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New Complex Hyperbolic Structures to the Lonngren-Wave Equation by Using Sine-Gordon Expansion Method

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Abstract

In this paper, a powerful sine-Gordon expansion method (SGEM) with aid of a computational program is used in constructing a new hyperbolic function solutions to one of the popular nonlinear evolution equations that arises in the field of mathematical physics, namely; longren-wave equation. We also give the 3D and 2D graphics of all the obtained solutions which are explaining new properties of model considered in this paper. Finally, we submit a comprehensive conclusion at the end of this paper.

Keywords: The sine-Gordon expansion method; the longren-wave equation; hyperbolic function solutions.
PACS codes: 02.60.Lj; 02.70.Wz; 02.90.+p; 03.65.-w.

1 Introduction

Over some decades, the field of nonlinear evolution equations (NEEs) has attracted the attention of many researchers. NEEs are broadly used to describe problems in science, engineering and mathematical physics such as fluid dynamics, plasma physics, hydro magnetic waves, optic fibers, solid state physics and many others. NEEs can also be used to describe the propagation of a nonlinear dispersive waves in inhomogeneous media [1,2]. It has become an important bottom-line to find the analytical solutions to these types of equations. Several methods for finding the solutions of various NEEs have been proposed and/or improved by many scholars [3-71].

The aim of this paper was to apply the sine-Gordon expansion method (SGEM) to find a new solutions to the Lonngren-wave equation [14].

$$(u_{xx} - \alpha u + \beta u^2)_{tt} + u_{xx} = 0, \tag{1}$$

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where α and β are real constants. The equation describes the electric signals in telegraph lines on the basis of the tunnel diode [15,16]. The Lonngren-wave equation was used as an example by Akcagil and Aydemir [14] to show the existence of strong connection between the $(\frac{G^{'}}{G})$-expansion method and the modified extended tanh method.

SGEM is a method for solving different nonlinear partial differential equations that is developed based on wave transformation and te sine-Gordon expansion method [17]. A new hyperbolic function solutions to the Davey-Stewartson equation with power-law nonlinearity was obtained in [18] by using SGEM. With the aid of symbolic computation, a new transformation was developed using the general sine-Gordon travelling wave reduction equation and a generalized transformation to obtain the solutions of various types of nonlinear differential equations [19]. A considerable investigation has been implemented by Yan [20] to sine-Gordon-type equations where the equations are systematically solved by using the Jacobi elliptic function expansion method.

The remaining parts of this paper are organized as follows: In Section 2, we discuss the general facts of the SGEM. In Section 3, we apply the SGEM to the Lonngren-wave equation given in Eq. (1). Section 4 is about results, discussion and some remarks. We finally, give the conclusion of this paper in Section 5.

2 General Facts of the SGEM

In this section we discuss the general facts of SGEM.

Consider the following sine-Gordon equation [17,21,22]:

$$u_{xx} - u_{tt} = m^2 sin(u), \tag{2}$$

where $u = u(x,\, t)$ and m is a real constant.

Applying the wave transformation $u = u(x,\, t) = U(\xi),\ \xi = \mu(x - ct)$ to Eq. (2), yields the following nonlinear ordinary differential equation (NODE):

$$U^{''} = \frac{m^2}{\mu^2(1-c^2)} sin(U), \tag{3}$$

where $U = U(\xi)$, ξ is the amplitude of the travelling wave and c is the velocity of the travelling wave. Reconsidering Eq. (3), we can write its full simplification as:

$$\left[\left(\frac{U}{2}\right)^{'}\right]^2 = \frac{m^2}{\mu^2(1-c^2)} sin^2\left(\frac{U}{2}\right) + K, \tag{4}$$

where K is the integration constant.

Substituting $K = 0$, $w(\xi) = \frac{U}{2}$ and $a^2 = \frac{m^2}{\mu^2(1-c^2)}$ in Eq. (4), gives:

$$w^{'} = asin(w), \tag{5}$$

Putting $a = 1$ in Eq. (5), we have:

$$w^{'} = sin(w), \tag{6}$$

Equation (6) is variables separable equation, we obtain the following two significant equations from solving it:

$$sin(w) = sin(w(\xi)) = \left.\frac{2pe^{\xi}}{p^2e^{2\xi}+1}\right|_{p=1} = sech(\xi), \tag{7}$$

$$cos(w) = cos(w(\xi)) = \frac{p^2e^{2\xi}-1}{p^2e^{2\xi}+1}\bigg|_{p=1} = tanh(\xi), \tag{8}$$

where p is the integral constant.

For the solution of the following nonlinear partial differential equation;

$$P(u,\, u_x,\, u_t,\, u_{xx},\, u_{tt},\, u_{xt}\ldots), \tag{9}$$

we consider,

$$U(\xi) = \sum_{i=1}^{n} tanh^{i-1}(\xi)\left[B_i sech(\xi) + A_i tanh(\xi)\right] + A_0, \tag{10}$$

Equation (10) can be rewritten according to Eqs. (7) and (8) as follows:

$$U(w) = \sum_{i=1}^{n} cos^{i-1}(w)\left[B_i sin(w) + A_i cos(w)\right] + A_0. \tag{11}$$

We determine the value n under the terms of NODE by the balance principle. Letting the coefficients of $sin^i(w)cos^j(w)$ to be all zero, yields a system of equations. Solving this system by using Wolfram Mathematica 9 gives the values of A_i, B_i, μ and c. Finally, substituting the values of A_i, B_i, μ and c in Eq. (10), we obtain the new travelling wave solutions to Eq. (9)

3 Applications

Consider the Lonngren-wave equation given in Eq. (1);

Applying the transformation $u = u(x,\, t) = U(\xi)$, $\xi = \mu(x - ct)$ to Eq. (1), we have:

$$c^2\mu^2U'' + (1 - c^2\alpha)U + c^2\beta U^2 = 0 \tag{12}$$

We obtain $n = 2$, by applying the balance principle on Eq. (12). With $n = 2$ and using Eq. (11), yields:

$$U(w) = B_1 sin(w) + A_1 cos(w) + B_2 cos(w) sin(w) + A_2 cos^2(w) + A_0. \tag{13}$$

Differentiating Eq. (13) twice, gives:

$$\begin{aligned} U''(w) = {} & B_1 cos^2(w)sin(w) - B_1 sin^3(w) - 2A_1 sin^2(w)cos(w) + B_2 cos^3(w)sin(w) \\ & -5B_2 sin^3(w)cos(w) - 4A_2 cos^2(w)sin^w(w) + 2A_2 sin^4(w), \end{aligned} \tag{14}$$

Putting Eqs. (13) and (14) in Eq. (12), yields;

$$\begin{aligned} & A_0 - c^2\alpha A_0 + c^2\beta A_0^2 + A_1 cos(w) - c^2\alpha A_1 cos(w) - 2c^2\mu^2A_1 sin^2(w)cos(w) + 2c^2\beta A_0A_1 cos(w) + \\ & c^2\beta A_1^2 cos^2(w) + A_2 cos^2(w) - c^2\alpha A_2 cos^2(w) \\ & -4c^2\mu^2A_2 cos^2(w)sin^2(w) + 2c^2\mu^2A_2 sin^4(w) + 2c^2\beta A_0A_2 cos^2(w) + \\ & 2c^2\beta A_1A_2 cos^3(w) + c^2\beta A_2^2 cos^4(w) + B_1 sin(w) - c^2\alpha B_1 sin(w) + \\ & c^2\mu^2B_1 cos^2(w)sin(w) - c^2\mu^2B_1 sin^3(w) + 2c^2\beta A_0B_1 sin(w) + \\ & 2c^2\beta A_1B_1 cos(w)sin(w) + 2c^2\beta A_2B_1 cos^2(w)sin(w) + c^2\beta B_1^2 sin^2(w) \\ & + B_2 cos(w)sin(w) - c^2\alpha B_2 cos(w)sin(w) + c^2\mu^2B_2 cos^3(w)sin(w) - \\ & 5c^2\mu^2B_2 sin^3(w)cos(w) + 2c^2\beta A_0B_2 cos(w)sin(w) + 2c^2\beta A_1B_2 cos^2 sin(w) \\ & + 2c^2\beta A_2B_2 cos^3 sin(w) + 2c^2\beta B_1B_2 sin^2(w)cos(w) + c^2\beta B_2^2 cos^2(w)sin^2(w) = 0. \end{aligned}$$

We collect a set of algebraic equations by equating each summation of the coefficients of the trigonometric terms of the same power to zero in the abovementioned equation. We solve the set of generated to obtained the values of the coefficients. To get the new solitary solutions, $u(x,\ t)$ to Eq. (1), we substitute in each case the obtained results of the coefficients into Eq. (10) along with $n = 2$.

Case-1: When we consider following coefficients:

$$A_0 = \frac{2\mu^2}{\beta}, A_1 = 0, B_1 = 0, A_2 = -\frac{6\mu^2}{\beta}, B_2 = 0, \alpha = \frac{1}{c^2} - 4\mu^2,$$

these produce new dark solution as:

$$u_1(x,\ t) = \frac{2\mu^2}{\beta}\left(1 - 3tanh[\mu(x - ct)]^2\right). \tag{15}$$

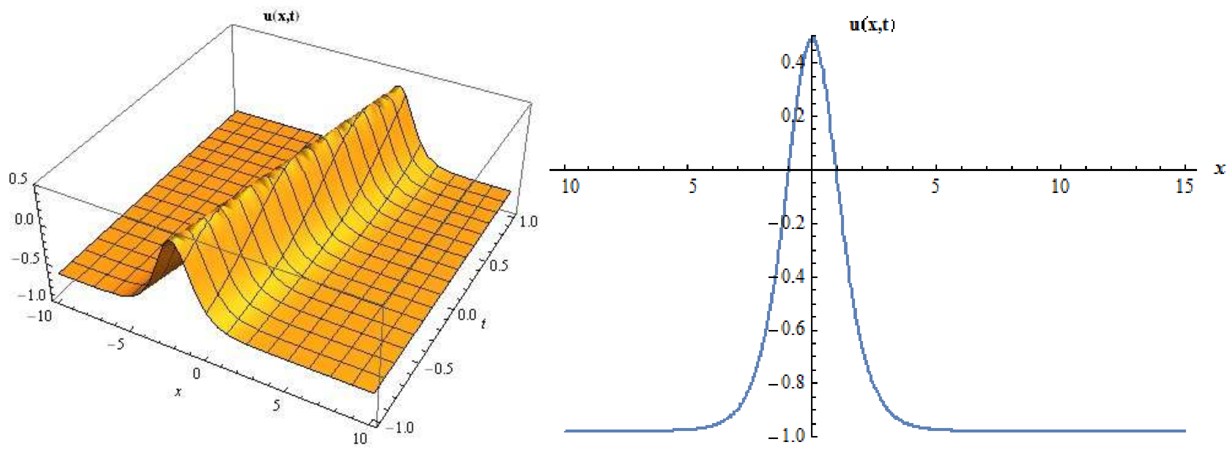

Fig. 1 The 3D and 2D surfaces of Eq. (15).

Case-2: If it is taken as

$$A_0 = \frac{6\mu^2}{\beta}, A_1 = 0, B_1 = 0, A_2 = -\frac{6\mu^2}{\beta}, B_2 = 0, \alpha = \frac{1}{c^2} + 4\mu^2,$$

they produce a new singular solution as:

$$u_2(x,\ t) = \frac{6\mu^2}{\beta} sech[\mu(x - ct)]^2. \tag{16}$$

Case-3: When we take

$$A_0 = \frac{2\mu^2}{\beta}, A_1 = 0, B_1 = 0, A_2 = -\frac{3\mu^2}{\beta}, B_2 = \frac{3i\mu^2}{\beta}, \alpha = \frac{1}{c^2} - \mu^2,$$

they give mixed complex singular solution as:

$$u_3(x,\ t) = \frac{\mu^2}{\beta}\left(-1 + \frac{3i}{i + sinh(\mu(x - ct))}\right). \tag{17}$$

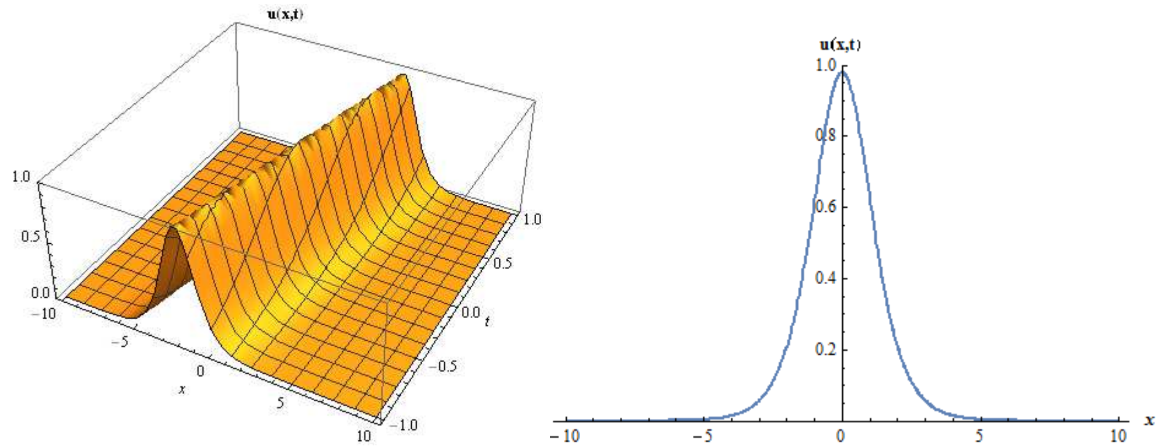

Fig. 2 The 3D and 2D surfaces of Eq. (16).

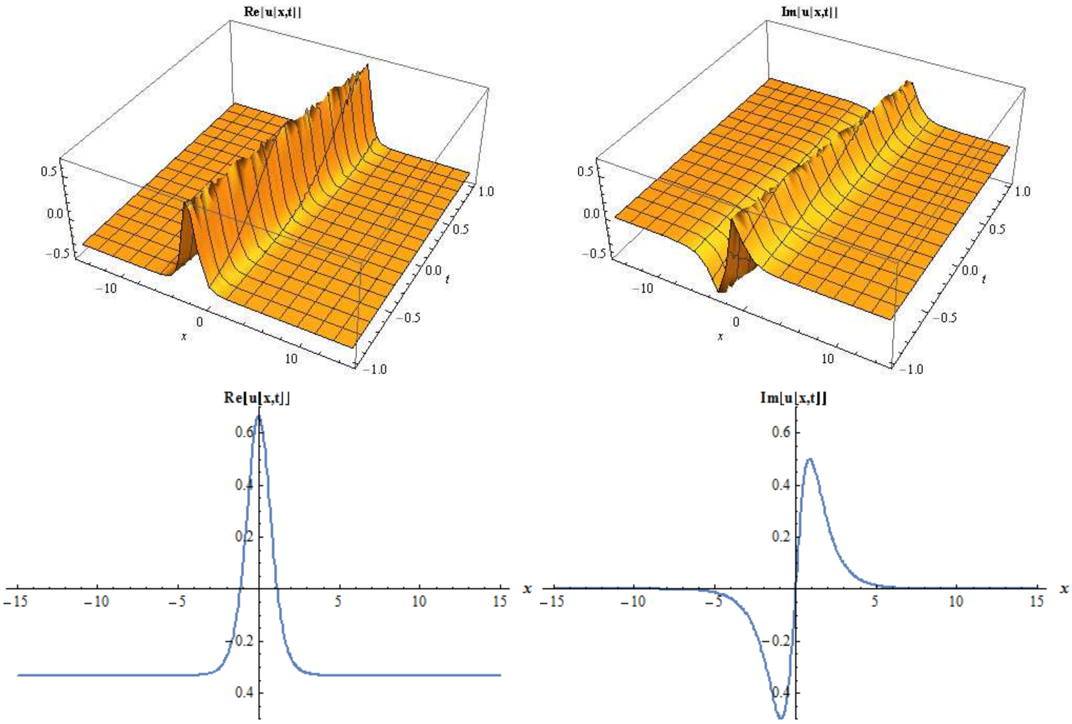

Fig. 3 The 3D and 2D surfaces of Eq. (17).

Case-4:

$$A_0 = \frac{3(c^2\alpha - 1)}{c^2\beta}, A_1 = 0, B_1 = 0, A_2 = \frac{3 - 3c^2\alpha}{c^2\beta}, B_2 = \frac{3i(c^2\alpha - 1)}{c^2\beta}, \mu = -\frac{\sqrt{c^2\alpha - 1}}{c},$$

give mixed complex rational solution as:

$$u_4(x,\, t) = \frac{3i(c^2\alpha - 1)}{c^2\beta\left(i - sinh[(\frac{\sqrt{c^2\alpha-1}}{c})(x - ct)]\right)}. \tag{18}$$

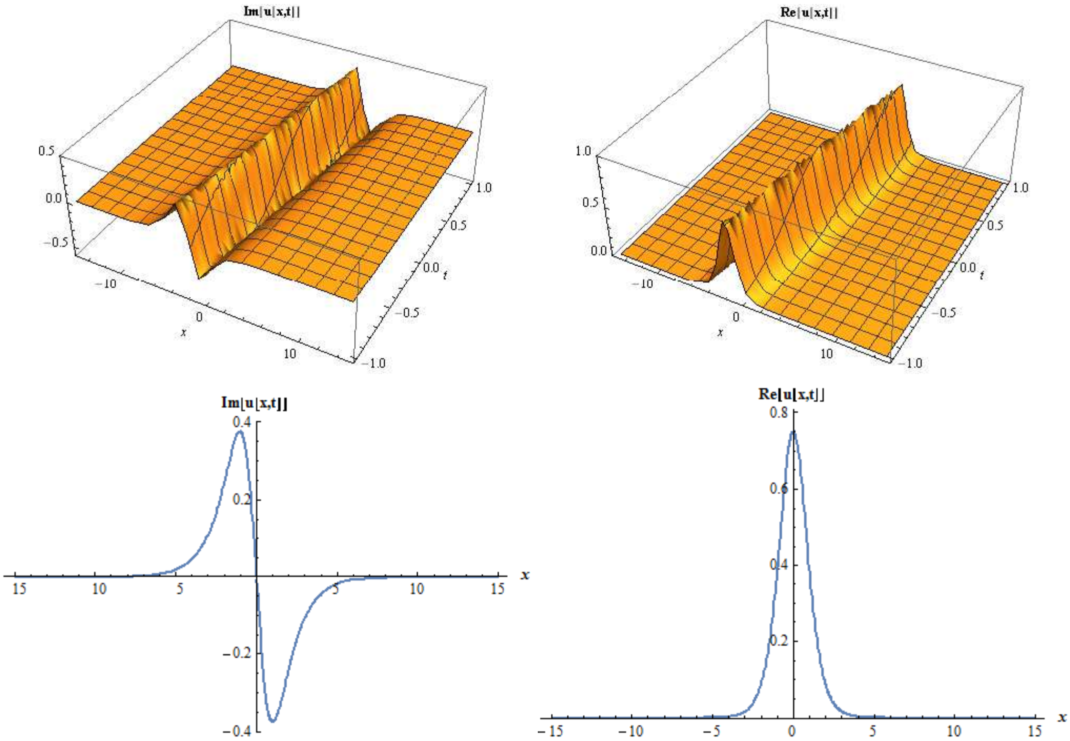

Fig. 4 The 3D and 2D surfaces of Eq. (18).

Case-5:

$$A_0=\frac{2-2c^2\alpha}{c^2\beta}, A_1=0, B_1=0, A_2=\frac{3(c^2\alpha-1)}{c^2\beta}, B_2=-\frac{3i(c^2\alpha-1)}{c^2\beta}, \mu=-\frac{i\sqrt{c^2\alpha-1}}{c},$$

which produces the following trigonometric travelling wave solution as:

$$u_5(x,\, t)=\frac{1}{c^2\beta}\left(c^2\alpha-1+\frac{3(c^2\alpha-1)}{-1+sin[(x-ct)\frac{\sqrt{c^2\alpha-1}}{c}]}\right). \tag{19}$$

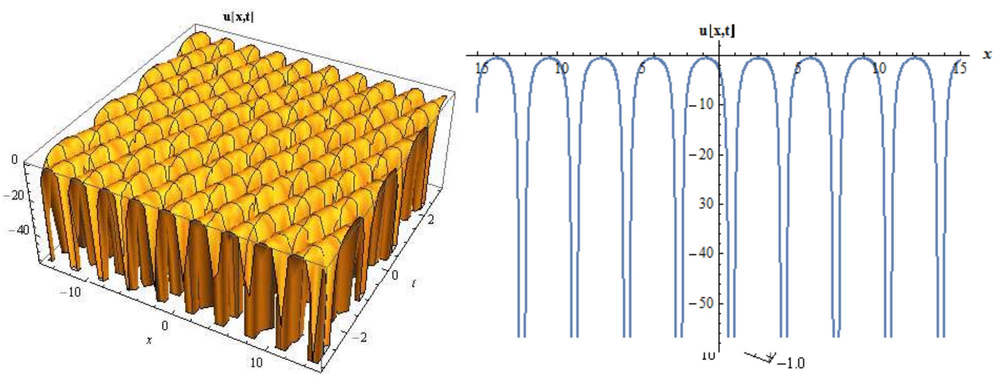

Fig. 5 The 3D and 2D surfaces of Eq. (19).

Case-6:

$$A_0 = \frac{3\mu^2}{\beta}, A_1 = 0, B_1 = 0, A_2 = -\frac{3\mu^2}{\beta}, B_2 = \frac{3i\mu^2}{\beta}, c = -\frac{1}{\sqrt{\alpha - \mu^2}},$$

which introduces the following complex mixed solution as:

$$\begin{aligned} u_7(x,\, t) = \frac{3\mu^2}{\beta} sech\left[\mu\left(x + \frac{t}{\sqrt{\alpha-\mu^2}}\right)\right]\Bigg(sech\left[\mu\left(x + \frac{t}{\sqrt{\alpha-\mu^2}}\right)\right] \\ + itanh\left[\mu\left(x + \frac{t}{\sqrt{\alpha-\mu^2}}\right)\right]\Bigg). \end{aligned} \tag{20}$$

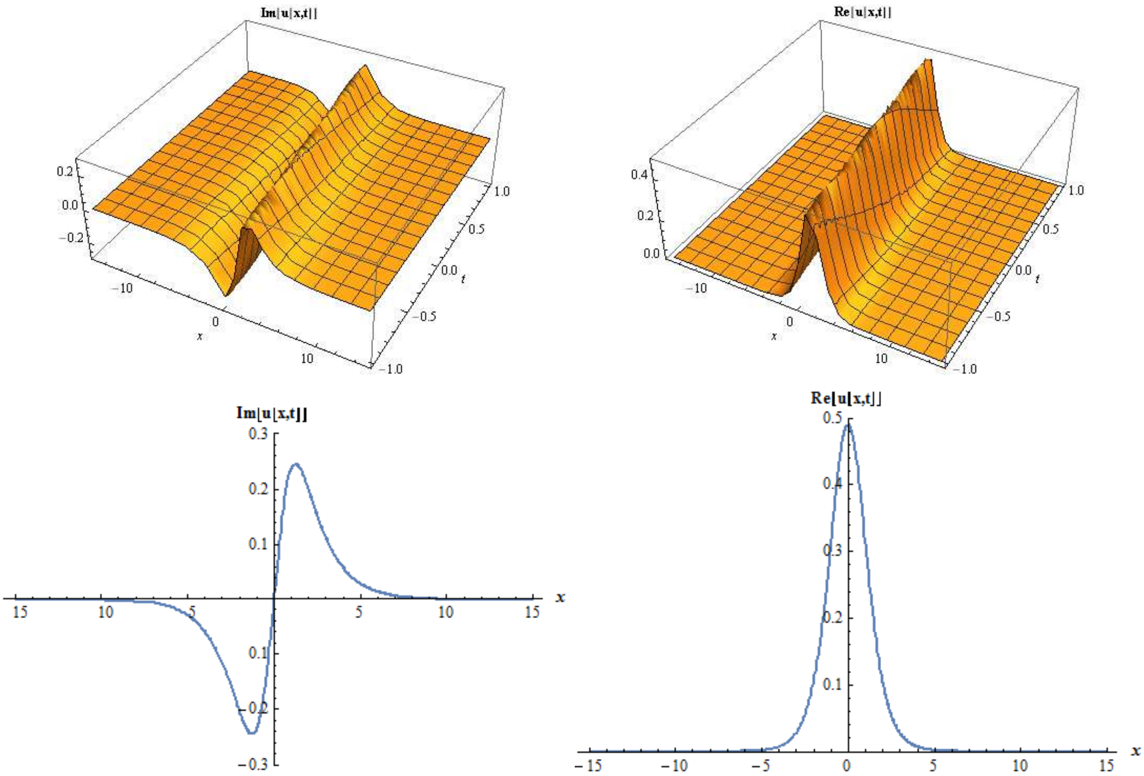

Fig. 6 The 3D and 2D surfaces of Eq. (20).

4 Results and Discussion

The powerful SGEM as one of the prominent methods for obtaining the some new travelling wave solutions to the nonlinear partial differential equations has been used in this paper. This method is based on both important properties of the sine-Gordon equation such as Eqs. (7) and Eq.(8). The SGEM includes trigonometric functions, which will be used later for obtaining novel solutions in Eq.(11). Many new solutions can be obtained by using the properties of these trigonometric functions. This is one of the main properties of SGEM. Therefore, it gives many coefficients to the considered model such as complex, exponential and trigonometric.

5 Conclusions

In this manuscript, by selecting of some of them, we have obtained the same solution, Eq.(15); moreover, we have found some entirely new complex, exponential, dark and hyperbolic solutions to the model considered when we compared the solutions obtained with the help of exp the (G'/G)-expansion method and the modified extended tanh method used in [14]. These solutions are new physical properties of model equation Eq.(1.1). The effectiveness and the simplicity of the method show that its powerful and reliable mathematical tool that can be applied in solving various NEEs. For computational calculations, we have used the packet programs for drawing graphical surfaces in this paper. To the best of our knowledge, the application of SGEM to the Lonngren-wave equation has not been submitted to the literature beforehand.

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