NEVANLINNA, SIEGEL, AND CREMER

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Abstract. We study an irrationally indifferent cycle of points or circles of a rational function, which is either Siegel or Cremer by definition. We invent a new argument from the viewpoint of the Nevanlinna theory. Using this argument, we give a clear interpretation of some Diophantine quantity associated with an irrationally indifferent cycle. This quantity turns out to be Nevanlinna-theoretical. As a consequence, we show that an irrationally indifferent cycle is Cremer if this Nevanlinna-theoretical quantity does not vanish.

1. Introduction

Let f be a rational function of degree ≥ 2 and $f^k := f^{\circ k}$ for $k \in \mathbb{N}$. The *Fatou set* $F(f)$ is defined by the set of all points of $\hat{\mathbb{C}}$ at which $\{f^n\}_{n\in\mathbb{N}}$ is normal in the Montel sense, and the *Julia set* $J(f)$ is defined by the complement of $F(f)$ in $\hat{\mathbb{C}}$. Both $F(f)$ and $J(f)$ are *completely invariant*, that is, their image and preimage by f equal themselves. $F(f)$ is open by definition, so $J(f)$ is closed. Furthermore $J(f)$ is non-empty and perfect.

We call a connected component of $F(f)$ a *Fatou component*. Every Fatou component is mapped to a Fatou one properly by f. A Fatou component D is *cyclic* if for some $n \in \mathbb{N}$, $f^n(D) = D$. Then the least such n is called the *period*, and $g := g_D := f^n|D$ is the *first return map* on D. On the other hand, a Fatou component is *preperiodic* if it is not cyclic but for some $n \in \mathbb{N}$, $f^n(D)$ is cyclic.

The classification of cyclic Fatou components is known: (g, D) is an *attractive basin* if $\{g^n\}_{n=1}^{\infty}$ converges to a point in D locally uniformly on D. The *parabolic basin* is similar, but $\{g^n\}_{n=1}^{\infty}$ converges to a point in the boundary of D locally uniformly on D. When g is a proper selfmap of D of degree ≥ 2, (g, D) is one of them. When g is a univalent selfmap of D, (g, D) is called a *singular domain* since people before doubted whether this case actually occurred. In this case, (g, D) is conformally conjugate to an irrational rotation on either a disk or an annulus, and called a *Siegel disk* or an *Herman ring* respectively. Hence the singular domain is also called the *rotation* one. For the details, see [11], [2], [4], [10].

Our main interest is an irrationally indifferent cycle of points or circles.

Definition 1.1 (irrationally indifferent cycle of points or circles)**.** A point z_0 in $\hat{\mathbb{C}}$ is *periodic* if for some $p \in \mathbb{N}$, $f^p(z_0) = z_0$. The least such p is the *period* of z_0, $\{f^n(z_0)\}_{n=1}^{p}$ is a *cycle* of points, and $\lambda := (f^p)'(z_0)$ is the *multiplier* of it. This cycle of *points* is *irrationally indifferent* if $\lambda = e^{2\pi i\alpha}$ for some $\alpha \in \mathbb{R} - \mathbb{Q}$.

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A topological circle $S \subset \hat{\mathbb{C}}$ is *periodic* if for some $p \in \mathbb{N}$, $f^p(S) = S$ *and* $f^p|S : S \to S$ is an orientation-preserving homeomorphism. The least such p is the *period* of S, $\{f^n(S)\}_{n=1}^p$ is a cycle of *circles*, and $\lambda := e^{2\pi i\alpha}$ is the *multiplier* of it, where $\alpha \in \mathbb{R}/\mathbb{Z}$ is the *rotation number* (cf. [6]) of a S^1-homeomorphism ϕ which is topologically conjugate to $f^p|S$. This cycle of *circles* is *irrationally indifferent* if α is irrational.

It is known that if irrationally indifferent cycles of points or circles intersect $F(f)$, then they are contained in some rotation domains, which are Fatou components.

Definition 1.2 (Siegel and Cremer cycles)**.** An irrationally indifferent cycle of points or circles is a *Siegel cycle* if it is contained in $F(f)$. Otherwise it is a *Cremer cycle.*

We study an unsolved problem with a *long* history: *Given an irrationally indifferent cycle of points or circles, how can we judge whether it is contained in the Fatou set or not?*

The following answers the problem in one direction.

Theorem 1.1 (Siegel[17] (1942), Rüssmann, Brjuno[3] (1972), Yoccoz[20] (1996))**.** *Let* $\lambda = e^{2\pi i\alpha}$ $(\alpha \in \mathbb{R} - \mathbb{Q})$*,* $f(z) = \lambda z + \cdots$ *be an analytic germ at the origin, and* $\{p_n/q_n\}_{n=0}^{\infty}$ *be the sequence of irreducible approximating fractions of* α *derived from its continued fraction expansion.*

If α *satisfies one of the following Diophantine conditions*:

(Si)
$$\sup_{n\geq 0} \frac{\log q_{n+1}}{\log q_n} < \infty,$$

a weaker one:

(Rü)
$$\sum_{n\geq 0} \frac{\log q_{n+1} \log\log q_{n+1}}{q_n} < \infty,$$

and the weakest one:

(Br)
$$\sum_{n\geq 0} \frac{\log q_{n+1}}{q_n} < \infty,$$

then f *is analytically linearizable at the origin, that is, the Schröder equation*:

$$h \circ f = R_\lambda \circ h,$$

where $R_\lambda(z) = \lambda z$ *is the linear term of* f*, holds for some analytic local coordinate* $h(z) = z + \cdots$ *around the origin.*

Corollary 1.1. *An irrationally indifferent cycle of points of a rational function is Siegel if its multiplier satisfies the condition* (Br)*.*

In the reverse direction,

Theorem 1.2 (Yoccoz[20] (1996), Okuyama[12] (2001))**.** *Let* P *be a quadratic polynomial. An irrationally indifferent cycles of points* (*of arbitrary period*) *of* P *is Cremer if its multiplier does not satisfy* (Br)*.*

Only for quadratic polynomials, the complete answer of the problem is known. The classical Cremer Theorem is a partial answer in the reverse direction.

Theorem 1.3 (Cremer[5] (1932))**.** *Let f be a rational function of degree $d \geq 2$, and $\mathcal{O}$ be an irrationally indifferent cycle of points of period p and of multiplier λ.*

$\mathcal{O}$ is Cremer if λ satisfies

$$\limsup_{n\to\infty} \frac{1}{d^{pn}} \log \frac{1}{|\lambda^n - 1|} = \infty. \tag{Cr}$$

It naturally arises:

Fundamental Question. How can we notice such complicate Diophantine conditions as (Si), (Rü), (Br) and (Cr)? What on earth are they?

We shall answer this Fundamental Question for the condition (Cr) and (Ok) below. The left hand side of them turns out to be *Nevanlinna-theoretical*. From this, it immediately follows that irrationally indifferent cycles of points or circles are Cremer if this Nevanlinna-theoretical quantity does not *vanish*:

Main Theorem 1 (Criterion for Cremer)**.** *Let f be a rational function of degree $d \geq 2$, and $\mathcal{O}$ be an irrationally indifferent cycle of points or circles of period p and of multiplier λ. If*

$$\limsup_{n\to\infty} \frac{1}{d^{pn}} \log \frac{1}{|\lambda^n - 1|} > 0, \tag{Ok}$$

then $\mathcal{O}$ is Cremer.

In Section 2, we shall study the Nevanlinna theory. We define the pointwise proximity function, the mean proximity and the *Valiron exceptionality*, and prove the *Fundamental Equality* for the Valiron exceptionality. This Fundamental Equality shows that the dynamics of a rational function is *homogeneous* on the *whole* Riemann sphere. In Section 3, by the Fundamental Equality, we shall prove the *Vanishing Theorem*, which states that the Valiron exceptionality *vanishes* for a rational function with non-empty Fatou set. In Section 4, we shall obtain the *Natural Equality*: the left hand side of (Ok) exactly equals the Valiron exceptionality. Main Theorem 1 is straightforward from both the Vanishing Theorem and the Natural Equality.

First of all, our method of studying this problem answers the Fundamental Question, and then naturally establishes a criterion for Cremer. Hence we find that our method is natural.

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2. Nevanlinna theory

Let $[p, q]$ be the chordal distance between $p, q \in \hat{\mathbb{C}}$ such that $[0, \infty] = 1$. For rational functions f and g, we define the *pointwise proximity function*:

$$w(g, f) := \log \frac{1}{[g, f]},$$

and the *mean proximity*:

$$m(g, f) := \int_{\hat{\mathbb{C}}} w(g, f) d\sigma,$$

where σ is the spherical area measure on $\hat{\mathbb{C}}$ such that $\sigma(\hat{\mathbb{C}}) = 1$.

Definition 2.1 ([18], cf. [8])**.** Let $\mathcal{F} = \{f_k\}_{k=1}^{\infty}$ be a sequence of rational functions such that $d_k := \deg f_k \uparrow \infty$ as $k \to \infty$. For a rational function g, we define the *Valiron exceptionality*:

$$\mathrm{VE}(g; \mathcal{F}) := \limsup_{k\to\infty} \frac{m(g, f_k)}{d_k}.$$

g is *Valiron exceptional* for $\mathcal{F}$ if $\mathrm{VE}(g; \mathcal{F}) > 0$.

Example 1. In the case that $\mathcal{F} = \{z^k\}$ and $g \equiv 0$, it holds that $\mathrm{VE}(g; \mathcal{F}) > 0$. Hence g is Valiron exceptional for $\mathcal{F}$.

Main Theorem 2 (Fundamental Equality)**.** *Let f be a rational function of degree $d \geq 2$. Then for every positive continuous function $\phi \not\equiv 0$ on $\hat{\mathbb{C}}$,*

$$\mathrm{VE}(\mathrm{Id}_{\hat{\mathbb{C}}}; \{f^k\}) = \limsup_{k\to\infty} \frac{\int_{\hat{\mathbb{C}}} \phi \cdot w(\mathrm{Id}_{\hat{\mathbb{C}}}, f^k) d\sigma}{d^k \cdot \int_{\hat{\mathbb{C}}} \phi d\sigma}.$$

Proof. Let $\mathbb{M}(\hat{\mathbb{C}})$ be the set of all Radon measures on $\hat{\mathbb{C}}$. For rational functions f and g, we define the *root measure*

$$(f-g)^*\delta_0 := \sum_{f\zeta = g\zeta} \delta_\zeta \in \mathbb{M}(\hat{\mathbb{C}}),$$

taking into account the multiplicities of roots, where $\delta_\zeta \in \mathbb{M}(\hat{\mathbb{C}})$ is the Dirac measure at $\zeta \in \hat{\mathbb{C}}$. For a rational function f and $\mu \in \mathbb{M}(\hat{\mathbb{C}})$, the *pullback* measure $f^*\mu \in \mathbb{M}(\hat{\mathbb{C}})$ is defined as

$$f^*\mu(U) = \int_{\hat{\mathbb{C}}} ((f-z)^*\delta_0)(U) d\mu(z),$$

where U is a Borel set in $\hat{\mathbb{C}}$. For $\mu \in \mathbb{M}(\hat{\mathbb{C}})$, we consider the *chordal logarithmic potential*

$$P_\mu(z) := \int_{\hat{\mathbb{C}}} \log \frac{1}{[w, z]} d\mu(w)$$

on $\hat{\mathbb{C}}$.

Lemma 2.1 (Riesz decomposition)**.** *For rational functions f and g,*

$$w(f, g) = P_{(f-g)^*\delta_0} - P_{f^*\sigma} - P_{g^*\sigma} + m(f, g).$$

Proof. By a Möbius conjugation, we assume $f(\infty) \neq g(\infty)$, $f'(\infty) \neq 0$, and $g'(\infty) \neq 0$ without loss of generality. Since $w(f, g)$ is δ-subharmonic on $\mathbb{C}$, it has the *Riesz decomposition* (cf. [13]):

$$w(f, g) = P_{(f-g)^*\delta_0} - P_{f^*\sigma} - P_{g^*\sigma} + c_{f,g},$$

where $c_{f,g}$ is harmonic on $\mathbb{C}$. Since $c_{f,g} = O(1)$ as $z \to \infty$, $c_{f,g}$ is a constant by the Liouville theorem. Since the σ-mean of $P_{(f-g)^*\delta_0} - P_{f^*\sigma} - P_{g^*\sigma}$ on $\hat{\mathbb{C}}$ vanishes, it follows that $c_{f,g} = m(f, g)$. □

Let ϕ be a continuous function on $\hat{\mathbb{C}}$. By Lemma 2.1, we have

$$\begin{aligned} &\frac{1}{d^k} \int_{\hat{\mathbb{C}}} \phi \cdot w(\mathrm{Id}_{\hat{\mathbb{C}}}, f^k) d\sigma \\ (*) \qquad &= \frac{1}{d^k} \int_{\hat{\mathbb{C}}} \phi \cdot (P_{(f^k - \mathrm{Id}_{\hat{\mathbb{C}}})^*\delta_0} - P_{(f^k)^*\sigma} - P_\sigma) d\sigma + \frac{m(\mathrm{Id}_{\hat{\mathbb{C}}}, f^k)}{d^k} \int_{\hat{\mathbb{C}}} \phi d\sigma, \end{aligned}$$

and by the Fubini theorem, the first term of (*) equals

$$\int_{\hat{\mathbb{C}}}\left(\int_{\hat{\mathbb{C}}}\phi(z)\log\frac{1}{[w,z]}d\sigma(z)\right)d\frac{(f^k-\mathrm{Id}_{\hat{\mathbb{C}}})^*\delta_0-(f^k)^*\sigma-\sigma}{d^k}(w).$$

Theorem 2.1 ([7] and [9])**.** *Both* $(f^k-\mathrm{Id}_{\hat{\mathbb{C}}})^*\delta_0/d^k$ *and* $(f^k)^*\sigma/d^k$ *converge to the same element of* $\mathbb{M}(\hat{\mathbb{C}})$ *as* $k\to\infty$ *weakly.*

Remark 2.1. The limit measure μ_f in Theorem 2.1 is said to be *balanced* since $f^*\mu_f/d=\mu_f$. It has many interesting dynamical properties.

From Theorem 2.1, we have that the first term of (*) converges to 0 as $k\to\infty$. It completes the proof of Main Theorem 2. □

3. The Fatou and Julia strategy

We are able to calculate the Valiron exceptionalities through the *Fatou and Julia strategy.*

Main Theorem 3 (Vanishing Theorem)**.** *Let* f *be a rational function of degree* ≥ 2 *such that* $F(f)\neq\emptyset$*. Then* $\mathrm{VE}(\mathrm{Id}_{\hat{\mathbb{C}}};\{f^k\})=0$.

Proof. It follows from $\deg f\geq 2$ that there exists a Fatou component either non-cyclic or non-singular. Hence there exist a positive continuous function $\phi\not\equiv 0$ and $r\in(0,1)$ such that for every $k\in\mathbb{N}$,

$$\inf\{[z,w];z\in\operatorname{supp}\phi,w\in f^k(\operatorname{supp}\phi)\}>r.$$

From this, it follows that

$$\int_{\hat{\mathbb{C}}}\phi\cdot w(\mathrm{Id}_{\hat{\mathbb{C}}},f^k)d\sigma\leq\log\frac{1}{r}\cdot\int_{\hat{\mathbb{C}}}\phi d\sigma,$$

which concludes that $\mathrm{VE}(\mathrm{Id}_{\hat{\mathbb{C}}};\{f^k\})=0$ from the Fundamental Equality. □

4. Siegel and Cremer cycles

Let f be a rational function of degree $d\geq 2$. From now on, cycles and Fatou components are always of f.

For a rotation domain D, there exists, by definition, a conformal map h from D onto either $\mathbb{D}$ or an annulus and $\lambda=e^{2\pi i\alpha}$ ($\alpha\in\mathbb{R}-\mathbb{Q}$) such that for the first return map $g=g_D$,

$$h\circ g=R_\lambda\circ h \tag{1}$$

on D. Here $R_\lambda(z)=\lambda z$ as Section 1. h is called a *linearizing map* of D and λ the *rotation number.*

Notation. $A\asymp B$ means $A/C<B<CA$ for some implicit constant C.

Remark 4.1. Our method of studying multipliers of irrationally indifferent cycles of *circles* dispenses with such quasiconformal surgeries as in [14].

The left hand side of (Ok) in Main Theorem 1 turns out to be the Valiron exceptionality.

Main Theorem 4 (Natural Equality). *Let $\mathcal{O}$ be an irrationally indifferent cycle of points or circles of period p and of multiplier λ. If $\mathcal{O}$ is Siegel, then*

$$\limsup_{k\to\infty}\frac{1}{d^{pk}}\log\frac{1}{|\lambda^k-1|}=\mathrm{VE}(\mathrm{Id}_{\hat{\mathbb{C}}};\{f^{pk}\}). \tag{2}$$

Proof. Let C be a connected component of $\mathcal{O}$. Then there exists a rotation domain D such that $D\supset C$. Let h be a linearizing map of D and $g=f^p|D$ the first return map. Clearly the rotation number of D equals the multiplier λ of $\mathcal{O}$.

By a Möbius conjugation, we assume $D\subset\mathbb{C}$ without loss of generality. There exists a positive continuous function $\phi\not\equiv 0$ such that $\operatorname{supp}\phi\subset D$ and $h(\operatorname{supp}\phi)\not\ni 0$. Since $\overline{\bigcup_{k\in\mathbb{N}}g^k(\operatorname{supp}\phi)}$ is compact in D, we have:

$$[g^k(z),z]\asymp|g^k(z)-z|\asymp|h\circ g^k(z)-h(z)|=|\lambda^k-1|\cdot|h(z)|,$$

where the implicit constants are independent of $k\in\mathbb{N}$ and $z\in\operatorname{supp}\phi$. Hence

$$\frac{\int_{\hat{\mathbb{C}}}\phi\cdot w(\mathrm{Id}_{\hat{\mathbb{C}}},f^{pk})d\sigma}{d^{pk}\cdot\int_{\hat{\mathbb{C}}}\phi d\sigma}=\frac{1}{d^{pk}}\log\frac{1}{|\lambda^k-1|}+\frac{\int_{\hat{\mathbb{C}}}\phi\cdot\log\frac{1}{|h|}d\sigma}{d^{pk}\cdot\int_{\hat{\mathbb{C}}}\phi d\sigma}+O(d^{-pk})$$

as $k\to\infty$. It easily follows from $h(\operatorname{supp}\phi)\not\ni 0$ that the second term tends to 0 as $k\to\infty$. Hence the proof is completed by the Fundamental Equality. □

Now the proof of Main Theorem 1 is straightforward:

Proof of Main Theorem 1. If $\mathcal{O}$ is Siegel, then $F(f)\neq\emptyset$. Hence from the Vanishing Theorem,

$$\mathrm{VE}(\mathrm{Id}_{\hat{\mathbb{C}}};\{f^{pk}\})\le\mathrm{VE}(\mathrm{Id}_{\hat{\mathbb{C}}};\{f^k\})=0.$$

It contradicts (Ok) by the Natural Equality. □

Remark 4.2. The Natural Equality in Main Theorem 4 is the very answer to the Fundamental Question in Section 1 for the condition (Ok). As we have just seen in the above, Main Theorem 1 is straightforward from this Natural Equality and the Vanishing Theorem.

In the case of polynomials, Pierre Tortrat showed in [19] a similar result to Main Theorem 1 by using a potential theoretical argument.

We also obtain *a priori* bounds of the rotation numbers of rotation domains.

Main Theorem 5. *The rotation numbers of no rotation domains satisfy* (Ok).

Proof. A rotation domain contains Siegel cycles of circles whose multipliers equal its rotation number. Hence they do not satisfy (Ok) by Main Theorem 1. □

Remark 4.3. When the rotation domain is an Herman ring, by quasiconformal surgery of it (cf. [14], [15] and [16]), we obtain a rational function $\tilde{f}$ whose degree is less than that of f and which has a Siegel disk with the same rotation number as the original Herman ring of f. Hence by applying Main Theorem 5 to $\tilde{f}$ rather than f, a stronger conclusion than Main Theorem 5 follows.

Finally, we note that Cremer cycles of circles do not always satisfy (Ok). A normalized cubic *critical* Blaschke product, e.g.,

$$f_\theta(z):=e^{i\theta}z^2\frac{1-\frac{1}{3}z}{z-\frac{1}{3}},$$

where θ is chosen such that S^1 is an irrationally indifferent cycle of circles of period one, is never Siegel since there exists a critical point on S^1. Other example is:

Main Theorem 6 (Jordan boundaries of rotation domains)**.** *Let $\mathcal{O}$ be an irrationally indifferent cycle of circles which are the boundary of a cycle $\mathcal{C}$ of rotation domains. Then the multiplier of $\mathcal{O}$ does not satisfy* (Ok).

Proof. Let D be a connected component of $\mathcal{C}$ and h its linearizing map. The Carathéodory theory (or the Ahlfors-Beurling extremal length method [1]) says that h extends to a homeomorphism from $\overline{D}$ to $\overline{h(D)}$. Hence by the equation (1), the multiplier of $\mathcal{O}$ equals the rotation number of $\mathcal{C}$. By Main Theorem 5, it does not satisfy (Ok). □

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