the depth habitat of many of those species which inhabit deeper water varies considerably from one region to another. Emiliani[13], in an early study, showed that the isotopic evidence was consistent with the assumption that water density was an important controlling factor. For this very reason, in his subsequent palaeotemperature work, he has relied exclusively on the surface-inhabiting species, which clearly cannot migrate upward during glacial phases.

When the problem of the isotopic composition of ocean water is better understood, it may be possible to use the isotopic analyses of Lidz *et al.* to determine the depth habitat of the various species during the glacial phases and thus perhaps to estimate the extent to which each species is able to compensate for changes in water density. It has been suggested that alteration in the density of pores in the test may be a possible mechanism achieving this in the species *Globigerina eggeri*, and Wiles[14] has shown that this parameter does vary with time in a manner closely related to the glacial-interglacial cycle. On the other hand, it should be clear from this discussion that to use the information in the way Lidz *et al.* do is quite unjustified in our present state of ignorance.

I thank Dr C. Emiliani for comments which have proved useful in the preparation of this communication; this should not be taken to imply that he is in agreement with my conclusions. I have also had discussions with Professor H. Godwin, Mr M. Fisher and Dr B. M. Funnel.

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PHYSICS

Neutron Diffraction on Piezoelectric Vibrating Resonators

THIS communication explains some experimental results published by Parkinson and Moyer[1,2]* by proving that the wave amplitude associated with neutrons reflected from a lattice of vibrating piezoelectric resonators corresponds to the amplitude of a frequency modulated wave[3,4].

In the Bragg formula

$$\lambda_n = \frac{\pi v_n}{\omega_n} = 2\, d \sin \vartheta \qquad (1)$$

we shall associate with the neutrons the wavelength λ_n given by the de Broglie relation $\lambda_n = h/m_n v_n$ and the angular frequency given by the formula $\omega_n = \pi h / m_n \lambda^2_n$.

As a modulator of the beam of neutrons diffracted on the vibrating crystal lattice, we shall assume the simplest type of piezoelectric resonator, cut from the crystal so that the resonator vibrates longitudinally in the direction of the lattice constant d—that is in the direction perpendicular to the lattice planes on which the diffraction of neutrons takes place. The periodic displacement can be expressed by the relationship

$$d = d_0\,(1 + q \cos \omega_d\, t) \qquad (2)$$

Substituting d from equation (2) in equation (1) we obtain

$$\omega_n \simeq \omega_{n,0} - q_0 \cos \omega_d\, t \qquad (3)$$

where $\omega_{n,0} = \pi h / 4 m_n d_0^2 \sin^2 \vartheta$ and $q_0 = 2q\, \omega_{n,0}$.

The wave function of the plane wave of reflected neutrons can be written in the form

$$\psi_n = \psi_0\,(\mathbf{r}) \exp\,(i\varphi_n(t)) \qquad (4)$$

in which

$$\varphi_n(t) = \int_0 \omega_n \mathrm{d}t = \omega_{n,0} t - \delta_0 \sin \omega_d t \qquad (5)$$

if we introduce equation (3) for ω_n in equation (5) and if $\delta_0 = q_0/\omega_d$, the modulation index. For $\varphi_n(t)$ given by equation (5) the wave function takes the form[5]

$$\psi_n = \psi_0 \exp i(\omega_{n,0} t - \delta_0 \sin \omega_d t)$$
$$= \psi_0 \sum_{k=-\infty}^{+\infty} J_k\,(\delta_0) \exp i(\omega_{n,0} + k\omega_d t) \qquad (6)$$

where $J_k(\delta_0)$ denotes the Bessel function of the first kind and of order k.

Equation (6) clearly shows that during the diffraction of neutrons on a vibrating resonator there arise side bands as well as the band of reflected neutrons with frequency $\omega_{n,0}$ and intensity proportional to $|\psi_0|^2\, J_0{}^2\,(\delta_0)$. These side bands of reflected neutrons have higher frequencies $(\omega_{n,0} + k\omega_d)$ and lower frequencies $(\omega_{n,0} - k\omega_d)$ and have intensities proportional to $|\psi_0|^2\, J_k^2\,(\delta_0)$ if the numbers of neutrons associated with the corresponding frequencies are determined.

In the case when the modulation index $\delta_0 < 1$, the $|\psi_n|^2$ of equation (6) can be written in the form

$$|\psi_n|^2 \simeq |\psi_0|^2\,[J_0^2(\delta_0) + 2J_1^2(\delta_0) + 2J_2^2(\delta_0) + 4J_0(\delta_0)J_2(\delta_0) \cos 2\,\omega_d t] \qquad (7)$$

if we use only five terms of equation (6). A further approximative form is

$$|\psi_n|^2 \simeq |\psi_0|^2 \left(1 + \frac{\delta_0^2}{2} - \frac{\delta_0^2}{2} \cos 2\,\omega_d t\right) \qquad (8)$$

This approximation can be taken as valid for the experiments[1,2] using quartz resonator frequencies, f_d, of 250, 400 or 500 kHz and neutron energies, E_n, of 0·02 to 0·20 eV. According to known experimental results[6,7], the amplitude $q = |d - d_0|/d_0$ can be expected to be of the order 10^{-6} or 10^{-7}, so the condition $q_0/\omega_d < 1$ is fulfilled and the intensity of the reflected neutron beam is proportional to $|\psi_n|^2$ given by equations (7) or (8). This fact explains the time analysis of the diffracted neutron beam[2] according to which the modulation of the neutron beam by a piezoelectric resonator oscillating with a frequency $f_d = \omega_d/2\pi = 500$ kHz occurred at the frequency of 1 MHz, twice the crystal oscillating frequency.

Ref. 4 describes a more detailed analysis, especially of the analogy to the spectral distribution in the Mössbauer effect on solid materials made to vibrate ultrasonically by piezoelectric resonators[8].

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