

Neutrino Masses in E_6 Unification

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We show that neutrino masses with large mixing between ν_μ and ν_τ are naturally reproduced in a supersymmetric E_6 grand unification model with an anomalous $U(1)_X$ symmetry. We propose a simple scenario which incorporates a novel mechanism called ‘ E -twisting’ by which all the characteristic features of the fermion mass matrices, not only the quark/lepton Dirac masses but also the neutrino’s Majorana masses, can be reproduced despite the fact that all the members in **27** of each generation are assigned a common $U(1)_X$ charge.

§1. Introduction

A recent report from Superkamiokande strongly indicates two important facts¹⁾ that have led us to unification groups including left-right symmetry. The first fact is the extremely small masses of the neutrinos. This is most naturally explained by the existence of right-handed neutrinos with their Majorana masses of order of 10^{13} GeV.²⁾ The existence of right-handed neutrinos implies that Nature prefers left-right symmetry, and we are led to a left-right symmetric unification group like $SO(10)$. The second fact is the large neutrino mixing angle, which is needed to explain the zenith angle dependence of the observed atmospheric muon neutrino spectrum. If one wishes to understand the mass matrices in unified theories larger than $SU(5)$, this large neutrino mixing angle is apparently unnatural compared with the ordinary quark lepton mass matrices. In $SO(10)$ GUT, for instance, quarks and leptons appear on the same footing, and Dirac-type masses for the neutrinos and charged leptons should be parallel to those for the up- and down-type quarks, respectively. Thus the most natural prediction would be that the neutrino mixing is also very small with hierarchical mass structure among generations. This naive prediction of parallelism between quark and lepton sectors, however, was found to be maximally violated in the neutrino sector. Thus the neutrino masses and mixings seem to require something beyond even $SO(10)$.^{*)}

Here we would like to remark also that the up and down quark mass matrices already violate the parallelism: The hierarchical structure of the up quark sector is

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*) If the right-handed neutrino Majorana mass matrix M_R can be chosen freely, then we can have any desired form for the left-handed neutrino Majorana mass matrix $M_\nu = M_D^T M_R^{-1} M_D$, whatever Dirac mass matrix M_D is used for the neutrino in the GUT. But the problem is whether the form of M_R can be derived in a given GUT framework.³⁾ In the scenarios which use the Frogatt-Nielsen mechanism, it seems that M_R always has a hierarchical structure.

far stronger than that of down quark sector. Also it has long been one of the most important problems why the top quark is far heavier than the bottom quark.

If one wishes to explain the inter-generation hierarchical mass structure, one natural possibility is the Frogatt-Nielsen mechanism⁴⁾ using an anomalous $U(1)_X$ charge. However, if the unified gauge group is $SO(10)$ or larger, all the fermion members in each generation have a common $U(1)_X$ charge quantum number, so that the naive application of the Frogatt-Nielsen mechanism predicts a completely parallel hierarchical mass matrix for all the quarks and leptons, contradicting observations. The naive prediction of this complete parallelism between quark and lepton mass matrices is, however, a result of our prejudice that the three generations of fermions are a mere repetition of the same structure, namely, a parallel family structure. If the parallelism between families is broken, then the parallelism between mass matrices of quarks and leptons can also be avoided. The importance of breaking parallel family structure was first emphasized by Yanagida.⁵⁾ Then the question is how the unification model, and what unification group, can avoid this parallel family structure. Among the simple groups larger than $SO(10)$, E_6 is essentially the unique group that admits complex representations in a manner consistent with the chiral structure of our low-energy fermions. The exceptional group E_6 has been investigated by many authors^{6)–11), 15), 12), 13)} as an attractive unification group, because of its desirable features: 1) it is automatically anomaly-free; 2) all the basic fermions of one generation belong to a single irreducible representation **27**; and 3) all the Higgs fields necessary for symmetry breaking are supplied by the fermion bilinears. On the other hand, 10 dimensional $E_8 \times E_8$ heterotic string theory has been thought to be very attractive as a unified theory including gravity from which the low energy $N = 1$ supersymmetric standard model may be derived.¹⁴⁾ The Calabi-Yau compactification into 4 dimensions may naturally produce E_6 gauge symmetry. In this kind of string model, there exists an anomalous $U(1)$ which is to be cancelled by the so-called Green-Schwarz mechanism.¹⁵⁾ In supersymmetric model this anomalous $U(1)$ was found to play an essential role for explaining the hierarchy of fermion masses via Frogatt-Nielsen mechanism.¹⁶⁾

The purpose of this paper is to show that there is a novel and natural mechanism in a supersymmetric E_6 grand unification model with an anomalous $U(1)_X$ symmetry, with which the parallel family structure can be avoided and all the characteristic features of the fermion mass matrices (not only the quark/lepton Dirac masses but also the neutrino's Majorana masses) can be reproduced. We propose and examine a very simple scenario which incorporates this novel mechanism called ' E -twisting'.

In §2 we present the basic framework of our E_6 model and point out that there is a freedom of an $SU(2)_E$ inner automorphism in E_6 in embedding $SO(10)$ such that $SU_{GG}(5) \subset SO(10) \subset E_6$. As a reflection of this $SU(2)_E$ symmetry, the fundamental representation **27** contains two $SU(5)$ **5*** and two **1** components, both giving spinor representations of $SU(2)_E$. This $SU(2)_E$ transformation plays an important role in explaining the large mixing angle of neutrinos while keeping very small family mixings in the quark sector. In §3 we determine the $U(1)_X$ charges assigned to the three generation matter superfields $\Psi_i(\mathbf{27})$ belonging to the **27** representations of E_6 . In §4 we discuss **5*** family structure; namely, we consider which three **5*** components

become the usual down quarks and charged leptons among the six $\mathbf{5}^*$ components in the three $\mathbf{27}$ representations. Describing three typical options, parallel family structure, nonparallel family structure, and E -twisted structure, we argue that the E -twisted structure gives the simplest option which can reproduce the down quark spectrum and the Cabibbo-Kobayashi-Maskawa (CKM) mixings, and, at the same time, the large lepton mixing angle. In §5 we examine the right-handed neutrino Majorana masses which naturally come from higher dimensional operators in the present scenario. Section 6 is devoted to the summary and discussion.

§2. E_6 unification model and E -symmetry

We consider here the simplest example of a supersymmetric E_6 unification model which reproduces the large neutrino mixing angle as well as the quark and lepton masses and mixings.

Aside from the E_6 gauge vector multiplet, we introduce several chiral multiplets which belong to the fundamental representation $\mathbf{27}$ or $\mathbf{27}^*$, or singlet $\mathbf{1}$, and carry a quantum number (denoted by X) of anomalous $U(1)_X$.

1. The quarks and the leptons of the three generations are included in three chiral multiplets Ψ_i ($i = 1, 2, 3$) of the $\mathbf{27}$ representation, to which we assign odd R -parity and $U(1)_X$ charges $X = f_i$. (The values f_i will be determined later.)
2. We introduce two pairs of Higgs fields, (H^h, \bar{H}^{-h}) and $(\Phi^x, \bar{\Phi}^{-x})$, of $\mathbf{27}$ and $\mathbf{27}^*$ chiral multiplets, which are R -parity even and carry the $U(1)_X$ charges $X = \pm h$ and $X = \pm x$, respectively. Their neutral components develop vacuum expectation values (VEVs) and give superheavy masses to the fermions other than the usual low-energy quarks and leptons and also the light masses to the ordinary fermions.
3. In addition to these, we introduce an E_6 singlet field Θ with $X = -1$ and R -parity even that supplements the Yukawa couplings so as to match the $U(1)_X$ charge.

Then the R -parity, $U(1)_X$ and E_6 invariant superpotentials giving Yukawa interactions are

$$\begin{aligned} W_Y(H) &= y \Psi_i(\mathbf{27}) \Psi_j(\mathbf{27}) H(\mathbf{27}) \left(\frac{\Theta}{M_P} \right)^{f_i+f_j+h}, \\ W_Y(\Phi) &= y' \Psi_i(\mathbf{27}) \Psi_j(\mathbf{27}) \Phi(\mathbf{27}) \left(\frac{\Theta}{M_P} \right)^{f_i+f_j+x}. \end{aligned} \quad (2.1)$$

The coupling constants y and y' may generally depend on (i, j) , but we assume they are all of order 1, and so we can suppress the (i, j) dependence in the present order of magnitude discussion. The hierarchical mass structure of fermions is explained via the Frogatt-Nielsen mechanism; that is, the powers of the E_6 singlet field Θ in this higher dimensional Yukawa coupling (2.1) are required by $U(1)_X$ quantum number matching, and the effective Yukawa couplings are suppressed by corresponding powers of $\lambda = \langle \Theta \rangle / M_P$. We assume $\lambda = \langle \Theta \rangle / M_P \sim 0.22$ henceforth.

Each fundamental representation $\Psi(\mathbf{27})$ is decomposed under $SO(10) \times U(1)_{V'} \subset$

E_6 as

$$\begin{aligned} \mathbf{27} &= \mathbf{16}_1 + \mathbf{10}_{-2} + \mathbf{1}_4, & (\alpha = 1, 2, \dots, 16) \\ \Psi_A : \quad \psi_\alpha & \quad \psi_M \quad \psi_0 & (M = 1, 2, \dots, 10) \end{aligned} \tag{2.2}$$

and under $SU(5) \times U(1)_V \subset SO(10)$ as

$$\begin{aligned} \mathbf{16} &= \mathbf{10}_{-1} + \mathbf{5}_3^* + \mathbf{1}_{-5}, \\ \psi_\alpha : \quad & \left[u^{ci}, \begin{pmatrix} u_i \\ d_i \end{pmatrix}, e^c \right] \quad (d^{ci}, e, -\nu) \quad \nu^c \end{aligned} \tag{2.3}$$

$$\begin{aligned} \mathbf{10} &= \mathbf{5}_2 + \mathbf{5}_{-2}^*, \\ \psi_M : \quad & \begin{pmatrix} D_i \\ E^c \\ -N^c \end{pmatrix} \quad (D^{ci}, E, -N) \end{aligned} \tag{2.4}$$

$$\begin{aligned} \mathbf{1} &= \mathbf{1}_0. \\ \psi_0 : \quad & S \end{aligned} \tag{2.5}$$

Note that the representations $\mathbf{5}^*$ and $\mathbf{1}$ of $SU(5)$ appear twice here, while $\mathbf{10}$ and $\mathbf{5}$ appear only once. This suggests that we can define a parity-like transformation, which we call ‘ E -parity’:

$$\begin{aligned} E\text{-parity transformation:} \quad \Psi(\mathbf{16}, \mathbf{5}^*) &\leftrightarrow \Psi(\mathbf{10}, \mathbf{5}^*), \\ \Psi(\mathbf{16}, \mathbf{1}) &\leftrightarrow \Psi(\mathbf{1}, \mathbf{1}). \end{aligned} \tag{2.6}$$

The other $SU(5)$ components, $\Psi(\mathbf{16}, \mathbf{10})$ and $\Psi(\mathbf{10}, \mathbf{5})$, remain intact. Actually, this E -parity turns out to be a π rotation of a certain $SU(2) \subset E_6$: Indeed, even if we fix the $SU(5)$ subgroup in E_6 as the usual $SU(5)_{\text{GG}}$ of Georgi-Glashow, the embedding of $SO(10)$ into E_6 such that $SU(5)_{\text{GG}} \subset SO(10) \subset E_6$ is not unique, but possesses a freedom of rotation of $SU(2)$.*)

To see this fact it would be easiest if we consider another maximal subgroup $SU(6) \times SU(2)_E \subset E_6$, where the former $SU(6)$ contains $SU(5)_{\text{GG}} \times U(1)_Z$ such that the first 5 entries of the fundamental representation $\mathbf{6}$ give the fundamental representation $\mathbf{5}$ of $SU(5)_{\text{GG}}$. Under this $SU(5)_{\text{GG}} \times SU(2)_E \times U(1)_Z \subset SU(6) \times SU(2)_E \subset E_6$, the $\mathbf{27}$ is decomposed into

$$\mathbf{27} = \underbrace{(\mathbf{5}^*, \mathbf{2})_{-1} + (\mathbf{1}, \mathbf{2})_5}_{(\mathbf{6}^*, \mathbf{2})} + \underbrace{(\mathbf{10}, \mathbf{1})_2 + (\mathbf{5}, \mathbf{1})_{-4}}_{(\mathbf{15}, \mathbf{1})}. \tag{2.7}$$

) The so-called “flipped $SU(5)$ ” gives another example similar to this situation. There, the embedding of $SU(5)$ into $SO(10)$ such that $SU(3)_c \times SU(2)_L \subset SU(5) \subset SO(10)$ holds, is not unique but possesses a freedom of rotation of $SU(2)$. This $SU(2)$ is in fact the usual $SU(2)_R$ of $SU(4)_{\text{Pati-Salam}} \times SU(2)_L \times SU(2)_R \subset SO(10)$. The $SU(5)$ group of the flipped $SU(5)$ model is given from the usual $SU(5)_{\text{GG}}$ by a π rotation of $SU(2)_R$. This can be understood by an argument very similar to that given in the next paragraph, by considering the decomposition of $SO(10)$ $\mathbf{16}$ under the subgroups $SU(3)_c \times SU(2)_L \times SU(2)_R \subset SU(4) \times SU(2)_L \times SU(2)_R$; $\mathbf{16} = (\mathbf{4}, \mathbf{2}_L, \mathbf{1}_R) + (\mathbf{4}^, \mathbf{1}_L, \mathbf{2}_R) = (\mathbf{3}_c, \mathbf{2}_L, \mathbf{1}_R) + (\mathbf{1}_c, \mathbf{2}_L, \mathbf{1}_R) + (\mathbf{3}_c^*, \mathbf{1}_L, \mathbf{2}_R) + (\mathbf{1}_c, \mathbf{1}_L, \mathbf{2}_R)$.

This decomposition of $\mathbf{27}$ clearly shows that the two $\mathbf{5}^*$ as well as $\mathbf{1}$ belong to a doublet of $SU(2)_E$, and hence they are rotated into each other as a spinor by this $SU(2)_E$. In view of the $U(1)_{V'}$ and $U(1)_V$ charges of $\mathbf{5}^*$ appearing in Eqs. (2.2) – (2.4) and the $U(1)_Z$ charge here, we can see that the following identifications are possible between these $U(1)$ charges and the third component E_3 of $SU(2)_E$ generators E_j ($j = 1, 2, 3$):

$$V = 5E_3 - \frac{1}{2}Z, \quad V' = 3E_3 + \frac{1}{2}Z. \quad (2.8)$$

Collecting the representation components carrying the same V' charge, we easily find that the $SO(10)$ multiplets $\mathbf{16}$, $\mathbf{10}$ and $\mathbf{1}$ are formed as follows:

$$\begin{aligned} \mathbf{16}_1 &= (\mathbf{5}^*, E_3 = +1/2)_{-1} + (\mathbf{1}, E_3 = -1/2)_5 + (\mathbf{10}, \mathbf{1})_2, \\ \mathbf{10}_{-2} &= (\mathbf{5}^*, E_3 = -1/2)_{-1} + (\mathbf{5}, \mathbf{1})_{-4}, \\ \mathbf{1}_4 &= (\mathbf{1}, E_3 = +1/2)_5. \end{aligned} \quad (2.9)$$

But this identification of $SO(10)$ must be possible even after performing an arbitrary $SU(2)_E$ rotation $\exp(i\theta^j E_j)$. Then the $SO(10)$ generators $T_{MN}(\boldsymbol{\theta})$ for the case of an angle $\boldsymbol{\theta}$ are given by $\exp(i\boldsymbol{\theta} \cdot \mathbf{E}) T_{MN} \exp(-i\boldsymbol{\theta} \cdot \mathbf{E})$ in terms of the original $SO(10)$ generators T_{MN} . This $\exp(i\boldsymbol{\theta} \cdot \mathbf{E})$ defines an inner automorphism of E_6 which gives a three parameter family of embeddings of $SO(10)$ into E_6 such that $SU(5)_{\text{GG}} \subset SO(10) \subset E_6$ holds. We may call this $\exp(i\boldsymbol{\theta} \cdot \mathbf{E})$ E -symmetry, and the E -parity transformation introduced above is merely a special case of $\theta = \pi$, $\exp(i\pi E_1)$ (or $\exp(i\pi E_2)$).*) Indeed, with this $\exp(i\pi E_1)$ rotation, the E_3 generator is rotated into $E'_3 = -E_3$, and hence the $E_3 = \pm 1/2$ eigenstates of the spinor representation are interchanged, implying that the two $\mathbf{5}^*$ states in $\mathbf{16}$ and $\mathbf{10}$, as well as the two $\mathbf{1}$ states in $\mathbf{16}$ and $\mathbf{1}$, in Eq. (2.9) are interchanged.

In Eq. (2.1) the Yukawa couplings for individual components are determined by the E_6 group relation: The E_6 invariant trilinear in $\mathbf{27}$ is given by a totally symmetric tensor Γ^{ABC} in the form⁸⁾

$$\begin{aligned} \Gamma^{ABC} \Psi_{1A} \Psi_{2B} \Psi_{3C} &= \psi_{1M} \psi_{2M} \psi_{30} + \psi_{1\alpha}^T \left(\psi_{2M} \frac{C\sigma_M}{\sqrt{2}} \right)^{\alpha\beta} \psi_{3\beta} \\ &+ [\text{cyclic permutations of } (1, 2, 3)], \end{aligned} \quad (2.10)$$

where ψ_{jM} , $\psi_{j\alpha}$ and ψ_{j0} are the $SO(10)$ vector, spinor and singlet components of Ψ_j defined in Eq. (2.2), and σ_M and C are the 16×16 γ -matrix and the charge conjugation matrix of $SO(10)$, respectively. (See the Appendix for a more explicit component expression of Eq. (2.10).) For our present purpose of discussing the fermion masses and mixings coming from the VEVs of the Higgs fields, we list here the explicit forms for the terms in $\Gamma^{ABC} \Psi_{1A} \Psi_{2B} H_C$ containing the neutral Higgs

*) It may worth noting that this E -symmetry group $SU(2)_E$ is identical with an $SU(2)$ subgroup of $SU(3)_R$ which appears in the famous maximal subgroup $SU(3)_c \times SU(3)_L \times SU(3)_R$ of E_6 . But $SU(2)_E$ is the $SU(2)$ subgroup of $SU(3)_R$ acting on the second and third entries of $\mathbf{3}$ of $SU(3)_R$, while the usual $SU(2)_R$ of left-right symmetric theory is another $SU(2)$ subgroup of $SU(3)_R$ acting on the first and second entries of $\mathbf{3}$. See the Appendix for details.

components $H(\mathbf{1}), H(\mathbf{16}, \mathbf{1}), H(\mathbf{16}, \mathbf{5}^*), H(\mathbf{10}, \mathbf{5}^*)$ and $H(\mathbf{10}, \mathbf{5})$, which correspond to the components $S, \nu^c, -\nu, -N$ and $-N^c$, respectively, of $\Psi(\mathbf{27})$ in Eqs. (2.2) – (2.4):

$$H(\mathbf{1}) \quad \left[\begin{array}{c} (D^{ci} E - N)_1 \begin{pmatrix} D_i \\ E^c \\ -N^c \end{pmatrix}_2 + (1 \leftrightarrow 2) \end{array} \right], \quad \mathbf{10}(\mathbf{5}^*) \times \mathbf{10}(\mathbf{5}) \quad (2.11)$$

$$H(\mathbf{16}, \mathbf{1}) \quad \left[\begin{array}{c} -(D_i E^c - N^c)_1 \begin{pmatrix} d^{ci} \\ e \\ -\nu \end{pmatrix}_2 + (1 \leftrightarrow 2) \end{array} \right], \quad \mathbf{10}(\mathbf{5}) \times \mathbf{16}(\mathbf{5}^*) \quad (2.12)$$

$$H(\mathbf{16}, \mathbf{5}^*) \quad \left[\begin{array}{c} -\nu_1^c N_2^c + (D_1^{ci} d_{i2} + E_1 e_2^c) + (1 \leftrightarrow 2) \end{array} \right], \quad \mathbf{16}(\mathbf{1}) \times \mathbf{10}(\mathbf{5}) \quad \mathbf{10}(\mathbf{5}^*) \times \mathbf{16}(\mathbf{10}) \quad (2.13)$$

$$H(\mathbf{10}, \mathbf{5}^*) \quad \left[\begin{array}{c} S_1 N_2^c - (d_1^{ci} d_{i2} + e_1 e_2^c) + (1 \leftrightarrow 2) \end{array} \right], \quad \mathbf{1} \times \mathbf{10}(\mathbf{5}) \quad \mathbf{16}(\mathbf{5}^*) \times \mathbf{16}(\mathbf{10}) \quad (2.14)$$

$$H(\mathbf{10}, \mathbf{5}) \quad \left[\begin{array}{c} S_1 N_2 - (u_1^{ci} u_{i2} + \nu_1 \nu_2^c) + (1 \leftrightarrow 2) \end{array} \right]. \quad \mathbf{1} \times \mathbf{10}(\mathbf{5}^*) \quad \mathbf{16}(\mathbf{10}) \times \mathbf{16}(\mathbf{10}) \quad \mathbf{16}(\mathbf{5}^*) \times \mathbf{16}(\mathbf{1}) \quad (2.15)$$

§3. Determination of $U(1)_X$ charges

In **27**, $SU(5)$ **10** and **5** components appear once, and hence the mass matrix of the up quark sector uniquely comes from the term $\Psi(\mathbf{16}, \mathbf{10})\Psi(\mathbf{16}, \mathbf{10})H(\mathbf{10}, \mathbf{5})$ in Eq. (2.15), which always yields a symmetric matrix. Let $H(\mathbf{10}, \mathbf{5})$ develop a VEV, $\langle H(\mathbf{10}, \mathbf{5}) \rangle = v_u$, giving the up sector masses. We can take the $U(1)_X$ charge of the Higgs fields H to be $h = 0$ without loss of generality. With this convention, we now determine the $U(1)_X$ charges f_i for the three families of $\Psi_i(\mathbf{27})$.

Experimentally, the up sector mass matrix is known to take the form

$$M_u = m_t \times \begin{matrix} & \mathbf{10}_1 & \mathbf{10}_2 & \mathbf{10}_3 \\ \mathbf{10}_1 & \left(\begin{array}{ccc} \lambda^7 & (\lambda^5) & (\lambda^3) \\ (\lambda^5) & \lambda^4 & (\lambda^2) \\ (\lambda^3) & (\lambda^2) & 1 \end{array} \right) & & \\ \mathbf{10}_2 & & & \\ \mathbf{10}_3 & & & \end{matrix}, \quad (3.1)$$

where $\lambda \sim 0.22$ is of the order of the Cabibbo angle, and the brackets indicate that the corresponding matrix element may be less than the order indicated, since the CKM angles represent the difference between the up and down sectors. We take $f_3 = -h/2$, which is zero by our convention for h . Then, in view of the diagonal 33, 22 and 11 elements of this mass matrix, we see that we must take $f_2 = 2$, and f_1 should be either 3 or 4.

In order to fix the $U(1)_X$ charge to either 3 or 4, we use information concerning the Cabibbo angle. But to discuss the Cabibbo angle, we need information regarding the down sector mass matrix. The mass matrix of the down sector has, however, a complication that it comes from the Yukawa couplings of two types, Eqs. (2.14) and (2.13) owing to the mixing of the two $\mathbf{5}^*$ representations in $\mathbf{27}$ both for the right-handed down quark d^c and the Higgs H . We discuss this problem in detail in the next section. The Cabibbo angle is given by $M_{u12}/M_{u22} - M_{d12}/M_{d22}$, where M_u and M_d are the mass matrices of the up and down quark sectors. We will see that, in the scenario discussed there, M_{d12}/M_{d22} as well as M_{u12}/M_{u22} is of the order of $\lambda^{f_1-f_2}$, determined by the $U(1)_X$ charge assignment of the first and second families. Thus the Cabibbo angle of order λ requires $f_1 - f_2 = 1$, from which we must uniquely choose $f_1 = 3$.

Now we have fixed the $U(1)_X$ charge as

$$X(H) = 0, \quad (X(\Psi_1), X(\Psi_2), X(\Psi_3)) = (3, 2, 0), \quad (3.2)$$

from which we have

$$M_u = m_t \times \begin{matrix} & \mathbf{10}_1 & \mathbf{10}_2 & \mathbf{10}_3 \\ \mathbf{10}_1 & \left(\begin{matrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{matrix} \right) & & \\ \mathbf{10}_2 & & & \\ \mathbf{10}_3 & & & \end{matrix}, \quad (3.3)$$

with $m_t \equiv yv_u$. The u quark mass of order λ^6 is a bit large, so that a small cancellation should occur in the computation of the correct eigenvalue for u .

§4. $\mathbf{5}^*$ family structure

Now that we have determined the $U(1)_X$ charges of the three families $\Psi_i(\mathbf{27})$, our next task is to look for a possible $\mathbf{5}^*$ family structure which can reproduce the down quark spectrum and CKM mixings, and, at the same time, the large lepton mixing angle.

As mentioned above, there are two $\mathbf{5}^*$ components in each $\mathbf{27}$. The Higgs doublet, which survives down to the low energy region, can in general be an arbitrary mixed state of the two $\mathbf{5}^*$ components in $H(\mathbf{27})$:

$$H(\mathbf{5}^*) = H(\mathbf{10}, \mathbf{5}^*) \cos \theta + H(\mathbf{16}, \mathbf{5}^*) \sin \theta. \quad (4.1)$$

This is just an $H(\mathbf{10}, \mathbf{5}^*)$ of the E -rotated $SO(10)$ by angle 2θ , corresponding to generators $T_{MN}(2\theta) = \exp(2i\theta E_2)T_{MN} \exp(-2i\theta E_2)$. Of course, it should be noted that this angle θ itself has no physical meaning, but becomes meaningful once the other down quarks or Higgs specify the direction of the $SO(10)$. In the conventional treatment, the low energy Higgs doublet is identified with $H(\mathbf{10}, \mathbf{5}^*)$, corresponding to the choice $\cos \theta = 1$, which implies that the $SO(10)$ direction is specified by the Higgs field. Here, in our scheme, the $SO(10)$ direction will be determined by the down quark sector.

Let us proceed to the three families of (right-handed) down quarks. They can in general be any three linear combinations of the six $\mathbf{5}^*$ components in the three Ψ_i (27). We here consider three typical scenarios.

1. Parallel family structure

Each family of low energy fermions $\mathbf{5}^*$ and $\mathbf{10}$ is contained in a $\mathbf{16}$ representation of (a fixed) $SO(10)$:

$$(\mathbf{5}_1^*, \mathbf{5}_2^*, \mathbf{5}_3^*) = (\Psi_1(\mathbf{16}, \mathbf{5}^*), \Psi_2(\mathbf{16}, \mathbf{5}^*), \Psi_3(\mathbf{16}, \mathbf{5}^*)) . \tag{4.2}$$

2. Nonparallel family structure

Inter-generation mixings are so large that the three $\mathbf{5}^*$ components do not come one from each Ψ_1, Ψ_2 and Ψ_3 . Such an example is

$$(\mathbf{5}_1^*, \mathbf{5}_2^*, \mathbf{5}_3^*) = (\Psi_1(\mathbf{16}, \mathbf{5}^*), \Psi_2(\mathbf{16}, \mathbf{5}^*), \Psi_2(\mathbf{10}, \mathbf{5}^*)) . \tag{4.3}$$

A structure similar to this was first proposed by Yanagida.^{5), 12), 17)}

3. E -twisted structure

Some of the $\Psi_i(\mathbf{16}, \mathbf{5}^*)$ are replaced by $\Psi_i(\mathbf{10}, \mathbf{5}^*)$ in the parallel family structure (4.2). For example the E -twisted version of the third family is given by

$$(\mathbf{5}_1^*, \mathbf{5}_2^*, \mathbf{5}_3^*) = (\Psi_1(\mathbf{16}, \mathbf{5}^*), \Psi_2(\mathbf{16}, \mathbf{5}^*), \Psi_3(\mathbf{10}, \mathbf{5}^*)) . \tag{4.4}$$

This structure implies that the third family falls into $\mathbf{16}$ of an $SO(10)$ E -twisted from that of the other two families.*)

Among these three possibilities, it is obvious that the first option (the parallel family structure) predicts completely parallel mass matrices for all the fermions, the up and down quarks, leptons and neutrinos. Thus it can reproduce neither the down quark masses nor the large neutrino mixing angle.

As for the second option (nonparallel family structure), it turns out that we need a higher representation Higgs field in order to make the other fermion components superheavy, as is seen as follows. We need to make $\Psi_1(\mathbf{10}, \mathbf{5}^*), \Psi_3(\mathbf{16}, \mathbf{5}^*)$ and $\Psi_3(\mathbf{10}, \mathbf{5}^*)$, as well as all the $\mathbf{5}$ components, $\Psi_i(\mathbf{10}, \mathbf{5})$, superheavy. This could be most economically achieved as shown in Fig. 1; namely, we first introduce a Higgs field $\Phi^{x=-3}$ with the $U(1)_X$ charge -3 such that its VEV $\langle \Phi^{x=-3}(\mathbf{1}) \rangle = M/y'$ in the $SO(10)$ singlet component can yield mass terms $M\Psi_3(\mathbf{10}, \mathbf{5}^*)\Psi_1(\mathbf{10}, \mathbf{5})$ and $M\Psi_1(\mathbf{10}, \mathbf{5}^*)\Psi_3(\mathbf{10}, \mathbf{5})$ via the Yukawa coupling in Eqs. (2.1) and (2.11). This VEV also yields other mass terms by the Yukawa interactions (2.1), which are generally allowed by supplementing powers of (Θ/M_P) to match the $U(1)_X$ quantum number:

$$M \times \begin{matrix} \Psi_1(\mathbf{10}, \mathbf{5}) \\ \Psi_2(\mathbf{10}, \mathbf{5}) \\ \Psi_3(\mathbf{10}, \mathbf{5}) \end{matrix} \begin{matrix} \Psi_1(\mathbf{10}, \mathbf{5}^*) & \Psi_2(\mathbf{10}, \mathbf{5}^*) & \Psi_3(\mathbf{10}, \mathbf{5}^*) \\ \left(\begin{matrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^2 & (\lambda) & 0 \\ 1 & 0 & 0 \end{matrix} \right) \end{matrix} . \tag{4.5}$$

*) In terms of the component notation in Eqs. (2.3) and (2.4), our E -twisted scenario is to take (d_1^c, d_2^c, D_3^c) for down quarks and (e_1, e_2, E_3) for leptons. Somewhat similar twisting in the down quark and lepton sectors was considered by Haba et al.¹³⁾ in the $SU(6)' \times SU(2)_R$ unified theory, in which they took essentially $(O(1)d_1^c + O(1)d_2^c, D_1^c, D_3^c)$ and $(e_1, E_1, O(1)E_2 + O(1)E_3)$.

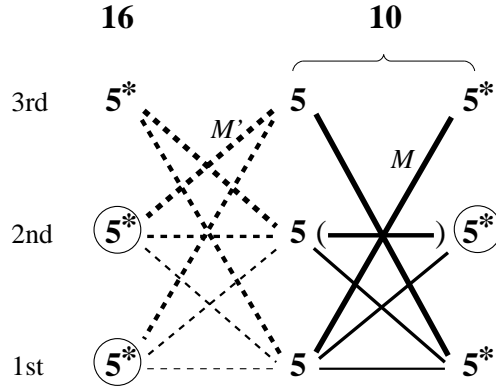


Fig. 1. Nonparallel family structure in which the $\mathbf{5}^*$ components enclosed by circles are the dominant components which remain light. The solid and dotted lines denote heavy mass terms given by $\langle \Phi^{x=-3}(\mathbf{1}) \rangle$ and $\langle \Phi^{x=-2}(\mathbf{16}, \mathbf{1}) \rangle$, respectively. The $U(1)_X$ -suppressed terms with higher powers of $\langle \Theta \rangle / M_P \sim \lambda$ are indicated by thinner lines. The bracketed line connecting $\mathbf{5}$ to $\mathbf{5}^*$ in the 2nd generation $\mathbf{10}$ should be absent for this scenario to work, which can be realized if $\Phi^{x=-3}$ is the $\mathbf{351}$ coupling to $(\mathbf{27} \times \mathbf{27})_A$ anti-symmetrically.

The 22 matrix element is bracketed here for the reason that becomes clear shortly. Next we add another Higgs field $\Phi^{x=-2}$ with $U(1)_X$ charge -2 such that its VEV $\langle \Phi^{x=-2}(\mathbf{16}, \mathbf{1}) \rangle = M'/y'$ in the $SO(10)$ $\mathbf{16}$ $SU(5)$ singlet component can give the mass term $M'\Psi_3(\mathbf{16}, \mathbf{5}^*)\Psi_2(\mathbf{10}, \mathbf{5})$ via the coupling (2.12). Of course, this coupling simultaneously yields an (unwanted) mass term $M'\Psi_2(\mathbf{16}, \mathbf{5}^*)\Psi_3(\mathbf{10}, \mathbf{5})$ with the same strength M' . Including also the subdominant terms suppressed by powers of Θ/M_P , this VEV gives the mass matrix

$$M' \times \begin{matrix} & \Psi_1(\mathbf{16}, \mathbf{5}^*) & \Psi_2(\mathbf{16}, \mathbf{5}^*) & \Psi_3(\mathbf{16}, \mathbf{5}^*) \\ \Psi_1(\mathbf{10}, \mathbf{5}) & \lambda^4 & \lambda^2 & \lambda \\ \Psi_2(\mathbf{10}, \mathbf{5}) & \lambda^2 & \lambda^2 & 1 \\ \Psi_3(\mathbf{10}, \mathbf{5}) & \lambda & 1 & 0 \end{matrix} \quad (4.6)$$

Suppose that $M \gg M'$; namely the breaking scale of E_6 down to $SO(10)$ is higher than the breaking scale of $SO(10)$ down to $SU(5)$. Then, first the pairs $\Psi_3(\mathbf{10}, \mathbf{5}^*)$ and $\Psi_1(\mathbf{10}, \mathbf{5})$, and $\Psi_1(\mathbf{10}, \mathbf{5}^*)$ and $\Psi_3(\mathbf{10}, \mathbf{5})$ acquire a superheavy mass M and decouple from the others. Next, we expect that the pair consisting of $\Psi_3(\mathbf{16}, \mathbf{5}^*)$ and $\Psi_2(\mathbf{10}, \mathbf{5})$ becomes superheavy by the VEV $\langle \Phi^{x=-2}(\mathbf{16}, \mathbf{1}) \rangle = M'/y'$. However, for the component $\Psi_2(\mathbf{10}, \mathbf{5})$, there is also another mass term $\lambda M\Psi_2(\mathbf{10}, \mathbf{5})\Psi_2(\mathbf{10}, \mathbf{5}^*)$ with strength λM coming from the coupling to the VEV $\langle \Phi^{x=-3}(\mathbf{1}) \rangle = M/y'$. In order to keep $\Psi_2(\mathbf{10}, \mathbf{5}^*)$ light while making the pair $\Psi_3(\mathbf{16}, \mathbf{5}^*)$ and $\Psi_2(\mathbf{10}, \mathbf{5})$ superheavy, the condition $M' \gg \lambda M$ should hold, but this is incompatible with the first assumption, $M \gg M'$. (Note that, without the condition $M \gg M'$, the presence of the mass term $M'\Psi_2(\mathbf{16}, \mathbf{5}^*)\Psi_3(\mathbf{10}, \mathbf{5})$ would have caused a large mixing between $\Psi_2(\mathbf{16}, \mathbf{5}^*)$ and $\Psi_1(\mathbf{10}, \mathbf{5}^*)$, contrary to our desire to keep the component $\Psi_2(\mathbf{16}, \mathbf{5}^*)$ light.) The only way to avoid this situation is therefore to forbid the problematic mass term $\lambda M\Psi_2(\mathbf{10}, \mathbf{5})\Psi_2(\mathbf{10}, \mathbf{5}^*)$, which would be realized if the first Higgs field

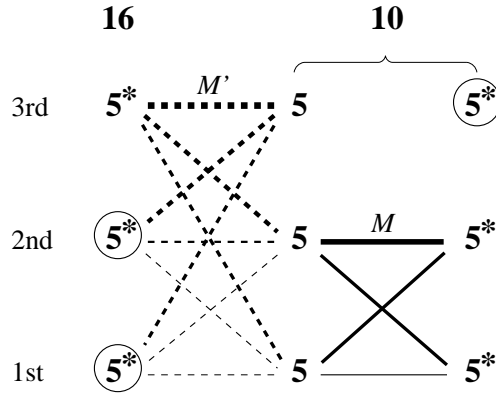


Fig. 2. E -twisted structure in which the 5^* components enclosed by circles are the dominant components that remain light. The solid and dotted lines denote heavy mass terms given by $\langle \Phi^{x=-4}(\mathbf{1}) \rangle$ and $\langle H(\mathbf{16}, \mathbf{1}) \rangle$, respectively. The $U(1)_X$ -suppressed terms with higher powers of $\langle \Theta \rangle / M_P \sim \lambda$ are indicated by thinner lines, as in Fig. 1.

$\Phi^{x=-3}$ with $\langle \Phi^{x=-3}(\mathbf{1}) \rangle = M/y'$ were not a $\mathbf{27}$ but rather $\mathbf{351}$ representation of E_6 whose Yukawa couplings to $\Psi_i(\mathbf{27})\Psi_j(\mathbf{27})$ are only antisymmetric in (i, j) .

The simplest and most attractive option is thus the third one, E -twisted structure, which we investigate in the following sections.

§5. Down sector masses in E -twisted structure

Let us investigate the case of E -twisted structure. In this case, we need only one Higgs field, $\Phi^{x=-4}$, in addition to the usual Higgs H , and we suppose that they develop the following VEVs:

$$\langle \Phi^{x=-4}(\mathbf{1}) = S_\Phi \rangle = M/y' , \quad \langle H(\mathbf{16}, \mathbf{1}) = \nu_H^c \rangle = M'/y . \tag{5.1}$$

These give the following superheavy mass terms via the Yukawa coupling (2.1) with Eqs. (2.11) and (2.12) (see Fig. 2):

$$\begin{matrix} & \Psi_1(\mathbf{16}, \mathbf{5}^*) & \Psi_2(\mathbf{16}, \mathbf{5}^*) & \Psi_3(\mathbf{16}, \mathbf{5}^*) & \Psi_1(\mathbf{10}, \mathbf{5}^*) & \Psi_2(\mathbf{10}, \mathbf{5}^*) & \Psi_3(\mathbf{10}, \mathbf{5}^*) \\ \Psi_1(\mathbf{10}, \mathbf{5}) & \left(\begin{matrix} \lambda^6 M' & \lambda^5 M' & \lambda^3 M' & \lambda^2 M & \lambda M & 0 \\ \lambda^5 M' & \lambda^4 M' & \lambda^2 M' & \lambda M & M & 0 \\ \lambda^3 M' & \lambda^2 M' & M' & 0 & 0 & 0 \end{matrix} \right) & & & & & \\ \Psi_2(\mathbf{10}, \mathbf{5}) & & & & & & \\ \Psi_3(\mathbf{10}, \mathbf{5}) & & & & & & \end{matrix} \tag{5.2}$$

Let us again assume that $M \gg M'$; i.e., the breaking scale of E_6 down to $SO(10)$ is higher than the breaking scale of $SO(10)$ down to $SU(5)$. This case has a very simple structure in which $\langle \Phi^{x=-4}(\mathbf{1}) \rangle = M/y'$ gives superheavy masses to $\Psi_i(\mathbf{10}, \mathbf{5}^*)$ and $\Psi_i(\mathbf{10}, \mathbf{5})$ with $i = 1$ and 2, while $\langle H(\mathbf{16}, \mathbf{1}) \rangle = M'/y$ gives a slightly smaller superheavy mass to $\Psi_3(\mathbf{16}, \mathbf{5}^*)$ and $\Psi_3(\mathbf{10}, \mathbf{5})$. This now leaves us with the following three light (massless at this stage) eigenstates:

$$\mathbf{5}_1^* = \Psi_1(\mathbf{16}, \mathbf{5}^*) + O(\lambda^3)\Psi_3(\mathbf{16}, \mathbf{5}^*) ,$$

$$\begin{aligned} \mathbf{5}_2^* &= \Psi_2(\mathbf{16}, \mathbf{5}^*) + O(\lambda^2)\Psi_3(\mathbf{16}, \mathbf{5}^*) , \\ \mathbf{5}_3^* &= \Psi_3(\mathbf{10}, \mathbf{5}^*) , \end{aligned} \tag{5.3}$$

up to unimportant minor mixings with heavier components. These three states are essentially $(\Psi_1(\mathbf{16}, \mathbf{5}^*), \Psi_2(\mathbf{16}, \mathbf{5}^*), \Psi_3(\mathbf{10}, \mathbf{5}^*))$, as desired. Then, taking the Higgs mixing (4.1) into account, the Yukawa couplings Eqs. (2.13) and (2.14) lead to the following form of the down sector mass matrix:

$$M_d^T = yv_d \times \begin{matrix} & \mathbf{5}_1^* & \mathbf{5}_2^* & \mathbf{5}_3^* \\ \mathbf{10}_1 & \left(\lambda^6 \cos \theta & \lambda^5 \cos \theta & \lambda^3 \sin \theta \right) \\ \mathbf{10}_2 & \left(\lambda^5 \cos \theta & \lambda^4 \cos \theta & \lambda^2 \sin \theta \right) \\ \mathbf{10}_3 & \left(\lambda^3 \cos \theta & \lambda^2 \cos \theta & \sin \theta \right) \end{matrix} , \tag{5.4}$$

where $v_d = \langle H(\mathbf{5}^*) \rangle$ and θ is the Higgs mixing angle. Our convention for the Dirac mass matrix M_{ij} is that the fermion mass term is given by $\mathcal{L}_m = -(\overline{\psi}_{Ri} M_{ij} \psi_{Lj} + \text{h.c.})$.*) Note that the mixing terms retained in Eq. (5.3), although small, also contribute the same order amounts as indicated in Eq. (5.4) to the $j1$ and $j2$ ($j = 1, 2, 3$) matrix elements. So they should be taken into account properly if we wish to calculate not only the orders of magnitude but also the coefficients of the matrix elements correctly. If we assume $\sin \theta \sim \lambda^2$, then

$$M_d^T = \lambda^2 yv_d \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix} . \tag{5.5}$$

Thus we have $\lambda^2 yv_d = m_b$. This factor λ^2 comes from the Higgs mixing $\sin \theta$ in the present E -twisted structure and provides a natural explanation for the reason that the bottom mass is very small compared with the top quark mass, in the small $\tan \beta \sim 1$ scenario.

This mass matrix M_d is common to the down quark and lepton sectors at the GUT scale in the present approximation, and it keeps almost the same form (aside from the overall normalization) down to our low energy regime, even after the renormalization group evolution; so we have $M_l \approx (m_\tau/m_b)M_d^T$. (The reason that the transposition T appears is because the left-handed and right-handed components are contained in the $SU(5)$ $\mathbf{5}^*$ and $\mathbf{10}$ in opposite ways for the lepton and down-quark cases.) It is important that the 32 and 33 elements have become of the same order. This gives a large mixing angle in the charged lepton sector,

$$M_l^\dagger M_l = (m_\tau/m_b)^2 M_d^* M_d^T = m_\tau^2 \times \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} , \tag{5.6}$$

*) The transpose T attached to M_d in Eq. (5.4) is due to this convention. Since $\psi \cdot \chi^c \equiv \psi^\alpha \chi_\alpha^c = \chi^c \cdot \psi$ in two-component notation is equivalent to $\overline{\chi_R} \psi_L$ in the usual four-component notation, our Dirac mass term $\overline{\psi_{Ri}} M_{ij} \psi_{Lj}$ equals $\psi_i^c \cdot M_{ij} \psi_j = \psi_i \cdot M_{ij}^T \psi_j^c$, and hence it corresponds to $\int d^2\theta (\phi_i^c M_{ij} \phi_j) = \int d^2\theta (\phi_i M_{ij}^T \phi_j^c)$ written in terms of chiral superfields ϕ and ϕ^c . This explains the transpose in Eq. (5.4).

and small mixing angles for down quark sector,

$$M_d^\dagger M_d = m_b^2 \times \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \tag{5.7}$$

The CKM and Maki-Nakagawa-Sakata (MNS) matrices¹⁸⁾ are given by

$$U_{\text{CKM}} = U_u U_d^\dagger, \quad U_{\text{MNS}} = U_l U_\nu^\dagger, \tag{5.8}$$

with U_u , U_d , U_l and U_ν which make the matrices $U_u(M_u^\dagger M_u)U_u^\dagger$, $U_d(M_d^\dagger M_d)U_d^\dagger$, $U_l(M_l^\dagger M_l)U_l^\dagger$ and $U_\nu^* M_\nu U_\nu^\dagger$ diagonal. The matrix M_ν is the Majorana mass matrix of the light (almost) left-handed neutrinos, which we discuss in the next section.

The remarkable fact is that the mass matrix in the down sector is not symmetric even in such a large unification group as E_6 and that it leads to quite different lepton and down quark mixing angles. This results from the E -twisted structure of the third generation. We have shown that the mass matrix form (5.5) surely reproduces the usual small down quark mixings on the one hand and very large lepton 2-3 mixing on the other. However, to show that the MNS matrix really gives the large 2-3 mixing, we must examine the the neutrino mass matrix M_ν . We now turn to this task.

§6. Neutrino masses in E -twisted structure

The Majorana mass matrix for the light neutrino is given by

$$M_\nu = M_D^T M_R^{-1} M_D, \tag{6.1}$$

where M_D is the neutrino Dirac mass matrix and M_R is the Majorana mass matrix for the right-handed neutrinos.²⁾

The neutrino Dirac masses come from the same Yukawa coupling Eq. (2.15) as the up quark masses:

$$\begin{aligned} \Psi_{27} \Psi_{27} H_{27} = H(\mathbf{10}, \mathbf{5}) & \left(\begin{array}{ll} \Psi(\mathbf{16}, \mathbf{10}) \Psi(\mathbf{16}, \mathbf{10}) & \rightarrow \text{up quark mass} \\ + \Psi(\mathbf{16}, \mathbf{5}^*) \Psi(\mathbf{16}, \mathbf{1}) & \rightarrow \nu \nu^c \\ + \Psi(\mathbf{10}, \mathbf{5}^*) \Psi(\mathbf{1}) & \rightarrow NS \end{array} \right). \end{aligned} \tag{6.2}$$

Since our three left-handed neutrinos are $\nu_1 \in \Psi_1(\mathbf{16}, \mathbf{5}^*)$, $\nu_2 \in \Psi_2(\mathbf{16}, \mathbf{5}^*)$ and $N_3 \in \Psi_3(\mathbf{10}, \mathbf{5}^*)$ in the present E -twisted structure,^{**)} the corresponding three right-handed neutrinos, which become their Dirac mass partners, are

$$(\nu_1^c, \nu_2^c, S_3). \tag{6.3}$$

^{**)} Precisely stated, there are $O(\lambda^3)$ and $O(\lambda^2)$ mixings of the component ν_3 to the first and second generation left-handed neutrinos, respectively, as indicated in Eq. (5.3). Here, however, we neglect this since the mixing only affects the coefficients and does not change the order of magnitude of the matrix elements for the same reason as with the down quark masses.

Thus the Dirac mass matrix is found to take the form

$$(\nu_1^c \ \nu_2^c \ S_3)M_D \begin{pmatrix} \nu_1 \\ \nu_2 \\ N_3 \end{pmatrix} \quad \text{with} \quad M_D = \tilde{m}_t \begin{pmatrix} \lambda^6 & \lambda^5 & 0 \\ \lambda^5 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (6.4)$$

which is in parallel with the up quark mass matrix, except for the 3rd generation, and exhibits a hierarchical structure. The coefficient \tilde{m}_t is $\sim m_t/3$, where the factor $1/3$ represents the effect of renormalization group flow.

Next we study the right-handed neutrino Majorana masses, which come from the higher dimensional interactions:

$$M_P^{-1} \Psi_i(\mathbf{27}) \Psi_j(\mathbf{27}) \bar{\Phi}^{x_k}(\overline{\mathbf{27}}) \bar{\Phi}^{x_l}(\overline{\mathbf{27}}) (\Theta/M_P)^{f_i+f_j+x_k+x_l}, \quad (6.5)$$

where $\bar{\Phi}^{x_k}(\overline{\mathbf{27}})$ stands for $\bar{\Phi}^{x=+4}(\overline{\mathbf{27}})$ and $\bar{H}^{h=0}(\overline{\mathbf{27}})$, the partners of our Higgs fields $\Phi^{x=-4}(\mathbf{27})$ and $H^{x=0}(\mathbf{27})$.

Suppose that the $SO(10)$ singlet component of $\bar{\Phi}^{x=4}$ and the $(\mathbf{16}, \mathbf{1})$ component of $\bar{H}^{x=0}$ develop VEVs:

$$\langle \bar{\Phi}^{x=4}(S) \rangle = M_\Phi, \quad \langle \bar{H}^{x=0}(\nu^c) \rangle = M_H. \quad (6.6)$$

All of the six right-handed neutrinos (ν_i^c, S_i) ($i = 1, 2, 3$) acquire superheavy masses from this interaction and mix with one another. Now define the 2×2 matrix

$$\begin{pmatrix} M_H^2 & M_H M_\Phi \lambda^4 \\ M_H M_\Phi \lambda^4 & M_\Phi^2 \lambda^8 \end{pmatrix} \equiv \begin{pmatrix} M_1^2 & M_1 M_2 \\ M_1 M_2 & M_2^2 \end{pmatrix}. \quad (6.7)$$

Then, the 6×6 Majorana mass matrix for the six right-handed neutrinos (ν_i^c, S_i) ($i = 1, 2, 3$) can be written in the following tensor product form:

$$\begin{aligned} M_R &= M_P^{-1} \times \begin{matrix} \nu^c & S \\ S & \end{matrix} \begin{pmatrix} M_1^2 & M_1 M_2 \\ M_1 M_2 & M_2^2 \end{pmatrix} \otimes \begin{matrix} 1 & 2 & 3 \\ 2 & \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \\ 3 & \end{matrix} \\ &= \frac{M_2^2}{M_P} \times \begin{matrix} \nu^c & S \\ S & \end{matrix} \begin{pmatrix} \alpha^2 & \alpha \\ \alpha & 1 \end{pmatrix} \otimes \begin{matrix} 1 & 2 & 3 \\ 2 & \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \\ 3 & \end{matrix}. \quad \left(\alpha \equiv \frac{M_1}{M_2} \right) \quad (6.8) \end{aligned}$$

The inverse of this matrix is given by

$$M_R^{-1} = \frac{M_P}{M_2^2} \times \begin{matrix} \nu^c & S \\ S & \end{matrix} \begin{pmatrix} \alpha^{-2} & \alpha^{-1} \\ \alpha^{-1} & 1 \end{pmatrix} \otimes \begin{matrix} 1 & 2 & 3 \\ 2 & \begin{pmatrix} \lambda^{-6} & \lambda^{-5} & \lambda^{-3} \\ \lambda^{-5} & \lambda^{-4} & \lambda^{-2} \\ \lambda^{-3} & \lambda^{-2} & 1 \end{pmatrix} \\ 3 & \end{matrix}, \quad (6.9)$$

so that the 3×3 submatrix for the three right-handed neutrinos (ν_1^c, ν_2^c, S_3) , which

are Dirac mass partners of our left-handed neutrinos (ν_1, ν_2, N_3) , reads

$$M_{R^{3 \times 3}}^{-1} = \frac{M_P}{M_2^2} \times \begin{matrix} \nu_1^c \\ \nu_2^c \\ S_3 \end{matrix} \begin{pmatrix} \alpha^{-2}\lambda^{-6} & \alpha^{-2}\lambda^{-5} & \alpha^{-1}\lambda^{-3} \\ \alpha^{-2}\lambda^{-5} & \alpha^{-2}\lambda^{-4} & \alpha^{-1}\lambda^{-2} \\ \alpha^{-1}\lambda^{-3} & \alpha^{-1}\lambda^{-2} & 1 \end{pmatrix}. \quad (6.10)$$

From this and Eq. (6.4), we find the induced left-handed Majorana mass matrix M_ν to be

$$M_\nu = M_D^T M_{R^{3 \times 3}}^{-1} M_D = \frac{\tilde{m}_t^2 M_P}{M_2^2} \times \begin{matrix} \nu_1 \\ \nu_2 \\ N_3 \end{matrix} \begin{pmatrix} \alpha^{-2}\lambda^6 & \alpha^{-2}\lambda^5 & \alpha^{-1}\lambda^3 \\ \alpha^{-2}\lambda^5 & \alpha^{-2}\lambda^4 & \alpha^{-1}\lambda^2 \\ \alpha^{-1}\lambda^3 & \alpha^{-1}\lambda^2 & 1 \end{pmatrix}. \quad (6.11)$$

If the parameter $\alpha = M_1/M_2$ is larger than λ^2 , this left-handed neutrino mass matrix M_ν exhibits hierarchical structure, implying small mixing angles in the left-handed neutrino sector. Therefore the large 2-3 mixing angle in the MNS matrix suggested in the recent atmospheric neutrino experiment can be explained for a wide range of parameters.

Finally, we add some numerology for the absolute values of light neutrino masses. Suppose that

$$\begin{aligned} M_2 &= M_\Phi \lambda^4 \sim 5 \times 10^{18} \text{ GeV} \times (2 \times 10^{-3}) = 10^{16} \text{ GeV}, \\ M_1 &= M_H = \alpha M_2 \sim \alpha \times 10^{16} \text{ GeV}, \end{aligned} \quad (6.12)$$

which correspond to the following reasonable orders of VEVs:

$$\begin{aligned} \langle \bar{\Phi}^{x=4}(S) \rangle &= M_\Phi \sim 5 \times 10^{18} \text{ GeV} \sim M_P, \\ \langle \bar{H}^{x=0}(\nu^c) \rangle &= M_H \sim \alpha \times 10^{16} \text{ GeV}. \end{aligned} \quad (6.13)$$

If we assume $\alpha \sim \lambda$, larger than λ^2 , then, with $M_P = 10^{19} \text{ GeV}$ and $\tilde{m}_t \sim m_t/3 \sim 60 \text{ GeV}$, this yields the left-handed Majorana mass matrix M_ν ,

$$M_\nu = \left(\frac{\tilde{m}_t^2 M_P}{M_2^2} \sim 4 \cdot 10^{-2} \text{ eV} \right) \times \begin{matrix} \nu_1 \\ \nu_2 \\ N_3 \end{matrix} \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^1 \\ \lambda^2 & \lambda^1 & 1 \end{pmatrix}, \quad (6.14)$$

with the eigenvalues

$$\begin{aligned} m_{\nu_3} &\sim 4 \times 10^{-2} \text{ eV}, \\ m_{\nu_2} &\sim \lambda^2 m_{\nu_3} \sim 2 \times 10^{-3} \text{ eV}, \\ m_{\nu_1} &\sim \lambda^4 m_{\nu_3} \sim 1 \times 10^{-4} \text{ eV}. \end{aligned} \quad (6.15)$$

These mass eigenvalues are consistent with the experimental data.

In the above, the $\alpha \sim \lambda$ was put by hand. There is another option which gives this factor more naturally. This is to adopt Higgs fields Φ^x and $\bar{\Phi}^{-x}$ that carry $x = -5$ instead of $x = -4$ as in the above scenario. This change causes essentially no change to the mass matrix structure for the down quarks and charged leptons, but it leads to some difference in the neutrino sector. In this case, we must interchange the roles of $\bar{\Phi}^{-x}$ and $\bar{H}^{x=0}$; namely, the $SO(10)$ $(\mathbf{16}, \mathbf{1})$ component of $\bar{\Phi}^{x=5}$ and the singlet component of $\bar{H}^{x=0}$ develop VEVs:

$$\begin{aligned} \langle \bar{\Phi}^{x=5}(\nu^c) \rangle &= M_\Phi \sim 5 \times 10^{18} \text{ GeV} \sim M_P, \\ \langle \bar{H}^{x=0}(S) \rangle &= M_H \sim 10^{16} \text{ GeV}. \end{aligned} \tag{6.16}$$

Then, the above 2×2 matrix in Eq. (6.7) is replaced by

$$\begin{array}{cc} & \begin{array}{cc} \nu^c & S \end{array} \\ \begin{array}{c} \nu^c \\ S \end{array} & \begin{pmatrix} M_\Phi^2 \lambda^{10} & M_H M_\Phi \lambda^5 \\ M_H M_\Phi \lambda^5 & M_H^2 \end{pmatrix} \end{array} \tag{6.17}$$

and leads to

$$\begin{aligned} M_2 &= M_H \sim 10^{16} \text{ GeV}, \\ M_1 &= M_\Phi \lambda^5 \sim 5 \times 10^{18} \text{ GeV} \times (2 \times 10^{-3}) \times \lambda = \lambda \times 10^{16} \text{ GeV}. \end{aligned} \tag{6.18}$$

These values of M_2 and M_1 are the same as those found before. Thus this also yields the same left-handed Majorana mass matrix M_ν as in the previous case. Note that $\alpha = M_1/M_2 \sim \lambda$ was supplied from one of the fifth power λ^5 , which came from the $U(1)_X$ charge assignment $x = 5$ to $\bar{\Phi}$. One problem in this scenario which seems unnatural is, however, that the $SO(10)$ $(\mathbf{16}, \mathbf{1})$ VEV of $\bar{\Phi}^{x=5}$ must be larger than the $SO(10)$ singlet VEV of $\bar{H}^{x=0}$, quite oppositely to the VEVs of their partner Higgs fields $H^{x=0}$ and $\Phi^{x=-5}$.

§7. Discussion and further problems

We have presented a supersymmetric E_6 GUT model with anomalous $U(1)_X$, which incorporates a novel mechanism for yielding nonparallel mass structures between up-quark and down-quark/charged-lepton sectors.

The hierarchical mass structure is basically explained by the Frogatt-Nielsen mechanism using the $U(1)_X$ quantum numbers. However, in large unification models with gauge groups larger than $SO(10)$, all members of each generation fall into a single multiplet and hence must carry a common value of the $U(1)_X$ charge. Therefore a straightforward application of the Frogatt-Nielsen mechanism would yield a *common* hierarchical mass matrix for all up-, down- and lepton sectors, in contradiction to observations. It seems that because of this ‘difficulty’ many authors who try to explain the mass hierarchy with the Frogatt-Nielsen mechanism adopt smaller unification gauge groups, or even discard the grand unification framework.

However, we have proposed a novel mechanism which can overcome this difficulty in an E_6 GUT framework. We pointed out that there is a freedom of the $SU(2)_E$

inner automorphism in E_6 in embedding $SO(10)$ in such a way that $SU_{GG}(5) \subset SO(10) \subset E_6$. As a reflection of this, the fundamental representation $\mathbf{27}$ contains two $SU(5)$ $\mathbf{5}^*$ (and $\mathbf{1}$) components, giving a spinor representation of $SU(2)_E$, and an arbitrary linear combination of these two together with $\mathbf{10}$ and a $\mathbf{1}$ can form a $\mathbf{16}$ multiplet of an $SU(2)_E$ -rotated $SO(10)$. Our low energy fermions plus a right-handed neutrino just give an $SO(10)$ $\mathbf{16}$ at each generation. However, the $SO(10)$ groups chosen by three generations need not be the same, but may be $SU(2)_E$ rotated from one another. Moreover, yet another $SU(2)_E$ rotation can also occur in the Higgs $\mathbf{27}$. Based on this observation, we examined the simplest and typical version in which the $SO(10)$ groups chosen by the first and second families coincide, but the $SO(10)$ group of the third family is rotated by an angle π from it.

It is remarkable that this simplest E -twisted scenario can reproduce all the characteristic features of the fermion masses, not only the Dirac masses but also the Majorana masses, by introducing only two Higgs fields $H(\mathbf{27})$ and $\Phi^{x=-4}(\mathbf{27})$, paired with $\bar{H}(\mathbf{27})$ and $\bar{\Phi}^{x=4}(\mathbf{27})$. The small $SU(2)_E$ rotation with $\sin \theta \sim \lambda^2$ in the Higgs H sector explains why the bottom quark is λ^2 times lighter than the top quark and why the mixing between the second and third generation neutrinos is very large. The large mass scales M and M' for the superheavy fermions are supplied by the two Higgs fields H and $\Phi^{x=-4}$, and their partners \bar{H} and $\bar{\Phi}^{x=4}$ at the same time give the right-handed neutrino masses.

Here we did not use any higher representations for Higgs fields. Note, however, that $\mathbf{27}$ of E_6 contains no $SU(5)$ nonsinglet component that is a standard gauge group singlet. Therefore, to break $SU(5)$ symmetry, we need at least one Higgs field of higher representations, as long as we stick to the conventional GUT framework. Another possibility is to use the symmetry breaking mechanism by Wilson lines, as suggested by string theory. We also did not discuss the detailed differences between the down-quark and charged-lepton mass matrices. If one wishes to utilize a Georgi-Jarlskog type mass matrix, one needs a $\mathbf{45}$ representation of $SU(5)$, which is not included in $\mathbf{27}$.

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Appendix A

— Representation **27** and Maximal Subgroups of E_6 —

The adjoint representation **78** of E_6 is decomposed under its maximal subgroup $SO(10) \times U(1)_{V'}$ as $\mathbf{78} = \mathbf{16}_{-3} + \overline{\mathbf{16}}_3 + \mathbf{45}_0 + \mathbf{1}_4$ (The suffices denote the values of the $U(1)_{V'}$ charge.), so that the E_6 generators are given by $SO(10)$ **16** Weyl-spinor generators E_α ($\alpha = 1, \dots, 16$) and their complex conjugates $\overline{E}^\alpha = (E_\alpha)^\dagger$ in addition to the 45 $SO(10)$ generators T_{MN} and one $U(1)$ generator T . The algebra is given by¹⁹⁾

$$\begin{aligned}
 [T_{MN}, T_{KL}] &= -i(\delta_{NK}T_{ML} + \delta_{ML}T_{NK} - \delta_{MK}T_{NL} - \delta_{NL}T_{MK}), \\
 [T_{MN}, \begin{pmatrix} E_\alpha \\ \overline{E}^\alpha \end{pmatrix}] &= - \begin{pmatrix} (\sigma_{MN})_\alpha^\beta & 0 \\ 0 & (-\sigma_{MN}^*)_{\alpha\beta} \end{pmatrix} \begin{pmatrix} E_\beta \\ \overline{E}^\beta \end{pmatrix}, \\
 [T, \begin{pmatrix} E_\alpha \\ \overline{E}^\alpha \end{pmatrix}] &= \frac{\sqrt{3}}{2} \begin{pmatrix} E_\alpha \\ -\overline{E}^\alpha \end{pmatrix}, \\
 [E_\alpha, \overline{E}^\beta] &= -\frac{1}{2}(\sigma_{MN})_\alpha^\beta T_{MN} + \frac{\sqrt{3}}{2}\delta_\alpha^\beta T. \tag{A.1}
 \end{aligned}$$

(The $U(1)_{V'}$ charge V' is related to T by $V' = 2\sqrt{3}T$, and T here is normalized in the same manner as the other charges: $\text{Tr}(T^2) = \text{Tr}(T_{MN}^2) = \text{Tr}(\overline{E}^\alpha E_\alpha)$, where M, N and α are not summed over.) The simplest representation **27** of E_6 is decomposed into $\mathbf{1}_4 + \mathbf{16}_1 + \mathbf{10}_{-2}$ under $SO(10) \times U(1)_{V'}$, and so it can be denoted $\Psi_A \equiv (\psi_0, \psi_\alpha, \psi_M)$, where α and M are $SO(10)$ (Weyl-)spinor and vector indices, respectively. The E_6 generators act on this representation as¹⁹⁾

$$\begin{aligned}
 & \left(\theta T + \frac{1}{2}\theta_{KL}T_{KL} + \bar{\epsilon}^\gamma E_\gamma + \overline{E}^\gamma \epsilon_\gamma \right) \begin{pmatrix} \psi_0 \\ \psi_\alpha \\ \psi_M \end{pmatrix} \\
 &= \begin{pmatrix} \frac{2}{\sqrt{3}}\theta & \bar{\epsilon}^\beta & 0 \\ \epsilon_\alpha & \frac{1}{2}\theta_{KL}(\sigma_{KL})_\alpha^\beta + \frac{1}{2\sqrt{3}}\theta\delta_\alpha^\beta & -\frac{1}{\sqrt{2}}(\sigma_N C \bar{\epsilon}^T)_\alpha \\ 0 & -\frac{1}{\sqrt{2}}(\epsilon^T C \sigma_M^\dagger)^\beta & -i\theta_{MN} - \frac{1}{\sqrt{3}}\theta\delta_{MN} \end{pmatrix} \begin{pmatrix} \psi_0 \\ \psi_\beta \\ \psi_N \end{pmatrix}, \tag{A.2}
 \end{aligned}$$

where $\sigma_{MN} \equiv (\sigma_M \sigma_N^\dagger - \sigma_N \sigma_M^\dagger)/4i$ is a 16×16 spinor representation matrix of the $SO(10)$ generators T_{MN} . Here σ_M and C are the 16×16 matrices with which the $SO(10)$ γ -matrices Γ_M and the charge conjugation matrix $C_{(10)}$, satisfying $\Gamma_M \Gamma_N + \Gamma_N \Gamma_M = 2\delta_{MN}$ and $C_{(10)}^{-1} \Gamma_M C_{(10)} = \Gamma_M^T$ and $C_{(10)} = C_{(10)}^T$, are given in the form

$$\Gamma_M = \begin{pmatrix} 0 & (\sigma_M)_{\alpha\beta} \\ (\sigma_M^\dagger)_{\alpha\beta} & 0 \end{pmatrix}, \quad C_{(10)} = \begin{pmatrix} 0 & C_{\alpha}^T \\ C_{\alpha} & 0 \end{pmatrix}. \tag{A.3}$$

Note that $C\sigma_M^\dagger$ and $\sigma_M C$ are symmetric matrices.

An E_6 invariant can be constructed using three $\mathbf{27}$ representations and is given by Eq. (2.10), $\Gamma^{ABC}\Psi_{1A}\Psi_{2B}\Psi_{3C}$, where Γ^{ABC} is a totally symmetric tensor:⁸⁾

$$\Gamma^{ABC} : \text{totally symmetric in } (A, B, C), \quad \begin{cases} \Gamma^{0MN} = \delta_{MN} , \\ \Gamma^{M\alpha\beta} = \frac{1}{\sqrt{2}}(C\sigma_M^\dagger)^{\alpha\beta} , \\ \text{otherwise } 0 . \end{cases} \quad (\text{A}\cdot 4)$$

By using a concrete representation for σ_M and C given in Ref. 8) in which the $SO(10)$ spinor $\mathbf{16}$, ψ_α , is represented in the form

$$\psi_\alpha(\mathbf{16}) = \begin{pmatrix} \begin{pmatrix} u_j \\ d_j \end{pmatrix} \\ \begin{pmatrix} \nu \\ e \end{pmatrix} \\ \begin{pmatrix} d^{cj} \\ -u^{cj} \end{pmatrix} \\ \begin{pmatrix} e^c \\ -\nu^c \end{pmatrix} \end{pmatrix}, \quad (\text{A}\cdot 5)$$

the explicit form of the $SO(10)$ invariant $\mathbf{16} \times \mathbf{10} \times \mathbf{16}$, $\psi_{1\alpha}^\dagger(C H_M \cdot \sigma_M)^{\alpha\beta}\psi_{2\beta}$, which appears in $\Gamma^{ABC}\Psi_{1A}\Psi_{2B}H_C$ (here $\Psi_{3C}(\mathbf{27})$ is replaced by $H_C(\mathbf{27})$ for clarity) is given as

$$\begin{aligned} & \psi_1^\dagger(\mathbf{16})C(H_M(\mathbf{10}) \cdot \frac{\sigma_M}{\sqrt{2}})\psi_2(\mathbf{16}) \\ &= \left[(u_1^{ci}H^\dagger\epsilon^\dagger + d_1^{ci}H^\dagger) \begin{pmatrix} u_i \\ d_i \end{pmatrix}_2 + (\nu_1^cH^\dagger\epsilon^\dagger + e_1^cH^\dagger) \begin{pmatrix} \nu \\ e \end{pmatrix}_2 \right. \\ & \quad + H_i(e^c - \nu^c)_1 \begin{pmatrix} u^{ci} \\ d^{ci} \end{pmatrix}_2 - \bar{H}'^i(e - \nu)_1 \begin{pmatrix} u_i \\ d_i \end{pmatrix}_2 \\ & \quad \left. + \varepsilon_{ijk}\bar{H}'^i u_1^{cj} d_2^{ck} - \varepsilon^{ijk} H_i u_{1j} d_{2k} \right] + (1 \leftrightarrow 2), \quad (\text{A}\cdot 6) \end{aligned}$$

where i, j and k denote $SU(3)$ color, and the components of $H_M(\mathbf{10})$ are denoted by

$$\begin{aligned} H(\mathbf{5}) &= \begin{pmatrix} H_i \\ H \end{pmatrix}, \quad \begin{cases} H_i = \frac{i}{\sqrt{2}}(H_{M=i+3} + iH_{M=i}), & (i = 1, 2, 3) \\ H = \begin{pmatrix} H_x \\ H_y \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -iH_7 - H_8 \\ H_0 + iH_9 \end{pmatrix}, \end{cases} \\ H(\mathbf{5}^*) &= \begin{pmatrix} \bar{H}'^i \\ \bar{H}' \end{pmatrix}, \quad \begin{cases} \bar{H}'^i = -\frac{i}{\sqrt{2}}(H_{M=i+3} - iH_{M=i}), & (i = 1, 2, 3) \\ \bar{H}' = \begin{pmatrix} \bar{H}'_x \\ \bar{H}'_y \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} iH_7 - H_8 \\ H_0 - iH_9 \end{pmatrix}. \end{cases} \quad (\text{A}\cdot 7) \end{aligned}$$

These $SU(5)$ $\mathbf{5}$ and $\mathbf{5}^*$ components, $H(\mathbf{5})$ and $H(\mathbf{5}^*)$, correspond to the decomposition $\mathbf{10} = \mathbf{5}_2 + \mathbf{5}^*_{-2}$ given in Eq. (2.4) for $\psi_M(\mathbf{10})$. Then the $SO(10)$ invariant

$H_M\psi_M$ is given

$$\sum_{M=1}^{10} H_M(\mathbf{10})\psi_M(\mathbf{10}) = (\bar{H}^i \quad H^{\dagger}) \begin{pmatrix} D_i \\ E^c \\ -N^c \end{pmatrix} + (D^{ci} \quad (E, -N)) \begin{pmatrix} H_i \\ H \end{pmatrix}. \tag{A.8}$$

In connection with this $SU(5)$ decomposition, we note that the above $SO(10)$ invariant $\mathbf{16} \times \mathbf{10} \times \mathbf{16}$ given in Eq. (A.6) is expressed as

$$\psi_1^T(\mathbf{16})C\left(H_M(\mathbf{10}) \cdot \frac{\sigma_M}{\sqrt{2}}\right)\psi_2(\mathbf{16}) = \frac{1}{2^2}\varepsilon^{ijklm}\psi_{1ij}(\mathbf{10})\psi_{2kl}(\mathbf{10})H_m(\mathbf{5}) + \left\{ \left(\psi_{1ij}(\mathbf{10})\bar{H}^i(\mathbf{5}^*)\psi_2^j(\mathbf{5}^*) - \nu_1^c H_i(\mathbf{5})\psi_2^i(\mathbf{5}^*) \right) + (1 \leftrightarrow 2) \right\} \tag{A.9}$$

in terms of the $SU(5)$ components $\psi_{1ij}(\mathbf{10})$, $\psi^i(\mathbf{5}^*)$ and $\nu^c(\mathbf{1})$ contained in $\psi_\alpha(\mathbf{16})$:

$$\psi_{ij}(\mathbf{10}) = \begin{pmatrix} \varepsilon_{ikj}u^{ck} & -u_i & -d_i \\ u_j & 0 & -e^c \\ d_j & e^c & 0 \end{pmatrix}, \quad \psi^i(\mathbf{5}^*) = \begin{pmatrix} d^{ci} \\ e \\ -\nu \end{pmatrix}. \tag{A.10}$$

Under a maximal subgroup $SU(3)_L \times SU(3)_R \times SU(3)_c \subset E_6$, $\Psi_A(\mathbf{27})$ is decomposed into the following three irreducible components

$$\psi_{iLj_c}(\mathbf{3}, \mathbf{1}, \mathbf{3}) = \begin{pmatrix} u_j \\ d_j \\ D_j \end{pmatrix}, \quad \psi^{icj_R}(\mathbf{1}, \mathbf{3}^*, \mathbf{3}^*) = (u^{ci} \ d^{ci} \ D^{ci}),$$

$$\psi_{iR}^{jL}(\mathbf{3}^*, \mathbf{3}, \mathbf{1}) = \begin{matrix} 1_L^* & 2_L^* & 3_L^* \\ 1_R & \begin{pmatrix} N^c & E^c & -e^c \\ -E & N & \nu^c \end{pmatrix} \\ 3_R & \begin{pmatrix} e & -\nu & -S \end{pmatrix} \end{matrix}, \tag{A.11}$$

where the suffices L, R, c are attached to the indices i, j to distinguish which of $SU(3)$ s the indices refer to, and the component fields are those defined in Eqs. (2.3) – (2.5). In terms of these $SU(3)_L \times SU(3)_R \times SU(3)_c$ component fields, the E_6 invariant (2.10), $\Gamma^{ABC}\Psi_{1A}\Psi_{2B}\Psi_{3C}$, trilinear in $\mathbf{27}$ can be written in the form:

$$\Gamma^{ABC}\Psi_{1A}\Psi_{2B}\Psi_{3C} = -\left(\psi_1^{icj_R}\psi_2^{kL}\psi_3^{kLi_c} + (3! - 1 \text{ permutations of } (1, 2, 3)) \right) - \varepsilon^{iLjLkL}\varepsilon^{lcmcn_c}\psi_{1iLl_c}\psi_{2jLm_c}\psi_{3kLn_c} + \varepsilon_{iRjRkR}\varepsilon_{lcmcn_c}\psi_1^{lciR}\psi_2^{mciR}\psi_3^{nckR} - \varepsilon^{iRjRkR}\varepsilon_{lLmLnL}\psi_{1iR}^{lL}\psi_{2jR}^{mL}\psi_{3kR}^{nL}. \tag{A.12}$$

Under another maximal subgroup $SU(6) \times SU(2)_E \subset E_6$ discussed in Eq. (2.7), the $\Psi_A(\mathbf{27})$ is decomposed into the following two irreducible components:

$$\psi_{ij}(\mathbf{15}, \mathbf{1}) = \begin{pmatrix} \varepsilon_{ikj}u^{ck} & -u_i & -d_i & -D_i \\ u_j & 0 & -e^c & -E^c \\ d_j & e^c & 0 & N^c \\ D_j & E^c & -N^c & 0 \end{pmatrix}, \quad \psi_a^i(\mathbf{6}^*, \mathbf{2}) = \begin{pmatrix} d^{ci} & -D^{ci} \\ e & -E \\ -\nu & N \\ -S & \nu^c \end{pmatrix}. \tag{A.13}$$

In terms of these, the same E_6 invariant (2·10) can be written in the form

$$\Gamma^{ABC}\Psi_{1A}\Psi_{2B}\Psi_{3C} = -\frac{1}{2^3}\varepsilon^{ijklmn}\psi_{1ij}\psi_{2kl}\psi_{3mn} + (\varepsilon^{ab}\psi_{1ij}\psi_{2a}^i\psi_{3b}^j + (\text{cyclic permutations of } (1, 2, 3))) . \quad (\text{A}\cdot 14)$$

It is clear from Eq. (A·13) that $SU(2)_E$ is an $SU(2)$ subgroup of $SU(3)_R$, which acts on the second and third entries of the fundamental representation $\mathbf{3}$ of $SU(3)_R$. Note also that $SU(5)_{GG}$ and $SU(3)_L$ are regular subgroups of this $SU(6)$ such that the first five entries and the last three entries of the $\mathbf{6}$ of $SU(6)$ are the fundamental representations $\mathbf{5}$ and $\mathbf{3}$ of $SU(5)_{GG}$ and $SU(3)_L$, respectively.

Another interesting $SU(6) \times SU(2)$ subgroup, discussed by Haba et al.,¹³⁾ is $SU(6)' \times SU(2)_R \subset E_6$, where $SU(6)' \supset SU(4)_{\text{Pati-Salam}} \times SU(2)_L$ and $SU(2)_R$ is an $SU(2)$ subgroup of $SU(3)_R$ which acts on the first and second entries of the fundamental representation $\mathbf{3}$ of $SU(3)_R$. Under this subgroup $SU(6)' \times SU(2)_R$, $\Psi_A(\mathbf{27})$ is decomposed into the following two irreducible components:

$$\psi_{ij}(\mathbf{15}, \mathbf{1}) = \begin{pmatrix} -\varepsilon_{ikj}D^{ck} & -D_i & -u_i & -d_i \\ D_j & 0 & \nu & e \\ u_j & -\nu & 0 & S \\ d_j & -e & -S & 0 \end{pmatrix}, \quad \psi_a^i(\mathbf{6}^*, \mathbf{2}) = \begin{pmatrix} d^{ci} & -u^{ci} \\ -e^c & \nu^c \\ N^c & -E \\ E^c & N \end{pmatrix}. \quad (\text{A}\cdot 15)$$

This subgroup $SU(6)' \times SU(2)_R$ can essentially be obtained from the above $SU(6) \times SU(2)_E$ by an inner automorphism by an element $\exp(-i\pi J_2^{R'})$, where $J_2^{R'}$ is the second generator of yet another $SU(2)$ subgroup of $SU(3)_R$ which acts on the first and third entries of the fundamental representation $\mathbf{3}$ of $SU(3)_R$. Indeed, under this rotation $\exp(-i\pi J_2^{R'})$, doublets $\mathbf{2}$ of this $SU(2)$ are changed as $(u, d) \rightarrow (-d, u)$, so that the $SU(3)_R$ non-singlet components in the $SU(3)_L \times SU(3)_R \times SU(3)_c$ decomposition Eq. (A·11) are transformed as

$$\begin{matrix} (u^{ci} & d^{ci} & D^{ci}) \\ 1_L^* & 2_L^* & 3_L^* \\ 1_R \begin{pmatrix} N^c & E^c & -e^c \\ -E & N & \nu^c \\ e & -\nu & -S \end{pmatrix} \end{matrix} \rightarrow \begin{matrix} (-D^{ci} & d^{ci} & u^{ci}), \\ 1_L^* & 2_L^* & 3_L^* \\ 1_R \begin{pmatrix} -e & \nu & S \\ -E & N & \nu^c \\ N^c & E^c & -e^c \end{pmatrix} \end{matrix}. \quad (\text{A}\cdot 16)$$

The field content in Eq. (A·15) is actually obtained from Eq. (A·13) with this replacement for the component fields together with a relabeling of the last three indices of $SU(6)$, $(4, 5, 6) \rightarrow (6, 4, 5)$. By this inner automorphism, it is clear that the E_6 invariant trilinear in $\mathbf{27}$ is given by the same formula, Eq. (A·14), also in this case. (If one wishes to have an $SU(4)_{\text{Pati-Salam}}$ which has (u_i, ν) and (d_i^c, e^c) as its $\mathbf{4}$ and $\mathbf{4}^*$ instead of $(u_i, -\nu)$ and $(d_i^c, -e^c)$ in Eq. (A·15), one can easily obtain this by using another suitable inner automorphism by an element of $SU(2) \subset SU(3)_L$, since $SU(2)$ spinors change sign under 2π rotation.)

Finally, as is now clear, yet another $SU(6) \times SU(2)$ subgroup,²⁰⁾ $SU(6)'' \times SU(2)'_R$, can be obtained from $SU(6) \times SU(2)_E$ by an inner automorphism by a

π rotation $\exp(-i\pi J_2^R)$ of $SU(2)_R$. Applying this transformation $\exp(-i\pi J_2^R)$ to Eq. (A.13), we find that $\Psi_A(\mathbf{27})$ is decomposed under this subgroup $SU(6)'' \times SU(2)'_R$ into

$$\psi_{ij}(\mathbf{15}, \mathbf{1}) = \begin{pmatrix} -\varepsilon_{ikj} d^{ck} & -u_i & -d_i & -D_i \\ u_j & 0 & -\nu^c & N \\ d_j & \nu^c & 0 & E \\ D_j & -N & -E & 0 \end{pmatrix}, \quad \psi^i_a(\mathbf{6}^*, \mathbf{2}) = \begin{pmatrix} u^{ci} & -D^{ci} \\ e & N^c \\ -\nu & E^c \\ -S & -e^c \end{pmatrix}. \quad (\text{A.17})$$

Clearly the $SU(5)$ subgroup of this $SU(6)''$, acting on the first five entries, is the flipped $SU(5)$.

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