

Neutrino Majorana Masses from String Theory Instanton Effects

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Abstract

Finding a plausible origin for right-handed neutrino Majorana masses in semirealistic compactifications of string theory remains one of the most difficult problems in string phenomenology. We argue that right-handed neutrino Majorana masses are induced by non-perturbative instanton effects in certain classes of string compactifications in which the $U(1)_{B-L}$ gauge boson has a Stückelberg mass. The induced operators are of the form $e^{-U} \nu_R \nu_R$ where U is a closed string modulus whose imaginary part transforms appropriately under $B-L$. This mass term may be quite large since this is not a gauge instanton and $Re U$ is not directly related to SM gauge couplings. Thus the size of the induced right-handed neutrino masses could be a few orders of magnitude below the string scale, as phenomenologically required. It is also argued that this origin for neutrino masses would predict the existence of R-parity in SUSY versions of the SM. Finally we comment on other phenomenological applications of similar instanton effects, like the generation of a μ -term, or of Yukawa couplings forbidden in perturbation theory.

1 Introduction

In recent years our experimental knowledge about neutrino masses has substantially improved. The evidence from solar, atmospheric, reactor and accelerator experiments indicates that neutrinos are massive. The observed structure of masses and (large) mixings of neutrinos is quite peculiar and different from their charged counterparts. The simplest explanation for the smallness of neutrino masses is the celebrated see-saw mechanism [1]. If there are right-handed neutrinos ν_R^a with large Majorana masses M_M and standard Dirac masses M_D , the lightest eigenvalues have masses of order

$$m_\nu \simeq \frac{M_D^2}{M_M}. \quad (1.1)$$

which are of order the experimental results for M_D of order of standard charged leptons and $M_M \propto 10^{10} - 10^{13}$ GeV.

Dirac masses are expected to be given by standard Yukawa couplings so from this point of view they are tied down to the usual flavor problem of the SM, the not yet understood structure of fermion masses or mixings. On the other hand the origin of the large Majorana masses for right-handed neutrinos is even more mysterious. A natural setting for such masses seems to be left-right extensions of the SM like $SO(10)$ unification. However in this case the appropriate Higgs fields leading to Majorana masses have dimension 126, making the models unattractive. Alternatively one may resort to non-renormalizable couplings to 16-plets of Higgs fields, but in SUSY models this generically breaks R-parity spontaneously, giving rise to Baryon- and Lepton-number violation (and hence fast proton decay) unless one invokes extra protecting symmetries.

The situation for Majorana masses in the case of string theory is worst (for a recent discussion see e.g. [2] and references therein) because there is less freedom to play around with models. One of the reasons is that Higgs fields with the appropriate quantum numbers to couple to the $\nu_R \nu_R$ bilinears at the renormalizable level do not appear in any of the semirealistic models constructed up to now. Although such couplings may appear at the non-renormalizable level, it is still typically problematic to obtain them without at the same time inducing (at least in the SUSY case) dangerous B/L-violating couplings. We think it is fair to say that there is at present no semirealistic model in which a large Majorana mass for the right-handed neutrinos appears in a natural way.

In this paper we present an elegant mechanism for the generation of right-handed neutrino masses in string theory. We claim that, in a (presumably large but) restricted class of string compactifications with semirealistic SM or MSSM light spectrum, there

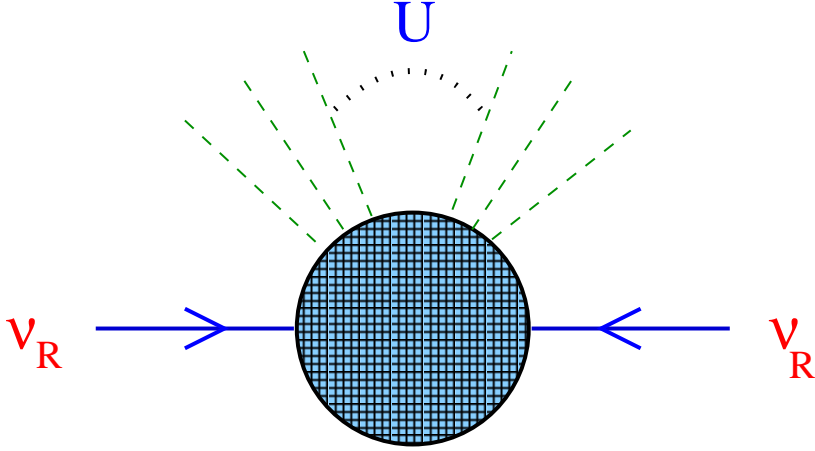


Figure 1: Instanton induced right-handed neutrino Majorana mass term.

exist string theory instanton effects which induce a Majorana mass term for the right-handed neutrino. They are of the form

$$e^{-\frac{1}{g^2(U)}} M_{String} c_{ab} \nu_R^a \nu_R^b \quad (1.2)$$

The instanton effect is depicted in Figure 1. Here U denotes a set of string moduli, on which the strength $g^2(U)$ of the non-perturbative effect depends. Here g^2 is *not* directly related to the SM gauge couplings so that these masses may be quite big although naturally suppressed with respect to the string scale M_{String} . A crucial ingredient for the mechanism to work is that the model should contain a gauged $B-L$ $U(1)$ symmetry beyond the SM gauge group, whose gauge boson gets a Stückelberg mass by combining with some scalar modulus field (e.g., a RR-scalar in Type II compactifications). Then under some conditions to be discussed below, certain (non-gauge) instanton effects analogous to those discussed in [3, 4] give rise to a right-handed Majorana mass term of the above form.

As we said, it is important that $U(1)_{B-L}$ gets a Stückelberg mass. It is a familiar fact in string models that $U(1)$ gauge fields with triangle anomalies canceled by the Green-Schwarz mechanism get Stückelberg masses. On the other hand, as emphasized in [5] (see also [6]) anomaly-free $U(1)$'s like $U(1)_{B-L}$ may also get Stückelberg masses, also by $B \wedge F$ couplings to suitable 2-form fields. For instance, a class of SM-like compactifications in which $U(1)_{B-L}$ gets a Stückelberg mass was provided in [5], based on models of intersecting D6-branes on an orientifold of type IIA on $T^2 \times T^2 \times T^2$ ¹. In this paper we use analogous examples to illustrate explicitly that (non-gauge)

¹These models are non-supersymmetric but a number of $N = 1$ SUSY models with MSSM-like

euclidean D2-brane instantons can give rise to Majorana mass terms as above.

On the other hand the mechanism is quite general and works in complete analogy in other string compactifications with D-branes (including non-geometric CFT compactifications like the models in [7]), or even in heterotic compactifications with $U(1)$ bundles. A difference in the heterotic case is that the effects can originate from world-sheet instantons, and are hence tree level in g_s (and non-perturbative in α').

This paper is organized as follows. In Section 2 we lay down the general idea of generating Majorana mass terms via non-perturbative instanton effects in string theory. In section 2.1 we motivate the proposal, and in section 2.2 we describe the instanton induced operator, its symmetry properties, and the microscopic mechanism in which it is generated. In section 2.3 we discuss some additional general aspects of the mechanism. Section 3 provides an explicit example of a semirealistic string theory D-brane model, where Majorana mass terms arise from D2-brane instantons. Section 4 discusses the use of similar non-perturbative instanton effects in generating other interesting operators, in particular the μ -term in supersymmetric models, or Yukawa couplings forbidden in perturbation theory. Section 5 contains our final comments.

Appendices contain technical details and additional examples. Appendix A describes instanton induced operators for completely general D-branes models (including type IIB with magnetized branes, D-branes at singularities, or even non-geometric CFT compactifications), and for heterotic models. Appendix B contains an additional class of semirealistic models allowing for instanton induced Majorana mass terms, while appendix C contains examples of a semirealistic SUSY model allowing for an instanton induced μ -term.

As this paper was ready for submission, ref. [8] appeared, which also discusses non-perturbative instanton effects in semirealistic string models.

2 The general scheme

2.1 General remarks

The discussion of the physics of neutrino masses in string models should clearly be carried out within the setup of semi-realistic string constructions reproducing structures close to the (possibly supersymmetric) Standard Model. It is interesting to point out that the presence of right-handed neutrinos is a quite generic feature within this class.

spectrum in which $U(1)_{B-L}$ gets a Stückelberg mass were reported in [7].

For instance, in type II compactifications with D-branes, right-handed neutrinos arise from open strings stretched between two stacks of $U(1)$ branes. They also appear in heterotic constructions with $U(1)$ bundles, but for concreteness we center our discussion on D-brane models.

The difficulty in obtaining Majorana masses for the right-handed neutrinos is manifest in this setup, since these fields carry non-trivial $U(1)$ charges. Typically these $U(1)$ gauge bosons become massive, by mixing with a RR closed string modulus, but the symmetries remain as global symmetries exact in perturbation theory. Hence it is natural to consider the corresponding non-perturbative effects, namely D-brane instantons, as the source for the corresponding terms.

The appearance of non-trivial field theory operators due to non-perturbative instanton effects is similar to the appearance of 't Hooft operators from gauge theory instantons in theories with mixed $U(1)$ anomalies. Namely, the operator arises from path integrating over zero modes of the instanton. However, in our setup there are important differences with respect to the field theory discussion. First and most importantly, $U(1)$ symmetries are actually gauged in string theory (although as mentioned, it is crucial that the $U(1)$ under which the Majorana mass term is charged becomes massive by coupling with a RR modulus). This implies that the exponential instanton amplitude in 1.2 should transform by a phase which cancels the transformation of the Majorana mass operator, yielding the instanton amplitude gauge-invariant. Secondly, the relevant instanton is *not* a gauge theory instanton. This has the nice consequence that the exponential factor need not lead to a large suppression, since it is not related to any SM gauge coupling.

The general description of D-brane instantons, their structure of fermionic zero modes, and the effective operators they induce, is carried out in complete generality in appendix A (in the absence of orientifold planes). It also contains the corresponding discussion for heterotic models.

In the coming sections we apply this kind of analysis, including orientifold planes, to the particular case of generating right-handed neutrino Majorana mass terms from string theory instantons, in the particular setup of type IIA models of intersecting D6-branes. It is however straightforward to rephrase the discussion in terms of other D-brane models or of heterotic compactifications.

2.2 Instantons and the right-handed neutrino Majorana mass operator

As we said, in order to discuss right-handed neutrino masses we need to work in the context of some semirealistic class of models with quark/lepton generations. For definiteness we are going to present our discussion in the context of type IIA orientifolds with D6-branes wrapping intersecting 3-cycles [9, 10] (for reviews see [11]). In this case the relevant instantons are D2-instantons wrapping 3-cycles on the compact space ² [20]. As mentioned above, the discussion in this section is a particular application (including orientifold planes) of the general discussion in appendix A.

Let us consider stacks of D6-branes a, b, c and d wrapping 3-cycles Π_a, Π_b, Π_c and Π_d on the CY orientifold, along with their orientifold images, denoted a^*, b^*, c^*, d^* branes. Let us denote their multiplicity by (N_a, N_b, N_c, N_d) . In the literature there are two main classes of semirealistic string models, with the chiral content of just the (possibly supersymmetric) Standard Model. They differ in the realization of the $SU(2)_L$ gauge factor of weak interactions, either as coming from a $U(2)$ in two overlapping D6-branes away from O6-planes ($N_b = 2$) [5], or from an $USp(2)$ group from one D6-brane ($N_b = 1$) overlapping with its orientifold image on top of and O6-plane [24]. The gauge group will be $SU(3) \times SU(2) \times U(1)_a \times U(1)_c \times U(1)_d$, with an additional Abelian $U(1)_b$ in the first case. This gauge group includes that of the SM.

In order to have the chiral fermion spectrum of the SM one has to ensure that the branes intersect the appropriate number of times. Thus e.g. left-handed quarks will come from the intersections of a and b, b^* branes and right-handed quarks from the intersections of a and c, c^* branes. Right-handed neutrinos will come from intersections of c and d^* branes (as discussed above, their charges under the $U(1)$ symmetries forbid Majorana mass terms in perturbation theory, although can be generated non-perturbatively as discussed below). A number of models of this type, with the SM gauge group and chiral spectrum, have been constructed in the last few years, and an explicit toroidal example will be described in the next section. An important point

²One could make an analogous discussion for orientifold compactifications of type IIB theory with $D(2p+1)$ -branes, like type I models with magnetized D9-branes [12, 9, 13], compactifications with D3- and D7-branes [14], models of D-branes at singularities [15, 16, 17], or non-geometric constructions like orientifolds of Gepner models [18, 19, 7]. The microscopic description of the corresponding D-brane instanton changes, but the physics of the four-dimensional theory remains identical. Also, one can make a similar discussion in the heterotic side with $U(1)$ bundles. As discussed in appendix A, in the heterotic the relevant operator could be induced simply by world-sheet instantons (hence tree-level in g_s and non-perturbative in α').

concerning the models we focus on is that they have the chiral matter content of exactly the SM. This implies that the discussion of global $U(1)$ symmetries and their anomalies is exactly as in the Standard model. For reference, the chiral fields and their $U(1)$ charges in our models are shown in table 1.

The $U(1)$ generators Q_a, Q_c, Q_d have interesting interpretations as SM global symmetries. For instance $\frac{1}{3}Q_a$ corresponds to baryon number Q_B , while Q_d corresponds to (minus) lepton number Q_L , and Q_c to the $U(1)_R$ generator Q_R of left-right symmetric models. Since $U(1)_b$ is not relevant in our discussion (and moreover is often absent in many interesting models) we do not include it in our discussion (see appendix B for models with $U(1)_b$). Note that $U(1)_B, U(1)_L$ and $U(1)_R$ are all tree-level symmetries of the SM (with right-handed neutrinos). There are three interesting orthogonal linear combinations that one can form

$$\begin{aligned} Q_{\text{anom}} &= 3Q_a - Q_d = 9Q_B + Q_L \\ Y &= \frac{1}{6}Q_a - \frac{1}{2}Q_c + \frac{1}{2}Q_d = \frac{1}{2}(Q_{B-L} - Q_R) \end{aligned} \quad (2.1)$$

$$Y' = \frac{1}{3}Q_a + Q_c + Q_d = Q_{B-L} + Q_R \quad (2.2)$$

The symmetry generated by Q_{anom} is anomalous (with anomaly canceled by the Green-Schwarz mechanism), while Y and Y' (equivalently $U(1)_{B-L}$ and $U(1)_R$) are anomaly free. The generator Y corresponds to the standard hypercharge, hence it should remain massless in order to have a realistic model. Finally, the generator Y' is an extra anomaly-free symmetry, to which, by a slight abuse of language, we refer to as the $B - L$ symmetry. It is crucial for our mechanism to work that this generator, even though it is non-anomalous, becomes massive by a Stückelberg coupling.

As we have argued, Majorana mass terms for right-handed neutrinos are forbidden in perturbation theory by the $U(1)_{Y'}$ and $U(1)_{\text{anom}}$ symmetries, but can be generated non-perturbatively as in (1.2) by non-perturbative instanton effects. Since the scalars making the $U(1)$'s massive are obtained from the RR 3-form integrated over 3-cycles, the relevant instantons are euclidean D2-branes wrapped on 3-cycles. Let us consider one such instanton corresponding to a D2-brane M (from Majorana) wrapping a 3-cycle Π_M in the compact space, and derive the constraints that it must satisfy to lead to operators of the form

$$e^{-S_{D2}} \nu_R \nu_R \quad (2.3)$$

Let us postpone for the moment the discussion of how the instanton generates the right-handed neutrino Majorana mass operator, and consider what are the symmetry properties of such an instanton amplitude. The right-handed neutrino Majorana mass

bilinears $\nu_R \nu_R$ have charge 2 under both $U(1)_{B-L}$ and $U(1)_R$ symmetries, and are neutral under hypercharge (and of course baryon number). The corresponding transformations under the $U(1)$ gauge symmetries should be canceled by a corresponding transformation of the exponential prefactor. This is the case if the imaginary part of the instanton action is given by a scalar field which shifts under the $U(1)$ symmetry in the appropriate way. Namely, as already mentioned, for the mechanism to work it is necessary that the relevant $U(1)$ gets a Stückelberg mass due to a $B \wedge F$ coupling to a closed string field.

Let us consider the general pattern of scalar shifts under the $U(1)$'s. Introduce the labels A, B, \dots for the different brane stacks, and their corresponding $U(1)$ gauge symmetries. The 3-cycles Π_A on which the $D6_A$ -brane wrap admit a decomposition in a basis $\{C_r\}$ of homology 3-cycles

$$\Pi_A = \sum_r p_{Ar} C_r \quad (2.4)$$

As discussed around (A.6), the coupling of the $D6_B$ -branes (and their orientifold images B^*) to the RR scalars a_r (obtained by integrating the RR 3-form over the 3-cycle D_r dual to C_r) implies that under $A_B \rightarrow A_B + d\Lambda_B$, the scalars shift as

$$a_r \rightarrow a_r + N_B (p_{Br} - p_{B^*r}) \Lambda_B \quad (2.5)$$

The action of the D2-brane instanton is given by DBI+ CS action, with the latter being responsible for its coupling to the RR scalars. Namely

$$\text{Im } S_{D2} = \sum_r c_r a_r = a_M \quad (2.6)$$

(for supersymmetric D2-branes, the complete action can be expressed holomorphically in terms of the complex structure moduli). Using (2.5) this quantity shifts under general $U(1)$ gauge transformations by the amount

$$-\sum_r c_r \sum_A N_A (p_{Ar} - p_{A^*r}) \Lambda_A = -\sum_A N_A (I_{MA} - I_{MA^*}) \Lambda_A \quad (2.7)$$

where $I_{MA} = \Pi_M \cdot \Pi_A$ is the intersection number, and similarly for I_{MA^*} . Hence the exponential amplitude for the instanton transforms as

$$e^{-S_{D2}} \rightarrow \exp\left(-i \sum_A N_A (I_{MA} - I_{MA^*}) \Lambda_A\right) e^{S_{D2}} \quad (2.8)$$

We thus have that for the exponential factor in (2.3) to cancel the transformation of the Majorana mass term, the intersection numbers of the D2-brane instanton and the

background D6-brane 3-cycles must satisfy ³

$$I_{Ma} - I_{Ma^*} = I_{Mb} - I_{Mb^*} = 0 \ ; \ I_{Mc} - I_{Mc^*} = I_{Md} - I_{Md^*} = 2 \quad (2.9)$$

Let us now consider the precise microscopic mechanism by which such an instanton generates the Majorana mass term insertion. In the presence of the D2-brane instanton, there are open strings stretching between the D2- and the background D6-branes. Quantization of these open strings shows that they lead to chiral fermions localized at the intersections between the D2- and the D6-branes. These are fermion zero modes of the instanton, over which one should integrate. For an instanton with the intersection numbers (2.9), we find two fermionic zero modes of each chirality, denoted α_i, γ_i , $i = 1, 2$ at the intersections of the D2-instanton and the d, c D6-branes carrying $U(1)_R$ and $U(1)_L$ gauge fields.

These fermion zero modes have $Mc-cd^*-d^*$ cubic couplings involving the D6-D6 fields in the cd^* sector, namely the right-handed neutrino multiplets, of the form

$$L_{cubic} \propto d_a^{ij} (\alpha_i \nu^a \gamma_j) \ , \ a = 1, 2, 3 \quad (2.10)$$

The coupling arises via a world-sheet disk amplitude, as shown ⁴ in Figure .

Upon integration over the fermion zero modes, the complete D2-brane instanton amplitude contains the additional contribution

$$\int d^2\alpha d^2\gamma e^{-d_a^{ij} (\alpha_i \nu^a \gamma_j)} \propto -\nu_a \nu_b \int d^2\alpha d^2\gamma \alpha_i \alpha_j \gamma_k \gamma_l d_a^{ik} d_b^{jl} = \nu_a \nu_b (\epsilon_{ij} \epsilon_{kl} d_a^{ik} d_b^{jl}) \quad (2.11)$$

giving rise to bilinears in the neutrino multiplets in the 4d effective action. Notice the role of the conditions (2.9) in both pieces of the instanton amplitude (2.3). It determines the number of fermion zero modes, and their charges, and hence the transformation of the monomial in the charged D6-D6 fields. On the other hand, it determines the amount by which the exponent S_{D2} shifts. The cancellation between the transformations of both pieces is a particular case of the self-consistency of these amplitudes, discussed in general in appendix A.

³ In addition, if any extra D6-brane X with $U(n)$ group is present beyond those of the SM, one should also impose $I_{MX} - I_{MX^*} = 0$ in order for the instanton-induced operator to be gauge invariant. See discussion in section 2.3

⁴We hope that the appearance of two kinds of instantons (the D2-brane instanton inducing a non-perturbative g_s correction in the 4d effective action, and the world-sheet instanton, inducing an α' effect on the D2-brane action) in the present IIA setup does not lead to confusion. Moreover, the cubic coupling arises as a pure α' tree-level effect in other D-brane constructions (namely in type IIB models).

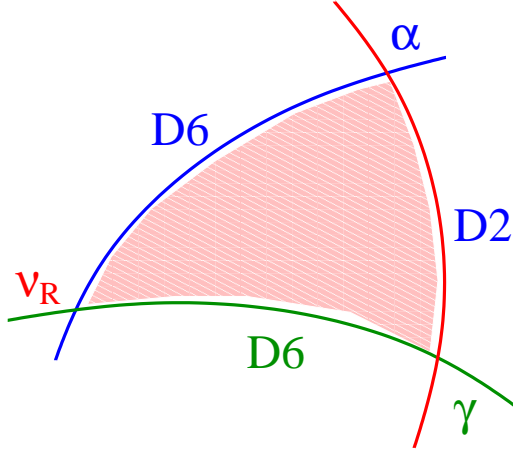


Figure 2: World-sheet disk amplitude inducing a cubic coupling on the D2-brane instanton action. The cubic coupling involves the right-handed neutrinos lying at the intersection of the c and d^* D6-branes, and the two instanton fermion zero modes α and γ from the D2-D6 intersections.

As we have tried to emphasize, the mechanism is rather general, and we only need to have a semirealistic compactification with the following ingredients:

- 1) The 4d theory should have the chiral content of the SM and additional right-handed neutrinos. There should be a gauged $U(1)_{B-L}$ gauge symmetry beyond the SM, under which the right-handed neutrinos are charged.
- 2) The $U(1)_{B-L}$ gauge boson should have a Stückelberg mass from a $B \wedge F$ coupling.
- 3) The compact manifold should admit D2-instantons yielding the two appropriate zero modes transforming under $U(1)_L$ and $U(1)_R$ (but no other symmetries in the theory) to yield neutrino bilinears.

Then the appropriate Majorana mass term will generically appear (see section 2.3 for some additional discussion on more detailed conditions on the instantons).

Note that the $e^{-S_{D2}}$ semiclassical factor will provide a suppression factor for this operator, but this suppression need not be large ⁵, since in general the field U is not directly related to the SM gauge coupling constants. Indeed, it is easy to see that U cannot appear in the gauge kinetic function for the SM gauge fields. The reason is that U transforms with a shift under an anomaly free $U(1)$. If it also had couplings to the

⁵One may worry that, if the exponential factor is not small, multiwrapped instantons may contribute with comparable strength, leading to a breakdown of the instanton expansion. However, the zero mode structure of the instanton is controlled by the intersection numbers of the overall cycle class, thus ensuring that only the single instanton we discuss contributes.

$F \wedge F$ SM gauge field operators a gauge anomaly would be created, which cannot be true for an anomaly-free gauge interaction. Thus U cannot appear in the SM gauge kinetic functions and hence there is no phenomenological constraint on the value of ReU . Thus the induced Majorana mass for right-handed neutrinos may be only a few order of magnitudes below the string scale, in agreement with phenomenological requirements. Note that in general a flavour structure will appear depending on the model dependent coefficients d_a^{ij} .

An important comment concerning discrete symmetries is in order. The mentioned instanton effects leading to the operator (2.3) break the $U(1)_{B-L}$ continuous symmetry. Notice however that a Z_2 group generated by $exp(i\pi Q_{B-L})$ remains unbroken (i.e. $exp(-U)$ has lepton charge -2). On the other hand it is well known that within the MSSM such a discrete Z_2 symmetry is equivalent to R-parity [25], the symmetry which guarantees the absence of dimension four operators violating Baryon and Lepton numbers in the MSSM. Then within the present scheme the existence of R-parity is automatic.

The present mechanism may be implemented in a way consistent with $SU(5)$ unification. The idea is having a $SU(5) \times U(1)_Z$ model with three chiral matter generations including 3 right-handed neutrinos, i.e.

$$3(10_1 + \bar{5}_{-3} + 1_5) \tag{2.12}$$

It is easy to check that the $U(1)_Z$ is anomaly-free generation by generation ⁶. An instanton with an action whose imaginary part X transforms under $U(1)_Z$ like

$$X \longrightarrow X + 10\Lambda_Z \tag{2.13}$$

would generate the operator $e^{-X} \nu_R^i \nu_R^j$, which is invariant under the gauge symmetry. It would be interesting to have some concrete $SU(5)$ example within string theory where this could be implemented.

2.3 Role of supersymmetry and additional zero modes

In our previous discussion we have focused on the relevant properties of the instanton to yield the effect we are interested in. These are essentially based on symmetry

⁶In fact one has $U(1)_Z = U(1)_Y + (5/2)U(1)_{Y'}$, with $U(1)_{Y'}$ the massive anomaly-free $U(1)$. Note also that the charge assignments are compatible with embedding the $SU(5) \times U(1)_Z$ into $SO(10)$. However, this enhancement would not be consistent with our mechanism, which requires the existence of a Stückelberg mass term for the relevant $U(1)$. Hence, after the instanton effect is taken into account, only the $SU(5) \times Z_2$ symmetry would be realized at low energies, with Z_2 being R-parity in the SUSY case.

arguments, and topological properties. In particular, the previous analysis ignores the discussion of other features, like the role of supersymmetry, the possible presence of additional instanton zero modes, etc. This section is devoted to filling this gap.

The role of supersymmetry

As with most physical effects in string theory compactifications, instanton corrections have usually been described in the supersymmetric setup. Indeed, beyond the usual advantages of ensuring stability of the vacuum, and that the wrapped brane is volume minimizing and thus a stationary point of the path integral, $N = 1$ supersymmetry provides a useful bookkeeping which facilitates the classification of the spacetime operators induced by the instanton. For instance, instanton corrections to the spacetime superpotential arise from instantons with two fermionic zero modes, which soak up the integration over half the superspace Grassman variables. These are usually generated by D-brane or world-sheet instantons, wrapped on rigid cycles, and preserving half of the supersymmetries. This ensures that the only zero modes are the Goldstinos of the two supersymmetries broken by the instanton, so that it generates a superpotential coupling.

However, it is clear that instanton effects exist in non-supersymmetric theories as well. Essentially the basic rule is that an instanton with a number of fermion zero modes leads to spacetime interactions with the appropriate number of spacetime fields to saturate the amplitude. Given this, we understand that our previous discussion of generation of Majorana mass terms from instantons can be carried out both in supersymmetric and non-supersymmetric string compactifications. In fact, our explicit examples in the coming sections are non-supersymmetric. In any event, it should be clear that a completely analogous analysis can be carried out for supersymmetric compactifications.

Additional zero modes

A physically more relevant issue is that in general an instanton may carry more zero modes than the minimum we require. More specifically, one may have instantons with the right topological intersection numbers, but with additional zero modes associated to deformations of the wrapped 3-cycle, (of for instantons that happen to break all the supersymmetries of a supersymmetric background, and hence have additional Goldstinos). These zero modes are uncharged under the D6-brane gauge factors, and therefore do not contribute to the structure of the spacetime operator in the charged fields (but rather to insertions of additional closed string fields). Since these zero modes do not really affect the Majorana mass term structure, their detailed discussion is ignored in

the present paper.

Such additional zero modes will however be present for the instantons we consider in our explicit examples. Indeed we present explicit examples in toroidal setups, where the 3-cycles wrapped by the D2-brane instantons are topologically T^3 , hence have three position plus Wilson line moduli, leading to additional zero modes. This may be considered a drawback, since as discussed leads to additional closed string field insertions, making the Majorana mass term structure be part of a higher-dimension operator (in particular, it would not be a superpotential coupling in supersymmetric cases). These models however illustrate the robust features in the generation of Majorana mass terms, and can be considered toy models of more realistic constructions (e.g. on CY threefolds) where instantons without the additional zero modes may exist.

Moreover, even the discussion of the toroidal setups may be useful by itself, in the following sense. It is well-known that additional bosonic and non-chiral fermionic instanton zero modes can be lifted by additional ingredients in the compactification. For instance, 3-form fluxes on type IIB orientifold models can lift certain fermion zero modes on euclidean D3-brane instantons [26] (and consequently, modify the topological conditions for an instanton to contribute to the superpotential). It is plausible to imagine that the additional zero modes in the models we discuss can be lifted by a similar mechanism upon introduction of a suitable set of fluxes, or generalization thereof (see [27] for a useful related discussion in the type IIA setup). Notice that in non-supersymmetric cases, this removal of the additional zero modes also involves stabilization of e.g. the deformation moduli of the wrapped cycle. This effect can in fact be crucial, since it underlies that fact that the wrapped brane is an stationary point of the path integral, and can thus be properly referred to as an instanton.

An additional possible source of additional zero modes is that the D2-brane may intersect other D6-branes beyond those involved in the SM sector. This possibility is fully encompassed by our general discussion in appendix A. Such additional zero modes would lead to insertions of new D6-D6 fields in the spacetime effective operator, thus spoiling in principle the Majorana mass term structure, and in general yielding a higher-dimension operator, involving hidden sector fields. Nevertheless, the right structure may be recovered if these new fields are allowed to get vevs, etc. Clearly the discussion then becomes very model-dependent. In our explicit neutrino mass models we will make sure that these additional zero modes are absent (see footnote 3).

There is one interesting exception to this last paragraph. Certain instantons contain zero modes arising from equal numbers of intersections between the D2-brane with a

given D6-brane A and its orientifold image A^* , namely $I_{MA} - I_{MA^*} = 0$. This leads to zero modes in the $(\square_M, \bar{\square}_A) + (\square_M, \square_A)$, and are hence vector-like with respect to the $U(N_A)$ gauge symmetry. From the discussion in previous sections, see e.g. 2.8 these zero modes do not affect the transformation of the exponential factor. Consistently with this, they do not have any cubic couplings with 4d fields, hence do not contribute insertions to the charged matter operator. These intersection zero modes are thus particularly inert, and we do not consider them in our discussion.

Moduli stabilization

We would like to conclude this section with a comment on the interplay of the wrapped brane instantons and the introduction of additional ingredients in the compactification, like fluxes. Indeed, since our mechanism involves the use of a shift for the imaginary part of a closed string modulus, one may fear that it is spoiled by the introduction of fluxes leading to closed string stabilization. In fact, this is guaranteed *not* to be the case. As discussed in [28] the Freed-Witten constraint [29] on the D6-branes guarantees that the superpotentials generated by the introduction of fluxes are fully compatible with the shift symmetries induced by the BF couplings due to the D6-branes. In other words, the modulus involved in the instanton amplitude is not affected by the flux superpotential, and the shift symmetry is intact.

Amusingly the converse result was shown in [30]. Namely, the Freed-Witten constraint on the D2-brane instanton guarantees that the superpotential induced by such instantons is compatible with the scalar shifts implicitly exploited by the fluxes (manifest in their description as gaugings of isometries of the scalar moduli space).

The bottomline is that flux stabilization mechanisms and instantons talk to different sets of moduli, and hence lead to no interference. This gives additional plausibility to our statements above concerning lifting of additional instanton zero modes, and hence motivates us to proceed with the construction of explicit models, even in toroidal setups.

3 An intersecting D6-brane example

To make the above described general mechanism explicit, we need semirealistic models (either supersymmetric or not) in which there is a gauged $U(1)_{B-L}$ symmetry getting a Stückelberg mass. These are a restricted subset of the semirealistic models in the literature. To our knowledge, the only models satisfying those requirements are the non-SUSY models constructed in [5], and a (small) subset of the SUSY CFT orientifolds

studied by Schellekens and collaborators [7]. In particular, there are no examples of toroidal/orbifold $N = 1$ SUSY constructions with massive $U(1)_{B-L}$ gauge bosons. In any event, and given the simplicity of toroidal constructions, our examples here will be analogous to the non-SUSY models in [5]. As just explained supersymmetry is not a crucial ingredient in our discussion, the relevant instantons and 't Hooft operators also exist in non-supersymmetric theories. In our case the relevant operator will be fermionic with a bilinear in right-handed neutrinos. It should be easy to implement a similar discussion in the supersymmetric models in [7].

As mentioned, ref.[5] provided a family of non-SUSY models with the required characteristics, i.e. SM spectrum and a massive $U(1)_{B-L}$. They are orientifolds of type IIA on $T^2 \times T^2 \times T^2$ modded by ΩR , with Ω being world-sheet parity and R the reflection $z_i \rightarrow \bar{z}_i$ of the three T^2 complex coordinates. There are four Standard Model D6-branes a, b, c, d (and their orientifold images a^*, b^*, c^*, d^*) in which the SM gauge group lives. The multiplicities are $N_a = 3$, $N_b = 2$, $N_c = N_d = 1$ so that, before some gauge bosons get Stückelberg masses, the full gauge group is $U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$. In the present section we will consider a slightly simpler class of models [31] in which the $SU(2)_L$ SM gauge group is realized in terms of a symplectic group $USp(2)$ rather than a unitary group $U(2)$ ⁷. This is obtained with $N_b = 1$ if the corresponding b-brane and its mirror sit on top of an orientifold plane. Then the initial gauge group is rather $SU(3) \times SU(2) \times U(1)_a \times U(1)_c \times U(1)_d$. Here $U(1)_a$ and $U(1)_d$ have the interpretation of gauged baryon and (minus)lepton numbers, whereas $U(1)_c$ behaves like the diagonal generator of right-handed weak isospin. Open strings at the intersections of the D6-branes lead to chiral fermions transforming like bifundamentals $(\square_a, \bar{\square}_b)$, and (\square_a, \square_b) for ab and ab^* intersections, respectively. The chiral fermion content reproduces the SM quarks and leptons if the D6-brane intersection numbers are given by

$$\begin{aligned} I_{ab} = I_{ab^*} = 3 & \quad ; \quad I_{ac} = I_{ac^*} = -3 \\ I_{db} = I_{db^*} = -3 & \quad ; \quad I_{cd} = -3 ; I_{cd^*} = 3 \end{aligned} \tag{3.1}$$

with the remaining intersections vanishing. As usual, negative intersection numbers denote positive multiplicities of the conjugate representation. The spectrum of chiral fermions is shown in Table 1. These correspond to three SM quark lepton generations. In addition there are three right-handed neutrinos ν_R whose presence is generic in this kind of constructions. At the intersections there are also complex scalar with the same

⁷See appendix B for the analogous discussion for the constructions in [5] which have a $U(2)_b$ gauge group.

Intersection	Matter fields		Q_a	Q_c	Q_d	Y	Y'	$3Q_a - Q_d$
(ab),(ab*)	Q_L	$3(3, 2)$	1	0	0	1/6	1/3	3
(ac)	U_R	$3(\bar{3}, 1)$	-1	1	0	-2/3	2/3	-3
(ac*)	D_R	$3(\bar{3}, 1)$	-1	-1	0	1/3	-4/3	-3
(bd),(b*d)	L	$3(1, 2)$	0	0	-1	-1/2	-1	1
(cd)	E_R	$3(1, 1)$	0	-1	1	1	0	-1
(cd*)	ν_R	$3(1, 1)$	0	1	1	0	2	-1

Table 1: Standard model spectrum and $U(1)$ charges in the realization in terms of D6-branes with intersection number (3.1)

charges as the chiral fermions. These are not necessarily massless (since the model may be non-supersymmetric), but by a judicious choice of the complex structure moduli one can generically avoid the presence of charged scalar tachyons [5].

One linear combination of the three $U(1)$'s, i.e.

$$Y = \frac{1}{6}(Q_a - 3Q_c + 3Q_d) \quad (3.2)$$

corresponds to the hypercharge generator. Another one, $(3Q_a - Q_d)$ is anomalous (with anomaly canceled by the Green-Schwarz mechanism) and becomes massive as usual. The remaining orthogonal linear combination Y' is anomaly free and will become massive or not depending on the structure of the couplings of the $U(1)$'s to the RR 2-forms in the given model. As we mentioned, the appearance of a Majorana mass term by our mechanism necessarily requires that this anomaly-free combination becomes massive, otherwise the term is forbidden by unbroken gauge interactions. As discussed in the previous section, in order for such mass terms to be generated there must exist a D2-brane instanton M with intersection numbers with the SM branes as in eq.(2.9). Taking into account that the helicities of the α and γ instanton zero modes have to match, this requires either

$$I_{Md} = 2 ; I_{Mc^*} = -2 ; I_{Mc} = I_{Md^*} = 0 \quad (3.3)$$

or else

$$I_{Mc} = 2 ; I_{Md^*} = -2 ; I_{Md} = I_{Mc^*} = 0 . \quad (3.4)$$

Thus in order to get a model with the SM chiral content and in addition right-handed Majorana masses both the conditions (3.1) and those above must be verified. The

latter conditions turn out to be rather restrictive in the present setup, and we suspect this to be valid in more general classes of models.

Let us make the discussion concrete and construct a specific class of models. Consider type IIA string theory compactified on $T^2 \times T^2 \times T^2$, with (x_i, y_i) parametrizing the i^{th} T^2 . We further mod out by ΩR , where Ω is world-sheet parity and R is the reflection of the three compact y_i coordinates. We consider D6-branes on factorizable 3-cycles and denote their wrapping numbers in the three (x_i, y_i) directions (n^1, m^1) , (n^2, m^2) , (n^3, m^3) . Consider a set of SM branes with wrapping numbers as shown in Table (2). Here $n_a^2, m_a^3, n_c^1, n_d^2, m_d^3$ are integers. It is easy to check that indeed these

N_i	(n_i^1, m_i^1)	(n_i^2, m_i^2)	(n_i^3, m_i^3)
$N_a = 3$	$(1, 0)$	$(n_a^2, 1)$	(n_g, m_a)
$N_b = 1$	$(0, 1)$	$(1, 0)$	$(0, -1)$
$N_c = 1$	$(n_c^1, 1)$	$(1, 0)$	$(0, 1)$
$N_d = 1$	$(1, 0)$	$(n_d^2, -n_g)$	$(1, m_d^3)$

Table 2: D6-brane wrapping numbers giving rise to a SM spectrum.

wrapping numbers give rise to the chiral spectrum of a SM with n_g quark/lepton generations. For more generality we have considered the case with a general number of generations n_g .

These models have in principle three $U(1)$ gauge fields. However generically two of them acquire Stückelberg masses due to the $B \wedge F$ couplings

$$B_2^i \wedge 2N_A m_A^i n_A^j n_A^k F_A \quad , \quad i \neq j \neq k \quad (3.5)$$

where A labels the D6-brane stacks and i, j, k run through the three 2-tori. The factor of N_A arises from the $U(1)$ normalization, and the factor of 2 arises from the coupling to the D6-brane and its orientifold image. Recall that in these toroidal models there are four massless 2-forms B_2^p , $p = 0, 1, 2, 3$ arising from integrating the type IIA 5-form over 3-cycles invariant under the orientifold action. For the model in table 2 the non-vanishing couplings are

$$\begin{aligned}
B_2^2 &\wedge 2n_g(3F^a - F^d) \\
B_2^3 &\wedge 2(3n_a^2 m_a^3 F^a + n_c^1 F^c + n_d^2 m_d^3 F^d)
\end{aligned}
\tag{3.6}$$

The RR fields B_2^0 and B_2^1 have no couplings to the $U(1)$'s. The existence of these couplings implies that the scalar fields a_i , 4d duals to the 2-forms B_i , transform under $U(1)$ gauge transformations with a shift

$$\begin{aligned}
a_{0,1} &\rightarrow a_{0,1} \\
a_2 &\rightarrow a_2 + 2n_g(3\Lambda(x)_a - \Lambda(x)_d) \\
a_3 &\rightarrow a_3 + 6n_a^2 m_a^3 \Lambda_a(x) + 2n_c^1 \Lambda_c(x) + 2n_d^2 m_d^3 \Lambda_d(x)
\end{aligned}
\tag{3.7}$$

Note that the a^i are the imaginary parts of the complex structure fields U^i in the case of SUSY type IIA orientifolds. There are a number of additional constraints to make the model realistic and to allow the non-perturbative appearance of right-handed neutrino Majorana masses:

i) There are three $U(1)$'s and only two RR-scalars, a^2 and a^3 , coupling to them. Hence necessarily one of the $U(1)$'s remains massless. In order for the model to be realistic, the standard hypercharge should be the massless generator. This is the case if

$$n_c^1 = n_a^2 m_a^3 + n_d^2 m_d^3 . \tag{3.8}$$

The other two linear combinations (in particular the $U(1)$ relevant for Majorana masses) are massive.

ii) In order for the model to be a consistent compactification, RR-tadpoles have to cancel. Tadpoles cancel in this simple model if

$$3m_a^3 = n_g m_d^3 . \tag{3.9}$$

In addition one should add $(3n_a^2 n_g + n_d^2 - 16)$ D6-branes (or antibranes, depending on the sign) along the orientifold plane. They have no intersection with the rest of the branes and do not modify the discussion in any way.

iii) Finally, the appearance of Majorana masses requires the existence of a D2-brane instanton M wrapping a 3-cycle on $T^2 \times T^2 \times T^2$ verifying the conditions (3.3) (conditions 3.4 lead to equivalent physics). For simplicity we focus on factorizable 3-cycles (see section 2.3 for discussion on our viewpoint on the extra zero modes that arise). It turns out that in this class of models there is a unique factorizable 3-cycle

with the required properties which is given by the wrapping numbers:

$$M = (n_1, m_1) (n_2, m_2) (n_3, m_3) = (1, 1) \left(\frac{n_a^2}{n_g}, 1 \right) (1, -m_a^3) \quad (3.10)$$

In addition in order to get (integer) quantized wrapping numbers for the instanton M , one must have $n_c^1 = 1$ and n_a^2 a multiple of n_g .

It is amusing that one can check that in this particular family of models all these three conditions i)-iii) are possible only for $n_g = 3$, i.e. only for three quark-lepton generations. This is mostly due to the condition from cancellation of RR-tadpoles. Although this probably will not be the case for other classes of models, it illustrates how restrictive the conditions to get Majorana masses may be in particular families of models. We expect this general lesson to extend to other classes of models. Another example of this is the number of Higgs multiplets. Pairs of Higgs doublets arise from open strings stretched between the b and c D6-branes, and the number of pairs is given by the intersection number of the two stack in the last two complex planes, namely n_c^1 in our class. Since the above conditions (in particular wrapping numbers for the instanton 3-cycle) required having $n_c^1 = 1$, they imply the requirement of having just one pair of Higgs doublets.

Let us verify that the above instanton has the correct transformation properties. The imaginary part of the action of the D2-brane instanton wrapping this 3-cycle is

$$\text{Im } S_{D2} = \frac{n_a^2}{n_g} a_0 - m_a^3 a_1 - \frac{n_a^2 m_a^3}{n_g} a_2 + a_3 \quad (3.11)$$

Now it is straightforward to check that the operator

$$\exp \left[- \left(\frac{n_a^2}{n_g} a_0 - m_a^3 a_1 - \frac{n_a^2 m_a^3}{n_g} a_2 + a_3 \right) \right] \nu_R^i \nu_R^j \quad ; \quad i, j = 1, 2, 3 \quad (3.12)$$

is gauge invariant under all three $U(1)$'s, when one takes into account the transformations (3.8).

As discussed in the previous section, the Majorana mass operator is generated as follows. At the intersections between the D2-brane instanton 3-cycle with the background D6-branes there are fermionic zero modes. Let us denote these modes as α_i, γ_i for the Mc^* and Md intersections respectively. These zero modes have cubic couplings

$$L_{cubic} \propto d_a^{ij} (\alpha_i \nu^a \gamma^j) \quad , \quad a = 1, 2, 3 \quad (3.13)$$

which are induced by disk world-sheet instantons. Here ν^a are the right-handed sneutrinos and d_a^{ij} are coefficients (which in general will also depend on the Kähler moduli

and open string moduli, like standard Yukawas). Upon integration over the fermion zero modes, one generates a contribution proportional to

$$\int d^2\alpha d^2\gamma e^{-d_a^{ij} (\alpha_i \nu^a \gamma_j)} \propto -\nu_a \nu_b \int d^2\alpha d^2\gamma \alpha_i \alpha_j \gamma_k \gamma_l d_a^{ik} d_b^{jl} = \nu_a \nu_b (\epsilon_{ij} \epsilon_{kl} d_a^{ik} d_b^{jl}) \quad (3.14)$$

Note that we get bilinears because we have two zero modes of each type α and γ .

The semiclassical contribution to the quantities d_a^{ij} may be explicitly computed in these toroidal models. Indeed these amplitudes are completely analogous to the Yukawa couplings computed in [24]. In each of the subtori the branes c^* , d and the instanton M intersect forming triangles. Being in a torus we have in fact a sum over triangles in the covering space. This computation was performed in [24] and it was found that the amplitudes may be written as products of Jacobi θ -functions with characteristics. In particular one finds

$$d_a^{ij} = \prod_{r=1}^3 \vartheta \left[\begin{matrix} \delta^{(r)} \\ \phi^{(r)} \end{matrix} \right] (\kappa^{(r)}), \quad (3.15)$$

where the product goes over the three tori. The dependence on i, j, a is contained in the arguments which are defined as

$$\begin{aligned} \delta^{(r)} &= \frac{i^{(r)}}{I_{Mc^*}^{(r)}} + \frac{j^{(r)}}{I_{dM}^{(r)}} + \frac{a^{(r)}}{I_{c^*d}^{(r)}} + \frac{\left(I_{Mc^*}^{(r)} \epsilon_d^{(r)} + I_{dM}^{(r)} \epsilon_{c^*}^{(r)} + I_{c^*d}^{(r)} \epsilon_M^{(r)} \right)}{I_{Mc^*}^{(r)} I_{dM}^{(r)} I_{c^*d}^{(r)}}, \\ \phi^{(r)} &= \left(I_{Mc^*}^{(r)} \theta_d^{(r)} + I_{dM}^{(r)} \theta_{c^*}^{(r)} + I_{c^*d}^{(r)} \theta_M^{(r)} \right), \\ \kappa^{(r)} &= \frac{J^{(r)}}{\alpha'} |I_{Mc^*}^{(r)} I_{dM}^{(r)} I_{c^*d}^{(r)}| \end{aligned} \quad (3.16)$$

Here e.g. $I_{Mc^*}^{(r)}$ denotes the intersection number of branes M and c^* in the r -th torus, and $i^{(r)}$ labels one particular intersection in each plane r . Note that in our case

$$I_{Mc^*}^{(1)} I_{Mc^*}^{(2)} I_{Mc^*}^{(3)} = I_{dM}^{(1)} I_{dM}^{(2)} I_{dM}^{(3)} = 2 \quad (3.17)$$

$$I_{c^*d}^{(1)} I_{c^*d}^{(2)} I_{c^*d}^{(3)} = 3. \quad (3.18)$$

The $J^{(r)}$ are the complex Kähler moduli for each tori, the $\epsilon^{(r)}$'s parametrize the position of each brane in each subtorus and the $\theta^{(r)}$ possible Wilson lines. These two degrees of freedom correspond to open string moduli which may be complexified as

$$\Phi_a^{(r)} = J \epsilon_a^{(r)} + \theta_a^{(r)}. \quad (3.19)$$

As discussed in section 2.3, the D2-brane moduli really correspond to instanton zero modes over which one should in principle integrate. However, our viewpoint is that the present model should be regarded either as a toy model of an improved setup, like

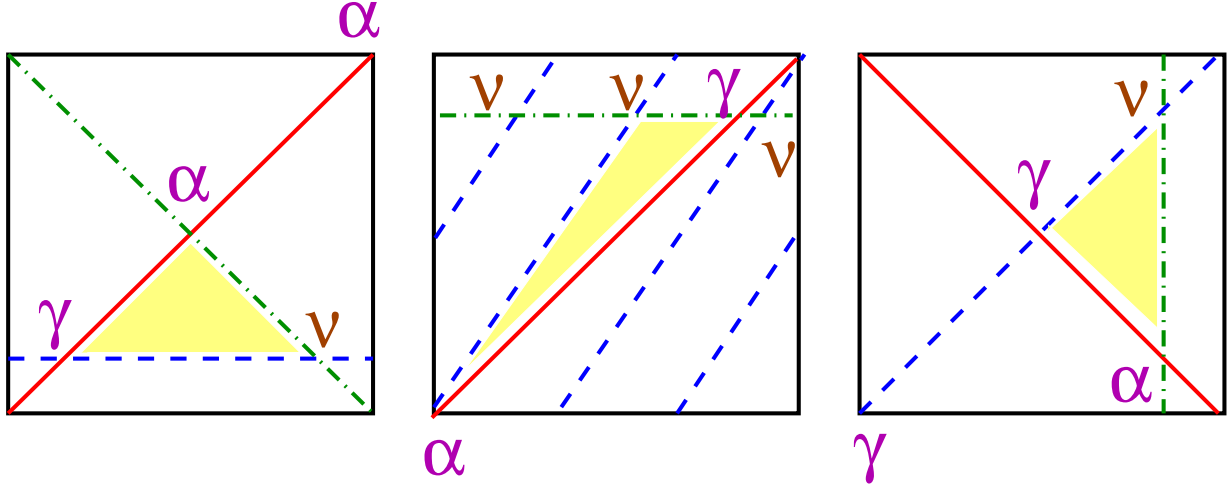


Figure 3: The figure shows the D2-brane instanton (continuous line) and the c^* and d D6-branes at whose intersections lie the right-handed neutrinos. The instanton zero modes from the D2-D6 open strings are denoted by α and γ . The yellow areas describe (the projections of) the open string disk inducing a cubic coupling on the D2-brane instanton action.

CY compactifications, where instantons without such zero modes exist, or as part of a construction with additional ingredients, like fluxes, which lift such zero modes. Either viewpoint is essentially mimicked by considering the D2-brane moduli as fixed numbers in the above formulae.

Let us provide a geometric picture of the instanton for a particular example. Consider the case

$$n_a^2 = 3 ; n_c^1 = 1 ; n_d^2 = -2 ; m_a^3 = m_d^3 = 1 ; n_g = 3 ; \epsilon = 1 \quad (3.20)$$

Then the relevant instanton M and branes c^* and d have wrapping numbers:

$$\begin{aligned} M & : (1, 1)(1, 1)(1, -1) \\ c^* & : (1, -1)(1, 0)(0, -1) \\ d & : (1, 0)(-2, -3)(1, 1) \end{aligned} \quad (3.21)$$

Then one can check $I_{dM^*} = I_{Mc} = 0$ so that there are no vector-like zero modes from extra intersections. These three 3-cycles are shown in figure 3. Note that they have the correct number of intersections and also that the expected triangle instanton contributions are indeed present.

The models discussed in this section come remarkably close to many of the features of the SM. On the other hand they are not fully realistic. In fact if the torus is factor-

izable (no off-diagonal Kahler moduli) the index dependence of the d_a^{ij} factorizes (i.e. $d_a^{ij} = d^i d_a d^j$) and the amplitude vanishes due to the contraction with antisymmetric indices in eq.(3.14). Thus only for non-diagonal Kahler moduli the mechanism may take place. Furthermore, being non-supersymmetric one expects the vacuum to get unstable unless the models are supplemented with some extra ingredient like RR/NS fluxes. Still we think they exemplify in very explicit detail how our proposed mechanism for the generation of right-handed neutrino Majorana masses works. We leave for the future the detailed study of the flavour patterns which may arise in this class of models.

4 Other instanton induced superpotentials. The μ -term.

It is clear that this kind of instanton effects may give rise to other interaction terms. This includes mass terms as well as certain higher dimensional superpotential couplings. Potentially the most relevant terms are those of dimension smaller than four like the case of right-handed neutrino masses just mentioned. In the context of the MSSM the only other (R-parity preserving) mass term which is allowed by SM gauge symmetries is a Higgs bilinear superpotential, i.e. the μ -term

$$W = \mu H \bar{H} . \quad (4.1)$$

One of the mysteries of the MSSM is the understanding of the reason why such a mass term, which in principle could be as large as the Planck scale, is so small, of the same order of magnitude of the electroweak scale. This is often called ‘the μ -problem’ [32]. A natural idea is to assume that such a coupling is forbidden by some $U(1)$ gauge interaction. If such a $U(1)$ gets a Stückelberg mass, then an operator of the general form

$$W_\mu = e^{-S_{ins}} H \bar{H} M_{string} \quad (4.2)$$

may be gauge invariant and be induced by some string instanton contribution. The exponential suppression could then perhaps provide a dynamical explanation for the smallness of the μ -term. The general idea can be described without need of a specific model. We consider again the case of a general Type IIA CY orientifold with intersecting D6-branes, although again a similar discussion can be carried out in other string constructions. Consider the case of four stacks of SM branes a, b, c, d (and their mirrors) leading to a general unitary group $U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$. The

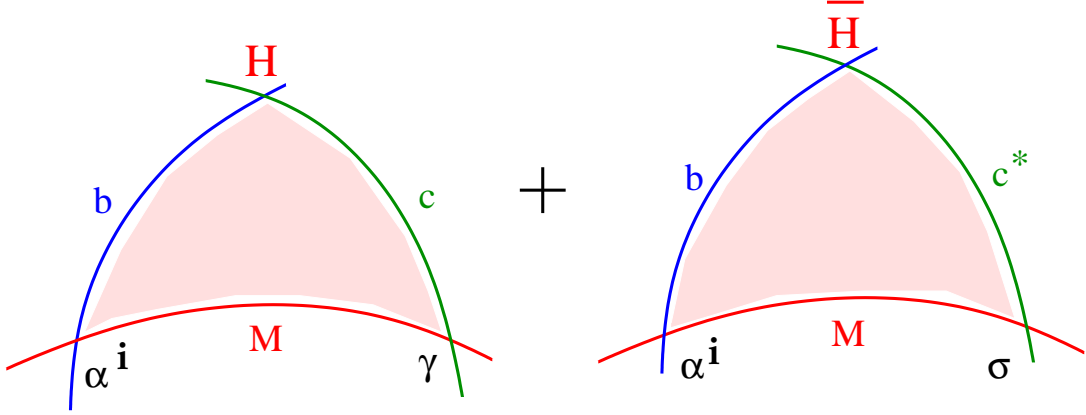


Figure 4: Disk amplitudes contributing cubic couplings between the D2-brane instanton fermion zero modes α^i , γ , σ , and the spacetime Higgs fields H , \bar{H} . Upon integration over the fermion zero modes, the induced effective operator is a μ -term.

Higgs fields H and \bar{H} will appear at the bc and bc^* intersections, respectively. The bilinear $H\bar{H}$ has $U(1)_b$ charge ± 2 , depending on the sign of the intersection number of both branes. Then the μ -term operator explicitly breaks $U(1)_b$ gauge invariance. In general $U(1)_b$ is anomalous and gets a Stückelberg mass as usual. Let us assume for definiteness that the $U(1)_b$ charge of the bilinear is -2 , and also that both H and \bar{H} come only in one copy, as in the MSSM. Then, if a D2-instanton M exists such that

$$I_{Mb} = -1 \quad I_{Mb^*} = 0 \quad ; \quad I_{Mc} = I_{Mc^*} = 1 \quad ; \quad I_{Mx} - I_{Mx^*} = 0 \quad (4.3)$$

with x any other brane in the model, then an operator of the required form appears. This means that there should be a doublet α^i of Mb zero modes, and in addition singlet zero modes γ , σ corresponding to Mc and Mc^* intersections. There is a cubic coupling of these modes to the spacetime Higgs fields, of the form

$$L_{cubic} \propto (\alpha H)\gamma + (\alpha \bar{H})\sigma \quad (4.4)$$

This is mediated by the world-sheet disk instanton amplitudes depicted in fig.(4).

Integration over the fermions zero modes gives a contribution proportional to

$$e^{-S_M} \int d^2\alpha d\gamma d\sigma e^{(\alpha H)\gamma + (\alpha \bar{H})\sigma} \propto e^{-S_M} H\bar{H} \quad (4.5)$$

We will illustrate this possibility in an explicit MSSM-like example in Appendix C. It turns out that if we want to construct an explicit RR-tadpole free SUSY model, extra new D6-branes beyond the SM ones have to be introduced. Then the actual operator

gets multiplied by a power of hidden sector fields. Still it provides an explicit SUSY example of this idea.

As should be clear, one can apply similar ideas to generate other interesting couplings forbidden in perturbation theory by some massive $U(1)$ symmetry. For instance, the Yukawa couplings $10 \cdot 10 \cdot 5$ in standard $SU(5)$ GUTs, which violates the $U(1)$ symmetry of $U(5)$ when realized in D-brane models. Or similarly, the quark Yukawa couplings in D-brane constructions where right-handed quarks are realized as anti-symmetric representations of $SU(3)$. Clearly, non-perturbative effects open up new possibilities for improving model building prospects of these constructions.

5 Discussion

We have found that neutrino Majorana masses are generated by D-brane instantons in certain general classes of string compactifications. We have discussed a set of necessary conditions for this mechanism to be allowed, like the presence of a massive (although obviously anomaly-free) $U(1)_{B-L}$ generator. This requirement is non-trivial and in fact it is not satisfied in most semirealistic constructions to date. Hence those models should be regarded as not fully realistic in the neutrino sector. The Majorana mass term constraint thus turns out to be a powerful new ingredient in model building requirements.

Although non-automatic, the requirements are however satisfied by a restricted but non-trivial subset of models. To our knowledge, only the examples [5] discussed above and some SUSY models built by Schellekens and collaborators [7] using CFT techniques in Type II orientifolds have this property. It would be very interesting and important to construct new models with this property using different string constructions. In the heterotic case that will require using $U(N)$ gauge bundles for compactification rather than $SU(N)$.

One point to remark is that, even within that class of models, the existence of the required instanton with the appropriate number of fermionic zero modes is also a strong constraint. For instance, in our toroidal orientifold example among the large class of three generation models which one can build with the wrapping numbers of Table 2, only those satisfying the constraints $|n_c^1| = 1, n_a^2 = \text{multiple of } 3$ have the appropriate zero modes to obtain Majorana neutrino masses. As we saw, this may have a bearing on the possible number of generations and of Higgs multiplets in this class of models. More generally, we expect that imposing the existence of the required D2 instanton in

generic constructions may give constraints on the number of generations and/or Higgs multiplets. It should be interesting to check for those constraints in general models.

The finding of this new source for the generation of Majorana neutrino masses opens the way to the study of the neutrino sector in string models. As in the case of the masses and mixings of quarks and charged leptons, they will depend on the details of the string compactification (brane geometry in the case of an intersecting brane setup). On the other hand the conditions that we have found for the generation of neutrino masses are topological in nature and hence are much easier to implement in a systematic search for a string vacuum consistent with phenomenological data. For example, one may consider the class of CFT MSSM-like models constructed in [7]. To begin with one should then concentrate on the limited set of models with a massive $U(1)_{B-L}$ and then look on whether one may find branes (corresponding to the instantons) with the appropriate intersection numbers (2.9) with the SM branes. That would single out a direction to go in the search for a fully realistic MSSM-like model.

As we argued, an additional important aspect of the proposed Majorana mass generation mechanism is that it is such that a Z_2 subgroup of the massive $U(1)_{B-L}$ generator remains unbroken. In the context of the MSSM this is equivalent to the existence of R-parity, which guarantees the absence of dimension four operators violating baryon or lepton number, a crucial ingredient of the MSSM which is imposed by hand in field theory. We now see that string theory would provide a rationale for the existence of this symmetry which is connected to the generation of neutrino masses.

String instantons may also give rise to other interesting superpotential terms in the low effective action. An important example that we have discussed in the text is the Higgs bilinear, the μ -term in the MSSM. Dimension four operators may also be obtained. For example, it is often the case in specific semirealistic D-brane models that some potential Yukawa coupling is forbidden by some anomalous $U(1)$ symmetry (like e.g. $U(1)_b$). Instanton effects may generate such couplings although the size of those terms will be generically exponentially suppressed compared to other allowed Yukawa couplings. This may be perhaps interesting in connection with the generation of hierarchies of quark/lepton masses.

Coming back to neutrino masses, it is clear that obtaining some specific prediction for the masses and mixings of neutrinos requires the study of concrete models. However one can argue that for a string instanton generation of neutrino Majorana masses it is natural to expect large neutrino mixing, having small mixings would be rather surprising. Indeed, at least from the intuition provided by string intersecting

brane models, one may perhaps understand qualitatively why the mixing among quark flavours given by the CKM matrix is relatively small. That may happen e.g. if there is some approximate left-right symmetry in the geometric distribution of D6-branes. The CKM matrix is related to the unitary matrices which diagonalize the quark masses and the Yukawa couplings depend on the geometry of the wrapping SM branes. The neutrino Dirac mass matrix M_ν^D will also depend on the geometry of the SM D-branes. However we have seen that the origin of the right-handed Majorana neutrino mass is totally different, not only depends on the geometry of some SM branes but also on that of the instanton M generating the coupling. Since the physically measured light neutrino masses depend on the Dirac neutrino mass matrix as well as on the right-handed Majorana mass, having both matrices totally distinct origin in our scheme, no particular correlation is expected which generically implies large neutrino mixing, as experimentally observed.

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A String theory instantons and effective operators

A.1 D-brane models

There are many discussions of brane instanton physics in string and M-theory in the literature (see e.g. [20, 21, 22, 23] among others). However they usually do not many deal with models with non-trivial gauge sectors and matter charged under them. Hence, the appearance of instanton induced operators of the kind we are interested in has not been much discussed, In this appendix we extend results in [3, 4] and discuss the microscopic mechanism for euclidean D-brane instantons to generate effective operators involving the charged fields in a string compactification. The language is completely general, and it is straightforward to particularize to any compactification (either in type IIA or type IIB) with D-branes. For simplicity we ignore orientifold projections, which can be easily incorporated (as done in the discussion in the main text). Despite of this and as discussed in the main text, we use supersymmetric language; in any event the relevant instanton physics is independent of supersymmetry. Also, as discussed in section 2.3, we focus on the relevant instanton zero modes, ignoring the possible presence of additional ones.

Consider a compactification of type IIA or IIB string theory with D-branes leading to four-dimensional gauge interactions and charged chiral fermions. This could be a type IIA compactification with D6-branes on intersecting 3-cycles [9, 10, 11], a type IIB compactification with magnetized D-branes [12, 9, 13], a type IIB model with D-branes at singularities [15, 16, 17], or even non-geometric compactifications like orientifolds of type II Gepner models [18, 19, 7]. In fact, since the ingredients are essentially topological, diverse dualities can be used to draw similar conclusion in other setups, like M-theory on G_2 holonomy manifolds, or F-theory on CY fourfold. The case of heterotic models is particularly interesting and will be discussed in section A.2.

As is by now familiar, we have several stacks, labeled by an index A , of N_A D-branes, denoted D_A branes. Each D-brane is characterized by a vector of RR charges Π_A . This corresponds to the homology charge of the D-brane in geometric compactifications, or to a suitable generalization in other models. These charge vectors admit a decomposition in a basis of D-brane charges C_r as follows

$$\Pi_A = \sum_r p_{Ar} C_r \tag{A.1}$$

where p_{Ar} are integers. The basis of D-brane charges C_r is associated to a basis of RR forms in the 4d theory, which in geometric compactification corresponds to cohomology

basis of the internal space (and to suitable generalizations in more general models). By abuse of language we use C_r to denote the basic D-brane charge and the corresponding cohomology class (or suitable generalization thereof).

The p_{Ar} correspond to the charge of the D_A -brane under the r^{th} RR 4-form of the four-dimensional effective theory. For instance, in geometric compactifications, such 4-form is given by KK reduction of a suitable RR form of the 10d theory over the basis cycle associated to C_r . The equations of motion for these 4d 4-forms imply the RR tadpole cancellation constraints (recall we do not include orientifold planes in the discussion).

$$\sum_A N_A \Pi_A = 0 \tag{A.2}$$

The 4d theory contains a gauge symmetry $\prod_A U(N_A)$. The AB open string sectors provide chiral fermions (chiral multiplets in susy cases), with a multiplicity I_{AB} , in bi-fundamental representations $(\square_A, \overline{\square}_B)$, hence with charges $+1, -1$ under $U(1)_A, U(1)_B$. As usual, opposite signs of I_{AB} indicate conjugate representations. The multiplicity is determined by a topological bilinear in the charge vectors

$$I_{AB} = \langle \Pi_A, \Pi_B \rangle \tag{A.3}$$

For type IIA intersecting brane models, this corresponds to the topological intersection number of the 3-cycle homology classes. For geometric type IIB models with magnetized branes, this is the index of the Dirac operator for a fermions coupled to the $U(1)_A \times U(1)_B$ bundle (or a suitable generalization to sheaves for lower dimensional branes). For D-branes at singularities, it is the adjacency matrix of the quiver diagram (which in the large volume limit corresponds to the just mentioned Dirac index). For abstract CFTs, it is the bilinear described in [33].

The $U(1)_A$ gauge bosons have non-trivial couplings to a set of basic 2-forms B_r (associated to the classes C_r) in the 4d theory, given by

$$S_{BF} = \sum_{A,r} p_{Ar} \int_{4d} B_r \wedge \text{tr} F_A = \sum_{A,r} N_A p_{Ar} \int_{4d} B_r \wedge F_A \tag{A.4}$$

where the factor of N_A arises from the $U(1)$ generator normalization. This implies that, upon $U(1)_B$ gauge transformations

$$A_B \rightarrow A_B + d\Lambda_B \tag{A.5}$$

the scalar a_r , which is the 4d dual to B_r , suffers a shift

$$a_r \rightarrow a_r + \sum_B N_B p_{Br} \Lambda_B \tag{A.6}$$

In geometric compactifications, the scalar a_r is obtained by integration over the cycle D_r dual to C_r of the RR form dual to that leading to B_r (and suitable generalizations for other non-geometric D-brane models).

Some of the $U(1)$'s with BF couplings are anomalous and this coupling is crucial in the Green-Schwarz cancellation of anomalies. However, as emphasized in the main text, we are particularly interested in non-anomalous $U(1)$'s which nevertheless also have these BF couplings.

Let us now consider an euclidean D-brane instanton, namely a D-brane, denoted M , which is localized in all 4d Minkowski dimensions, and has a RR charge vector Π_M . Just as for 4d spacetime filling D-branes, this corresponds to an euclidean D2-brane on a 3-cycle on type IIA geometric models, to a (possibly magnetized) D($2p+1$)-brane on a ($2p+1$)-cycle in geometric type IIB models, or to suitable generalizations in other setups. Expanding Π_M on the dual basis

$$\Pi_M = \sum_r q_{M,r} D_r \quad (\text{A.7})$$

the usual amplitude of the instanton is

$$e^{-S_{inst}} = \exp(- (V_{\Pi_M} + i \sum_r q_{M,r} a_r)) \quad (\text{A.8})$$

This would seem puzzling, since this amplitude is not invariant under $U(1)_a$ gauge transformations

$$e^{-S_{inst}} \rightarrow \exp(-i \sum_{A,r} N_A q_{M,r} p_{A,r} \Lambda_A) e^{-S_{inst}} = \exp(-i \sum_A N_A I_{M,A} \Lambda_A) e^{-S_{inst}} \quad (\text{A.9})$$

The puzzle is solved by the fact that the instanton in general has fermionic zero modes over which one should integrate. Namely, the sector of open strings stretching between the euclidean D-brane and the A^{th} background D-brane leads to $I_{M,A}$ fermionic zero modes for the instanton (here $I_{M,a} = \langle \Pi_M, \Pi_a \rangle$), transforming in the bi-fundamental $(\square_M, \bar{\square}_A)$. It is convenient to split the set of D_A -branes into two subsets, labeled by indices P, Q , with $I_{M,P}$ and $I_{M,Q}$ positive and negative respectively. We also denote $\alpha_{i_P}^P$, $\beta_{j_Q}^Q$ the corresponding instanton fermion zero modes, with the indices $i_P = 1, \dots, I_{M,P}$, $j_Q = 1, \dots, |I_{M,Q}|$ labeling the multiplicity in a given sector. Notice that due to the RR tadpole cancellation (A.2), the numbers N_α , N_β of positive and negative chirality fermion zero modes are equal

$$N_\alpha - N_\beta = \sum_P N_P I_{M,P} - \sum_Q N_Q I_{M,Q} = \sum_A N_A I_{M,A} = \langle \Pi_M, \sum_A N_A \Pi_A \rangle = 0 \quad (\text{A.10})$$

In general, the instanton zero modes have non-trivial cubic couplings with fields $\Phi_{k_{PQ}}^{PQ}$, with $k_{PQ} = 1, \dots, I_{PQ}$ in the PQ open string sector of the background D-branes, of the form

$$S_{z.m.} = d_{i_P j_Q k_{PQ}} \alpha_{i_P}^P \beta_{j_Q}^Q \Phi_{k_{PQ}}^{PQ} \quad (\text{A.11})$$

Integration over the Grassman variables α, β in the instanton path integral, leads to a term proportional to the determinant of the $N \times N$ matrix Φ , with $N = \sum_P N_P I_{M,P} = \sum_Q N_Q I_{M,Q}$. This term, which we denote $(\det \Phi)$ for short, is a prefactor that accompanies the exponential (A.8) in the complete instanton amplitude. Since it is roughly an order N polynomial in fields in the AB sector, under the $U(1)$ gauge transformations, it transforms as

$$\det \Phi \rightarrow \exp\left(i \sum_P N_P I_{M,P} - i \sum_Q N_Q I_{M,Q}\right) \det \Phi = \exp\left(i \sum_A N_A I_{M,A} \Lambda_A\right) \det \Phi$$

which precisely cancels the transformation of the exponential, leading to a gauge invariant 4d interaction.

A.2 Heterotic models

One can carry out a similar discussion for heterotic models (see [34] for early discussions). In fact, compactifications of the heterotic strings with $U(N)$ gauge bundles (as opposed to $SU(N)$ bundles) lead to 4d theories with a structure of gauge factors (and most notably of $U(1)$ factors) similar to that in D-brane models in the previous section. This has been discussed in [35]. This is nicely consistent with S-duality of type I and the $SO(32)$ heterotic models. In the following we focus on instanton effects on such geometric $SO(32)$ heterotic constructions⁸.

Focusing on abelian bundles, the backgrounds are most simply described by regarding each of the Cartan generators Q_A of $SO(32)$ as an antisymmetric 2×2 block, which plays a role similar to a D9-brane and its orientifold image in a type I compactification. Hence a non-trivial abelian field strength 2-form

$$F = \sum_{A=1}^{16} F_A \quad (\text{A.12})$$

(where F represents an abelian $SO(32)$ matrix and F_a is a $SO(32)$ matrix with entries only in the a^{th} $SO(2)$ block) is completely similar to turning on a field strength F_a in the a^{th} D9-brane (and $-F_a$ in its image) in a type I model.

⁸One can also consider the $E_8 \times E_8$ theory, which is conceptually similar, with differences only at the group-theoretical level

Moreover, the structure of 2-forms and their dual scalars in the 4d-theory is also similar to that of type I models. In the KK reduction of the heterotic string, the 10d 2-form leads to a universal 4d 2-form B_0 . In addition, one can integrate the 10d 6-form over the $h_{1,1}$ independent 4-cycles C_r in the Calabi-Yau to obtain further 4d 2-forms

$$B_r = \int_{C_r} B_6 \quad (\text{A.13})$$

These 2-forms couple to the 4d $U(1)$ gauge fields. These couplings arise from the 10d Chern-Simons couplings

$$\begin{aligned} S_{CS1} &= \int_{10d} B_2 \wedge \text{tr}(F \wedge F \wedge F \wedge F) \\ S_{CS2} &= \int_{10d} B_6 \wedge \text{tr}(F \wedge F) \end{aligned} \quad (\text{A.14})$$

which are crucial for the 10d Green-Schwarz mechanism. Namely, the first leads upon KK reduction to

$$S_{CS1,4d} = N_{AP_0} \int_{4d} B_0 \wedge F_A \quad (\text{A.15})$$

with $p_{A0} = \int_{CY} \text{tr} F_A^3$. On the other hand, from the second kind of 10d coupling we obtain the 4d couplings

$$S_{CS2,4d} = N_A p_{Ai} \int_{4d} B_r \wedge F_A \quad (\text{A.16})$$

where

$$\int_{D_r} F_A = p_{Ar} \quad (\text{A.17})$$

and D_r is the 2-cycle dual to C_r . As usual the N_a factor arises from the $U(1)$ normalization.

The dual scalars a_r , therefore suffer a shift under $U(1)_A$ gauge transformations, given by (A.6). For the scalars dual to the 2-forms B_r , $r \neq 0$, the coupling to the $U(1)$ factors can be recovered in a language more familiar in the heterotic literature, as follows. The field strength for the 10d 2-form roughly has the structure

$$H_{MNP} = \partial_{[M} B_{NP]} + A_{[M} F_{NP]} \quad (\text{A.18})$$

with the additional piece required to yield the anomalous Bianchi identity. The mixed terms in the 10d kinetic term for H_{MNP} lead to 10d couplings

$$\int_{10d} d^{10}x \partial_{[M} B_{NP]} A_{[M} F_{NP]} \quad (\text{A.19})$$

which upon KK reduction lead to the 4d couplings

$$N_B p_{B_r} \int_{4d} d^4x \partial_\mu a_r A_{B,\mu} \quad (\text{A.20})$$

where $a_r = \int_{D_r} B_2$ is the scalar dual to B_r above. These couplings are equivalent to (A.16), and imply the mentioned scalar shift.

The shift of the scalars under $U(1)$ gauge transformations renders the exponential amplitudes of certain instantons naively non-gauge-invariant. Specifically, the generic such instanton will be a bound state of an euclidean NS5-brane wrapped on the whole Calabi-Yau and euclidean fundamental strings wrapped on 2-cycles. This bound state admits an explicit realization as a magnetized NS5-branes, namely NS5-branes with a non-trivial background for its worldvolume symplectic gauge field. The discussion is however insensitive to this detailed realization, and only depends on the vector of charges $(q_0; q_i)$ of the bound state (where q_0 denotes the NS5-brane charge and q_i the charge of fundamental strings wrapped on D_i). The naive amplitude of such instanton clearly shifts as

$$e^{-S_{inst}} \rightarrow \exp\left(-i \sum_{A,r} N_A q_{M,r} p_{a,r} \Lambda_A\right) e^{-S_{inst}} \quad (\text{A.21})$$

which is in fact identical to (A.9).

This phase will in fact be canceled by the appearance of spacetime charged fields, due to integration over zero modes of the instantons. The microscopic description of instantons with $q_0 \neq 0$ is not available, since it involves heterotic NS5-branes. The results for this can nevertheless be derived by simply dualizing results from type I models, which we leave as an exercise. We rather focus on the case $q_0 = 0$ which is in fact very interesting since it corresponds to a world-sheet instanton on the curve $D = \sum_r q_r D_r$, for which one has a microscopic description. Thus we can directly compute the field-dependent prefactor and verify the gauge invariance of the complete instanton amplitude.

In the fermionic world-sheet formulation of the $SO(32)$ heterotic, there are 32 2d fermions in the fundamental of $SO(32)$, which couple to the spacetime gauge field A in the adjoint as $A\lambda\lambda$. Let us label as λ^B, λ^{B^*} , $a = 1, \dots, 16$, and split the $SO(32)$ adjoint field accordingly. Then the coupling becomes

$$A^{BC} \lambda^B \lambda^C + A^{B^*C} \lambda^{B^*} \lambda^C + A^{BC^*} \lambda^B \lambda^{C^*} + A^{B^*C^*} \lambda^{B^*} \lambda^{C^*} \quad (\text{A.22})$$

The analogy with an euclidean D1-brane instanton in a type I model should be clear at this point. In fact, the KK reduction of the $SO(32)$ gauge field in a given sector

e.g. AB , lead to $4d$ chiral fields $\phi_{k_{AB}}^{AB}$ in the corresponding AB sector. Here the label $k_{AB} = 1, \dots, I_{AB}$ takes into account the multiplicity I_{AB} given by the index of the Dirac operator for a field with charges ± 1 under $U(1)_A \times U(1)_B$. Also, the KK reduction of the 2d fermions e.g. λ_A lead to $I_{M,A}$ instanton fermionic zero modes, where $I_{M,A}$ is the index of the Dirac operator for a field with charges $+1$ under $U(1)_A$. Hence the instanton contains cubic couplings among the I_{MA}, I_{MB} fermion zero modes α, β and the spacetime fields ϕ . The integration over fermion zero modes leaves a determinant in the latter, whose transformation cancels the phase of the exponential, as should be familiar by now.

An interesting point to emphasize is that, for $q_0 = 0$, the relevant instantons are world-sheet instantons, hence they are not suppressed by g_s . Rather they are tree-level in the string coupling, but non-perturbative in α' .

B Some further intersecting brane examples

In this appendix we describe how neutrino Majorana mass terms may appear in the family of models considered in [5]. We refer to that paper for notation and details. One important difference with the family of models in the main text is that the $SU(2)_L$ gauge group comes from a $U(2)_b$ group and the number of generations is fixed to three from the start. The wrapping numbers of the SM D6-branes in this family of models are given by Table 3. The models are parametrized by a phase $\epsilon = \pm 1$, four integers

N_i	(n_i^1, m_i^1)	(n_i^2, m_i^2)	(n_i^3, m_i^3)
$N_a = 3$	$(1/\beta^1, 0)$	$(n_a^2, \epsilon\beta^2)$	$(1/\rho, 1/2)$
$N_b = 2$	$(n_b^1, -\epsilon\beta^1)$	$(1/\beta^2, 0)$	$(1, 3\rho/2)$
$N_c = 1$	$(n_c^1, 3\rho\epsilon\beta^1)$	$(1/\beta^2, 0)$	$(0, 1)$
$N_d = 1$	$(1/\beta^1, 0)$	$(n_d^2, -\beta^2\epsilon/\rho)$	$(1, 3\rho/2)$

Table 3: D6-brane wrapping numbers giving rise to a SM spectrum as in ref.[5].

$n_a^2, n_b^1, n_c^1, n_d^2$ and a parameter $\rho = 1, 1/3$. In addition $\beta^i = 1 - b^i = 1, 1/2$ depending

on whether the corresponding tori are tilted or not. Such classes of models have in principle up to four $U(1)$ gauge fields, but generically three of them acquire Stückelberg masses due to the $B \wedge F$ couplings. In particular one has

$$\begin{aligned}
B_2^1 &\wedge \frac{-4\epsilon\beta^1}{\beta^2} F^b \\
B_2^2 &\wedge \frac{(2\epsilon\beta^2)}{\rho\beta^1} (3F^a - F^d) \\
B_2^3 &\wedge \frac{1}{\beta^2} \left(\frac{3\beta^2 n_a^2}{\beta^1} F^a + 6\rho n_b^1 F^b + 2n_c^1 F^c + \frac{3\rho\beta^2 n_d^2}{\beta^1} F^d \right)
\end{aligned} \tag{B.1}$$

The four scalar fields a_i transform with a shift under $U(1)$ transformations as

$$\begin{aligned}
a_0 &\rightarrow a_0 \\
a_1 &\rightarrow a_1 - \frac{4\epsilon\beta^1}{\beta^2} \Lambda(x)_b \\
a_2 &\rightarrow a_2 + \frac{2\epsilon\beta^2}{\rho\beta^1} (3\Lambda(x)_a - \Lambda(x)_d) \\
a_3 &\rightarrow a_3 + \frac{3n_a^2}{\beta^1} \Lambda_a(x) + \frac{6\rho n_b^1}{\beta^2} \Lambda_b(x) + \frac{2n_c^1}{\beta^2} \Lambda_c(x) + \frac{6\rho n_d^2}{2\beta^1} \Lambda_d(x)
\end{aligned} \tag{B.2}$$

There is just one linear combination of $U(1)$'s which remains massless. That linear combination is precisely the standard hypercharge $U(1)_Y$ as long as one has the constraint

$$n_c^1 = \frac{\beta^2}{2\beta^1} (n_a^2 + 3\rho n_d^2) \tag{B.3}$$

It is easy to check that there is a unique factorizable 3-cycle which may be wrapped by a D2-instanton with the required zero modes:

$$(n_1, m_1), (n_2, m_2), (n_3, m_3) = (3\rho n_b^1, -\beta^1), (\rho n_a^2, \beta^2), (-\epsilon, 1/2) \tag{B.4}$$

as long as

$$\beta^1 n_c^1 = 1 \quad , \quad \rho n_a^2 = \text{integer} \tag{B.5}$$

Such an instanton would generate a coupling

$$e^{-V_{\Pi M}} e^{\frac{1}{2}(6\epsilon\rho^2 n_b^1 n_a^2 a_0 - 3\rho n_b^1 \beta^2 a_1 + \rho n_a^2 \beta^1 a_2 - 2\epsilon\beta^1 \beta^2 a_3)} \mathcal{V}_R^i \mathcal{V}_R^j \quad ; \quad i, j = 1, 2, 3 \tag{B.6}$$

which is fully gauge invariant. As an example consider the choice of parameters

$$n_a^2 = 3 \quad ; \quad n_c^1 = n_d^2 = \beta^1 = 1 \quad ; \quad n_b^1 = -1 \quad ; \quad \rho = 1/3 \quad ; \quad \beta^2 = 1/2 \quad ; \quad \epsilon = 1 \tag{B.7}$$

Then the relevant instanton M and branes c^* and d have wrapping numbers:

$$M : (1, 1)(1, 1/2)(1, -1/2) \quad (\text{B.8})$$

$$c^* : (1, -1)(2, 0)(0, -1) \quad (\text{B.9})$$

$$d : (1, 0)(1, -3/2)(1, 1/2) \quad (\text{B.10})$$

and $I_{Md} = -I_{Mc^*} = 2$, $I_{Md^*} = I_{Mc} = 0$, as required.

C A μ -term example

Here we will provide an explicit MSSM-like example in which the conditions (4.3) for the generation of a Higgs bilinear are met. We will consider the MSSM-like model in section 6.1 of [36], we refer the reader to that paper and the references therein for more details. This is a toroidal $Z_2 \times Z_2$ orientifold with D6-branes with wrapping numbers as in table (4). In order to cancel RR-tadpoles one can add a single coisotropic D8-brane

N_i	(n_i^1, m_i^1)	(n_i^2, m_i^2)	(n_i^3, m_i^3)
$N_a = 6 + 2$	(1, 0)	(3, 1)	(3, -1/2)
$N_b = 4$	(1, 1)	(1, 0)	(1, -1/2)
$N_c = 2$	(0, 1)	(0, -1)	(2, 0)
$N_X = 4$	(-2, 1)	(-3, 1)	(-3, 1/2)
$N_O = 6$	(1, 0)	(1, 0)	(1, 0)

Table 4: D6-brane wrapping numbers giving rise to the MSSM-like model in the text.

stack as in [36] or else two D6-brane stacks as in table 4, it is not relevant for the present discussion. The gauge group after one takes into account those $U(1)$'s getting Stückelberg masses is that of the SM plus an additional $B - L$ (and a $SU(2)_X$ 'hidden sector' group). Since in this model $B - L$ is massless the generation of a Majorana neutrino mass operator is not possible. The spectrum may be found in table 5 of [36]

(the multiplicity of the G, \bar{G} states in that table is 6 instead of 5 if we use D6-branes X,O instead of a D8-brane). It corresponds to a three generation MSSM-like spectrum with a minimal set of Higgs fields H, \bar{H} plus additional chiral fields transforming under the electroweak and the $SU(2)_X$ 'hidden group'. It can be checked that this model preserves $N = 1$ SUSY and all RR-tadpoles cancel. It is easy to check that a D2 instanton wrapping the 3-cycle M

$$M = 2(1, 0)(1, -1)(1, 1/2) \tag{C.1}$$

has the appropriate intersection numbers in eq.(4.3), i.e. $I_{Mb} = -1, I_{Mb^*} = 0, I_{Mc} = I_{Mc^*} = 1$. In addition it preserves the same $N = 1$ SUSY as the D6-branes. One has $\text{Im } S_{D2} = a_0 - \frac{1}{2}a_1$ so that under a $U(1)_b$ gauge transformation of parameter $\Lambda_b(x)$ one has

$$S_{D2} \longrightarrow S_{D2} - i2\Lambda_b(x) \tag{C.2}$$

so that indeed the operator $\exp(-S_{D2})H\bar{H}$ is gauge invariant under $U(1)_a, U(1)_b$, and $U(1)_c$ as expected. However one can check that in the present example there are non-vanishing intersections $I_{MX} = -4, I_{MX^*} = 4$ between the instanton and the auxiliary branes X (or their coisotropic D8-brane analogues) which are added to cancel RR-tadpoles. This implies that $\exp(-S_{D2})$ is also charged under $U(1)_X$ and that the actual operator which could be induced will have an additional factor involving fields charged under the 'hidden sector' gauge group $U(2)_X$.

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