UCRL - 76301
PREPRINT
Conf-741215--1

LAWRENCE LIVERMORE LABORATORY
University of California/Livermore, California

NEUTRINO FLOW AND THE COLLAPSE OF STELLAR CORES

James R. Wilson

December 27, 1974

This paper was prepared for submission to
The Seventh Texas Symposium on
Relativistic Astrophysics

NEUTRINO FLOW AND THE COLLAPSE OF STELLAR CORES*

James R. Wilson

Introduction

Neutral current theories of neutrino interactions predict a scattering cross section for heavy nuclei proportional to th square of the nuclear mass (Freedman 1974). Wilson (1974) has shown that this high cross section may make it possible for the hot neutron star newly formed in stellar collapse to emit enough neutrinos to blow off the iron envelope which would surround the newly formed neutron star. The present work is an effort to repeat the above calculations with a better model for the neutrino interactions.

Arnett and Scramm (1975) and Mazurek (1974) have criticized Wilson (1974) for not including degeneracy effects of electrons and neutrinos and for not imposing the conservation of lepton number. The neutrino cross sections in Wilson (1974), while of the correct magnitude, did not fit to any particular neutrino model.

Equation of State

The material is assumed to be composed to two principle species: heavy nuclei of weight A and free baryons. The number and weight of the free baryons is determined as follows. From the density of material A_{eq} and Z_{eq} (the A and Z are the atomic weight and charge for material in its lowest energy state at zero temperature) are selected. A and Z are then found as a function of time from

$$\dot{A} = \frac{1}{\tau}(A_{eq}-A)$$
$$\dot{Z}_A = \frac{1}{\tau}(Z_{eq}-Z_A) \qquad (1)$$

* This work was performed under the auspices of the United States Atomic Energy Commission.

Using the values of A and Z_A found from equation (1) and the energy of formation of the nucleus of weight A_{eq} a Saha type equation is solved for the fraction f_A of baryons in heavy nuclei. The mean charge, Z, per free baryon is found from the neutrino transport equations and will be discussed in the next section. The pressure and energy are calculated by assuming all the particles are perfect Fermi gases.

Neutrino equations

In the capture of neutrinos by heavy nuclei given by equation (1) the emitted neutrinos are assumed to have an energy equal to the chemical potential, ϕ, of the electrons. These neutrinos are added to the neutrino field, F_e, and no temperature change is made.

Only three other neutrino absorption processes are considered at this time: $P + e^- \leftrightarrow n + \nu$, $n + e^+ \leftrightarrow P + \bar{\nu}$, and $e^+ + e^- \leftrightarrow \nu + \bar{\nu}$ (plasma neutrino production is included in the latter process). The energy exchange between neutrinos and electrons from elastic scattering is treated by a Fokker-Planck type equation. This treatment assumes $\sqrt{\overline{(\Delta\nu/\nu)^2}} << 1$. In the present circumstances $\sqrt{\overline{(\Delta\nu/\nu)^2}} \approx 1/2$. The diffusion coefficient uses a transport mean free path, λ, that includes the above processes (except pair production) together with the nuclear scattering which is assumed to have a cross section proportional to A^2. The neutrino degeneracy effects in the pair production formula are not quite correct; however, they are of such form that neutrino states will never be over filled.

The equation for density of electron neutrino energy is:

$$\dot{F}_e(r,\nu,t) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_e \frac{\partial F_e}{\partial r}\right) - \dot{Z}_A\,\delta(\nu-\phi) \tag{2}$$
$$+ n_b\,\sigma_{p+e^-}\left\{ Z n_{e^-}\nu - 2F_e(1-Z)\left(1 - \frac{1}{e^{\frac{\nu-\phi}{T}}+1}\quad\frac{1-2Z}{1-Z}\right)\right\}$$

$$+\left(\frac{\dot{E}^{e}_{pair}}{T^{9}}\right)\left\{\frac{\nu^4T^4}{e^{\nu/T}+1} - F_e\nu\left(\frac{1}{1+e^{-\nu/T}}\int\frac{\bar{F}_e d\bar{\nu}}{6} + \frac{T^4}{e^{\nu}/_{T}+1}\right)\right\}\frac{1}{48}$$

$$+\nu\frac{\partial}{\partial\nu}\left\{\sqrt{\left(\frac{\Delta\bar{\nu}}{\nu}\right)^2}\frac{1}{\tau_c}\left(T\left(\frac{\partial F_e}{\partial\nu} - 3F_e\right) + F_e\left(1-\frac{F_e}{\nu^3}\right)\right)\right\}$$

$$+\frac{V D_e}{\lambda_e}\frac{\partial F_e}{\partial r} - \frac{1}{r^2}\frac{\partial}{\partial r}\left(Vr^2 F_e\right)$$

Where D_e is diffusion coefficient, ν is neutrino energy, ϕ is electron chemical potential, n_b is density of free baryons, M_e is density of electrons of energy ν, $\bar{F}_e$ is density of electron antineutrion energy, $\dot{E}^{e}_{pair}$ is the energy production rate for the emission of electron neutrino pairs, T is the material temperature, τ_{sc} is the electron-neutrino scattering time, and V is the material velocity. The energy difference between neutrons and protons is ignored. Note all numerical factors have been suppressed. The diffusion coefficient is given by:

$$D = \frac{\lambda}{3 + \lambda\left|\frac{\partial F}{\partial r}/F\right|}$$

It has been assumed that all pairs are emitted in the form $\frac{\dot{E}_{pair}}{T^9}\frac{\nu^4T^4}{e^{\nu}/T+1}$ where $\dot{E}_{pair}$ is the total energy emission rate. The readsorption is then taken so that detailed balance will obtain. Hansen (1968) fits to pair production are used for $\dot{E}_{pair}$ for the old C_{VA} theory and then these fits are modified by the prescriptions of Dicus (1974), Arnett and Schramm (1975).

Similarly the equation for antineutrino density F_e is given by

$$(3)\quad \dot{\bar{F}}_e(r,\nu,t) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\bar{D}_e\frac{\partial\bar{F}_e}{\partial r}\right) + \frac{\bar{D}_e}{\bar{\lambda}_e}\frac{\partial\bar{F}_e}{\partial r} - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\bar{F}_e\right)$$

$$+ n_b \sigma_{n+e+} \left|(1-z)\ n_e + \nu - 2\bar{F}_e\ z \left(1 + \frac{1}{e^{\frac{\nu+\phi}{\tau}} + 1} \left(\frac{1-2z}{z}\right)\right)\right|$$

$$+ \left(\frac{\dot{E}^e_{pair}}{T^9}\right)\left|\frac{\nu^4 T^4}{e^{\nu}/T + 1} - \bar{F}_e \nu \left(\frac{1}{1 + e^{-\nu/T}} \int \frac{F_e \phi_\nu}{6} + \frac{T^4}{e^{\nu}/T + 1}\right)\right| \frac{1}{48}$$

$$+ \nu \frac{\partial}{\partial_\gamma} \left(\sqrt{(\Delta\nu/_\nu)^2}\ \frac{1}{\tau_{sc}} \left(T\left(\frac{\partial \bar{F}_e}{\partial_\nu} - 3\frac{\bar{F}_e}{\nu}\right) + \bar{F}_e\left(1 - \bar{F}_e/\nu^3\right)\right)\right)$$

Mu neutrinos are only made by pair production so the equations for mu neutrinos are simply:

$$\dot{F}_\mu = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_\mu \frac{\partial F_\mu}{\partial r}\right) + \frac{V D_\mu}{\gamma_\mu} \frac{\partial F_\mu}{\partial r} - \frac{1}{r^2} \frac{\partial}{\partial r} (\nabla r^2 F_\mu)$$

$$+ \left(\frac{\dot{E}^\mu_{pair}}{T^9}\right)\left\{\frac{\nu^4 T^4}{e^{\nu}/T + 1} - F_\mu \left(\frac{1}{\lambda + e^{-2}/_T} \int \frac{\bar{F}_\mu \phi_\nu}{6} + \frac{T^4}{e^{\nu}/T + 1}\right)\right\}$$

Dicus (1972) is used to obtain the relation of $\dot{E}^\mu$ to $\dot{E}^e$ for pairs.

The above equations for neutrino changes lead to the following temperature changes.

$$\rho\ C_V \dot{T} = \int \left[\sigma_{P + e^-} \left\{ zn_e - \nu - (1-z)\ 2F_e \left(1 - \frac{1}{e^{\frac{\nu-\phi}{T}} + 1}\left(\frac{1-2z}{1z}\right)\right)\right\} (\phi - \nu)\right.$$

$$- \sigma_{n+e+} \left|(1-z)\ n_e + \nu - z\ 2\bar{F}_e \left(1 + \frac{1}{e^{\frac{\nu+\phi}{T}} + 1}\left(\frac{1-2z}{z}\right)\right)\right| (\phi + \nu)$$

$$- \left(\frac{\dot{E}^e_{pair}}{48\ T^9}\right)\left\{\frac{2\ \nu^4 T^4}{e^{\nu}/T + 1} - \frac{\left(F_e + \bar{F}_e\right) T^4}{e^{\nu}/T + 1} - \frac{1}{6\ (1 + e^{-\nu}/T)}\left(F_e \int \bar{F}_e d_{\nu'} + \bar{F}_e \int F_e d_{\nu'}\right)\right\}$$

$$\left. - \left(\frac{\dot{E}^\mu_{pair}}{48\ T^9}\right)\left\{\frac{2\ \nu^4 T^4}{e^{\nu}/T + 1} - \frac{\left(F_\mu + \bar{F}_\mu\right)}{e^{\nu}/T + 1} T^4 - \frac{1}{6\ (1 + e^{-\nu}/T)}\left(F_\mu \int \bar{F}_\mu d_{\nu'} + \bar{F}_\mu \int F_\mu d_{\nu'}\right)\right\}\right] d\nu$$

$$+ \left(P + \frac{\partial E}{\partial v}\right) \dot{\rho}/\rho^2$$

$$+ \int \nu \frac{\partial}{\partial \nu}\left\{\sqrt{\overline{(\Delta\nu/\nu)^2}}\ 1/\tau_{sc}\left(T\left(\frac{\partial F_e}{\partial \nu} - \frac{3F_e}{\nu}\right) + F_e\left(1 - \frac{F_e}{\nu^3}\right)\right)\right\} d\nu$$

$$+ \int \nu \frac{\partial}{\partial \nu}\left\{\sqrt{\overline{(\Delta\nu/\nu)^2}}\ 1/\tau_{sc}\left(T\left(\frac{\partial \overline{F}_e}{\partial \nu} - \frac{3\overline{F}_e}{\nu}\right) + \overline{F}_e\left(1 - \frac{\overline{F}_e}{\nu^3}\right)\right)\right\} d\nu$$

and the following composition changes

$$\dot{z} = -\int \Bigg[\sigma_{P + e}\left\{z\, n_{e-}\, \nu - (1 - z)\, 2\, F_e\left(1 - \frac{1}{e^{\frac{\nu - \phi}{T}} + 1}\left(\frac{1 - 2z}{1 - z}\right)\right)\right\}$$

$$- \sigma_{n + e^+}\left\{(1 - z)\, n_{e+}\, \nu - 2z\, \overline{F}_e\left(1 + \frac{1}{e^{\frac{\nu + \phi}{T}} + 1}\left(\frac{1 - 2z}{z}\right)\right)\right\}\Bigg] \frac{d\nu}{\nu}$$

These equations together with the hydrodynamic equations are solved on a computor using finite difference approximations. Cross sections are from Weinberg theory, Dicus (1972) and Freedman (1974).

Results

The initial stellar model used is a model given by Barkat (same as in Wilson (1974). It consists of an iron core of 1.5 solar masses with silicon and carbon envelope of low density. The temperatures are taken a little under equilibrium values so that collapse starts immediately. Two calculations have been made so far. In the first the neutrino couplings were determined by the Weinberg theory with a coupling constant $a_o = .45$. In the second calculation all cross sections remained the same except the nuclear scattering cross section was raised a factor of 6. In neither calculation did an explosion result. From figures 1a and b it can be seen that shock wave from the bounce at a time of .23 seconds does not proceed out very far. The bounce was considered important in previous calculations for starting the material outward.

Figures 2 a, b show the central temperatures, densities, compositions (Z and f_A), chemical potentials and the liminosities at the outside. The density and temperature at bounce are a low. The previous calculations (Wilson 1974) bounced with central densities of about 3×10^{13} gm/cc and temperatures of 30 MeV. The luminosity is less than half the luminosity of the previous calculations. The spectrum is about 30% cooler than before, see figures 3 a, b. With a scattering cross section of $1.7 \times 10^{-44}\ \nu^2_{MeV}\ A^2\ cm^2$ and a mean neutrino energy of 14 MeV and A = 56 as found in these calculations the luminosity necessary to overcome gravity (Eddington limit) is 6.6×10^{53} ergs/sec. From figure 2 we can see that the luminosity is too low to produce a radiation driven blow off. At early times the electron neutrinos from the electron captures are the dominant luminosity source (see figure 4) later antineutrinos become important but still considerably less important than neutrinos (see figure 5). At late times the core is very opaque so that mu neutrinos with their smaller cross sections are becoming the dominate source of luminosity. While only 10% of the neutron star binding energy has been emitted at the end of the calculations, since the luminosity is not rising rapidly, it has been assumed nothing will happen later.

Discussion

These calculations were performed just before the meeting and so there is still a chance of error. These results should be considered preliminary. Since an earlier model gave an explosion and the present calculation came close to producing an explosion, it is necessary to do the calculations very carefully.

It also should be remembered that at present the theory of neutral currents allow several possibilities. The model of Freedman (1974) yields the weakest nuclear scattering. For comparison, Freedman's model requires a luminosity of about 1 x 10^{54} ergs/sec to overcome gravity. A model proposed by Sakurai (1974) would require about one quarter that luminosity. Adler's (1974) would require about half again that luminosity. Barshay's (1974) light scaler meson coupling model yields the same luminosity requirement as Freedman for gradient coupling and about L = 10^{50} ergs/sec for simple coupling. Bernabeu (1974) has a model in which the required luminosity is only 10^{52} ergs/sec. This seems like a large variety of models. However, as the nuclear scattering cross section is increased a point is reached where further increases do no good since the luminosity of the neutron core will be limited by nuclear scattering. Future calculations should find this limit.

Work was done in collaboration with R. Couch, S. Cochran, and J. Le Blanc, with the initial stellar model provided by Z. Barkat.

Figure Captions

1 a, b - The lines are the radius time trajectories of every fifth zone in the calculation. a) is example with $a_o^2 = .2$. b) is example with $a_o^2 = .2$ for all cross sections except nuclear scattering which uses $a_o^2 = 1.2$ in Freedman's formula for coherent scattering.

2 a, b - Central temperature T, central density, ρ, central fraction of heavy nuclei, f_A, central charge of free baryons z, and luminosity L at outside of star as functions of time. a and b have same meaning as in figure 1.

3 a, b - Time integrated neutrino fluxes outside the star as functions of the neutrino energy. a and b have same meaning as in figure 1.

4 - The specta of the various neutrinos in the central zone at a time of .226 seconds for example a of figure 1.

References

Adler, S. L., Preprint to be published in Physical Review D (1975).

Arnett, D. and Schramm, D., Preprint to be published in Ap. J. (1975).

Barshay, S., Phy. Rev. D 10, 2313 (1974).

Bernabeu, J., Lettere Nuovo Cimento 10 (1974) 329.

Dicus, D. A., Phys. Rev. D, Vol. 6, p. 941, (1972).

Freedman, D. Z., Phys. Rev. D 5, 1389, (1974).

Hansen, Astrophys. and Sp. Sci. 1 (1968) p. 499.

Mazurek, T., Astrophys. and Sp. Sci. (in press).

Sakurai, J. J. and Urrutia, L. F., UCLA report 74/TEP/16 (1974).

Wilson, J. R. Phy. Rev. Lett. 32, 849, (1974).

Figure 1a

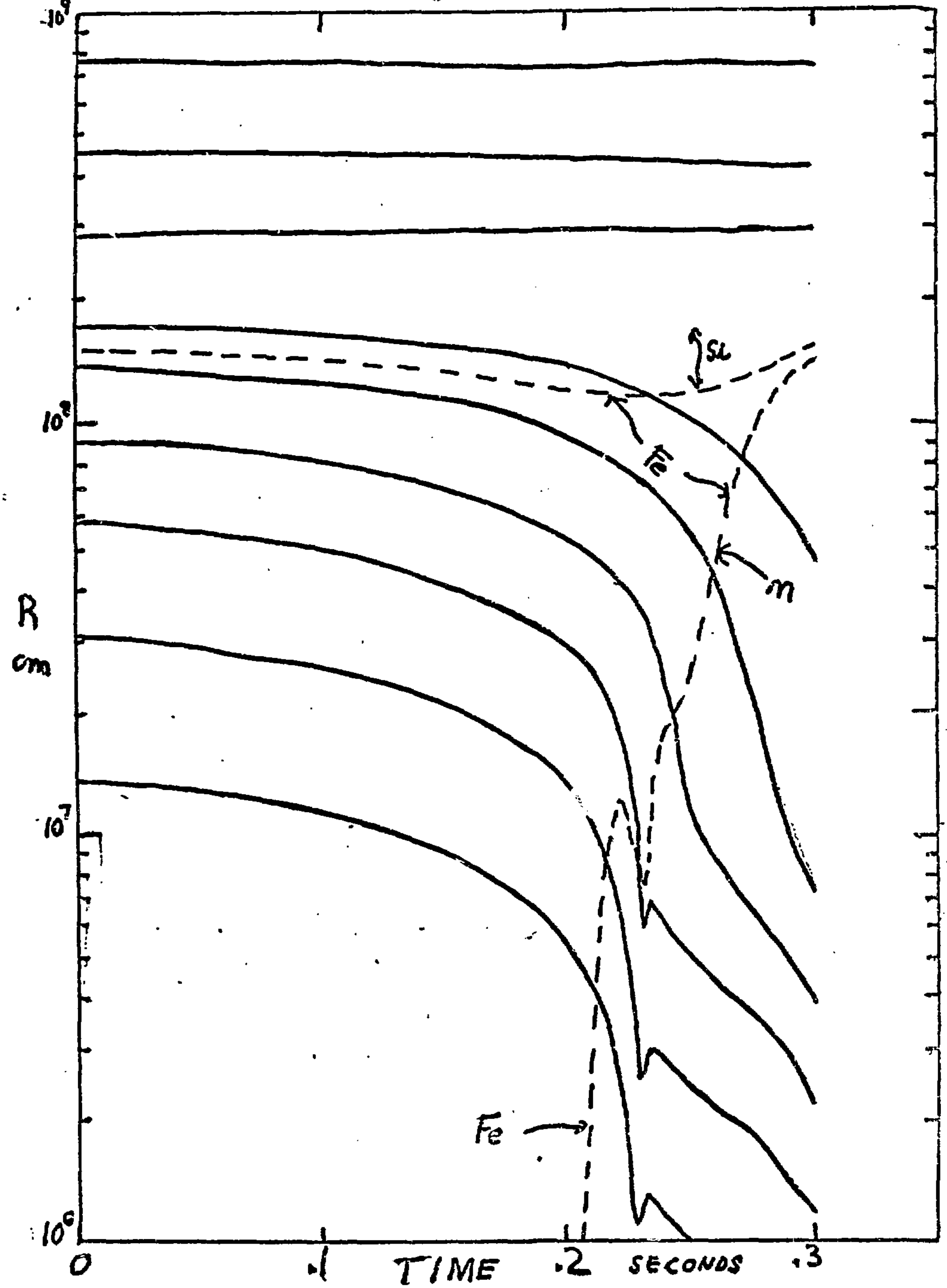
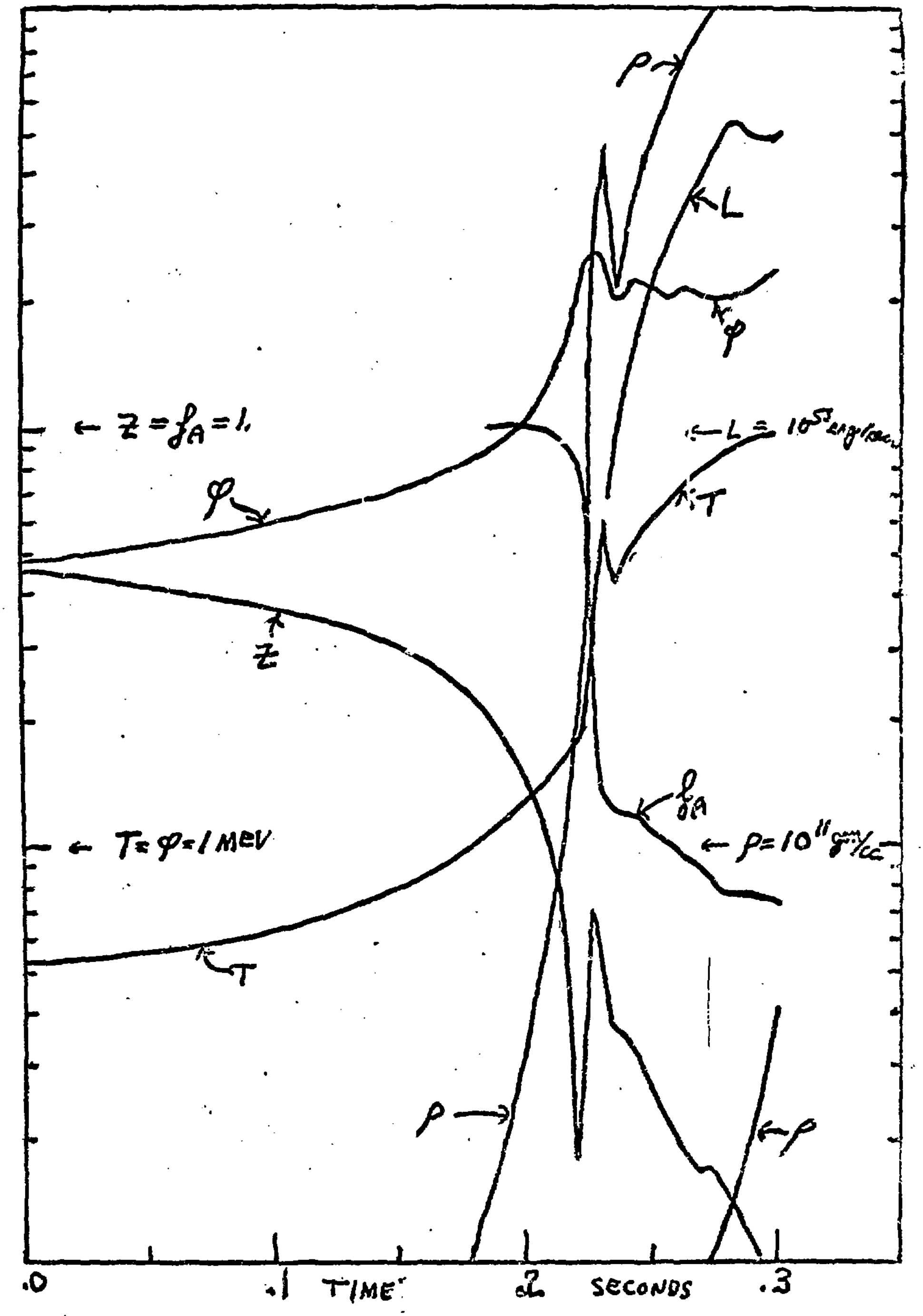
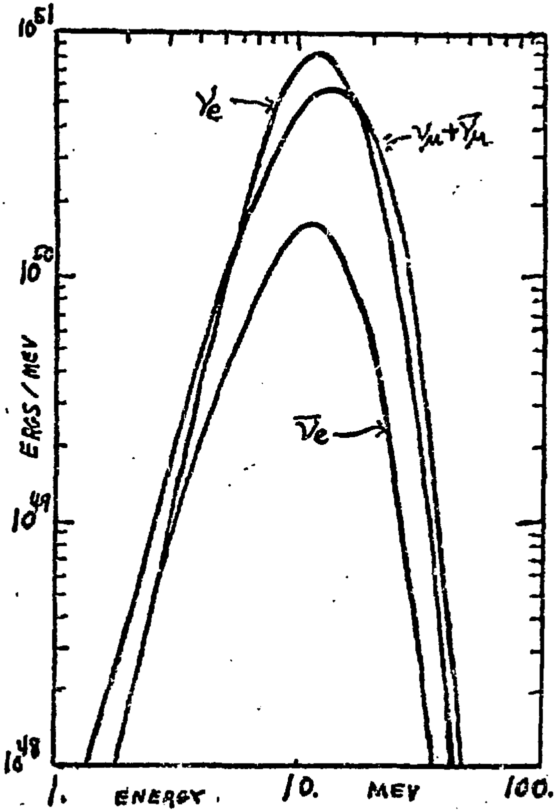


Figure 2a



ρ→
←L
←φ
← Z = f_A = 1.
←L = 10^53 erg/sec
φ→
←T
↑ Z
f_A
← T = φ = 1 MEV
← ρ = 10^11 gm/cc
←T
ρ→
←ρ
.0
.1 TIME
.2
SECONDS
.3

Figure 3a



10^{51}
10^{50}
10^{49}
10^{48}
ERGS/MEV
ν_e
$\nu_\mu + \bar{\nu}_\mu$
$\bar{\nu}_e$
1.
ENERGY
10.
MEV
100.

Figure 1b

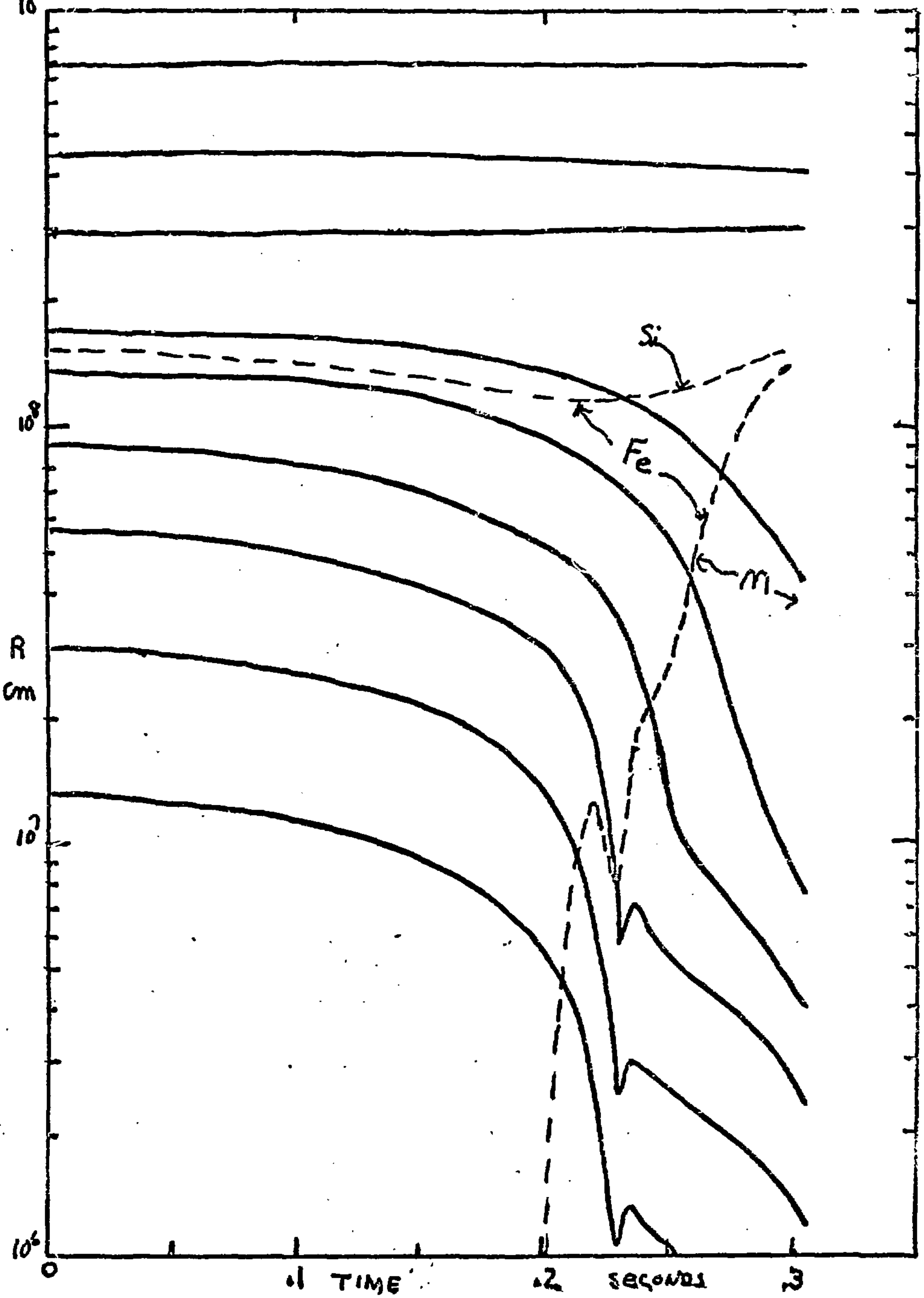


Figure 2b

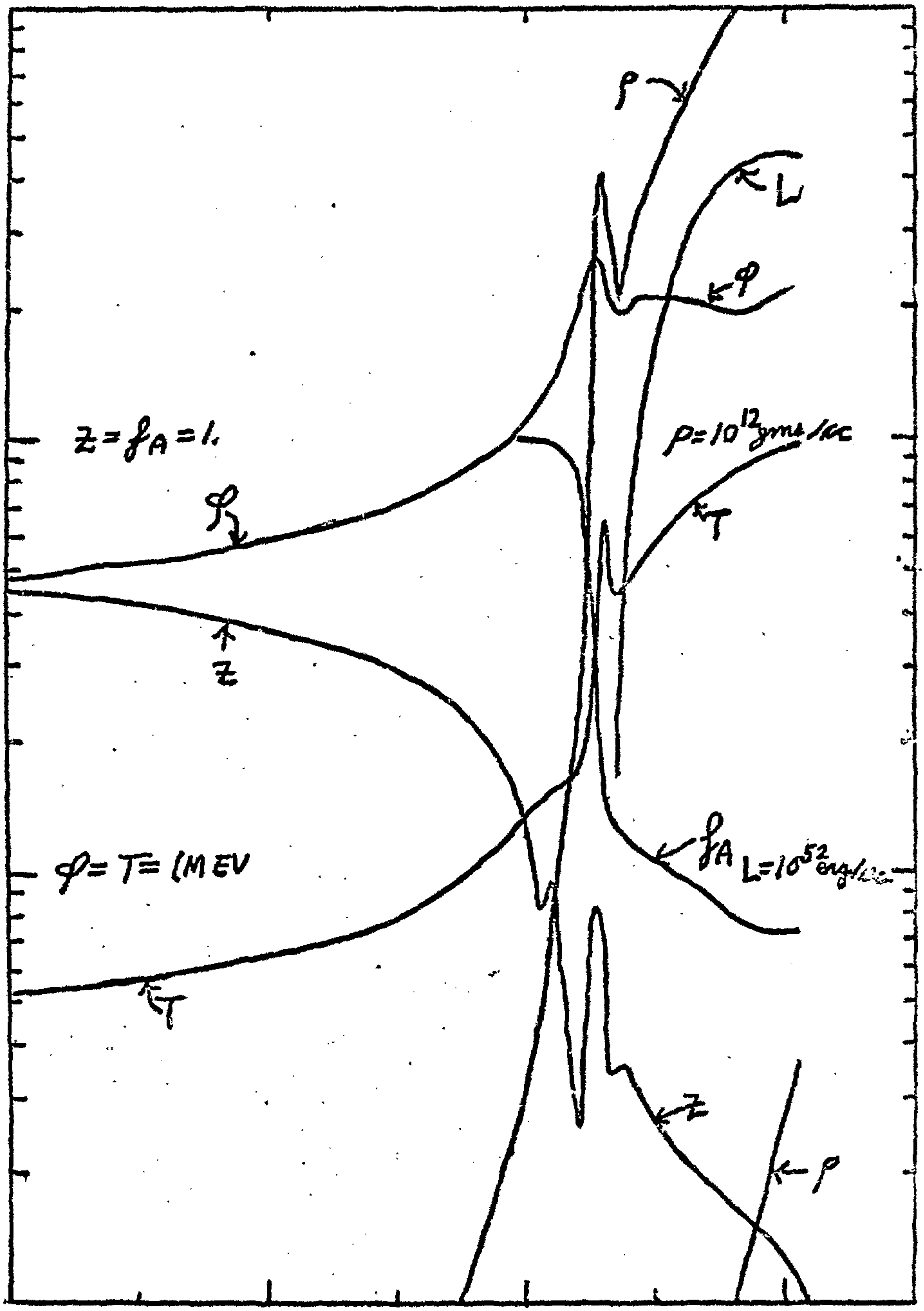
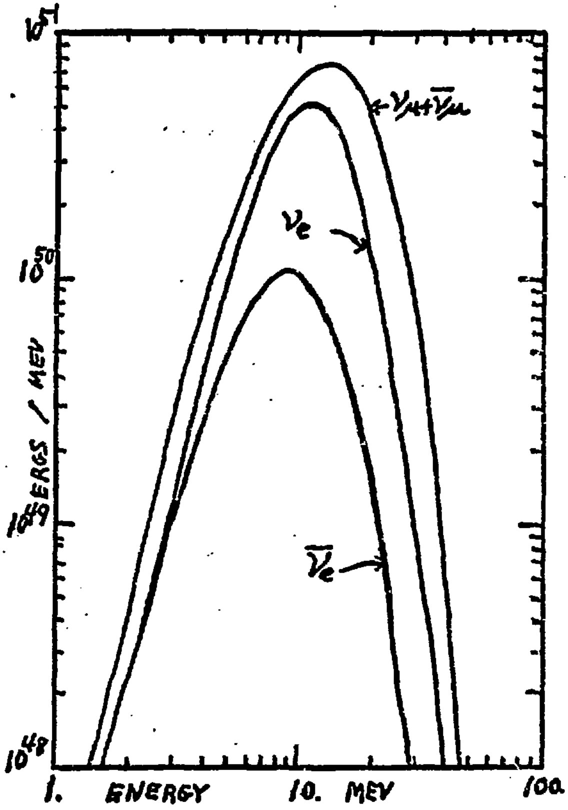
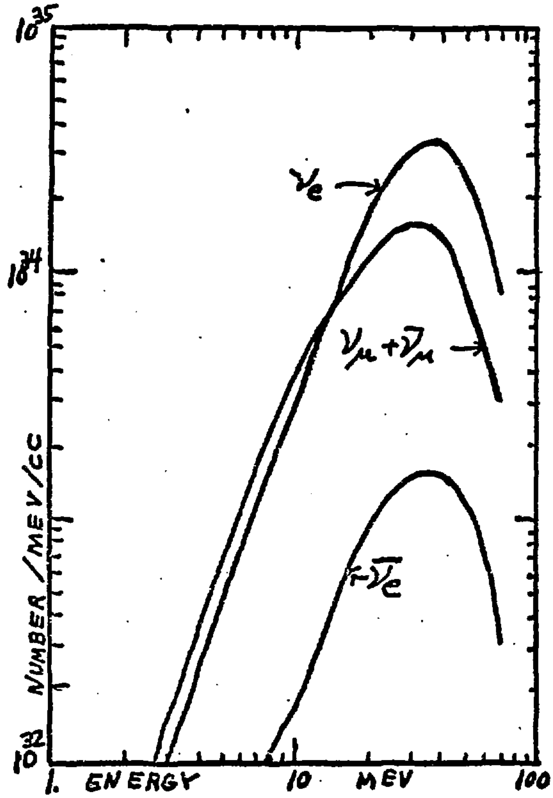


Figure 3b



10^{51}
10^{50}
10^{49}
10^{48}
ERGS / MEV
$\nu_\mu + \bar{\nu}_\mu$
ν_e
$\bar{\nu}_e$
1.
ENERGY
10.
MEV
100.

Figure 4



10^{35}
10^{34}
10^{32}
NUMBER /MEV/CC
1.
ENERGY
10
MEV
100
ν_e
$\nu_\mu + \bar{\nu}_\mu$
$\bar{\nu}_e$