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New approach for solving intuitionistic fuzzy multi-objective transportation problem

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Abstract. Multi-objective transportation problem (MOTP) under intuitionistic fuzzy (IF) environment is analysed in this paper. Due to the fluctuation of market scenario, we assume that the transportation cost, the supply and the demand parameters are not always precise. Hence, the parameters are imprecise, i.e., they are IF numbers. Considering the specific cut interval, the IF transportation cost matrix is converted to interval cost matrix in our proposed problem. Again, using the same concept, the IF supply and the IF demand of the MOTP are reduced to the interval form. Then the proposed MOTP is changed into the deterministic MOTP, which includes interval form of the objective functions. Two approaches, namely intuitionistic fuzzy programming and goal programming, are used to derive the optimal solutions of our proposed problem, and then the optimal solutions are compared. A numerical example is included to illustrate the feasibility and the applicability of the proposed problem. Finally, we present the conclusions with the future scopes of our study.

Keywords. Transportation problem; multi-objective decision making; intuitionistic fuzzy programming; interval programming; goal programming.

1. Introduction

The classical transportation problem (TP) is mainly concerned with distributing any homogeneous product from a group of supply centres, called sources, to any group of receiving centres, called destinations, in such a way as to minimize the total transportation cost, where the transportation cost per unit product is constant regardless of the amount transported. In the TP, we minimize the cost for single objective function, i.e., the total transportation cost. However, in real-life situation, the TPs are not designed as single objective function. The TP that deals with multiple objective functions is called a multi-objective transportation problem (MOTP). The MOTP is a special type of multi-objective linear programming problem in which objective functions conflict with each other. Normally, there does not exist an optimal solution that would simultaneously satisfy all the criteria. Hence, we seek the best compromise solution. In this context, two approaches, namely intuitionistic fuzzy programming (IFP) and goal programming (GP), are chosen to find the compromise (optimal) solution of the MOTP in our study.

In traditional TP, it is considered that all the transportation parameters like supply, demand and transportation cost, are precise. However, in real-life situations, these parameters are imprecise due to incomplete information and uncertainty in various potential suppliers and environments. Uncertainty may occur due to the following uncontrollable factors:

1. Decision maker has no notion related to the transportation cost when an item is to be delivered at the beginning. Hence, some uncertainty may occur in connection with the transportation cost.
2. Nowadays, market situation is always unstable due to large competitions. Hence, the demand of the newly launched items is totally unpredictable.
3. There may be some sort of uncertainty in connection with the availability of items at a source because of various facts. During the time of delivery, the required amount of items may not be attainable. If a demander needs more number of items, supplier is not certain about the distribution of the items. Demander may vary this order through e-mail/mobile within a few seconds.

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To deal quantitatively with such uncertain information, many researchers studied the MOTP in fuzzy environment, which was introduced by Zadeh [1]. Zimmermann [2] applied fuzzy optimization technique with the linear membership function to solve the linear programming problem with several objective functions. Das *et al* [3] proposed a solution procedure of the MOTP, where all the parameters are expressed in terms of an interval by the decision maker. Li and Lai [4] presented a fuzzy compromise programming approach to solve the MOTP. Ammar and Youness [5] investigated the efficient solutions and stability of the MOTP with fuzzy parameters. Roy [6] described and solved the TP with multi-choice cost and demand and stochastic supply. Liu [7] developed a technique to find the value of the objective function in fuzzy to solve solid TP with fuzzy parameters. Roy and Mahapatra [8] solved the MOTP with interval-valued parameters in probabilistic environment. Roy *et al* [9] investigated multi-choice TP involving exponential distribution. Mahapatra *et al* [10] studied multi-choice stochastic TP involving extreme value distribution. Maity and Roy [11] solved the MOTP under multi-choice environment using utility function approach. Also, Maity and Roy [12] proposed another approach to solve the MOTP with nonlinear cost and multi-choice demand. Rani and Gulati [13] discussed uncertain multi-objective multi-product solid TPs. Maity *et al* [14] described MOTP with cost reliability under uncertain environment. Kocken *et al* [15] proposed a compensatory fuzzy approach to solve multi-objective linear TP with fuzzy parameters. Roy *et al* [16] depicted conic scalarization approach to solve multi-choice MOTP with interval goal. Rani *et al* [17] presented a method for solving unbalanced TPs in fuzzy environment. Roy and Maity [18] solved minimizing cost and time through a single objective function in multi-choice interval-valued TP. Gupta and Kumar [19] depicted a new method for solving linear MOTP where all the parameters are interval-valued fuzzy numbers. Ebrahimnejad [20] considered fuzzy linear programming approach for solving TPs with interval-valued trapezoidal fuzzy numbers. Ebrahimnejad [21] discussed a new method for solving fuzzy TPs with LR flat fuzzy numbers. Roy *et al* [22] described multi-objective two-stage grey TP using a utility function with goals.

In fuzzy optimization, the degrees of acceptance of objective functions and constraints are considered. Fuzzy set theory has also been developed in many areas and its different modifications and generalization forms have appeared. One of the generalization form of fuzzy set theory is an intuitionistic fuzzy set (IFS), which was introduced by Atanassov [23]. The concept of IFS is an alternative approach to define fuzzy set in the case where available information is not sufficient for the definition of an imprecise concept by means of the conventional fuzzy set. The new concept of optimization under intuitionistic fuzzy (IF) environment was introduced by Angelov [24]. In an IF optimization, degree of acceptance (membership) and degree of non-acceptance (non-membership) of objective functions and constraints are considered simultaneously so that the sum of both values is always less than or equal to one. Recently, many researchers introduced IF optimization technique in different fields. Jana and Roy [25] proposed a technique to solve multi-objective IF linear programming problem and applied it in a capacitated TP. Garg *et al* [26] used IF optimization technique to solve multi-objective reliability optimization in interval environment. Chakraborty *et al* [27] developed a new approach to solve multi-objective multi-choice multi-item Atanassov's IF TP using a chance operator.

GP has been widely used to solve multi-objective decision making problem. The basic concept of GP was introduced by Charnes and Cooper [28]. They modelled GP for linear programming problem in which conflicting goals were incorporated in the constraints. It has been further improved by Lee [29] and later by Ignizio [30]. Aenaida and Kwak [31] applied the GP approach to solve TP. Many researchers used GP approach to solve multi-objective optimization problem in various uncertain environments. Abd El-Wahed and Lee [32] used interactive fuzzy GP to solve the MOTP. Zangiabadi and Maleki [33] used fuzzy GP technique to solve the MOTP by considering non-linear membership functions.

Though many investigations have been performed on TP under different environments by several researchers, there are some gaps in TP that occur in real-life situations, where traditional fuzzy environment is not adequate to tackle the situation. Based on this consideration, we incorporate IF environment in our discussed TP. The main contributions of the proposed study are as follows:

(1) In our proposed MOTP, all the parameters of TP are considered as intuitionistic fuzzy numbers (IFNs) due to fluctuation of market scenario. (2) In our proposed approach, we define (α, β) cut to convert IF transportation cost into an interval, and demand and supply into inequalities. Values of α and β are chosen by the decision maker according to his/her choice. (3) We deduce a crisp mathematical model with interval-valued objective function, from the proposed intuitionistic fuzzy multi-objective transportation problem (IFMOTP). (4) Two approaches, namely IFP and GP, are considered to solve the interval-valued MOTP and the obtained solutions are also solutions of the primary IFMOTP at (α, β) cut level. (5) Different values of α and β provide different solutions and the decision maker has a freedom to choose a better solution.

The rest of the paper is designed as follows. In section 2, the basic preliminaries in connection with IFS and interval number are briefly summarized. Section 3 contains the mathematical model of the IFMOTP with a conversion technique for IFMOTP into the crisp model. In section 4, drawbacks of the existing methods are discussed. Section 5 depicts two approaches, namely IFP and GP, which are used to solve the crisp model. Section 6 discusses the advantages of our proposed study. In section 7, a numerical example is provided to justify our proposed problem and the results are discussed. Section 8 contains the conclusion of the paper with the future studies.

2. Preliminaries

In this section, we include some basic definitions and arithmetic operations on IFNs and interval numbers.

Definition 2.1 ([23]) Let X denote a universe of discourse; then an IFS $\widetilde{A}^I$ in X is given by a set of ordered triplet as follows:

$$\widetilde{A}^I = \{\langle x, \mu_{\widetilde{A}^I}(x), \gamma_{\widetilde{A}^I}(x)\rangle : x \in X\},$$

where $\mu_{\widetilde{A}^I}(x)$, $\gamma_{\widetilde{A}^I}(x)$: X $\rightarrow$ [0,1] are functions such that $0 \leq \mu_{\widetilde{A}^I}(x) + \gamma_{\widetilde{A}^I}(x) \leq 1 \forall x \in X$. For each x, $\mu_{\widetilde{A}^I}(x)$ and $\gamma_{\widetilde{A}^I}(x)$ represent the degree of membership and degree of non-membership functions, respectively. Again the function $\pi_{\widetilde{A}^I}(x) = 1 - \mu_{\widetilde{A}^I}(x) - \gamma_{\widetilde{A}^I}(x)$ is called *"degree of hesitation"* of the element x in the set A. If $\pi_{\widetilde{A}^I}(x) = 0 \forall x \in X$, then the IFS reduces to a fuzzy set.

Definition 2.2 ([23]) Let X be a non-empty set; $\widetilde{A}^I$ and $\widetilde{B}^I$ are two IFSs in X given by $\widetilde{A}^I = \{\langle x, \mu_{\widetilde{A}^I}(x), \gamma_{\widetilde{A}^I}(x)\rangle : x \in X\}$ and $\widetilde{B}^I = \{\langle x, \mu_{\widetilde{B}^I}(x), \gamma_{\widetilde{B}^I}(x)\rangle : x \in X\}$, respectively. Then the following properties hold:

(i) $\widetilde{A}^I \subseteq \widetilde{B}^I$ if and only if $\mu_{\widetilde{A}^I}(x) \leq \mu_{\widetilde{B}^I}(x)$ and $\gamma_{\widetilde{A}^I}(x) \geq \gamma_{\widetilde{B}^I}(x) \forall x \in X$,

(ii) $\widetilde{A}^I \cap \widetilde{B}^I = \{\langle x, \mu_{\widetilde{A}^I}(x) \wedge \mu_{\widetilde{B}^I}(x), \gamma_{\widetilde{A}^I}(x) \vee \gamma_{\widetilde{B}^I}(x)\rangle : x \in X\} = \{\langle x, \min(\mu_{\widetilde{A}^I}(x), \mu_{\widetilde{B}^I}(x)), \max(\gamma_{\widetilde{A}^I}(x), \gamma_{\widetilde{B}^I}(x))\rangle : x \in X\}$,

(iii) $\widetilde{A}^I \cup \widetilde{B}^I = \{\langle x, \mu_{\widetilde{A}^I}(x) \vee \mu_{\widetilde{B}^I}(x), \gamma_{\widetilde{A}^I}(x) \wedge \gamma_{\widetilde{B}^I}(x)\rangle : x \in X\} = \{\langle x, \max(\mu_{\widetilde{A}^I}(x), \mu_{\widetilde{B}^I}(x)), \min(\gamma_{\widetilde{A}^I}(x), \gamma_{\widetilde{B}}(x))\rangle : x \in X\}$.

Definition 2.3 An IFS $\widetilde{A}^I$ is said to be normal if there exists x_0 such that $\mu_{\widetilde{A}^I}(x_0) = 1$ and $\gamma_{\widetilde{A}^I}(x_0) = 0$.

Definition 2.4 Support of an IFS $\widetilde{A}^I$ with universal set X is denoted by Support $(\widetilde{A}^I)$ and is defined by Support $(\widetilde{A}^I) = \{x : \mu_{\widetilde{A}^I}(x) > 0 \text{ and } \gamma_{\widetilde{A}^I}(x) \leq 1, x \in X\}$.

Definition 2.5 (α, β)-cut of an IFS $\widetilde{A}^I$ is denoted by $\widetilde{A}^I_{(\alpha,\beta)}$ and is defined by $\widetilde{A}^I_{(\alpha,\beta)} = \{x : \mu_{\widetilde{A}^I}(x) \geq \alpha$ and $\gamma_{\widetilde{A}^I}(x) \leq \beta, \alpha + \beta \leq 1, x \in X\}$ where $\alpha, \beta \in (0, 1]$.

Definition 2.6 An IFN $\widetilde{A}^I$ is an IF subset of real numbers with the following results:

(i) The IFS $\widetilde{A}^I$ is normal.

(ii) The IFS $\widetilde{A}^I$ is convex for the membership function $\mu_{\widetilde{A}^I}(x)$, i.e., $\mu_{\widetilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\widetilde{A}^I}(x_1), \mu_{\widetilde{A}^I}(x_2)\}$ for $x_1, x_2 \in \mathbb{R}, \lambda \in [0, 1]$.

(iii) The IFS A is concave for the non-membership function $\gamma_{\widetilde{A}^I}(x)$, i.e., $\gamma_{\widetilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \leq \max\{\gamma_{\widetilde{A}^I}(x_1), \gamma_{\widetilde{A}^I}(x_2)\}$ for $x_1, x_2 \in \mathbb{R}, \lambda \in [0, 1]$.

Definition 2.7 An IFN of the form $\widetilde{A}^I = (a_1, a_2, a_3)(\overline{a}_1, a_2, \overline{a}_3)$ where $\overline{a}_1 \leq a_1 \leq a_2 \leq a_3 \leq \overline{a}_3$ is said to be triangular IFN if its membership and non-membership functions, respectively, are defined as follows:

$$\mu_{\widetilde{A}^I}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if} \quad a_1 \leq x \leq a_2, \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{if} \quad a_2 \leq x \leq a_3, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\gamma_{\widetilde{A}^I}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - \overline{a}_1} & \text{if} \quad \overline{a}_1 \leq x \leq a_2, \\ \dfrac{x - a_2}{\overline{a}_3 - a_2} & \text{if} \quad a_2 \leq x \leq \overline{a}_3, \\ 1 & \text{otherwise.} \end{cases}$$

Triangular IFN is depicted in figure 1.

Arithmetic operations on triangular IFNs: Let $A = (a_1, a_2, a_3)(\overline{a}_1, a_2, \overline{a}_3)$ and $B = (b_1, b_2, b_3)(\overline{b}_1, b_2, \overline{b}_3)$ represent two triangular IFSs; then addition, subtraction, multiplication and scalar multiplication of the numbers are stated as follows:

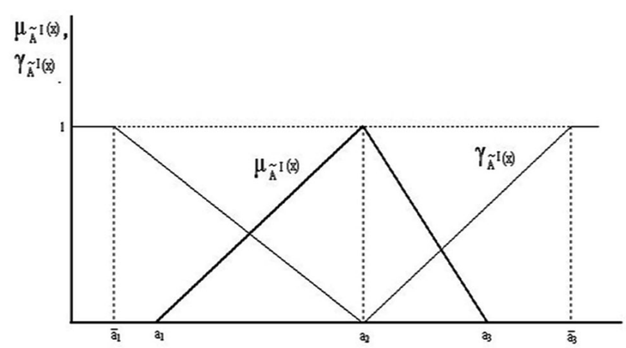

Figure 1. Triangular intuitionistic fuzzy number.

$$\begin{aligned}
\textbf{Addition}: \quad & A+B=(a_1+b_1,a_2+b_2,a_3+b_3) \\
& (\overline{a}_1+\overline{b}_1,a_2+b_2,\overline{a}_3+\overline{b}_3). \\
\textbf{Subtraction}: \quad & A-B=(a_1-b_3,a_2-b_2,a_3-b_1) \\
& (\overline{a}_1-\overline{b}_3,a_2-b_2,\overline{a}_3-\overline{b}_1). \\
\textbf{Multiplication}: \quad & A.B=[\min\{a_1b_1,a_1b_3,a_3b_1,a_3b_3\}, \\
& a_2b_2,\max\{a_1b_1,a_1b_3,a_3b_1,a_3b_3\}] \\
& [\min\{\overline{a}_1\overline{b}_1,\overline{a}_1\overline{b}_3,\overline{a}_3\overline{b}_1,\overline{a}_3\overline{b}_3\},a_2b_2, \\
& \max\{\overline{a}_1\overline{b}_1,\overline{a}_1\overline{b}_3,\overline{a}_3\overline{b}_1,\overline{a}_3\overline{b}_3\}].
\end{aligned}$$

Scalar multiplication: For any real k

$$kA=\begin{cases}(ka_1,ka_2,ka_3)(k\overline{a}_1,ka_2,k\overline{a}_3) & \text{if} \quad k\geq 0,\\ (ka_3,ka_2,ka_1)(k\overline{a}_3,ka_2,k\overline{a}_1) & \text{if} \quad k<0.\end{cases}$$

The (α,β) cut of a triangular IFN is shown in figure 2.

Definition 2.8 (α,β)-cut of a triangular intuitionistic fuzzy number $\widetilde{A}^I=(a_1,a_2,a_3)(\overline{a}_1,a_2,\overline{a}_3)$ is the set of all x whose degree of membership is greater than or equal to α and degree of non-membership is less than or equal to β, i.e., $\widetilde{A}^I_{(\alpha,\beta)}=\{x:\mu_{\widetilde{A}^I}(x)\geq\alpha$ and $\gamma_{\widetilde{A}^I}(x)\leq\beta,\alpha+\beta\leq 1,x\in X\}$.

Now, $\mu_{\widetilde{A}^I}(x)\geq\alpha$

$$\Rightarrow \frac{x-a_1}{a_2-a_1}\geq\alpha \text{ and } \frac{a_3-x}{a_3-a_2}\geq\alpha, \text{ or } x\geq a_1+\alpha(a_2-a_1) \text{ and } x\leq a_3-\alpha(a_3-a_2).$$

Again, $\gamma_{\widetilde{A}^I}(x)\leq\beta$

$$\Rightarrow \frac{a_2-x}{a_2-\overline{a}_1}\leq\beta \text{ and } \frac{x-a_2}{\overline{a}_3-a_2}\leq\beta \text{ or } x\geq a_2-\beta(a_2-\overline{a}_1) \text{ and } x\leq a_2+\beta(\overline{a}_3-a_2).$$

Let $A_L=\max\{a_1+\alpha(a_2-a_1),a_2-\beta(a_2-\overline{a}_1)\}$, and $A_U=\min\{a_3-\alpha(a_3-a_2),a_2+\beta(\overline{a}_3-a_2)\}$.

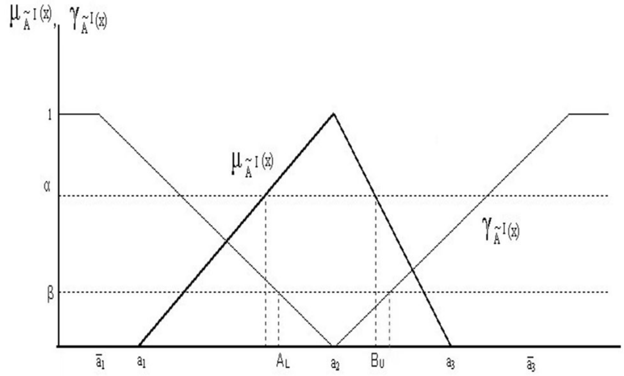

Figure 2. (α,β) cut of a triangular intuitionistic fuzzy number.

Then (α,β)-cut of a triangular IFN can be reduced into an interval form of $\widetilde{A}^I_{(\alpha,\beta)}$ as $[A_L,A_U]$ where A_L and A_U are, respectively, lower and upper limits of the interval.

Definition 2.9 ([26]) Let $\mathbb{R}$ be the set of all real numbers; then the interval number $\overline{C}$ is a closed interval denoted by $\overline{C}=[c_L,c_R]$ and is defined as follows:

$$\overline{C}=[c_L,c_R]=\{x:c_L\leq x\leq c_R,c_L,c_R\in\mathbb{R}\},$$

where c_L and c_R are, respectively, lower and upper limits of the interval $\overline{C}$. If $c_L=c_R$ then $\overline{C}$ reduces to a real number. An interval $\overline{C}$ can also be written as

$$\overline{C}=<c_c,c_w>=\{x:c_c-c_w\leq x\leq c_c+c_w,x\in\mathbb{R}\},$$

where c_c and c_w are, respectively, the centre and the width of the interval $\overline{C}$ and $c_c=\frac{c_L+c_R}{2}$ and $c_w=\frac{c_R-c_L}{2}$. An interval can also be presented as an order triplet $\bar{C}=[c_L,c_C,c_R]$.

Interval arithmetic: Let $\overline{C}=[c_L,c_R]$ and $\overline{D}=[d_L,d_R]$ be two intervals; then addition, subtraction, multiplication, division and scalar multiplication of interval numbers are described as follows:

$$\begin{aligned}
\textbf{Addition}: \quad & \overline{C}+\overline{D}=[c_L+d_L,c_R+d_R]. \\
\textbf{Subtraction}: \quad & \overline{C}-\overline{D}=[c_L-d_R,c_R-d_L]. \\
\textbf{Multiplication}: \quad & \overline{C}.\overline{D}=[\min(c_Ld_L,c_Ld_R,c_Rd_L,c_Rd_R), \\
& \max(c_Ld_L,c_Ld_R,c_Rd_L,c_Rd_R)]. \\
\textbf{Division}: \quad & \overline{C}/\overline{D}=[\min(c_L/d_L,c_L/d_R,c_R/d_L,c_R/d_R), \\
& \max(c_L/d_L,c_L/d_R,c_R/d_L,c_R/d_R)] \\
& \text{provided } 0\notin\overline{D}.
\end{aligned}$$

Scalar multiplication: For any real k

$$k\overline{C}=\begin{cases}[kc_L,kc_R] & \text{if} \quad k\geq 0,\\ [kc_R,kc_L] & \text{if} \quad k<0.\end{cases}$$

Order relation between intervals: We assume the following definition, which is mainly used for comparing intervals involving many practical applications.

Definition 2.10 ([3]) The order relation $\leq_{LR}$ between $\overline{C}=[c_L,c_R]$ and $\overline{D}=[d_L,d_R]$ is defined as $\overline{C}\leq_{LR}\overline{D}$ iff $c_L\leq d_L$ and $c_R\leq d_R$, $\overline{C}<_{LR}\overline{D}$ iff $\overline{C}\leq_{LR}\overline{D}$ and $\overline{C}\neq\overline{D}$.

This order relation represents decision maker's preference for selecting the alternative with minimum cost and maximum cost, i.e., if $\overline{C}<_{LR}\overline{D}$, then $\overline{C}$ is preferred to $\overline{D}$.

3. Model formulation

TP with a single objective function is hardly applicable to design many practical problems. To overcome this difficulty, we choose multiple objective functions (with conflicting and non-commensurable nature) into the TP and here it is referred to as MOTP. The main aim of the MOTP is to calculate an optimal plan for transporting a homogeneous commodity from m sources to n destinations in such a way that all the objective functions are optimized simultaneously. Let there be K number of objective functions $Z_1, Z_2, \ldots, Z_K$. For each objective function Z_k, a transportation cost c_{ij}^k is associated with transporting one unit of commodity from the ith source to the jth destination. Let a_i be the total availability of the product at the ith source and b_j be the total demand of the product at the jth destination. Let x_{ij} be the unknown quantity transported from the ith source to the jth destination so as to minimize the objective functions. The mathematical model of the MOTP is described as follows:

Model 1

$$\text{minimize } Z_k(x_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij} \tag{3.1}$$

$$\text{subject to } \sum_{j=1}^{n} x_{ij} \le a_i \quad (i = 1, 2, \ldots, m), \tag{3.2}$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j \quad (j = 1, 2, \ldots, n), \tag{3.3}$$

$$x_{ij} \ge 0 \quad \forall \quad i \text{ and } \quad j. \tag{3.4}$$

3.1 *IFMOTP*

In real-life situations, the transportation parameters (transportation cost, supply and demand) are not precise due to incomplete information of various potential suppliers and environments. To deal quantitatively with such imprecise information, we consider the MOTP in IF environment. Here, we assume transportation cost $(\widetilde{c_{ij}^k}^I)$, supply $(\widetilde{a}_i^I)$ and demand $(\widetilde{b}_j^I)$ as the IFNs whose membership and non-membership functions are supplied. The MOTP with the IF parameters is treated here as an IFMOTP. The mathematical model of the IFMOTP is shown in Model 2 as follows:

Model 2

$$\text{minimize} \quad \widetilde{Z}_k^I(x_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n} \widetilde{c_{ij}^k}^I x_{ij} \tag{3.5}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \le \widetilde{a}_i^I \quad (i = 1, 2, \ldots, m), \tag{3.6}$$

$$\sum_{i=1}^{m} x_{ij} \ge \widetilde{b}_j^I \quad (j = 1, 2, \ldots, n), \tag{3.7}$$

$$x_{ij} \ge 0 \quad \forall \quad i \text{ and } \quad j, \tag{3.8}$$

where $\widetilde{Z}_k^I(x_{ij})$ is the kth objective function in IF form. Now, the membership function $\mu_{\widetilde{c_{ij}^k}^I}(x)$ and non-membership function $\gamma_{\widetilde{c_{ij}^k}^I}(x)$ of transportation cost $\widetilde{c^k}^I_{ij}$ ($\forall i, j$ and k) are given as follows:

$$\mu_{\widetilde{c_{ij}^k}^I}(x) = \begin{cases} \dfrac{x - c_{ij1}^k}{c_{ij2}^k - c_{ij1}^k} & \text{if} \quad c_{ij1}^k \le x \le c_{ij2}^k, \\ \dfrac{c_{ij3}^k - x}{c_{ij3}^k - c_{ij2}^k} & \text{if} \quad c_{ij2}^k \le x \le c_{ij3}^k, \\ 0 & \text{otherwise}, \end{cases} \tag{3.9}$$

and

$$\gamma_{\widetilde{c_{ij}^k}^I}(x) = \begin{cases} \dfrac{c_{ij2}^k - x}{c_{ij2}^k - \overline{c}_{ij1}^k} & \text{if} \quad \overline{c}_{ij1}^k \le x \le c_{ij2}^k, \\ \dfrac{x - c_{ij2}^k}{\overline{c}_{ij3}^k - c_{ij2}^k} & \text{if} \quad c_{ij2}^k \le x \le \overline{c}_{ij3}^k, \\ 0 & \text{otherwise}. \end{cases} \tag{3.10}$$

Similarly the membership and non-membership functions of supply $\widetilde{a_i}^I$ $(i = 1, 2, \ldots, m)$ are defined as follows:

$$\mu_{\widetilde{a_i}}^I(x) = \begin{cases} 1 & \text{if} \quad x \le a_{i1}, \\ \dfrac{a_{i2} - x}{a_{i2} - a_{i1}} & \text{if} \quad a_{i1} \le x \le a_{i2}, \\ 0 & \text{if} \quad x \ge a_{i2}, \end{cases} \tag{3.11}$$

and

$$\gamma_{\widetilde{a_i}^I}(x) = \begin{cases} 0 & \text{if} \quad x \le a_{i1} + d_i, \\ \dfrac{x - a_{i1} - d_i}{a_{i2} - a_{i1} - d_i} & \text{if} \quad a_{i1} + d_i \le x \le a_{i2}, \\ 1 & \text{if} \quad x \ge a_{i2}, \end{cases} \tag{3.12}$$

where d_i is the acceptable limit of non-membership function of $\widetilde{a_i}^I$.

Also the membership and non-membership functions of demand $\widetilde{b_j}^I$ $(j = 1, 2, \ldots, n)$ are given as follows:

$$\mu_{\widetilde{b_j}^I}(x) = \begin{cases} 1 & \text{if} \quad x \ge b_{j2}, \\ \dfrac{x - b_{j1}}{b_{j2} - b_{j1}} & \text{if} \quad b_{j1} \le x \le b_{j2}, \\ 0 & \text{if} \quad x \le b_{j1}, \end{cases} \tag{3.13}$$

and

$$\gamma_{\widetilde{b_j}^I}(x) = \begin{cases} 0 & \text{if} \quad x \ge b_{j2} - p_j, \\ \dfrac{b_{j2} - p_j - x}{b_{j2} - p_j - b_{j1}} & \text{if} \quad b_{j1} \le x \le b_{j2} - p_j, \\ 1 & \text{if} \quad x \le b_{j1}, \end{cases} \tag{3.14}$$

where p_j is also the acceptable limit of non-membership function of $\widetilde{b}_j{}^I$.

The degree of acceptance (membership) and degree of rejection (non-membership) both are described simultaneously; however, they are not complementary to each other. IFS can be used in a more general way for defining this fuzziness. It is possible to represent the objective functions and constraints by the IFS, i.e., a pair of membership and non-membership functions.

Now (α, β)-level interval or (α, β)-cut of the IF cost coefficients are given by $\widetilde{A}^I_{(\alpha,\beta)} = \{\langle x, \mu_{\widetilde{c}^{k\,I}_{ij}}(x), \gamma_{\widetilde{c}^{k\,I}_{ij}}(x)\rangle : \mu_{\widetilde{c}^{k\,I}_{ij}}(x) \geq \alpha,\ \gamma_{\widetilde{c}^{k\,I}_{ij}}(x) \leq \beta, \alpha, \beta \in (0, 1]\}$, where α and β are fixed numbers such that $\alpha + \beta \leq 1$. This is a set of elements that belong to the set at least to the degree α and do not belong to the set at most to the degree β. The values of α and β are prescribed by the decision maker according to his/her choice. Now, for transportation cost, we consider $\mu_{\widetilde{c}^{k\,I}_{ij}}(x) \geq \alpha, \gamma_{\widetilde{c}^{k\,I}_{ij}}(x) \leq \beta$.

Hence, we have

$$\frac{x - c^k_{ij1}}{c^k_{ij2} - c^k_{ij1}} \geq \alpha, \frac{c^k_{ij1} - x}{c^k_{ij2} - c^k_{ij1}} \leq \beta \text{ or } x \geq c^k_{ij1} + \alpha(c^k_{ij2} - c^k_{ij1}),$$
$$x \geq c^k_{ij2} - \beta(c^k_{ij2} - c^k_{ij1})$$

and

$$\frac{c^k_{ij3} - x}{c^k_{ij3} - c^k_{ij2}} \geq \alpha, \frac{x - c^k_{ij2}}{c^k_{ij3} - c^k_{ij2}} \leq \beta \text{ or } x \leq c^k_{ij3} - \alpha(c^k_{ij3} - c^k_{ij2}),$$
$$x \leq c^k_{ij2} + \beta(c^k_{ij3} - c^k_{ij2}).$$

We assign c^k_{ijL} and c^k_{ijU} in the following way:

$$c^k_{ijL} = \max\{c^k_{ij1} + \alpha(c^k_{ij2} - c^k_{ij1}), c^k_{ij2} - \beta(c^k_{ij2} - c^k_{ij1})\}$$

and

$$c^k_{ijU} = \min\{c^k_{ij3} - \alpha(c^k_{ij3} - c^k_{ij2}), c^k_{ij2} + \beta(c^k_{ij3} - c^k_{ij2})\} \forall \quad i, j, k.$$

Hence, we derive the interval for each IF cost parameter in the form $[c^k_{ijL}, c^k_{ijU}]$. Then the objective function (3.5) can be rewritten as follows:

$$\text{minimize} \quad \overline{Z}_k(x_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n}[c^k_{ijL}, c^k_{ijU}]x_{ij} \quad (k = 1, 2, \ldots, K),$$

where $\overline{Z}_k(x_{ij})$ is the kth objective function in interval form. Similarly, for the supply parameter $\widetilde{a}^I_i$, we have $\mu_{\widetilde{a}_i{}^I}(x) \geq \alpha$ and $\gamma_{\widetilde{a}_i{}^I}(x) \leq \beta$.

$$\text{Hence} \quad \frac{a_{i2} - x}{a_{i2} - a_{i1}} \geq \alpha \quad \text{and} \quad \frac{x - a_{i1} - d_i}{a_{i2} - a_{i1} - d_i} \leq \beta,$$
$$\text{i.e.,} \quad x \leq a_{i2} - \alpha(a_{i2} - a_{i1}) \text{ and } x \leq (a_{i1} + d_i) + \beta(a_{i2} - a_{i1} - d_i).$$

Then the supply constraints (3.6) can be described as follows:

$$\sum_{j=1}^{n} x_{ij} \leq A_i \quad \forall \quad i, \tag{3.15}$$

where $A_i = \min\{a_{i2} - \alpha(a_{i2} - a_{i1}), (a_{i1} + d_i) + \beta(a_{i2} - a_{i1} - d_i)\}$.

In a similar way, for demand parameter $\widetilde{b}_j{}^I$, we have $\mu_{\widetilde{b}_j{}^I}(x) \geq \alpha$ and $\gamma_{\widetilde{b}_j{}^I}(x) \leq \beta$.

$$\text{Hence} \quad \frac{x - b_{j1}}{b_{j2} - b_{j1}} \geq \alpha \quad \text{and} \quad \frac{b_{j2} - p_j - x}{b_{j2} - p_j - b_{j1}} \leq \beta,$$
$$\text{i.e.,} \quad x \geq b_{j1} + \alpha(b_{j2} - b_{j1}) \quad \text{and } x \geq (b_{j2} - p_j) - \beta(b_{j2} - p_j - b_{j1}).$$

Hence the demand constraints (3.7) can be written as follows:

$$\sum_{i=1}^{m} x_{ij} \geq B_j \quad \forall \quad j, \tag{3.16}$$

where $B_j = \max\{b_{j1} + \alpha(b_{j2} - b_{j1}), (b_{j2} - p_j) - \beta(b_{j2} - p_j - b_{j1})\}$.

Finally Model 2 is converted to Model 3 as follows:

Model 3

$$\text{minimize} \quad \overline{Z}_k(x_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n}[c^k_{ijL}, c^k_{ijU}]x_{ij} \quad (k = 1, 2, \ldots, K)$$
$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq A_i \quad (i = 1, 2, \ldots, m),$$
$$\sum_{i=1}^{m} x_{ij} \geq B_j \quad (j = 1, 2, \ldots, n),$$
$$x_{ij} \geq 0 \quad \forall \quad i, j.$$

Here, we choose α and β in such a manner that $\sum_{i=1}^{m} A_i \geq \sum_{j=1}^{n} B_j$. This is the necessary condition for feasible region of this problem.

Then the optimization problem, i.e., Model 3 can be rewritten as follows:

Model 3.1

$$\text{minimize } [\overline{Z}_1, \overline{Z}_2, \ldots, \overline{Z}_K]$$
$$\text{subject to } X \in S,$$

where $\overline{Z}_k = [Z_{Lk}, Z_{Rk}] = [\sum_{i=1}^{m}\sum_{j=1}^{n} c^k_{ijL}x_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} c^k_{ijU}x_{ij}]$ $(k = 1, 2, \ldots, K)$, and $S = \{X : \sum_{j=1}^{n} x_{ij} \leq A_i\ (i = 1, 2, \ldots, m)$ and $\sum_{i=1}^{m} x_{ij} \geq B_j\ (j = 1, 2, \ldots, n)\}$.

Here $X \in S \subset \mathbb{R}$ is the decision vector and S is the feasible region. Z_{Lk} and Z_{Rk} are the left and right limits of the objective function Z_k at decision vector X. Now the centre of the objective function is given by $Z_{Ck} = \frac{Z_{Lk} + Z_{Rk}}{2}$. Then, the problem, i.e., Model 3.1 is transformed into mathematical programming with ordered triplet as follows:

Model 4

$$\begin{aligned} &\text{minimize } [Z_{Lk}, Z_{Ck}, Z_{Rk}] \quad (k = 1, 2, \ldots, K) \\ &\text{subject to } \sum_{j=1}^{n} x_{ij} \le A_i \quad (i = 1, 2, \ldots, m), \\ &\sum_{i=1}^{m} x_{ij} \ge B_j \quad (j = 1, 2, \ldots, n), \\ &x_{ij} \ge 0 \quad \forall \quad i, j. \end{aligned}$$

Theorem 3.1 *Model 4 is solvable as well as feasible if* $\sum_{i=1}^{m} A_i \ge \sum_{j=1}^{n} B_j$ *where* A_i *and* B_j *are given by Eqs.* (3.15) *and* (3.16), *respectively.*

Proof Straightforward.

Definition 3.1 A feasible solution vector $X^* \in S$ is said to be the Pareto-optimal solution (efficient or non-dominated) of the MOTP if there does not exist any other vector $X \in S$ such that $\overline{Z}_k(X) \le_{LR} \overline{Z}_k(X^*)$ for $k = 1, 2, \ldots, K$ and $\overline{Z}_k(X) <_{LR} \overline{Z}_k(X^*)$ for at least one k.

Definition 3.2 A feasible solution vector $X^* \in S$ is an optimal compromise solution for the MOTP if it is preferred by the decision maker to all other feasible solutions, taking into consideration all criteria contained in the multi-objective functions.

4. Drawbacks of the existing methods

From the past literature review, researchers have developed various procedures for solving intuitionistic fuzzy transportation problem (IFTP). Hence we mention the main deviations of the existing methods as follows:

1. There are few research papers available in the literature to solve the MOTP in IF environment.
2. The existing method (Jana and Roy [25]) can be applied to solve the MOTP only where demand and supply are IF in nature. It cannot be used to solve the MOTP where all the parameters are IF in nature.
3. Jana and Roy [25] considered membership function as a hyperbolic function and non-membership function as a parabolic function, which are nonlinear functions and they are very complicated to handle for solving MOTP in IF environment.
4. Kumar and Hussain [34] utilized the ranking function for solving fully IFTP. However, the method has no new concept for solving fully IFTP when compared to the existing literature.

5. Solution procedure

We consider two approaches for solving Model 4 and they are as follows:

- IFP approach
- GP approach

In the next subsections, we briefly describe the solution concepts of these approaches. Utilizing the approaches, we extract the optimal solutions of our proposed problem.

5.1 *IFP approach*

For each objective function, i.e., Z_{Lk}, Z_{Ck} and Z_{Rk}, we first find the lower bounds L_{Lk}, L_{Ck} and L_{Rk} (best values) and the upper bounds U_{Lk}, U_{Ck} and U_{Rk} (worst values) where L_{Lk}, L_{Ck} and L_{Rk} are the aspired levels of achievement and U_{Lk}, U_{Ck} and U_{Rk} are the highest acceptable levels of achievement for the objective functions Z_{Lk}, Z_{Ck} and Z_{Rk}, respectively. When the aspired level and the acceptance level for each objective function are specified, we formulate a crisp model. The algorithm of this approach is given as follows.

Algorithm

Step 1: Solve the MOTP by considering one objective function at a time and ignoring all others; collect the obtained solutions. Repeat this process K times if there are K number of objective functions.

Step 2: Determine the corresponding cost for every objective function at each obtained solution.

Step 3: To each objective function, find best values $[L_{Lk}, L_{Ck}, L_{Rk}]$ and worst values $[U_{Lk}, U_{Ck}, U_{Rk}]$. Define membership and non-membership functions for each objective function as follows:

$$\mu(Z_{mk}) = \begin{cases} 1 & \text{if} \quad Z_{mk} \le L_{mk}, \\ \dfrac{U_{mk} - Z_{mk}}{U_{mk} - L_{mk}} & \text{if} \quad L_{mk} \le Z_{mk} \le U_{mk}, \\ 0 & \text{if} \quad Z_{mk} \ge U_{mk}. \end{cases}$$
$$k = 1, 2, \ldots, K; \quad m = L, C, R,$$

and

$$\gamma(Z_{mk}) = \begin{cases} 0 & \text{if} \quad Z_{mk} \le L_{mk}, \\ \dfrac{Z_{mk} - L_{mk}}{U_{mk} - L_{mk}} & \text{if} \quad L_{mk} \le Z_{mk} \le U_{mk}, \\ 1 & \text{if} \quad Z_{mk} \ge U_{mk}. \end{cases}$$
$$k = 1, 2, \ldots, K; \quad m = L, C, R,$$

Step 4: After this, the intuitionistic optimization model can be changed into Model 5 as follows.

Model 5

$$\begin{aligned} &\text{maximize} \quad (\theta - \delta) \\ &\text{subject to} \quad \mu(Z_{mk}) \ge \theta, \gamma(Z_{mk}) \le \delta, m = L, C, R; \\ &k = 1, 2, \ldots, K, \\ &\sum_{j=1}^{n} x_{ij} \le A_i \quad (i = 1, 2, \ldots, m), \end{aligned}$$

$$\sum_{i=1}^{m} x_{ij} \geq B_j \quad (j = 1, 2, \ldots, n),$$
$$\theta \geq \delta, \theta + \delta \leq 1, \theta, \delta \in [0, 1],$$
$$x_{ij} \geq 0 \quad \forall \quad i, j,$$

where θ and δ are the aspiration levels for membership and non-membership functions of the objective functions, respectively.

Step 5: Now Model 5 can be solved using LINGO software; let X^* be the optimal solution. Then we calculate left, centre and right limits of each objective function at X^*; hence we get the optimal compromise solutions of the objective functions in triplet form as $[Z_{Lk}(X^*), Z_{Ck}(X^*), Z_{Rk}(X^*)], \quad k = 1, 2, \ldots, K.$

5.2 *GP approach*

GP is used to solve multi-objective decision making problems. The basic concept of the GP is to minimize the sum of the deviations of the objective functions from their respective goals (target values), which is determined by the decision maker. The proposed problem is solved using the GP approach through the following steps.

Step 1: Solve the MOTP by choosing only one objective function at a time and omitting others; store the derived solutions. Continue this process K times if there are K number of objective functions.

Step 2: Compute the corresponding cost for every objective function at each derived solution. Thereafter, find the best value of Z_{mk} as L_{mk} and worst value of Z_{mk} as U_{mk}, $(m = L, C, R; k = 1, 2, \ldots, K)$ for each objective function.

Step 3: Take goal at most Z_{mk}^g to every objective function as $Z_{mk}^g = \frac{L_{mk}+U_{mk}}{2}, m = L, C, R; \quad k = 1, 2, \ldots, K.$

Step 4: Formulate the mathematical model (Model 6) using the GP as follows.

Model 6

$$\text{minimize} \quad \sum_{k=1}^{K} (d_{Lk}^+ + d_{Ck}^+ + d_{Rk}^+)$$
$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq A_i \quad (i = 1, 2, \ldots, m),$$
$$\sum_{i=1}^{m} x_{ij} \geq B_j \quad (j = 1, 2, \ldots, n),$$
$$Z_{mk} - d_{mk}^+ + d_{mk}^- = Z_{mk}^g, \quad m = L, C, R,$$
$$k = 1, 2, \ldots, K,$$
$$x_{ij} \geq 0 \quad \forall \quad i, j,$$

where d_{mk}^+ and d_{mk}^- $\quad (m = L, C, R; \quad k = 1, 2, \ldots, K)$ are positive and negative deviations of the objective functions.

Step 5: Model 6 can be solved with the help of LINGO software; let X^* be the optimal solution. Calculate left, centre and right limits of each objective function at X^*; hence, we derive the optimal compromise solutions of the objective functions in triplet form as $[Z_{Lk}(X^*), Z_{Ck}(X^*), Z_{Rk}(X^*)], \quad k = 1, 2, \ldots, K.$

6. Advantages of our proposed method

In this section, we explore the main advantages of our proposed method over the existing methods.

1. In our proposed method, all the transportation parameters are IFNs, which are not considered in the existing methods.
2. In our formulated method, we use linear membership and non-membership functions, which are easy to tackle with less computational burden for solving the proposed MOTP.
3. The decision maker has freedom to choose the values of α and β with the condition $\alpha + \beta \leq 1$. Different values of α and β provide a variety of solutions, which have a wide spectrum to select a better solution by the decision maker.

7. Application example

A renowned company collects baby food products [35] from three production sources and then supplies to four destination centres, in which the transportation cost, the supply and the demand are IFNs. The decision maker lays emphasis on criteria such as minimization of total transportation cost, transportation time (delivery time) and loss during the transportation through the given route (i, j) $(i = 1, 2, 3; \ j = 1, 2, 3, 4)$. Here, Z_1, Z_2 and Z_3 represent the total transportation cost in hundred dollar($) per unit, transportation time in days per unit and loss during the transportation in dollar, respectively, from each production source to each destination centre. Without loss of generality, the transportation cost, the transportation time and loss during the transportation are assumed as triangular IFN; the cost matrix corresponding to the objective functions Z_1, Z_2 and Z_3 are specified, respectively, in tables 1, 2 and 3.

The decision maker is interested in transporting the baby food products from the ith source to jth destination so as to satisfy the following availability and demand constraints:

$$x_{11} + x_{12} + x_{13} + x_{14} = \widetilde{a}_1^I,$$
$$x_{21} + x_{22} + x_{23} + x_{24} = \widetilde{a}_2^I,$$
$$x_{31} + x_{32} + x_{33} + x_{34} = \widetilde{a}_3^I,$$
$$x_{11} + x_{21} + x_{31} = \widetilde{b}_1^I,$$
$$x_{12} + x_{22} + x_{32} = \widetilde{b}_2^I,$$
$$x_{13} + x_{23} + x_{33} = \widetilde{b}_3^I,$$
$$x_{14} + x_{24} + x_{34} = \widetilde{b}_4^I.$$

Table 1. Transportation cost for IFMOTP.

	D_1	D_2	D_3	D_4
S_1	(6, 8, 10)(5, 8, 11)	(7, 9, 10)(6, 9, 10.5)	(6, 7, 8)(5, 7, 9)	(1, 2, 3)(0.5, 2, 3.5)
S_2	(3, 5, 7)(2.5, 5, 7.5)	(4, 6, 8)(3, 6, 9)	(2.5, 4, 5)(2, 4, 6)	(6, 7, 8)(5, 7, 9)
S_3	(2, 3, 4)(1, 3, 4.5)	(6, 7, 8)(5, 7, 9)	(6, 7, 8)(5, 7, 9)	(3, 5, 7)(2.5, 5, 7.5)

Table 2. Transportation time for IFMOTP.

	D_1	D_2	D_3	D_4
S_1	(1, 2, 3)(0.5, 2, 3.5)	(7, 9, 10)(6, 9, 10.5)	(6, 8, 10)(5, 8, 11)	(.5, 1, 1.5)(0, 1, 2)
S_2	(2.5, 4, 5)(2, 4, 6)	(2, 3, 4)(1, 3, 4.5)	(4, 6, 8)(3, 6, 9)	(6, 7, 8)(5, 7, 9)
S_3	(3, 5, 7)(2.5, 5, 7.5)	(1, 2, 3)(0.5, 2, 3.5)	(6, 8, 10)(5, 8, 11)	(1, 2, 3)(0.5, 2, 3.5)

Table 3. Loss during the transportation for IFMOTP.

	D_1	D_2	D_3	D_4
S_1	(1, 2, 3)(0.5, 2, 3.5)	(2.5, 4, 5)(2, 4, 6)	(6, 7, 8)(5, 7, 9)	(2, 3, 4)(1, 3, 4.5)
S_2	(4, 6, 8)(3, 6, 9)	(2.5, 4, 5)(2, 4, 6)	(6, 8, 10)(5, 8, 11)	(2.5, 4, 5)(2, 4, 6)
S_3	(6, 8, 10)(5, 8, 11)	(1, 2, 3)(0.5, 2, 3.5)	(3, 5, 7)(2.5, 5, 7.5)	(.5, 1, 1.5)(0, 1, 2)

The membership functions $\mu_{\widetilde{a_1}^I}(x),\ \mu_{\widetilde{a_2}^I}(x)$ and $\mu_{\widetilde{a_3}^I}(x)$ and non-membership functions $\gamma_{\widetilde{a_1}^I}(x),\ \gamma_{\widetilde{a_2}^I}(x)$ and $\gamma_{\widetilde{a_3}^I}(x)$ of corresponding supplies $\widetilde{a_i}^I$ $(i = 1, 2, 3)$ are designed based on the data prescribed by the decision maker as follows:

$$\mu_{\widetilde{a_1}^I}(x) = \begin{cases} 1 & \text{if} \quad x \le 8, \\ \dfrac{13 - x}{13 - 8} & \text{if} \quad 8 \le x \le 13, \\ 0 & \text{if} \quad x \ge 13, \end{cases}$$

$$\mu_{\widetilde{a_2}^I}(x) = \begin{cases} 1 & \text{if} \quad x \le 11, \\ \dfrac{15 - x}{15 - 11} & \text{if} \quad 11 \le x \le 15, \\ 0 & \text{if} \quad x \ge 15, \end{cases}$$

$$\mu_{\widetilde{a_3}^I}(x) = \begin{cases} 1 & \text{if} \quad x \le 14, \\ \dfrac{18 - x}{18 - 14} & \text{if} \quad 14 \le x \le 18, \\ 0 & \text{if} \quad x \ge 18. \end{cases}$$

and

$$\gamma_{\widetilde{a_1}^I}(x) = \begin{cases} 0 & \text{if} \quad x \le 9, \\ \frac{x-9}{13-9} & \text{if} \quad 9 \le x \le 13, \\ 1 & \text{if } x \ge 13, \end{cases}$$

$$\gamma_{\widetilde{a_2}^I}(x) = \begin{cases} 0 & \text{if} \quad x \le 13, \\ \frac{x-13}{15-13} & \text{if} \quad 13 \le x \le 15, \\ 1 & \text{if} \quad x \ge 15, \end{cases}$$

$$\gamma_{\widetilde{a_3}^I}(x) = \begin{cases} 0 & \text{if} \quad x \le 14.5, \\ \dfrac{x - 14.5}{18 - 14.5} & \text{if} \quad 14.5 \le x \le 18, \\ 1 & \text{if} \quad x \ge 18. \end{cases}$$

Also the membership functions $\mu_{\widetilde{b_1}^I}(x),\ \mu_{\widetilde{b_2}^I}(x),\ \mu_{\widetilde{b_3}^I}(x)$ and $\mu_{\widetilde{b_4}^I}(x)$ and non-membership functions $\gamma_{\widetilde{b_1}^I}(x),\ \gamma_{\widetilde{b_2}^I}(x),\ \gamma_{\widetilde{b_3}^I}(x)$ and $\gamma_{\widetilde{b_4}^I}(x)$ of corresponding demands $\widetilde{b_j}^I$ $(j = 1, 2, 3, 4)$ are described with the concept of the decision maker as follows:

$$\mu_{\widetilde{b_1}^I}(x) = \begin{cases} 1 & \text{if} \quad x \ge 12, \\ \dfrac{x - 7}{12 - 7} & \text{if} \quad 7 \le x \le 12, \\ 0 & \text{if} \quad x \le 7, \end{cases}$$

$$\mu_{\widetilde{b_2}^I}(x) = \begin{cases} 1 & \text{if} \quad x \ge 9, \\ \dfrac{x - 5}{9 - 5} & \text{if} \quad 5 \le x \le 9, \\ 0 & \text{if} \quad x \le 5, \end{cases}$$

$$\mu_{\widetilde{b_3}^I}(x) = \begin{cases} 1 & \text{if} \quad x \ge 9, \\ \dfrac{x - 4}{9 - 4} & \text{if} \quad 4 \le x \le 9, \\ 0 & \text{if} \quad x \le 4, \end{cases}$$

$$\mu_{\widetilde{b_4}^I}(x) = \begin{cases} 1 & \text{if} \quad x \ge 7, \\ \dfrac{x - 3}{7 - 3} & \text{if} \quad 3 \le x \le 7, \\ 0 & \text{if} \quad x \le 3. \end{cases}$$

and

$$\gamma_{\widetilde{b_1}^I}(x) = \begin{cases} 0 & \text{if} \quad x \ge 10, \\ \dfrac{10 - x}{10 - 7} & \text{if} \quad 7 \le x \le 10, \\ 1 & \text{if} \quad x \le 7, \end{cases}$$

$$\gamma_{\tilde{b}_2{}'}(x) = \begin{cases} 0 & \text{if} \quad x \geq 7, \\ \dfrac{7-x}{7-5} & \text{if} \quad 5 \leq x \leq 7, \\ 1 & \text{if} \quad x \leq 5, \end{cases}$$

$$\gamma_{\tilde{b}_3{}'}(x) = \begin{cases} 0 & \text{if} \quad x \geq 8, \\ \dfrac{8-x}{8-4} & \text{if} \quad 4 \leq x \leq 8, \\ 1 & \text{if} \quad x \leq 4, \end{cases}$$

$$\gamma_{\tilde{b}_4{}'}(x) = \begin{cases} 0 & \text{if} \quad x \geq 6, \\ \dfrac{6-x}{6-3} & \text{if} \quad 3 \leq x \leq 6, \\ 1 & \text{if} \quad x \leq 3. \end{cases}$$

If the decision maker chooses the values of α and β as 0.7 and 0.2, respectively, the corresponding objective functions are derived as follows:

$$\begin{aligned} Z_{L1} &= 7.4x_{11} + 8.4x_{12} + 6.7x_{13} + 1.7x_{14} + 4.5x_{21} + 5.4x_{22} \\ &\quad + 3.6x_{23} + 6.7x_{24} + 2.3x_{31} + 6.7x_{32} + 6.7x_{33} + 4.5x_{34}, \\ Z_{C1} &= 8x_{11} + 8.85x_{12} + 7x_{13} + 2x_{14} + 5x_{21} + 6x_{22} \\ &\quad + 3.95x_{23} + 7x_{24} + 3x_{31} + 7x_{32} + 7x_{33} + 5x_{34}, \\ Z_{R1} &= 8.6x_{11} + 9.3x_{12} + 7.3x_{13} + 2.3x_{14} + 5.5x_{21} + 6.6x_{22} \\ &\quad + 4.3x_{23} + 7.3x_{24} + 3.3x_{31} + 7.3x_{32} + 7.3x_{33} + 5.5x_{34}, \\ Z_{L2} &= 1.7x_{11} + 8.4x_{12} + 7.4x_{13} + .85x_{14} + 3.6x_{21} + 2.7x_{22} \\ &\quad + 5.4x_{23} + 6.7x_{24} + 4.5x_{31} + 1.7x_{32} + 7.4x_{33} + 1.7x_{34}, \\ Z_{C2} &= 2x_{11} + 8.95x_{12} + 8x_{13} + x_{14} + 3.95x_{21} + 3x_{22} \\ &\quad + 6x_{23} + 7x_{24} + 5x_{31} + 2x_{32} + 8x_{33} + 2x_{34}, \\ Z_{R2} &= 2.3x_{11} + 9.3x_{12} + 8.6x_{13} + 1.15x_{14} + 4.3x_{21} \\ &\quad + 3.3x_{22} + 6.6x_{23} + 7.3x_{24} + 5.5x_{31} + 2.3x_{32} \\ &\quad + 8.6x_{33} + 2.3x_{34}, \\ Z_{L3} &= 1.7x_{11} + 3.6x_{12} + 6.7x_{13} + 2.7x_{14} + 5.4x_{21} + 3.6x_{22} \\ &\quad + 7.4x_{23} + 3.6x_{24} + 7.4x_{31} + 1.7x_{32} + 4.5x_{33} + .85x_{34}, \\ Z_{C3} &= 2x_{11} + 3.95x_{12} + 7x_{13} + 3x_{14} + 6x_{21} + 3.95x_{22} \\ &\quad + 8x_{23} + 3.95x_{24} + 8x_{31} + 2x_{32} + 5x_{33} + x_{34}, \\ Z_{R3} &= 2.3x_{11} + 4.3x_{12} + 7.3x_{13} + 3.3x_{14} + 6.6x_{21} + 4.3x_{22} \\ &\quad + 8.6x_{23} + 4.3x_{24} + 8.6x_{31} + 2.3x_{32} + 5.5x_{33} + 1.15x_{34}. \end{aligned}$$

Also, the supply and the demand constraints are calculated as follows:

$$\begin{aligned} & x_{11} + x_{12} + x_{13} + x_{14} \leq 9.5, \\ & x_{21} + x_{22} + x_{23} + x_{24} \leq 12.2, \\ & x_{31} + x_{32} + x_{33} + x_{34} \leq 15.2, \\ & x_{11} + x_{21} + x_{31} \geq 10.5, \\ & x_{12} + x_{22} + x_{32} \geq 7.8, \\ & x_{13} + x_{23} + x_{33} \geq 7.5, \\ & x_{14} + x_{24} + 34 \geq 5.8. \end{aligned}$$

Finally, the proposed model of the application example is transformed into an interval optimization in triplet as follows.

Model 7

$$\begin{aligned} \text{minimize} \quad & [Z_{Lk}, Z_{Ck}, Z_{Rk}] \quad (k = 1, 2, 3) \\ \text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} \leq 9.5, \\ & x_{21} + x_{22} + x_{23} + x_{24} \leq 12.2, \\ & x_{31} + x_{32} + x_{33} + x_{34} \leq 15.2, \\ & x_{11} + x_{21} + x_{31} \geq 10.5, \\ & x_{12} + x_{22} + x_{32} \geq 7.8, \\ & x_{13} + x_{23} + x_{33} \geq 7.5, \\ & x_{14} + x_{24} + 34 \geq 5.8, \\ & x_{ij} \geq 0 \quad \forall \quad i, j. \end{aligned}$$

7.1 *Results and discussion*

We apply two approaches to solve Model 7 with the help of LINGO software. The IF approach provides the optimal (compromise) solution with the degree of membership as 0.5324982 and the degree of non-membership as 0.4675018; the solution is $x_{11} = 5.826373$, $x_{14} = 3.673627$, $x_{23} = 7.5, x_{31} = 4.673627$, $x_{32} = 7.8$, $x_{34} = 2.126373$. The solution is derived using GP approach as $x_{11} = 5.843116$, $x_{14} = 3.656884$, $x_{23} = 7.432055$, $x_{24} = 1.093637$, $x_{31} = 4.656884$, $x_{32} = 7.8$, $x_{33} = .06794452$, $x_{34} = 1.049479$. A comparison between the optimal (compromise) solutions calculated from both approaches to each objective function is presented in table 4.

Comparing the results from table 4, we conclude that IFP produces a better result (from Def. 2.10) than the GP approach in our proposed study. One can consider any type of example on TP for efficiency of the proposed study. Both the solutions satisfy Model 2 at (α, β) cut level. The IF costs are obtained from IFP approach as follows:

Table 4. Comparison between the results obtained from two approaches.

Objective function	Intuitionistic fuzzy programming	Goal programming
$\overline{Z}_1$	[150.8078, 162.8360, 174.8642]	[153.5500, 165.3644, 177.1789]
$\overline{Z}_2$	[91.43357, 103.5473, 115.6609]	[97.0050, 109.1178, 121.2307]
$\overline{Z}_3$	[124.9759, 137.7890, 150.6021]	[127.6599, 140.6775, 153.6950]

$$\widetilde{Z_1}^I = (119.908238, 163.210984, 202.76373)\ (94.958238, 163.210984, 229.1269165),$$
$$\widetilde{Z_2}^I = (61.6104405, 103.547254, 145.4840675)\ (42.0604405, 103.547254, 165.0340675),$$
$$\widetilde{Z_3}^I = (95.0785755, 137.789016, 180.4994565)\ (71.3549485, 137.789016, 202.38627).$$

IF costs are calculated from GP approach as follows:

$$\widetilde{Z_1}^I = (120.3789746, 160.4886386, 196.8822752)\ (95.3729236, 160.4886386, 230.5355035),$$
$$\widetilde{Z_2}^I = (66.13991912, 107.0188812, 147.8978432)\ (46.5678406, 107.0188812, 170.3957085),$$
$$\widetilde{Z_3}^I = (96.42844406, 139.6826666, 182.3900706)\ (73.2802723, 139.6826666, 205.845993).$$

8. Conclusion and future study

We have analysed the MOTP in IF environment in our paper. More specifically, the parameters of the MOTP such as the transportation cost, the supply and the demand have been treated as IFNs. Considering the specific cut interval, the IF transportation cost of cost matrix is converted to the interval cost matrix in the proposed problem. Again, using the specific cut interval, the IF supply and the IF demand of the MOTP are reduced to the inequality interval form. The values of α and β are prescribed by the decision maker according to her/his choice in the proposed problem. Each objective function is represented in triplet with left, right and centre interval form, and hence the proposed problem becomes a multi-objective interval optimization problem. To solve the formulated problem, we have constructed the linear membership and the non-membership functions to each parameter. Then the proposed MOTP is solved by two approaches, namely the IFP and the GP. Thereafter, we have derived the optimal (compromise) solutions for each approach with specific instance and the solutions are compared. Finally, we have concluded that the IFP approach produces a better solution compared with GP approach of the proposed problem. Our proposed method is not applicable when the transportation parameters are not IF in nature. Also, our proposed method is not suitable for arbitrary choice of the values of (α, β).

We must emphasize that in relation to this paper, there are other lines of work of absolute relevance and importance that we have not raised because they are outside the objectives initially set; however, in future investigation, one can analyse the MOTP with parameters that are interval-valued IFNs and study the effect of variation in solution of the MOTP. In the same way, possibility of using fuzzy MCDM methods [36] is an interesting line to be explored in forthcoming paper(s). Another scope to consider is the non-linear membership function and non-linear non-membership function like exponential, hyperbolic, etc., instead of linear membership and non-membership functions for solving the MOTP. Besides, it is of upmost importance to think about real world problems in this context (cf. [37–41]), to see that we have problems with large dimensions where it is not possible to apply the algorithms presented here. In this regard, a line of research that we intend to explore in the future is the application of meta-heuristic algorithms to solve such problems. Nature-inspired meta-heuristic algorithms, such as Genetic Algorithms, Ant Colony Optimization, Simulated Annealing, etc., seem more than appropriate to successfully solve these problems, and will be the grounds for research work in the near future.

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