P H Y S I C A L   R E V I E W   L E T T E R S

Effect of Pair Breaking on Mesoscopic Persistent Currents Well above the Superconducting Transition Temperature

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We consider the mesoscopic normal persistent current (PC) in a very low-temperature superconductor with a bare transition temperature T_c^0 much smaller than the Thouless energy E_c. We show that in a rather broad range of pair-breaking strength, $T_c^0 \lesssim \hbar/\tau_s \lesssim E_c$, the transition temperature is renormalized to zero, but the PC is hardly affected. This may provide an explanation for the magnitude of the average PC's in the noble metals, as well as a way to determine their T_c^0's.

Introduction.—The magnitude of the equilibrium averaged persistent currents (PC's) [1,2] in normal metals has been a long-standing puzzle. Experiments [3–5] produce a current larger by at least 2 orders of magnitude than the theoretical prediction for noninteracting electrons [6–8] and seem to indicate that the low-flux response is diamagnetic. The average PC of a diffusive system with interactions was calculated first in this connection [9] in Refs. [10,11]. The resulting PC was found to be much larger than that of a noninteracting system, but nevertheless not large enough to explain the experiments.

Repulsive electron-electron interactions [11] result in a paramagnetic response (at small magnetic fluxes) whose magnitude is smaller than the experiment by about a factor of 5. This disagreement is due to the downward renormalization of the interaction [12,13]. Attractive interactions [10] result in a diamagnetic response, whose magnitude (due to the very low superconducting transition temperature), is again smaller by a factor of order five than the measured one. This is in spite of the renormalization upward of the attractive interaction. Attractive interactions, at low energies, imply (with no pair breaking) a transition into a superconducting state, and the PC of such an interacting system depends on its transition temperature. These temperatures are very low [14] for the noble metals used in the PC experiments—hence the too small predicted values for the PC.

Here we consider attractive interactions. We show that the presence of a very small amount of pair breakers, e.g., magnetic impurities (which seem to be very difficult to avoid in these metals [15]), may change the picture profoundly. Obviously, one may consider other pair breakers, such as a two-level systems [16] or simply a magnetic field [17]. In this Letter we treat specifically the case of magnetic impurities. We find that within a significant range of the pair-breaking strength, the magnetic impurities suppress the transition temperature down to immeasurable values, leaving concomitantly the PC almost unchanged.

The physical reason for this remarkable observation is that the PC is determined by the interaction on the scale of the Thouless energy $E_c = \hbar D/L^2$ (~ 20 mK for a typical experimental system), while the bare transition temperature, T_c^0, is much smaller. (The circumference of the ring is denoted by L and D is the diffusion coefficient.) This gives rise to a rather wide range of pair-breaking strengths, presented here by the spin-scattering time τ_s,

$$T_c^0 \lesssim \hbar/\tau_s \lesssim E_c, \qquad (1)$$

in which the actual transition temperature T_c will drop to zero [18], but the PC will be hardly affected. As a result, it is the bare transition temperature of the system without the magnetic impurities, T_c^0, as opposed to T_c, which dominates the expression for PC, see Fig. 1. We conclude here [19] on the experimental results of Ref. [3]. In order to explain them, it is necessary to assume a T_c^0 in the 1 mK range for copper. Our basic assertion is that this may indeed be the correct order of magnitude of T_c^0 for ideally clean copper, but that it is knocked down to zero or to a very low value by a minute, $\lesssim$ppm, amount of unwanted [15] pair breakers. We emphasize, however, that our result concerning the fundamentally different sensitivities of T_c and PC to pair breaking in the range given by Eq. (1),
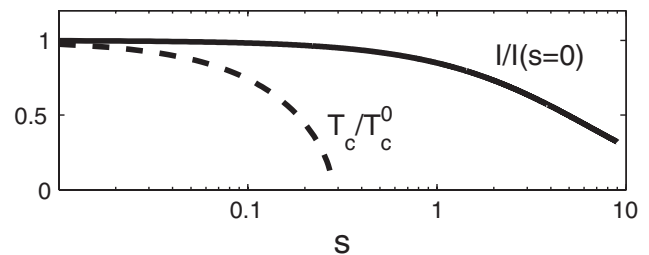

FIG. 1. The first flux harmonic [$m = 1$; see Eq. (2)], in units of $I(s = 0)$, of the PC at $T = E_c$ (full line) and T_c/T_c^0 (dashed line) as functions of the pair-breaking strength, $s = 1/(\pi T_c^0 \tau_s)$, displayed on a logarithmic scale.

remains valid regardless of the situation in specific materials. The Kondo screening of the spins is not considered here. Other effects of magnetic impurities have previously been considered in Ref. [20].

Results.—The expression we obtained for the PC in a diffusive ring with magnetic impurities can be expressed as a sum over the harmonics of the magnetic flux through the ring ϕ, in units of the flux quantum h/e,

$$I = -8eE_c \sum_{m=1}^{\infty} \frac{\sin(4\pi m\phi)}{m^2}$$

$$\times \sum_{\nu} \int_0^{\infty} dx \frac{x\sin(2\pi x)\Psi'(F(x,\nu))}{\ln(T/T_c^0) + \Psi(F(x,\nu)) - \Psi(\frac{1}{2})},$$

$$F(x,\nu) = \frac{1}{2} + \frac{|\nu| + 2/\tau_s}{4\pi T} + \frac{\pi E_c x^2}{m^2 T}, \tag{2}$$

(using $\hbar = 1$). Here ν denotes the bosonic Matsubara frequency [21], Ψ and Ψ' are the digamma function and its derivative, and T is the temperature. Our expression (2) generalizes the result of Ref. [10] for the case where spin-scattering is present: the Matsubara frequency $|\nu|$ is shifted by $2/\tau_s$. However, the superconducting transition temperature (which appears formally in the denominator of the integrand) is not the one modified by the pair breakers, but retains its bare (magnetic impurities free) value. Interestingly enough, it follows that by measuring the PC one may determine T_c^0 (which would be directly measurable only if all low-temperature pair breaking could be eliminated).

In Fig. 2 the PC is plotted using Eq. (2). At the critical pair-breaking time $1/\tau_s \simeq T_c^0$, corresponding to $s = 1/\pi\tau_s T_c^0 \simeq 1/\pi$, the transition temperature vanishes [18], while the PC is hardly affected. The measured PC in the copper samples of Ref. [3] is $I(T \lesssim E_c) \simeq -eE_c$. The curve with $s = 1$ in the upper panel, taken with $T_c^0 = 1.5$ mK and $E_c = 15$ mK (the value for the samples of Ref. [3]) gives a PC lower by only 25%. A better fit is possible by changing the parameters somewhat, but we do not regard this as crucial at the present stage. Likewise, we can qualitatively explain the result of Ref. [4]. The high frequency results of Ref. [5] require a separate discussion [5]. The PC is reduced significantly once $1/\tau_s \gtrsim E_c$, or $L_s \equiv \sqrt{D\tau_s} \lesssim L$. For $T_c^0/E_c = 0.1(0.01)$, the condition for $E_c\tau_s \sim 1$ is $s = 10(100)$.

Derivation.—For completeness, we outline below the derivation of the PC in the presence of magnetic scattering [19]. The PC, Eq. (2), is obtained by differentiating the free energy with respect to the flux. Our system is described by the Hamiltonian [18]

$$\mathcal{H} = \int d\mathbf{r}\Big(\psi_\alpha^\dagger(\mathbf{r})[(\mathcal{H}_0 + u_1(\mathbf{r}))\delta_{\alpha\gamma} + u_2(\mathbf{r})\mathbf{S}\cdot\boldsymbol{\sigma}^{\alpha\gamma}]\psi_\gamma(\mathbf{r})$$

$$- \frac{g}{2}\psi_\alpha^\dagger(\mathbf{r})\psi_\gamma^\dagger(\mathbf{r})\psi_\gamma(\mathbf{r})\psi_\alpha(\mathbf{r})\Big), \tag{3}$$

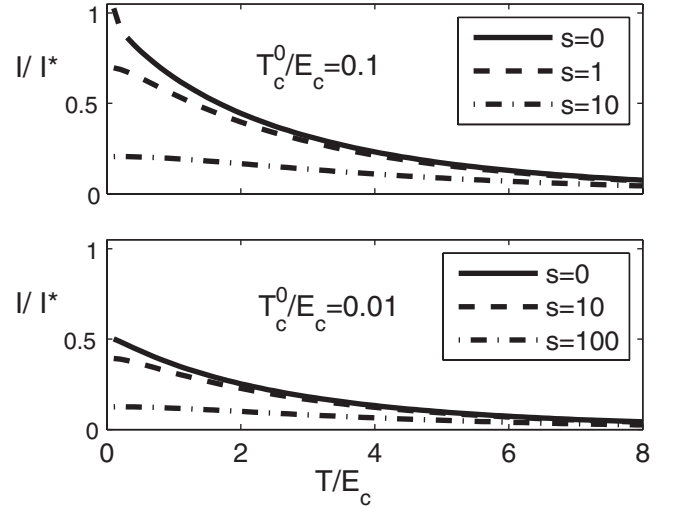in which the last term is the attractive interaction, of

FIG. 2.   The first flux harmonic of the PC in units of $I^* = -eE_c$ as a function of the temperature, for two values of T_c^0/E_c and several values of s. Keeping, at $T \lesssim E_c$, up to the 100 lowest values of $|\nu|$, was necessary for convergence. Note that the $s = 0$ curve in the upper panel is valid only for $T/T_c \gtrsim 1 + $ Gi, where Gi is the Ginzburg parameter (Gi ~ 0.1 for the samples of Ref. [3]).

coupling g. The spin components are α and γ, $\boldsymbol{\sigma}$ is the vector of the Pauli matrices, and $\mathcal{H}_0 = (-i\boldsymbol{\nabla} - e\mathbf{A})^2/2m - \mu$ (μ is the chemical potential and $\mathbf{A}$ is the vector potential describing the flux through the ring). The scattering, both nonmagnetic and magnetic, is assumed to result from N_i pointlike impurities, such that

$$u_1(\mathbf{r}) + u_2(\mathbf{r})\mathbf{S}\cdot\boldsymbol{\sigma} \equiv \sum_{i=1}^{N_i}\Big(\delta(\mathbf{r} - \mathbf{R}_i) - \frac{1}{V}\Big)(u_1 + u_2\mathbf{S}_{\mathbf{R}_i}\cdot\boldsymbol{\sigma}), \tag{4}$$

where V is the system volume. In averaging over the impurity disorder one assumes that the impurity locations, $\mathbf{R}_i$, are random, and so are their classical spins, such that $\langle\mathbf{S}_{\mathbf{R}_i}\rangle = 0$, and $\langle\mathbf{S}_{\mathbf{R}_i}\cdot\mathbf{S}_{\mathbf{R}_j}\rangle = \delta_{ij}S(S+1)$.

The partition function, $\mathcal{Z}$, is calculated by the method of Feynman path integrals [22], combined with the Grassman algebra of many-body fermionic coherent states in terms of the variables $\psi_\alpha(\mathbf{r},\tau)$ ($\bar\psi_\alpha(\mathbf{r},\tau)$). Introducing the bosonic fields $\Delta(\mathbf{r},\tau)$ via the Hubbard-Stratonovich transformation leads to the partition function $\mathcal{Z} = \int D(\psi(\mathbf{r},\tau), \bar\psi(\mathbf{r},\tau))D(\Delta(\mathbf{r},\tau), \Delta^*(\mathbf{r},\tau))e^{-\mathcal{S}}$ with

$$\mathcal{S} = \int d\mathbf{r}\int_0^\beta d\tau\Big(\frac{|\Delta(\mathbf{r},\tau)|^2}{g} - \frac{1}{2}\bar\Psi(\mathbf{r},\tau)G^{-1}_{\mathbf{r},\mathbf{r};\tau,\tau}\Psi(\mathbf{r},\tau)\Big), \tag{5}$$

where $\bar\Psi = (\bar\psi_\uparrow, \bar\psi_\downarrow, \psi_\uparrow, \psi_\downarrow)$. The inverse Green function G^{-1} (at equal positions $\mathbf{r}$ and equal imaginary times τ) is given by

$$G^{-1}_{\mathbf{r}=\mathbf{r}';\tau=\tau'} = \begin{bmatrix} -\partial_\tau - h^\phi_\uparrow & -2u_2 S_- & 0 & \Delta \\ -2u_2 S_+ & -\partial_\tau - h^\phi_\downarrow & -\Delta & 0 \\ 0 & -\Delta^* & -\partial_\tau + h^{-\phi}_\uparrow & 2u_2 S_+ \\ \Delta^* & 0 & 2u_2 S_- & -\partial_\tau + h^{-\phi}_\downarrow \end{bmatrix} \equiv \begin{bmatrix} \hat{G}^{-1}_p & \hat{\Delta} \\ \hat{\Delta}^\dagger & \hat{G}^{-1}_h \end{bmatrix}, \quad (6)$$

where $h^{\pm\phi}_\alpha = \mathcal{H}_0(\pm\mathbf{A}) + u_1 + \text{sgn}(\alpha)S_z u_2$, and $S_\pm = (S_x \pm iS_y)/2$.

The integration over the fermionic part of the action (5) yields

$$Z = \int D(\Delta(\mathbf{r}, \tau), \Delta^*(\mathbf{r}, \tau)) \exp\left(\frac{1}{2}\text{Tr}\ln(\beta G^{-1}) - \int d\mathbf{r} \int_0^\beta d\tau \frac{|\Delta(\mathbf{r}, \tau)|^2}{g}\right). \quad (7)$$

In order to treat the boson fields Δ, we expand $\text{Tr}\ln(\beta G^{-1})$ up to second order in Δ. This expansion is valid for temperatures well above the transition temperature, and, strictly speaking, above the Ginzburg critical region. The zeroth order is omitted as it leads to the tiny magnitude PC of noninteracting, grand-canonical, normal metal rings [6]. The result in Fourier space reads (the dependence on the magnetic flux is specified below)

$$\text{Tr}\ln(\beta G^{-1})|^{2\text{nd}} = -\sum_{\mathbf{q}_1,\mathbf{q}_2,\nu}\sum_{\mathbf{k}_1,\mathbf{k}_2,\omega} \text{Tr}[\hat{G}_p(\mathbf{k}_1 + \mathbf{q}_1, \mathbf{k}_2 + \mathbf{q}_2, \omega + \nu)\hat{\Delta}(\mathbf{q}_2, \nu)\hat{G}_h(\mathbf{k}_2, \mathbf{k}_1, -\omega)\hat{\Delta}^\dagger(\mathbf{q}_1, \nu)]. \quad (8)$$

The resulting expression for the partition function may be simplified considerably. First, the terms that survive the disorder-average in the sum of Eq. (8) are those for which [23] $\mathbf{q}_1 = \mathbf{q}_2$. Second, the particle and the hole Green functions, $\hat{G}_p$ and $\hat{G}_h$, [see Eq. (6)] are related,

$$\hat{G}_h(\mathbf{k}, \mathbf{k}'; \omega) = -\hat{G}^t_p(-\mathbf{k}, -\mathbf{k}', \omega), \quad (9)$$

where the superscript t denotes the transposed. Carrying out the integration in Eq. (7),

$$Z = \prod_{\mathbf{q},\nu} \mathcal{N}(0)\left(\frac{V}{g} - T\Pi(\mathbf{q}, \nu)\right)^{-1}, \quad (10)$$

where $\mathcal{N}(0)$ denotes the extensive density of states at the Fermi level. The polarization is

$$\Pi(\mathbf{q}, \nu) = \frac{1}{2}\sum_\omega \varepsilon_{\alpha\gamma}K_{\omega\alpha\gamma}(\mathbf{q}, \nu) \quad (11)$$

with

$$K_{\omega\alpha\gamma}(\mathbf{q}, \nu) = \sum_{\mathbf{k}_1,\mathbf{k}_2} \langle G_{\alpha\alpha'}(\mathbf{k}_1 + \mathbf{q}, \mathbf{k}_2 + \mathbf{q}, \omega + \nu)$$
$$\times \varepsilon_{\alpha'\gamma'}G_{\gamma\gamma'}(-\mathbf{k}_2, -\mathbf{k}_1, -\omega)\rangle. \quad (12)$$

Here ε is the antisymmetric tensor, $\varepsilon_{\alpha\alpha} = 0$, and $\varepsilon_{\uparrow\downarrow} = -\varepsilon_{\downarrow\uparrow} = 1$, and G denotes the particle Green function.

In Ref. [18] $K(0, 0)$ was calculated using a Dyson equation. We generalize their calculation to obtain $K(\mathbf{q}, \nu)$ and, consequently, the polarization becomes [19]

$$\frac{T}{\mathcal{N}(0)}\Pi(\mathbf{q}, \nu) = \Psi\left(\frac{1}{2} + \frac{\omega_D}{2\pi T} + \frac{|\nu| + D\mathbf{q}^2}{4\pi T}\right)$$
$$- \Psi\left(\frac{1}{2} + \frac{D\mathbf{q}^2 + |\nu| + 2/\tau_s}{4\pi T}\right). \quad (13)$$

Here ω_D is the cutoff frequency on the attractive interaction, and the pair-breaking time τ_s is given by

$$\frac{1}{\tau_s} = 2\pi\mathcal{N}(0)N_i S(S + 1)u_2^2. \quad (14)$$

The transition temperature of the system in the absence of pair breakers, T_c^0, is obtained from the $\mathbf{q} = 0$, $\nu = 0$ pole of Z, upon setting $1/\tau_s = 0$,

$$\frac{V}{g\mathcal{N}(0)} = \Psi\left(\frac{1}{2} + \frac{\omega_D}{2\pi T_c^0}\right) - \Psi\left(\frac{1}{2}\right). \quad (15)$$

(Note that the same procedure in the presence of the pair breaking reproduces the decrease in the transition temperature T_c, as found in Ref. [18].) Since $\omega_D \gg T_c^0, T$ we may use the asymptotic expansion of the digamma function. In this way we obtain

$$Z = \prod_{\mathbf{q},\nu}\left[\ln\left(\frac{T}{T_c^0}\right) + \Psi\left(\frac{1}{2} + \frac{D\mathbf{q}^2 + |\nu| + 2/\tau_s}{4\pi T}\right) - \Psi\left(\frac{1}{2}\right)\right]^{-1}. \quad (16)$$

Finally, the PC is given by $I = (e/h)\partial T \ln Z/\partial\phi$. In our ring geometry, the flux enters the longitudinal component, $q_\parallel$, of the vector $\mathbf{q}$ as

$$q_\parallel = \frac{2\pi}{L}(n + 2\phi), \quad (17)$$

where n is an integer. Only the zero transverse momentum contributes significantly to the current. Our result (2) is obtained upon inserting Eq. (16) into the definition of the current and employing the Poisson summation formula. It then follows from Eq. (2) that values of τ_s which are detrimental to T_c, may hardly affect the PC (see Fig. 1).

We conclude by further explaining the physical argument behind our result. Very roughly, the renormalization of the dimensionless attractive interaction λ (>0) from a higher frequency scale $\omega_>$ to a lower one, $\omega_<$, is given by $\lambda(\omega_<) = \frac{\lambda(\omega_>)}{1-\lambda(\omega_>)\ln(\frac{\omega_>}{\omega_<})}$. At T_c^0 and $1/\tau_s = 0$, the attractive

interaction should diverge. Using this to eliminate $\lambda(\omega_D)$ ($\equiv gN(0)/V$), we obtain that for $T_c^0 \lesssim \omega \ll \omega_D$, $\lambda(\omega) \curvearrowright 1/\ln(\omega/T_c^0)$, which around the Thouless scale, is close to the value found in Ref. [10]. The pair breaking stops the renormalization at $1/\tau_s$, but does not significantly change the interaction on the much larger scale of E_c. Our prediction can also be tested with very small rings made of known low T_c superconductors.

We point out that the mechanism suggested by Kravtsov and Altshuler [24], relating extrinsic dephasing to an enhanced PC, is different than ours, since it relies on the rectification of the noise.

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