

## NEUTROSOPHIC CLASSICAL EVENTS AND ITS PROBABILITY

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### ABSTRACT

In this paper we introduce a new type of classical set called the neutrosophic classical set. After given the fundamental definitions of neutrosophic classical set operations, we obtain several properties, and discussed the relationship between neutrosophic classical sets and others. Finally, we generalize the classical probability to the notion of neutrosophic probability. This kind of probability is necessary because it provides a better representation than classical probability to uncertain events. Possible applications to computer sciences are touched upon.

**KEYWORDS:** Neutrosophic Probability, Neutrosophic Set, Probability Theory

### INTRODUCTION

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory. After the introduction of the neutrosophic set concepts in [1, 2, 3, 4, 5]. In this paper we introduce definitions of neutrosophic classical sets. After given the fundamental definitions of neutrosophic classical set operations, we obtain several properties, and discussed the relationship between neutrosophic classical sets and others. Finally, we generalize the classical probability to the notion of neutrosophic probability. This kind of probability is necessary because it provides a better representation than classical probability to uncertain events neutrosophic.

### TERMINOLOGIES

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [1, 2, 3], and Salama et al. [4,5]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where  $]^{-}0, 1^{+}[$  is nonstandard unit interval.

**Definition 1** [1, 2, 3]

Let T, I, F be real standard or nonstandard subsets of  $]^{-}0, 1^{+}[$ , with

$$\text{Sup}_T = t_{\text{sup}}, \text{inf}_T = t_{\text{inf}}$$

$$\text{Sup}_I = i_{\text{sup}}, \text{inf}_I = i_{\text{inf}}$$

$$\text{Sup}_F = f_{\text{sup}}, \text{inf}_F = f_{\text{inf}}$$

$$n\text{-sup} = t_{\text{sup}} + i_{\text{sup}} + f_{\text{sup}}$$

$$n\text{-inf} = t_{\text{inf}} + i_{\text{inf}} + f_{\text{inf}},$$

T, I, F are called neutrosophic components

**Definition 2** [4, 5]

Let  $X$  be a non-empty fixed set. A neutrosophic set (NS for short)  $A$  is an object having the form  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  Where  $\mu_A(x)$ ,  $\sigma_A(x)$  and  $\gamma_A(x)$  which represent the degree of member ship function (namely  $\mu_A(x)$ ), the degree of indeterminacy (namely  $\sigma_A(x)$ ), and the degree of non-member ship (namely  $\gamma_A(x)$ ) respectively of each element  $x \in X$  to the set  $A$ .

**Definition 3** [4, 5] The NSS  $0_N$  and  $1_N$  in  $X$  as follows:

$0_N$  may be defined as:

$$(0_1) \quad 0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$$

$$(0_2) \quad 0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$$

$$(0_3) \quad 0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$$

$$(0_4) \quad 0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$$

$1_N$  may be defined as:

$$(1_1) \quad 1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$$

$$(1_2) \quad 1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$$

$$(1_3) \quad 1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$$

$$(1_4) \quad 1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$$

**NEUTROSOPHIC CLASSICAL SETS**

We shall now consider some possible definitions for basic concepts of the neutrosophic classical sets and its operations.

**Definition 1**

Let  $X$  be a non-empty fixed set. A neutrosophic classical set (NCS for short)  $A$  is an object having the form  $A = \langle X, A_1, A_2, A_3 \rangle$  where  $A_1, A_2$  and  $A_3$  are subsets of  $X$  satisfying  $A_1 \cap A_2 \cap A_3 = \emptyset$ . The set  $A_1$  is called the set of member of  $A$ ,  $A_2$  is called indeterminacy of  $A$  and  $A_3$  is called non-members of  $A$ .

**Remark 1**

A neutrosophic classical set  $A = \langle X, A_1, A_2, A_3 \rangle$  can be identified to an ordered triple  $\langle A_1, A_2, A_3 \rangle$  are subsets on  $X$ , and every classical set in  $X$  is obviously an NCS having the form  $\langle A_1, A_2, A_3 \rangle$ , and one can define several relations and operations between NCSs.

Since our purpose is to construct the tools for developing neutrosophic classical set, we must introduce the NCS  $\phi_N, X_N$  in  $X$  as follows:

1.  $\phi_N$  may be defined as
  - $\phi_N = \langle X, \phi, \phi, X \rangle$ , or
  - $\phi_N = \langle X, \phi, X, X \rangle$ , or
  - $\phi_N = \langle X, \phi, X, \phi \rangle$ , or
  - $\phi_N = \langle X, \phi, \phi, \phi \rangle$
2.  $X_N$  may be defined as
  - i)  $X_N = \langle X, X, \phi, \phi \rangle$ ,
  - ii)  $X_N = \langle X, X, X, \phi \rangle$ ,
  - $X_N = \langle X, X, X, \phi \rangle$ ,

Every classical set  $A$  a non-empty set  $X$  is obviously on NCS having the form  $A = \langle X, A_1, A_2, A_3 \rangle$

### Definition 2

Let  $A = \langle X, A_1, A_2, A_3 \rangle$  a NCS on  $X$ , then the complement of the set  $A$  ( $C(A)$ , for short) maybe defined as three kinds of complements

- $(C_1) A^c = \langle X, A^c_1, A^c_2, A^c_3 \rangle$ ,
- $(C_2) A^c = \langle X, A_3, A_2, A_1 \rangle$
- $(C_3) A^c = \langle X, A_3, A^c_2, A_1 \rangle$

One can define several relations and operations between NCS as follows:

### Definition 3

Let  $X$  be a non-empty set, and NCSS  $A$  and  $B$  in the form  $A = \langle X, A_1, A_2, A_3 \rangle, B = \langle X, B_1, B_2, B_3 \rangle$ , then we may consider two possible definitions for subsets ( $A \subseteq B$ )

$(A \subseteq B)$  may be defined as:

- $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \subseteq B_2$  and  $A_3 \supseteq B_3$  or
- $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \supseteq B_2$  and  $A_3 \supseteq B_3$

**Proposition 1**

For any neutrosophic classical set  $A$  the following are holds

- $\phi_N \subseteq A, \phi_N \subseteq \phi_N$ .
- $A \subseteq X_N, X_N \subseteq X_N$ .

**Definition 4**

Let  $X$  be a non-empty set, and NCSS  $A$  and  $B$  in the form  $A = \langle X, A_1, A_2, A_3 \rangle, B = \langle X, B_1, B_2, B_3 \rangle$  are NCSS Then

1.  $A \cap B$  may be defined as:

- $A \cap B = \langle X, A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$  or
- $A \cap B = \langle X, A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$

2.  $A \cup B$  may be defined as:

- $A \cup B = \langle X, A_1 \cup B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$  or
- $A \cup B = \langle X, A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$

3.  $[ ]A = \langle X, A_1, A_2, A_3^c \rangle$ .

4.  $\langle \rangle A = \langle X, A_3^c, A_2, A_1 \rangle$ .

**Proposition 2**

For all two neutrosophic classical sets  $A$  and  $B$  on  $X$ , then the following are true

- $(A \cap B)^c = A^c \cup B^c$ .
- $(A \cup B)^c = A^c \cap B^c$ .

We can easily generalize the operations of intersection and union in definition 3.2 to arbitrary family of neutrosophic classical subsets as follows:

**Proposition 3**

Let  $\{A_j : j \in J\}$  be arbitrary family of neutrosophic classical subsets in  $X$ , then

1.  $\bigcap A_j$  may be defined as :

- $\bigcap A_j = \langle X, \bigcap A_{j_1}, \bigcap A_{j_2}, \bigcup A_{j_3} \rangle$ , or
- $\bigcap A_j = \langle X, \bigcap A_{j_1}, \bigcup A_{j_2}, \bigcup A_{j_3} \rangle$ .

2.  $\cup A_j$  may be defined as :

- $\cup A_j = \langle X, \cup A_{j_1}, \cap A_{j_2}, \cap A_{j_3} \rangle$  or
- $\cup A_j = \langle X, \cup A_{j_1}, \cup A_{j_2}, \cap A_{j_3} \rangle$ .

### Definition 5

The product of two neutrosophic classical sets A and B is a neutrosophic classical set  $A \times B$  given by

$$A \times B = \langle X, A_1 \times B_1, A_2 \times B_2, A_3 \times B_3 \rangle.$$

## NEUTROSOPHIC PROBABILITY

Neutrosophic probability is a generalization of the classical probability in which the chance that event  $A = \langle X, A_1, A_2, A_3 \rangle$  occurs is  $P(A_1)$  true,  $P(A_2)$  indeterminate,  $P(A_3)$  false on a space X, then  $NP(A) = \langle X, P(A_1), P(A_2), P(A_3) \rangle$

Neutrosophic probability space the universal set, endowed with a neutrosophic probability defined for each of its subset, from a neutrosophic probability space.

### Definition 1

Let A and B be a neutrosophic events on a space X, then  $NP(A) = \langle X, P(A_1), P(A_2), P(A_3) \rangle$  and  $NP(B) = \langle X, P(B_1), P(B_2), P(B_3) \rangle$  their neutrosophic probabilities.

Neutrosophic probability is necessary because it provides a better representation than classical probability to uncertain events.

### Definition 2

Let A and B be a neutrosophic events on a space X, and  $NP(A) = \langle X, P(A_1), P(A_2), P(A_3) \rangle$  and  $NP(B) = \langle X, P(B_1), P(B_2), P(B_3) \rangle$  are neutrosophic probabilities. Then we define

- $NP(A \cap B) = \langle X, P(A_1 \cap B_1), P(A_2 \cap B_2), P(A_3 \cap B_3) \rangle$
- $NP(A \cup B) = \langle X, P(A_1 \cup B_1), P(A_2 \cup B_2), P(A_3 \cup B_3) \rangle$
- $NP(A^c) = \langle X, P(A_1^c), P(A_2^c), P(A_3^c) \rangle$

Since our main purpose is to construct the tools for developing neutrosophic probability, we must introduce the following

- $NP(\phi_N)$  may be defined as
- $NP(\phi_N) = \langle X, P(\phi), P(\phi), P(X) \rangle$  or

- $NP(\phi_N) = \langle X, P(\phi), P(X), P(X) \rangle$  or
- $NP(\phi_N) = \langle X, P(\phi), P(\phi), P(\phi) \rangle$  or
- $NP(\phi_N) = \langle X, P(\phi), P(X), P(\phi) \rangle$
- $NP(X_N)$  may be defined as
- $NP(X_N) = \langle X, P(X), P(\phi), P(\phi) \rangle$  or
- $NP(X_N) = \langle X, P(X), P(X), P(\phi) \rangle$ .

**Remark 1**

$NP(X_N) = 1_N$ ,  $NP(\phi_N) = O_N$ . Where  $1_N, O_N$  are in Definition 2.3.

**Proposition 1**

For all two neutrosophic classical events A and B on X, then the following are true

- $NP(A \cap B)^c = NP(A^c) \cup NP(B^c)$ .
- $NP(A \cup B)^c = NP(A^c) \cap NP(B^c)$ .

**Example 1**

Let  $X = \{a, b, c, d\}$  and A, B two events on X defined by  $A = \langle X, \{a\}, \{b, c\}, \{c, d\} \rangle$ ,  $B = \langle X, \{a, b\}, \{a, c\}, \{c\} \rangle$ , then see that  $NP(A) = \langle X, 0.25, 0.5, 0.5 \rangle$ ,  $NP(B) = \langle X, 0.5, 0.5, 0.25 \rangle$ , one can compute all probabilities from definitions.

**Example 2**

Let us consider a neutrosophic set a collection of possible locations (position) of particle x and Let A and B two neutrosophic sets.

One can say, by language abuse, that any particle x neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between  $^-0$  and  $^+1$ . For example :x(0.5,0.2,0.3) belongs to A (which means, the probability of 50% particle x is in a position of A, with a probability of 30% x is not in A, and the rest is undecidable); or y(0,0,1) belongs to A (which normally means y is not for sure in A); or z(0,1,0) belongs to A (which means one does know absolutely nothing about z affiliation with A). More general,  $x((0.2-0.3), (0.4-0.45) \cup [0.50-0.51, \{0.2, 0.24, 0.28\}])$  belongs to the set, which means:

- With a probability in between 20-30% particle x is in a position of A ( one cannot find an exact approximate because of various sources used );

- With a probability of 20% or 24% or 28% x is not in A;

- The indeterminacy related to the appurtenance of  $x$  to  $A$  is in between 40-45% or between 50-51% (limits included). The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and  $n\text{-sup}=30\%+51\%+28\%>100$  in this case

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