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Neutron Stars in the Scalar-Tensor Theory of Gravitation

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We have several theories of gravitation other than Einstein's theory of general relativity. Distinction among these theories might be possible from the study of superdense objects. The best-studied superdense object is the so-called neutron star. Its mass, radius and other properties can be determined from the equation of state for the stellar matter and the theory of gravitation. Although the general relativity has been mainly used as the theory of gravitation, Salmona[1] applied the scalar-tensor theory of Brans and Dicke.[2] In his calculation he assumed the equation of state for a noninteracting Fermi gas neglecting the effects of nuclear forces. In this letter we take into account the nuclear forces using the Kodama-Yamada mass formula for compressible nuclei.[3]

As for the model of neutron star we assume, for simplicity, that the star is static and spherically symmetric. We also assume that the star is composed of neutrons only. Then, the energy density ε is written as

$$\varepsilon(\rho)=M_n c^2\rho+f(\rho), \qquad (1)$$

where M_n is the rest mass of neutron and ρ is the neutron number density. The first term is the reast-mass term. The second term is the energy density due to the kinetic energies of neutrons and the nuclear interactions among them. This term is explicitly written as[3]

$$f(\rho)=[a_1(\xi)+a_3(\xi)]\rho, \qquad (2)$$

where

$$a_1(\xi)=2203.649+191.7527\xi - \frac{4870.351}{\sqrt{4-\xi}}+\frac{3591.238}{(4-\xi)^2}\ \mathrm{MeV},$$

$$a_3(\xi)=-700.9167 - 88.08083\xi+\frac{1415.981}{\sqrt{4-\xi}}\ \mathrm{MeV},$$

$$\xi=\sqrt[3]{\frac{4}{3}\pi\rho}\ \mathrm{fm}^{-1}.$$

The first term in the square brackets of Eq. (2) is the so-called volume term and the second is the symmetry term.

By the use of Eq. (1) as the equation of state, the field equations in the scalar-tensor theory can be solved numerically under appropriate initial values at $r=0$. In doing so we follow Salmona's method.

Figure 1 shows the relation between the gravitational masses M and the central neutron number densities ρ_c, and in Fig. 2 the relation between the radius R and the central density ρ_c is shown. For the Dicke coupling constant ω we use the values $\omega=\infty$, 20, 6, 2.5. In the limit $\omega=\infty$, the scalar-tensor theory reduces to the general relativity and our results agree with those of Kodama and Yamada.[4] From

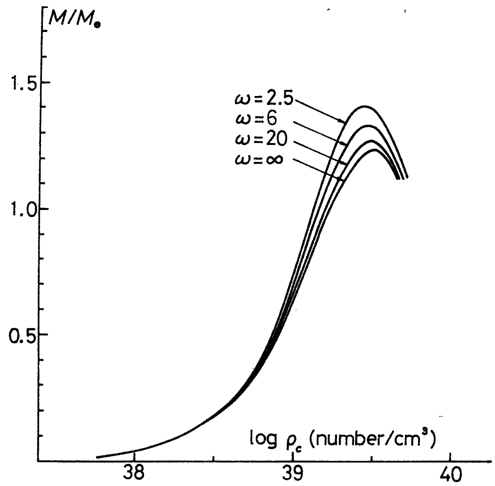

Fig. 1. Dependense of the gravitational mass M of a neutron star upon the central number density ρ_c and the coupling constant ω.

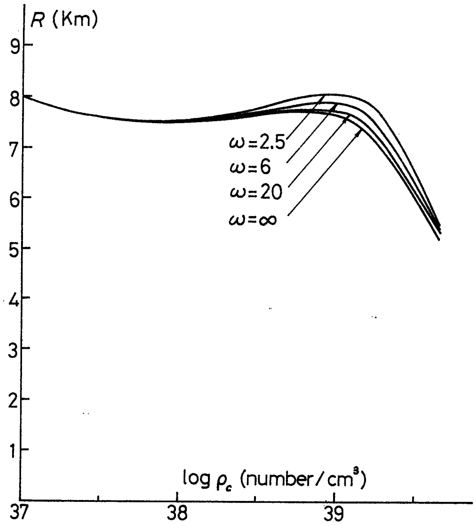

Fig. 2. Relation between the radius R and the central number density ρ_c.

comparison with observations it is shown that $\omega \geq 6$. From those figures we conclude, in qualitative agreement with Salmona's conclusion, that the difference between the two gravitational theories is rather small in comparison with uncertainties caused by various equations of state.

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1) A. Salmona, Phys. Rev. **154** (1967), 1218.
2) C. Brans and R. H. Dicke, Phys. Rev. **124** (1961), 925.
3) T. Kodama and M. Yamada, Prog. Theor. Phys. **45** (1971), 1763.
4) T. Kodama and M. Yamada, Prog. Theor. Phys. **47** (1972), 444.