Neutral currents and Glashow-Iliopoulos-Maiani mechanism in $SU(3)_L \otimes U(1)_N$ models for electroweak interactions

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We study the Glashow-Iliopoulos-Maiani mechanism for flavor-changing neutral-current suppression in both the gauge and Higgs sectors, for models with $SU(3)_L \otimes U(1)_N$ gauge symmetry. The models differ from one another only with respect to the representation content. The main features of these models are that in order to cancel the triangle anomalies the number of families must be divisible by three (the number of colors) and that the lepton number is violated by some lepton-gauge bosons and lepton-scalar interactions.

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I. INTRODUCTION

It is a well known fact that flavor-changing neutral currents (FCNC's) are very suppressed with respect to the charged-current weak interactions. This follows from experimental data on decays such as $K_L^0 \to \mu^+\mu^-$, $D^0 \to \mu^+\mu^-$, and $B^0 \to \mu^+\mu^-$ for transitions $s \leftrightarrow d$, $c \leftrightarrow u$, and $b \leftrightarrow d$ respectively. Experimentally it is found that [1]

$$\Gamma(K_L \to \mu^-\mu^-)/\Gamma(K_L \to \text{all}) = (7.3 \pm 0.4) \times 10^{-9}, \tag{1}$$

$$\Gamma(D^0 \to \mu^+\mu^-)/\Gamma(D^0 \to \text{all}) < 1.1 \times 10^{-5}, \tag{2}$$

$$\Gamma(B^0 \to \mu^+\mu^-)/\Gamma(B^0 \to \text{all}) < 1.2 \times 10^{-5}. \tag{3}$$

The change of flavor by two units as $|\Delta S| = 2$ and $|\Delta B| = 2$ also is very suppressed. In the standard model this occurs only in second-order weak interactions. The two examples for which this has been measured are the K^0-$\bar{K}^0$ and B^0-$\bar{B}^0$ mass differences [1]:

$$m_{K_S} - m_{K_L} = (3.522 \pm 0.016) \times 10^{-12}\,\text{MeV}, \tag{4}$$

$$|m_{B_1^0} - m_{B_2^0}| = (3.6 \pm 0.7) \times 10^{-10}\,\text{MeV}. \tag{5}$$

On the other hand, no evidence exists for D^0-$\bar{D}^0$ mixing. Next, from the experimental data we infer that FCNC effects in the s-d, c-u, and d-b systems are smaller than $O(\alpha G_F)$, but we do not know if the same occurs in the s-b sector and in other systems involving the as yet undiscovered top quark.

In the standard model, which is based on the $SU(2)_L \otimes U(1)_Y$ gauge symmetry [2], the Glashow-Iliopoulos-Maiani (GIM) mechanism accounts for the suppression of neutral processes in which there is a change of flavor to order G_F or αG_F [3].

The problem of how to implement such a suppression of FCNC effects in $SU(2)_L \otimes U(1)_Y$ models in a *natural* way was considered a long time ago by Glashow and Weinberg [4]. The term *natural* means that the conservation of flavor in neutral currents follows from the group structure and representation content of the theory; that is, the suppression of FCNC's is valid for all values of the parameters of the theory. The necessary and sufficient conditions are that all quarks of fixed charge and helicity must (i) transform according to the same irreducible representation of the SU(2) group, (ii) have the same values of weak I_3, and (iii) gain mass from a single source.

As the main points are valid for a general kind of model we revised them briefly. The neutral currents have in general the form [4]

$$J_Z^\mu = \bar{q}_L \gamma^\mu Y_L q_L + \bar{q}_R \gamma^\mu Y_R q_R, \tag{6}$$

where q denotes a column vector with all quark fields (phenomenological states) and Y_L, Y_R are the matrices

$$\begin{aligned} Y_L &\propto I_{3L} - 2\sin^2\theta_W Q, \\ Y_R &\propto I_{3R} - 2\sin^2\theta_W Q, \end{aligned} \tag{7}$$

where θ_W is the electroweak mixing angle, I_{3L} (I_{3R}) are the matrices corresponding to the third component of the SU(2) group for the left-handed (right-handed) fields, and Q is the charge operator. On the other hand, the mass term of the Lagrangian has the form

$$-\bar{q}_L M^Q q_R - \bar{q}_R M^{Q\dagger} q_L, \tag{8}$$

where the mass matrix for the charge-Q sector, M^Q, is in general neither Hermitian nor diagonal. It is possible to redefine the quark fields as

$$Q_L = V_L^Q q_L, \quad Q_R = V_R^Q q_R, \tag{9}$$

with V_L^Q, V_R^Q unitary matrices in the flavor space. In the $Q_{L,R}$ basis

$$M'^Q = V_L^Q M^Q V_R^{Q^{-1}}, \tag{10}$$

with M'^Q diagonal. The physical states ($Q_{L,R}$ fields) are defined as eigenstates of the quark mass matrix.

In order to have natural suppression of FCNC's it is necessary that in Eq. (7) all quarks with the same charge have the same values of the third component of I_L and I_R, which implies that these components are functions of the electric charge [4]:

$$I_{3L} = f_L(Q), \qquad I_{3R} = f_R(Q). \tag{11}$$

The conditions (11) imply the GIM mechanism in each charge sector. In $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge theories αG neutral couplings induced at the one-loop level are suppressed if all quarks with the same charge have the same value of $\mathbf{I}_L^2$ and $\mathbf{I}_R^2$.

Later Georgi and Pais [5] have generalized the GIM mechanism to systems of more than four quarks and leptons in a different way. They have expanded the quark and lepton systems in the "vertical" direction by enlarging the gauge group to $\mathrm{SU}(3) \otimes \mathrm{U}(1)$. They called "horizontal mixings" the particle mixtures at equivalent position within equivalent representation of the gauge group, otherwise the particle mixtures are called "vertical mixings."

In this work we will consider the group $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ [6–8] as the gauge symmetry with several representation contents. Some FCNC's are naturally suppressed in the horizontal mixings in the gauge and in the Higgs sectors, but vertical mixings are associated with FCNC's. This happens in the Higgs sector because quarks with the same charge but in nonequivalent representations gain mass from different sources.

In all these models there are two real neutral bosons; however, currents coupled to the lightest Z^0 neutral boson implement the GIM mechanism naturally. Notwithstanding, FCNC are induced by the heaviest neutral boson Z'^0, but, as it gains mass from the Higgs field which breaks the $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ symmetry into the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ one, its mass is arbitrarily large.

In this kind of model the lepton number is a gauge symmetry which is spontaneously broken in the interactions of leptons with gauge and scalar bosons. This is so because we have both particles and antiparticles in the same multiplets [6–8].

The organization of this paper is as follows. In Sec. II we consider three representation contents for the same gauge symmetry. In Sec. III we give the Yukawa Lagrangian for the matter field representations given in the previous section. The vector-boson sector for one of the models is presented in detail in Sec. IV. The charged and neutral currents are given in Sec. V. Section VI is devoted to show explicitly that the GIM mechanism is in fact implemented because quarks of the same charge have the same coupling with the Z^0 neutral boson. In one of the models (model I) it is necessary to add some discrete symmetries which ensure that the Higgs fields give a quark mass matrix of the tensor product form. Our conclusions are summarized in the last section. In the Appendix we give the $\bar{\lambda}$'s (λ's) matrices for the antitriplet (triplet) representation that we have used in this work.

II. $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ MODELS

As we have mentioned in the previous section, we will consider an $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ gauge theory for the weak interactions of quarks and leptons. We assume that the strong interactions are described by the unbroken color $\mathrm{SU}(3)_c$ but we have suppressed the color indices. Notwithstanding, they are considered in order to cancel anomalies. In fact, the sort of models we will consider have the interesting feature that the number of families must be divisible by the number of colors, three in the present case, in order to make the model anomaly-free. This happens only if we have an equal number of triplets and antitriplets taking into account the color degree of freedom and requiring the sum of all fermions charges to vanish [6–8]. This means that one of the quark multiplets transforms identically to the leptons under $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ and the other two-quark multiplets transform similarly to each other but differently from the leptons. The phenomenology depends somewhat on the choice of which quark multiplet transforms in the same representation of the leptons but we will not investigate this issue here.

This sort of model has become an interesting possibility for an extension of the standard model since the measurements [1, 9] of the Z^0 width at the CERN e^+e^- collider indicated that the number of sequential families is three, and this feature has no explanation within the standard model; otherwise these models are indistinguishable from the standard model up to the current energies achieved experimentally.

A. Model I

This is a model with leptons, ν_l^c, ν_l, $l = e, \mu, \tau$ and nine quarks, four with charge $\frac{2}{3}$ and five with charge $-\frac{1}{3}$. Let us start by defining the electric charge operator as

$$\frac{Q}{e} = \frac{\lambda_3}{2} + \frac{1}{\sqrt{3}}\frac{\lambda_8}{2} + N. \tag{12}$$

The left-handed leptons are assumed to belong to the following antitriplets $(\mathbf{3}^*, -\frac{1}{3})$ [10]:

$$\psi_{lL} = \begin{pmatrix} \nu_l^c \\ \nu_l \\ l^- \end{pmatrix}_L, \tag{13}$$

where $-\frac{1}{3}$ denotes the $\mathrm{U}_N(1)$ quantum number, and $l = e, \mu, \tau$.

In the quark sector, the first and second "generations" are in triplets $(\mathbf{3}, 0)$, and the third one in an antitriplet $(\mathbf{3}^*, +\frac{1}{3})$:

$$\begin{aligned} Q_{iL} &= \begin{pmatrix} u_i \\ d_i \\ d_i' \end{pmatrix}_L \sim (\mathbf{3}, 0), \quad i = 1, 2, \\ Q_{3L} &= \begin{pmatrix} u_3 \\ u_4 \\ d_3 \end{pmatrix}_L \sim (\mathbf{3}^*, +\tfrac{1}{3}). \end{aligned} \tag{14}$$

All right-handed charged fermions are taken to be $\mathrm{SU}(3)_L$ singlets. The representations above are symmetry eigenstates and they are related to the mass eigenstates by Cabibbo-like angles as we will discuss in Sec. III. In fact it does not matter which generation transforms similarly to the leptons because the three generations are well defined only after the symmetry breaking.

We will introduce the following Higgs multiplets:

$$\eta = \begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^- \end{pmatrix} \sim (\mathbf{3}, -\tfrac{2}{3}),$$
$$\rho = \begin{pmatrix} \rho^+ \\ \rho_1^0 \\ \rho_2^0 \end{pmatrix}, \sigma = \begin{pmatrix} \sigma^+ \\ \sigma_1^0 \\ \sigma_2^0 \end{pmatrix} \sim (\mathbf{3}, +\tfrac{1}{3}). \tag{15}$$

In order to avoid additional FCNC effects we will also assume that the vacuum expectation value (VEV) of the ρ and σ fields lie in the second and third component, respectively:

$$\langle \rho_1^0 \rangle \neq 0,\ \langle \rho_2^0 \rangle = 0;\ \langle \sigma_1^0 \rangle = 0,\ \langle \sigma_2^0 \rangle \neq 0\,. \tag{16}$$

Let us briefly comment about the question of whether the choice of VEV's in Eq. (16) is a *natural* one. We can see this by considering that the more general gauge-invariant potential involving the Higgs triplets η, σ, ρ is

$$\begin{aligned} V(\eta,\sigma,\rho) = {} & \lambda_1(\eta^\dagger\eta - v_\eta^2)^2 + \lambda_2(\rho^\dagger\rho - v_\rho^2)^2 + \lambda_3(\sigma^\dagger\sigma - v_\sigma^2)^2 \\ & + \lambda_4[(\eta^\dagger\eta - v_\eta^2) + (\rho^\dagger\rho - v_\rho^2) + (\sigma^\dagger\sigma - v_\sigma^2)]^2 + \lambda_5[(\rho^\dagger\rho)(\sigma^\dagger\sigma) - (\rho^\dagger\sigma)(\sigma^\dagger\rho)] \\ & + \lambda_6(\rho^\dagger\eta)(\eta^\dagger\rho) + \lambda_7(\sigma^\dagger\eta)(\eta^\dagger\sigma) + \lambda_8(\rho^\dagger\sigma)^2 + f\epsilon^{ijk}\eta_i\rho_j\sigma_k + \text{H.c.}, \end{aligned} \tag{17}$$

where $\lambda_i > 0$, $i = 1, \ldots, 6$, and ϵ^{ijk} is the totally antisymmetric symbol. Notice that in Eq. (17) there are linear terms in σ_1^0 and ρ_2^0. These terms and the Yukawa interactions in Eqs. (24) and (25) will induce divergent loop corrections and a counterterm will be necessary; hence, it would be impossible to maintain $\langle \sigma_1^0 \rangle = \langle \rho_2^0 \rangle = 0$. However, the trilinear terms f are forbidden by the appropriate discrete symmetries [see Eq. (59) below] and the choice of the VEV's in (16) is in fact a natural one.

B. Model II

This model has already been studied in detail in Refs. [6, 11]; hence, we will reexamine it briefly here. In this model we define the charge operator as

$$\frac{Q}{e} = \frac{1}{2}\left(\lambda_3 - \sqrt{3}\lambda_8\right) + N. \tag{18}$$

The leptons are in triplets $(\mathbf{3}, 0)$:

$$\psi_{lL} = \begin{pmatrix} \nu_l \\ l \\ l^c \end{pmatrix}_L \tag{19}$$

with $l = e, \mu, \tau$. The quarks belong to one triplet and two antitriplets:

$$Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ J_1 \end{pmatrix}_L \sim (\mathbf{3}, +\tfrac{2}{3}),\ Q_{iL} = \begin{pmatrix} J_i \\ u_i \\ d_i \end{pmatrix}_L \sim (\mathbf{3}^*, -\tfrac{1}{3}),$$
$$i = 2, 3. \tag{20}$$

All right-handed fields are in singlets and the introduction of right-handed neutrinos is optional. Notice that we have to introduce exotic quarks of charge $\frac{5}{3}$ and $-\frac{4}{3}$. The model also has exotic gauge bosons Y^+, U^{++} which are very massive and this is the case for the neutral boson Z'^0.

In this model it is necessary to introduce three Higgs triplets $(\mathbf{3}, 0)$, $(\mathbf{3}, -1)$, and $(\mathbf{3}, 1)$ in order to give mass to all the quarks. Charged leptons obtain mass when we introduce a sextet $(\mathbf{6}, 0)$ [11].

One of the Higgs, the one transforming as $(\mathbf{3}, -1)$, is responsible for the first symmetry breaking $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N \to \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. As the exotic quarks and gauge bosons and Z'^0 obtain a contribution to their masses from this triplet, they must be very heavy. The exotic quark masses are in fact arbitrary but the exotic gauge bosons and Z'^0 must have masses larger than 4 TeV and 40 TeV respectively [6].

C. Model III

This model is analogous to the previous one, but the leptons have heavy charged partners,

$$\begin{pmatrix} \nu_l \\ e^- \\ E^+ \end{pmatrix}_L, \begin{pmatrix} \nu_l \\ \mu^- \\ M^+ \end{pmatrix}_L, \begin{pmatrix} \nu_l \\ \tau^- \\ T^+ \end{pmatrix}_L, \tag{21}$$

and now we have to introduce right-handed singlets for e_R^-, μ_R^-, τ_R^- and E_R^+, M_R^+, T_R^+. Only the Higgs triplets of model II are required. The quark sector is the same of model II and the introduction of right-handed neutrinos is also optional.

III. YUKAWA LAGRANGIAN

A. Model I

The Yukawa Lagrangian in terms of the symmetry eigenstates [we have suppressed $\mathrm{SU}(3)_L$ indices] is

$$-\mathcal{L}_Y = G_{lm}\bar{\psi}_{lL}R_m\eta^* + \bar{Q}_{iL}G^u_{i\alpha}U_{\alpha R}\eta + \bar{Q}_{3L}(G^t_{3\alpha}\rho^* + G'^t_{3\alpha}\sigma^*)U_{\alpha R} + \bar{Q}_{iL}(G^d_{i\beta}\rho + G'^d_{i\beta}\sigma)D_{\beta R} + \bar{Q}_{3L}G^b_{3\beta}D_{\beta R}\eta^* + \text{H.c.}, \tag{22}$$

where $i = 1,2$, $\alpha = 1,2,3,4$, $\beta = 1,2,3,4,5$, and $U_{1,2,3,4R} = u_{1R}, u_{2R}, u_{3R}, u_{4R}$, $D_{1,2,3,4,5R} = d_{1R}, d_{2R}, d_{3R}, d'_{1R}, d'_{2R}$. For the leptons $R_m = e_R, \mu_R, \tau_R$. Summation is assumed in the repeated indices.

From Eq. (22) we obtain the mass terms

$$-\bar{U}_{\alpha L}M^U_{\alpha\alpha'}U_{\alpha' R} - \bar{D}_{\beta L}M^D_{\beta\beta'}D_{\beta' R} + \text{H.c.}, \tag{23}$$

where the indices (α,α') and (β,β'), run over all quarks with charge $+\frac{2}{3}$ and $-\frac{1}{3}$ respectively. Notice that the M^U is a 4×4 matrix and M^D a 5×5 one. We will denote the mass eigenstates obtained after the diagonalization of the matrix in Eq. (23) u, c, t, t' and d, s, b, d', s' for the charge $\frac{2}{3}$ and $-\frac{1}{3}$ sectors respectively. The primed quarks are new heavy quarks.

Explicitly, we can write the Yukawa interactions for the leptons in Eq. (22) as

$$-\mathcal{L}_{lY} = G_l(\bar{\nu}^c_{lL} l_R \eta_1^+ + \bar{\nu}_{lL} l_R \eta_2^+ + \bar{l}_L l_R \bar{\eta}^0 + \text{H.c.}). \tag{24}$$

We have also the Yukawa term

$$-\mathcal{L}'_{lL} = \frac{1}{2}\sum_{l,m}\epsilon^{ijk}h_{lm}\psi_{il}C^{-1}\psi_{jm}\eta^*_k \tag{25}$$

with $h_{lm} = -h_{ml}$. Equation (25) implies an antisymmetric 3×3 mass matrix for the neutrinos, and it is well known that for this kind of matrix one of the eigenvalues is zero and the other two are degenerated. One way to obtain an arbitrary mass matrix for the neutrinos is to introduce a symmetric sextet $S: (\mathbf{6}, +\frac{2}{3})$ or other Higgs multiplets with the same quantum number of η in order to give a mass to the neutrinos by radiative corrections [12, 13]. However, Eq. (25) can be forbidden by introducing an appropriate discrete symmetry.

The sextet has the charge assignment

$$S = \begin{pmatrix} S^0_1 & S^0_2 & S^-_1 \\ S^0_2 & S^0_3 & S^-_2 \\ S^-_1 & S^-_2 & S^{--} \end{pmatrix}. \tag{26}$$

The $\langle S^0_2\rangle \neq 0$ gives a Dirac mass term for neutrinos. In order to avoid a Majorana mass term we have to choose $\langle S^0_{1,3}\rangle = 0$. The Yukawa Lagrangian of the leptons with the sextet is

$$\mathcal{L}_{lS} = \sum_{lm} G_{lm}\bar{\psi}^c_{il}\psi_{jm}S^{ij} + \text{H.c.} \tag{27}$$

In Eq. (27) i,j denote SU(3) indices and $l, m = e, \mu, \tau$ and $G_{lm} = G_{ml}$. As we have not assigned a lepton number to either the η or to the sextet S Higgs multiplets, we have lepton-number violations in Eqs. (24) and (27).

B. Models II and III

As both models have the same quark content and they differ only in the leptonic sector we will consider them together. We also will write down the Yukawa interactions for the quarks with charge $\frac{2}{3}, -\frac{1}{3}$. Exotic quarks of charge $-\frac{4}{3}$ will mix with one another but not with the one with charge $\frac{5}{3}$. Considering only the quarks with charge $\frac{2}{3}$ and $-\frac{1}{3}$ the quark-Higgs interaction is

$$\mathcal{L}_Y = \bar{Q}_{1L}(G^u_{1\alpha}U_{\alpha R}\eta + G^d_{1\alpha}D_{\alpha R}\rho) + \bar{Q}_{i\alpha L}(F^u_{i\alpha}U_{\alpha R}\rho^* + F^d_{i\alpha}D_{\alpha R}\eta^*) + \text{H.c.}, \tag{28}$$

where $\alpha = 1,2,3$, $i = 2,3$, and $U_{\alpha R} = u_{1R}, u_{2R}, u_{3R}$, $D_{\alpha R} = d_{1R}, d_{2R}, d_{3R}$. The mass terms from Eq. (28) are

$$-\bar{U}_{\alpha' L}M^U_{\alpha'\alpha}U_{\alpha R} - \bar{D}_{\alpha' L}M^D_{\alpha'\alpha}D_{\alpha R} + \text{H.c.} \tag{29}$$

In Eq. (29) $U_{\alpha' L} = u_{1L}, u_{2L}, u_{3L}$, $D_{\alpha' L} = d_{1L}, d_{2L}, d_{3L}$. M^U and M^D are arbitrary 3×3 matrices. For the quantum numbers of the Higgs multiplet η and ρ see Ref. [6].

IV. GAUGE BOSONS

In this section, we will only consider the model I. For models II and III which are similar in the gauge sector, see Ref. [6].

The gauge bosons of this theory consist of an octet W^a_μ associated with $\text{SU}(3)_L$ and a singlet B_μ associated with $\text{U}(1)_N$. Considering only the triplets of Higgs bosons, the covariant derivatives are

$$\mathcal{D}_\mu\varphi_j = \partial_\mu\varphi_j + ig\left(\mathbf{W}_\mu\cdot\frac{\boldsymbol{\lambda}}{2}\right)^k_j\varphi_k + i\frac{g'}{2}N_\varphi B_\mu\varphi_j, \tag{30}$$

where N_φ denotes the N charge for the φ Higgs multiplet, $\varphi = \eta, \rho, \sigma$.

We will denote $v_\rho = \langle\rho^0_1\rangle$, $v_\sigma = \langle\sigma^0_2\rangle$, and $v_\eta = \langle\eta^0\rangle$ the vacuum expectation values of the corresponding Higgs multiplet. Using them in Eq. (30) we obtain the symmetry breaking pattern

$$\begin{array}{c} \text{SU}(3)_L \otimes \text{U}(1)_N \\ \downarrow \langle\sigma\rangle \\ \text{SU}(2)_L \otimes \text{U}(1)_Y \\ \downarrow \langle x\rangle \\ \text{U}(1)_{\text{em}}, \end{array} \tag{31}$$

where $\langle x\rangle = \langle\eta\rangle, \langle\rho\rangle$ and the $\text{SU}(3)_c$ of color remaining unbroken.

The non-Hermitian gauge bosons $\sqrt{2}W^+ \equiv W^1 - iW^2$, $\sqrt{2}V^+ \equiv W^4 - iW^5$ and $\sqrt{2}X^0 \equiv W^6 - iW^7$ have the following masses:

$$M_W^2 = \frac{1}{4}g^2\left(v_\eta^2 + v_\rho^2\right), \quad M_V^2 = \frac{1}{4}g^2\left(v_\eta^2 + v_\sigma^2\right),$$
$$M_X^2 = \frac{1}{4}g^2\left(v_\rho^2 + v_\sigma^2\right). \tag{32}$$

Notice that even if $v_\eta = v_\rho \approx v/\sqrt{2}$, where v is the usual vacuum expectation value for the Higgs boson in the standard model, the VEV v_σ must be large enough in order to leave the new gauge bosons sufficiently heavy to keep consistency with low energy phenomenology.

Notice that there are two gauge bosons forming an $\mathrm{SU}(2)_L$ doublet,

$$\begin{pmatrix} V^+ \\ X^0 \end{pmatrix}, \quad \begin{pmatrix} \bar{X}^0 \\ V^- \end{pmatrix}, \tag{33}$$

consisting of a charged gauge boson V^+ and a neutral one X^0.

On the other hand, the neutral (Hermitian) gauge bosons have the mass matrix $\frac{1}{4}g^2M^2$ in the (W^3, W^8, B) basis, where M^2 is given by

$$\begin{pmatrix} v_\eta^2 + v_\sigma^2 & \frac{1}{\sqrt{3}}(v_\eta^2 - v_\sigma^2) & -\frac{2}{3}t(2v_\eta^2 + v_\sigma^2) \\ \frac{1}{\sqrt{3}}(v_\eta^2 - v_\sigma^2) & \frac{1}{3}(v_\eta^2 + v_\sigma^2 + 4v_\rho^2) & -\frac{2}{3\sqrt{3}}t(2v_\eta^2 - v_\sigma^2 + 2v_\rho^2) \\ -\frac{2}{3}t(2v_\eta^2 + v_\sigma^2) & -\frac{2}{3\sqrt{3}}t(2v_\eta^2 - v_\sigma^2 + 2v_\rho^2) & \frac{4}{9}t^2(4v_\eta^2 + v_\sigma^2 + v_\rho^2) \end{pmatrix}, \tag{34}$$

with $t = g'/g$. Since $\det M^2 = 0$ we must have a photon after the symmetry breaking [14]. The introduction of the sextet S spoils the fact that the determinant of Eq. (34) vanishes. However this can be achieved by imposing fine-tuning of the VEV of the sextet $\langle S_2^0\rangle$ with the VEV's of the triplets. In fact, this implies that $\langle S_2^0\rangle$ is of the same order of magnitude of v_σ which is the highest VEV in the model. This is not phenomenologically interesting nor natural since $\langle S_2^0\rangle$ will give mass to the neutrinos. For this reason we will not introduce the sextet and, since the interaction in Eq. (25) will be forbidden by a discrete symmetry (see Sec. IV), in the present model the neutrinos remain massless at the tree level. Lepton-number-violating mass terms can arise from calculable radiative corrections mediated by gauge or scalar bosons but here we will not consider this issue [7, 12, 13]. We must stress that in model II the introduction of a sextet $(\mathbf{6}, 0)$ does not spoil the fact that $\det M^2 = 0$ without any restriction on the VEV's [11].

The eigenvalues of the matrix in Eq. (34) are

$$M_A^2 = 0, \qquad M_Z^2 \simeq \frac{g^2}{4}\frac{3+4t^2}{3+t^2}(v_\eta^2 + v_\rho^2),$$
$$M_{Z'}^2 \simeq \frac{g^2}{3}(1 + \tfrac{1}{3}t^2)v_\sigma^2, \tag{35}$$

where we have used $v_\sigma \gg v_{\rho,\eta}$ for the case of M_Z and $M_{Z'}$. Then, we have in this approximation

$$\frac{M_Z^2}{M_W^2} = \frac{3+4t^2}{3+t^2}. \tag{36}$$

In order to obtain the usual relation $\cos^2\theta_W M_Z^2 = M_W^2$, we must have

$$t^2 = \frac{s_W^2}{1 - \frac{4}{3}s_W^2}, \tag{37}$$

where $s_W^2 \equiv \sin^2\theta_W$. Hence, we can identify Z^0 as the neutral gauge boson of the standard model.

The neutral physical states in the (W^3, W^8, B) basis are

$$A_\mu = \frac{1}{(3+4t^2)^{\frac{1}{2}}}(\sqrt{3}t, t, \sqrt{3}),$$
$$Z_\mu \simeq \frac{-1}{(3+t^2)^{\frac{3}{2}}(3+4t^2)^{\frac{1}{2}}}\left(\frac{9-3t^2-2t^4}{2}, \frac{27+27t^2+6t^4}{2\sqrt{3}}, -3t(3+t^2)\right), \tag{38}$$
$$Z'_\mu \simeq \frac{1}{(3+t^2)^{\frac{1}{2}}}\left(-\frac{3}{2}, \frac{3}{2\sqrt{3}}, t\right).$$

V. CHARGED AND NEUTRAL CURRENTS

The interactions among the gauge bosons and fermions are read off from

$$\mathcal{L}_F = \bar{R}i\gamma^\mu\left(\partial_\mu + \frac{ig'}{2}B_\mu N_R\right)R + \bar{L}i\gamma^\mu\left(\partial_\mu + \frac{ig'}{2}B_\mu N_L + \frac{ig}{2}\boldsymbol{\lambda}\cdot\mathbf{W}_\mu\right)L, \tag{39}$$

where R represents any right-handed singlet and L any left-handed triplets or antitriplets and $N_R(N_L)$ is the $U(1)_N$ charge of the right-handed (left-handed) fermions. Here also we will consider only the model I.

A. Charged currents

Concerning the charged leptons, we obtain the electromagnetic interaction by identifying the electric charge e by

$$|e| = g\sin\theta_W. \tag{40}$$

Concerning the interactions with the charged vector fields we have

$$\mathcal{L}_l = -\frac{g}{\sqrt{2}}\left(-\bar{\nu}^c_{lL}\gamma^\mu l_L V^+_\mu + \bar{\nu}_{lL}\gamma^\mu l_L W^+_\mu\right) + \text{H.c.}, \tag{41}$$

where $l = e, \mu, \tau$. There are also currents coupled to the non-Hermitian neutral boson, $\bar{\nu}^c_{lL}\gamma^\mu \nu_{lL} X^0_\mu$.

For the first two quark "generations" we have

$$\mathcal{L}_{Q_{1,2}} = -\frac{g}{\sqrt{2}}\left(\bar{U}_{iL}\gamma^\mu D_{iL} W^+_\mu - \bar{U}_{iL}\gamma^\mu D'_{iL} V^+_\mu\right) + \text{H.c.}, \tag{42}$$

where $i = 1, 2$, $U_i = u_1, u_2$, $D_i\,(D'_i) = d_1, d_2\,(d'_1, d'_2)$. For the third quark "generation" we have

$$\mathcal{L}_{Q_3} = -\frac{g}{\sqrt{2}}\left(-\bar{u}_{3L}\gamma^\mu d_{3L} V^+_\mu + \bar{u}_{4L}\gamma^\mu d_{3L} W^+_\mu\right) + \text{H.c.} \tag{43}$$

As D'_i and u_3 are $SU(2)_L$ singlets the interaction with V^+ violates the lepton number [see Eq. (41)] and the weak isospin (see Eq. (42)). In this case the interactions with the neutral boson X^0 are proportional to

$$-(\bar{D}_{iL}\gamma^\mu D'_{iL} - \bar{u}_{3L}\gamma^\mu u_{4L})X^0_\mu.$$

Notice that the interactions with the X^0 boson also violate the lepton number and weak isospin. Recall that we have not attributed lepton number to the gauge bosons; however, as can be seen from Eq. (32) the masses of the vector bosons V^+, X^0 are proportional to the VEV of the σ, and therefore should be heavier than W^+, Z^0.

B. Neutral currents

Similarly, we have the neutral currents coupled to both Z^0 and Z'^0 massive vector bosons. For neutrinos,

$$\mathcal{L}^{\rm NC}_\nu = -\frac{g}{2c_W}\sum_l\left[-\bar{\nu}^c_{lL}\gamma^\mu\nu^c_{lL}\left(Z^0_\mu + \frac{1-2s^2_W}{(3-4s^2_W)^{\frac{1}{2}}}Z'^0_\mu\right) - \frac{2(1-s^2_W)}{(3-4s^2_W)^{\frac{1}{2}}}\bar{\nu}_{lL}\gamma^\mu\nu_{lL}Z'^0_\mu\right]. \tag{44}$$

Using $\bar{\nu}^c_L\gamma^\mu\nu^c_L = -\bar{\nu}_R\gamma^\mu\nu_R$, we see that, in this model, the magnitude of the neutral couplings of the right-handed neutrinos coincide with that of the left-handed neutrinos in the standard model. In the last equation neutrinos are still symmetry eigenstates.

Concerning the charged leptons we have

$$\mathcal{L}^{\rm NC}_l = -\frac{g}{2c_W}\sum_l\left(-\bar{l}_L\gamma^\mu l_L\left[(1-2s^2_W)Z_\mu + \frac{1-2s^2_W}{(3-4s^2_W)^{\frac{1}{2}}}Z'^0_\mu\right] + 2s^2_W\bar{l}_R\gamma^\mu l_R\left[-Z^0_\mu + \frac{1}{(3-4s^2_W)^{\frac{1}{2}}}Z'^0\right]\right). \tag{45}$$

Next, let us consider the quark sector. The electromagnetic current for quarks is the usual one

$$Q_q e\bar{q}\gamma^\mu q A_\mu, \tag{46}$$

where q is any of the quarks with $Q_q = +\frac{2}{3}, -\frac{1}{3}$ and e was defined in Eq. (40).

The neutral currents coupled to the Z^0 boson are

$$\begin{gathered}
-\frac{e}{6}\frac{1}{c_W s_W}(3-4s^2_W)\bar{U}_{iL}\gamma^\mu U_{iL} + \frac{2}{3}e\tan\theta_W\bar{U}_{iR}\gamma^\mu U_{iR}\,,\\
-\frac{e}{3}\tan\theta_W\bar{D}_{iL}\gamma^\mu D_{iL} - \frac{e}{3}\tan\theta_W\bar{D}_{iR}\gamma^\mu D_{iR}\,,\\
\frac{e}{6}\frac{1}{c_W s_W}(3-2s^2_W)\bar{D}'_{iL}\gamma^\mu D'_{iL} - \frac{1}{3}e\tan\theta_W\bar{D}'_{iR}\gamma^\mu D'_{iR}\,,\\
-\frac{e}{6}\frac{1}{c_W s_W}(3-4s^2_W)\bar{u}_{3L}\gamma^\mu u_{3L} + \frac{2}{3}e\tan\theta_W\bar{u}_{3R}\gamma^\mu u_{3R}\,,\\
\frac{2}{3}e\tan\theta_W\bar{u}_{4L}\gamma^\mu u_{4L} + \frac{2}{3}e\tan\theta_W\bar{u}_{4R}\gamma^\mu u_{4R}\,,\\
\frac{e}{6}\frac{1}{c_W s_W}(3-2s^2_W)\bar{d}_{3L}\gamma^\mu d_{3L} - \frac{1}{3}e\tan\theta_W\bar{d}_{3R}\gamma^\mu d_{3R}.
\end{gathered} \tag{47}$$

The interactions with the Z'^0 neutral boson are via the currents

$$\begin{gathered}
\frac{e}{6}\frac{(3-4s^2_W)^{\frac{1}{2}}}{c_W s_W}\bar{U}_{iL}\gamma^\mu U_{iL} - \frac{2}{3}e\frac{\tan\theta_W}{(3-4s^2_W)^{\frac{1}{2}}}\bar{U}_{iR}\gamma^\mu U_{iR}\,,\\
-\frac{e}{3}\frac{(3-4s^2_W)^{\frac{1}{2}}}{c_W s_W}\bar{D}_{iL}\gamma^\mu D_{iL} + \frac{e}{3}\frac{\tan\theta_W}{(3-4s^2_W)^{\frac{1}{2}}}\bar{D}_{iR}\gamma^\mu D_{iR}\,,\\
\frac{e}{6}\frac{(3-4s^2_W)^{\frac{1}{2}}}{c_W s_W}\bar{D}'_{iL}\gamma^\mu D'_{iL} + \frac{e}{3}\frac{\tan\theta_W}{(3-4s^2_W)^{\frac{1}{2}}}\bar{D}'_{iR}\gamma^\mu D'_{iR}\,,\\
-\frac{e}{6c_W s_W}\frac{(3-2s^2_W)}{(3-4s^2_W)^{\frac{1}{2}}}\bar{u}_{3L}\gamma^\mu u_{3L}\\
-\frac{2}{3}e\frac{\tan\theta_W}{(3-4s^2_W)^{\frac{1}{2}}}\bar{u}_{3R}\gamma^\mu u_{3R}\,,\\
\frac{e}{3c_W s_W}\frac{(3-5s^2_W)}{(3-4s^2_W)^{\frac{1}{2}}}\bar{u}_{4L}\gamma^\mu u_{4L}\\
-\frac{2}{3}e\frac{\tan\theta_W}{(3-4s^2_W)^{\frac{1}{2}}}\bar{u}_{4R}\gamma^\mu u_{4R}\,,\\
-\frac{e}{6c_W s_W}\frac{(3-2s^2_W)}{(3-4s^2_W)^{\frac{1}{2}}}\bar{d}_{3L}\gamma^\mu d_{3L}\\
+\frac{1}{3}e\frac{\tan\theta_W}{(3-4s^2_W)^{\frac{1}{2}}}\bar{d}_{3R}\gamma^\mu d_{3R}.
\end{gathered} \tag{48}$$

Here $U = u_1, u_2$, $D = d_1, d_2$, and $D' = d'_1, d'_2$.

VI. GIM MECHANISM

In this section we will study the GIM mechanism in the models we have considered before. In the gauge sector, the neutral currents have the form shown in Eq. (6) or

$$-\frac{g}{2c_W}[a_L(f)\bar{f}\gamma^\mu(1-\gamma_5)f + a_R(f)\bar{f}\gamma^\mu(1+\gamma_5)f]Z^0_\mu \tag{49}$$

for the Z^0, and

$$-\frac{g}{2c_W}[a'_L(f)\bar{f}\gamma^\mu(1-\gamma_5)f + a'_R(f)\bar{f}\gamma^\mu(1+\gamma_5)f]Z'^0_\mu \tag{50}$$

for the Z'^0, where f denotes any fermion.

A. Model I

For leptons, the neutral currents appear in Eqs. (44) and (45):

$$a_L(\nu_l) = 0, \quad a_R(\nu_l) = 1. \tag{51}$$

In the standard model one has $a_L(\nu_l) = 1$, $a_R(\nu_l) = 0$. For the couplings of neutrinos with Z'^0 we have

$$a'_L(\nu_l) = 0, \quad a'_R(\nu_l) = 2\frac{1-x}{g(x)} \tag{52}$$

with $l = e, \mu, \tau$. The couplings of the charged leptons with the Z^0 are

$$a_L(l) = -(1-2x), \quad a_R(l) = 2x, \tag{53}$$

and those with the Z'^0,

$$a'_L(l) = \frac{1-2x}{g(x)}, \quad a'_R(l) = 2x\frac{1}{g(x)}, \tag{54}$$

where we have defined $x = \sin^2\theta_W$ and $g(x) = (3-4x)^{\frac{1}{2}}$. Even if neutrinos were massive we can see from Eqs. (44) and (45) [or (51) and (53)] that there is no FCNC at the tree level in the leptonic sector since all lepton representations transform similarly. This is the case also in models II and III if we had introduced right-handed neutrinos.

For quarks, recall that the fields are still symmetry eigenstates, we have

$$\begin{aligned}
&a_L(u_1) = a_L(u_2) = a_L(u_3) = \frac{1}{6}g^2(x),\\
&a_R(u_1) = a_R(u_2) = a_R(u_3) = -\frac{2}{3}x,\\
&a_L(d_1) = a_L(d_2) = \frac{1}{3}x, \quad a_R(d_1) = a_R(d_2) = \frac{1}{3}x,\\
&a_L(d'_1) = a_L(d'_2) = a_L(d_3) = -\frac{1}{6}(3-2x),\\
&a_R(d'_1) = a_R(d'_2) = a_R(d_3) = \frac{1}{3}x,\\
&a_L(u_4) = -\frac{2}{3}x, \quad a_R(u_4) = -\frac{2}{3}x.
\end{aligned} \tag{55}$$

Similarly for the case of the Z'^0 we have

$$\begin{aligned}
&a'_L(u_1) = a'_L(u_2) = -\frac{1}{6}g(x),\\
&a'_R(u_1) = a'_R(u_2) = \frac{2}{3}\frac{x}{g(x)},\\
&a'_L(d_1) = a'_L(d_2) = \frac{1}{3}g(x),\\
&a'_R(d_1) = a'_R(d_2) = -\frac{1}{3}\frac{x}{g(x)},\\
&a'_L(d'_1) = a_L(d'_2) = -\frac{1}{6}g(x),\\
&a'_R(d'_1) = a_R(d'_2) = -\frac{1}{3}\frac{x}{g(x)},\\
&a'_L(u_3) = \frac{1}{6}\frac{3-2x}{g(x)}, \quad a'_R(u_3) = \frac{2}{3}\frac{x}{g(x)},\\
&a'_L(u_4) = -\frac{1}{3}\frac{3-5x}{g(x)}, \quad a'_R(u_4) = \frac{2}{3}\frac{x}{g(x)},\\
&a'_L(d_3) = \frac{1}{3}\frac{3-2x}{g(x)}, \quad a'_R(d_3) = -\frac{1}{3}\frac{x}{g(x)}.
\end{aligned} \tag{56}$$

Notice that as quarks of the same charge gain mass from different sources, in Eq. (22) we have FCNC at the tree level. In fact, the quark mass matrices in Eq. (23) arising from (22) are 4×4 and 5×5 for the charge $\frac{2}{3}$ and $-\frac{1}{3}$ respectively. After diagonalizing these matrices we obtain mixing among all quarks of the same charge. Since the coupling, appearing in Eqs. (55) do not satisfy the condition (11) the GIM mechanism does not work. One way, but probably not the only one, to overcome this trouble is the introduction of appropriate discrete symmetries. We can see from (55) that if the quarks u_1, u_2, u_3 do not couple to the quark u_4, and the quarks d'_1, d'_2, d_3 do not couple to the quarks d_1, d_2 the mass matrix will have the form

$$\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \text{ for the charge } -\tfrac{1}{3} \text{ sector}, \tag{57}$$

$$\begin{pmatrix} M_3 & 0 \\ 0 & 1 \end{pmatrix} \text{ for the charge } \tfrac{2}{3} \text{ sector}, \tag{58}$$

where M_1, M_3 are 3×3 matrices and M_2 a 2×2 matrix. To constrain the quark mass matrix to have the form given in Eqs. (57) and (58) we impose the following discrete symmetries on the Yukawa Lagrangian of Eq. (22):

$$\begin{gathered}
(\eta,\sigma) \to (\eta,\sigma),\\
\rho \to -\rho,\\
(\psi_{lL}, R_l) \to (\psi_{lL}, R_l),\\
(u_{1R}, u_{2R}, u_{3R}) \to (u_{1R}, u_{2R}, u_{3R}),\\
u_{4R} \to -u_{4R},\\
(d'_{1R}, d'_{2R}, d_{3R}) \to (d'_{1R}, d'_{2R}, d_{3R}),\\
(d_{1R}, d_{2R}) \to -(d_{1R}, d_{2R}).
\end{gathered} \tag{59}$$

Taking into account these symmetries we can rewrite the quark sector of Eq. (22) as

$$\begin{aligned}
-\mathcal{L}_{QY} = {} & \bar{Q}_{iL}G^u_{i\alpha}U_{\alpha R}\eta + \bar{Q}_{3L}G^b_{3\alpha}D_{\alpha R}\eta\\
&+ \bar{Q}_{iL}\left(G^d_{i\alpha}D_{\alpha R}\sigma + G'^d_{i\alpha}D_{\alpha R}\rho\right)\\
&+ \bar{Q}_{3L}\left(G'^t_{3\alpha}U_{\alpha R}\sigma^* + u_{4R}\rho^*\right) + \text{H.c.},
\end{aligned} \tag{60}$$

where $i = 1,2$, $\alpha = 1,2,3$, $a = 4,5$, and $U_{\alpha R} = u_{1R}, u_{2R}, u_{3R}$, $D_{\alpha R} = d'_{1R}, d'_{2R}, d_{3R}$, and $D_{aR} = d_{1R}, d_{2R}$. The mass matrices have the tensor product form in (57) and (58) next, they can be diagonalized with unitary matrices which are themselves tensor product of unitary matrices.

We see that, by imposing the symmetries (59), we have the GIM mechanism in the neutral currents coupled to the Z^0, separately, in the following sectors: (u_1,u_2,u_3), (d_1,d_2), and (d'_1,d'_2,d_3). The u_4 quark does not mix at all. Notice that it is the sector (u_1,u_2,u_3) which has the same neutral couplings as the u,c,t quarks of the standard model. The same occurs for the (d'_1,d'_2,d_3) which has the same neutral couplings as the d,s,b of the standard model. Notice also that d_1,d_2 have only vector currents coupled to Z^0.

On the other hand, there are FCNC's in the scalar sector since quarks with the same charge still gain mass from different Higgs fields. The Yukawa couplings of the charge $\frac{2}{3}$ quarks (u_1,u_2,u_3) to the neutral Higgs η^0 and σ^0, defined as

$$\eta^0 = v_\eta + \zeta \equiv v_\eta + \zeta_1 + i\zeta_2$$

and

$$\sigma^0 = v_\sigma + \zeta' \equiv v_\sigma + \zeta'_1 + i\zeta'_2,$$

can be written as

$$\left(1+\frac{\zeta}{v_\eta}\right)\bar{U}_\alpha M^u_{\alpha\alpha'}U_{\alpha'} + \left(\zeta' - \frac{v_\sigma}{v_\eta}\zeta\right)\bar{u}_{3L}G'^{t}_{3\alpha}U_{\alpha R}. \quad (61)$$

We recall that all fields in Eq. (61) are still phenomenological states, that is, linear combinations of the mass eigenstates. We will denote mass eigenstates u,c,t' in the u_1,u_2,u_3 sector. Then, from Eq. (61) we see that we will have $u \leftrightarrow c$ transitions.

This situation also occurs in the charge $-\frac{1}{3}$ sector composed of d'_1,d'_2,d_3 (with the respective mass eigenstates denoted d',s',b):

$$\left(1+\frac{\zeta'}{v_\sigma}\right)\bar{D}_\alpha M^d_{\alpha\alpha'}D_{\alpha'} + \left(\zeta - \frac{v_\eta}{v_\sigma}\zeta'\right)\bar{d}_{3L}G^b_{3\alpha}D_{\alpha R}. \quad (62)$$

We see that there are flavor changes in both charge sectors induced by the physical scalars.

However, we must notice that as the fields $\zeta_{1,2}$ and $\zeta'_{1,2}$ are linear combinations of mass eigenstates, say $\zeta_i = V_{ij}\phi_j$ and similarly with the ζ'_i fields. Next, suppose we choose one of the mass eigenstates ϕ_1. From Eq. (62) we will obtain terms such as

$$\left(V_{i1} - \frac{v_\sigma}{v_\eta}V'_{i1}\right)V_{bd'}\bar{b}_L d'_R\phi_1 + \text{H.c.}, \quad (63)$$

with i fixed. In the last equation all fields are already mass eigenstates, the matrix $V_{bd'}$ is a typical set of mixing angles among the charge $-\frac{1}{3}$ sector (d',s',b).

We see that interactions in Eq. (63) produce FCNC's in the (d',s',b) system, but not one of them produces the transition appearing in Eqs. (1)–(5). In the charge $\frac{2}{3}$ sector there are also interactions such as those in Eq. (63). In addition, these interactions do not imply strong constraints on the masses of the neutral scalars if conditions such as $V_{i1} - \frac{v_\sigma}{v_\eta}V'_{i1} \simeq 0$ are valid.

The quarks d_1,d_2 have the GIM mechanism in the coupling with Z^0, Z'^0 and also in the scalar sector. The respective mass eigenstate are denoted by d,s.

B. Models II and III

With respect to model II the respective couplings are given in Ref. [6]. In this case we have the same couplings for each charge sector; that is, we have the relation in Eq. (7) valid and when we redefine the quark fields as in Eq. (9) FCNC's appear with the coupling to the Z'^0 but not with those to the Z^0. The same is true for the case of model III. Here we will rewrite the coefficients calculated in Ref. [6]. Couplings with Z^0:

$$\begin{aligned}
&a_L(\nu)=1, \quad a_R(\nu)=0,\\
&a_L(l)=-1+2x, \quad a_R(l)=2x,\\
&a_L(u_1)=a_L(u_2)=a_L(u_3)=\frac{1}{6}g^2(x),\\
&a_R(u_1)=a_R(u_2)=a_R(u_3)=-\frac{2}{3}x,\\
&a_L(d_1)=a_L(d_2)=a_L(d_3)=-\frac{1}{6}(3-2x),\\
&a_R(d)=a_R(s)=a_R(b)=\frac{1}{3}x.
\end{aligned} \quad (64)$$

Couplings with Z'^0:

$$\begin{aligned}
&a'_L(\nu_l)=-\frac{1}{2\sqrt{3}}h(x), \quad a'_R(\nu_l)=0,\\
&a'_L(l)=-\frac{h(x)}{2\sqrt{3}}, \quad a'_R(l)=-\frac{1}{\sqrt{3}}h(x),\\
&a'_L(u_1)=-\frac{1}{2\sqrt{3}h(x)}, \quad a'_R(u_1)=-\frac{2}{\sqrt{3}}\frac{x}{h(x)},\\
&a'_L(d_1)=-\frac{1}{2\sqrt{3}h(x)}, \quad a'_R(d_1)=\frac{2x}{\sqrt{3}h(x)},\\
&a'_L(u_2)=a'_L(u_3)=\frac{1}{4\sqrt{3}}h(x),\\
&a'_R(u_2)=a'_R(u_3)=-\frac{2}{\sqrt{3}h(x)}x,\\
&a'_L(d_2)=a'_L(d_3)=\frac{1-2x}{\sqrt{3}h(x)},\\
&a'_R(d_2)=a'_R(d_3)=\frac{2x}{\sqrt{3}h(x)},
\end{aligned} \quad (65)$$

where $h(x)=(1-4x)^{\frac{1}{2}}$.

We see from Eqs. (64) and (65) that there is a GIM mechanism in the couplings with Z^0 but not in the couplings with Z'^0 as was stressed in Ref. [6]. It is also necessary in model II to introduce discrete symmetries if we want to prevent neutrinos from gaining mass but this will not be considered in this paper [15].

As in the case of model I [see Eq. (61)], we can verify that there are scalars coupled to $\bar{u}_R G^u_{1\alpha} U_{\alpha R}$ implying mixing among u, c, t and also scalars coupled to $\bar{d}_L G^d_{1\alpha} D_{\alpha R}$ producing mixing among d, s, b; but, as it will also appear in factors such as $V_{i1} - \frac{v_\sigma}{v_\eta} V'_{i1}$, this does not impose necessarily a strong lower bound on the neutral scalar masses.

VII. CONCLUSIONS

Following the generalization of the GIM mechanism of Ref. [5], we have considered three different representation contents in a theory of electroweak interactions with $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ gauge symmetry and we have studied in detail the neutral currents. However, it should be noticed that, in all these models we have considered, fields of the same charge belong to different representations and for this reason the mixings are neither pure "horizontal" nor pure "vertical" in the sense of Ref. [5]. For example there are mixings in the system d', s', b and these mass eigenstates are linear combinations of two SU(2) singlets, d'_1, d'_2 and one member of an SU(2) doublet d_3. We must recall that this choice of the representation content was necessary in order to avoid anomalies in the theory.

Here we will revise and comment on the FCNC effects and GIM mechanism in this kind of model. As in the standard model, all neutral couplings depend on the weak angle θ_W, which can be determined from several neutral currents processes, the W^+ and Z^0 masses and also from Z-pole observables such as the width and some asymmetries. It is well known that these experiments have such a level of precision that complete $O(\alpha)$ radiative corrections are mandatory. In particular some radiative corrections modify the tree-level expressions for neutral currents processes.

Let us start with model I. Table I summarizes the situation.

Notice that the mass eigenstates u, c, t' and d', s', b have the same neutral current couplings as the u, c, t and d, s, b, respectively, in the electroweak standard model. However, one quark with charge $\frac{2}{3}$, say t', and two quarks with charge $-\frac{1}{3}$, say d', s', can be chosen to have their masses depending mainly on the v_σ which is the VEV that it is responsible of the first symmetry breakdown; hence these quarks may be heavier than the others, as demanded by experimental data.

However in the charged currents the mixing is not the usual one. For example, in terms of the mass eigenstates, we have the interactions with the W^+ vector boson (which coincide with the charged vector boson of the standard model):

$$-\frac{g}{\sqrt{2}}(\bar{u}\,\bar{c}\,\bar{t}) \begin{pmatrix} U & 0 \\ 0 & 1 \end{pmatrix} \gamma^\mu \begin{pmatrix} d \\ s \\ b_\theta \end{pmatrix} W^+_\mu + \text{H.c.}, \tag{66}$$

where U is a 2×2 Cabibbo-like mixing matrix and $b_\theta = c_1 d' + c_2 s' + c_3 b$, c_i being appropriate combinations of mixing angles in the d', s', b sector. In the present model, we have also the charged vector boson V^+,

$$-\frac{g}{\sqrt{2}}(\bar{u}\,\bar{c}\,\bar{t}')U'\gamma^\mu \begin{pmatrix} d' \\ s' \\ b \end{pmatrix} V^+_\mu + \text{H.c.}, \tag{67}$$

and U' is an arbitrary 3×3 unitary matrix.

In this model the expression for the couplings a_L and a_R of quarks u, c, t' and d', s', b coincide with the respective a_L and a_R of the standard model but the quarks d and s have pure (at the tree level) vector neutral currents.

We can verify that this model is not inconsistent with present day experiments.

As an example, let us consider the deep inelastic neutrino scattering from approximately isoscalar targets. Experimentally the ratio $R_\nu \equiv \sigma^{\rm NC}_{\nu N}/\sigma^{\rm CC}_{\nu N}$ was measured where NC and CC denote neutral and charged current respectively. In the simplest approximation,

$$R_\nu = g_L^2 + g_R^2 r. \tag{68}$$

Assuming that there is a contribution from the d' quark other than those of the u, d and using the constants $a_{L,R}$ defined in Eq. (55), at the zeroth-order approximation, one has [16–18]

$$g_L^2 \equiv a_L^2(u) + a_L^2(d') + a_L^2(d) = \frac{1}{2} - x + \frac{2}{3}x^2 \simeq 0.304, \tag{69}$$

$$g_R^2 \equiv a_R^2(u) + a_R^2(d') + a_R^2(d) = \frac{2}{3}x^2 \simeq 0.036, \tag{70}$$

where we have used $x \equiv \sin^2\theta_W \simeq 0.232$. In (68) $r \equiv \sigma^{\rm CC}_{\bar{\nu}N}/\sigma^{\rm CC}_{\nu N}$ is the ratio of $\bar{\nu}$ and ν charged current cross section. Assuming the value of the parton model, $r \simeq 0.440$ we obtain that, in model I, $R_\nu \simeq 0.320$ which lies in the range of the measured values from several isoscalar targets [16].

Similarly, it is easy to see that, if only the quark d' gives an additional contribution to the Z^0 width, the partial width for the Z^0 to decay into massless fermions is

$$\Gamma(Z \to \bar{\Psi}_i \Psi_i) = \frac{C G_F M_Z^3}{6\sqrt{2}\pi}(V_i^2 + A_i^2).$$

For leptons $C = 1$ while for quarks $C = 3$ without QCD corrections, and $V^i = a_L^i + a_R^i$, $A^i = a_L^i - a_R^i$ with a_L and a_R defined in Eqs. (55). Considering contributions of leptons and of the quarks u, c, d, d', s, and b we obtain $\Gamma_Z \sim 2.600$ GeV (which is not too different from the value in the standard model without radiative corrections). The experimental value is $\Gamma_Z \simeq 2.487 \pm 0.010$ GeV [1]. The small discrepancy could be explained by taking into account radiative corrections and uncertainties coming from QCD. The forward-backward asymme-

TABLE I. FCNC effects in model I. Z^0 and Z'^0 are the lightest and the heaviest neutral boson and H^0 is a typical neutral scalar. The "yes" and "no" in the entries denote whether or not the FCNC exists.

Quark sector	Z^0	Z'^0	H^0
u, c, t'	No	Yes	Yes
t	No	No	No
d, s	No	No	No
d', s', b	No	Yes	Yes

try of quark pairs measured in e^+e^- processes also is sensitive to the weak angle, but in this case (as in the case of the width) it is necessary for use in the calculations of the effective weak angle at the Z^0 mass. Hence, radiative corrections in this model must be calculated.

Summarizing, we have shown that in model I, by imposing the discrete symmetry given in Eq. (59), we have separated the quark sector in two charge $\frac{2}{3}$, u, c, t', and t, and two charge $-\frac{1}{3}$ ones, d', s', b and d, s. In this situation the GIM mechanism works in each separate sector in the neutral currents coupled to the lightest neutral gauge bosons Z^0. This means that the new quark d' must not be too heavy since its contributions to the R_ν and Γ_Z are necessary to fit experimental data.

Another possibility is that the symmetry breaking of the SU(3)⊗U(1) symmetry occurs at an energy scale which is not too different from the Fermi scale. In this case all exotic gauge bosons are heavier but not much more than the $W^\pm, Z^0$ gauge bosons of the standard model implying a rich phenomenology. However, our expressions are not valid in this situation.

Next, in models II and III, the FCNC in the gauge sector are restricted mainly to the Z'^0 exchange. See Eqs. (64) and (65). This boson also gains mass from the VEV which induce the first symmetry breaking. In this case, however, constraints on the FCNC coming from the K^0-$\bar{K}^0$ system imply that this energy scale is greater than 8 TeV; that is, $M_{Z'} > 40$ TeV [6]. In these models since there are only new quarks with charge $\frac{5}{3}$, $-\frac{4}{3}$, the phenomenology of the usual charge sectors, i.e., $\frac{2}{3}, -\frac{1}{3}$, decouples from that of the exotic sector and it is automatically consistent with the present observation if the VEV which is in control of the breaking of the $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ symmetry are fixed by the FCNC contributions of the Z'^0 neutral boson.

In the three models, FCNC's also appear in the Yukawa interactions; however, these involve some of the new quarks; therefore there are no strong constraints coming from experimental data. In addition, the suppression factors as in Eq. (63) imply that the masses of the neutral Higgs scalars are not necessarily very high in order to have the appropriate suppression of the FCNC.

The new gauge bosons, such as V^+, X^0 in model I, only couple one of the light (u, c, t, b) to one of the heavy (d', s', t') quarks; they cannot be produced in current experiments. FCNC's induced by box diagrams by the V^+ boson involve also the heavy quarks, and for this reason it does not imply strong constraints on its masss. Interfamily lepton number violations provide weaker bounds on these masses (and on that of the X^0 boson) also. The same happens with Z'^0 and exotic gauge bosons in models II and III [6]. Even the neutrino counting experiment $e^+e^- \to \nu^c\nu^c\gamma$ is not so restrictive, and since the GIM mechanism works in the leptonic sector, even if the neutrinos become massive, processes such as $\mu \to e\nu_e^c\nu_\mu^c$ will be very suppressed. In fact, these processes imply masses of all the new gauge bosons of the order of two or three times the mass of the charged gauge boson in the standard electroweak model.

We think that these models, being consistent with low energy data and because of the color degree of freedom and the number of families that are related in order to be anomaly-free models, could imply unforeseen options in model building. New physics can arise at not too high energies.

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APPENDIX

As we have used $\bar{\lambda}$ matrices, for the **3*** antitriplet representation of the SU(3) group, which are not usually found in the literature, we will consider them explicitly in this Appendix [19].

The up and down operators in the triplet representation are given by

$$E_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad E_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{A1}$$

$$E_{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad E_{-2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$E_{-3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{A2}$$

with the commutation relations

$$\begin{aligned} &[E_1, E_3] = [E_1, E_{-2}] = [E_2, E_3] = 0, \\ &[E_1, E_{-3}] = -E_{-2}, \quad [E_1, E_2] = E_3, \quad [E_2, E_{-3}] = E_{-1}. \end{aligned} \tag{A3}$$

The Cartan subalgebra generators are represented by

$$H_1^\lambda = \mathrm{diag}(1, -1, 0), \quad H_2^\lambda = \mathrm{diag}(0, 1, -1). \tag{A4}$$

The SU(3) algebra is generated by a real linear combination of the matrices H_k, $(E_l + E_{-l})$, and $i(E_l - E_{-l})$; $k = 1, 2; l = 1, 2, 3$. Then the so-called Gell-Mann matrices λ are

$$
\begin{aligned}
\lambda_1 &\equiv E_1 + E_{-1} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & 0 \end{pmatrix}, \\
\lambda_2 &\equiv -i(E_1 - E_{-1}) = \begin{pmatrix} \sigma_2 & 0 \\ 0 & 0 \end{pmatrix}, \\
\lambda_3 &\equiv H_1^\lambda = \begin{pmatrix} \sigma_3 & 0 \\ 0 & 0 \end{pmatrix}, \\
\lambda_4 &\equiv E_3 + E_{-3} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\
\lambda_5 &\equiv -i(E_3 - E_{-3}) = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \\
\lambda_6 &\equiv E_2 + E_{-2} = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_1 \end{pmatrix}, \\
\lambda_7 &\equiv -i(E_2 - E_{-2}) = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_2 \end{pmatrix}, \\
\lambda_8 &\equiv \frac{1}{\sqrt{3}}(H_1^\lambda + 2H_2^\lambda) = \mathrm{diag}\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}}\right),
\end{aligned} \tag{A5}
$$

where σ_i are the usual 2×2 Pauli matrices. The $\lambda_a, a = 1, 2, ..., 8$ have the normalization

$$\mathrm{tr}(\lambda_a \lambda_b) = 2\delta_{ab}, \tag{A6}$$

and the closed algebra

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = i f_{abc} \frac{\lambda_c}{2} \tag{A7}$$

with the following nonvanishing values for the totally antisymmetric structure constant f_{abc}:

$$
\begin{aligned}
& f_{123} = 1, \\
& f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2}, \\
& f_{458} = f_{678} = \frac{\sqrt{3}}{2}.
\end{aligned} \tag{A8}
$$

As the SU(3) is a rank-2 group, the diagonal matrices λ_3 and λ_8 are such that

$$\left[\frac{\lambda_3}{2}, \frac{\lambda_8}{2}\right] = 0, \tag{A9}$$

and $\lambda_{1,2,3}$ are a representation of the subgroup SU(2) algebra.

The other fundamental representation of SU(3) of dimension 3, i.e., the antitriplet representation $\bar{\lambda}$ can be obtained by the substitution

$$
\begin{aligned}
E_{\pm 1} &\longrightarrow E_{\pm 2}, \\
E_{\pm 2} &\longrightarrow E_{\pm 1}, \\
E_{\pm 3} &\longrightarrow -E_{\pm 3},
\end{aligned} \tag{A10}
$$

and

$$H_1^{\bar{\lambda}} = H_2^\lambda, \quad H_2^{\bar{\lambda}} = H_1^\lambda. \tag{A11}$$

We have explicitly

$$
\begin{aligned}
& \bar{\lambda}_1 = E_2 + E_{-2} = \lambda_6, \; \bar{\lambda}_2 = -i(E_2 - E_{-2}) = \lambda_7, \\
& \bar{\lambda}_3 = H_1^{\bar{\lambda}} = \mathrm{diag}(0, 1, -1), \; \bar{\lambda}_4 = -(E_3 + E_{-3}) = -\lambda_4, \\
& \bar{\lambda}_5 = i(E_3 - E_{-3}) = -\lambda_5, \; \bar{\lambda}_6 = E_1 + E_{-1} = \lambda_1, \\
& \bar{\lambda}_7 = -i(E_1 - E_{-1}) = \lambda_2, \\
& \bar{\lambda}_8 = \frac{1}{\sqrt{3}}(H_1^{\bar{\lambda}} + 2H_2^{\bar{\lambda}}) = \mathrm{diag}\left(\frac{2}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right).
\end{aligned} \tag{A12}
$$

Using Eqs. (A12) in Eq. (12) we can verify that the leptons are in antitriplets and that the Higgs antitriplets are

$$
\begin{aligned}
& \eta^* = \begin{pmatrix} \eta_1^+ \\ \eta_2^+ \\ \bar{\eta}^0 \end{pmatrix} \sim (\mathbf{3}^*, \tfrac{2}{3}), \quad \rho^* = \begin{pmatrix} \bar{\rho}_2^0 \\ \bar{\rho}_1^0 \\ \rho^- \end{pmatrix}, \\
& \sigma^* = \begin{pmatrix} \bar{\sigma}_2^0 \\ \bar{\sigma}_1^0 \\ \sigma^- \end{pmatrix} \sim (\mathbf{3}^*, -\tfrac{1}{3}).
\end{aligned} \tag{A13}
$$

It is well known that the anomaly is proportional to [20]

$$\mathcal{A}(\mathbf{3}) \propto \mathrm{Tr}(\{\lambda_a, \lambda_b\}, \lambda_c)$$

for the triplet, and

$$\mathcal{A}(\mathbf{3}^*) \propto \mathrm{Tr}(\{\bar{\lambda}_a, \bar{\lambda}_b\}, \bar{\lambda}_c)$$

for the antitriplet representation. Using the λ's and $\bar{\lambda}$'s given above it is straightforward to verify that

$$\mathcal{A}(\mathbf{3}) = -\mathcal{A}(\mathbf{3}^*). \tag{A14}$$

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