Network planning by the extended metra potential method (EMPM)

NETWORK PLANNING BY THE EXTENDED METRA POTENTIAL METHOD

(EMPM)

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1 *Introduction*

The METRA Potential Method (MPM), like Precedence, is a network planning method of the type: "activity on node" [7]. MPM offers some major advantages compared with other methods of network planning.

Certain objections can however be made to MPM:

1. The limited practival usefulness.
   In the original version of MPM only relations of the "begin-begin" type are permitted.
2. The deficiency of a good algorithm (efficient and adequate error detection) to perform the network computations.

In this paper an extended version of MPM is presented, which we shall refer to as: "Extended MPM", or EMPM in abbreviated form. This extension of MPM is fairly simple to implement. It consists of the following:

All the relations which exist in Precendence are possible in EMPM. The MPM concept of a "negative arrow" and a negative "cycle" are retained within EMPM. This implies, that a number of relations are possible in EMPM, which are not allowed in Precedence.

These extra features of EMPM demand a basically different algorithm. The algorithms published up to now [3, 6, 7] all are based on [2]. They are strongly iterative.

A new algorithm is discussed in [4] with the following properties:

- only one iteration is performed when there are no cycles;
- just a few iterations are performed when (negative) cycles are present.

2 *MPM*

2.1 History of MPM

The MPM method was developed in France in 1958 by the advisory bureau SEMA [6]. The method was used for the first time at the construction of nuclear power plants on the Loire.

"Metra" originates from a group of advisory bureaux (including SEMA) of the same name which offers its clients MPM as a planning method. The method is called: "potential method" because of the analogy which exists - from a mathematical point of view - with the system of potentials in an electrical network.

2.2 Concise description of MPM

MPM is a network planning method of the type "activity on node" as stated in the introduction.

However we prefer to speak of "activity on bar", as the activities in the bar diagram are not represented by nodes but by bars.

2.2.1 *Activity represented by bar*

The project consists of activities, which are represented in MPM by bars.
An example:

activity I

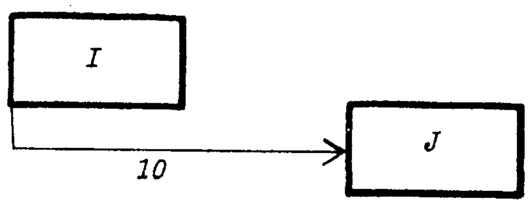
2.2.2 *Relation represented by arrow*

We shall first state some difinitions in order to simplify the further description:

- I and J are both activities;
- $B(I)$ is the moment at which activity I starts;
- $L(I,J)$ is the duration of the relation between I and J.
  In other words the length of the arrow between I and J.

In MPM only relations, which refer to the moments at which activities start, are permitted. In the terminology defined above this will mean, that we only can represent relations between $B(I)$ and $B(J)$.

An example of such a relation: "J can only start 10 time units after I has started". This relation in formula is: $B(J) \geq B(I) + 10$. In a MPM bar diagram this relation is given as follows:



Let an arrow point from activity I to activity J. We call I the *source* and J the *sink* of the arrow.
Let the relation between I and J be given in the following standard form:

$$B(J) \geq B(I) + L(I,J)$$

in which:

- the sink activity is stated left of the ≥ sign
- the source activity is stated right of the ≥ sign
- the length $L(I,J)$ of the arrow is also stated right of the ≥ sign.

Once given the relation in standard form, we can construct a bar diagram with the aid of the rules stated above. It appears that there is a unique representation of a relation in a bar diagram.

2.2.3 *Negative arrows*

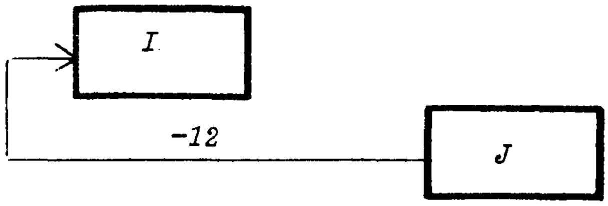
Suppose we should wish to represent the following relation: "J must start within 12 time units after I has started".
We can formulate this as follows:

$$B(J) \leq B(I) + 12$$

In standard form:

$$B(I) \geq B(J) - 12.$$

We now can draw the relation, as described in 2.2.2. in a network.



From the above it appears that in this concept the length *L(I,J)* of an arrow may be negative.
In permitting this kind of relations MPM really is distinct from all other known network methods offering the user the possibility of formulating a more realistic model of a project.

In practice one is often faced with relations of the following kind: "*A* must start within so many days after *B*", "*C* must start before a certain date etc.". This kind of relations cann't be used in the model (= bar diagram) when dealing with other network planning methods.
It is out of the question that such omissions in the model may lead to incorrect results and decisions.

2.2.4 *Negative cycles*

In MPM also combinations are permitted of the relations, described in 2.2.2. and 2.2.3. For instance: "*J* must start between the 10th and the 12th time unit after *I has starte*

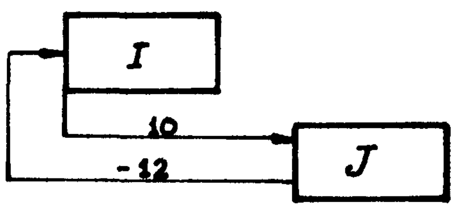
In formula: $B(J) \geq B(I) + 10$

and $B(J) \leq B(I) + 12$

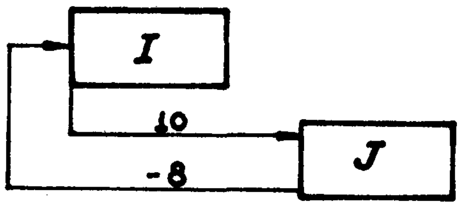
in standard form:

$$B(I) \geq B(J) - 12.$$

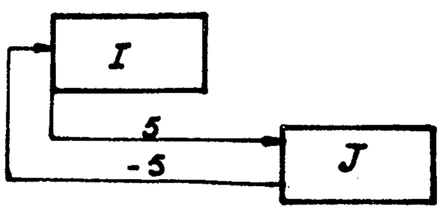
We then find the bar diagram:



In such a case we speak of a "cycle" (loop). The length of the cycle is the sum of the lengths of the arrows composing the cycle.
In the example mentioned above the length of the cycle is therefore 10-12 = -2. The length of a cycle should be less than or equal to zero. If this condition is not met, we are dealing with a kind of vicious circle:

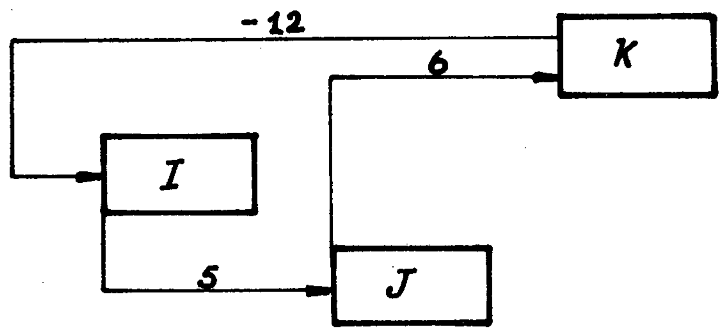


This means that:
- *J* must start within 8 time units after *I* has started;
- *J* can only start 10 time units after *I* has started.
These two relations contradict. In this network they form a "positive cycle".
Negative cycles can be used in many ways in practical network planning. For instance, the relation: "*J* must start exactly 5 time units after *I* has started" can be represented as follows in a MPM bar diagram:

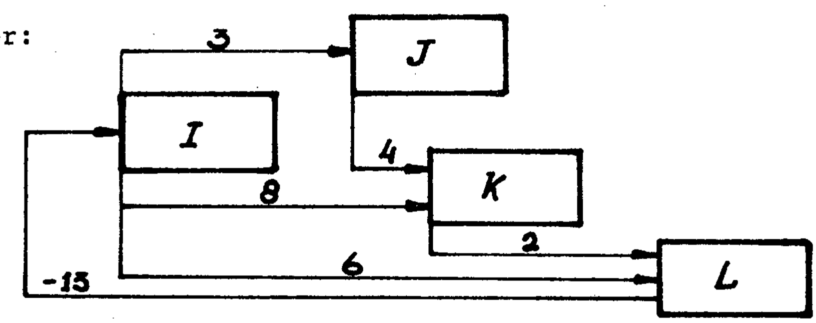


In a network far more complicated cycles may be found.

For example:



or:



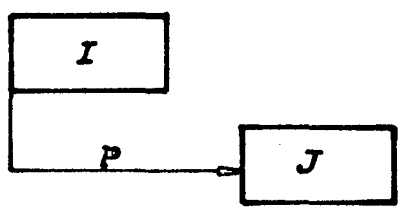
In practice the cycles often are fairly simple.
It is however essential, that the algorithm and the computational program correctly can handle the most complicated cycles and this makes the problem mathematically interesting.

2.3 *Transformation of a MPM bar diagram into a directed graph*

After a project has been split up into activities and the relations between these activities have been established, computations must be performed on the produced bar diagram. In other words, planning data such as: "earliest start", "latest start", etc. have to be determined. The bar diagram must be transformed into a directed graph (network), before these computations can start. Note, that this also holds for Precedence. This transformation may be part of the program and is necessary due to the fact, that there exists no "bar diagram theory"; we however can use graph theory. This graph theory provides us with concepts, theorems and algorithms. The transformation is very simple in the MPM case, because only relations are permitted between the start moments of activities. The start moments are considered to be the nodes of the graph, and the relations the arrows.

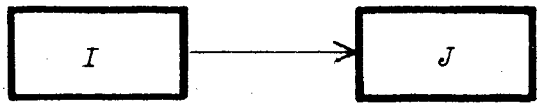
2.4 *Practical objection to MPM*

As a method of planning MPM is difficult to handle. The reason for this is that only relations between start moments are permitted. For example suppose we wish to represent the relation: "activity *J* may only start when activity *I* has finished". This can be depicted as follows:



Here the duration of activity *I* determines the length of the arrow. For the one who produces the bar diagram this involves a lot of extra work. In addition, if the duration of *I* is changed, the length of the arrow must be adjusted accordingly. The chance of making mistakes becomes considerably greater.

If we wish to represent the same relation in Precendence, we find:



This representation is much more logical and straightforward.

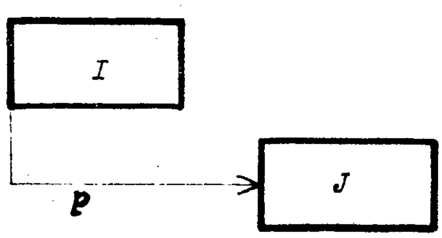
3 *Extended MPM*

3.1 *Extension of the permitted relations*

The objection to MPM mentioned above is removed in a quite simple way in EMPM. In EMPM the following relations are permitted:

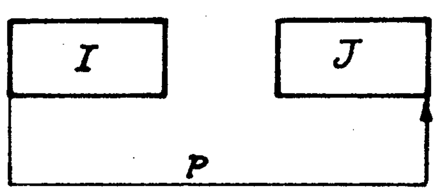
3.1.1 *The begin-begin relation*

This is in the form of:
"activity *J* only may start *p* time units after activity *I* has started".
In diagram:



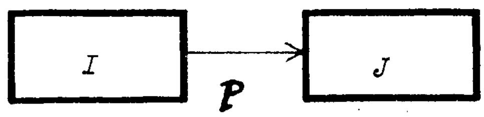
3.1.2 *The begin-end relation*

"Activity *J* only may finish *p* time units after activity *I* has started". In diagram:



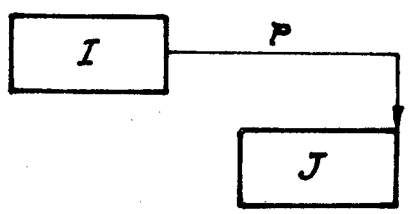
3.1.3 *The end-begin relation*

"Activity *J* only may start *p* time units after activity *I* has finished".
In diagram:



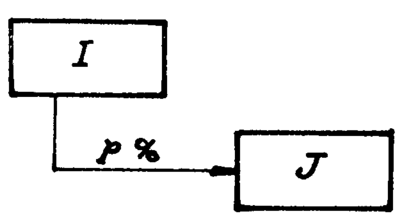
3.1.4 *The end-end relation*

"Activity *J* only may finish *p* time units after activity *I* has finished".



3.1.5 *The percentage relation*

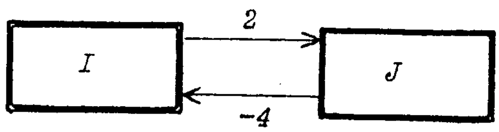
"Activity *J* may only start if the duration of *I* has expired by *p*%".
In diagram:



3.1.6 *Comparison of relations in Precedence and EMPM*

The relations described above are analogous to those found in the Precedence method except for the begin-end relation, although this relation does have a practival value.

For example, the relation: "Activity J must start between the 2nd and the 4th time unit after activity I has finished" can be represented in EMPM as follows:



The arrow from J to I, which is -4 in length, is a begin-end relation.

3.2 *Transformation of an EMPM bar diagram into a directed graph*

The transformation is more complicated in the case of an EMPM bar diagram. There are two possibilities, which we shall discuss now.

3.2.1 *Fixed duration of activities*

The first possibility assumes the estimated duration of an activity being a hard fact. We shall show that the transformation of an EMPM bar diagram into a directed graph roughly occurs in the same way as in the case of a MPM bar diagram. For this purpose, we define:

- I and J are two activities;
- $D(I)$ is the duration of I;
- $B(I)$ is the start moment of I;
- $E(I)$ is the finish moment of I.

The duration of the activity is considered to be fixed; therefore, the following applies:

$E(I) = B(I) + D(I)$ and $E(J) = B(J) + D(J)$.

With the aid of these two relationships, we can transform the EMPM relations of *3.1.2* through *3.1.5* in a simple way into relations of the begin-begin type *(3.1.1)*. For example, we take an end-end relation.

In formula: $E(J) \geq E(I) + p$

or $B(J) + D(J) \geq B(I) + D(I) + p$, in standard form:

$$B(J) \geq B(I) + D(I) - D(J) + p$$

We now have created a begin-begin relation, with a duration of:

$D(I) - D(J) + p$.

By transforming all the relations in a EMPM bar diagram into begin-begin relations, we have in fact constructed a MPM network. We already showed in *2.3* how we can interpret this as a directed graph.

3.2.2 *Second method: variable activity duration*

In this method, we assume a certain amount of variability in the duration of an activity. This is only of value if:

- the activity can be interrupted
- the activity can be temporized, e.g. by using less resources.

The "normal" duration of an activity is taken to be the lower bound of the duration. This means the number of time units required for the execution of the activity, using "normal" capacity, "normal" pace of work, etc. In other words, the activity duration according to the first method *(3.2.1)*.

The upper bound of the activity duration is the number of time units related to the use of minimum capacity or interruption.

Example:

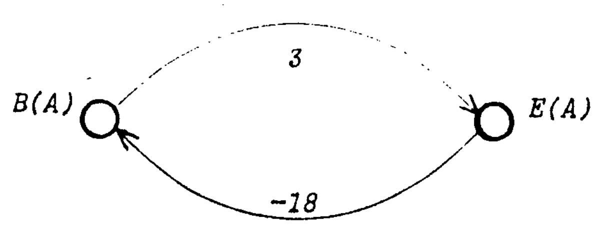
Activity *A* can be completed by 5 men in 3 days, or by 1 man in 18 days. Intermediate possibilities also are permitted. The lower bound of the activity duration then would be 3, and the upperbound 18.

Therefore: $3 \leq D(A) \leq 18$.

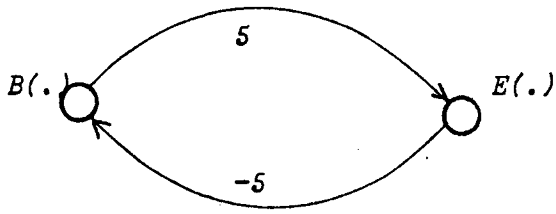
A bar diagram of such a type can be transformed as follows into a directed graph:

- the begin and end points of the activities are represented by nodes;
- arrows represent the relations supplemented with one pair of arrows per activity, which connect the begin and end point of the activity.

Activity *A* can be represented as follows:



Note, that if the lowerbound equals the upperbound, we are dealing with an activity of fixed duration, e.g. an activity which lasts exactly 5 time units.



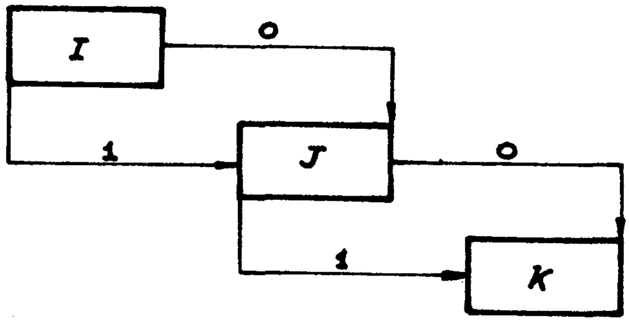
The method described under *3.2.1* may be considered as a special case of the method mentioned here; however, the percentage relation is not allowed when using a variable activity duration.
Note, that it is no longer necessary to transform the relations into begin-begin relations.

3.2.3 *Difference between both methods demonstrated by means of an example*

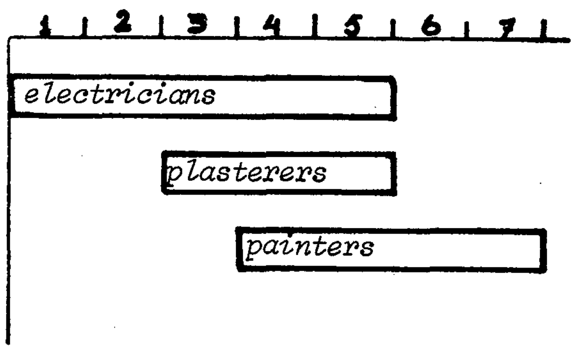
In order to make the difference between the two methods evident, we shall give an example.
A project consists of the following three activities:
I (electricians' work) which takes 5 days;
J (plasterers' work) which takes 3 days;
K (painters' work) which takes 4 days.
The three activities can be carried out simultaneously, on condition that:
- the eclectricians begin at least one day before the plasterers begin;
- the plasterers do not complete the work before the electricians have finished.

These same restrictions also apply to the painters' work in relation to the plasterers' work. We can denote the relations in formula as follows:
$B(J) \geq B(I) + 1$, $E(J) \geq E(I)$, $B(K) \geq B(J) + 1$, $E(K) \geq E(J)$.

In diagram:

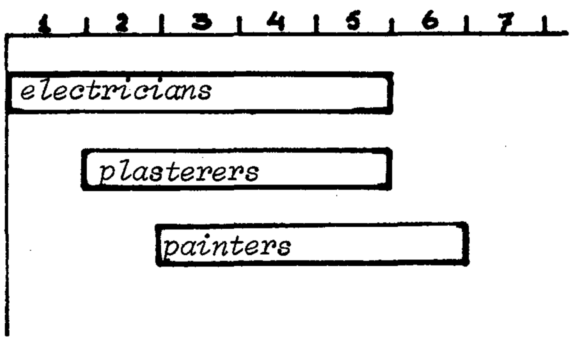


Using the first method, we assume, that *I*, *J* and *K* have durations of exactly 5, 3 and 4 days respective. We represent the results of the calculations by means of a Gantt-chart.



Therefore, the total duration of the project is 7 days.

Using the second method, we assume for instance, that activity *J* can be extended up to 10 days. This could happen if less plasterers were used.
The result of the second method then would be:



Therefore, the duration of the project is 6 days.

The gain of 1 day is attained by starting *J* on the second day and extending the total duration to 4 days. This means, that *J* must be executed at a slower pace.
This sounds like a paradox: if we slow down on an activity, the total duration of the project is shortened. The explanation for this is, that the three activities can be executed simultaneously, for a longer period. Sometimes time can be gained by starting earlier and working at a more moderate pace.

3.3 *EMPM in practice*

A computer program has been developed dealing with EMPM networks, which is called ANNETTE (*A*ctivity in *N*ode *Net*workanalysis University of *T*echnology *E*indhoven) [5]. The term "dealing with" should be taken in the broadest sense of the word; this comprises not only the transformation into a graph and the performing of network computations, but also all the administrative computations like file handling, report printing, etc. This makes the program not only suitable for the initial planning, but also for continuous project control. We are adopting the method, described under *3.2.2* for the transformation of a bar diagram into a graph. In the building industry we are using and testing EMPM on certain projects concerning planning and project control. Recently EMPM was adopted at an oil refinery maintainance project. The results reached so far are very satisfactory.

4 *Final Conclusion*

EMPM is a network planning method combining the advantages of MPM and Precedence. The advantages for the EMPM user are:

- the variety of relations, which makes Precedence easy to deal with;
- the possibility of constructing a more realistic model by using negative arrows and cycles.

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