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Neural Network Control of a Rehabilitation Robot by State and Output Feedback

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Abstract — In this paper, neural network control is presented for a rehabilitation robot with unknown system dynamics. To deal with the system uncertainties and improve the system robustness, adaptive neural networks are used to approximate the unknown model of the robot and adapt interactions between the robot and the patient. Both full state feedback control and output feedback control are considered in this paper. With the proposed control, uniform ultimate boundedness of the closed loop system is achieved in the context of Lyapunov's stability theory and its associated techniques. The state of the system is proven to converge to a small neighborhood of zero by appropriately choosing design parameters. Extensive simulations for a rehabilitation robot with constraints are carried out to illustrate the effectiveness of the proposed control.

Index Terms — Adaptive neural network control, Full state feedback control, Lyapunov's direct method, Output feedback control, Rehabilitation robot.

1 Introduction

More than two-thirds of stroke patients with the limb impairment have significant weakness to conduct activities of daily living [1]. Due to the increase of aged population and decline in number of caregivers, the robotic device is in urgent need to fulfill these training tasks for those stroke patients. Advantages of robot aided rehabilitation include the ability to document and store motion and force parameters, the ability to achieve thousands of repetitions per treatment session, and increased biofeedback through the incorporation

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of training tasks [2, 3]. Moreover, robotic rehabilitation could potentially improve the productivity of stroke rehabilitation, reduce cost and improve precision [4]. Therefore, it is essential for the robot to be able to perform efficient control in unknown dynamical interactions with patients [5]. In addition, a model-free method, which requires little model information, is always favorable to mimic high DOF behaviors due to its simplicity.

In recent years, the control problem of the constrained robot has gained much attention [6, 7, 8, 9]. The control of an uncertain robot in interactions with the patient is difficult to handle in the control design and stability analysis due to the unknown system dynamics and unknown external force. In the literatures, impedance control is widely used to handle the interaction between the constrained robots and the human, as it can achieve a stable position and force control by tracking a target impedance model [10]. Under impedance control, the position tracking is realized during the robot's free motion, while the position and force are indirectly controlled during the robot's constrained motion [11]. Based on the Lyapunov's direct method, impedance control of the constrained robot has been presented in [9]. The hybrid force/position control is also widely used for the constrained robotic systems. In [11], adaptive position/force control is proposed for an uncertain constrained flexible joint robot for ensuring the position tracking and the boundedness of force errors. In [12], robust adaptive position/force control is investigated for a nonholonomic robot under holonomic constraints.

Neural networks based approach is considered to be an effective method in a number of research fields, which requires relatively less information of the system dynamics. It has been proven that artificial neural networks are able to approximate a wide range of nonlinear functions to any desired degree of accuracy under certain conditions [13, 14]. Artificial neural networks have been widely used for the control design of uncertain nonlinear systems [15, 16, 17, 18, 19, 20, 21, 22, 23]. The relevant applications for this approach based on the Lyapunov's stability theory include [24, 25, 26, 27, 28, 29, 30, 31]. However, most of above papers considers the constrained force as a part of the system uncertainties, and neural networks are used to approximate those uncertainties for achieving the control objective. This method may influence the system performance of the proposed control when the constrained force is large. In order to overcome the drawbacks, the unknown constrained force of the robotic system needs to be efficiently handled to guarantee the closed-loop control performance. To improve the compensation performance for the external force, a signum term is employed to deal with the unknown constrained force.

In this paper, we consider an n-link constrained rehabilitation robot with unknown system dynamics in interactions with the human user. To deal with the system uncertainties and improve the robustness of the system, adaptive neural networks are used to approximate the unknown model of the robot and adapt interactions between robots and humans. This work is well motivated by the control problem of the constrained rehabilitation robot with uncertainties. Compared to the existing work, the main contributions of the paper include:

(i) Adaptive neural network control with both full state feedback and output feedback is proposed to compensate for the system uncertainties and improve the robustness of the closed-loop system.

(ii) Output feedback neural network control is proposed via the high gain observer when the full states information is not available in some practical applications.

(iii) With the proposed control, uniform ultimate boundedness of the system is proved via the Lyapunov's direct method. The closed-loop system states will eventually converge to a compact set and the control performance of the system is guaranteed by suitably choosing the design parameters.

The rest of the paper is organized as follows. The preliminaries and the dynamics of an n-link rehabilitation robotic system are given in Section 2. Adaptive neural network control via the Lyapunov's direct method is discussed for system uncertainties in Section 3, where it is shown that the uniform boundedness of the closed-loop system can be achieved by the proposed control. Simulations are carried out to illustrate the performance of the proposed control in Section 4. The conclusion of this paper is presented in Section 5.

2 Preliminaries and Problem Formulation

2.1 Function Approximation

A class of linearly parameterized neural networks can be used to approximate the continuous function $f_i(Z) : \mathbb{R}^q \to \mathbb{R}$,

$$f_i(Z) = W_i^T S_i(Z), \;\; i = 1, 2, ..., n, \tag{1}$$

where the input vector $Z = [Z_1, Z_2, \ldots, Z_q]^T \in \Omega_Z \subset \mathbb{R}^q$, weight vector $W_i \in \mathbb{R}^l$, the neural network node number $l > 1$ and $S_i(Z) = [s_1, s_2, \ldots, s_l]^T \in \mathbb{R}^l$. Universal approximation results indicate that, if l is chosen sufficiently large, $W_i^T S_i(Z)$ can approximate any continuous function, $f_i(Z)$, to any desired accuracy over a compact set $\Omega_Z \subset \mathbb{R}^q$. This is achieved as

$$f_i(Z) = W_i^{*T} S_i(Z) + \epsilon_i(Z), \quad \forall Z \in \Omega_Z \subset \mathbb{R}^q, \;\; i = 1, 2, ..., n, \tag{2}$$

where W_i^* is the ideal constant weight vector, and $\epsilon_i(Z)$ is the approximation error which is bounded over the compact set, i.e. $|\epsilon_i(Z)| \leq \bar{\epsilon}_i$, $\forall Z \in \Omega_Z$ with $\bar{\epsilon}_i > 0$ as an unknown constant. The ideal weight vector W_i^* is an "artificial" quantity required for analytical purposes. W_i^* is defined as the value of W_i that minimizes $|\epsilon_i|$ for all $Z \in \Omega_Z \subset \mathbb{R}^q$, i.e.

$$W_i^* = \arg \min_{W_i \in R^l} \{ \sup_{Z \in \Omega_Z} |f_i(Z) - W_i^T S_i(Z)| \}. \tag{3}$$

The Radial Basis Function (RBF) neural network is a particular network architecture which uses l Gaussian functions of the form

$$s_k(Z) = \exp\left[\frac{-(Z-\mu_k)^T (Z-\mu_k)}{\eta_k^2} \right], \quad k = 1, 2, ..., l, \tag{4}$$

where $\mu_k = [\mu_{k1}, \mu_{k2}, ..., \mu_{kq}]^T$ is the center of the receptive field and η_k is the width of the Gaussian function [32].

2.2 Useful Technical Lemmas and Definitions

Lemma 1 *[33] For bounded initial conditions, if there exists a C^1 continuous and positive definite Lyapunov function $V(x)$ satisfying $\kappa_1(\|x\|) \le V(x) \le \kappa_2(\|x\|)$, such that $\dot{V}(x) \le -\rho V(x) + c$, where κ_1 and κ_2 : $\mathbb{R}^n \to \mathbb{R}$ are class $\mathcal{K}$ functions and c is a positive constant, then the solution $x(t)$ is uniformly bounded.*

Lemma 2 *[34] Consider the basis functions of Gaussian RBF neural network (4) with $\hat{Z}$ being the input vector, if $\hat{Z} = Z - \epsilon\bar{\psi}$, where $\bar{\psi}$ is a bounded vector and constant $\epsilon > 0$, then we have*

$$\begin{aligned} s_j(\hat{Z}) &= \exp\left[\frac{-(\hat{Z}-\mu_j)^T (\hat{Z}-\mu_j)}{\eta_j^2} \right], \quad j = 1, 2, ..., l, \\ S(\hat{Z}) &= S(Z) + \epsilon S_t, \end{aligned} \tag{5}$$

where S_t is a bounded vector function.

Definition 1 ***(SGUUB)*** *[34] The solution $X(t)$ of a system is semi-globally uniformly ultimately bounded (SGUUB) if, for any compact set Ω_0 and all $X(t_0) \in \Omega_0$, there exists an $\mu > 0$ and $T(\mu, X(t_0))$ such that $\|X(t)\| \le \mu$ for all $t \ge t_0 + T$.*

Definition 2 *The operator "$\odot$" is defined as follows:*

$$\boldsymbol{a} \odot \boldsymbol{b} = [a_1, a_2, ...a_n]^T \odot [b_1, b_2, ...b_n]^T = [a_1 b_1, a_2 b_2, ...a_n b_n]^T, \ \forall \boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}^n, \tag{6}$$

where $\boldsymbol{a} = [a_1, a_2, ...a_n]^T$ *and* $\boldsymbol{b} = [b_1, b_2, ...b_n]^T$ *are two n-dimensional vectors.*

Lemma 3 *[35] Suppose that a system output* $y(t)$ *and its first* n *derivatives are bounded such that* $|y^{(k)}| < Y_K$ *with positive constants* Y_K*, we can consider the following linear system:*

$$\begin{aligned} \varepsilon\dot{\pi}_i &= \pi_{i+1}, \quad i = 1, ..., n-1, \\ \varepsilon\dot{\pi}_n &= -\bar{\lambda}_1\pi_n - \bar{\lambda}_2\pi_{n-1} - ... - \bar{\lambda}_{n-1}\pi_2 - \pi_1 + x_1(t), \end{aligned} \tag{7}$$

where ε *is any small positive constant and the parameters* $\bar{\lambda}_1$ *to* $\bar{\lambda}_{n-1}$ *are chosen such that the polynomial* $s^n + \bar{\lambda}_1 s^{n-1} + ... + \bar{\lambda}_{n-1}s + 1$ *is Hurwitz. Then, the following property holds:*

$$\xi_k = \frac{\pi_k}{\varepsilon^{k-1}} - x_1^{(k-1)} = -\varepsilon\psi^{(k)}, \quad k = 1, ..., n-1, \tag{8}$$

where $\psi = \pi_n + \bar{\lambda}_1\pi_{n-1} + ... + \bar{\lambda}_{n-1}\pi_1$ *with* $\psi^{(k)}$ *denoting the* k^{th} *derivative of* ψ*. Also, there exist positive constants* t^* *and* h_k *such that* $\forall t > t^*$*, we have* $||\xi_k|| \leq \varepsilon h_k$*,* $k = 1, 2, 3, ..., n$*.*

Lemma 4 *[36, 37, 38] Rayleigh-Ritz theorem: Let* $A \in \mathbb{R}^{n\times n}$ *be a real, symmetric, positive-definite matrix; therefore, all the eigenvalues of A are real and positive. Let* $\lambda_{\min}$ *and* $\lambda_{\max}$ *denote the minimum and maximum eigenvalues of A, respectively; then for* $\forall x \in \mathbb{R}^n$*, we have*

$$\lambda_{\min}||x||^2 \leq x^T Ax \leq \lambda_{\max}||x||^2, \tag{9}$$

where $||\cdot||$ *denotes the standard Euclidean norm.*

2.3 Problem Formulation

The dynamics of an n-link rehabilitation robotic system are described by [9]

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau(t) - J^T(q)f(t), \tag{10}$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ are the position, velocity and acceleration vectors respectively, $\tau \in \mathbb{R}^n$ is the input torque, $M(q) \in \mathbb{R}^{n\times n}$ is a symmetric positive definite inertia matrix, $C(q, \dot{q})\dot{q} \in \mathbb{R}^n$ denotes the Centripetal and Coriolis force, $G(q) \in \mathbb{R}^n$ is the gravitational force, $J(q)$ is the Jacobian matrix which is assumed to be nonsingular, and $f(t) \in \mathbb{R}^n$ is the vector of constrained force exerted by the user.

Let $x_1 = q$ and $x_2 = \dot{q}$, we have the description of the robot dynamics as

$$\dot{x}_1 = x_2, \tag{11}$$

$$\dot{x}_2 = M^{-1}(x_1)[\tau - J^T(x_1)f - C(x_1, x_2)x_2 - G(x_1)]. \tag{12}$$

The control objective is to design control torques such that the system variable x_1 tracks the given reference trajectory $x_d(t) = [q_{d1}(t), q_{d2}(t), ...q_{dn}(t)]^T$, while ensuring that all closed loop signals are bounded.

Property 1 *[15] The matrix $M(q)$ is symmetric and positive definite.*

Property 2 *[15] The matrix $\dot{M}(q) - 2C(q, \dot{q})$ is skew-symmetric.*

Property 3 *[15] The left-hand side of the dynamic equation can be linearly parameterized as*

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Y(q, \dot{q}, \ddot{q})\theta, \tag{13}$$

where $\theta \in R^p$ contains the system parameters, and $Y(q, \dot{q}, \ddot{q}) \in R^{n\times p}$ is the regression matrix, which contains known functions of the signals $q(t)$, $\dot{q}(t)$ and $\ddot{q}(t)$.

Assumption 1 *We assume that the constrained force $f(t)$ is uniformly bounded, i.e., there exists a constant $\bar{f} \in \mathbb{R}^+$, such that $|f(t)| \leq \bar{f}$, $\forall t \in [0, \infty)$.*

Remark 1 *This is a reasonable assumption as the time-varying constrained force $f(t)$ is bounded from an engineering point of view. Thus, the knowledge of the exact value for $f(t)$ is not required, and the force sensors mounted at the interaction points are not needed as well. As such, different constrained forces up to various levels of interactions can be applied without affecting the control design or analysis.*

3 Control Design

In this paper, two cases are investigated for the constrained robotic system: (i) full-state feedback control design, i.e. x_1 and x_2 are all known; and (ii) output feedback control design, i.e. only x_1 is known. For the first case, adaptive neural network control is introduced for approximating the unknown model of the robot and adapt interactions. For the second case where x_2 cannot be directly measured, the high-gain observer is designed to estimate x_2 and ensure the control performance.

3.1 Adaptive Neural Network Control with Full-State Feedback

We first consider the case where full state information, x_1 and x_2, is available. Define a generalized tracking error as $z_1(t) = x_1(t) - x_d(t)$ and have $\dot{z}_1(t) = x_2(t) - \dot{x}_d(t)$. We introduce a virtual control $\alpha_1(t)$ and define a second error variable as $z_2(t) = x_2(t) - \alpha_1(t)$. We choose

$$\alpha_1 = -K_1 z_1 + \dot{x}_d, \tag{14}$$

where the gain matrix $K_1 = K_1^T > 0$, and we have

$$\dot{z}_1 = z_2 + \alpha_1 - \dot{x}_d = z_2 - K_1 z_1. \tag{15}$$

Differentiating z_2 with respect to time, we have

$$\dot{z}_2 = M^{-1}(x_1)[\tau - J^T(x_1)f - C(x_1, x_2)x_2 - G(x_1)] - \dot{\alpha}_1(t). \tag{16}$$

Considering a Lyapunov function candidate as

$$V_1 = \frac{1}{2} z_1^T z_1, \tag{17}$$

and taking its time derivative along Eq. (15), we have

$$\dot{V}_1 = -z_1^T K_1 z_1 + z_1^T z_2. \tag{18}$$

Then, we consider the Lyapunov function candidate as

$$V_2 = \frac{1}{2} z_1^T z_1 + \frac{1}{2} z_2^T M(x_1) z_2. \tag{19}$$

Differentiating Eq. (19) with respect to time leads to

$$\dot{V}_2 = -z_1^T K_1 z_1 + z_1^T z_2 + z_2^T[\tau - J^T(x_1)f - C(x_1, x_2)\alpha_1(t) - G(x_1) - M(x_1)\dot{\alpha}_1(t)]. \tag{20}$$

From Property 2, we know that $\frac{1}{2}(\dot{M}(x_1) - 2C(x_1, x_2))$ is a skew-symmetric matrix. From definition of the skew-symmetric matrix, we know that for any skew-symmetric matrix $A \in \mathbb{R}^{n \times n}$, it satisfies the condition $-A = A^T$. Then, we have $-z_2^T A z_2 = z_2^T A^T z_2 = z_2^T A z_2$. We further obtain $z_2^T A z_2 = 0$. Hence, we can state $\frac{1}{2}z_2^T(\dot{M}(x_1) - 2C(x_1, x_2))z_2 = 0$. For this reason, $\frac{1}{2}(\dot{M}(x_1) - 2C(x_1, x_2))z_2$ disappears in the above equation. When $M(x_1)$, $C(x_1, x_2)$, $G(x_1)$, f are known, we design the model-based control as

$$\tau_0 = -z_1 - K_2 z_2 + J^T(x_1)f + C(x_1, x_2)\alpha_1(t) + G(x_1) + M(x_1)\dot{\alpha}_1(t), \tag{21}$$

where the gain matrix $K_2 = K_2^T > 0$. Substituting Eq. (21) into Eq. (20), we have

$$\dot{V}_2 = -z_1^T K_1 z_1 - z_2^T K_2 z_2. \tag{22}$$

Since the uncertainties exist in $M(x_1)$, $C(x_1, x_2)$, $G(x_1)$, f, the model-based control design may not be realizable. Thus, the above model-based control is not applicable for the robots with uncertainties. To overcome the challenge, neural networks based control is utilized to approximate the uncertainties and improve the performance of the system via the online estimation.

We propose the following control as

$$\tau = -z_1 - K_2 z_2 - \text{sgn}(z_2^T) \odot J^T(x_1)\bar{f} + \hat{W}^T S(Z), \tag{23}$$

where $\text{sgn}(\bullet)$ returns a vector with the signs of the corresponding elements of the vector $(\bullet)$, $\odot$ is the operator defined as in Definition 2, $\hat{W}$ are the weights of neural networks, and $S(Z)$ is the basis function. The neural networks $\hat{W}^T S(Z)$ approximate $W^{*T} S(Z)$ defined by

$$W^{*T} S(Z) = C(x_1, x_2)\alpha_1(t) + G(x_1) + M(x_1)\dot{\alpha}_1(t) - \epsilon(Z), \tag{24}$$

where $Z = [x_1^T, x_2^T, \alpha_1^T, \dot{\alpha}_1^T]$ are the input variables to the adaptive neural networks and $\epsilon(Z) \in \mathbb{R}^n$ is the approximation error.

The adaptation law is designed as

$$\dot{\hat{W}}_i = -\Gamma_i[S_i(Z)z_{2,i} + \sigma_i \hat{W}_i], \tag{25}$$

where Γ_i is the constant gain matrix, and $\sigma_i > 0$, $i = 1, 2, ...n$, are small positive constants.

Remark 2 *The σ modification term in Eq.* (25) *is introduced to improve the robustness of the closed-loop system when the system is subjected to bounded disturbance [39, 40]. Without such a modification term, the estimate $W^{*T} S(Z)$ might drift to a very large value, which will result in a variation of a high-gain control* *scheme [34].*

Remark 3 *In this paper, we deal with a more challenging problem by considering the values of $M(x_1)$, $C(x_1, x_2)$ and $G(x_1)$ as fully unknown. If the individual term is known exactly, the terms can be excluded from the approximations in Eq.* (24) *and rewritten explicitly as a part of the adaptive neural control Eq.* (23).

Theorem 1 *For the system dynamics described by Eq.* (10), *under Assumption 1, and the control Eq.* (23) *with the adaptation laws Eq.* (25)*, given that the full state information is available, for each compact set Ω_0 where ($x_1(0)$, $x_2(0)$, $\hat{W}_1(0)$, $\hat{W}_2(0)$ ... $\hat{W}_n(0)$) $\in \Omega_0$, i.e., the initial conditions are bounded, the trajectories of the closed-loop system are semiglobally uniformly bounded. The closed-loop error signals z_1, z_2 and $\tilde{W}$ will remain within the compact sets Ω_{z_1}, Ω_{z_2} and Ω_W, respectively, defined by*

$$\Omega_{z_1} : = \left\{ z_1 \in R^n \mid \|z_1\| \leq \sqrt{D} \right\}, \tag{26}$$

$$\Omega_{z_2} : = \left\{ z_2 \in R^n \mid \|z_2\| \leq \sqrt{\frac{D}{\lambda_{\min}(M)}} \right\}, \tag{27}$$

$$\Omega_W : = \left\{ \tilde{W} \in R^{l \times n} \mid \|\tilde{W}\| \leq \sqrt{\frac{D}{\lambda_{\min}(\Gamma^{-1})}} \right\}, \tag{28}$$

where $D = 2\,(V(0) + C/\rho)$, ρ and C are two positive constants.

Proof: Consider the following Lyapunov function candidate

$$V = \frac{1}{2} z_1^T z_1 + \frac{1}{2} z_2^T M z_2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i, \tag{29}$$

where $\tilde{W}_i = \hat{W}_i - W_i^*$, and $\tilde{W}_i$, $\hat{W}_i$ and W_i^* are the neural network weight error, estimate and actual value, respectively. Differentiating Eq. (29), we obtain

$$\begin{aligned} \dot{V} \leq\ & -z_1^T K_1 z_1 + z_1^T z_2 + z_2^T [\tau - J^T(x_1) f - C(x_1, x_2)\alpha_1(t) - G(x_1) - M(x_1)\dot{\alpha}_1(t)] \\ & + \sum_{i=1}^{n} \tilde{W}_i^T \Gamma_i^{-1} \dot{\hat{W}}_i. \end{aligned} \tag{30}$$

Applying the approximation Eq. (24), we have

$$\dot{V} \leq -z_1^T K_1 z_1 + z_1^T z_2 + z_2^T \left[-W^{*T} S(Z) - \epsilon(Z) + \tau - J^T(x_1) f \right] + \sum_{i=1}^{n} \tilde{W}_i^T \Gamma_i^{-1} \dot{\hat{W}}_i. \tag{31}$$

Substituting the control Eq. (23) and adaptation law Eq. (25) into Eq. (31) yields

$$\begin{aligned}\dot{V} &\leq -z_1^T K_1 z_1 - z_2^T (K_2 - \frac{1}{2} I_{n\times n}) z_2 - \sum_{i=1}^{n} \frac{\sigma_i}{2} \|\tilde{W}_i\|^2 + \sum_{i=1}^{n} \frac{\sigma_i}{2} \|W_i^*\|^2 + \frac{1}{2}\|\bar{\epsilon}\|^2 \\ &\leq -\rho V + C,\end{aligned} \tag{32}$$

where

$$\rho = \min\Big(2\lambda_{\min}(K_1), \frac{2\lambda_{\min}(K_2 - \frac{1}{2} I_{n\times n})}{\lambda_{\max}(M)}, \min_{i=1,2,...n}\Big(\frac{\sigma_i}{\lambda_{\max}(\Gamma_i^{-1})}\Big)\Big), \tag{33}$$

$$C = \sum_{i=1}^{n} \frac{\sigma_i}{2} \|W_i^*\|^2 + \frac{1}{2}\|\bar{\epsilon}\|^2, \tag{34}$$

where $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and maximum eigenvalues of matrix A, respectively. To ensure $\rho > 0$, the control gains K_1 and K_2 are chosen to satisfy the following conditions:

$$\lambda_{\min}(K_1) > 0, \quad \lambda_{\min}(K_2 - \frac{1}{2} I) > 0. \tag{35}$$

From the above analysis, it is straightforward to show that the signals $z_1,\ z_2$, and $\tilde{W}_i (i = 1, 2, ...n)$ are semiglobally uniformly bounded. From the boundedness of x_{1d} in Assumption 1, we know that x is bounded. Since $\dot{x}_{1d}$ is also bounded, it follows that α_1 is bounded and in turn x_2 is bounded. As $W_i^* (i = 1, 2, ...n)$ are constants, we know that $\hat{W}_i (i = 1, 2, ...n)$ are also bounded. For completeness, the details of the proof, similar to [33], are provided here. Multiplying Eq. (32) by $e^{\rho t}$ yields

$$\frac{d}{dt}(V e^{\rho t}) \leq C e^{\rho t}. \tag{36}$$

Integrating the above inequality, we obtain

$$V \leq \Big(V(0) - \frac{C}{\rho}\Big) e^{-\rho t} + \frac{C}{\rho} \leq V(0) + \frac{C}{\rho}. \tag{37}$$

Then, we have

$$\frac{1}{2}||z_1||^2 \leq V(0) + \frac{C}{\rho}. \tag{38}$$

Hence, z_1 converges to the compact set Ω_{zs}. Bounds for z_2 and $\tilde{W}_i$ can be similarly shown and this concludes the proof. ■

Remark 4 *The stability result proposed is semiglobal in the sense that if the number of neural network nodes* l *is chosen large enough such that the approximation holds in* $\Omega_{z1},\ \Omega_{z2}$ *and* Ω_W *, then the closed-loop stability can be guaranteed for bounded initial states and neural network weights. The exact sizes of the compact sets* $\Omega_{z1},\ \Omega_{z2}$ *and* Ω_W *are not available as they depend on the unknown parameters* W^* *and* ϵ*.*

Remark 5 *It is easily seen that the increase in the control gains,* K_1 *and* K_2*, and the adaptive gains* $\Gamma_i(i = 1, 2, 3, ...n)$*, will result in a better control performance. Therefore, we can conclude that the tracking errors,* z_1 *and* z_2*, will eventually converge to a small neighborhood around zero by appropriately choosing design parameters.*

3.2 Adaptive Neural Network Control with Output Feedback

The proposed control Eq. (23) requires full states, $x_1(t)$ and $x_2(t)$, to be implemented. In the absence of velocity sensors, we introduce a high-gain observer to estimate $x_2(t)$ through the certainty equivalence property and separation principle.

From Lemma 3, $\frac{\pi_{k+1}}{\varepsilon^k}$ converges asymptotically to $x_1^{(k)}$, and the derivative of x_1 to the kth order, i.e. ξ_k converges to zero with a small time constant (due to the high-gain $1/\varepsilon$) provided that x_1 and its k derivatives are bounded. Then, we can state that π_{k+1}/ε^k is proper as an observer for estimating the output signals up to the nth order. The observer for system Eq. (10) is considered with $n = 2$ and the estimate of the unmeasurable state vector z_2 is designed as

$$\hat{z}_2 = \pi_2/\varepsilon - \alpha_1, \tag{39}$$

where the dynamics of π_2 are described as

$$\varepsilon\dot{\pi}_1 = \pi_2, \tag{40}$$

$$\varepsilon\dot{\pi}_2 = -\bar{\lambda}_1\pi_2 - \pi_1 + x_1. \tag{41}$$

From the full state feedback control design, we rewrite the control law Eq. (23) and adaptation law Eq. (25) to obtain the control and adaptation law for output feedback control as

$$\tau = -z_1 - K_2\hat{z}_2 + \hat{W}^T S(\hat{Z}), \tag{42}$$

$$\dot{\hat{W}}_i = -\Gamma_i(S_i(\hat{Z})\hat{z}_{2,i} + \sigma_i\hat{W}_i), \tag{43}$$

where K_2 is the control gain, Γ_i is the constant gain matrix, and $\sigma_i > 0$, $i = 1, 2, ...n$, are small positive constants.

Theorem 2 *For the system dynamics described by Eq.* (10)*, under Assumption 1, and the control Eq.* (42) *with the adaptation law Eq.* (43)*, given that the output information is available, for each compact set* Ω_0 *where (*$x_1(0)$*,* $x_2(0)$*,* $\hat{W}_1(0)$*,* $\hat{W}_2(0)$ *...* $\hat{W}_n(0)$*)* $\in \Omega_0$*, i.e., the initial conditions are bounded, the trajectories of the closed-loop system are semiglobally uniformly bounded. The closed-loop error signals* z_1*,* z_2 *and* $\tilde{W}$ *will remain within the compact sets* Ω_{z_1}*,* Ω_{z_2} *and* Ω_W*, respectively, defined by*

$$\Omega_{z_1} : \; = \; \left\{ z_1 \in R^n \mid \|z_1\| \leq \sqrt{D} \right\}, \tag{44}$$

$$\Omega_{z_2} : \; = \; \left\{ z_2 \in R^n \mid \|z_2\| \leq \sqrt{\frac{D}{\lambda_{\min}(M)}} \right\}, \tag{45}$$

$$\Omega_W : \; = \; \left\{ \tilde{W} \in R^{l \times n} \mid \|\tilde{W}\| \leq \sqrt{\frac{D}{\lambda_{\min}(\Gamma^{-1})}} \right\}, \tag{46}$$

where $D = 2\left(V_2(0) + C/\rho\right)$*, and* ρ *and* C *are two positive constants.*

Proof: Consider the following Lyapunov function candidate

$$V_2 = \frac{1}{2} z_1^T z_1 + \frac{1}{2} z_2^T M z_2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i, \tag{47}$$

where $\tilde{W}_i = \hat{W}_i - W_i^*$, and $\tilde{W}_i$, $\hat{W}_i$ and W_i^* are the neural network weight error, estimate and actual value, respectively. The time derivative of the Lyapunov function candidate V_2 along the closed loop trajectory with Eqs. (42) and (43) yields

$$\begin{aligned} \dot{V}_2 \;\; \leq \;\; & -z_1^T K_1 z_1 - z_2^T (K_2 - \frac{1}{2} I) z_2 - z_2^T K_2 \tilde{z}_2 \\ & + \sum_{i=1}^{n} z_{2,i} [\hat{W}_i^T S_i(\hat{Z}) - W_i^{*T} S_i(Z)] \\ & - \sum_{i=1}^{n} [\tilde{W}_i^T S_i(\hat{Z}) \hat{z}_{2,i} + \sigma_i \tilde{W}_i^T \hat{W}_i] + \frac{1}{2} \|\bar{\epsilon}\|^2. \end{aligned} \tag{48}$$

According to Lemma 3, we have

$$\xi_2 = \frac{\pi_2}{\varepsilon} - \dot{x}_1 = -\varepsilon \psi^{(2)}, \tag{49}$$

$$\tilde{z}_2 = \hat{z}_2 - z_2 = \frac{\pi_2}{\varepsilon} - \alpha_1 - \dot{x}_1 + \alpha_1 = \xi_2. \tag{50}$$

where ε is any small constant, $\psi = \pi_2 + \bar{\lambda}_1 \pi_1$, and there exist positive constants t^* and h_2 such that $\forall t > t^*$, we have $\|\xi_2\| \leq \varepsilon h_2$. Thus, we can use $\frac{\pi_2}{\varepsilon}$ to estimate $\dot{x}_1$, then x_2 and z_2 can be estimated as follows

$$\hat{x}_2 = \frac{\pi_2}{\varepsilon}, \tag{51}$$

$$\hat{z}_2 = \frac{\pi_2}{\varepsilon} - \alpha_1. \tag{52}$$

From Lemma 2, using the properties, we have

$$-\sigma_i \tilde{W}_i^T \hat{W}_i \leq \frac{\sigma_i}{2}(||W_i^*||^2 - ||\tilde{W}_i||^2), \tag{53}$$

$$||S_i(\hat{Z})||^2 \leq l_i, \tag{54}$$

$$\begin{aligned} \hat{W}_i^T S_i(\hat{Z}) &= W_i^{*T}(S_i(Z) + \epsilon S_{ti}) + \tilde{W}_i^T S_i(\hat{Z}) \\ &= W_i^{*T} S_i(Z) + W_i^{*T} \epsilon S_{ti} + \tilde{W}_i^T S_i(\hat{Z}), \end{aligned} \tag{55}$$

where l_i and ϵ are two positive constants, and S_t is a bounded vector function.

Substituting Eqs. (53) and (55) to Ineq. (48) leads to

$$\begin{aligned} \dot{V}_2 \leq & - z_1^T K_1 z_1 - z_2^T (K_2 - \frac{1}{2} I) z_2 - z_2^T K_2 \tilde{z}_2 - \sum_{i=1}^n \tilde{W}_i^T S_i(\hat{Z}) \tilde{z}_{2,i} \\ & - \sum_{i=1}^n \frac{\sigma_i}{2} \|\tilde{W}_i\|^2 + \frac{1}{2}\|\bar{\epsilon}\|^2 + \sum_{i=1}^n z_{2,i} W_i^{*T} \epsilon S_{ti} + \sum_{i=1}^n \frac{\sigma_i}{2} \|W_i^*\|^2. \end{aligned} \tag{56}$$

Substituting $\sum_{i=1}^n z_{2,i} W_i^{*T} \epsilon S_{ti} \leq \frac{1}{2} z_2^T z_2 + \sum_{i=1}^n \frac{\|W_i^*\|^2 \epsilon^2 \|S_{ti}\|^2}{2}$ and $-\sum_{i=1}^n \tilde{W}_i^T S_i(\hat{Z}) \tilde{z}_{2,i} = -\sum_{i=1}^n \frac{\sqrt{\sigma_i} \tilde{W}_i^T}{\sqrt{2}} \frac{\sqrt{2} S_i(\hat{Z}) \tilde{z}_{2,i}}{\sqrt{\sigma_i}} \leq \sum_{i=1}^n \frac{\sigma_i \|\tilde{W}_i\|^2}{4} + \sum_{i=1}^n \frac{2||S_i(\hat{Z})||^2}{\sigma_i} \frac{1}{2} \tilde{z}_2^T \tilde{z}_2$ to the above equation, we can obtain

$$\begin{aligned} \dot{V}_2 \leq & - z_1^T K_1 z_1 - z_2^T (K_2 - I) z_2 - z_2^T K_2 \tilde{z}_2 + \sum_{i=1}^n \frac{\sigma_i \|\tilde{W}_i\|^2}{4} + \sum_{i=1}^n \frac{2||S_i(\hat{Z})||^2}{\sigma_i} \frac{1}{2} \tilde{z}_2^T \tilde{z}_2 \\ & - \sum_{i=1}^n \frac{\sigma_i}{2} \|\tilde{W}_i\|^2 + \frac{1}{2}\|\bar{\epsilon}\|^2 + \sum_{i=1}^n (\frac{\epsilon^2 \|S_{ti}\|^2}{2} + \frac{\sigma_i}{2}) \|W_i^*\|^2. \end{aligned} \tag{57}$$

Using Eq. (50), Ineq. (54) and $z_2^T K_2 \tilde{z}_2 \leq \frac{1}{2} z_2^T z_2 + \frac{1}{2}(K_2 \tilde{z}_2)^T (K_2 \tilde{z}_2)$, we further have

$$\begin{aligned} \dot{V}_2 \leq & - z_1^T K_1 z_1 - z_2^T (K_2 - \frac{3}{2} I) z_2 + \frac{1}{2}\|\bar{\epsilon}\|^2 - \sum_{i=1}^n \frac{\sigma_i}{4} \|\tilde{W}_i\|^2 \\ & + \lambda_{\max}(K_2^T K_2 + \mathrm{diag}[2 l_i / \sigma_i]) \frac{1}{2} \xi_2^T \xi_2 + \frac{1}{2} \sum_{i=1}^n (\epsilon^2 \|S_{ti}\|^2 + \sigma_i) \|W_i^*\|^2. \end{aligned} \tag{58}$$

From Lemma 3, applying $\frac{1}{2}\xi_2^T\xi_2 \le \frac{1}{2}\varepsilon^2 h_2^2$, we have

$$\begin{aligned}
\dot{V}_2 &\le -z_1^T K_1 z_1 - z_2^T\Big(K_2 - \frac{3}{2}I\Big)z_2 + \frac{1}{2}||\bar{\epsilon}||^2 - \sum_{i=1}^{n}\frac{\sigma_i}{4}||\tilde{W}_i||^2 \\
&\quad +\lambda_{\max}(K_2^T K_2 + \mathrm{diag}[2l_i/\sigma_i])\frac{1}{2}\varepsilon^2 h_2^2 + \frac{1}{2}\sum_{i=1}^{n}\Big(\epsilon^2||S_{ti}||^2 + \sigma_i\Big)\|W_i^*\|^2 \\
&\le -\rho V_2 + C, \qquad (59)
\end{aligned}$$

where ρ and C are two constants defined as

$$\rho = \min\Big(2\lambda_{\min}(K_1), \frac{2\lambda_{\min}(K_2 - \frac{3}{2}I)}{\lambda_{\max}(M)}, \min_{i=1,2,\dots,n}\Big(\frac{\sigma_i}{2\lambda_{\max}(\Gamma_i^{-1})}\Big)\Big), \qquad (60)$$

$$C = \frac{1}{2}\sum_{i=1}^{n}\Big(\epsilon^2||S_{ti}||^2 + \sigma_i\Big)\|W_i^*\|^2 + \lambda_{\max}(K_2^T K_2 + \mathrm{diag}[2l_i/\sigma_i])\frac{1}{2}\varepsilon^2 h_2^2 + \frac{1}{2}||\bar{\epsilon}||^2. \qquad (61)$$

To ensure that $\rho > 0$, the control gains K_1 and K_2 are chosen to satisfy the following conditions:

$$\lambda_{\min}(K_1) > 0, \quad \lambda_{\min}(K_2 - \frac{3}{2}I) > 0. \qquad (62)$$

■

Remark 6 *In this section, we have assumed that the position measurements are perfect and designed a rigorous theoretical treatment of the output feedback problem using a high-gain observer corresponding to a non-model-based approach. If the output measurements are contaminated with zero mean Gaussian white noise within tolerance, careful implementation is necessary by designing ϵ to be sufficiently small. A saturation function can be used to overcome the peaking phenomenon of the high-gain observer following the procedure detailed in [41].*

Remark 7 *The tracking error has been shown to converge and remain within a small neighborhood of the origin. If the residual error is desired to be lower, it can be reduced such that C/ρ in both Theorems 1 and 2 decreases. The reduction is achieved by increasing K_1, K_2, the approximation accuracy of the neural networks, and the high-gain $1/\varepsilon$ of the state observer.*

4 Simulation

Considering a rehabilitation robot with two revolute joints in the vertical plane as shown in Figs. 1 and 2, simulations are carried out to verify the effectiveness of the proposed control. Let m_i and l_i be the mass and length of link i, l_{ci} be the distance from joint $i-1$ to the center of mass of link i, as indicated in the figure, and I_i be the moment of inertia of link i about an axis coming out of the page passing through the center of mass of link i, $i = 1, 2$.

Figure 1: A 2-DOF rehabilitation robotic system.

We define

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}. \tag{63}$$

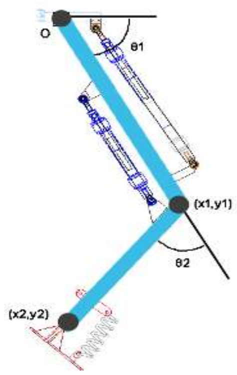

Figure 2: The schematic of the 2-DOF knee rehabilitation robotic system.

Then, we have the kinetic energy given as

$$\begin{aligned}\mathcal{K}(q,\dot{q}) &= \frac{1}{2}m_1 l_{c1}^2\dot{q}_1^2 + \frac{1}{2}I_1\dot{q}_1^2 + \frac{1}{2}m_2 l_1^2\dot{q}_1^2 + m_2 l_1 l_{c2}\dot{q}_1(\dot{q}_1+\dot{q}_2)\cos q_2 + \frac{1}{2}m_2 l_{c2}^2(\dot{q}_1+\dot{q}_2)^2 \\ &\quad + \frac{1}{2}I_2(\dot{q}_1+\dot{q}_2)^2. \end{aligned} \tag{64}$$

The potential energy is written as

$$\mathcal{P}(q) = m_1 g l_{c2} \sin q_1 + m_2 g [l_1 \sin q_1 + l_{c2} \sin(q_1 + q_2)]. \tag{65}$$

Using the Lagrange's equation $\frac{d}{dt}\frac{\partial(\mathcal{K}-\mathcal{P})}{\partial \dot{q}} - \frac{\partial(\mathcal{K}-\mathcal{P})}{\partial q} = 0$, the dynamics of the robot can be expressed as Eq. (10), where

$$G(q) = \begin{bmatrix} (m_1 l_{c2} + m_2 l_1) g \cos q_1 + m_2 l_{c2} g \cos(q_1 + q_2) \\ m_2 l_{c2} g \cos(q_1 + q_2) \end{bmatrix}, \tag{66}$$

$$M(q) = \begin{bmatrix} m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} \cos q_2) + I_1 + I_2 & m_2(l_{c2}^2 + l_1 l_{c2} \cos q_2) + I_2 \\ m_2(l_{c2}^2 + l_1 l_{c2} \cos q_2) + I_2 & m_2 l_{c2}^2 + I_2 \end{bmatrix}, \tag{67}$$

$$C(q, \dot{q}) = \begin{bmatrix} -m_2 l_1 l_{c2} \dot{q}_2 \sin q_2 & -m_2 l_1 l_{c2} (\dot{q}_1 + \dot{q}_2) \sin q_2 \\ m_2 l_1 l_{c2} \dot{q}_1 \sin q_2 & 0 \end{bmatrix}. \tag{68}$$

The kinematics of the robot and the Jacobian matrix are written as

$$\phi(q) = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos(q_1 + q_2) \\ l_1 \sin q_1 + l_2 \sin(q_1 + q_2) \end{bmatrix}, \tag{69}$$

$$J(q) = \begin{bmatrix} -(l_1 \sin q_1 + l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1 + q_2) \\ l_1 \cos q_1 + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \end{bmatrix}, \tag{70}$$

$$J^{-1}(q) = \frac{1}{l_1 l_2 \sin q_2} \begin{bmatrix} l_2 \cos(q_1 + q_2) & l_2 \sin(q_1 + q_2) \\ -[l_1 \cos q_1 + l_2 \cos(q_1 + q_2)] & -[l_1 \sin q_1 + l_2 \cos(q_1 + q_2)] \end{bmatrix}. \tag{71}$$

Parameters of the robot are listed in the table below.

Table 1: Parameters of the robot

Parameter	Description	Value
m_1	Mass of link 1	2.00 kg
m_2	Mass of link 2	0.85 kg
l_1	Length of link 1	0.35 m
l_2	Length of link 2	0.31 m
I_1	Moment of inertia of link 1	$\frac{1}{4} m_1 l_1^2$ kgm^2
I_2	Moment of inertia of link 2	$\frac{1}{4} m_2 l_2^2$ kgm^2

The initial positions of the robot are given as

$$q_1(0) = q_2(0) = \dot{q}_1(0) = \dot{q}_2(0) = 0. \tag{72}$$

The desired trajectory tracking a circular path is given as $q_d = [0.14\sin(0.5t), 0.14\cos(0.5t)]^T$, where $t \in [0, t_f]$ and $t_f = 40$s.

We consider the robot under external disturbance composed by the Gaussian white noise. $f(t)$ is given as $[\sin(t) + 1 + d(t), 2\cos(t) + 0.5 + d(t)]^T$, where $d(t) \sim \mathcal{N}(0, 1)$ is a white Gaussian noise of power 0dBW. As $-0.783 \leq d(t) \leq 0.818$, we have $f(t) \leq [2.818 \;\; 3.3180]^T$, and we choose $\bar{f} = [3 \;\; 3.4]^T$. Three different cases are evaluated for the simulation studies. Firstly, we examine the model-based control designed in Eq. (21). Secondly, the proposed adaptive neural network control with the full state feedback Eq. (23) is considered. Thirdly, the adaptive neural network control with the output feedback Eq. (42) is evaluated.

For the model-based control, the control parameters are chosen as $K_1 = \mathrm{diag}[10, 10]$, $K_2 = \mathrm{diag}[8, 8]$. For the approximation-based control, a number of 256 nodes are used for each $S_i(Z)$ with centers chosen in the area of $[-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1]$. Variance of centers is set as $\eta^2 = 1$. The initial weights $\hat{W}_{1,i} = 0$, $\hat{W}_{2,i} = 0$, $(i = 1, 2, ...256)$. For adaptive neural network control with the state feedback, the control parameters are chosen as $K_1 = \mathrm{diag}[50, 50]$, $K_2 = \mathrm{diag}[30, 30]$, $\sigma_1 = 0.02$, $\sigma_2 = 0.02$, $\Gamma_1 = 10I_{256\times256}$, $\Gamma_2 = 10I_{256\times256}$, which satisfy the conditions in Eq. (35). For adaptive neural network control with the output feedback, the control parameters are set as $K_1 = [200 \;\; 0; 0 \;\; 80]$, $K_2 = [60 \;\; 0; 0 \;\; 20]$, $\sigma_1 = 0.02$, $\sigma_2 = 0.02$, $\Gamma_1 = 10I_{256\times256}$, $\Gamma_2 = 10I_{256\times256}$, $\eta^2 = 1$. The initial weights $\hat{W}_{1,i} = 0$, $\hat{W}_{2,i} = 0$, $(i = 1, 2, ...256)$. The high-gain observer in Eq. (42) is used to obtain the output feedback controller, with $n = 2$, $\bar{\lambda}_1 = [4, 2]^T$, and $\epsilon = 0.001$. The initial conditions of the observer are set as $\pi_1 = \pi_2 = \dot{\pi}_1 = \dot{\pi}_2 = [0, 0]^T$.

The tracking performance of the closed-loop system for the robot with two revolute joints are given in Figs. 3 and 4. From the two figures, we can state that all the three kinds of control (21), (23) and (42) can successfully track the desired trajectory, where the system error is converging to a small value close to zero. However, to achieve the control objective, the model-based control requires the fully-known system dynamics, which is difficult to obtain in practice. The simulation results also show that neural networks are able to approximate the unknown system dynamics and ensure the control performance. The tracking error reduces corresponding to an increase in the adaptation gains, Γ_1 and Γ_2. Simulation results show the boundedness of the adaptation gains where larger Γ_1 and Γ_2 will increase the convergence rate.

The angular velocities of the joints are given in Fig. 5. The corresponding control inputs are given in Fig.

6. The control inputs are affected by the Gaussian noise, where the oscillation appears in the control inputs. Especially, the high-gain observer does not bring the increase of the control inputs, which is an effective design for the output feedback control. The control inputs are varying from -5Nm to 14Nm, which are implementable by using motors in practice. Even though the robotic system is subjected to the Gaussian noise, the control performance is not affected, and the proposed control still can work well. Fig. 7(a) demonstrates the boundedness of the adaptation weights, $\hat{W}_1(t)$ and $\hat{W}_2(t)$. The proposed adaptive neural network control produces a low norm of the error as shown in Fig. 7(b). Both the norms of the errors for the full state feedback control and the output feedback control converge to a small constant.

Meanwhile, we consider the changes in model parameters to illustrate the compensation for system uncertainties in the simulation part. In the range of $0 \sim 20$ seconds, we set the mass of the link 1 and the link 2 in accordance with Table 1. In the range of $20 \sim 40$ seconds, m_1 and m_2 are changed to 3.00 kg and 1.50 kg respectively with all other parameters fixed. By using the same control gains as in the previous case, the tracking performance of the closed-loop system for the robot with two revolute joints are given in Figs. 8 and 9. From the two figures, we can state that although the error has an abrupt charge at the time of 20 second, all the three kinds of control (21), (23) and (42) can successfully track the desired trajectory too, and the system error is converging to a small value close to zero. This is to say that the proposed control is able to compensate for the system uncertainties. The angular velocities of the joints are given in Fig. 10. The corresponding control inputs are given in Fig. 11. Fig. 12(a) gives the boundedness of the adaptation weights, $\hat{W}_1(t)$ and $\hat{W}_2(t)$. The proposed adaptive neural network control produces a low norm of the error as shown in Fig. 12(b). Both the norms of the two errors also converge to a small constant.

Furthermore, to better illustrate the tracking performance for the proposed control, we use a step reference in the desired trajectory $q_d = [0.14, 0.14]^T$ $(t > 0)$. The tracking performance for the robot with the parameters in Table 1 is shown in Figs. 13 and 14. We can state that the proposed control can also track the step reference successfully. The corresponding control inputs are given in Fig. 15.

5 Conclusion

In this paper, adaptive neural network control has been developed for a rehabilitation robot with unknown dynamics. Two cases are investigated for the robot: (i) full-state feedback control design; and (ii) output feedback control design. For the first case, adaptive neural network control has been introduced for approximating the unknown model of the robot. For the second case where x_2 cannot be directly measured, the high-gain observer has been designed to estimate x_2 and ensure the control performance. The adaptive neural networks aim to compensate for the uncertainties of the dynamic model of the robot. All the signals of the closed-loop system have been proved to be uniformly ultimately bounded by tuning the weights of the neural networks and the control gains. The simulation results have illustrated that the proposed control is able to track the desired trajectory with a good performance.

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References

[1] C. Wolfe, "The impact of stroke," *British Medical Bulletin*, vol. 56, no. 2, pp. 275–286, 2000.

[2] N. Hogan, H. Krebs, B. Rohrer, J. Palazzolo, L. Dipietro, S. Fasoli, J. Stein, R. Hughes, W. Frontera, D. Lynch, *et al.*, "Motions or muscles? some behavioral factors underlying robotic assistance of motor recovery," *Journal of rehabilitation research and development*, vol. 43, no. 5, p. 605, 2006.

[3] R. Riener, L. Lünenburger, and G. Colombo, "Human-centered robotics applied to gait training and assessment," *Journal of rehabilitation research and development*, vol. 43, no. 5, pp. 679–694, 2006.

[4] C. Koh, T. Hoffmann, S. Bennett, and K. McKenna, "Management of patients with cognitive impairment after stroke: A survey of australian occupational therapists," *Australian occupational therapy journal*, vol. 56, no. 5, pp. 324–331, 2009.

[5] W. He, S. S. Ge, Y. Li, E. Chew, and Y. S. Ng, "Impedance control of a rehabilitation robot for interactive training," in *Proceedings of the 2012 International Conference on Social Robotics, Chengdu, China*, pp. 526–535, 2012.

[6] C. Yang, G. Ganesh, S. Haddadin, S. Parusel, A. Albu-Schaeffer, and E. Burdet, "Human-like adaptation of force and impedance in stable and unstable interactions," *IEEE Transactions on Robotics*, vol. 27, no. 5, pp. 918–930, 2011.

[7] Z. Li, S. S. Ge, and A. Ming, "Adaptive robust motion/force control of holonomic-constrained nonholonomic mobile manipulators," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 37, no. 3, pp. 607–616, 2007.

[8] Z. Li, J. Li, and Y. Kang, "Adaptive robust coordinated control of multiple mobile manipulators interacting with rigid environments," *Automatica*, vol. 46, no. 12, pp. 2028–2034, 2010.

[9] Y. Li, S. S. Ge, and C. Yang, "Learning impedance control for physical robot–environment interaction," *International Journal of Control*, vol. 85, no. 2, pp. 182–193, 2012.

[10] N. Hogan, "Impedance control: An approach to manipulation: Part iii applications," *Journal of dynamic systems, measurement, and control*, vol. 107, no. 2, pp. 1–24, 1985.

[11] L. Huang, S. S. Ge, and T. H. Lee, "Position/force control of uncertain constrained flexible joint robots," *Mechatronics*, vol. 16, no. 2, pp. 111–120, 2006.

[12] Z. Li, S. S. Ge, M. Adams, and W. Wijesoma, "Robust adaptive control of uncertain force/motion constrained nonholonomic mobile manipulators," *Automatica*, vol. 44, no. 3, pp. 776–784, 2008.

[13] K. S. Narendra and K. Parthasarathy, "Identification and control of dynamical systems using neural networks," *IEEE Transactions on Neural Networks*, vol. 1, no. 1, pp. 4–27, 1990.

[14] A. U. Levin and K. S. Narendra, "Control of nonlinear dynamical systems using neural networks. ii. observability, identification, and control," *IEEE Transactions on Neural Networks*, vol. 7, no. 1, pp. 30–42, 1996.

[15] S. S. Ge, T. H. Lee, and C. J. Harris, *Adaptive Neural Network Control of Robotic Manipulators*. London, UK: World Scientific, 1998.

[16] F. L. Lewis, S. Jagannathan, and A. Yeildirek, *Neural network control of robot manipulators and nonlinear systems*. London: Taylor & Francis, 1999.

[17] C. Wang and D. J. Hill, *Deterministic learning theory for identification, recognition, and control*. CRC Press, 2009.

[18] Y. Li, C. Yang, S. S. Ge, and T. H. Lee, "Adaptive output feedback nn control of a class of discrete-time MIMO nonlinear systems with unknown control directions," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 41, no. 2, pp. 507–517, 2011.

[19] B. Ren, S. S. Ge, C.-Y. Su, and T. H. Lee, "Adaptive neural control for a class of uncertain nonlinear systems in pure-feedback form with hysteresis input," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 39, no. 2, pp. 431–443, 2009.

[20] M. Chen, S. S. Ge, and B. Ren, “Adaptive tracking control of uncertain mimo nonlinear systems with input constraints,” *Automatica*, vol. 47, no. 3, pp. 452–465, 2011.

[21] Y.-J. Liu, C. P. Chen, G.-X. Wen, and S. Tong, “Adaptive neural output feedback tracking control for a class of uncertain discrete-time nonlinear systems,” *IEEE Transactions on Neural Networks*, vol. 22, no. 7, pp. 1162–1167, 2011.

[22] Y.-J. Liu, S.-C. Tong, D. Wang, T.-S. Li, and C. Chen, “Adaptive neural output feedback controller design with reduced-order observer for a class of uncertain nonlinear siso systems,” *IEEE Transactions on Neural Networks*, vol. 22, no. 8, pp. 1328–1334, 2011.

[23] S.-L. Dai, C. Wang, and M. Wang, “Dynamic learning from adaptive neural network control of a class of nonaffine nonlinear systems,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 25, no. 1, pp. 111–123, 2014.

[24] S.-L. Dai, C. Wang, and F. Luo, “Identification and learning control of ocean surface ship using neural networks,” *IEEE Transactions on Industrial Informatics*, vol. 8, no. 34, pp. 801–810, 2012.

[25] Z. Li and C.-Y. Su, “Neural-adaptive control of single-master-multiple-slaves teleoperation for coordinated multiple mobile manipulators with time-varying communication delays and input uncertainties,” *IEEE Transactions on Neural Networks and Learning System*, vol. 24, no. 9, pp. 1400–1413, 2013.

[26] F. Sun, Z. Sun, Y. Zhu, and W. Lu, “Stable neuro-adaptive control for robots with the upper bound estimation on the neural approximation errors,” *Journal of Intelligent & Robotic Systems*, vol. 26, no. 1, pp. 91–100, 1999.

[27] F. Sun and Z. Sun, “Stable sampled-data adaptive control of robot arms using neural networks,” *Journal of Intelligent & Robotic Systems*, vol. 20, no. 2-4, pp. 131–155, 1997.

[28] W. He, S. S. Ge, B. V. E. How, and Y. S. Choo, *Dynamics and Control of Mechanical Systems in Offshore Engineering*. London, UK: Springer, 2014.

[29] R. Selmic and F. Lewis, “Deadzone compensation in motion control systems using neural networks,” *IEEE Transactions on Automatic Control*, vol. 45, no. 4, pp. 602–613, 2000.

[30] C. Yang, Z. Li, and J. Li, “Trajectory planning and optimized adaptive control for a class of wheeled inverted pendulum vehicle models,” *IEEE Transactions on Cybernetics*, vol. 43, no. 1, pp. 24–36, 2013.

[31] R. Cui, B. Ren, and S. S. Ge, “Synchronised tracking control of multi-agent system with high order dynamics,” *IET Control Theory & Applications*, vol. 6, no. 5, pp. 603–614, 2012.

[32] R. M. Sanner and J. E. Slotine, “Gaussian networks for direct adaptive control,” *IEEE Transactions on Neural Networks*, vol. 3, no. 6, pp. 837–863, 1992.

[33] S. S. Ge and C. Wang, “Adaptive neural network control of uncertain MIMO non-linear systems,” *IEEE Transactions on Neural Networks*, vol. 15, no. 3, pp. 674–692, 2004.

[34] S. S. Ge, C. C. Hang, T. H. Lee, and T. Zhang, *Stable Adaptive Neural Network Control*. Boston, USA: Kluwer Academic, 2001.

[35] S. Behtash, “Robust output tracking for nonlinear system,” *International Journal of Control*, vol. 51, pp. 1381–1407, 1990.

[36] R. Horn and C. Johnson, *Matrix analysis*. Cambridge, UK: Cambridge University Press, 1990.

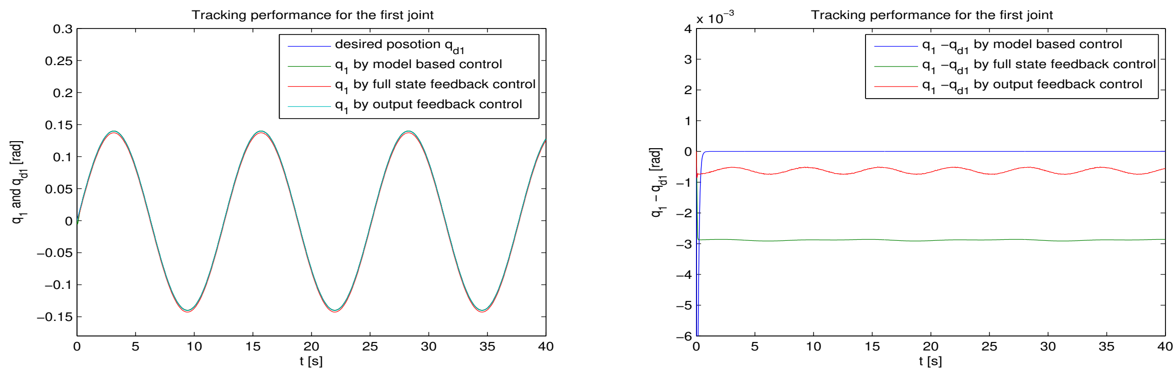
[37] W. He and S. S. Ge, “Robust adaptive boundary control of a vibrating string under unknown time-varying disturbance,” *IEEE Transactions on Control Systems Technology*, vol. 20, no. 1, pp. 48–58, 2012.

[38] W. He, S. S. Ge, B. V. E. How, Y. S. Choo, and K.-S. Hong, “Robust adaptive boundary control of a flexible marine riser with vessel dynamics,” *Automatica*, vol. 47, no. 4, pp. 722–732, 2011.

[39] P. Ioannou and J. Sun, *Robust Adaptive Control*. Eaglewood Cliffs, New Jersey: Prentice-Hall, 1996.

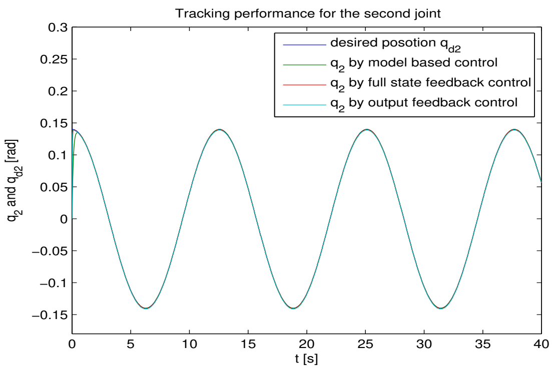
[40] W. He, S. S. Ge, and S. Zhang, “Adaptive boundary control of a flexible marine installation system,” *Automatica*, vol. 47, no. 12, pp. 2728–2734, 2011.

[41] H. K. Khalil, *Nonlinear Systems*. New Jersey, USA: Prentice Hall, 2002.

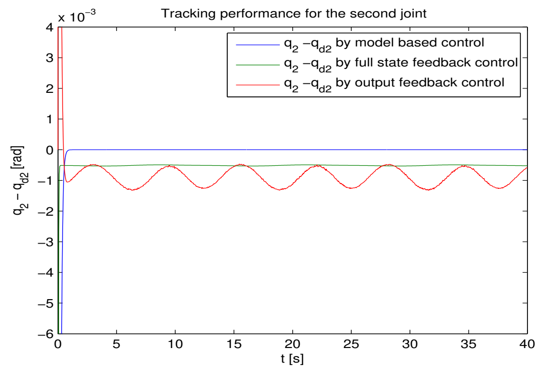


(a) q_1 (b) $q_1 - q_{d1}$

Figure 3: Tracking performance and the error of the closed-loop system for the first joint.

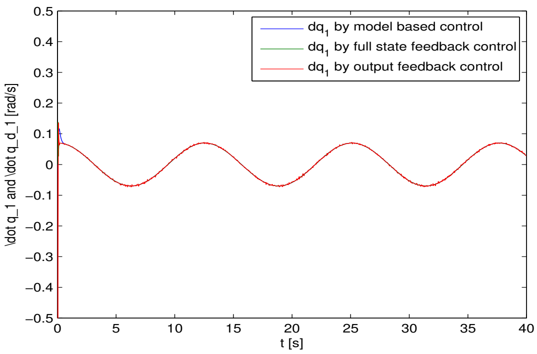


(a) q_2

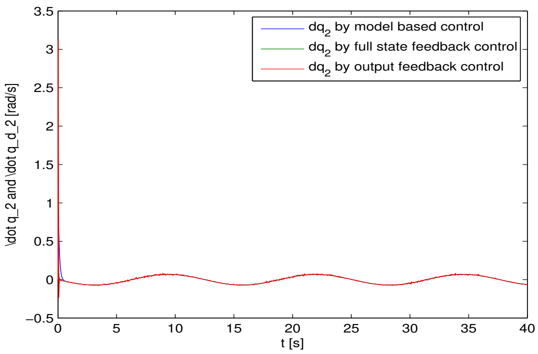


(b) $q_2 - q_{d2}$

Figure 4: Tracking performance and the error of the closed-loop system for the second joint.

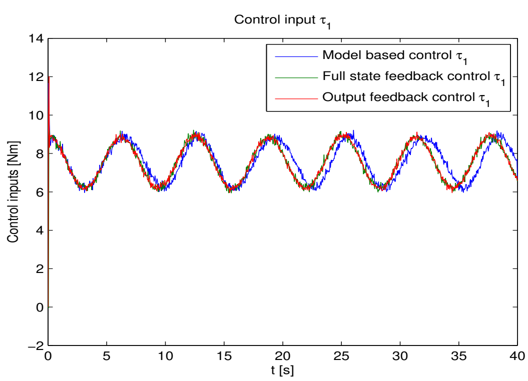


(a) $\dot{q}_1$

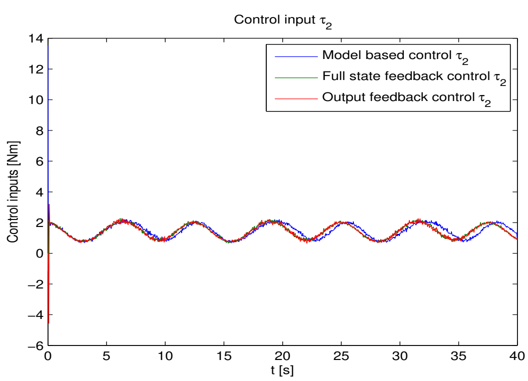


(b) $\dot{q}_2$

Figure 5: Performance of the $\dot{q}_1$ and $\dot{q}_2$.

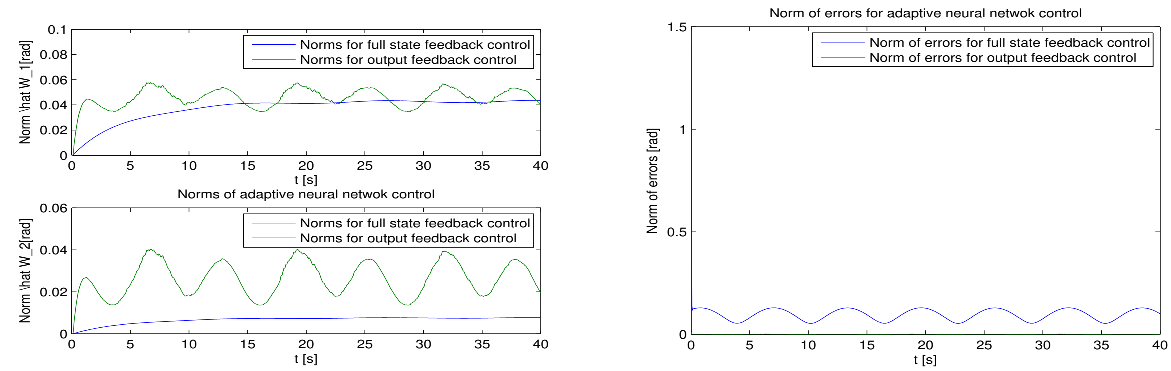


(a) Control input τ_1



(b) Control input τ_2

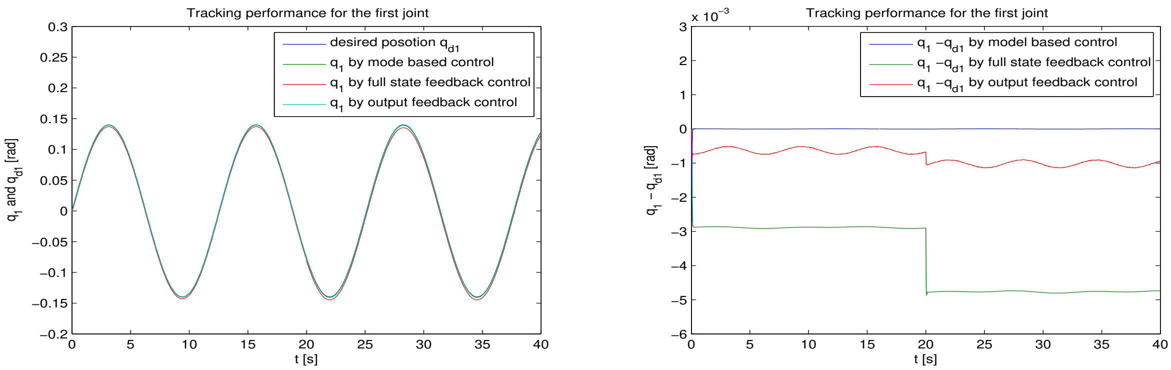
Figure 6: Control inputs τ_1 and τ_2.



(a) Norms of $||\hat{W}_1||$ and $||\hat{W}_2||$

(b) Norms of the errors $||z_1||$

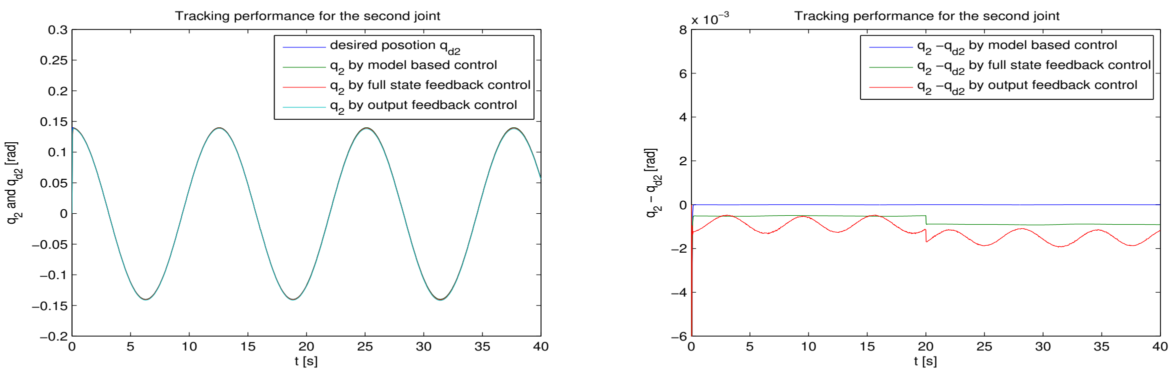
Figure 7: Norms of the adaptation weights and errors.



(a) q_1

(b) $q_1 - q_{d1}$

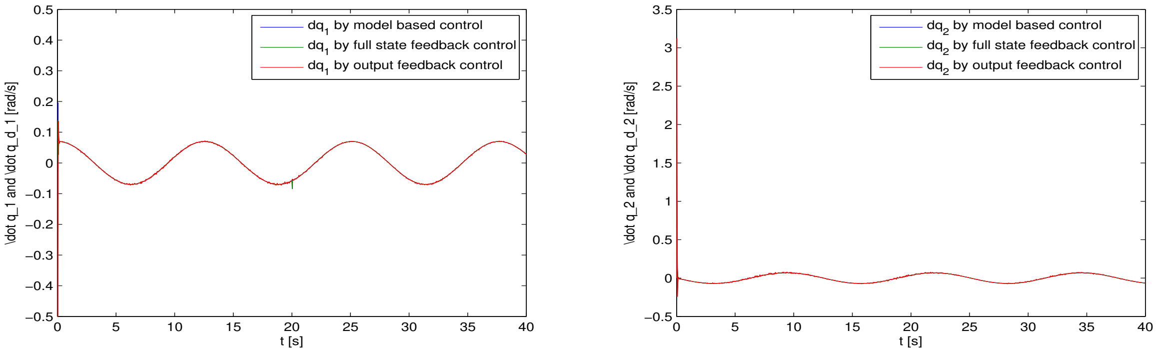
Figure 8: Tracking performance and the error of the closed-loop system for the first joint.



(a) q_2

(b) $q_2 - q_{d2}$

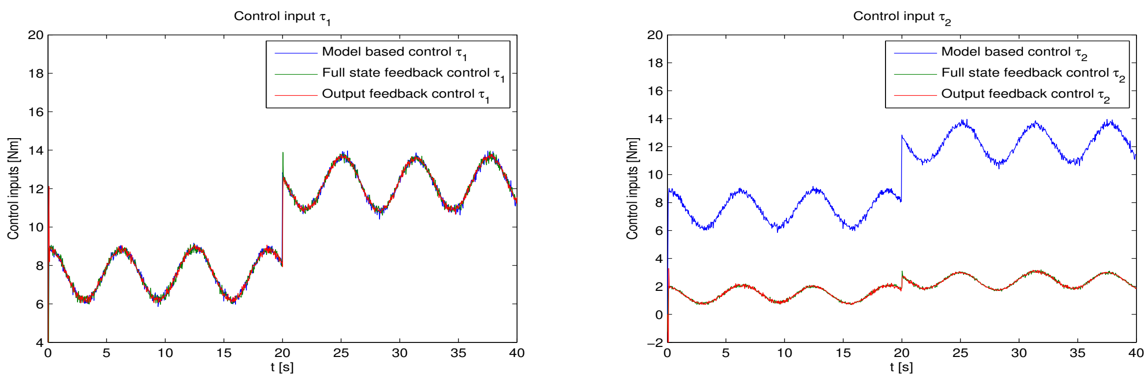
Figure 9: Tracking performance and the error of the closed-loop system for the second joint.



(a) $\dot{q}_1$

(b) $\dot{q}_2$

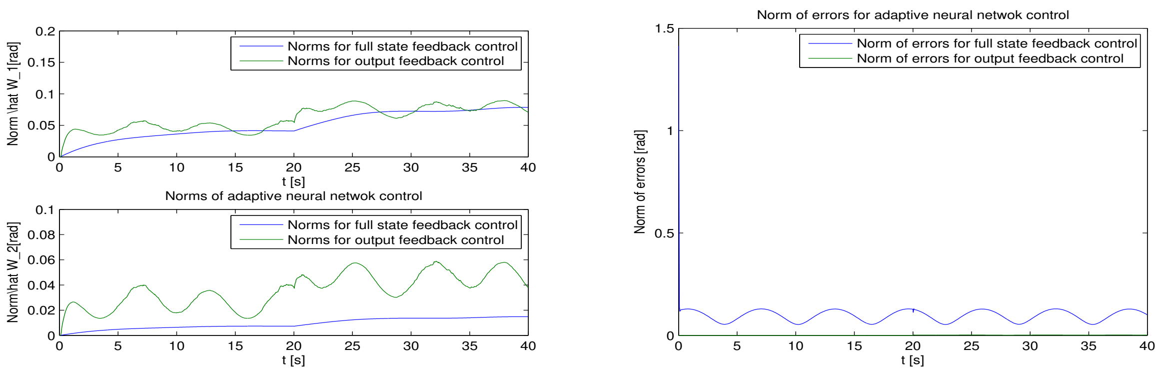
Figure 10: Performance of the $\dot{q}_1$ and $\dot{q}_2$.



(a) Control input τ_1

(b) Control input τ_2

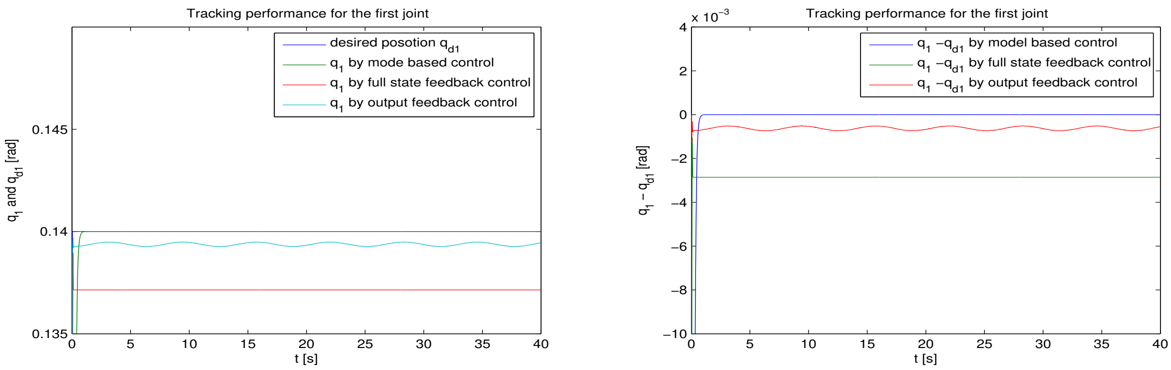
Figure 11: Control inputs τ_1 and τ_2.



(a) Norms of $||\hat{W}_1||$ and $||\hat{W}_2||$

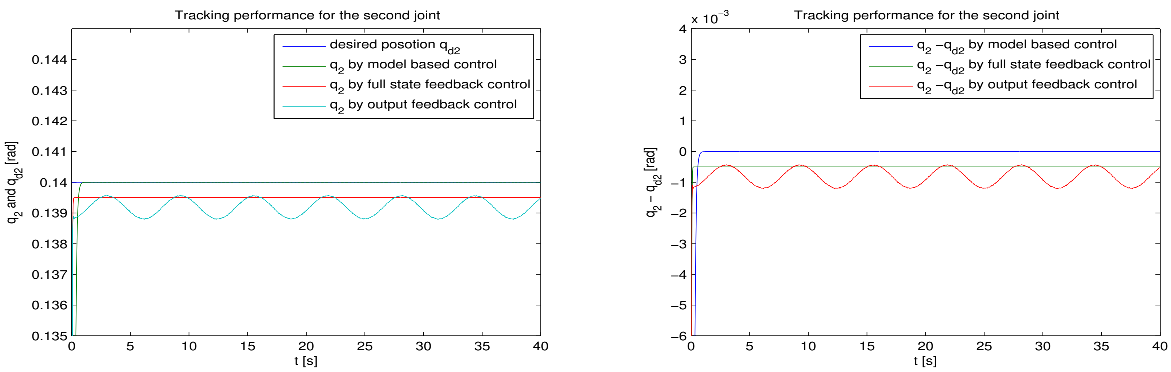
(b) Norms of the errors $||z_1||$

Figure 12: Norms of the adaptation weights and errors.



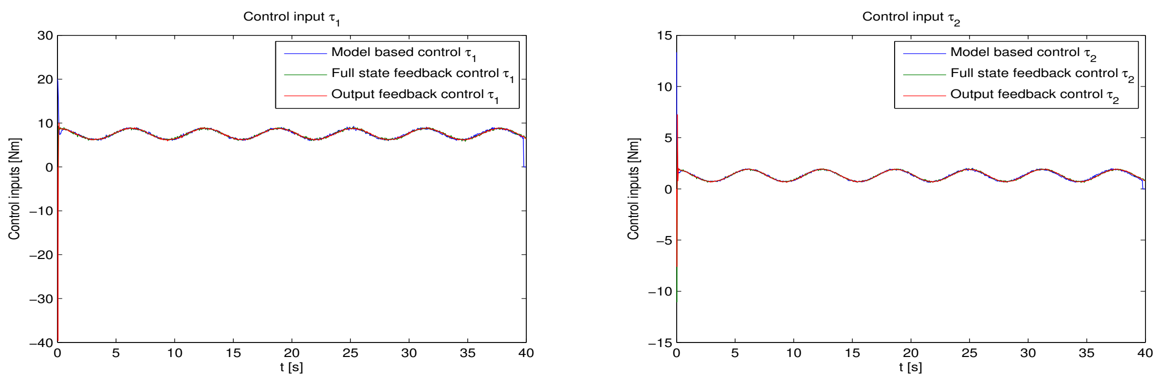
(a) q_1 (b) $q_1 - q_{d1}$

Figure 13: Tracking performance and the error of the closed-loop system for the first joint.



(a) q_2 (b) $q_2 - q_{d2}$

Figure 14: Tracking performance and the error of the closed-loop system for the second joint.



(a) control input τ_1 (b) control input τ_2

Figure 15: Control inputs τ_1 and τ_2.