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ORIGINAL ARTICLE

Neutrosophic analysis of variance: application to university students

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Abstract

The existing analysis of variance (ANOVA) test cannot be applied when the sample is selected from the population having come imprecise, fuzzy and uncertain observations. The neutrosophic statistics will be applied to analyze the data having uncertain observations or the parameters. In this paper, we will introduce the neutrosophic analysis of variance (NANONA). The NANONA is an extension of the classical ANOVA. We presented the NANONA table. We performed the NANONA to test teaching methods using data collected from the university students.

Keywords Classical statistics · Neutrosophic statistics · ANOVA · NANONA · Uncertainty

Abbreviations

ANOVA	Analysis of variance
NANONA	Neutrosophic analysis of variance
NS	Neutrosophic statistics
NSQC	Neutrosophic statistical quality control
NRV	Neutrosophic random variable
NND	Neutrosophic normal distribution
NSD	Neutrosophic standard deviation
NSS	Neutrosophic sum of square
NMS	Neutrosophic mean square
ndf	Neutrosophic degree of freedom
SS	Sum of square
df	Degree of freedom
MS	Mean square
y_{Nij}	The *j*th neutrosophic sample observation from *i*th neutrosophic population
n_{Ni}	The number of neutrosophic values chosen from *i*th neutrosophic population
n_{NT}	The total neutrosophic sample size
$\bar{y}_{N..}$	The neutrosophic average from *i*th neutrosophic population
$S^2_{NB} = SSB_N$	Neutrosophic sum of squares between groups
$S^2_{NW} = SSW_N$	Neutrosophic sum of squares within groups
NTSS	Total sum of squares
$\mu_{N1} = \mu_{N2} = \mu_{N3} = \cdots = \mu_{Nt}$	Neutrosophic means to *t*-neutrosophic populations
F_N	Neutrosophic *f*-distribution
α	Level of significance
p_N-value	Neutrosophic *p* value

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Introduction

Statistics is the branch of the mathematical sciences, which deals with the collection, analysis and interpretation of the data. The hypothesis or statement is established, accepted, and rejected based on sample information. Among many tests, the analysis of variance (ANOVA) is an important test, which is applied, for the testing of the hypothesis that the population means of various groups or methods and population are same or not. The ANOVA is an extension of Student t-test, which is applied for the testing of the hypothesis that two populations have the same means. The analyses based on ANOVA have been widely used in a variety of fields. Armstrong et al. [9] used the ANOVA for analysis of the optometry data. Ulusoy [29] applied the ANONA for the analysis of talc particles. Tarrío-Saavedra et al. [28] used to analyze Chemometrics data. Niedoba and Pięta [22] used it in the mineral processing. More applications of the ANOVA can be seen in [13, 23, 30].

The ANONA available in the literature is designed under classical statistics. Therefore, the existing ANONA technique can only be applied when all observations in the population or selected sample are determined. The existing ANOVA cannot be applied to make analysis when some observations in the population or the sample are determined. The fuzzy logic is applied when the exact measurement of the variable is not possible. Several authors worked on the ANOVA under the fuzzy approach. González-Rodríguez et al. [18] applied ANOVA on fuzzy functional data. Jiryaei et al. [19] presented the fuzzy ANOVA. Lin et al. [21] applied the ANOVA on fuzzy consumer data. More details on fuzzy ANOVA can be seen in [12, 16, 17, 20, 21, 24, 25].

A neutrosophic logic is the generalization of the fuzzy logic. The first one considers the measure of truth, false and indeterminacy. Smarandache [26] claimed that the neutrosophic logic is more efficient than the fuzzy logic. The neutrosophic statistics (NS) is introduced by Smarandache [27] which is the extension of classical statistics. The NS is applied when the sample is drawn from the population having uncertain observations. Chen et al. [14, 15] introduced the neutrosophic numbers for rock engineering issues. Aslam [10] introduced the neutrosophic statistical quality control (NSQC) the first time. Aslam [11] provided a table to explain the difference between fuzzy statistics, the NS and classical statistics. We provide the difference, advantages and limitations of classical statistics, fuzzy statistics, intuitionistic fuzzy statistics and the NS as below.

More details about the neutrosophic theory can be seen in [1–8].

By exploring the literature and according to the best of our knowledge, there is no work on ANONA under the NS. We will introduce the ANONA under the NS, called NANOVA. The proposed NANOVA can be applied under the uncertainty environment. We will present NANOVA table. It is expected that the proposed NANONA will be more flexible, effective and informative than ANOVA under classical statistics for the testing of means of various groups under the uncertainty environment. We will discuss the proposed NANOVA with the help of real data collected from university students.

The neutrosophic ANOVA

Some notations for the neutrosophic ANOVA (NANOVA) is given as

- $y_{\mathrm{N}ij}$ The jth neutrosophic sample observation from ith neutrosophic population
- $n_{\mathrm{N}i}$ The number of neutrosophic values chosen from ith neutrosophic population
- n_{NT} The total neutrosophic sample size
- $\bar{y}_{\mathrm{N}..}$ The neutrosophic average from ith neutrosophic population
- $\mathrm{SSB_N}$ Neutrosophic sum of squares between groups
- $\mathrm{SSW_N}$ Neutrosophic sum of squares within groups
- NTSS Total sum of squares.

Suppose that a neutrosophic random variable (NRV) $X_{\mathrm{N}} \in [X_{\mathrm{L}}, X_{\mathrm{U}}]$ is selected from a neutrosophic normal distribution

Classical statistics	Fuzzy statistics	Intuitionistic fuzzy statistics	Neutrosophic statistics
Classical statistics is applied for the analysis of the data when all observations/parameters in the sample or the population are precise and determined	The fuzzy statistics is applied for the analysis of the data having imprecise, uncertain and fuzzy observations/parameters. The statistics are based on fuzzy statistics does not consider the measure of indeterminacy	Intuitionistic fuzzy (IF) is the extension of the classical fuzzy logic. The IF is considered membership and non-membership, which belong to the real unit interval. Therefore, the statistics are based on IF will be the extension of fuzzy statistics	The neutrosophic statistics (NS) is based on the idea of neutrosophic logic. The neutrosophic logic is the extension of the fuzzy logic and considered the measure of indeterminacy. Therefore, the NS is the extension of classical statistics which deals to analyze under uncertainty
Limitations			
Classical statistics can be only applied when all data or parameters are determined and precise	The fuzzy statistics will be applied only when some observations/parameters are fuzzy	The IF will be applied only when membership and non-membership, which belong to the real unit interval	The NS is applied under the uncertainty environment. It reduces to classical statistics when all observations/parameters are determined

Table 1 NANOVA table

Source	NSS	ndf	NMS	F_N
Between samples	$SSB_N = SSB + ub\ I$	$t_N - 1$	$S^2_{NB} = SSB_N/t_N - 1$	$\frac{S^2_{NB}}{S^2_{NW}}$
Within samples	$SSW_N = SSW + uw\ I$	$n_{NT} - t_N$	$S^2_{NW} = SSW_N/n_{NT} - t_N$	
Totals		$n_{NT} - 1$		

Table 2 The analysis of variance for various concepts

Analysis of variance	Fuzzy analysis of variance	Intuitionistic fuzzy analysis of variance	Neutrosophic analysis of variance
This analysis of variance is applied for the testing of means under classical statistics when all the observations are determined	This analysis of variance is applied for the testing of means under fuzzy statistics when some or all observations are fuzzy and uncertain	This analysis of variance is applied for the testing of means when membership and non-membership, which belong to the real unit interval	This analysis of variance is applied for the testing of means when observations/parameters are not fuzzy, in-interval. The analysis of variance is the extension of the analysis of variance under classical and fuzzy statistics

(NND) with neutrosophic mean, say $\mu_N \in [\mu_L, \mu_U]$ and neutrosophic standard deviation (NSD), say $\sigma_N \in [\sigma_L, \sigma_U]$, respectively. Like the ANONA in classical statistics, the ANONA under the neutrosophic statistics can be applied to test more than one neutrosophic population means simultaneously. The one way ANOVA under the neutrosophic statistics is explained as follows:

Step-1: State the null hypothesis and an alternative hypothesis that the neutrosophic population means are equal:

$$H_{N0} : \mu_{N1} = \mu_{N2} = \mu_{N3} = \cdots = \mu_{Nt}. \quad (1)$$

H_{N1} At least one of the t_N-neutrosophic populations means are not equal

Step-2: Establish the neutrosophic F test:

$$F_N = \frac{S^2_{NB}}{S^2_{NW}}; \quad F_N \in [F_L, F_U] \quad (2)$$

where $S^2_{NB} = SSB_N/t_N - 1$; $S^2_{NB} \in [S^2_{LB}, S^2_{UB}]$, $S^2_{NW} = SSW_N/n_{NT} - t_N$; $S^2_{NW} \in [S^2_{LW}, S^2_{UW}]$. Note here that $SSB_N = \sum_{i=1}^{t_N} n_{Ni}(\bar{y}_{Ni} - \bar{y}_{N..})$; $SSB_N \in [SSB_L, SSB_U]$ and $SSW_N = \sum_{i=1}^{t_N}\sum_{j=1}^{n_{Ni}}(y_{Nij} - \bar{y}_{Ni.})$; $SSW_N \in [SSW_L, SSW_U]$. Finally, neutrosophic total sum of squares (NTSS) is given as follows

$$NTSS = S^2_{NB} + S^2_{NW}. \quad (3)$$

Assumptions

The assumptions for the NANOVA are given as follows

1. To perform the NANOVA, it is assumed that the neutrosophic random sample is selected from the neutrosophic population.
2. Further, it is assumed that neutrosophic variance is the same in neutrosophic population.

The structure of the proposed NANOVA is given in Table 1.

The difference and advantages between analysis of variance, fuzzy analysis of variance, intuitionistic fuzzy analysis of variance and neutrosophic analysis of variance is explained in the following Table 2

Table 3 The ALT scores

Method	Test score				Total
1	[80, 81]	[92, 92]	[87, 88]	[83, 83]	[342, 344]
2	[70, 70]	[81, 82]	[78, 78]	[74, 75]	[303, 305]
3	[63, 64]	[76, 76]	[70, 71]	[80, 81]	[289, 292]
Total					[934, 941]

Application

To minimize the hostility levels among the university students, a clinical psychologist is interested to perform NANONA to compare three methods. He is interested to test either the population means of all groups are equal or not. He applied the HLT test to measure the data from various students and a high HLT score shows the great hostility

Table 4 The NANOVA table for data

Source	NSS	ndf	NMS	F_N	p_N-value
Between samples	[377.2, 366.2]	[2, 2]	[188.58, 183.08]	[5.397, 5.33]	[0.0288, 0.0296]
Within samples	[314.5, 308.7]	[9, 9]	[34.94, 34.31]		
Totals	[691.7, 6749]	[11, 11]			

Table 5 The ANOVA table for data

Source	SS	*df*	MS	*F*	*p*-value
Between samples	366.2	2	183.08	5.33	0.0296
Within samples	308.7	9	34.31		
Totals	6749	11			

levels. The twelve students are selected for this test. Four students are randomly selected and placed in a group and treated with method 1. Four students from eight students are again selected at random and treated with method 2. Four students from four students are again selected at random and treated with method 3. While measuring HLT scores, the clinical psychologist is uncertain in some scores. Under this situation, he recorded data in the neutrosophic interval. Therefore, the ANOVA under classical statistics cannot be applied for the testing of means among groups. The alternative test is NANONA for the testing of means. The data is shown in Table 3. For the data given in Table 3, the existing ANOVA under classical statistics cannot be applied for testing either three methods are equal in means or not. Therefore, the proposed NANOVA is a reasonable alternative for this data.

The sum of squares between samples, within sample and totals, as shown in Table 4. Smarandache [27] discussed the rules of the p value for the neutrosophic data. Let p_N-value denotes the p value for the neutrosophic statistics. According to rules, the null hypothesis that three means are equal will be rejected if $\max\{p_N\text{-value}\} \leq \alpha$, where α is a level of significance. For this real data, $\max\{p_N\text{-value} = 0.0295\} \leq 0.05$, we reject the hypothesis that the means of three methods are the same. Table 4 presented all sum of squared in indeterminacy interval. The ANOVA under classical statistics is shown in Table 5. From Table 5, we note that all values in ANOVA table are determined. By comparing both tables, it is concluded that the proposed NANONA is a more adequate method than the methods under classical statistics.

Conclusions

In this paper, an ANOVA under the NS is presented. The proposed NANOVA is the generalization of the existing ANOVA under classical statistics. The proposed methods have the ability to be applied effectively than the existing under uncertainty. A NANOVA table from a real example shows that sum squares were in indeterminacy interval. We recommend applying the proposed ANONA for testing of the means in a variety of fields under uncertainty. The post hoc tests under the neutrosophic NANOVA can be considered as future research.

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