

# Neural-Symbolic Learning and Reasoning: Contributions and Challenges

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## Abstract

The goal of neural-symbolic computation is to integrate robust connectionist learning and sound symbolic reasoning. With the recent advances in connectionist learning, in particular deep neural networks, forms of representation learning have emerged. However, such representations have not become useful for reasoning. Results from neural-symbolic computation have shown to offer powerful alternatives for knowledge representation, learning and reasoning in neural computation. This paper recalls the main contributions and discusses key challenges for neural-symbolic integration which have been identified at a recent Dagstuhl seminar.

## 1. Introduction

In order to respond to one of the main challenges of Artificial Intelligence (AI), that is, the effective integration of learning and reasoning (Valiant 2008), both symbolic inference and statistical learning need to be combined in an effective way. However, over the last three decades, statistical learning and symbolic reasoning have been developed largely by distinct research communities in AI (but see below for exceptions). More recently, developments in deep learning have been connected strongly with and have contributed novel insights into representational issues. So far these representations have been *low level*, and have not been integrated with the high-level symbolic representations used in knowledge representation. It is exactly in this area that neural-symbolic learning and reasoning has been relevant for over two decades, having addressed many relevant representational issues, e.g. the binding problem (Feldman, 2013; Sun, 1994). Neural-Symbolic Learning and Reasoning seeks to integrate principles from neural-networks learning and logical reasoning. It is an interdisciplinary field involving components of knowledge representation, neuroscience, machine learning and cognitive science. This note briefly overviews some of the achievements in neural-symbolic computation and outlines some

key challenges and opportunities. These challenges have been identified at a recent Dagstuhl seminar on Neural-Symbolic Learning and Reasoning, in Wadern, Germany (September 2014), which marked the tenth anniversary of the workshop series on Neural-Symbolic Learning and Reasoning, which started at IJCAI 2005 in Edinburgh. For details about the seminar presentations, please visit: <http://www.dagstuhl.de/14381>. For more information about the workshop series, please visit [www.neural-symbolic.org](http://www.neural-symbolic.org). Another area that is relevant to Valiant’s challenge is that of statistical relational learning and probabilistic logic learning (Getoor et al., 2007; De Raedt et al., 2007), which aim at integrating probabilistic graphical models rather than connectionist methods with logical and relational reasoning.

The integration of the symbolic and connectionist paradigms of AI has been pursued by a relatively small research community over the last two decades and has yielded several significant results. Over the last decade, neural-symbolic systems have been shown capable of overcoming the so-called *propositional fixation* of neural networks, as McCarthy (1988) put it in response to Smolensky (1988); see also (Hinton, 1990). Neural networks were shown capable of representing modal and temporal logics (d’Avila Garcez and Lamb, 2006) and fragments of first-order logic (Bader, Hitzler, Hölldobler, 2008; d’Avila Garcez, Lamb, Gabbay, 2009). Further, neural-symbolic systems have been applied to a number of problems in the areas of bioinformatics, control engineering, software verification and adaptation, visual intelligence, ontology learning, and computer games (Borges, d’Avila Garcez, and Lamb, 2011; de Penning et al., 2011; Hitzler, Bader and d’Avila Garcez, 2005). Most of the work on knowledge representation and learning in neural networks has focused on variable-free logic fragments. However, one should note that several approaches have dealt with alternative formalizations of variable binding, and the representation of relations (Bader, Hitzler, and Hölldobler, 2008; d’Avila Garcez, Lamb, and Gabbay, 2009; Pinkas, Lima, and Cohen, 2012; Franca, d’Avila Garcez and Zaverucha, 2014).

In deep learning (Hinton, Osindero, and Teh, 2006), the generalization of abstract representations from raw data may be a fundamental objective, but how it happens is not fully understood (Tran and d'Avila Garcez, 2013). Deep architectures seek to manage complex issues of representation abstraction, modularity, and the trade-off between distributed and localist representations. Several techniques developed under the umbrella of neural-symbolic computation can be useful towards this goal. For instance, *fibring neural networks* offer the expression of levels of symbolic abstraction. Connectionist modal logics are modular by construction (d'Avila Garcez, Lamb and Gabbay, 2007).

In what follows, challenges and opportunities for neural-symbolic integration are outlined, as a summary of the discussions held at the Dagstuhl seminar. In a nutshell: (i) the mechanisms for structure learning remain to be fully understood, whether they consist of hypothesis search at the concept level, including (probabilistic) Inductive Logic Programming (ILP) and statistical AI approaches, or iterative adaptation processes such as Hebbian learning and contrastive divergence; (ii) the learning of generalizations of symbolic rules is a crucial process and not well understood - the adoption of neural networks that can offer degrees of modularity, such as deep networks, and the neural-symbolic methods for knowledge insertion and extraction from neural networks may help shed light into this question; (iii) effective knowledge extraction from large-scale networks remains a challenge - computational complexity issues and the provision of compact, expressive descriptions continue to be a barrier for explanation, lifelong learning and transfer learning. Items (i)-(iii) above open up a number of research opportunities, to be discussed next.

## 2. State-of-The-Art Results and Challenges

**Representation:** Most of the work on neural-symbolic learning and reasoning has focused on propositional logics. Early approaches were based essentially on the connectionist representation of propositional logic, a line of research which has since been substantially extended to other finitary logics (d'Avila Garcez, Lamb, Gabbay 2009). Some primary proposals for overcoming the propositional fixation of neural networks include (Gust, Kühnberger, and Geibel, 2007) which leverages variable-free representations of predicate logic using category-theoretic Topoi, (Bader, Hitzler, and Hölldobler, 2008) for predicate Horn logic programs (with function symbols) which utilizes an encoding of logic as vectors of real numbers mediated by the Cantor set, and (Guillame-Bert, Broda, and d'Avila Garcez, 2010) for learning first-order rules based on term encoding also as vectors. These systems have been shown to work in limited proof-of-concept settings or small examples, and attempts to achieve useful performance in practice have so far not been successful. To advance, it may well be necessary to consider logics of intermediate expressiveness, such as description logics (Krötzsch, Ru-

dolph, and Hitzler, 2013), so-called propositionalization methods, as used by ILP (Blockeel et. al, 2011; França, Zaverucha, and d'Avila Garcez, 2014) and answer-set programming (Lifschitz, 2002), and modal logics (d'Avila Garcez, Lamb and Gabbay, 2007), known to be more expressive than propositional logic and decidable. More recent results regarding the integration of description logics and rules (Krisnadhi, Maier, and Hitzler, 2011, Krötzsch et al., 2011) indicate the feasibility of the approach w.r.t. neural-symbolic integration. The variable binding problem, though, and how neural networks reason with variables remain central to this enterprise (d'Avila Garcez, Broda, Gabbay 2002; Feldman, 2006; Pinkas, Lima, and Cohen, 2012).

There has been much successful work in the neural-symbolic computation community on extracting logical expressions from trained neural networks, and using this extracted knowledge to seed learning in further tasks (see d'Avila Garcez, Lamb, and Gabbay (2009) for an overview). Meanwhile, there has been some suggestive recent work showing that neural networks can learn entire sequences of actions, thus amounting to "mental simulation" of some concrete, temporally extended activity. There is also a very well developed logical theory of action, for instance related to the basic propositional logic of programs PDL (Harel, Kozen, and Tiuryn, 2001), capturing what holds true after various combinations of actions. A natural place to extend the aforementioned work would be to explore extraction from a trained network exhibiting this kind of simulation behavior, a PDL expression capturing a high-level description of that sequence of actions. As argued by Feldman (2006), if the brain is not a network of neurons that represent things, but a network of neurons that do things, action models should be playing a central role.

As regards knowledge representation in the brain, one of the key challenges is to understand how neural activations, which are widely distributed and sub-symbolic, give rise to behavior that is symbolic, such as language and logical reasoning. Recent advances in fMRI and MEG analysis make it possible to develop and test such theories. For instance, formal concept analysis (Ganter and Wille, 1999; Endres and Foldiak, 2009) leads to characterization of semantic structures in the brain, and conceptual attribute representations (Binder and Desai 2011) make it possible to model how semantics concepts map to brain areas. A major challenge for the future is to understand how such semantics are constructed and affected by context, such as a sequence of words in a sentence.

**Consolidation:** Learning to Reason (L2R) is a framework that makes learning an integral part of the reasoning process (Khardon and Roth, 1997). L2R studies the entire process of learning a knowledge base (KB) representation from examples of the truth-table of a logical expression, and then reasoning with that knowledge base by querying

it with similar examples. Learning is done specifically for the purpose of reasoning with the learned knowledge in the Probably Approximately Correct (PAC) sense. This work has close connections to the *neuroidal* model developed by Valiant (2000) that examines computationally tractable learning and reasoning given PAC constraints. These constraints consider limiting the agent’s environment via a probability distribution over the input space and relaxing performance bounds on learning and reasoning (Khardon and Roth, 1999). Despite interesting early findings (Valiant, 2008; Juba, 2013), there is much work to be done to make this a practical approach. A major question is how a L2R agent can develop a complete KB over time when examples of the logical expressions arrive with values for only part of the input space.

This suggests that a Lifelong Machine Learning (LML) approach is needed that can consolidate the knowledge of individual examples over many learning episodes (Silver, 2013a; Fowler, 2011). The consolidation of learned knowledge facilitates the efficient and effective retention and transfer of knowledge (e.g. rapid and beneficial inductive bias) when learning a new task (Silver, 2013b). It is also a challenge because of the computational complexity of knowledge extraction and the need for compact representations that would enable efficient reasoning about what has been learned. Deep networks, however, represent knowledge at different levels of abstraction in a modular way. This may be related to the representation of modal logics, which are intrinsically modular (d’Avila Garcez, Lamb, and Gabbay, 2007) and decidable, offering a *sweet spot* in the complexity-expressiveness landscape (Vardi, 1996). Modularity of deep networks seem suitable to relational knowledge extraction, which may reduce the computational complexity of extraction (d’Avila Garcez, Broda, and Gabbay, 2001).

**Transfer:** Knowledge transfer between, at first site, unrelated domains is a characteristic feature and crucial cornerstone of human learning. As, for example, evidenced by the work in (Gentner, Holyoak, and Kokinov, 2001), analogy is considered essential for learning abstract concepts or procedures and for adapting existing knowledge to newly encountered scenarios and contexts. Whilst most of the prominent computational models of analogy, such as the Structure Mapping Engine (Falkenhainer, Forbus, and Gentner 1989) or Heuristic-Driven Theory Projection (Schmidt, Krumnack, Gust, Kühnberger, 2014) are logic-based, recent developments in structure learning in a neural-symbolic paradigm may open the way for an application of analogy on a sub-symbolic level. The expected gain is enormous: instead of having to retrain a network model on a new domain, already obtained insights could meaningfully be transferred between different networks, giving subsequent models a *head start*. Still, many questions have to be answered with two of the most foundational ones

being: How can the knowledge-level notion of analogical transfer practically be implemented in connectionist architectures? How can possible analogies between different domains be discovered on a sub-symbolic level in the first place? Some work on heterogeneous transfer learning has been directed at these questions (Yang et al, 2009).

**Application:** From a practical perspective, neural-symbolic integration has been applied to training and assessment in simulators, normative reasoning, rule learning, integration of run-time verification and adaptation, action learning and description in videos (Borges, d’Avila Garcez, Lamb, 2011; de Penning et al., 2011). Future application areas that seem promising include the analysis of complex networks, social robotics and health informatics, and multimodal learning and reasoning combining video and audio data with metadata. Overall, neural-symbolic integration seems suitable in application areas where large amounts of heterogeneous data are available and knowledge descriptions are needed, such as e.g. when video and audio data are tagged with ontological metadata (de Penning, 2011, Tran and d’Avila Garcez, 2013), including robot navigation and communication, health, genomics, hardware/software specification, multimodal data fusion for information retrieval, big data understanding and, ultimately, language understanding. Several features illustrate the advantages of neural-symbolic computation when it comes to specific applications: its explanation capacity, no a priori assumptions, its comprehensive cognitive models integrating symbolic and statistical learning with sound logical reasoning. Ultimately, however, in each of the above application areas, measurable criteria for comparison should include accuracy and efficiency measures, knowledge readability.

### 3. Conclusions

Neural-symbolic computation reaches out to two communities and seeks to achieve the fusion of competing views, when such fusion can be beneficial. In doing so, it sparks new ideas and promotes cooperation. Further, neural-symbolic computation brings together an integrated methodological perspective, as it draws from both neuroscience and cognitive systems. Methodologically, it bridges gaps, as new ideas can emerge through *changes of representation*. In summary, neural-symbolic computation is a promising approach, both from a methodological and computational perspective to answer positively to the need for effective knowledge representation, reasoning and learning systems. Both its representational generality (the ability to represent, learn and reason about several symbolic systems) and its learning robustness can open interesting opportunities leading to adequate forms of knowledge representation, be they purely symbolic, or hybrid combinations involving probabilistic or numerical representations.

**Acknowledgements:** We are grateful to Jerry Feldman for useful discussions which took place during Dagstuhl Seminar 14381 and have helped shape the challenges presented in this paper.

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