c_S^{pref,i}$ and the amount of modifications introduced to each buyer's order proposal captured by

$$D^{\max,i}_{k,j} = \sum_{t \in T} (d^{+*}_{k,j,t} + d^{-*}_{k,j,t}) \qquad \forall k \in K, j \in JS_k \qquad (3.25)$$

Finally, the actual compromise counter-proposal is obtained by goal programming. Since the supplier serves several buyers, it is now however useful not only to trade-off a total deviation measure with cost savings, but *to balance the amount of modifications proposed to each individual buyer* in order to share the burden of cost increases among the SC partners. Thus, we define a total deviation measure per buyer as

$$d_k = \sum_{j \in JS_k} w_{k,j} \frac{\sum_{t \in T} (d^{+}_{k,j,t} + d^{-}_{k,j,t})}{D^{\max,i}_{k,j}} \qquad \forall k \in K \qquad (3.26)$$

In the goal programming model, the buyer-specific deviation measures all receive target values of zero (d_k*=0) and are pursued simultaneously to minimizing costs (c*=$c_S^{pref,i}$).

Just as above, appropriate scaling is required in the objective function to ensure a fair trade-off between the separate goals. Here the situation once more differs from that of the buyers because the supplier can use additional information to generate “good” compromises. Recapping the previous section, we know that each buyer incurs a cost increase above his locally optimal outcome by complying with a supply pattern proposed by the supplier. Now, assuming that the supplier can gain knowledge of these additional costs, expected buyer cost increases can be *anticipated* in the goal programming model and the compromise generation be guided towards introducing more or fewer modifications to supply patterns of individual buyers depending on their expected cost increases.

The anticipation can be realized by scaling each buyer’s total deviation d_k by an estimate ΔP_k of the cost increase that would follow from suggesting the preferred solution $C_S^{pref,i}$ as counter-proposal. In that way we assume that the buyer’s cost increase grows proportionally with d_k from zero to $\Delta P_{k.}$ (a way to estimate the parameter is laid out below).

The resulting goal program for the supplier is given in model 5.

Model 5 Supplier Goal Programming

Data

$C_S^{pref,i}$	*preferred cost of order pattern i*
ΔP_k	*estimated cost increase for buyer k associated with C^{pref}-solution*
$D_{k,j}^{max,i}$	*maximum deviation in supply units of item j from order pattern i of buyer k*
$w_{k,j}$	*weight of operation (item) j in total deviation measure of buyer k*

Variables

Δ	*deviation from minimum cost*
d_k	*percentage distance in supply pattern to buyer k*

Formulation

min $$\Delta + \sum_{k \in K} \Delta \mathrm{P_k}\, d_\mathrm{k} \qquad (3.27)$$

s.t. (2.2), (2.4) - (2.9), (3.19), (3.21) - (3.24)

$$d_k = \sum_{j \in JO_k} \frac{w_{k,j}}{D_{k,j}^{\max,i}} \sum_{t \in T} (d_{k,j,t}^{+} + d_{k,j,t}^{-}) \qquad \forall k \in K \qquad (3.28)$$

$$c - \Delta = C_S{}^{pref,i} \qquad (3.29)$$

In model 5 the cost increase above $C_S^{pref,i}$ (Δ) contributes with its full magnitude to the objective function, while total deviations per buyer are scaled by the corresponding cost increase estimate ΔP_k as explained above. In result, local cost savings are balanced with estimated cost increases to the buyers. Also, modifications are first introduced to order patterns of buyers with low expected cost increases, which improves chances to obtain lower total SC costs.

A final question at this point is how to determine the parameters ΔP_k. The exact way to obtain them is by sending the supply patterns which correspond to the $C_S^{pref,i}$ solution to the buyers for evaluation. However, to avoid such a direct inquiry, the buyers' likely cost increases are estimated from the effect of the *previous* compromise proposal. Assuming that the previous compromise proposal led to cost increases per buyer of ΔP_k^{act} and displayed total deviations d_k^{act}, the estimates are obtained as

$$\Delta P_k = \Delta P_k^{act} / d_k^{act} \qquad \forall k \in K \tag{3.30}$$

The actual cost effects ΔP_k^{act} are reported by the buyers during the negotiation process, as we will see shortly in the next section. The optimal cost c^* to model 5 is abbreviated by $C_S^{comp,i}$.

3.3 Total negotiation process

Combining the above decision situation of the buyers and the supplier one can construct a negotiation process which installs a synchronized MPS for the entire SC and subsequently improves SC-wide cost.

Assuming that the buyers can ultimately chose the supply quantities they like to procure, the negotiation will naturally start with the upstream planning scheme. I.e., each buyer determines his locally optimal plan and transmits associated order quantities to the supplier who plans based on the received order requests by applying model 4. Resulting is the upstream planning solution. At this point, the supplier can initiate the actual negotiation by generating a counter-proposal of supply quantities as described in 3.2 (as no information on buyer cost increases is available yet, the cost parameters ΔP_k are initialized by $(C_S^{prop,1} - C_S^{pref,1})/\|K\|$ for every buyer k). The associated supply quantities (resulting values of variables $xo_{k,j,t}$ in model 5) are then transmitted to the buyers for inspection.

Each buyer can analyze the received supply proposal by solving model 2. Also, in reply to the supplier's proposal, the buyers can generate new counter-proposals by following the steps described in 3.1. The order quantities of the new counter-proposal can again be submitted to the supplier who can analyze their impact on local planning by solving model 4 based on the new quantities $XO_{k,j,t}$.

At the buyers' side, the resulting cost of complying with the supplier's first proposal ($C_{B,k}^{prop,1}$) as well as the cost associated with the compromise counter-proposal ($C_{B,k}^{comp,2}$) will usually be higher than that of the initial locally optimal plan. On the other hand, the supplier cost of his first proposal ($C_S^{comp,1}$) and the cost if complying to the buyers' first counter-proposal ($C_S^{prop,2}$) will usually be lower than the initial upstream planning outcome. Therefore, in order to see whether the negotiation yields a cost improvement for the SC as a whole, the cost effects of the buyers and the supplier need to be summed up. This evaluation could be carried out by a neutral third party receiving reports on the cost effects of all SC partners. Alternatively, the supplier himself is well positioned to obtain the total cost effect, as he is in communication with all the buyers anyway.

Therefore, we assume that, each time the buyers generate a new counter-proposal, they announce two cost effects along with the new order quantities to the supplier. First, the cost increase above each buyer's locally optimal plan of accepting the supplier's last supply proposal ($C_{B,k}^{prop,i-1}$) and second the cost increase associated with their current counter-proposal ($C_{B,k}^{comp,i}$). These cost effects reported by the buyers can be interpreted as minimum compensation needs for accepting the respective compromise solution as final pattern of supply quantities.

Based on this information, the supplier can determine the total cost effects. Generally, i.e. in any iteration of the negotiation process, the supplier obtains the total cost effect of his previous compromise proposal as

$$\Delta_{Total}^{i,1} = (C_S^{prop,1} - C_S^{comp,i}) - \sum_{k \in K} (C_{B,k}^{prop,i} - C_{B,k}^{\min}) \tag{3.31}$$

The first term of the RHS of (3.31) represents the cost savings accruing to the supplier as compared to his initial upstream outcome ($C_S^{prop,1}$), while the second term contains the compensation needs of all the buyers associated with the compromise.

The total effect of the buyers' counter-proposal is determined similarly as

$$\Delta_{Total}^{i,2} = (C_S^{prop,1} - C_S^{prop,i}) - \sum_{k \in K} (C_{B,k}^{comp,i} - C_{B,k}^{\min}) \tag{3.32}$$

Given the two total cost effects, the supplier can conclude whether the new compromise proposals have brought an improved total cost outcome, i.e.

$$\max\{\Delta_{Total}^{i,1}, \Delta_{Total}^{i,2}\} > \max\{\Delta_{Total}^{l,1}, \Delta_{Total}^{l,2}\} \qquad \forall l = 1..(i-1) \tag{3.33}$$

and select the best solution and associated supply pattern detected so far in the process.

As long as the supplier's previous proposal or the buyers' current counter-proposal yields an improved overall cost outcome, the negotiation process can naturally continue and the supplier can generate a new compromise proposal. The buyers always analyze the supplier proposal, generate a new counter-proposal and submit the new order patterns as well as the associated cost effects to the supplier.

Even in the case that no additional cost improvement is reached in an iteration of the negotiation, the negotiation process can still continue, as temporary degradations can give way for further cost improvements at later stages. Therefore, a probability-based decision rule adapted from Simulated Annealing (e.g. Downsland (1996)) can be used here in order to decide about the continuation in case of a degradation of total cost (see Dudek / Stadtler (2004) for details).

During the negotiation process, the buyer cost increase parameters ΔP_k are calculated by the supplier from the cost increases reported by the buyers, i.e.

$$\Delta P_k = \Delta P_k^{act} / d_k^{act} = \frac{1}{d_k^{act}} \left(C_{B,k}^{comp,i-2} - C_{B,k}^{prop,i-1}\right) \qquad \forall k \in K \tag{3.34}$$

where d_k represents the value of the deviation measure in the previous solution to model 5 (in iteration i=2 $C_{B,k}^{comp,0}$ takes the initial value $C_{B,k}^{\min}$).

An overview of the total resulting negotiation process is depicted in figure 2.

Once the negotiation is terminated, the best solution detected during the negotiation (maximum $\Delta_{Total}^{i,1/2}$) and the associated supply pattern can be installed as the final outcome. Also, the compensation requests of the best solution represent the minimum reward the supplier needs to grant to the buyers in order to offset the cost increases they face. In addition, a share of the supplier's remaining net savings, e.g. a fixed reward for joining the negotiation, should be spread among the buyers for ensuring that each SC partner gains a true advantage. Examples of other savings sharing agreements are given e.g. by Fleischmann (1999), Corbett / DeCroix (2001), Wu / Kleindorfer / Zhang (2002). The compensation and savings share can be incorporated into given supply contracts in the form of a bonus rendered to each buyer, if he truly complies with the agreed to order quantities.

Given the financial implications of the negotiation process, i.e. buyer compensation and savings sharing, a final issue of concern is the question of opportunistic behavior by the SC partners. Since the bonus rendered to each buyer is directly derived from the compensation requested during the negotiation, each buyer can principally increase his payoff by reporting overly high cost increases. However, requesting inflated compensation does not necessarily lead to an increased payoff, because, based on inflated compensation requests, a given compromise may not be considered as an improved solution by the supplier. Hence, assuming perfect information, where each buyer would know the compensation needs of all the other buyers and the cost saving accruing to the supplier, the buyers' compensation claim would represent a bargaining game with simultaneous bids by all players (e.g. Rasmusen (1994)): Only, if all buyers' claims sum up to less than the saving achieved on the supplier's side, a positive payoff accrues to each buyer; otherwise the "game" ends with a zero-payoff to all parties. Although the game lacks a clear equilibrium, a typical outcome is the focal point where all parties claim a compensation yielding an equal share of the corresponding net savings. However, the situation here is further complicated by asymmetric information, as each individual buyer has neither knowledge of the compensation needs of other buyers nor of the gross savings accruing to the supplier. Therefore, the extent of opportunistic behavior by individual buyers depends on their attitude towards risk on the one hand (i.e. claiming an inflated compensation even though it may be discarded) and on the other their ability to predict the supplier's cost savings and other buyers' compensation needs.

From the supplier's perspective, incentives for opportunistic behavior concern the sharing of net savings. If a fixed negotiation reward is paid to the buyers, incentives for cheating do not exist at the supplier side.

Given the complexity of the planning situation and models in place, which comes close to practice applications of APS, it is unfortunately hardly possible to show the schemes cost improvement potential in an analytic way. Therefore, in the next section we discuss numerical results obtained by applying the negotiation scheme to a set of test problems.

4 Computational Results

The performance of the negotiation scheme is explored with an automated version of the negotiation process. The lot-sizing models were implemented in CLPEX and augmented by valid inequalities in order to speed up the search for optimal integer solutions (see Pochet / Wolsey (1995)).

The structure of input parameters considered is taken from Tempelmeier / Derstroff (1996). Resource capacities are constrained; however can be expanded up to 20% of each period's regular capacity. The planning horizon covers 12 periods for all problems.

Four classes of test instances with 2 and 3 buyers, respectively, are considered as shown in Table 1. Six demand series were randomly generated for each test class based on a constant, weakly seasonal and strongly seasonal demand curve and a coefficient of variation of 0.1 or 0.2. Also, three cost structures were considered for each class based on the average ratio between holding and setup costs at the buyers and supplier (constant, high for buyers / low at supplier, and low for buyers / high at supplier).

Finally, seven capacity utilization profiles were regarded as shown in Table 2. Available resource capacity, which is input to the planning models (constraints (2.4)), is calculated from the average capacity requirement based on the final demand series and the utilization factors given in Table 2.

Resulting are 126 (6*3*7) test instances for each class, giving a total of 504 test problems.

Two benchmarks are used for evaluating the performance of the negotiation scheme. First, upstream planning results are considered as the starting point of negotiations and thus an upper bound on total cost. Second, the solutions to a single, centralized planning model are considered. They represent a lower bound on total cost, as long as the global model can be solved to optimality, which is the case in 314 out of the 504 test problems (based on a time limit of 1200 sec.). For the remaining 190 test instances the best solution detected after 1200 sec. is used as reference value.

An overview of the test results is given in Table 3. With regard to upstream planning, the number of capacity infeasible solutions is listed first, i.e. cases with a capacity requirement at the supplier above regular capacity and maximal overtime (20%). Secondly, the solution quality of both, remaining upstream solutions as well as negotiation outcomes, is assessed based upon percentage gaps vs. the central planning solution C^{cent}, i.e.

$(C^{UP} - C^{cent})/C^{cent}$ and $(C^{NEGO} - C^{cent})/C^{cent}$.

Also, the number of iterations carried out during the negotiation process is presented (where an iteration is defined to cover the generation of a compromise proposal by both tiers, i.e. the buyer and the supplier).

The top row shows the total result over all test instances. As can be seen, there is a considerable number of capacity infeasible solutions to upstream planning (104 in total), which mainly go back to test classes 2B-2 and 3B. The remaining upstream solutions deviate substantially from central planning with an average gap of about 40%. The high standard deviation (64%) suggests that individual results are thereby spread over a large interval.

In contrast, the results obtained with the negotiation scheme deviate on average by 6.2% from central planning. Capacity infeasible solutions do not occur at all, and hence are not reported in the table. Also, given a standard deviation of 14.3%, the majority of results falls into the vicinity of the central planning solutions. The number of iterations comes to 4.1, implying that about 8 order / supply proposals (one per iteration and tier) are exchanged during the process.

Comparing the results of classes 2-B1 to 2B-3 (two buyers) with class 3B (three buyers) reveals that the solution quality of the negotiation scheme is lower in the latter case. Nonetheless, the negotiation scheme successfully brings total costs towards the global optimum in both settings, yielding an average gap of 9.0% for class 3B compared to upstream planning results with gaps of 47.8%.

In order to examine the gaps to central planning more closely, Figure 3 shows cumulated frequency distributions of the gaps, i.e. the number of test instances with gaps less or equal to the corresponding x-axis value. The dashed curve represents upstream, the solid negotiation solutions.

It can be observed that with upstream planning only about 3% of the results exhibit gaps of 1% or less, about 10% of the results fall into the 0-5% interval, and about 50% of the test cases have optimality gaps of maximal 12%. This suggests that upstream planning yields good results in some instances, but a rather low solution quality in many others.

This situation clearly alters after negotiations. Here, more than 20% of instances display optimality gaps of less than one percent. Accordingly, 55% fall into the 0-3% interval, and about 90% exhibit a gap to central planning of less than 12%. Only a small fraction of 2% remains with gaps of 30% or higher.

As a final aspect of the solution quality of the negotiation scheme Table 4 shows an overview of the "remaining gaps" of negotiation outcomes. The remaining gap measures the difference between negotiation and central planning solution relative to the difference of the initial upstream outcome to central planning, i.e.

$$(C^{NEGO} - C^{cent}) / (C^{UP} - C^{cent})$$

Thus, it represents an indicator of the improvement potential delivered by the negotiation process.

As can be seen, an average gap of 29.3% remains in total, indicating that about 70% of the gap between upstream and central planning is closed by the negotiations. The standard deviation of 36.5% however implies that values of individual test problems vary strongly. This variation is primarily caused by test instances whose upstream planning solution is already relatively close the lower benchmark; e.g. about 20% have gaps to central planning of less than 5% as shown in Figure 3. Additional, substantial improvements of the cost outcome here are difficult to realize, so that remaining gaps likely take relatively high values.

Also, remaining gaps of all three test classes with two buyers take similar values of around 30%, and class 3B with three buyers even results in a smaller average gap of 24.4%. Thus, despite the fact that the average cost gap of negotiations to central planning is particularly high in class 3B as shown in Table 3, the negotiation scheme performs constantly well (or even better in case of 3B) in bringing the initial upstream result closer to the benchmark solution of central planning.

5 Conclusions

In summary, this paper describes a negotiation-based scheme for collaborative planning in two-tier SCs comprising a single supplier and several buyers. It rests on the approach developed for two SC partners in Dudek / Stadtler (2004) and extends the negotiation mechanism to cover multiple buyers. Also, the amount of information exchanged between the partners is reduced such, that only the respective order / supply proposals are transmitted between the planning partners and compensation needs are requested by the buyers, required to offset cost increases above the initial outcome.

In terms of financial implications, the supplier needs to render the requested compensation to each buyer. In addition, the buyers should receive a portion of the actual, net savings accruing to the supplier in order to establish a win-win situation for all parties. Compensation and savings share can be incorporated into given contract terms as a bonus, granted when a buyer complies with negotiated order quantities. The scheme leaves a limited opportunity for opportunistic behavior, especially at the buyers' side. The buyers can principally report inflated compensation needs in order to gain an additional share of the generated savings. However, requesting inflated compensation can turn an actually improved compromise solution into an unfavorable, second-best outcome, and prevent that any additional savings are shared. Buyers will therefore cheat only to a limited extent, depending on their ability to predict the savings generated in a negotiation round and their attitude towards risk. In addition, other influences may affect the partners' behavior in negotiations, such as cultural habits (see. e.g. Ahlert (1999)), which are not considered by the model.

Computational tests suggest that the negotiation scheme leads to favorable results in the multi-buyer setting considered here. On average, resulting total costs deviate from the benchmark solution to central planning by 6.2% compared to initial cost gaps of upstream planning of 40% and are reached within about four iterations on average.

A corresponding negotiation logic can be devised for two-tier SCs with a single buyer but several suppliers with limited additional adaptations. Here, all suppliers realize cost savings and compensate the buyer's cost increases. The contribution of individual suppliers to the compensation constitutes another, interesting negotiation issue.

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Table 1 Structure of test classes

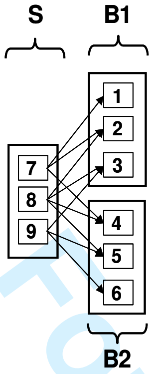
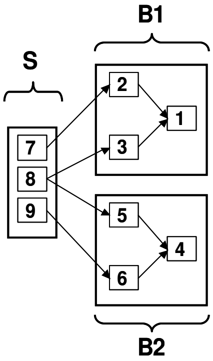
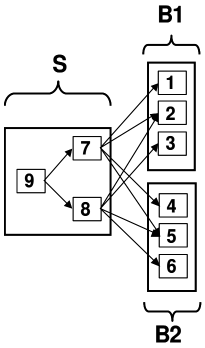
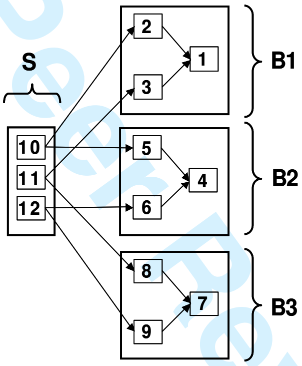
2 **Buyers**	 **2B-1**  **2B-2**  **2B-3**
3 **Buyers**	**3B** 

Table 2 Capacity utilization profiles

Profile	Buyers Util.	Supplier Util.
1	90 %	90 %
2	70 %	70 %
3*	90 % (1-3,10-12), 70 % (4-9)	70 % (1-3,10-12), 90 % (4-9)
4*	70 % (1-3,10-12), 90 % (4-9)	90 % (1-3,10-12), 70 % (4-9)
5	90 %	70 %
6	70 %	90 %
7	50 %	50 %

* Utilization varies over time; numbers in brackets refer to respective periods.

Table 3 Test results overview

Class	Upstream Planning			Negotiations		
	Cap. infeasible	Gaps to central pl.		Gaps to central pl.		# Iterations
	#	Av.	Std. dev.	Av.	Std. dev.	Av.
Total	104	40.3%	64.0%	6.2%	14.3%	4.1
2B-1	20	34.6%	60.2%	4.2%	4.0%	3.6
2B-2	33	52.7%	79.3%	7.9%	20.9%	4.4
2B-3	16	29.1%	52.0%	3.7%	8.4%	4.0
3B	35	47.8%	64.7%	9.0%	16.6%	4.2

Table 4 Remaining gaps of negotiation outcomes to central planning

	Average	Std. Dev.
Total	29.3%	36.5%
2B-1	31.7%	32.2%
2B-2	32.5%	37.8%
2B-3	28.4%	35.3%
3B	24.4%	41.1%

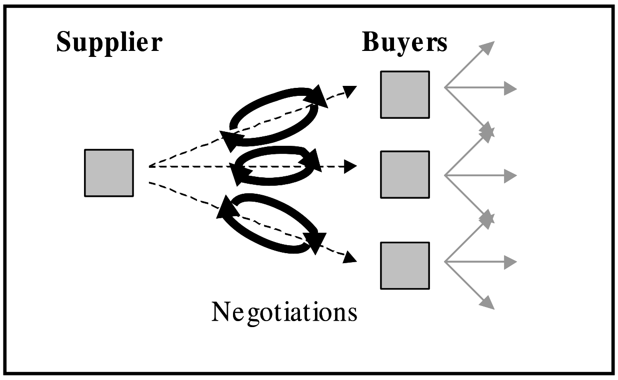


Figure 1 Supply chain structure

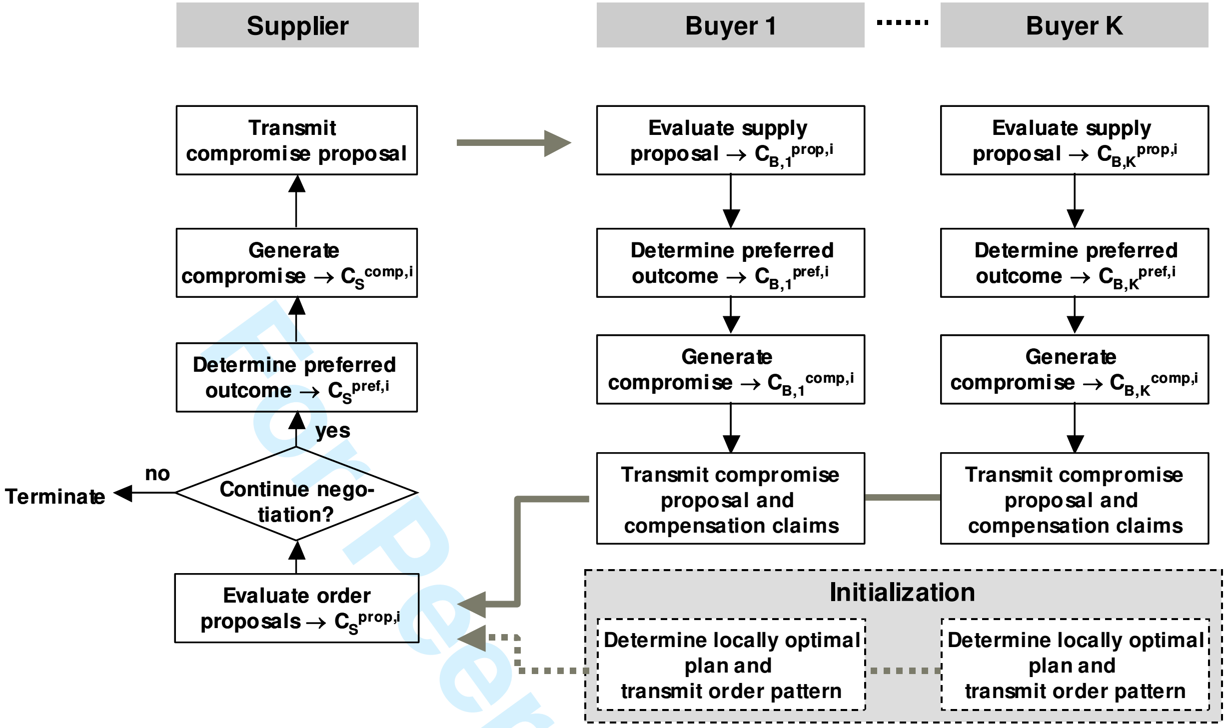


Figure 2 Total negotiation process

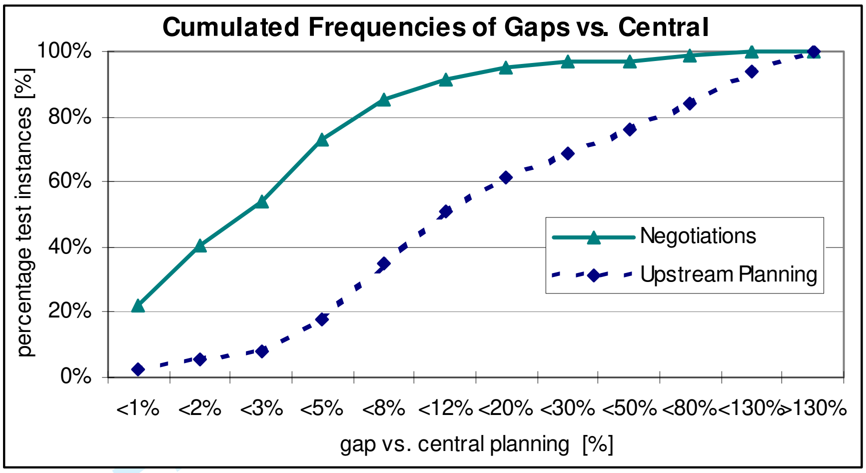


Figure 3 Cumulated frequency distribution of gaps to central planning

Ahlert (1999), 19

Bilgic (2003), 5

Cachon (2003), 4, 5

Cachon / Fisher (2000), 5

Chen (1999), 4

Chen / Federgruen / Zheng (2001), 5

Corbett / DeCroix (2001), 19

Downsland (1996), 18

Dudek / Stadtler (2004), 4, 8

Erengüc / Simpson / Vakharia (1999), 4

Fleischmann (1999), 19

Fransoo / Wouters / de Kok (2001), 5

Kilger / Reuter (2005), 3

Lariviere / Porteus (2001), 4

Lee / Padmanabhan / Whang (1997), 4

Lee / Whang (1999), 4

Monahan (1984), 4

Pochet / Wolsey (1995), 20

Rohde / Meyr / Wagner (2000), 3

Shapiro (1999), 3

Silver / Pyke / Peterson (1998), 3

Simpson / Erengüc (1998), 10

Simpson / Erenguc (2001), 5

Stadtler (2003), 6

Stadtler (2005), 2

Tempelmeier / Derstroff (1996), 20

Thomas / Griffin (1996), 4

Tsay (1999), 4

Tsay / Nahmias / Agrawal (1999), 4

Wheng (1995), 4

Wu / Kleindorfer / Zhang (2002), 19