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Boston – Delft

Foundations and Trends® in Networking

Published, sold and distributed by:
now Publishers Inc.
PO Box 1024
Hanover, MA 02339
USA
Tel. +1-781-985-4510
www.nowpublishers.com
sales@nowpublishers.com

Outside North America:
now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is C. Fragouli and E. Soljanin, Network Coding Fundamentals, Foundation and Trends® in Networking, vol 2, no 1, pp 1–133, 2007

Printed on acid-free paper

ISBN: 978-1-60198-032-8

Foundations and Trends® in Networking
Volume 2 Issue 1, 2007

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Foundations and Trends® in Networking, 2007, Volume 2, 4 issues. ISSN paper version 1554-057X. ISSN online version 1554-0588. Also available as a combined paper and online subscription.

Foundations and Trends® in
Networking
Vol. 2, No. 1 (2007) 1–133

DOI: 10.1561/1300000003

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Abstract

Network coding is an elegant and novel technique introduced at the turn of the millennium to improve network throughput and performance. It is expected to be a critical technology for networks of the future. This tutorial addresses the first most natural questions one would ask about this new technique: how network coding works and what are its benefits, how network codes are designed and how much it costs to deploy networks implementing such codes, and finally, whether there are methods to deal with cycles and delay that are present in all real networks. A companion issue deals primarily with applications of network coding.

Contents

1

Introduction

Networked systems arise in various communication contexts such as phone networks, the public Internet, peer-to-peer networks, ad-hoc wireless networks, and sensor networks. Such systems are becoming central to our way of life. During the past half a century, there has been a significant body of research effort devoted to the operation and management of networks. A pivotal, inherent premise behind the operation of all communication networks today lies in the way information is treated. Whether it is packets in the Internet, or signals in a phone network, if they originate from different sources, they are transported much in the same manner as cars on a transportation network of highways, or fluids through a network of pipes. Namely, independent information streams are kept separate. Today, routing, data storage, error control, and generally all network functions operate on this principle.

Only recently, with the advent of network coding, the simple but important observation was made that in communication networks, we can allow nodes to not only forward but also process the incoming independent information flows. At the network layer, for example, intermediate nodes can perform binary addition of independent bitstreams, whereas, at the physical layer of optical networks, intermediate nodes can superimpose incoming optical signals. In other words, data streams that are independently produced and consumed do not necessarily need to be kept separate when they are transported throughout the network: there are ways to combine and later extract independent information. Combining independent data streams allows to better tailor the information flow to the network environment and accommodate the demands of specific traffic patterns. This shift in paradigm is expected to revolutionize the way we manage, operate, and understand organization in networks, as well as to have a deep impact on a wide range of areas such as reliable delivery, resource sharing, efficient flow control, network monitoring, and security.

This new paradigm emerged at the turn of the millennium, and immediately attracted a very significant interest in both Electrical Engineering and Computer Science research communities. This is an idea whose time has come; the computational processing is becoming cheaper according to Moore's law, and therefore the bottleneck has shifted to network bandwidth for support of ever-growing demand in applications. Network coding utilizes cheap computational power to dramatically increase network throughput. The interest in this area continues to increase as we become aware of new applications of these ideas in both the theory and practice of networks, and discover new connections with many diverse areas (see Figure 1.1).

Throughout this tutorial we will discuss both theoretical results as well as practical aspects of network coding. We do not claim to exhaustively represent and reference all current work in network coding; the presented subjects are the problems and areas that are closer to our interests and offer our perspective on the subject. However, we did attempt the following goals:

(1) to offer an introduction to basic concepts and results in network coding, and
(2) to review the state of the art in a number of topics and point out open research directions.

We start from the main theorem in network coding, and proceed to discuss network code design techniques, benefits, complexity require-

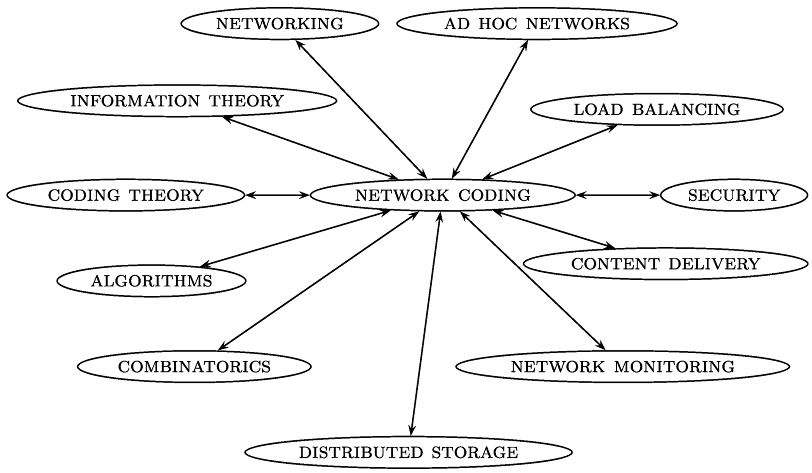

Fig. 1.1 Connections with other disciplines.

ments, and methods to deal with cycles and delay. A companion volume is concerned with application areas of network coding, which include wireless and peer-to-peer networks.

In order to provide a meaningful selection of literature for the novice reader, we reference a limited number of papers representing the topics we cover. We refer a more interested reader to the webpage www.networkcoding.info for a detailed literature listing. An excellent tutorial focused on the information theoretic aspects of network coding is provided in [49].

1.1 Introductory Examples

The following simple examples illustrate the basic concepts in network coding and give a preliminary idea of expected benefits and challenges.

1.1.1 Benefits

Network coding promises to offer benefits along very diverse dimensions of communication networks, such as throughput, wireless resources, security, complexity, and resilience to link failures.

Throughput

The first demonstrated benefits of network coding were in terms of throughput when multicasting. We discuss throughput benefits in Chapter 4.

Example 1.1. Figure 1.2 depicts a communication network represented as a directed graph where vertices correspond to terminals and edges correspond to channels. This example is commonly known in the network coding literature as the butterfly network. Assume that we have slotted time, and that through each channel we can send one bit per time slot. We have two sources S_1 and S_2, and two receivers R_1 and R_2. Each source produces one bit per time slot which we denote by x_1 and x_2, respectively (unit rate sources).

If receiver R_1 uses all the network resources by itself, it could receive both sources. Indeed, we could route the bit x_1 from source S_1 along the path $\{AD\}$ and the bit x_2 from source S_2 along the path $\{BC, CE, ED\}$, as depicted in Figure 1.2(a). Similarly, if the second receiver R_2 uses all the network resources by itself, it could also receive both sources. We can route the bit x_1 from source S_1 along the path $\{AC, CE, EF\}$, and the bit x_2 from source S_2 along the path $\{BF\}$ as depicted in Figure 1.2(b).

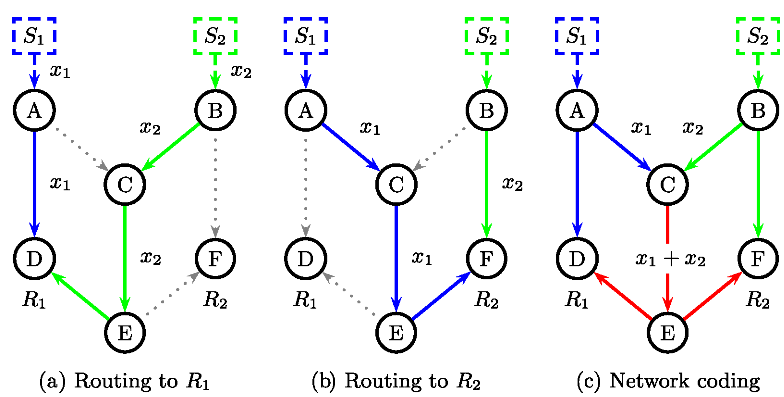

Fig. 1.2 The Butterfly Network. Sources S_1 and S_2 multicast their information to receivers R_1 and R_2.

Now assume that both receivers want to simultaneously receive the information from both sources. That is, we are interested in multicasting. We then have a "contention" for the use of edge CE, arising from the fact that through this edge we can only send one bit per time slot. However, we would like to simultaneously send bit x_1 to reach receiver R_2 and bit x_2 to reach receiver R_1.

Traditionally, information flow was treated like fluid through pipes, and independent information flows were kept separate. Applying this approach we would have to make a decision at edge CE: either use it to send bit x_1, or use it to send bit x_2. If for example we decide to send bit x_1, then receiver R_1 will only receive x_1, while receiver R_2 will receive both x_1 and x_2.

The simple but important observation made in the seminal work by Ahlswede *et al.* is that we can allow intermediate nodes in the network to process their incoming information streams, and not just forward them. In particular, node C can take bits x_1 and x_2 and `xor` them to create a third bit $x_3 = x_1 + x_2$ which it can then send through edge CE (the `xor` operation corresponds to addition over the binary field). R_1 receives $\{x_1, x_1 + x_2\}$, and can solve this system of equations to retrieve x_1 and x_2. Similarly, R_2 receives $\{x_2, x_1 + x_2\}$, and can solve this system of equations to retrieve x_1 and x_2.

The previous example shows that if we allow intermediate node in the network to combine information streams and extract the information at the receivers, we can increase the throughput when multicasting. This observation is generalized to the main theorem for multicasting in Chapter 2.

Wireless Resources

In a wireless environment, network coding can be used to offer benefits in terms of battery life, wireless bandwidth, and delay.

Example 1.2. Consider a wireless ad-hoc network, where devices A and C would like to exchange the binary files x_1 and x_2 using device B as a relay. We assume that time is slotted, and that a device can

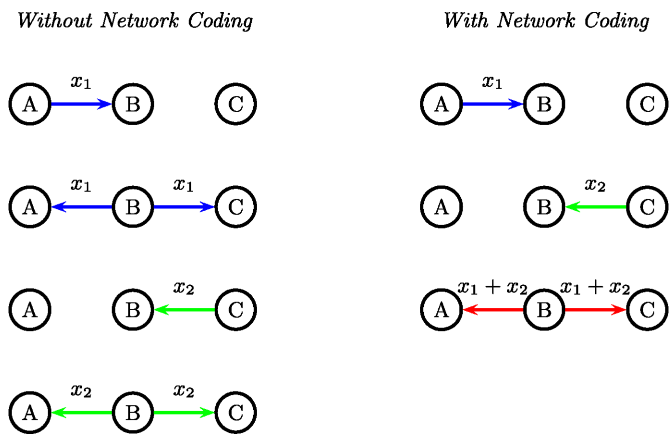

Fig. 1.3 Nodes A and C exchange information via relay B. The network coding approach uses one broadcast transmission less.

either transmit or receive a file during a timeslot (half-duplex communication). Figure 1.3 depicts on the left the standard approach: nodes A and C send their files to the relay B, who in turn forwards each file to the corresponding destination.

The network coding approach takes advantage of the natural capability of wireless channels for broadcasting to give benefits in terms of resource utilization, as illustrated in Figure 1.3. In particular, node C receives both files x_1 and x_2, and bitwise `xor`s them to create the file $x_1 + x_2$, which it then broadcasts to both receivers using a common transmission. Node A has x_1 and can thus decode x_2. Node C has x_2 and can thus decode x_1.

This approach offers benefits in terms of energy efficiency (node B transmits once instead of twice), delay (the transmission is concluded after three instead of four timeslots), wireless bandwidth (the wireless channel is occupied for a smaller amount of time), and interference (if there are other wireless nodes attempting to communicate in the neighborhood).

The benefits in the previous example arise from that broadcast transmissions are made maximally useful to all their receivers. Network coding for wireless is examined in the second part of this review. As we will discuss there, $x_1 + x_2$ is nothing but some type of binning or hashing for the pair (x_1, x_2) that the relay needs to transmit. Binning is not a new idea in wireless communications. The new element is that we can efficiently implement such ideas in practice, using simple algebraic operations.

Security

Sending linear combinations of packets instead of uncoded data offers a natural way to take advantage of multipath diversity for security against wiretapping attacks. Thus systems that only require protection against such simple attacks, can get it "for free" without additional security mechanisms.

Example 1.3. Consider node A that sends information to node D through two paths ABD and ACD in Figure 1.4. Assume that an adversary (Calvin) can wiretap a single path, and does not have access to the complementary path. If the independent symbols x_1 and x_2 are sent uncoded, Calvin can intercept one of them. If instead linear combinations (over some finite field) of the symbols are sent through the different routes, Calvin cannot decode any part of the data. If for example he retrieves $x_1 + x_2$, the probability of his guessing correctly x_1 equals 50%, the same as random guessing.

Similar ideas can also help to identify malicious traffic and to protect against Byzantine attacks, as we will discuss in the second part of this review.

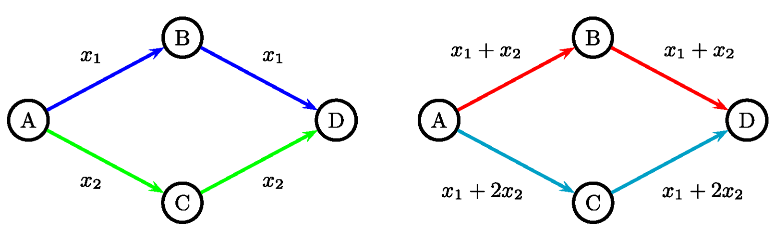

Fig. 1.4 Mixing information streams offers a natural protection against wiretapping.

1.1.2 Challenges

The deployment of network coding is challenged by a number of issues that will also be discussed in more detail throughout the review. Here we briefly outline some major concerns.

Complexity

Employing network coding requires nodes in the network to have additional functionalities.

Example 1.4. In Example 1.2, Figure 1.3, node B has additional memory requirements (needs to store file x_1 instead of immediately broadcasting it), and has to perform operations over finite fields (bitwise `xor` x_1 and x_2). Moreover, nodes A and C need to also keep their own information stored, and then solve a system of linear equations.

An important question in network coding research today is assessing the complexity requirements of network coding, and investigating tradeoffs between complexity and performance. We discuss such questions in Chapter 7.

Security

Networks where security is an important requirement, such as networks for banking transactions, need to guarantee protection against sophisticated attacks. The current mechanisms in place are designed around the assumption that the only eligible entities to tamper with the data are the source and the destination. Network coding on the other hand requires intermediate routers to perform operations on the data packets. Thus deployment of network coding in such networks would require to put in place mechanisms that allow network coding operations without affecting the authenticity of the data. Initial efforts toward this goal are discussed in the second part of the review.

Integration with Existing Infrastructure

As communication networks evolve toward an ubiquitous infrastructure, a challenging task is to incorporate the emerging technologies such as network coding, into the existing network architecture. Ideally, we would like to be able to profit from the leveraged functionalities network coding can offer, without incurring dramatic changes in the existing equipment and software. A related open question is, how could network coding be integrated in current networking protocols. Making this possible is also an area of current research.

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