

# Neutrosophic triplet group

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**Abstract** Groups are the most fundamental and rich algebraic structure with respect to some binary operation in the study of algebra. In this paper, for the first time, we introduced the notion of neutrosophic triplet which is a group of three elements that satisfy certain properties with some binary operation. These neutrosophic triplets highly depends on the defined binary operation. Further, in this paper, we utilized these neutrosophic triplets to introduce the innovative notion of neutrosophic triplet group which is completely different from the classical group in the structural properties. A big advantage of neutrosophic triplet is that it gives a new group (neutrosophic triplet group) structure to those algebraic structures which are not group with respect to some binary operation in the classical group theory. In neutrosophic triplet group, we apply the fundamental law of Neutrosophy that for an idea  $A$ , we have neutral of  $A$  denoted as  $neut(a)$  and anti of  $A$  denoted as  $anti(A)$  to capture this beautiful picture of neutrosophic triplet group in algebraic structures. We also studied some interesting properties of this newly born structure. We further defined neutro-homomorphisms for neutrosophic triplet groups. A neutro-homomorphism is the generalization of the classical homomorphism with two extra conditions. As a further generalization, we gave rise to a new field or research called Neutrosophic Triplet

Structures (such as neutrosophic triplet ring, neutrosophic triplet field, neutrosophic triplet vector space, etc.). In the end, we gave main distinctions and comparison of neutrosophic triplet group with the classical Molaei's generalized group as well as the possible application areas of the neutrosophic triplet groups.

**Keywords** Groups · Homomorphism · Neutrosophic triplet · Neutrosophic triplet group · Neutro-homomorphism

## 1 Introduction

Neutrosophy is a new branch of philosophy which studies the nature, origin and scope of neutralities as well as their interaction with ideational spectra. Florentin Smarandache [8] in 1995, first introduced the concept of neutrosophic logic and neutrosophic set where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset  $T$ , the percentage of indeterminacy in a subset  $I$ , and the percentage of falsity in a subset  $F$  so that this neutrosophic logic is called an extension of fuzzy logic especially to intuitionistic fuzzy logic. In fact neutrosophic set is the generalization of classical sets [9], fuzzy set [12], intuitionistic fuzzy set [1, 9] and interval valued fuzzy set [9], etc. This mathematical tool is used to handle problems consisting uncertainty, imprecision, indeterminacy, inconsistency, incompleteness and falsity. By utilizing the idea of neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache studied neutrosophic algebraic structures in [4–6] by inserting an indeterminate element “ $I$ ” in the algebraic structure and then combine “ $I$ ” with each element of the structure with respect to corresponding binary operation  $*$ . They call it neutrosophic element, and the

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generated algebraic structure is then termed as neutrosophic algebraic structure. They further study several neutrosophic algebraic structures such as neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids and neutrosophic bigroupoids and so on.

Groups [2, 3, 11] are so much important in algebraic structures as they play the role of back bone in almost all algebraic structures theory. Groups are thought as old algebra due to its rich structure than any other notion. In many algebraic structures, groups provide concrete foundation such as, rings, fields, vector spaces, etc. Groups are also important in many other areas like physics, chemistry, combinatorics, biology, etc., to study the symmetries and other behavior among their elements. The most important aspect of a group is group action. There are many types of groups such as permutation groups, matrix groups, transformation groups, lie groups, etc., which are highly used as a practical perspective in our daily life. Generalized groups [7] are important in this aspect.

In this paper, for the first time, we introduced the idea of neutrosophic triplet. The newly born neutrosophic triplets are highly dependable on the proposed binary operation. These neutrosophic triplets have been discussed by Smarandache and Ali in Physics [10]. Moreover, we utilized these neutrosophic triplets to introduce neutrosophic triplet group which is different from the classical group both in structural and foundational properties from all aspects. Furthermore, we gave some interesting and fundamental properties and notions with illustrative examples. We also introduced a new type of homomorphism called as neutro-homomorphism which is in fact a generalization of the classical homomorphism under some conditions. We also study neutro-homomorphism for neutrosophic triplet groups. The rest of the paper is organized as follows. After the literature review in Sect. 1, we introduced neutrosophic triplets in Sect. 2. Section 3 is dedicated to the introduction of neutrosophic triplet groups with some of its interesting properties. In Sect. 4, we developed neutro-homomorphism, and in Sect. 5, we gave distinction and comparison of neutrosophic triplet group with the classical Molaei's generalized group. We also draw a brief sketch of the possible applications of neutrosophic triplet group in other research areas. Conclusion is given in Sect. 6.

## 2 Neutrosophic triplet

**Definition 2.1** Let  $N$  be a set together with a binary operation  $*$ . Then,  $N$  is called a neutrosophic triplet set if

for any  $a \in N$ , there exist a neutral of “ $a$ ” called  $neut(a)$ , different from the classical algebraic unitary element, and an opposite of “ $a$ ” called  $anti(a)$ , with  $neut(a)$  and  $anti(a)$  belonging to  $N$ , such that:

$$a * neut(a) = neut(a) * a = a,$$

and

$$a * anti(a) = anti(a) * a = neut(a).$$

The elements  $a$ ,  $neut(a)$  and  $anti(a)$  are collectively called as neutrosophic triplet, and we denote it by  $(a, neut(a), anti(a))$ . By  $neut(a)$ , we mean *neutral* of  $a$  and apparently,  $a$  is just the first coordinate of a neutrosophic triplet and not a neutrosophic triplet. For the same element “ $a$ ” in  $N$ , there may be more neutrals to it  $neut(a)$  and more opposites of it  $anti(a)$ .

**Definition 2.2** The element  $b$  in  $(N, *)$  is the second component, denoted as  $neut(\cdot)$ , of a neutrosophic triplet, if there exist other elements  $a$  and  $c$  in  $N$  such that  $a * b = b * a = a$  and  $a * c = c * a = b$ . The formed neutrosophic triplet is  $(a, b, c)$ .

**Definition 2.3** The element  $c$  in  $(N, *)$  is the third component, denoted as  $anti(\cdot)$ , of a neutrosophic triplet, if there exist other elements  $a$  and  $b$  in  $N$  such that  $a * b = b * a = a$  and  $a * c = c * a = b$ . The formed neutrosophic triplet is  $(a, b, c)$ .

*Example 2.4* Consider  $Z_6$  under multiplication modulo 6, where

$$Z_6 = \{0, 1, 2, 3, 4, 5\}$$

Then, 2 gives rise to a neutrosophic triplet because  $neut(2) = 4$ , as  $2 \times 4 = 8 \equiv 2 \pmod{6}$ . Also  $anti(2) = 2$  because  $2 \times 2 = 4$ . Thus,  $(2, 4, 2)$  is a neutrosophic triplet. Similarly 4 gives rise to a neutrosophic triplet because  $neut(4) = anti(4) = 4$ . So  $(4, 4, 4)$  is a neutrosophic triplet. 3 does not give rise to a neutrosophic triplet as  $neut(3) = 5$ , but  $anti(3)$  does not exist in  $Z_6$ , and last but not the least 0 gives rise to a trivial neutrosophic triplet as  $neut(0) = anti(0) = 0$ . The trivial neutrosophic triplet is denoted by  $(0, 0, 0)$ .

**Theorem 2.5** If  $(a, neut(a), anti(a))$  form a neutrosophic triplet, then

1.  $(anti(a), neut(a), a)$  also form a neutrosophic triplet, and similarly
2.  $(neut(a), neut(a), neut(a))$  form a neutrosophic triplet.

*Proof* We prove both 1 and 2.

1. Of course,  $anti(a) * a = neut(a)$ .

We need to prove that:  $anti(a) * neut(a) = anti(a)$ . Multiply by  $a$  to the left, and we get:

$$a * anti(a) * neut(a) = a * anti(a)$$

or

$$[a * anti(a)] * neut(a) = neut(a)$$

or

$$neut(a) * neut(a) = neut(a).$$

Again multiply by  $a$  to the left and we get:

$$a * neut(a) * neut(a) = a * neut(a)$$

or

$$[a * neut(a)] * neut(a) = a$$

or

$$a * neut(a) = a.$$

2. To show that  $(neut(a), neut(a), neut(a))$  is a neutrosophic triplet, it results from the fact that  $neut(a) * neut(a) = neut(a)$ .

### 3 Neutrosophic triplet group

**Definition 3.1** Let  $(N, *)$  be a neutrosophic triplet set. Then,  $N$  is called a neutrosophic triplet group, if the following conditions are satisfied.

- (1) If  $(N, *)$  is well-defined, i.e. for any  $a, b \in N$ , one has  $a * b \in N$ .
- (2) If  $(N, *)$  is associative, i.e.  $(a * b) * c = a * (b * c)$  for all  $a, b, c \in N$ .

The neutrosophic triplet group, in general, is not a group in the classical algebraic way.

We consider, as the neutrosophic neutrals replacing the classical unitary element, and the neutrosophic opposites as replacing the classical inverse elements.

*Example 3.2* Consider  $(Z_{10}, \#)$ , where  $\#$  is defined as  $a\#b = 3ab \pmod{10}$ . Then,  $(Z_{10}, \#)$  is a neutrosophic triplet group under the binary operation  $\#$  with the following table (Tables 1, 2).

It is also associative, i.e.

$$(a\#b)\#c = a\#(b\#c)$$

Now take L. H. S to prove the R. H. S, so

$$\begin{aligned} (a\#b)\#c &= (3ab)\#c, \\ &= 3(3ab)c = 9abc, \\ &= 3a(3bc) = 3a(b\#c), \\ &= a\#(b\#c). \end{aligned}$$

For each  $a \in Z_{10}$ , we have  $neut(a)$  in  $Z_{10}$ . That is  $neut(0) = 0, neut(1) = 7, neut(2) = 2, neut(3) = 7, neut(4) = 2$ , and so on.

**Table 1** Cayley table of neutrosophic triplet group  $(Z_{10}, \#)$

#	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	3	6	9	2	5	8	1	4	7
2	0	6	2	8	4	0	6	2	8	4
3	0	9	8	7	6	5	4	3	2	1
4	0	2	4	6	8	0	2	4	6	8
5	0	5	0	5	0	5	0	5	0	5
6	0	8	6	4	2	0	8	6	4	2
7	0	1	2	3	4	5	6	7	8	9
8	0	4	8	2	6	0	4	8	2	6
9	0	7	4	1	8	5	2	9	6	3

**Table 2** Cayley table of a non-commutative neutrosophic triplet group  $(Z_{10}, *)$

*	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	5	6	7	8	9	0	1	2	3	4
2	0	1	2	3	4	5	6	7	8	9
3	5	6	7	8	9	0	1	2	3	4
4	0	1	2	3	4	5	6	7	8	9
5	5	6	7	8	9	0	1	2	3	4
6	0	1	2	3	4	5	6	7	8	9
7	5	6	7	8	9	0	1	2	3	4
8	0	1	2	3	4	5	6	7	8	9
9	5	6	7	8	9	0	1	2	3	4

Similarly, for each  $a \in Z_{10}$ , we have  $anti(a)$  in  $Z_{10}$ . That is  $anti(0) = 0, anti(1) = 9, anti(2) = 2, anti(3) = 3, anti(4) = 1$ , and so on. Thus,  $(Z_{10}, \#)$  is a neutrosophic triplet group with respect to  $\#$ .

**Definition 3.3** Let  $(N, *)$  be a neutrosophic triplet group. Then,  $N$  is called a commutative neutrosophic triplet group if for all  $a, b \in N$ , we have  $a * b = b * a$ .

*Example 3.4* Consider  $(Z_{10}, *)$ , where  $*$  is defined as  $a * b = 5a + b \pmod{10}$  for all  $a, b \in Z_{10}$ . Then,  $(Z_{10}, *)$  is the neutrosophic triplet group which is given by the following Table 2: Then,  $(Z_{10}, *)$  is a non-commutative neutrosophic triplet group.

**Theorem 3.5** Every idempotent element gives rise to a neutrosophic triplet.

*Proof* Let  $a$  be an idempotent element. Then, by definition  $a^2 = a$ . Since  $a^2 = a$ , which clearly implies that  $neut(a) = a$  and  $anti(a) = a$ . Hence  $a$  gives rise to a neutrosophic triplet

$$(a, a, a).$$

**Theorem 3.6** There are no neutrosophic triplets in  $Z_n$  with respect to multiplication if  $n$  is a prime.

*Proof* It is obvious.

**Remark 3.7** Let  $(N, *)$  be a neutrosophic triplet group under  $*$  and let  $a \in N$ . Then,  $neut(a)$  is not unique in  $N$ , and also  $neut(a)$  depends on the element  $a$  and the operation  $*$ . To prove the above remark, let's take a look to the following example.

**Example 3.8** Let  $N = \{0, 4, 8, 9\}$  be a neutrosophic triplet group under multiplication modulo 12 in  $(Z_{12}, \times)$ . Then  $neut(4) = 4$ ,  $neut(8) = 4$  and  $neut(9) = 9$ . This shows that  $neut(a)$  is not unique.

**Remark 3.9** Let  $(N, *)$  be a neutrosophic triplet group with respect to  $*$  and let  $a \in N$ . Then,  $anti(a)$  is not unique in  $N$  and also  $anti(a)$  depends on the element  $a$  and the operation  $*$ . To prove the above remark, let's take a look to the following example.

**Example 3.10** Let  $N$  be a neutrosophic triplet group in above example. Then,  $anti(4) = 4$ ,  $anti(8) = 8$  and  $anti(9) = 9$ .

**Proposition 3.11** Let  $(N, *)$  be a neutrosophic triplet group with respect to  $*$  and let

$$a, b, c \in N.$$

then

- (1)  $a * b = a * c$  if and only if  $neut(a) * b = neut(a) * c$ .
- (2)  $b * a = c * a$  if and only if  $b * neut(a) = c * neut(a)$ .

*Proof*

1. Suppose that  $a * b = a * c$ . Since  $N$  is a neutrosophic triplet group, so  $anti(a) \in N$ . Multiply  $anti(a)$  to the left side with  $a * b = a * c$ .

$$anti(a) * a * b = anti(a) * a * c$$

$$[anti(a) * a] * b = [anti(a) * a] * c$$

$$neut(a) * b = neut(a) * c$$

Conversely suppose that  $neut(a) * b = neut(a) * c$ . Multiply  $a$  to the left side, we get:

$$a * neut(a) * b = a * neut(a) * c$$

$$[a * neut(a)] * b = [a * neut(a)] * c$$

$$a * b = a * c$$

2. The proof is similar to 1.

**Proposition 3.12** Let  $(N, *)$  be a neutrosophic triplet group with respect to  $*$  and let  $a, b, c \in N$ .

1. If  $anti(a) * b = anti(a) * c$ , then  $neut(a) * b = neut(a) * c$ .

2. If  $b * anti(a) = c * anti(a)$ , then  $b * neut(a) = c * neut(a)$ .

*Proof*

1. Suppose that  $anti(a) * b = anti(a) * c$ . Since  $N$  is a neutrosophic triplet group with respect to  $*$ , so  $a \in N$ . Multiply  $a$  to the left side with  $anti(a) * b = anti(a) * c$ , we get:

$$a * anti(a) * b = a * anti(a) * c$$

$$[a * anti(a)] * b = [a * anti(a)] * c$$

$$neut(a) * b = neut(a) * c.$$

2. The proof is same as (1).

**Theorem 3.13** Let  $(N, *)$  be a commutative neutrosophic triplet group with respect to  $*$  and  $a, b \in N$ . Then  $neut(a) * neut(b) = neut(a * b)$ .

*Proof* Consider left hand side,  $neut(a) * neut(b)$ . Now multiply to the left with  $a$  and to the right with  $b$ , we get:

$$a * neut(a) * neut(b) * b = [a * neut(a)] * [neut(b) * b]$$

$$= a * b.$$

Now consider right hand side, we have  $neut(a * b)$ . Again multiply to the left with  $a$  and to the right with  $b$ , we get:  $a * neut(a * b) * b = [a * b] * [neut(a * b)]$ , as  $*$  is associative.

$$= a * b.$$

This completes the proof.

**Theorem 3.14** Let  $(N, *)$  be a commutative neutrosophic triplet group with respect to  $*$  and  $a, b \in N$ . Then  $anti(a) * anti(b) = anti(a * b)$ .

*Proof* Consider left hand side,  $anti(a) * anti(b)$ . Multiply to the left with  $a$  and to the right with  $b$ , we get:

$$a * anti(a) * anti(b) * b = [a * anti(a)] * [anti(b) * b]$$

$$= neut(a) * neut(b)$$

$$= neut(a * b), \text{ By above theorem.}$$

Now consider right hand side, which is  $anti(a * b)$ .

Multiply to the left with  $a$  and to the right with  $b$ , we get:  $a * anti(a * b) * b = [a * b] * [anti(a * b)]$ , since  $*$  is associative.

$$= neut(a * b).$$

This shows that  $anti(a) * anti(b) = anti(a * b)$  is true for all  $a, b \in N$ .

**Theorem 3.15** Let  $(N, *)$  be a commutative neutrosophic triplet group under  $*$  and

$$a, b \in N.$$

then

1.  $neut(a) * neut(b) = neut(b) * neut(a).$
2.  $anti(a) * anti(b) = anti(b) * anti(a).$

*Proof*

1. Consider right hand side  $neut(b) * neut(a).$  Since by Theorem 3, we have

$$neut(b) * neut(a) = neut(b * a)$$

$= neut(a * b),$  as  $N$  is commutative.  $= neut(a) * neut(b),$  again by theorem 3. Hence

$$neut(a) * neut(b) = neut(b) * neut(a).$$

2. On similar lines, one can easily obtained the proof of (2).

**Definition 3.16** Let  $(N, *)$  be a neutrosophic triplet group under  $*$ , and let  $H$  be a subset of  $N.$  Then,  $H$  is called a neutrosophic triplet subgroup of  $N$  if  $H$  itself is a neutrosophic triplet group with respect to  $*$ .

*Example 3.17* Consider  $(Z_{10}, \#)$  be a neutrosophic triplet group in Example 3.2, and  $H = \{0, 2, 4, 6, 8\}$  be a subset of  $Z_{10}.$  Then, clearly  $H$  is a neutrosophic triplet subgroup of  $Z_{10}.$

**Proposition 3.18** Let  $(N, *)$  be a neutrosophic triplet group and  $H$  be a subset of  $N.$  Then  $H$  is a neutrosophic triplet subgroup of  $N$  if and only if the following conditions hold.

1.  $a * b \in H$  for all  $a, b \in H.$
2.  $neut(a) \in H$  for all  $a \in H.$
3.  $anti(a) \in H$  for all  $a \in H.$

*Proof* The proof is straightforward.

**Definition 3.19** Let  $N$  be a neutrosophic triplet group and let  $a \in N.$  A smallest positive integer  $n \geq 1$  such that  $a^n = neut(a)$  is called neutrosophic triplet order. It is denoted by  $nto(a).$

*Example 3.20* Let  $N$  be a neutrosophic triplet group under multiplication modulo 10 in  $(Z_{10}, \times),$  where

$$N = \{0, 2, 4, 6, 8\}.$$

then

$$nto(2) = 4, nto(4) = 2,$$

$$nto(6) = 2, nto(8) = 4.$$

**Theorem 3.21** Let  $(N, *)$  be a neutrosophic triplet group with respect to  $*$  and let  $a \in N.$  Then

$$1. \quad neut(a) * neut(a) = neut(a).$$

In general  $(neut(a))^n = neut(a),$  where  $n$  is a nonzero positive integer.

$$2. \quad neut(a) * anti(a) = anti(a) * neut(a) = anti(a).$$

*Proof*

1. Consider  $neut(a) * neut(a) = neut(a).$

Multiply  $a$  to the left side, we get;

$$a * neut(a) * neut(a) = a * neut(a)$$

$$[a * neut(a)] * neut(a) = [a * neut(a)]$$

$$a * neut(a) = a$$

$$a = a.$$

On the same lines, we can see that  $(neut(a))^n = neut(a)$  for a nonzero positive integer  $n.$

2. Consider  $neut(a) * anti(a) = anti(a).$

Multiply to the left with  $a,$  we get

$$a * neut(a) * anti(a) = a * anti(a)$$

$$[a * neut(a)] * anti(a) = neut(a)$$

$$a * anti(a) = neut(a)$$

$$neut(a) = neut(a).$$

Similarly  $anti(a) * neut(a) = anti(a).$

**Definition 3.22** Let  $N$  be a neutrosophic triplet group and  $a \in N.$  Then,  $N$  is called neutro-cyclic triplet group if  $N = \langle a \rangle.$  We say that  $a$  is a generator part of the neutrosophic triplet.

*Example 3.23* Let  $N = \{2, 4, 6, 8\}$  be a neutrosophic triplet group with respect to multiplication modulo 10 in  $(Z_{10}, \times).$  Then, clearly  $N$  is a neutro-cyclic triplet group as  $N = \langle 2 \rangle.$  Therefore, 2 is the generator part of the neutrosophic triplet  $(2, 6, 8).$

**Theorem 3.24** Let  $N$  be a neutro-cyclic triplet group and let  $a$  be a generator part of the neutrosophic triplet. Then

1.  $\langle neut(a) \rangle$  generates neutro-cyclic triplet subgroup of  $N.$
2.  $\langle anti(a) \rangle$  generates neutro-cyclic triplet subgroup of  $N.$

*Proof* Straightforward.

### 4 Neutro-homomorphism

In this section, we introduced neutron-homomorphism for the neutrosophic triplet groups. We also studied some of their properties. Further, we defined neutro-isomorphisms.

**Definition 4.1** Let  $(N_1, *_1)$  and  $(N_2, *_2)$  be two neutrosophic triplet groups. Let

$$f : N_1 \rightarrow N_2$$

be a mapping. Then,  $f$  is called neutro-homomorphism if for all  $a, b \in N_1$ , we have

1.  $f(a *_1 b) = f(a) *_2 f(b)$ ,
2.  $f(\text{neut}(a)) = \text{neut}(f(a))$   
and
3.  $f(\text{anti}(a)) = \text{anti}(f(a))$ .

*Example 4.2* Let  $N_1$  be a neutrosophic triplet group with respect to multiplication modulo 6 in  $(Z_6, \times)$ , where

$$N_1 = \{0, 2, 4\}.$$

And let  $N_2$  be another neutrosophic triplet group with respect to multiplication modulo 10 in  $(Z_{10}, \times)$ , where

$$N_2 = \{0, 2, 4, 6, 8\}.$$

Let  $f: N_1 \rightarrow N_2$  be a mapping defined as

$$f(0) = 0, f(2) = 4, f(4) = 6.$$

Then, clearly  $f$  is a neutro-homomorphism because conditions (1), (2) and (3) are satisfied easily.

**Proposition 4.3** Every neutro-homomorphism is a classical homomorphism by neglecting the unity element in classical homomorphism.

*Proof* First we neglect the unity element that classical homomorphism maps unity element to the corresponding unity element. Now suppose that  $f$  is a neutro-homomorphism from a neutrosophic triplet group  $N_1$  to a neutrosophic triplet group  $N_2$ . Then, by condition (1), it follows that  $f$  is a classical homomorphism.

**Definition 4.4** A neutro-homomorphism is called neutro-isomorphism if it is one–one and onto.

## 5 Distinctions and comparison

The distinctions between Molaei's Generalized Group [7] and Neutrosophic Triplet Group are:

1. in MGG for each element there exists a unique neutral element, which can be the group neutral element, while in NTG each element may have many neutral

elements, and also the neutral elements have to be different from the unique group neutral element;

2. in MGG the associativity applies, and in NTG the associativity is not required;
3. in MGG there exists a unique inverse of an element, while in NTG there may be many inverses for the same given element;
4. MGG has a weaker structure than NTG.

So far the applications of neutrosophic triplet sets are in  $Z$ , modulon,  $n \geq 2$ . But new applications can be found, for example in social science: One person  $\langle A \rangle$  that has an enemy  $\langle \text{anti}(A_{d_1}) \rangle$  (enemy in a degree  $d_1$  of enemy city), and a neutral person  $\langle \text{neut}(A_{d_1}) \rangle$  with respect to  $\langle \text{anti}(A_{d_1}) \rangle$ . Then, another enemy  $\langle \text{anti}(A_{d_2}) \rangle$  in a different degree of enemy city, and a neutral  $\langle \text{anti}(A_{d_2}) \rangle$ , and so on. Hence one has the neutrosophic triplets:

$$(A, \langle \text{neut}(A_{d_1}) \rangle, \langle \text{anti}(A_{d_1}) \rangle),$$

$$(A, \langle \text{neut}(A_{d_2}) \rangle, \langle \text{anti}(A_{d_2}) \rangle), \text{ and so on.}$$

Then, we take another person  $B$  in the same way...

$$(B, \langle \text{neut}(B_{d_1}) \rangle, \langle \text{anti}(B_{d_1}) \rangle),$$

$$(B, \langle \text{neut}(B_{d_2}) \rangle, \langle \text{anti}(B_{d_2}) \rangle)$$

etc.

More applications can be found, if we deeply think about cases where we have neutrosophic triplets  $(A, \langle \text{neut}(A) \rangle, \langle \text{anti}(A) \rangle)$  in technology and in science.

## 6 Conclusion

Inspiring from the Neutrosophic philosophy, we defined neutrosophic triplet. Basically A neutrosophic triplet in a set is a group of certain elements which satisfy certain conditions that highly depends upon the proposed binary operation. The main theme of this paper is first to introduced the neutrosophic triplets which are completely new notions and then utilize these neutrosophic triplets to introduce the neutrosophic triplet groups. This neutrosophic triplet group has several extraordinary properties as compared to the classical group. We also studied some interesting properties of this newly born structure. We further defined neutro-homomorphisms for neutrosophic triplet groups. A neutro-homomorphism is the generalization of the classical homomorphism with two extra conditions. As a further generalization, we gave rise to a new field or research called Neutrosophic Triplet Structures (such as neutrosophic triplet ring, neutrosophic triplet field, neutrosophic triplet vector space, etc.). In the end, we gave main distinctions and comparison of neutrosophic triplet group with the classical Molaei's generalized group

as well as the possible application areas of the neutrosophic triplet groups.

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