Neutrosophic-simplified-TOPSIS

Multi-Criteria Decision-Making using combined Simplified-TOPSIS method and Neutrosophics

Azeddine Elhassouny
Team RIITM
Dept. Engineering Software, ENSIAS,
MOHAMMED V UNIVERSITY IN RABAT
Rabat, Morocco
elhassounyphd@gmail.com

Florentin Smarandache
Dept. Mathematics
University of New Mexico
Gallup, New Mexico, USA
smarand@unm.edu

***Abstract*—In this paper, (1) one simplified the standard TOPSIS to new Multi-Criteria Decision-Making (MCDM) called Simplified-TOPSIS. Simplified-TOPSIS gives the same results and simplifies the calculation of the classical TOPSIS. An example is presented distinctions between Simplified-TOPSIS and classical TOPSIS are underlined. (2) extend the new Simplified-TOPSIS method to Neutrosophic-simplified-TOPSIS using single valued Neutrosophic information. An example showing the interest of Neutrosophic-simplified-TOPSIS to manipulate the uncertainty linked to information presented in Multi-Criteria Decision-Making.**

Keywords—Simplified TOPSIS; Neutrosophic; MCDM; Neutrosophic-simplified-TOPSIS

I. INTRODUCTION

Standard TOPSIS, the Technique for Order Preference by Similarity to Ideal Solution method is a multi-criteria decision-making approach, was introduced by Hwang and Yoon [1]. The classical TOPSIS is one of sophisticated MCDM for solving problems with respect to crisp numbers, often involving complicated steps of calculation algorithms that are difficult to learn and apply.

In the real MCDM problems, the attribute values are always be expressed with imperfect information, however, decision-makers may prefer to use an easy, simple technique and give same result rather than complex algorithm. The objective of this paper, we present, firstly, simplified-TOPSIS, a new MCDM method that simplifies the calculation and gives the same results of traditional TOPSIS. Secondly, we introduce a hybrid method to resolve real MCDM problems with imperfect information based on Neutrosophic and simplified-TOPSIS method (Neutrosophic-simplified-TOPSIS).

Smarandache [2,3] proposed a generalization of the Intuitionistic Fuzzy Set (IFS), called Neutrosophic Set (NS) which based on three values (truth, indeterminacy and falsity) and able to handle incomplete information (such as uncertainty, imprecise, incomplete and inconsistent information)[4].

Wang and Smarandache [5] defined single valued Neutrosophic Set (SVNS). Broumi and Smarandache [4,6,7] offered different operators such as distance and similarity measures over the single valued Neutrosophic Set and their basic properties were studied.

Mumtaz and Smarandache [8] introduced complex Neutrosophic Set. Mumtaz et al. [9] proposed and applied the theory of Neutrosophic cubic Sets in pattern recognition area.

Bahramloo and Hoseini [10] used MCDM method in Intuitionist Fuzzy Sets, which extended by Smarandache [2] to Neutrosophic Set, for raking alternatives.

Biswas [11] summarized the definition given by Wang and Smarandache [5] of single valued Neutrosophic Set as well as the definition of some aggregation operators such as aggregated single valued Neutrosophic, weighted Neutrosophic to solve MCDM problems using extended TOPSIS.

Broumi [7] studied multiple attribute decision making by using interval Neutrosophic uncertain linguistic variables.

Peng [12] also developed a Multi-criteria decision making method based on aggregation operators and TOPSIS in multi hesitant fuzzy environment. Furthermore, Deli et al. [13] applied bipolar Neutrosophic Sets on MCDM problems.

The paper is organized as follows. In the next section we present TOPSIS method. Section 3 will focus on the proposed method Simplified-TOPSIS. Afterwards, the Neutrosophic-TOPSIS in section 4. In section 5 a Neutrosophic-simplified-TOPSIS is introduced and it is shown how it can be applied for ranking preferences. In the final section, conclusions are drawn.

II. TOPSIS METHOD

Let us assume that $\mathrm{C} = \{\mathrm{C}_1, \mathrm{C}_2, \cdots, \mathrm{C}_n\}$ is a set of Criteria, withn ≥ 2, $\mathrm{A} = \{\mathrm{A}_1, \mathrm{A}_2, \cdots, \mathrm{A}_n\}$ is the set of Preferences (Alternatives), withm ≥ 1, a_{ij} the score of preferencei with respect to criterion j, and let ω_{i} weight of criteria C_{i}.

Using a_{ij}we construct the decision matrix denoted by

$$D = \left(a_{ij}\right)_{m\times n} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix}$$

TOPSIS method summarizes as follow :

Step 1: The normalized decision matrix is obtained, which is given here with r_{ij}.

$$r_{ij} = a_{ij} / \left(\sum_{i=1}^{m} a_{ij}^2 \right)^{0.5}; \; j = 1, 2, \cdots, n; i = 1, 2 \cdots, m$$

Step 2: Obtain the weighted normalized decision matrix v_{ij}:

Multiply each column of the normalized decision matrix by its associated weight.

$$v_{ij} = w_j r_{ij}; \; j = 1, 2, \cdots, n; i = 1, 2 \cdots, m$$

Step 3: Determine the ideal and negative ideal solutions.

$$A^+ = (v_1^+, v_2^+, \cdots, v_n^+) = \left\{ \begin{array}{l} \left(max_i \left\{ v_{ij} \mid j \in B \right\} \right), \\ \left(min_i \left\{ v_{ij} \mid j \in C \right\} \right) \end{array} \right\}$$

$$A^- = (v_1^-, v_2^-, \cdots, v_n^-) = \left\{ \begin{array}{l} \left(min_i \left\{ v_{ij} \mid j \in B \right\} \right), \\ \left(max_i \left\{ v_{ij} \mid j \in C \right\} \right) \end{array} \right\}$$

Where sets B and C are associated with the benefit and cost attribute sets, respectively.

Step 4: Calculate the separation measures for each alternative from the positive (negative) ideal solution.

The separation from the positive ideal alternative is

$$S_i^+ = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^+)^2 \right\}^{0.5}; \; i = 1, 2 \cdots, m$$

Similarly, the separation from the negative ideal alternative is

$$S_i^- = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^-)^2 \right\}^{0.5}; \; i = 1, 2 \cdots, m$$

Step 5: The relative closeness to the ideal solution of each alternative is calculated as.

$$T_i = \frac{S_i^-}{(S_i^+ + S_i^-)}; \; i = 1, 2 \cdots, m$$

A set of alternatives can now be ranked according to the descending order of the value of T_i.

A. Numerical example

In the examples below we used TOPSIS to rank the four alternatives.

The table (Table I) below contains the weights of criteria (three criteria C_1, C_2 and C_3) and the decision matrix summarized by the score of preference A_i (A_1, A_2, A_3 and A_4) with respect to criterion C_i.

TABLE I. DECISION MATRIX

a_{ij}	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	7	9	9
A_2	8	7	8
A_3	9	6	8
A_4	6	7	8

Calculate $\sum_{i=1}^{n} a_{ij}$ for each column, we get (Table II).

TABLE II. MULTIPLE DECISION MATRIX

a_{ij}^2	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	49	81	81
A_2	64	49	64
A_3	81	36	64
A_4	36	49	64
$\sum_{i=1}^{n} a_{ij}$	230	215	273

Divide each column by $(\sum_{i=1}^{n} a_{ij}^2)^{1/2}$ to get r_{ij} (Table III).

TABLE III. NORMALIZED DECISION MATRIX

r_{ij}	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	0.4616	0.6138	0.5447
A_2	0.5275	0.4774	0.4842
A_3	0.5934	0.4092	0.4842
A_4	0.3956	0.4774	0.4842
$\sum_{i=1}^{n} a_{ij}$	230	215	273

Multiply each column by w_j to get v_{ij} (Table IV).

TABLE IV. WEIGHTED DECISION MATRIX

v_{ij}	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	0.3462	0.1151	0.0340
A_2	0.3956	0.0895	0.0303
A_3	0.4451	0.0767	0.0303
A_4	0.2967	0.0895	0.0303
v_{max}	0.4451	0.1151	0.0340
v_{min}	0.2967	0.0767	0.0303

The distance values from the positive and negative ideal solution and the final rankings for decision matrix are showed in Table V.

TABLE V. DISTANCE MEASURE AND RANKING COEFFICIENT

Alternative	S_i^+	S_i^-	T_i
A_1	0.0989	0.0627	0.3880
A_2	0.0558	0.0997	0.6412
A_3	0.0385	0.1484	**0.7938**
A_4	0.1506	0.0128	0.0783

According to values of ranking measure coefficients, the Table V indicates that better alternative is A_3 and preferences are classified as $A_3 > A_1 > A_4 > A_2$.

III. SIMPLIFIED-TOPSIS METHOD (OUR PROPOSED METHOD)

Let consider $C = \{C_1, C_2, \cdots, C_n\}$ is a set of Criteria, with $n \geq 2$, $A = \{A_1, A_2, \cdots, A_n\}$ is the set of Preferences (Alternatives), with $m \geq 1$, a_{ij} the score of preference i with respect to criterion j, and let ω_i weight of criteria C_i.

$$D = (a_{ij})_{m\times n} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix}$$

Our proposed MCDM method called Simplified-TOPSIS can be described in following steps:

Step 1: Calculate weighted decision matrix v_{ij}.

Multiply each column of the normalized decision matrix by its associated weight

$$v_{ij} = w_j a_{ij};\ j = 1,2,\cdots,n; i = 1,2\cdots,m$$

In our method we have not normalized the decision matrix (step1 of classical TOPSIS (section II)), but we calculate directly the weighted decision matrix v_{ij} by multiplying a_{ij} with ω_j.

Step 2: Determine the maximum (largest) ideal solution (LIS) and minimum (smallest) ideal solution (SIS).

$$A^+ = (v_1^+, v_2^+, \cdots, v_m^+) = \left(max_i \left\{v_{ij} \mid j = 1,2,\cdots,n\right\}\right)$$

$$A^+ = (v_1^-, v_2^-, \cdots, v_m^-) = \left(\min_i \left\{v_{ij} \mid j = 1,2,\cdots,n\right\}\right)$$

Step 3: Calculate the sums for each line, by subtracting each number from LIS (from SIS).

$$S_i^+ = \left\{\sum_{j=1}^{n}(v_{ij} - v_j^+)^2\right\}^{0.5};\ i = 1,2\cdots,m$$

Similarly, we compute the sums for each line, by subtracting each number from SIS.

$$S_i^- = \left\{\sum_{j=1}^{n}(v_{ij} - v_j^-)^2\right\}^{0.5};\ i = 1,2\cdots,m$$

Classifying these sums which one is closer to the maximum (or is further from the minimum)

A set of alternatives can now be ranked according to the descending order of the value of sums (S_i^+) or (S_i^-).

Step 4(facultative): We can compute T_i, though the previous steps enough to rank the alternatives.

$$T_i = \frac{S_i^-}{(S_i^+ + S_i^-)};\ i = 1,2\cdots,m$$

A. Numerical example

In order to compare the result with classical TOPSIS we use the same numerical examples used in classical TOPSIS.

TABLE VI. DECISION MATRIX

a_{ij}	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	7	9	9
A_2	8	7	8
A_3	9	6	8
A_4	6	7	8

One multiplies on columns with the weights $12/16$, $3/16$, and $1/16$ respectively, and one gets:

TABLE VII. WEIGHTED DECISION MATRIX

$\omega_j a_{ij}$	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	84/16	27/16	9/16
A_2	96/16	21/16	8/16
A_3	108/16	18/16	8/16
A_4	72/16	21/16	8/16

We compute the sums for each line, by subtracting each number from the largest one:

$$S_{1+} = |84/16 - 108/16| + |21/16 - 27/16| + |9/16 - 9/16| = 1.5000$$

$$S_{2+} = |96/16 - 108/16| + |27/16 - 27/16| + |9/16 - 9/16| = 1.1875$$

$$S_{3+} = |108/16 - 108/16| + |18/16 - 27/16| + |8/16 - 9/16| = 0.6250$$

$$S_{4+} = |72/16 - 108/16| + |21/16 - 27/16| + |8/16 - 9/16| = 2.6875$$

Classifying these sums we get them on places: S_{3+}, S_{2+}, S_{1+}, S_{4+} in the order of which one is closer to the maximum.

We compute the sums for each line, by subtracting each number from the smaller one:

$$S_{1-} = |84/16 - 72/16| + |27/16 - 18/16| + |9/16 - 8/16| = 1.3750$$

$$S_{2-} = |96/16 - 72/16| + |21/16 - 18/16| + |8/16 - 8/16| = 1.6875$$

$$S_{3-} = |108/16 - 72/16| + |18/16 - 18/16| + |8/16 - 8/16| = 2.2500$$

$$S_{4-} = |72/16 - 72/16| + |21/16 - 18/16| + |8/16 - 8/16| = 0.1875$$

Classifying these sums we get them on places: S_{3-}, S_{2-}, S_{1-}, S_{4-} in the order of which one is further from the minimum.

If we compute T_i, we get the same ordering of classical TOPSIS:

$$T_1 = (S_{1-})/[(S_{1-}) + (S_{1+})] = 0478261$$

$$T_2 = (S_{2-})/[(S_{2-}) + (S_{2+})] = 0.586957$$

$$T_3 = (S_{3-})/[(S_{3-}) + (S_{3+})] = 0.782609$$

$$T_4 = (S_{4-})/[(S_{4-}) + (S_{4+})] = 0.065217$$

The following table (Table VIII) summarized previous calculations

TABLE VIII. DISTANCE MEASURE AND RANKING COEFFICIENT

Alternative	S_i^+	S_i^-	T_i
A_1	1.5000	1.3750	0.478261
A_2	1.1875	1.6875	0.586957
A_3	0.6250	2.2500	**0.782609**
A_4	2.6875	0.1875	0.065217

By applying Simplified-TOPSIS, we get for T_3(0.782609), T_2(0.586957), T_1(0.478261) and T_4(0.065217), and we got with classical TOPSIS T_3(0.7938), T_2(0.6412), T_1(0.3880) and T_4(0.0783).

Hence the order obtained with our approach simplified-TOPSIS is the same of classical TOPSIS: T_3, T_2, T_1 and T_4, with little change in values between both approaches.

IV. NEUTROSOPHIC TOPSIS [11]

The MCDM Neutrosophic TOPSIS approach is explained in the following steps.

Step 1: Construction of the aggregated single valued Neutrosophic decision matrix based on decision makers assessments

$$D = (d_{ij})_{\substack{1\le i\le n \\ 1\le j\le m}} = (T_{ij}, I_{ij}, F_{ij})_{\substack{1\le i\le n \\ 1\le j\le m}}$$

Where T_{ij} denote truth, I_{ij} indeterminacy and F_{ij} falsity membership score of preference i with respect to criterion j in single valued Neutrosophic.

$w = (\omega_1, \omega_2, \cdots, \omega_n)$ with ω_i a single valued Neutrosophic weight of criteria (so $\omega_i = (a_i, b_i, c_i)$).

Example 1: For compare the results obtained by our approach Neutrosophic-simplified-TOPSIS (will be presented afterwards) with those obtained with Neutrosophic-TOPSIS, we use the example introduced by Biswas [11].

Let (DM_1, DM_2, DM_3, DM_4) fours decisions makers aims to select an alternative $A_i(A_1, A_2, A_3, A_4)$ with respect six criteria $(C_1, C_2, C_3, C_4, C_5, C_6)$.

The Neutrosophic weight of each criterion (Table IX) and Neutrosophic decision matrix (Table X) presented respectively.

TABLE IX. CRITERIA WEIGHTS

	C_1	C_2	C_3
ω_i	(0.755,0.222,0.217)	(0.887,0.113,0.107)	(0.765,0.226,0.182)
	C_4	C_5	C_6
ω_i	(0.692,0.277,0.251)	(0.788,0.200,0.180)	(0.700,0.272,0.244)

TABLE X. NEUTROSOPHIC DECISION MATRIX

	C_1	C_2	C_3
A_1	(0.864,0.136,0.081)	(0.853,0.147,0.:092)	(0.800,0.200,0.150)
A_2	(0.667,0.333,0.277)	(0.727,0.273,0.219)	(0.667,0.333,0.277)
A_3	(0.880,0.120,0.067)	(0.887,0.113,0.064)	(0.834,0.166,0.112)
A_4	(0.667,0.333,0.277)	(0.735,0.265,0.195)	(0.768,0.232,0.180)
	C_4	C_5	C_6
A_1	(0.704,0.296,0.241)	(0.823,0.177,0.123)	(0.864,0.136,0.081)
A_2	(0.744,0.256,0.204)	(0.652,0.348,0.293)	(0.608,0.392,0.336)
A_3	(0.779,0.256,0.204)	(0.811,0.189,0.109)	(0.850,0.150,0.092)
A_4	(0.727,0.273,0.221)	(0.791,0.209,0.148)	(0.808,0.192,0.127)

Step 2: Aggregation of the weighted Neutrosophic decision matrix

$$D^w = D \otimes W = (d_{ij}^w)_{\substack{1\le i\le n \\ 1\le j\le m}} = (T_{ij}^w, I_{ij}^w, F_{ij}^w)_{\substack{1\le i\le n \\ 1\le j\le m}}$$

Step 3: Determination of the relative Neutrosophic positive ideal solution (RNPIS) and the relative negative ideal solution (RNIS) for SVNSs.

$$T_j^{w+} = \left\{\left(max_i\left\{T_{ij}^{w_i}|j \in B\right\}\right), \left(min_i\left\{T_{ij}^{w_i}|j \in C\right\}\right)\right\}$$

$$Q_N^+ = (d_1^{w+}, d_2^{w+}, \cdots, d_n^{w+})$$

$$T_j^{w+} = \left\{\left(max_i\left\{T_{ij}^{w_j}|j \in B\right\}\right), \left(min_i\left\{T_{ij}^{w_j}|j \in C\right\}\right)\right\}$$

$$I_j^{w+} = \left\{\left(min_i\left\{I_{ij}^{w_j}|j \in B\right\}\right), \left(max_i\left\{I_{ij}^{w_j}|j \in C\right\}\right)\right\}$$

$$F_j^{w+} = \left\{\left(min_i\left\{F_{ij}^{w_j}|j \in B\right\}\right), \left(max_i\left\{F_{ij}^{w_j}|j \in C\right\}\right)\right\}$$

$$Q_N^- = (d_1^{w-}, d_2^{w-}, \cdots, d_n^{w-})$$

$$T_j^{w-} = \left\{\left(min_i\left\{T_{ij}^{w_j}|j \in B\right\}\right), \left(max_i\left\{T_{ij}^{w_j}|j \in C\right\}\right)\right\}$$

$$I_j^{w-} = \left\{\left(max_i\left\{I_{ij}^{w_j}|j \in B\right\}\right), \left(min_i\left\{I_{ij}^{w_j}|j \in C\right\}\right)\right\}$$

$$F_j^{w-} = \left\{\left(max_i\left\{F_{ij}^{w_j} | j \in B\right\}\right), \left(min_i\left\{F_{ij}^{w_j} | j \in C\right\}\right)\right\}$$

Where sets B and C are associated with the benefit and cost attribute sets, respectively

Step 4: Determination of the distance measure of each alternative from the RNPIS and the RNNIS for SVNSs.

$$D_{Eu}^{i+}\left(d_{ij}^{wj}, d_{ij}^{w+}\right) = \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left\{\begin{array}{l}\left(T_{ij}^{wj}(x) - T_{ij}^{w+}(x)\right)^2 + \\ \left(I_{ij}^{wj}(x) - I_{ij}^{w+}(x)\right)^2 + \\ \left(F_{ij}^{wj}(x) - F_{ij}^{w+}(x)\right)^2\end{array}\right\}}$$

with $i = 1, 2 \cdots, m$

And

$$D_{Eu}^{i-}\left(d_{ij}^{wj}, d_{ij}^{w-}\right) = \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left\{\begin{array}{l}\left(T_{ij}^{wj}(x) - T_{ij}^{w-}(x)\right)^2 + \\ \left(I_{ij}^{wj}(x) - I_{ij}^{w-}(x)\right)^2 + \\ \left(F_{ij}^{wj}(x) - F_{ij}^{w-}(x)\right)^2\end{array}\right\}}$$

Step 5: Determination of the relative closeness coefficient to the Neutrosophic ideal solution for SVNSs.

$$C_i^* = \frac{NS_i^-}{(NS_i^+ + NS_i^-)};\ i = 1, 2 \cdots, m$$

A set of alternatives can now be ranked according to the descending order of the value of C_i^*.

Table below (Table XI) shows the results obtained by Neutrosophic-TOPSIS.

TABLE XI. CLOSENESSCOEFFICIENT

Alternative	C_i^*
A_1	0.8190
A_2	0.1158
A_3	**0.8605**
A_4	0.4801

Based on the values of closeness coefficient, the four alternatives are classified as $A_3 > A_1 > A_4 > A_2$. Then, the alternative A_3 is the best solution.

V. NEUTROSOPHIC-SIMPLIFIED-TOPSIS (OUR PROPOSED METHOD)

Step 1: Building of the SVNS decision matrix and SVNS weight of each criterion.

$$D = (d_{ij})_{\substack{1\le i\le n\\ 1\le j\le m}} = (T_{ij}, I_{ij}, F_{ij})_{\substack{1\le i\le n\\ 1\le j\le m}}$$

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & \cdots & \cdots & d_{mn} \end{pmatrix}\end{array}$$

Where T_{ij} denote truth, I_{ij} indeterminacy and N_{ij} falsity membership score of preference i with respect to criterion j in single valued Neutrosophic.

$w = (\omega_1, \omega_2, \cdots, \omega_n)$ with ω_i a single valued Neutrosophic weight of criteria (so $\omega_i = (a_i, b_i, c_i)$)

Step 2: Calculate SVNS weighted decision matrix

$$D^w = D \otimes W = (d_{ij}^w)_{\substack{1\le i\le n\\ 1\le j\le m}} = \omega_j \otimes d_{ij}^w = (T_{ij}^w, I_{ij}^w, F_{ij}^w)_{\substack{1\le i\le n\\ 1\le j\le m}}$$

$$\omega_j \otimes d_{ij} = \left(a_j T_{ij}, b_j + I_{ij} - b_j I_{ij}, c_j + F_{ij} - c_j F_{ij}\right)$$

Step 3: Determine the maximum (larger) Neutrosophic ideal solution (LNIS) and minimum (smaller) Neutrosophic ideal solution (SNIS).

$$A_N^+ = (d_1^{w+}, d_2^{w+}, \cdots, d_n^{w+})$$

$$d_j^{\omega+} = \left(T_j^{\omega+}, I_j^{\omega+}, F_j^{\omega+}\right)$$

$$T_j^{w+} = \left\{\left(max_i\left\{T_{ij}^{w_j} | j = 1, \cdots, n\right\}\right)\right\}$$

$$I_j^{w+} = \left\{\left(min_i\left\{I_{ij}^{w_j} | j = 1, \cdots, n\right\}\right)\right\}$$

$$F_j^{w+} = \left\{\left(min_i\left\{F_{ij}^{w_j} | j = 1, \cdots, n\right\}\right)\right\}$$

$$A_N^- = (d_1^{w-}, d_2^{w-}, \cdots, d_n^{w-})$$

$$d_j^{\omega-} = \left(T_j^{\omega-}, I_j^{\omega-}, F_j^{\omega-}\right)$$

$$T_j^{w-} = \left\{\left(min_i\left\{T_{ij}^{w_j} | j = 1, \cdots, n\right\}\right)\right\}$$

$$I_j^{w-} = \left\{\left(max_i\left\{I_{ij}^{w_j} | j = 1, \cdots, n\right\}\right)\right\}$$

$$F_j^{w-} = \left\{\left(max_i\left\{F_{ij}^{w_j} | j = 1, \cdots, n\right\}\right)\right\}$$

Step 4: Calculate the Neutrosophic separation measures for each alternative from LNIS and from SNIS.

In this case we have introduced à new distance measure (definition 1) between two single-valued Neutrosophic (SVNs) using Manhattan distance [14] instead of the Euclidean distance used to calculate similarity measure between two

SVNs in literature and in Neutrosophic-TOPSIS method, the defined distance is used to calculate distance measure.

Definition 1. Let $X_1 = (x_1, y_1, z_1)$ and $X_2 = (x_2, y_2, z_2)$ be a SVN numbers. Then the separation measure between X_1 and X_2 based on Manhattan distance is defined as follows:

$$D_{Manh}(X_1, X_2) = |x_1 - x_2| + |y_1 - y_2| + |z_1 - x_2|$$

The separation from the maximum Neutrosophic ideal solution is :

$$D_{Manh}^{j+}\left(d_{ij}^{wj}, d_{ij}^{w+}\right) = \left\{\begin{array}{l} \left|T_{ij}^{wj}(x) - T_{ij}^{w+}(x)\right| + \\ \left|I_{ij}^{wj}(x) - I_{ij}^{w+}(x)\right| + \\ \left|F_{ij}^{wj}(x) - F_{ij}^{w+}(x)\right| \end{array}\right\}$$

with $j = 1, 2 \cdots, n$

$$NS_i^+ = \sum_{j=1}^{n} D_{Manh}^{j+}\left(d_{ij}^{wj}, d_{ij}^{w+}\right) \text{ with } i = 1, 2 \cdots, m$$

Similarly, the separation from the minimum Neutrosophic ideal solution is:

$$D_{Manh}^{j-}\left(d_{ij}^{wj}, d_{ij}^{w-}\right) = \left\{\begin{array}{l} \left|T_{ij}^{wj}(x) - T_{ij}^{w-}(x)\right| + \\ \left|I_{ij}^{wj}(x) - I_{ij}^{w-}(x)\right| + \\ \left|F_{ij}^{wj}(x) - F_{ij}^{w-}(x)\right| \end{array}\right\}$$

with $j = 1, 2 \cdots, n$

$$NS_i^- = \sum_{j=1}^{n} D_{Manh}^{j-}\left(d_{ij}^{wj}, d_{ij}^{w-}\right) \text{ with } i = 1, 2 \cdots, m$$

Ranking the alternatives according to the values of NS_i^- or according to $1/\mathrm{NS}_i^+$

Step 5: The measure ranking coefficient is calculated as.

$$NT_i = \frac{NS_i^-}{(NS_i^+ + NS_i^-)}; \; i = 1, 2 \cdots, m$$

A set of alternatives can now be ranked according to the descending order of the value of NT_i

A. Numerical example

Step 1: Building of the SVNS decision matrix and SVNS weight of each criterion.

Let $A_i(A_1, A_2, A_3, A_4)$ a set of alternative and $C_i(C_1, C_2, C_3, C_4, C_5, C_6)$ a set of criteria.

Let considers the following Neutrosophic weights of criteria (Table XII) and Neutrosophic decision matrix (Table XIII) respectively (used in above example 1).

TABLE XII. CRITERIA NEUTROSOPHIC WEIGHTS

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
$\boldsymbol{\omega_i}$	(0.755,0.222,0.217)	(0.887,0.113,0.107)	(0.765,0.226,0.182)
	$\mathbf{C_4}$	$\mathbf{C_5}$	$\mathbf{C_6}$
$\boldsymbol{\omega_i}$	(0.692,0.277,0.251)	(0.788,0.200,0.180)	(0.700,0.272,0.244)

TABLE XIII. NEUTROSOPHIC DECISION MATRIX

$\mathbf{d_{ij}}$	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
$\mathbf{A_1}$	(0.864,0.136,0.081)	(0.853,0.147,0.092)	(0.800,0.200,0.150)
$\mathbf{A_2}$	(0.667,0.333,0.277)	(0.727,0.273,0.219)	(0.667,0.333,0.277)
$\mathbf{A_3}$	(0.880,0.120,0.067)	(0.887,0.113,0.064)	(0.834,0.166,0.112)
$\mathbf{A_4}$	(0.667,0.333,0.277)	(0.735,0.265,0.195)	(0.768,0.232,0.180)
	$\mathbf{C_4}$	$\mathbf{C_5}$	$\mathbf{C_6}$
$\mathbf{A_1}$	(0.704,0.296,0.241)	(0.823,0.177,0.123)	(0.864,0.136,0.081)
$\mathbf{A_2}$	(0.744,0.256,0.204)	(0.652,0.348,0.293)	(0.608,0.392,0.336)
$\mathbf{A_3}$	(0.779,0.256,0.204)	(0.811,0.189,0.109)	(0.850,0.150,0.092)
$\mathbf{A_4}$	(0.727,0.273,0.221)	(0.791,0.209,0.148)	(0.808,0.192,0.127)

Step 2: Calculate SVNs weighted decision matrix

$$D^w = (d_{ij}^w)_{\substack{1 \le i \le n \\ 1 \le j \le m}} = (T_{ij}^w, I_{ij}^w, F_{ij}^w)_{\substack{1 \le i \le n \\ 1 \le j \le m}}$$

$$d_{ij}^w = \left(a_j T_{ij}, b_j + I_{ij} - b_j I_{ij}, c_j + F_{ij} - c_j F_{ij}\right)$$

One multiplies each columns of Neutrosophic decision matrix with the weights of criteria, and one gets:

$$T_{11}^{\omega} = 0.864 \times 0.755 = 0.6523$$

$$I_{11}^{\omega} = 0.136 + 0.222 - 0.136 \times 0.222 = 0.328$$

$$F_{11}^{\omega} = 0.081 + 0.217 - 0.081 \times 0.217 = 0.280$$

TABLE XIV. WEIGHTED DECISION MATRIX

$\mathbf{d_{ij}^w}$	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
$\mathbf{A_1}$	(0.6523,0.328,0.28)	(0.757,0.243,0.189)	(0.612,0.381,0.305)
$\mathbf{A_2}$	(0.504,0.481,0.434)	(0.645,0.355,0.303)	(0.510,0.484,0.409)
$\mathbf{A_3}$	(0.664,0.315,0.269)	(0.787,0.213,0.164)	(0.638,0.354,0.274)
$\mathbf{A_4}$	(0.504,0.481,0.434)	(0.652,0.348,0.281)	(0.588,0.406,0.329)
	$\mathbf{C_4}$	$\mathbf{C_5}$	$\mathbf{C_6}$
$\mathbf{A_1}$	(0.487,0.491,0.432)	(0.649,0.342,0.281)	(0.605,0.371,0.305)
$\mathbf{A_2}$	(0.515,0.462,0.404)	(0.514,0.478,0.420)	(0.426,0.557,0.498)
$\mathbf{A_3}$	(0.539,0.462,0.404)	(0.639,0.351,0.269)	(0.595,0.381,0.314)
$\mathbf{A_4}$	(0.503,0.474,0.417)	(0.623,0.367,0.301)	(0.566,0.412,0.340)

a. Numbers are rounded to three decimal place.

Step 3: Determine the maximum (larger) Neutrosophic ideal solution (LNIS) and minimum (smaller) Neutrosophic ideal solution (SNIS).

TABLE XV. MAXIMUM (LARGE) NEUTROSOPHIC IDEAL SOLUTION(LNIS)

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
$\mathbf{d_j^{\omega+}}$	(0.664,0.315,0.269)	(0.887,0.213,0.264)	(0.638,0.354,0.274)
	$\mathbf{C_4}$	$\mathbf{C_5}$	$\mathbf{C_6}$
$\mathbf{d_j^{\omega+}}$	(0.539,0.462,0.404)	(0.649,0.341,0.294)	(0.605,0.371,0.305)

TABLE XVI. MINIMUM (SMALLER) NEUTROSOPHICIDEAL SOLUTION (SNIS)

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
$\mathbf{d_j^{\omega-}}$	(0.504,0.481,0.434)	(0.645,0.355,0.303)	(0.510,0.484,0.409)
	$\mathbf{C_4}$	$\mathbf{C_5}$	$\mathbf{C_6}$
$\mathbf{d_j^{\omega-}}$	(0.487,0.491,0.432)	(0.514,0.478,0.420)	(0.426,0.557,0.498)

Step 4: Calculate the Neutrosophic separation measures for each alternative from the LNIS and from SNIS.

We compute the sums for each line, by subtracting each alternative from the larger one and by subtracting each alternative from the smaller one.

TABLE XVII. NEUTROSOPHIC SEPARATION MEASURES AND NEUTROSOPHIC MEASURE RANKING

	$\mathbf{NS_i^+}$	$\mathbf{NS_i^-}$	$\mathbf{NT_i}$
$\mathbf{A_1}$	0,324	2,07	0,86459295
$\mathbf{A_2}$	2,31	0,084	0,03521102
$\mathbf{A_3}$	0,047	2,347	**0,98021972**
$\mathbf{A_4}$	1,293	1,101	0,45987356

Based on the values of coefficients of decreasing rank, four alternatives are ranked as $A_3 > A_1 > A_4 > A_2$ as in Table XVII. Then, the alternative A_3 is also the best solution.

Hence, we get the same rank of Neutrosophic-TOPSIS.

VI. CONCLUSION

In this paper, we have presented two new MCDM methods, the first is simplified-TOPSIS, that simplifies the calculation of classical TOPSIS to a simple formulas easy to applying and a reduced number of steps and give same results of classical TOPSIS. The second is MCDM method in Neutrosophic environment, which is too simplifies the Neutrosophic-TOPSIS, extending the Simplified-TOPSIS using single valued Neutrosophic information. Maximum larger) Neutrosophic Ideal Solution (LNIS) and Minimum (smaller) Neutrosophic Ideal Solution (SNIS) are defined from weighted decision matrix. Manhattan distance Neutrosophic measure is defined and used to determine the distances of each alternative from maximum as well as minimum Neutrosophic ideal solutions, which used to calculate the measure ranking coefficient of each alternative.

REFERENCES

[1] L. Hwang and K. Yoon, Multiple Attribute Decision Making Methods and Applications, Springer, Heidelberg, Germany, 1981.

[2] Smarandache, F. (1999). A unifying field in logics. Neutrosophy: Neutrosophic probability, set and logic, American Research Press, Rehoboth.

[3] Smarandache, F. (2005). A generalization of the intuitionistic fuzzy set. International journal of Pure and Applied Mathematics, 24, 287-297.

[4] Broumi, S., Deli, I., & Smarandache, F. (2014). Distance and similarity measures of interval Neutrosophic soft sets. Neutrosophic Theory and Its Applications. 79.

[5] Wang, H., Smarandache, F., Zhang, Y., & Sunderraman, R. (2010). SINGLE VALUED NEUTROSOPHIC SETS. Review of the Air Force Academy, 17(1).

[6] Said Broumi, and Florentin Smarandache, Several Similarity Measures of Neutrosophic Sets" ,Neutrosophic Sets and Systems,VOL1 ,2013,54 62.

[7] Broumi, S., Ye, J., & Smarandache, F. (2015). An Extended TOPSIS Method for Multiple Attribute Decision Making based on Interval Neutrosophic Uncertain Linguistic Variables. Neutrosophic Sets & Systems, 8.

[8] M. Ali, and F. Smarandache, Complex Neutrosophic Set, Neural Computing and Applications, Vol. 25, (2016), 1-18. DOI: 10.1007/s00521-015-2154-y.

[9] M. Ali, I. Deli, F. Smarandache, The Theory of Neutrosophic Cubic Sets and Their Applications in Pattern Recognition, Journal of Intelligent and Fuzzy Systems, (In press), 1-7, DOI:10.3233/IFS-151906.

[10] Bahramloo, M & Hoseini, M. (2013). A multiple criteria decision making for raking alternatives using preference relation matrix based on intuitionistic fuzzy sets. Decision Science Letters , 2(4), 281-286.

[11] Biswas, P., Pramanik, S., & Giri, B. C. (2015). TOPSIS method for multi-attribute group decision-making under single-valued Neutrosophic environment. Neural Computing and Applications, 1-11.

[12] Peng, J. J., Wang, J. Q., Zhou, H., & Chen, X. H. (2015). A multi-criteria decision-making approach based on TODIM and Choquet integral within a multiset hesitant fuzzy environment. Applied Mathematics & Information Sciences, 9(4), 2087-2097.

[13] Deli, M. Ali, and F. Smarandache, Bipolar Neutrosophic Sets And Their Application Based On Multi-Criteria Decision Making Problems. (Proceeding of the 2015 International Conference on Advanced Mechatronic Systems, Beijing, China, August 22-24, 2015. IEEE Xplore, DOI: 10.1109/ICAMechS.2015.7287068

[14] Paul E. Black, "Manhattan distance", in Dictionary of Algorithms and Data Structures.