

# *New Computational Paradigms*

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Computational Paradigms

*Changing Conceptions  
of What is Computable*

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 Springer

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## Preface

June 2005 saw the coming together, in Amsterdam, of the first meeting of a new research community, which sought to renew, in the new century, the ground-breaking legacy of Alan Turing. *Computability in Europe* (CiE) originated with a 2003 proposal for EU funding but rapidly developed into a Europe-wide network of over 400 researchers from 17 countries, around 70 institutions, and a number of different research disciplines (mathematics, computer science, physics, biology, philosophy, and logic). This book of invited (and rigorously refereed) articles showcases the diversity, excitement, and scientific innovation of that first meeting, and the powerful multidisciplinary that it injected into computational research.

Many of the contributions to be found here reflect the necessity to deal with computability in the real world—computing on continuous data, biological computing, physical computing, etc.—which has focused attention on new paradigms of computation, based on biological and physical models. This book looks at new developments in the theory and practice of computation from a mathematical and predominantly logical perspective, with topics ranging from classical computability to complexity, biocomputing, and quantum computing. Traditional topics in computability theory are also covered as well as relationships among proof theory, computability and complexity theory, and new paradigms of computation arising from biology and quantum physics and issues related to computability with/on the real numbers. The book is addressed to researchers and graduate students in mathematics, philosophy, and computer science with a special interest in foundational issues. Logicians and theoretical physicists will also benefit from this book.

Since that first conference, CiE has become more than the sum of its parts, reasserting an older tradition of scientific research. This more thoughtful approach is what this 1944 quotation from Einstein<sup>1</sup> seems to refer to:

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<sup>1</sup> A. Einstein to R. A. Thornton, unpublished letter dated Dec. 7, 1944; in Einstein Archive, Hebrew University, Jerusalem.

So many people today—and even professional scientists—seem to me like someone who has seen thousands of trees but has never seen a forest. A knowledge of the historical and philosophical background gives that kind of independence from prejudices of his generation from which most scientists are suffering. This independence created by philosophical insight is—in my opinion—the mark of distinction between a mere artisan or specialist and a real seeker after truth.

There is a parallel between the competitive hyperactive specialism of parts of computer science (and logic) and that of the string theory community that Lee Smolin<sup>2</sup> focuses on in his recent book. He pinpoints:

... a more brash, aggressive, and competitive atmosphere, in which theorists vie to respond quickly to new developments ... and are distrustful of philosophical issues. This style supplanted the more reflective, philosophical style that characterized Einstein and the inventors of quantum theory, and it triumphed as the center of science moved to America and the intellectual focus moved from the exploration of fundamental new theories to their application.

This book embodies what is special about what CiE is trying to do in taking computational research beyond the constraints of “normal science,” while building a cohesive research community around fundamental issues of computability.

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<sup>2</sup> L. Smolin, *The Trouble With Physics: The Rise of String Theory, the Fall of a Science and What Comes Next*, Houghton Mifflin, 2006.



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