New Condition of Stabilization for Continuous Takagi-Sugeno Fuzzy System based on Fuzzy Lyapunov Function

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Abstract— This paper try to give a new stabilization condition of continuous Takagi-Sugeno fuzzy models. Using non-quadratic Lyapunov function, the new condition of stabilization are used in terms of linear matrix inequalities LMIs. To verify the robustness of this new condition, a numeric example is used.

Index Terms: Takagi-Sugeno fuzzy system, Fuzzy Lyapunov Function, Linear Matrix Inequalities LMIs, Parallel Distributed Controller PDC

Introduction

Non linear systems are difficult to describe. Takagi-Sugeno fuzzy model is a multimodel approach very used to modelize non linear sytems by construction with identification of input-output data. Many mechanical systems are modeling with T-S fuzzy system.

In this paper, a new stabilization conditions for continuous Takagi Sugeno fuzzy models based on the use of fuzzy Lyapunov function are discussed. This condition was reformulated into a linear matrix inequality problem (LMIs). [1],[2]

To use the Takagi-Sugeno fuzzy models, we must analyze the stabilization of this system. Two classes of lyapunov functions are used in the literature : quadratic and non quadratic lyapunov functions. Which the second is less conservative then the first [3],[4].

The organization of the paper is as follows. First, T-S fuzzy modeling is discussed. Second we discuss the proposed approach to stabilize a T-S fuzzy system with Parallel Distributed Controller (PDC). Third, simulation results show the robustness of the proposed fuzzy control approach. Finally we make conclusion.

I. System description and preliminaries

The continuous Takagi-Sugeno fuzzy model for a nonlinear system is described as follows:[5]

Si $z_1(t)$ is M_{i1} and…*and* $z_p(t)$ *is* M_{ip} then $\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases}$ **(1)**

Where :

M_{ij}	:	fuzzy set
r	:	nombre of model rules
$u \in \Re^{q\times n}$	:	input vector
$y \in \Re^{q}$	:	output vector
$A_i \in \Re^{n\times n}$	:	states matrices
$B_i \in \Re^{n\times m}$	:	controls matrices

The T-S fuzzy model is written as follow:

$$\begin{cases} \dot{x}(t) = \dfrac{\sum_{i=1}^{r} w_i(z(t))\left(A_i x(t) + B_i u(t)\right)}{\sum_{i=1}^{r} w_i(z(t))} \\ y(t) = \dfrac{\sum_{i=1}^{r} w_i(z(t)) C_i x(t)}{\sum_{i=1}^{r} w_i(z(t))} \end{cases} \quad \textbf{(2)}$$

Consider:

$$\begin{cases} w_i(z(t)) = \prod_{j=1}^{r} M_{ij}(z_j(t)) \\ h_i(z(t)) = \dfrac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))} \qquad i = 1,2,\dots,r \end{cases} \quad \textbf{(3)}$$

r: is the number of model rules

The term $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}

Since $\begin{cases} \sum_{i=1}^{r} w_i(z(t)) > 0 \\ w_i(z(t)) \geq 0 \qquad i = 1, 2, \dots, r \end{cases}$ **(4)**

We have

$$\begin{cases} 0 < h_i(z(t)) < 1 \\ \sum_{i=1}^{r} h_i(z(t)) = 1 \qquad i = 1, 2, \dots, r \end{cases} \quad \textbf{(5)}$$

The final outputs can be also rewritten in the following form

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum_{i=1}^{r} h_i(z(t)) C_i x(t) \end{cases} \quad \textbf{(6)}$$

Consider the following open-loop system:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) A_i x(t) \quad \textbf{(7)}$$

The closed -loop system is given as follow :

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) h_i(z(t)) G_{ii} x(t) + 2\sum_{i=1}^{r} \sum_{i<j} h_i(z(t)) h_j(z(t)) \left\{ \frac{G_{ij} + G_{ji}}{2} \right\} x(t), \quad \textbf{(8)}$$

The objective of this section is to find sufficient conditions for stabilization of Takagi-Sugeno closed loop fuzzy system by using Lyapunov theory.

II. Stabilization with PDC Controller:

Consider the closed loop system is written by the following equation:

$$\dot{x}(t)=\sum_{i=1}^{r}h_i(z(t))h_i(z(t))G_{ii}x(t) \\ +2\sum_{i=1}^{r}\sum_{i\prec j}h_i(z(t))h_j(z(t))\left\{\frac{G_{ij}+G_{ji}}{2}\right\}x(t), \quad (9)$$

Where $G_{ij}=A_i-B_iF_j$, $G_{ii}=A_i-B_iF_i$

The PDC fuzzy controller is represented as follows:

$$u(t)=-\frac{\sum_{i=1}^{r}w_i(z(t))F_ix(t)}{\sum_{i=1}^{r}w_i(z(t))}=-\sum_{i=1}^{r}h_i(z(t))F_ix(t) \quad (10)$$

The design of the PDC back to determine the gains returns. In this section defines the Lyapunov function and sufficient condition for stabilization.

Théorèm 1[6]:

The system of fuzzy Takagi-Sugeno is stable if there exist matrices P_k , $k = 1, 2, \dots, r$ and R, symmetric and positive definite matrices and matrices such that they satisfy the following LMI:

$$P_k+R\succ 0,\quad k\in\{1,\dots,r\} \quad (11)$$

$$P_j+\mu R\geq 0,\quad j=1,2,\dots,r \quad (12)$$

$$P_\phi+\{G_{ii}^T(P_k+\mu R)+(P_k+\mu R)G_{ii}\}\prec 0, i,k\in\{1,\dots,r\} \quad (13)$$

$$\left\{\frac{G_{ij}+G_{ji}}{2}\right\}^T(P_k+\mu R)+(P_k+\mu R)\left\{\frac{G_{ij}+G_{ji}}{2}\right\}\prec 0, \quad (14)$$

for $i,j,k=1,2,\dots,r$ where $i\prec j$

Where

$$G_{ij}=A_i-B_iF_j, G_{ii}=A_i-B_iF_i \\ P_\phi=\sum_{k=1}^{r}\phi_k(P_k+R),\ 0\leq\varepsilon\leq 1 \quad (15)$$

III. Numerical Examples:

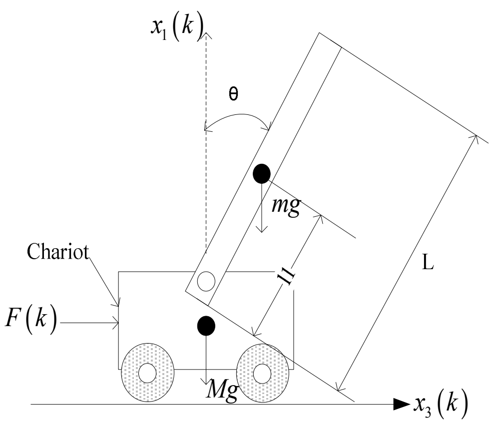

Figure 1 : inverted pendulum

Parameters that characterize the inverted pendulum are:

M	:	the carriage mass,
$F(t)$	:	the force applied to the carriage,
$x_3(t)$	:	moving the carriage,
m	:	the pendulum mass,
$\theta(t)$	:	the angle to the vertical,
$x_1(t)$	:	the lead angle.

Inverted pendulum movements are represented by the following dynamic equations.

$$\begin{cases}\dot{x}_1(t)=x_2(t)\\ \dot{x}_2(t)=\frac{1}{a}(a_{21}x_1(t)+a_{22}x_2(t)+a_{24}x_4(t)+b_2u(t))\\ \dot{x}_3(t)=x_4(t)\\ \dot{x}_4(t)=\frac{1}{a}(a_{41}x_1(t)+a_{42}x_2(t)+a_{44}x_4(t)+b_4u(t))\end{cases} \quad (16)$$

$$\begin{cases}a=(M+m)(J+ml^2)-m^2l^2\cos^2(x_1(t))\\ a_{21}=\frac{1}{a}\frac{\sin(x_1(t))}{x_1(t)}((M+m)m\,g\,l-m^2l^2x_2^2(t)\cos(x_1(t)))\\ a_{22}=-\frac{1}{a}f_1(M+m)\\ a_{24}=\frac{1}{a}f_0\,m\,l\cos(x_1(t))\\ b_2=-\frac{1}{a}m\,l\cos(x_1(t))\\ a_{41}=-\frac{1}{a}\frac{\sin(x_1(t))}{x_1(t)}(m^2gl^2\cos(x_1(t))-(J+ml^2)m\,l\,x_2^2(t))\\ a_{42}=\frac{1}{a}f_1\,m\,l\cos(x_1(t))\\ a_{44}=-\frac{1}{a}f_0(J+ml^2)\\ b_4=J+ml^2\end{cases} \quad (17)$$

These equations can be put in the form of a state space representation:

$$\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ y(t)=Cx(t)\end{cases} \quad (18)$$

where :

$x(t)=\begin{pmatrix}x_1(t)\\x_2(t)\\x_3(t)\\x_4(t)\end{pmatrix}$: the state vector characterizing the system

Where

$x_1(rad)$	:	the lead angle of the pendulum relative to the vertical,
$x_2(rad/s)$	:	the angular velocity of the pendulum
$x_3(m)$	:	the movement of the carriage,
$x_4(m/s)$	:	the speed of movement of the carriage.

Its characteristics matrices are written as follows:

$$A=\begin{pmatrix}0&1&0&0\\a_{21}&a_{22}&0&a_{24}\\0&0&0&1\\a_{41}&a_{42}&0&a_{44}\end{pmatrix}\ B=\begin{pmatrix}0\\b_2\\0\\b_4\end{pmatrix}\ C=\begin{pmatrix}1&0&0&0\\0&0&1&0\end{pmatrix}$$

The matrix A_i et B_i are written in another way:

$$A_i = \frac{1}{a}\begin{pmatrix} 0 & 1 & 0 & 0 \\ a_{21i} & a_{22i} & 0 & a_{24i} \\ 0 & 0 & 0 & 1 \\ a_{41i} & a_{42i} & 0 & a_{44i} \end{pmatrix},\ B_i = \begin{pmatrix} 0 \\ b_{2i} \\ 0 \\ b_{4i} \end{pmatrix}$$

The characteristic parameters of the inverted pendulum is given by the following table

Table 1 : Characteristic parameters of the inverted pendulum

Parameters	Numerical value	Designation
$g(m/s^2)$	9.8	gravitational constant
$M(Kg)$	1.3282	Mass of carriage
$m(Kg)$	0.22	mass of pendulum
$f_0(N/m/s)$	22.915	coefficient of friction of the carriage
$f_1(N/rad/s)$	0.007056	coefficient of friction of pendulum
$l(m)$	0.304	half- length of the pendulum
$J(Kgm^2)$	0.004963	moment of inertia of the pendulum

To avoid getting a big number of rules, simplifications can be made and which lead to a fuzzy model consists of four rules. In fact, a fuzzy model can be obtained accurately representing the equations by using 32 rules. Because we have five nonlinear terms.

$$(\cos^2(x_1(t)), x_2^2(t)\cos(x_1(t)), \cos(x_1(t)), \frac{\sin(x_1(t))}{x_1(t)}, x_2^2(t))$$

The study of the control of the inverted pendulum for angles close to 0, some simplifications are allowed:

$m^2 l^2 \cos^2(x_1(t))$ is negligible $(M+m)(J+ml^2)$.

$m^2 l^2 x_2^2(t)\cos(x_1(t))$ is negligible $(M+m)mgl$.

$(J+ml^2)ml\,x_2^2(t)$ is negligible $m^2 g l^2 \cos(x_1(t))$.

By making these simplifications there is two nonlinear terms $\frac{\sin(x_1(t))}{x_1(t)}, \cos(x_1(t))$ giving a model of the pendulum in 4 fuzzy rules.

The simplified equations model of the pendulum becomes:

$$\begin{cases} a = (M+m)(J+ml^2) \\ a_{21} = \frac{1}{a}\frac{\sin(x_1(t))}{x_1(t)}((M+m)mgl) \\ a_{22} = -\frac{1}{a} f_1 (M+m) \\ a_{24} = \frac{1}{a} f_0\, ml\cos(x_1(t)) \\ b_2 = -\frac{1}{a} ml\cos(x_1(t)) \\ a_{41} = -\frac{1}{a}\frac{\sin(x_1(t))}{x_1(t)}(m^2 g l^2 \cos(x_1(t))) \\ a_{42} = \frac{1}{a} f_1\, ml\cos(x_1(t)) \\ a_{44} = -\frac{1}{a} f_0 (J+ml^2) \\ b_4 = J+ml^2 \end{cases} \quad (19)$$

To deal with the nonlinearity, and delay in the premise of the fuzzy model, we must take into account function $f(x)$, which is bounded. $f_{\min} \le f(x) \le f_{\max}$ for $x \in [-x_0, x_0], x_0 \le \pi$ can be writing in:

$$f(x) = F_1^1.\alpha + F_1^2.\beta \quad (20)$$

Consider the function $f(x) = \cos(x)$

$$\cos(x) = \frac{\cos(x) - \cos(x_0)}{1-\cos(x_0)}.1 + \frac{1-\cos(x)}{1-\cos(x_0)}.\cos(x_0) \quad (21)$$

$$\begin{cases} \alpha = f_{\max} = 1 \\ \beta = f_{\min} = \cos(x_0) \end{cases} \quad (22)$$

$$F_1^1 = \frac{\cos(x)-\cos(x_0)}{1-\cos(x_0)}, \quad F_1^2 = \frac{1-\cos(x)}{1-\cos(x_0)} \quad (23)$$

Consider the function $f(x) = \frac{\sin(x)}{x}$ for all $x \in [-x_0, x_0], x_0 \le \pi$ can write as :

$$\frac{\sin(x)}{x} = \frac{x_0\sin(x) - x\sin(x_0)}{x(x_0-\sin(x_0))}.1 + \frac{x_0(x-\sin(x))}{x(x_0-\sin(x_0))}.\frac{\sin(x_0)}{x_0} \quad (24)$$

Where

$$F_1^1 = \frac{x_0\sin(x) - x\sin(x_0)}{x(x_0-\sin(x_0))}\ F_1^2 = \frac{x_0(x-\sin(x))}{x(x_0-\sin(x_0))} \quad (25)$$

The inverted pendulum is not controllable around $-\frac{\pi}{2}$ and $\frac{\pi}{2}$

So we chose $x1 \in [-88°, 88°]$

$$F_1^1 = \frac{\cos(x_1)-0.0348}{1-0.0348}, F_1^2 = \frac{1-\cos(x_1)}{1-0.0349} \quad (26)$$

$$F_2^1 = \frac{1.5359\sin(x_1) - x_1}{x_1(1.5359-1)}, F_2^2 = \frac{1.5359(x_1-\sin(x_1))}{x_1(1.5359-1)} \quad (27)$$

The weights $h_i(x_1(t)), i = 1,2,3,4$ are obtained from the products F_1^1, F_1^2, F_2^1 et F_2^2 :

$$h_1(x_1) = F_1^1.F_2^1\quad h_3(x_1) = F_1^2.F_2^1$$
$$h_2(x_1) = F_1^1.F_2^2\quad h_4(x_1) = F_1^2.F_2^2 \quad (28)$$

The Inverted pendulum modeling and gives the following matrices:

$$A_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 25.91161 & -0.2789 & 0 & 39.71095 \\ 0 & 0 & 0 & 1 \\ -1.1193 & 0.01222 & 0 & -14.801 \end{pmatrix},\ A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0.294096 & -0.2789 & 0 & 39.71095 \\ 0 & 0 & 0 & 1 \\ -0.012704 & 0.01222 & 0 & -14.801 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 25.91161 & -0.2789 & 0 & 1.3819 \\ 0 & 0 & 0 & 1 \\ 0.038951 & 0.0004252 & 0 & -14.801 \end{pmatrix},\ A_4 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0.294096 & -0.2789 & 0 & 1.3819 \\ 0 & 0 & 0 & 1 \\ 0.0004421 & 0.0004252 & 0 & -14.801 \end{pmatrix},$$

$B_1 = \begin{pmatrix} 0 \\ 1.7078 \\ 0 \\ 0.02529 \end{pmatrix}$, $B_2 = \begin{pmatrix} 0 \\ 0.05943 \\ 0 \\ 0.02529 \end{pmatrix}$, $B_3 = \begin{pmatrix} 0 \\ 1.7078 \\ 0 \\ 0.02529 \end{pmatrix}$ et $B_4 = \begin{pmatrix} 0 \\ 0.05943 \\ 0 \\ 0.02529 \end{pmatrix}$.

$Rule1: if\ x_1\ is\ F_1^1\ then \begin{cases} \dot{x}(t) = A_1 x(t) + B_1 u(t) \\ y(t) = C_1 x(t) \end{cases}$

$Rule2: if\ x_1\ is\ F_1^2\ then \begin{cases} \dot{x}(t) = A_2 x(t) + B_2 u(t) \\ y(t) = C_2 x(t) \end{cases}$

$Rule3: if\ x_1\ is\ F_2^1\ then \begin{cases} \dot{x}(t) = A_3 x(t) + B_3 u(t) \\ y(t) = C_3 x(t) \end{cases}$

$Rule4: if\ x_1\ is\ F_2^2\ then \begin{cases} \dot{x}(t) = A_4 x(t) + B_4 u(t) \\ y(t) = C_4 x(t) \end{cases}$

The four initial models are unstable, they have positive eigenvalues, Table 2

Table 2 : Summary table of the eigenvalues of matrices Ai

$eig(A_1)$	$eig(A_2)$	$eig(A_3)$	$eig(A_4)$
0 5.2339 -4.9099 -14.8461	0 5.1806 -4.8379 -14.8647	0 1.6152 -1.2863 -14.8510	0 1.6655 -1.3742 -14.8134

1. Stabilization control law PDC:

Consider the closed loop system written by the following equation

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) h_i(z(t)) G_{ii} x(t) + 2\sum_{i=1}^{r}\sum_{i<j} h_i(z(t)) h_j(z(t)) \left\{\frac{G_{ij} + G_{ji}}{2}\right\} x(t), \quad \textbf{(29)}$$

where $G_{ij} = A_i - B_i F_j$ et $G_{ii} = A_i - B_i F_i$

PDC control law is represented as follows:

$$u(t) = -\frac{\sum_{i=1}^{r} w_i(z(t)) F_i x(t)}{\sum_{i=1}^{r} w_i(z(t))} = -\sum_{i=1}^{r} h_i(z(t)) F_i x(t) \quad \textbf{(30)}$$

The closed-loop system is globally asymptotically stable if there exists symmetric matrices and positive definite $P_k, k = 1, 2, \ldots, r$ et $R = R^T$, and matrices $F_1, \ldots, F_r$ such that they satisfy the following LMI: .**[4]**

$$P_k + R \succ 0, \quad k \in \{1, \ldots, r\} \quad \textbf{(31)}$$

$$P_j + \mu R \geq 0, \quad j = 1, 2, \ldots, r \quad \textbf{(32)}$$

$$P_\phi + \left\{G_{ii}^T (P_k + \mu R) + (P_k + \mu R) G_{ii}\right\} \prec 0, i, k \in \{1, \ldots, r\} \quad \textbf{(33)}$$

$$\left\{\frac{G_{ij} + G_{ji}}{2}\right\}^T (P_k + \mu R) + (P_k + \mu R)\left\{\frac{G_{ij} + G_{ji}}{2}\right\} \prec 0, \quad \textbf{(34)}$$

For $i, j, k = 1, 2, \ldots, r$ where $i \prec j$
where

$$G_{ij} = A_i - B_i F_j, G_{ii} = A_i - B_i F_i$$
$$P_\phi = \sum_{k=1}^{r} \phi_k (P_k + R)\ and\ 0 \leq \varepsilon \leq 1 \quad \textbf{(35)}$$

We chose $\phi_k = 0.85, \mu = 0.5$

The results of the stability analysis and the shaping of LMIs are:

$$P1 = \begin{pmatrix} 104.7472 & -115.6169 & 11.9122 & -3.9113 \\ -115.6169 & 413.1766 & -0.7102 & -14.7651 \\ 11.9122 & -0.7102 & 4.953 & -1.2836 \\ -3.9113 & -14.7651 & -1.2836 & 18.407 \end{pmatrix}$$

$$P2 = \begin{pmatrix} 103.9682 & -119.8725 & 11.9138 & -4.5012 \\ -119.8725 & 416.846 & -0.949 & -8.0854 \\ 11.9138 & -0.949 & 4.954 & -1.2713 \\ -4.5012 & -8.0854 & -1.2713 & 17.9878 \end{pmatrix}$$

$$P3 = \begin{pmatrix} 104.4175 & -114.8835 & 11.9012 & -3.8319 \\ -114.8835 & 419.5671 & -0.5652 & -16.6687 \\ 11.9012 & -0.5652 & 4.9529 & -1.2837 \\ -3.8319 & -16.6687 & -1.2837 & 18.4179 \end{pmatrix}$$

$$P4 = \begin{pmatrix} 115.0851 & -69.6047 & 12.2236 & -5.1634 \\ -69.6047 & 177.2886 & -1.1119 & -11.3246 \\ 12.2236 & -1.1119 & 4.954 & -1.2536 \\ -5.1634 & -11.3246 & -1.2536 & 19.0259 \end{pmatrix}$$

$$R = \begin{pmatrix} -98.2618 & 110.8835 & -11.6188 & 1.2030 \\ 110.8835 & -36.9477 & 3.9139 & -18.936 \\ -11.6188 & 3.9139 & -3.8708 & -7.9774 \\ 1.2030 & -18.936 & -7.9774 & 124.1508 \end{pmatrix}$$

$$F_1 = [41.2123 \quad 28.1726 \quad -76.1726 \quad 119.1816]$$
$$F_2 = [23.544 \quad 6.5301 \quad -32.6125 \quad 34.2956]$$
$$F_3 = [22.8407 \quad 8.8906 \quad -45.7892 \quad 122.6646]$$
$$F_4 = [14.6584 \quad 11.7044 \quad -17.0676 \quad 53.4315]$$

The results of the stability analysis and the shaping of LMIs are state matrices closed-loop of four basic models are given by:
$G_{ii} = A_i - (B_i F_i)$ and matrices
$G_{ijji} = G_{ij} + G_{ji} = A_i - (B_i F_j) + A_j - (B_j F_i)$

$$G_{11} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -44.4714 & -47.8342 & 130.2358 & -164.4037 \\ 0 & 0 & 0 & 1 \\ -1.0932 & -0.7005 & 1.9286 & -17.8151 \end{pmatrix}$$

$$G_{22} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -14.2975 & -10.8732 & 55.6956 & -39.0028 \\ 0 & 0 & 0 & 1 \\ -1.1550 & -0.1591 & 0.8248 & -15.6683 \end{pmatrix}$$

$$G_{33} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -17.1458 & -7.3128 & 39.0994 & -65.0323 \\ 0 & 0 & 0 & 1 \\ -0.6794 & -0.2128 & 1.1580 & -17.9032 \end{pmatrix}$$

$$G_{44} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -10.1589 & -9.7155 & 14.5740 & -26.0579 \\ 0 & 0 & 0 & 1 \\ -0.4216 & -0.29 & 0.4316 & -16.1523 \end{pmatrix}$$

$$G_{1221} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ -58.7690 & -58.7074 & 185.9314 & -203.4064 \\ 0 & 0 & 0 & 2 \\ -2.2482 & -0.8596 & 2.7534 & -33.4834 \end{pmatrix}$$

$$G_{1331} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ -45.9296 & -38.6822 & 143.3167 & -232.4101 \\ 0 & 0 & 0 & 2 \\ -1.7726 & -0.9133 & 3.0866 & 35.7183 \end{pmatrix}$$

$$G_{1441} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ -31.9560 & -43.4876 & 94.2659 & -134.3176 \\ 0 & 0 & 0 & 2 \\ -1.5148 & -0.9905 & 2.3602 & -33.9674 \end{pmatrix}$$

$$G_{2332} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ -30.8427 & -20.2017 & 106.0466 & -179.4934 \\ 0 & 0 & 0 & 2 \\ -1.8345 & -0.3720 & 1.9828 & -33.5715 \end{pmatrix}$$

$$G_{2442} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ -16.8690 & -25.0071 & 56.9958 & -81.4008 \\ 0 & 0 & 0 & 2 \\ -1.5766 & -0.4492 & 1.2564 & -31.8206 \end{pmatrix}$$

$$G_{3443} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ -27.3047 & -17.0283 & 53.6734 & -91.0902 \\ 0 & 0 & 0 & 2 \\ -1.1011 & -0.5028 & 1.5896 & -34.0555 \end{pmatrix}$$

Table 3: Summary table of the eigenvalues of stabilizing gains

$eig(G_{11})$	$eig(G_{22})$	$eig(G_{33})$	$eig(G_{44})$	$eig(G_{1221})$
-50.4318 -14.3813 -0.7291 -0.1070	-16.3119 -9.0832 -0.5732 + 0.1613i -0.5732 - 0.1613i	-18.9134 -3.1384 + 1.9635i -3.1384 - 1.9635i -0.0259	-17.0752 -7.6179 -1.1631 -0.0116	-62.52 -27.97 -1.21 -0.48

Table 4 : Summary table of the eigenvalues of stabilizing gains

$eig(G_{1331})$	$eig(G_{1441})$	$eig(G_{2332})$	$eig(G_{2442})$	$eig(G_{3443})$
-50.5298 -21.6285 -2.0408 -0.2014	-50.1811 - 25.9454 - 1.1483 - 0.1803	- 36.4793 - 14.7689 - 2.0393 - 0.4856	-34.7308 -0.5371 + 0.2962i -0.5371 - 0.2962i -21.0227	-36.1573 - 10.6302 - 4.2580 - 0.0384

All the eigenvalues of stabilizing gains are negative real parts then inverted pendulum is stable in closed loop.
To properly check the stability of inverted pendulum closed loop applies the basic commands to the system. Figure 2 shows the simulation results.

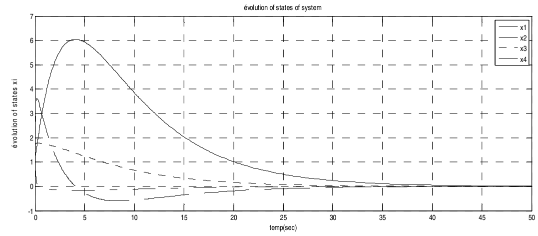

Figure 2 : Evolution of state variables of the inverted pendulum

The figure 2 shows that all states of the inverted pendulum x_1, x_2, x_3, x_4 converge to a steady state which is $(x_e = 0)$ starting from the initial state $x_0 = \begin{pmatrix} 0.5 & 0 & 0.3 & 0 \end{pmatrix}$.Which verifies the stability of the closed loop system.

The shapes of the two outputs y_1 et y_2 are given in Figure 3:

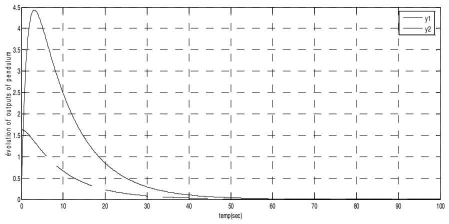

Figure 3 : Evolution outputs inverted pendulum

Let the initial condition $x_0 = (0.5 \quad 0 \quad 0.3 \quad 0)$, we check the convergence of the states of each subsystem to the steady state $(x_e = 0)$

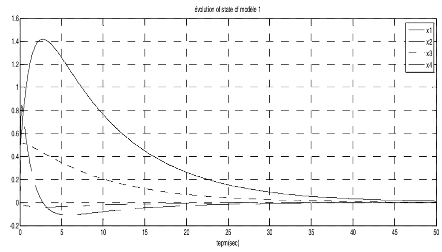

Figure 4 : Evolution of the state variables of model 1

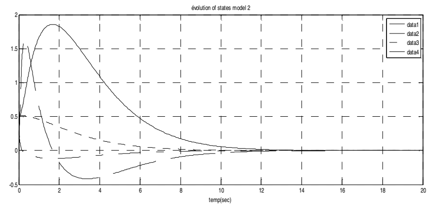

Figure 5 : Evolution of the state variables of model 2

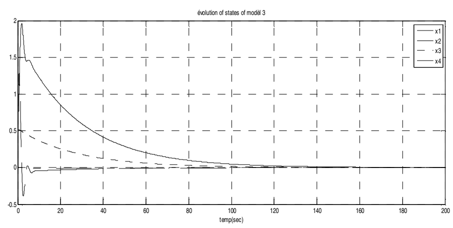

Figure 6 : Evolution of the state variables of model 3

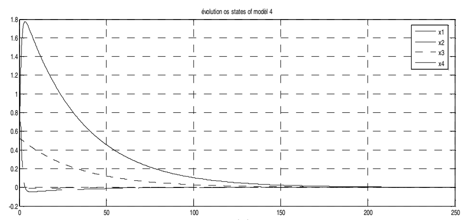

Figure 7 : Evolution of the state variables of model 4

The shapes of the two outputs y_1 et y_2 of each subsystem are presented in the figure below :

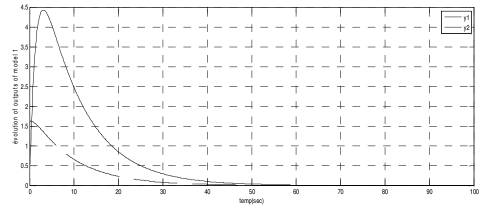

Figure 8 : Evolution of model outputs 1

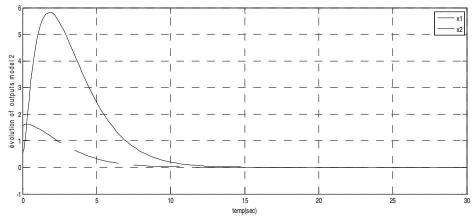

Figure 9 : Evolution of model outputs 2

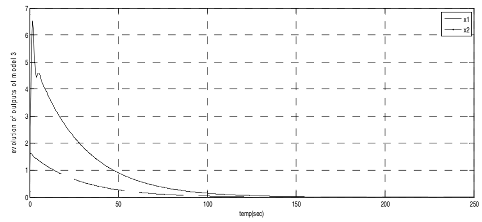

Figure 10 : Evolution of model outputs 3

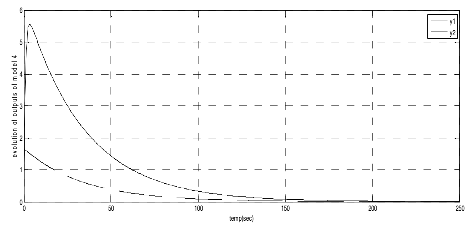

Figure 11: Evolution of model outputs 4

IV. Conclusion :

This paper introduces the modeling of the inverted pendulum . We studied the stabilisation of the inverted pendulum via linear matrix inequality. The stabilisation conditions are verified by PDC control law, simulation results verify the convergence of the states of the closed loop system and outputs of each sub-systems to the point of stable equilibrium.

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