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# Network Virtualization over Elastic Optical Networks with Different Protection Schemes

K.D.R. Assis, S. Peng, R. C. Almeida Jr, H. Waldman, A. Hammad, A. F. Santos and D. Simeonidou

**Abstract**— With network virtualization, the physical infrastructure can be partitioned into multiple parallel virtual networks for sharing purpose. However, different transport technologies or Quality of Service (QoS) levels may impact in both the requested amount of resources and the characteristics of different virtual instances that can be built on top of a single physical infrastructure. In this paper we propose a novel Mixed Integer Linear Programming (MILP) formulation for different schemes of protection in scenarios where multiple virtual topologies run over an elastic optical network. The proposed MILP formulation uses the concept of bandwidth squeezing to guarantee a minimum bandwidth for surviving virtual topologies. It achieves a high level of survivability for the traffic that is subject to a different committed service profile for each virtual topology. Case studies are carried out in order to analyze the basic properties of the formulation in small networks, and three heuristics are proposed for larger networks.

**Index Terms**— Elastic Optical Networks, Optimization, Routing, Survivability, Virtualization.

## I. INTRODUCTION

CURRENTLY service providers are focused on offering services on top of the infrastructures which they own and manage. The end users have no control over these services and the provider-consumer relationship is far from being automated [1]. For different users, each of these applications has its own specific access and network resource usage pattern as well as quality of service (QoS) and dynamicity requirements. Therefore, dedicated and application-specific optical networks services are desired to support each application category. However, as applications evolve, the current technical and operational complexities will limit the ability of network operators to set up and configure dedicated optical networks for each application type [2],[3]. Furthermore, demands are becoming more and more sporadic. Driven by user behaviours, these new requirements are difficult to be accommodated with the existing, rigid telecommunication operation models. Through network

virtualization, we are able to allocate isolated instances from a given resource to different users or applications and manipulate them logically, before inferring changes to the real resource [4]-[6].

Following the variability concept, several authors [7]-[13] have also pointed out that it is possible to increase the spectrum efficiency of WDM optical networks if one assumes a more elastic method of spectrum allocation and make it “Gridless”. The “Gridless” network architecture is called Elastic Optical Network (EON) in the literature and was originally proposed in [7]. Therefore, VONs on EON networks supports more sporadic demand. Fig. 1 shows an example of 3 Virtual Optical Networks (VONs) ( $z^1$ ,  $z^2$  and  $z^3$ ) that can be allocated over a single physical topology (substrate).

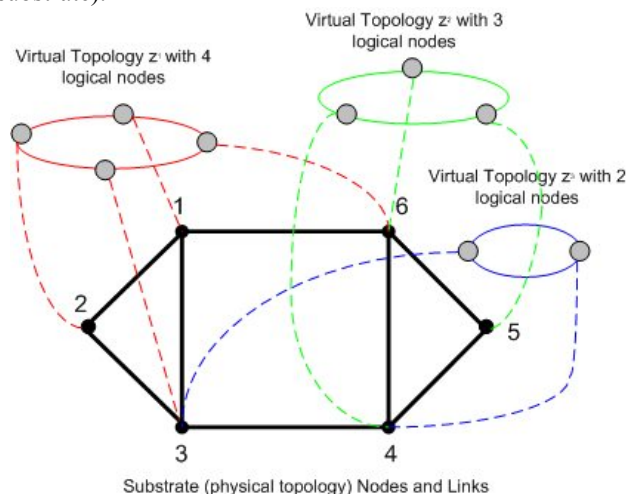


Fig. 1. 6-node network and 3 virtual topologies

In addition, it is well known that a network (regardless of the technology) is subject to natural disasters (e.g., earthquake) and intentional human attacks (e.g., link cuts or malicious attacks) [3]. This is aggravated by the fact that disruptions in optical networks affect huge amount of data. Therefore, it is essential to provision survivability when mapping virtual topologies over the optical physical substrate (EON or traditional WDM). Since each VON may present different types of service, it is expected that a different protection scheme may be used for each VON.

In this work, we investigate how to efficiently map the VONs over the substrate of EON networks with different survivability schemes against any single physical link failure. In particular, we take, for each virtual topology, one of these two different protection schemes: Squeezing Protection [14],

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[15] (with different squeezing rates for each virtual topology) and Dedicated Protection. No Protection is also assessed for comparison purposes.

We formulate the problem as a MILP, in order to minimize the link utilization when different protection schemes for each VON are applied over the EON physical topology. A series of experiments are carried out in order to demonstrate the validity of the proposed formulation, as well as demonstrate the bandwidth economy in each topology with the use of the different protection schemes in each VON. We also propose heuristics and show their efficient bandwidth economy for a small and a large network.

The remainder of this paper is structured as follows: We first discuss the related works in Section II and introduce the problem and methodology that we have used in the paper in Section III. Then, we present the MILP formulation in Section IV and three heuristics for larger networks in Section V. In Section VI, we compare the performance of our proposed method with traditional methods and discuss the obtained results. Section VII concludes the paper.

## II. RELATED WORKS

The virtual topology mapping problem has been extensively studied in both optical “Grid” and “Gridless” networks. The authors in [1]-[6] studied the mapping problem in optical networks, taking into consideration some optical layer constraints, such as the transmission reach constraint and the spectral continuity/conflict constraints. Some results experiments can be seen in [16] and [17]. The authors in [18] study the problem of survivable VON mapping in a flexible grid optical network with programmable regenerators that are capable of converting spectrum and modulation format. In [19], the letter shows experiments in VONs, and proposed multi-layer protection schemes, so the results showed that the proposed schemes can provide differentiated protection for virtual transport network services with multiple operators and diversified services. The recent work [3] focused on investigating how to provision topology mapping with survivability criteria against physical node or link failures. The authors in [20] proposed an efficient link protection scheme that relies on constructing an enhanced topology with survivability in the virtual layer. A recently proposed restoration scheme is Squeezed Restoration [14]. It is a type of recovery scheme where the backup path is established with a bandwidth reduced in relation to the working path’s bandwidth and may reach a required minimum amount considering the client requirement (which is known as “bandwidth squeezing”). This generates cost-effective restoration in terms of spectral resource utilization, which increases the number of surviving paths for the mission-critical data when there are insufficient backup resources in a disastrous failure situation. The authors in [15] developed a scheme similar to [14], but aimed to protection purpose, which is referred to as Partial Protection: after any single link failure, the flow can drop to the partial protection requirement, where a fraction of the demand is guaranteed to remain available between the source and destination after any failure. However,

these works have not addressed the problem of providing different protection characteristics for each virtual topology. That is important because different topologies can demand different protection requirements.

Notice that the terms Squeezed Restoration and Partial Protection were originally used, respectively, for restoration and protection mechanisms. Although in this paper we deal with protection mechanisms, we preferred to use the term Squeezed Protection.

## III. METHODOLOGIES

In optical networks, a connection is routed through many nodes in the network between its source and destination, and there are many elements along its path that can fail. The only practical way of obtaining good availability is to make the network survivable, that is, able to continue providing service in the presence of failures. Protection switching is the key technique used to ensure survivability. Its protection techniques involve providing some redundant capacity within the network and automatically rerouting traffic around the failure using the redundant capacity. Below, we present some basic concepts as well as alternative forms of protection mechanisms that can be efficiently used in EONs. Such explanation will help on the reading of the paper.

### A. Survivability Design in Optical Networks with/without traffic Partitioning and Squeezing

To illustrate the different protection schemes that can be applied to the set of connections and make them resilient to a single link failure in the context of EONs, Fig. 2a shows a simple network topology where a lightpath is set up between nodes 1 and 3. Let us assume that the lightpath is transporting 100 Gb/s of traffic. To protect such lightpath against a link failure, it is possible to find another lightpath, including route and available spectrum, for the same 100 Gb/s of capacity (Fig. 2b). In the event of a failure, the disrupted lightpath is obviously restored using the backup path. This is the normal protection scheme that has been traditionally used in optical networking and is known as Dedicated Path Protection (DPP).

As discussed before, the squeezed protection scheme can be applied as a new type of service-recovery class besides the conventional Dedicated Path Protection. With Partial Protection, the traffic of disrupted lightpaths at failure time may be reduced in comparison to the previously running working traffic. This case is named in this paper as DPP with squeezing capability (DPP+S) and is illustrated in Fig.2c. Note that if, under a link failure, the original 100 Gb/s of traffic may be squeezed to 50% of its normal operation bitrate, just an extra of 50 Gb/s has to be reserved for protection purpose, requiring from the network 150 Gbps, i.e., much less capacity than with DPP.

Another possibility to reserve link capacity efficiently and still restore the original bitrate of the disrupted lightpath was proposed in [21]. The idea is to use some disjoint lightpaths, each of which conveying part of the total bitrate, with the aggregated traffic lower than twice the required. The idea is to save bandwidth when compared to DPP and guarantee the

total committed bitrate under a link failure. This is shown in Fig.2d, where a total capacity equivalent to 150 Gbps, partitioned in three link-disjoint lightpaths of 50 Gbps each, is reserved to transmit the required 100 Gbps. Notice that, since the lightpaths are disjoint, a failure in any of their links will maintain the committed 100 Gbps of traffic. This scheme has been named as Partitioning DPP (PDPP). Notice that PDPP has some advantages over both DPP and DPP+S, since it saves spectrum when compared to DPP at the same time that it is able, with the same total reserved bandwidth (in the example, 150 Gbps), to keep the committed bitrate on the event of a failure, unlike to DPP+S. Finally, it is interesting to perceive that bandwidth squeezing can still be used with PDPP to form PDPP+S. This alleviates even more the amount of extra bandwidth required by PDPP. For instance, suppose that we assume a bandwidth squeezing of only 20%. In this case, three link-disjoint lightpaths with 40 Gbps each would require just 120 Gbps, and guarantee 80 Gbps after a link failure. This example illustrates the huge advantage of using PDPP and PDPP+S when compared to the previously proposed DPP and DPP+S. Although for that very reason network operators prefer not using multipath for provisioning, it is clear that PDPP and PDPP+S can be exploited to improve restorability, provided that the number of parallel lightpaths is kept limited.

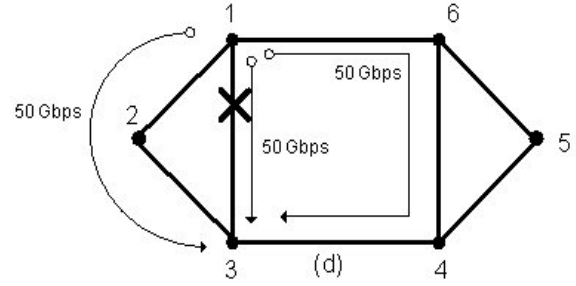


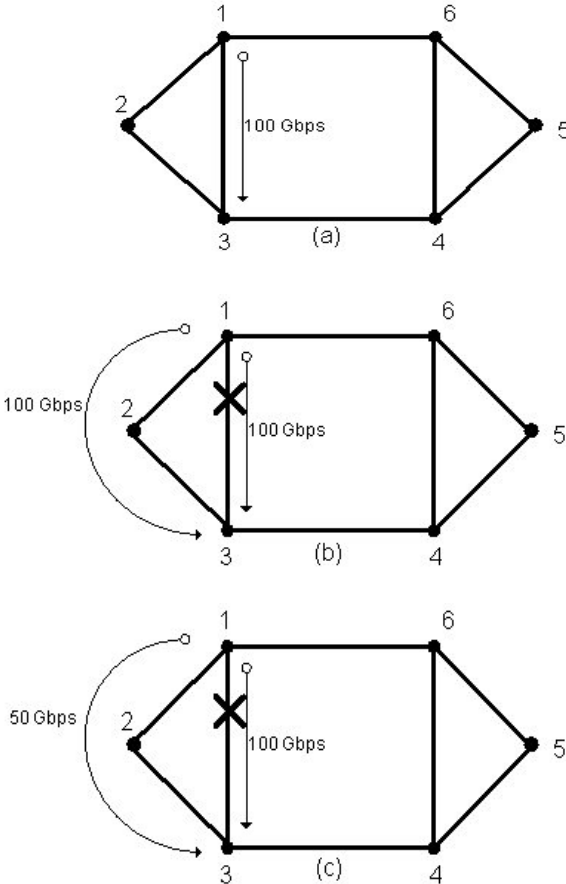
Fig. 2. Protection Mechanism Examples for a source-destination traffic transmission: (a) Normal condition, (b) DPP, (c) DPP+S, (d) PDPP

### B. The Equations for the Protection Mechanisms

The core idea of our proposed partitioning protection mechanism with different squeezed bandwidth over multiple VONs is to reduce the amount of extra bandwidth necessary for the protection at the same time that, under a link failure, the working traffic between any source-destination pairs on the  $z^{\text{th}}$  VON,  $\Lambda^{\text{sd},z}$ , can be kept above or equal to a minimum value agreed by its corresponding Service Level Agreement (SLA).

Let  $\alpha(z)$  be the relation between the reserved bandwidth for protection on demand s-d and its working bandwidth ( $\Lambda^{\text{sd},z}$ ) on the virtual topology  $z$ . In case of conventional DPP,  $\alpha(z)$  is equal to 1 as the total amount of traffic must be reserved for protection purpose. In partitioning DPP we assume  $0 < \alpha(z) \leq 1$ , so that the total amount of reserved bandwidth for the working traffic on s-d over  $z$  is  $[1 + \alpha(z)]\Lambda^{\text{sd},z} \leq 2\Lambda^{\text{sd},z}$ , i.e., lower than if conventional DPP were used. Considering that at most  $\alpha(z)\Lambda^{\text{sd},z}$  can be routed in any link of the substrate network, we can guarantee that, in the event of a link failure, the reserved bandwidth for any demand s-d on  $z$  will be at least  $[1 + \alpha(z)]\Lambda^{\text{sd},z} - \alpha(z) \cdot \Lambda^{\text{sd},z} = \Lambda^{\text{sd},z}$ , i.e., the same as if conventional DPP were used, but with less total reserved capacity. Notice that our strategy may still be referred to as DPP, since the traffic will be fully active in the network. Our objective is to keep the value of  $\alpha(z)$  as small as possible so that the minimum amount of extra bandwidth  $[\alpha(z)\Lambda^{\text{sd},z}]$  is reserved for protection to the demands s-d on  $z$ .

Instead of increasing  $\alpha(z)$  and approaching the large bandwidth utilization of conventional DPP, let's assume that, in the event of a failure, the SLA of demands s-d on  $z$  allows the traffic to be squeezed to at most an agreed fraction of  $\Lambda^{\text{sd}}$ , say  $[1 - \beta(z)]\Lambda^{\text{sd},z}$ , where  $0 \leq \beta(z) < 1$ . In order to guarantee such a constraint, the traffic on each link is relaxed, since each link now may transport at most  $[\alpha(z) + \beta(z)]\Lambda^{\text{sd},z}$  of the total source-destination traffic on s-d over  $z$ :  $[1 + \alpha(z)]\Lambda^{\text{sd},z}$ . This can be understood by the fact that a failure in any link of the network will make the total source-destination traffic on s-d over  $z$  to be dropped from  $[1 + \alpha(z)]\Lambda^{\text{sd},z}$  to at least  $[1 + \alpha(z)]\Lambda^{\text{sd},z} - [\alpha(z) + \beta(z)]\Lambda^{\text{sd},z} = [1 - \beta(z)]\Lambda^{\text{sd},z}$ , as agreed in the SLA of the demand. Under this condition the minimum required node degree will be



$\lceil [1 + \alpha(z)] / [\alpha(z) + \beta(z)] \rceil$ . Moreover, the physical topology must have enough connectivity to provide this number of link-disjoint paths between source and destination nodes.

Therefore, depending on the value of  $\beta(z)$  and the network nodes' degree, it might be possible to select  $\alpha$  considerably smaller than 1 (i.e., considerably smaller than as required in the conventional dedicated protection) and therefore keep the traffic as close as  $\Lambda^{sd,z}$  in the event of a link failure. In the worst situation in which traffic is not allowed to be squeezed (i.e.  $\beta(z) = 0$ ) and the node degree is just 2, we may set  $\alpha(z) = 1$  as in conventional dedicated protection.

For demands s-d on z in the network, if  $0 < \alpha(z) < 1$  and  $\beta(z) = 0$ , the problem reduces to PDPP. However, if  $0 < \alpha(z) < 1$  and  $\beta(z) > 0$ , the working traffic will not be totally protected, which corresponds to PDPP+S. The value of  $\beta(z)$  can therefore be used to reduce the amount of extra reserved bandwidth,  $\alpha(z)\Lambda^{sd}$ , while guaranteeing the SLA (i.e. the minimum amount of traffic) after a link failure. It is easy to see that, from PDPP+S, one can obtain PDPP by imposing  $\beta(z) = 0$ , DPP+S by setting  $\beta(z) > 0$  and  $\alpha(z) = 1 - \beta(z)$ , as well as conventional DPP by imposing  $\alpha(z) = 1$  and  $\beta(z) = 0, \forall z$ . Therefore, a single model will be derived and all such distinct situations can be evaluated.

#### IV. MILP FORMULATION

In most works related to EON networks, such as in [12][13], it is assumed that the usable bandwidth of an optical fiber can be discretized into multiple slots and the bandwidth requested by a demand can be converted into a number of slots. Therefore, in this work the demand and bandwidth will be treated by their number of slots. Compared to the traditional Routing and Wavelength Assignment (RWA) problem, the Routing and Spectrum Allocation (RSA) problem is subject to two unique constraints: The constraint of *spectrum contiguity* requires all the slots that make up an optical channel to be contiguous. In addition, the constraint of *spectrum continuity* requires that the assigned contiguous spectra on all the fiber links traversed by a lightpath be the same when none of nodes in the network is capable of spectrum conversion. Below, we describe the proposed MILP formulation for mapping VONs over shared EON taking into account survivability, grooming and RSA.

##### A. Notations

- $N$ : set of nodes,  $E$  set of links and  $D$  set of VONs.
- $z$ : a VON  $\in D$
- $sd, z$  denote the source and destination nodes of the traffic demands on virtual topology  $z$ .  $s$  and  $d \in z$ .
- $ij, z$  denote originating and terminating nodes of a variable bandwidth lightpath on virtual topology  $z$ .
- $m$  and  $n$  denote endpoints of a physical link in the substrate network. The physical link  $m-n \in E$ .

##### B. Inputs

- Traffic matrix element for each VON:  $\Lambda^{sd,z}$ , which denotes the traffic intensity (in number of slots) from

source node  $s$  to destination node  $d$  in virtual topology  $z$ .

- Filter Guard Band: FGB, which is the minimum spectrum width between wavebands (in number of slots).
- Virtual degree per node for each VON:  $\Delta$
- Maximum squeezed bandwidth ratio:  $\beta(z)$ , where  $[1 - \beta(z)]$  is the minimum admitted bandwidth fraction after a link failure, as agreed in the SLA on  $z$ .
- Expansion traffic factor:  $\alpha(z)$ , where  $\alpha(z)\Lambda^{sd,z}$  is the amount of extra traffic reserved to a source-destination node pair in virtual topology  $z$ .
- A large number  $M$ .

##### C. Variables

- Lightpath bandwidth  $V_{ij,z}$ : bandwidth of an elastic lightpath from node  $i$  to node  $j$  in virtual topology  $z$ .
- Lightpath indicator  $b_{ij,z}$ : a binary variable used to indicate whether there is an elastic lightpath from node  $i$  to node  $j$  in virtual topology  $z$ .
- Traffic routing  $\lambda_{ij}^{sd,z}$ : traffic flow from source node  $s$  to destination node  $d$ , using the lightpath from node  $i$  to node  $j$  in virtual topology  $z$ .
- A binary variable  $B_{ij}^{sd,z}$ : used to indicate whether a fraction of the traffic from node  $s$  to node  $d$  is routed through a lightpath from node  $i$  to node  $j$  in virtual topology  $z$ .  $B_{ij}^{sd,z}$  equals to 1 if  $\lambda_{ij}^{sd,z} > 0$ ; equals to 0 if  $\lambda_{ij}^{sd,z} = 0$ .
- Physical topology route  $P_{mn}^{ij,z}$ : amount of bandwidth that a lightpath from node  $i$  to node  $j$  in virtual topology  $z$  uses in a fiber link  $m - n$ .
- A binary variable  $A_{mn}^{ij,z}$ : used to indicate whether the lightpath in virtual topology  $z$  from node  $i$  to node  $j$  passes through a link  $m - n$ .  $A_{mn}^{ij,z}$  equals to 1 if  $P_{mn}^{ij,z} > 0$ ; equals to 0 if  $P_{mn}^{ij,z} = 0$ .
- Number of frequency slots among all the fiber links:  $C$ .

In the proposed problem optimization, since it is required that all traffic demand is attended in the network, the objective function has been chosen to minimize the link utilization when different protection schemes for each VON are applied over the EON physical topology.

##### D. Mathematical Formulation

The formulation presented in this section does not impose a constraint on the spectrum continuity and contiguity. Therefore, the output of the formulation is equivalent to as assuming spectrum conversion in any node of the network, which provides a lower bound on the number of required slots. However, such results will be used as input to the formulation provided at Section IV.F, which will perform the spectrum assignment taking into account both the continuity and contiguity constraints to adjust the simplification assumed

below. Such assumption alleviates temporarily the spectrum continuity constraint in order to reduce processing time while minimizing the number of slots effectively used by the connections ( $C$ ). This has been intentionally assumed since  $C$  is not only a lower bound to the required number of slots used by the network, which is the final objective, but is also strictly related to it. Consequently, the required processing time is severely reduced at the same time that the minimization of the total number of slots is possibly acquired or minimally affected.

The following objective function and constraints will be used in the MILP:

**Minimize:**  $C$  (4.1)

$$\sum_j \lambda_{ij}^{sd,z} - \sum_j \lambda_{ji}^{sd,z} = \begin{cases} [1 + \alpha(z)] \cdot \Lambda^{sd,z} & \text{if } i = s \\ -[1 + \alpha(z)] \cdot \Lambda^{sd,z} & \text{if } i = d \\ 0 & \text{if } i \neq d \end{cases} \quad (4.2)$$

$$\sum_{sd} \lambda_{ij}^{sd,z} = V_{ij,z} \quad \forall i-j \in z \text{ and } \forall z \in D \quad (4.3)$$

$$\sum_n P_{mn}^{ij,z} - \sum_n P_{nm}^{ij,z} = \begin{cases} V_{ij,z} & m = i \\ -V_{ij,z} & m = j \\ 0 & m \neq i, j \end{cases} \quad (4.4)$$

$$\sum_{ij,z} (P_{mn}^{ij,z} + FGB \cdot A_{mn}^{ij,z}) \leq C \quad \forall m-n \in E \quad (4.5)$$

$$A_{mn}^{ij,z} \geq P_{mn}^{ij,z} / M \quad \forall i,j \in N, \forall m-n \in E \text{ and } \forall z \in D \quad (4.6)$$

$$b_{ij,z} \geq V_{ij,z} / M \quad \forall i,j \in N \text{ and } \forall z \in D \quad (4.7)$$

$$\sum_i b_{ij,z} \leq \Delta \quad \forall j \in N \text{ and } \forall z \in D \quad (4.8)$$

$$\sum_j b_{ij,z} \leq \Delta \quad \forall i \in N \text{ and } \forall z \in D \quad (4.9)$$

**SLA constraints:**

$$\lambda_{ij}^{sd,z} \leq [\alpha(z) + \beta(z)] \cdot \Lambda^{sd,z} \quad \forall i,j,s,d \text{ and } \forall z \in D \quad (4.10)$$

$$B_{ij}^{sd,z} \geq \lambda_{ij}^{sd,z} / M \quad \forall i,j,s,d \text{ and } \forall z \in D \quad (4.11)$$

$$\sum_{ij} B_{ij}^{sd,z} \cdot A_{mn}^{ij,z} \leq 1 \quad \forall s,d,m,n \text{ and } \forall z \in D \quad (4.12)$$

In MILP formulation, the Traffic-Grooming approach (4.1 to 4.3) has the task of establishing bandwidth-variable lightpaths for each virtual topology, whereas the SLA constraints (4.10 to 4.12) denote the type of protection scheme on each virtual topology.

Since network cost and power consumption are related to the number of slots at the fiber links, the objective in this study is to minimize the maximum slot index among all fibers, as can be seen in (4.1). Equation (4.2) is the flow conservation constraints of flows on each virtual topology (grooming layer). Equation (4.3) denotes that low-speed traffic flows are groomed into bandwidth-variable lightpaths. Equation (4.4) is the flow conservation constraints of routing at the optical layer. Equation (4.5) denotes that the utilized bandwidth of all VONs (including Filter Guard Band) should not exceed the spectrum capacity of the fiber. Equation (4.6) is used to count the amount of FGB overheads. The constraint 4.7 determines the existence or absence of a lightpath between nodes  $i$  and  $j$  on the virtual topology  $z$ . The constraints (4.8) and (4.9) ensure that the designed topology has no more than  $\Delta$  transceivers in each node. In SLA constraints, (4.10) is the bandwidth squeezed definition. Equation (4.11) is used to count the amount of virtual hops. Equation (4.12) denotes that if traffic from a source-destination pair is routed on multiple lightpaths, such lightpaths must not use a common physical links.

### E. Linearization

Unfortunately, the constraint (4.12) is non-linear, but it is formed by the multiplication of two binary variables. Therefore, we can replace (4.12) by some constraints, which linearizes the problem (see [20], Section III.E).

### F. Spectrum Allocation Phase.

The spectrum allocation is similar to the formulation presented in [22], but now, the set  $P$  of pre-calculated paths, as defined in [22], is fed with the set of paths calculated in the routing phase of the MILP formulation, where  $P_{ij}$  is the path of each source-destination node pair  $i$ - $j$  and  $P = \{P_{ij}\}$  is the set of paths of all lightpaths. The returned  $A_{mn}^{ij,z}$  denotes the optical links included in the path  $P_{ij}$ . Thus, for each lightpath, one path is included in  $P$ .

The ILP spectrum allocation has as objective function the minimization of the total number of allocated slots. Then, the spectrum allocation phase minimizes the maximum slot index,  $F_{max}$ , among all links. Therefore  $F_{max} \geq C$ , since  $C$  is the lower bound on the number of slots as stated before. The proximity between  $C$  and  $F_{max}$  indicates the efficiency of the spectrum allocation phase for the paths found by the MILP. The ILP spectrum formulation is omitted for brevity purposes.

## V. HEURISTIC ALGORITHMS

Due to the complexity of the problem for large networks, the complete strategy presented in Section IV (D-F) may be very time consuming. For instance, to run the MILP formulation for the network shown in Fig. 1 in an Intel Intel i3 2.27GHz 2GB machine, it was necessary 7 minutes. If we add an additional node connected to nodes 1 and 6, the required simulation time increases to 1h. Finally, if we include a new node between nodes 3 and 4, we observed a simulation time of around 5,5 hours, showing that we rapidly need a heuristic model to deal with moderate to large networks. To reduce such complexity, we propose to decompose the problem into two sub-problems. First, the lightpaths of each VON will be defined by adopting the protection criteria studied in this paper and the constraints imposed by the virtual topologies; subsequently, an RSA sub-problem will be solved to find the route and set of contiguous and continuous slots in the physical topology. Notice that, by solving the sub-problems in sequence and combining their solutions, we may end up not finding a solution as good as the one provided by the fully integrated problem, but the processing time may be substantially reduced while still acquiring a good solution. Obviously, a good solution will depend on the use of efficient strategies for each of the sub-problems. The two sub-problems together with their solving strategies are described below:

### A. Sub-Problem 1: Lightpath Definition on each VON

The aim of this sub-problem is to determine the VONs that will be composed on the physical topology, i.e., find all lightpaths for each VON in terms of their source and destination nodes together with their necessary number of slots. This is shown below:

Given the traffic matrix demands  $\{\Lambda^{sd,z}\}$ ,  $\Delta$ ,  $D$  and desired values for  $\alpha(z)$  and  $\beta(z)$ , find the virtual links  $(b_{ij,z})$ , their corresponding bandwidth  $(V_{ij,z})$  and the flow routes  $(B_{ij}^{sd,z})$  with the objective function of minimizing the maximum lightpaths' load along all fibers ( $C'$ ) with the sub-formulation 1 below:

$$\text{Minimize } C' \quad (5.1)$$

$$V_{ij,z} \leq C' \quad (5.2)$$

$$\lambda_{ij}^{sd,z} \leq b_{ij,z}[\alpha(z) + \beta(z)] \cdot \Lambda^{sd,z} \quad \forall i,j,s,d \text{ and } \forall z \quad (5.3)$$

$$\in D$$

In addition to the above three equations, the constraints imposed by Eqs. (4.2), (4.3), (4.7), (4.8), (4.9) and (4.11) should also be included in the sub-formulation 1.

### B. Sub-Problem 2: RSA Execution

Here, three alternatives of solving the RSA problem are presented. The first is based on a simplified ILP formulation and the other two are algorithm-based. It is important to say that the lightpaths found in the sub-problem 1 for each VON will be used as a common step for the three proposed RSA schemes described below.

#### B.1) VON-SF

The idea here is to use part of the equations of the MILP (IV.D) and spectrum allocation phase (IV.F) to solve the RSA problem in the physical topology with the lightpaths found in the Sub-Problem 1. Notice that the RSA subproblem is then based on an ILP and therefore is also referred to as ILP-RSA, as shown below.

*Step 1:* Solve the sub-problem 2 using the equations (4.4)-(4.6) and (4.12) with the objective function (4.1), but using the values from  $V_{ij,z}$  and  $B_{ij}^{sd,z}$  found in Sub-Problem 1 as inputs.

*Step 2:* Solve the Spectrum Allocation Phase with the paths found on step 1 above as the input set P.

The two heuristic-algorithms described below are based on solving the RSA problem over several VONs.

#### B.2) Shortest Path with Maximum Spectrum Reuse with Protection (SPSR-P)

Let us assume that a given set of path requests on  $z$  is given with their amount of requested slots. Intuitively, the higher is the slot reuse, the higher is the reduction in the maximum number of required slots. In this section, we propose the shortest path with maximum spectrum reuse with protection (SPSR-P) algorithm, which combines the shortest path routing with the maximum reuse spectrum allocation (MRSA) algorithm shown in Algorithm 1 from [13]. In this approach, the spectrum path requests from every VON are first sorted according to the number of required slots, since the slots consecutiveness constraint makes it harder to find available consecutive slots for demands for larger number of slots. After that, spectrum paths are selected and assigned by following the order as they were sorted. Shortest path routing and first-fit spectrum assignment are used during the RSA

process. Note that only fiber-disjoint spectrum paths may reuse the same slots.

#### B.3) Balanced Load Spectrum Allocation with Protection (BLSA-P)

In this subsection, we propose another method, namely, Balanced Load Spectrum Allocation with Protection (BLSA-P), which determines the routing by balancing the load within the network to potentially minimize the maximum number of used slots. As shown in the following 3 steps of the heuristic, BLSA-P also employs the First Fit.

*Step 1: Path generation.* In this stage, we use the  $k$ -shortest path algorithm to generate  $k$  ( $k \geq 1$ ) path(s) for each pair of nodes  $i$ - $j$  where there is a demanded spectrum path, namely  $P_h^{i,j}$ , where  $h = 1, 2, \dots, k$ .

*Step 2: Path selection.* In this stage, we decide the path for each spectrum path with the goal of balancing the load among all fibers within the substrate network. The load of a fiber  $j$ ,  $L_j$ , is estimated using Eq. (12) from [13]. The goodness of a path is evaluated by calculating the maximum fiber load  $C = \max L_j$  of the substrate network if the path is used to serve the demand. The candidate path that produces the lowest value of  $C$  is used as the routing path for the corresponding spectrum path. More specifically, starting from the spectrum path with the largest traffic demand, assign one of the  $k$  paths to it while minimizing  $C$ , until all the node-pairs with nonzero traffic demands are considered. After the path is selected,  $L_j$  is updated according to Eq. (12) from [13]. The protection path routing is performed so that it is disjoint with the working path.

*Step 3: Spectrum allocation.* In this stage, we use the First Fit to accommodate all the spectrum paths for each VON and to find the required number of slots,  $F_{max}$ , of the network.

## VI. DISCUSSIONS AND RESULTS

For evaluating the effectiveness of the proposed optimization, we analyzed two different network topologies (one small and another large). We used IBM ILOG CPLEX v.11.0 [23] on an Intel i3 2.27GHz 2GB computer to solve the formulation. We specified an upper limit of 3600 seconds as the maximum allowed computation time for solving the MILP formulation. We noted that the simulations using the complete formulation (Sections IV.D – VI.F) needed considerably less time than the upper limit to solve the problem for the small network (around 7 minutes). We first describe the performance of the small network in detail and then summarize the results of the large network with the proposed heuristics.

### A. Small Network

We use a 6-node and 8-link topology (Fig. 1) to evaluate the performance of the proposed formulation and the three heuristic algorithms for different traffic demands on the virtual topologies. We assume that there is one pair of bidirectional fiber on each link and  $FGB = 1$  slot. We simulate 4 cases.

Each case assumes 3 virtual topologies ( $z^1$ ,  $z^2$  and  $z^3$ ) and random traffic demand (4, 8 or 16 slots, as shown in Table I) for each of them, where three different combinations of protection criteria, as described in Table II, have been used.

TABLE I. RANDOM TRAFFIC DEMANDS (IN SLOTS) - SMALL NETWORK

Case I ( $z^1$ , $z^2$ and $z^3$ )	Case II ( $z^1$ , $z^2$ and $z^3$ )	Case III ( $z^1$ , $z^2$ and $z^3$ )	Case IV ( $z^1$ , $z^2$ and $z^3$ )
$\begin{bmatrix} - & 0 & 4 & 0 & 0 & 4 \\ 0 & - & 0 & 0 & 0 & 0 \\ 0 & 8 & - & 0 & 0 & 8 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 16 & 0 & 4 & 0 & 0 & - \end{bmatrix}$	$\begin{bmatrix} - & 0 & 0 & 0 & 0 & 0 \\ 0 & - & 8 & 16 & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 \\ 0 & 4 & 16 & - & 0 & 0 \\ 0 & 4 & 0 & 4 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & - \end{bmatrix}$	$\begin{bmatrix} - & 0 & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 16 & 8 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 8 & 0 & - & 16 \\ 0 & 0 & 16 & 0 & 16 & - \end{bmatrix}$	$\begin{bmatrix} - & 16 & 0 & 0 & 0 & 16 \\ 8 & - & 0 & 0 & 0 & 8 \\ 0 & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 16 & 16 & 0 & 0 & 0 & - \end{bmatrix}$
$\begin{bmatrix} - & 0 & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & - & 4 & 0 & 8 \\ 0 & 4 & 0 & - & 0 & 0 \\ 0 & 0 & 4 & 0 & - & 16 \\ 0 & 0 & 4 & 8 & 0 & - \end{bmatrix}$	$\begin{bmatrix} - & 16 & 0 & 8 & 0 & 0 \\ 0 & - & 0 & 0 & 0 & 0 \\ 8 & 0 & - & 8 & 0 & 0 \\ 16 & 8 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & - \end{bmatrix}$	$\begin{bmatrix} - & 0 & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & - & 4 & 16 \\ 0 & 0 & 0 & 8 & - & 8 \\ 0 & 0 & 0 & 4 & 8 & - \end{bmatrix}$	$\begin{bmatrix} - & 0 & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & - & 16 & 16 & 0 \\ 0 & 4 & 0 & - & 8 & 0 \\ 0 & 8 & 0 & 8 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & - \end{bmatrix}$
$\begin{bmatrix} - & 0 & 0 & 0 & 0 & 0 \\ 0 & - & 4 & 0 & 8 & 0 \\ 0 & 4 & - & 0 & 8 & 0 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 8 & 4 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & - \end{bmatrix}$	$\begin{bmatrix} - & 0 & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & - & 16 & 8 & 0 \\ 0 & 0 & 16 & - & 8 & 0 \\ 0 & 0 & 16 & 16 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & - \end{bmatrix}$	$\begin{bmatrix} - & 0 & 16 & 0 & 0 & 8 \\ 0 & - & 0 & 0 & 0 & 0 \\ 4 & 0 & - & 0 & 0 & 4 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 4 & 0 & 8 & 0 & 0 & - \end{bmatrix}$	$\begin{bmatrix} - & 0 & 0 & 0 & 0 & 0 \\ 0 & - & 16 & 4 & 0 & 0 \\ 0 & 16 & - & 8 & 0 & 0 \\ 0 & 8 & 4 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & - \end{bmatrix}$

TABLE II. PROTECTION SCHEMES

Protection Schemes	VON	$\alpha(z)$	$\beta(z)$
<b>DP</b> (Dedicated Protection) DPP for $z^1$ , $z^2$ and $z^3$	$z^1$	1	0
	$z^2$	1	0
	$z^3$	1	0
<b>MP</b> (Multiple Protection) DPP for $z^1$ , DPP+S for $z^2$ and Non-Protection for $z^3$	$z^1$	1	0
	$z^2$	0	0.5
	$z^3$	0	1
<b>NP</b> (Non Protection) Non Protection for $z^1$ , $z^2$ and $z^3$	$z^1$	0	1
	$z^2$	0	1
	$z^3$	0	1

TABLE III. MAXIMUM NUMBER OF FREQUENCY SLOTS,  $F_{MAX}$ , AMONG ALL FIBER LINKS IN THE SUBSTRATE NETWORK

CASES	MILP Formulation			HEURISTICS								
	NP	MP	DP	VON-SF			BLSA-P			SPSR-P		
<b>I</b>	21	41	59	27	48	72	30	64	120	41	68	120
	Avr. Gap:			5,3%			20,6%			31,3%		
<b>II</b>	40	57	91	41	68	111	42	68	120	64	108	180
	Avr. Gap:			13%			15,3%			44%		
<b>III</b>	24	58	74	24	58	76	26	68	90	40	68	90
	Avr. Gap:			1%			10%			24%		
<b>IV</b>	36	68	103	37	73	111	42	72	110	42	96	150
	Avr. Gap:			4,2%			9%			27%		

Table III shows the performance in terms of the maximum number of frequency slots,  $F_{MAX}$ , among all fiber links in the substrate network for the complete MILP formulation as well as for VON-SF, BLSA-P and SPSR-P under the four traffic-demand considered cases. As it can be seen, in all reported cases, MP provides less spectrum resource than DP. In addition, NP, as expected, is the case with the lowest spectrum requirement, but subject to not providing any link-failure

protection. When the heuristics are investigated and compared to the MILP formulation, one can see that, among the proposed heuristics, VON-SF produces the best results, and these are close to the optimal MILP solution (no more than 13% in average to the cases considered). Among the Algorithm-based heuristics, BLSA-P outperformed SPSR-P in almost all analysed cases, except in four of them, where the same performance was observed.

In Fig. 3, the x-axis shows each physical link in the small network, whereas the y-axis represents the number of slots used in each referred link using MILP for the case III traffic-demand scenario. Fig. 3 indicates that MP can indeed achieve load balancing (in terms of number of slots) in the network.

Just for a quick comparison, we have also used another MP protection scheme, as shown in Table IV and referred to as MP2, with different squeezing factors on each VON. Again, comparing the mixed schemes MP and MP2 against DP, we observe, for the same traffic profile, a clear gain in terms of saved bandwidth.

TABLE IV. PROTECTION SCHEME MP2

Protection Schemes	VON	$\alpha(z)$	$\beta(z)$
<b>MP2</b> (Multiple Protection) PDPP+S	$z^1$	0.5	0.25
	$z^2$	0.5	0.5
	$z^3$	0.5	0.75

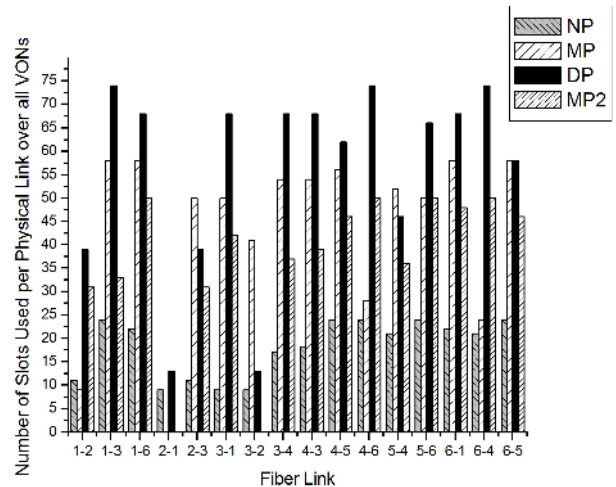


Fig. 3. Number of slots per fiber-link.

### B. Larger Network (NSFNET)

Due to the complexity of solving the MILP formulation for large networks, we performed experiments just with the algorithm-based heuristics for a moderate large network (NSFNET, [15]) with 14 nodes and 21 links. We use DP and MP protection schemes with the four cases described in Table V to compare their performance.

First of all, we recognize that it may be impractical to use the proposed VON-SF heuristic with networks of reasonable size. In our simulations, it could take more than 1h to find the solution of NSFNET with DP scheme. This occurs because sub-problem 1 combined with sub-problem 2 are still hard to be solved. But we expect our VON-SF to act as a benchmark for other heuristics.



TABLE V. PROTECTION SCHEMES (NSFNET)

Protection Schemes	VON	$\alpha(z)$	$\beta(z)$
<b>DP</b> (Dedicated Protection- Case I) DPP for $z^1, z^2$ and $z^3$	$z^1$	1	0
	$z^2$	1	0
	$z^3$	1	0
<b>MP</b> (Multiple Protection- Case II) PDPP+S for $z^1, z^2$ and $z^3$	$z^1$	0.5	0.3
	$z^2$	0.5	0.4
	$z^3$	0.5	0.5
<b>MP</b> (Multiple Protection- Case III) PDPP+S for $z^1, z^2$ and $z^3$	$z^1$	0.5	0.6
	$z^2$	0.5	0.7
	$z^3$	0.5	0.8
<b>MP</b> (Multiple Protection- Case IV) PDPP+S for $z^1, z^2$ and $z^3$	$z^1$	0.5	0.3
	$z^2$	0.5	0.5
	$z^3$	0.5	0.8

Next figures show the simulation results for the 14-node NSFNET [20] under three virtual topologies for the four cases described in Table V (other combinations which show the same pattern are omitted here). We use only SPSR-P and BLSA-P. Fig.4 shows the maximum number of slots used in any fiber in the network,  $F_{MAX}$ , for BLSA-P and SPSR-P when the four protection schemes described in Table V are employed. When we compare the protection cases described in Table V), it can be seen that BLSA-P used less slots than SPSR-P. For instance, for the first and fourth cases, a reduction of about 17% and 23%, respectively, was achieved with the use of BLSA-P instead of SPSR-P.

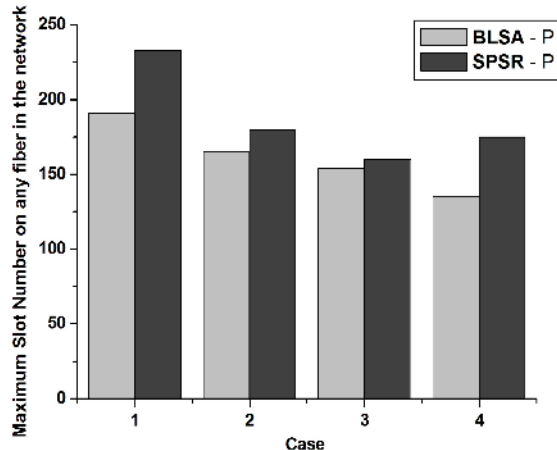


Fig.4. Maximum slot index,  $F_{max}$ , when different cases of protection SLAs (Table V) are employed

In all analyzed cases, we can confirm that SPSR-P does not balance the load as efficiently as BLSA, which implies additional overhead and thus requiring more slots for every case. Therefore, the comparison between BLSA-P and SPSR-P indicates that BLSA-P can indeed achieve load balancing in the network, as summarized in Fig.4.

In Figs. 5, 6, 7 and 8, the  $y$ -axis is the ID of each fiber link in the 14-node network and the  $x$ -axis represents the number of slots used on each fiber link for each of the heuristics. One can see that, for the case I (dedicated protection), the required

number of slots is around 230 for SPSR-P and 190 for BLSA-P, while for all other cases, the required number of slots is around 180 for SPSR-P and 160 for BLSA-P. In general, we can conclude that protection with balanced routing results in smaller  $F_{max}$  when compared to the shortest path routing.

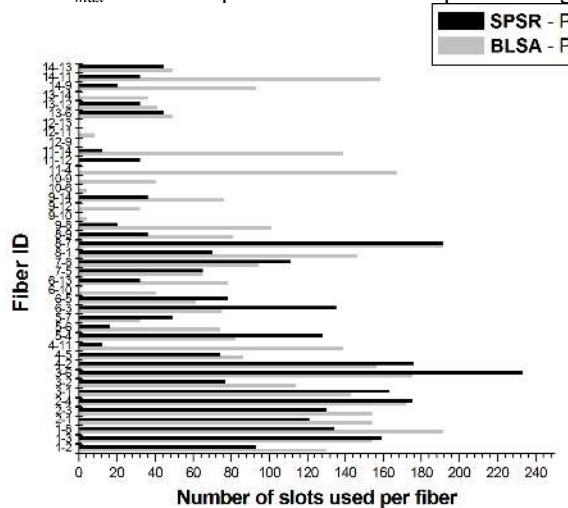


Fig. 5. Case I (maximum number of slots per fiber)

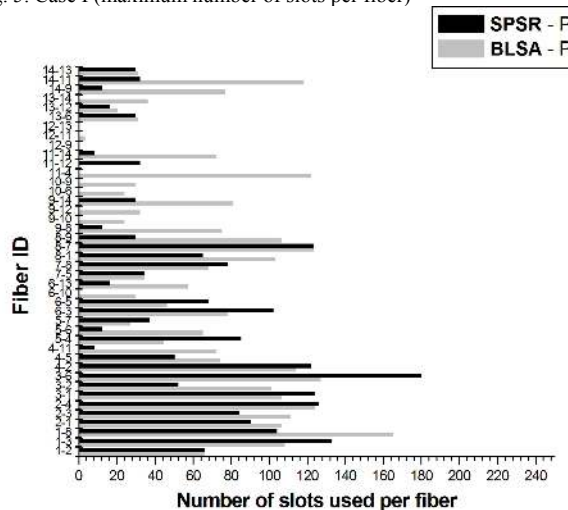


Fig. 6. Case II (maximum number of slots per fiber)

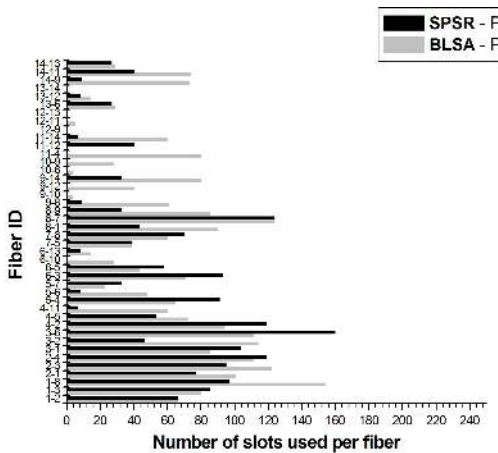


Fig. 7. Case III (maximum number of slots per fiber)

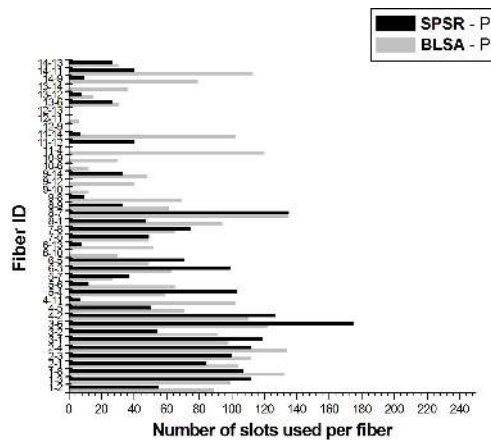


Fig. 8. Case IV (maximum number of slots per fiber)

## VII. CONCLUSIONS

In this paper, a novel and unified MILP formulation for new and distinct protection concepts in EON networks with multiple virtual topologies was proposed. The proposed formulation provides different survivability levels for traffic demands subject to committed service profiles, including bandwidth squeezing, which can increase the number of surviving paths in the network at the price of reducing the traffic bandwidth under a link failure. Using extensive simulation experiments, we have demonstrated the effectiveness of our MILP formulation. The performance obtained in terms of objective value and protection is very good. We also proposed three heuristics for large networks. We noticed that VON-SF, by using an ILP formulation, still takes a long time to find a solution with DP scheme. This processing time burden can be alleviated with the use of BLSA-P and SPSR-P with still good results, remarkably to BLSA-P.

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## REFERENCES

- [1] A. Pages Perelló, J. ; Spadaro, S. ; Garcia-Espin, J.A. ; Riera, J.F. ; Figuerola, S "Optimal allocation of virtual optical networks for the future internet," in Proc. of ONDM 2012, pp. 1–6, Apr. 2012.
- [2] P. Khodashenas et al "Using spectrum fragmentation to better allocate time-varying connections in elastic optical networks". Vol 6. No 5/may 2014/J. Opt. Commun.Netw.
- [3] Z Ye, A. N Patel, PN Ji, C Qiao "Survivable Virtual Infrastructure Mapping with Dedicated Protection in Transport Software-Defined Networks [Invited]". J. Opt. Commun. Netw. Vol. 7, No. 2, Feb 2015.
- [4] R. Nejabati, E. Escalona, S. Peng, and D. Simeonidou, "Optical network virtualization," in Proc. of ONDM 2011, pp. 1–5, Feb. 2011.
- [5] Juzi Zhao, Subramaniam, S. ; Brandt-Pearce, M. "Virtual Topology Mapping in Elastic Optical Networks" IEEE ICC 2013.
- [6] S Zhang, L Shi, CSK Vadrevu, B Mukherjee "Network virtualization over WDM and flexible-grid optical networks."Optical Switching and Networking, 10 (2013) 291-300.
- [7] M. Jinno, H. Takara, B. Kozicki, Y. Tsukishima, Y. Sone, and S. Matsuoka, "Spectrum-Efficient and Scalable Elastic Optical Path Network: Architecture, Benefits, and Enabling Technologies," IEEE Comm. Mag., vol.47, pp. 66-73, 2009.
- [8] B. Kozicki, H. Takara, T. Yoshimatsu, K. Yonenaga, and M. Jinno, "Filtering Characteristics of Highly-Spectrum Efficient Spectrum-Sliced Elastic Optical Path (SLICE) Network," Proc. OFC/NFOEC'09, 2009.
- [9] Y. Zhang, X. Zheng, Q. Li, N. Hua, Y. Li, and H. Zhang, "Traffic Grooming in Spectrum-Elastic Optical Path Networks," in Optical Fiber Communication Conference, OSA Technical Digest (CD) (Optical Society of America, 2011), paper OTu11.
- [10] B. C. Chatterjee, N. Sarma and E. Oki "Routing and spectrum allocation in elastic optical networks: A tutorial" IEEE Comm. Surveys Tuts., vol. 17, no. 3, pp. 1776-1800, May, 2015.
- [11] L. Velasco, M. Klinkowski, M. Ruiz, and J. Comellas, "Modeling the Routing and Spectrum Allocation Problem for Flexgrid Optical Networks," Springer Photonic Network Communications, vol. 24, pp. 177-186, 2012.
- [12] K. Christodoulopoulos Tomkos, I. ; Varvarigos, E.A. "Routing and spectrum allocation in OFDM-based optical networks with elastic bandwidth allocation," IEEE Globecom, Dec. 2010.
- [13] Yang Wang, Xiaojun Cao, Yi Pan. "A Study of the Routing and Spectrum Allocation in Spectrum-sliced Elastic Optical Path Networks" IEEE Infocom 2011.
- [14] Y. Sone, A. Watanabe, W. Imajuku, Y. Tsukishima, B. Kozicki, H. Takara, and M. Jinno "Bandwidth Squeezed Restoration in Spectrum-Sliced Elastic Optical Path Networks (SLICE)," J. Optical Communications and Networking, Vol. 3, No. 3, pp. 223-233, 2011.
- [15] K.D.R. Assis, R.C. Almeida Jr, H. Waldman, "MILP formulation for Squeezed Protection in Spectrum-Sliced Elastic Optical Path Networks. SPECTS 2012.
- [16] Ou, Y.; Yan, S.; Hammad, A.; Guo, B.; Peng, S.; Nejabati, R.; Simeonidou, D."Demonstration of Virtualize-able and Software Defined Optical Transceiver" Journal of Lightwave Technology (Volume:PP , Issue: 99 ) DOI: 10.1109/JLT.2015.2510819 (dez 2015).
- [17] Ou, Yanni, et al. "Online and Offline Virtualization of Optical Transceiver."Journal of Optical Communications and Networking 7.8 (2015): 748-760.
- [18] Khandaker, F.A. ; Erik Jonsson et al. "Survivable Virtual Optical Network mapping in spectrum and modulation format convertible flexible grid optical networks" Optical Fiber Communications Conference and Exhibition (OFC), 2015.
- [19] Wei Wang et al. "First Demonstration of Virtual Transport Network Services With Multi-layer" IEEE Communications Letters, Vol. 20, No 2, February 2016.
- [20] G.Kuperman, E. Modiano A, N. Tam. Network Protection with Multiple Availability Guarantees. ICC 2012.
- [21] F. Paolucci, A. Castro, F. Cugini, L. Velasco, and P. Castoldi, "Multipath Restoration and Bitrate Squeezing in SDN-based Elastic Optical Networks," (Invited Paper) Springer Photonic Network Communications, vol. 28, pp. 45-57, 2014.
- [22] A. Hammad, R. Nejabati and D. Simeonidou "Novel approaches for composition of online virtual optical networks utilizing O-OFDM technology" Optical Communication (ECOC 2013).
- [23] "IBM ILOG CPLEX: high-performance mathematical programming engine," <http://www.ibm.com/>, 2011.