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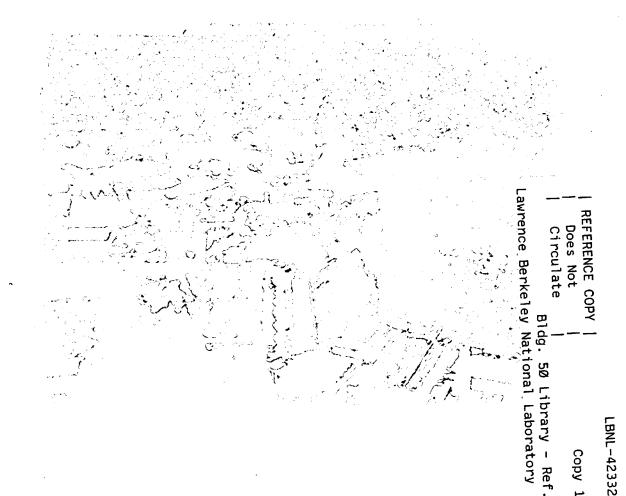**Publication Date**
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New Confining $N = 1$ Supersymmetric Gauge Theories[*]

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Abstract

We examine $N = 1$ supersymmetric gauge theories which confine in the presence
of a tree-level superpotential. We show the confining spectra which satisfy the 't
Hooft anomaly matching conditions and give a simple method to find the confining
superpotential. Using this method we fix the confining superpotentials in the simplest
cases, and show how these superpotentials are generated by multi-instanton effects in
the dual theory. These new type of confining theories may be useful for model building,
since the size of the matter content is not restricted by an index constraint. Therefore,
one expects that a large variety of new confining spectra can be obtained using such
models.

1 Introduction

Confining theories are the simplest asymptotically free $N = 1$ supersymmetric gauge theories. In such theories the low-energy effective theory is simply given by a Wess-Zumino model for the composite gauge singlets. The first example of such a confining theory has been found by Seiberg [1]. Later several other confining theories have been found [2, 3, 4, 5, 6, 7, 8] and classified (for a recent review see [9]). Some of these confining theories have been used for constructing models which explain the flavor hierarchy [10], but the limited number of known confining models strongly limits their applications for model building. All of the confining theories mentioned above are based on examples with simple gauge groups which confine without the presence of a tree-level superpotential. However, it has been noted by Kutasov, Schwimmer and Seiberg in Ref. [11] that certain theories might be confining if a suitable tree-level superpotential is added to the theory. In [11], the dualities of $SU(N)$ with an adjoint field X and F flavors were examined in the presence of a tree-level superpotential $\text{Tr} X^{k+1}$. They noted that for a certain number of colors ($N = kF - 1$) the dual gauge group reduces to $SU(1)$, and identified a set of composites which satisfy the 't Hooft anomaly matching conditions.

In this paper, we show that the theory is indeed confining for all values of k. These theories can be thought of as generalizations of the well-known s-confining theories [6], since in the case when the tree-level superpotential reduces to a mass term ($k = 1$ in the above example) one always obtains an s-confining theory. However, contrary to the ordinary s-confining theories, the matter content of these examples is not restricted by an index constraint. Therefore, we expect that there are many more confining theories of this sort exhibiting a large variety of global symmetries and confining spectra, some of which may be useful for composite model building.

In this paper we will give a method to find the confining superpotential of these theories, which reproduces the classical constraints once the F-flatness conditions arising from the tree-level superpotential $\text{Tr} X^{k+1}$ is taken into account. Once this superpotential is established, one can integrate out flavors in order to find dynamically generated Affleck-Dine-Seiberg-type [12] (ADS) superpotentials. However, the confining superpotentials obtained in this paper are not the most general ones, since the form of the tree-level superpotential is assumed to be $\text{Tr} X^{k+1}$, and the relations resulting from the requirement of the vanishing of the F-terms are used in constructing the superpotential. Therefore superpotential perturbations along directions other than mass terms for some flavors will not be correctly reproduced by the superpotentials presented here. In order to reproduce such perturbations as well, one would need to find the confining superpotential in the presence of the most general tree-level superpotential, which we leave for future investigation.

In all solutions presented in this paper, we find that the confining superpotentials or the ADS superpotentials are always due to multi-instanton effects, and not due to a one-instanton effect. As expected, the coupling of the tree-level superpotential also appears in

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the dynamically generated superpotentials, which diverge in the limit when this coupling is turned off.

This paper is organized as follows. In Section 2 we first discuss the confining theories based on $SU(N)$ with an adjoint and fundamentals. We examine the $k = 2$ case in detail, show how to find the confining superpotential and how it arises from a two-instanton effect in the dual theory. Then we consider integrating out a flavor from the $k = 2$ theory. Next we examine the $k > 2$ theories. We show that there are no additional branches in this theory, and thus just as in the $k = 2$ case they are confining at the origin. We show the confining spectrum and write down the form of the confining superpotential (without fixing the coefficients of the individual terms). In Section 3 we consider generalizations of the theories presented in Section 2 to theories with more complicated gauge group and/or matter content. We find the confining spectrum for these theories, but in most examples leave the determination of the superpotentials for future work. We conclude in Section 4.

2 The $SU(N)$ theory with an adjoint and fundamentals

2.1 The $k = 2$ Theories

Consider $SU(N)$ with an adjoint and F flavors, and a superpotential $\mathrm{Tr}X^3$ for the adjoint. The global symmetries of the theory are given by

$$
\begin{array}{c|c|cccccc}
 & SU(N) & SU(F) & SU(F) & U(1) & U(1)_R & Z_{3F} \\
\hline
X & Adj & 1 & 1 & 0 & 2/3 & F \\
Q & \square & \square & 1 & 1 & 1 - \frac{2N}{3F} & -N \\
\bar{Q} & \bar{\square} & 1 & \square & -1 & 1 - \frac{2N}{3F} & -N
\end{array}
\tag{2.1}
$$

This is the special case of the theories considered in [11, 13, 14] for the theory with an adjoint and the superpotential

$$
W = h\mathrm{Tr}X^{k+1}
$$

for $k = 2$. Here, h is a coupling constant, dimensionless for $k = 2$, and of dimension $k - 2$ in general. For $2F - N > 1$ it has been shown in [11, 13, 14] that the theory has a dual description in terms of the gauge group $SU(2F - N)$, an adjoint Y, dual quarks $q, \bar{q}$ and mesons M_1, M_2. The field content and superpotential of the dual theory are summarized below:

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	$SU(2F-N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$	Z_{3F}
Y	Adj	1	1	0	$2/3$	F
q	$\square$	$\bar{\square}$	1	1	$1 - \frac{2(2F-N)}{3F}$	$N+F$
$\bar{q}$	$\bar{\square}$	1	$\bar{\square}$	-1	$1 - \frac{2(2F-N)}{3F}$	$N+F$
M_1	1	$\square$	$\square$	0	$2 - \frac{4N}{3F}$	$-2N$
M_2	1	$\square$	$\square$	0	$\frac{8}{3} - \frac{4N}{3F}$	$F - 2N$

$$(2.2)$$

$$W_{magn} = -h\,\mathrm{Tr}Y^3 + \frac{h}{\mu^2}\left(M_1 \bar{q} Y q + M_2 \bar{q} q\right). \tag{2.3}$$

The coefficients of the superpotential terms were fixed to be $-h$ and h by to the analysis in [11]. However, for the case $2F - N = 1$ the theory is no longer in the non-abelian Coulomb phase (or the free magnetic phase), but rather confining. The spectrum can be obtained by adding a superpotential term M_1 to the magnetic $SU(3)$ theory of $2F - N = 3$ (which corresponds to integrating out a single flavor from the electric theory). The confining spectrum is given by:

	$SU(2F-1)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$	Z_{3F}
X	Adj	1	1	0	$2/3$	F
Q	$\square$	$\square$	1	1	$1 - \frac{2(2F-1)}{3F}$	$1+F$
$\bar{Q}$	$\bar{\square}$	1	$\square$	-1	$1 - \frac{2(2F-1)}{3F}$	$1+F$
$M_1 = (\bar{Q}Q)$		$\square$	$\square$	0	$2 - \frac{4(2F-1)}{3F}$	$2 - F$
$M_2 = (\bar{Q}XQ)$		$\square$	$\square$	0	$\frac{8}{3} - \frac{4(2F-1)}{3F}$	2
$B = (Q^F(XQ)^{F-1})$		$\bar{\square}$	1	$2F-1$	$1 - \frac{2}{3F}$	-1
$\bar{B} = (\bar{Q}^F(X\bar{Q})^{F-1})$		1	$\bar{\square}$	$-2F+1$	$1 - \frac{2}{3F}$	-1

$$(2.4)$$

This anomaly matching for the continuous global symmetries $SU(F)^3$, $SU(F)^2U(1)$, $SU(F)^2U(1)_R$, $U(1)^3$, $U(1)^2U(1)_R$, $U(1)U(1)_R^2$, $U(1)_R^3$, $U(1)$ and $U(1)_R$ has been noted in [11], and can be extended to the discrete symmetries $SU(F)^2Z_{3F}$, Z_{3F}, Z_{3F}^3, $U(1)^2Z_{3F}$, $U(1)Z_{3F}^2$, $U(1)_R^2Z_{3F}$, $U(1)_RZ_{3F}^2$, $U(1)U(1)_RZ_{3F}$ as well. We will argue that the theory is indeed s-confining, that is it is described by these gauge singlets everywhere on the moduli space. For this in the next section we will analyze the classical limit of the theory, that is the classical constraints satisfied by these operators. This will completely determine the form of the confining superpotential.* Then we will show that instanton effects in the dual magnetic theory indeed do generate this confining superpotential term.

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*It has been recently argued [16], that a sufficient and necessary condition for the continuous 't Hooft anomaly matching conditions to be satisfied by the holomorphic gauge invariants is that the classical constraints be derivable from a superpotential. Below we find this superpotential whose existence is guaranteed by the above quoted theorem, and argue that this is indeed the full confining superpotential of the theory.

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2.1.1 Analysis in the Electric Theory

In this section we will give a method to analyze the classical constraints of this theory and show the superpotential which can reproduce these constraints. First we note that the analysis is different than in the ordinary s-confining theories, since there is a tree-level superpotential present in this theory, and the resulting F-flatness conditions have to be taken into account when analyzing the constraints. The tree-level superpotential is $\text{Tr}X^3$, and the resulting F-flatness condition is

$$X^2 - \frac{1}{N}\text{Tr}X^2 = 0, \qquad (2.5)$$

that is $X^2 \propto 1$. One can use the complexified gauge group to transform X to a Jordan normal form, *e.g.*,

$$X = \begin{pmatrix} a & 1 & & & & & \\ & a & 1 & & & & \\ & & a & & & & \\ & & & b & 1 & & \\ & & & & b & & \\ & & & & & c & \\ & & & & & & \ddots \\ & & & & & & & z \end{pmatrix} \qquad (2.6)$$

In the case of a completely diagonal X, one can easily see that $a = b = \cdots = z = 0$ as follows. The F-flatness condition $X^2 \propto 1$ forces all the diagonal elements to be $\pm v$ while there are odd number $(2F - 1)$ of diagonal elements, and hence their sum can not be zero, unless $v = 0$. The diagonal elements $a, b, \cdots, z$ have to vanish in the case of the general Jordan normal form as well, since the diagonal elements will still be $a^2, a^2, a^2, b^2, b^2, c^2, \cdots, z^2$. Thus the only possible form of the matrix X is diagonal entries with the eigenvalue 0 and a certain number of non-vanishing 2×2 blocks of the form

$$\begin{pmatrix} 0 & 1 \\ & 0 \end{pmatrix} \qquad (2.7)$$

to guarantee $X^2 = 0$. The most general form then is:

$$X = \begin{pmatrix} 0 & & & & & & & \\ & \ddots & & & & & & \\ & & 0 & & & & & \\ & & & 0 & 1 & & & \\ & & & & 0 & & & \\ & & & & & \ddots & & \\ & & & & & & 0 & 1 \\ & & & & & & & 0 \end{pmatrix} \qquad (2.8)$$

This classical analysis immediately justifies the fact that $\text{Tr} X^2$ (and all other invariants of the form $\text{Tr} X^p$) is not among the confining degrees of freedom, since for all configurations satisfying F-flatness $\text{Tr} X^2 = 0$.

Next we will identify the classical constraints among the gauge-invariant polynomials M_1, M_2, B and $\bar{B}$. For this, we introduce F *dressed flavors*, $XQ, X\bar{Q}$, in addition to the original F flavors $Q, \bar{Q}$. Thus we consider the enlarged flavor space $\mathcal{Q} = (Q, XQ)$, $\bar{\mathcal{Q}} = (\bar{Q}, X\bar{Q})$. Treating all $2F$ "flavors" independently, we find the same classical constraints as in an $SU(2F-1)$ theory with the $2F$ flavors. The classical constraints among meson and baryon operators in this case are well-known from the analysis of the $SU(N)$ theories with $N+1$ flavors [1].[†]

The meson matrix of the theory with dressed flavors is given by

$$\mathcal{M} = \bar{\mathcal{Q}}\mathcal{Q} = \begin{pmatrix} \bar{Q}Q & \bar{Q}XQ \\ \bar{Q}XQ & \bar{Q}X^2Q \end{pmatrix}. \tag{2.9}$$

However, we know that due to the F-flatness conditions $X^2 = 0$, and we obtain

$$\mathcal{M} = \begin{pmatrix} M_1 & M_2 \\ M_2 & 0 \end{pmatrix}. \tag{2.10}$$

Similarly, we can construct the baryons for the enlarged flavor space:

$$\mathcal{B} = (Q^{F-1}(XQ)^F, Q^F(XQ)^{F-1}), \qquad \bar{\mathcal{B}} = (\bar{Q}^{F-1}(X\bar{Q})^F, \bar{Q}^F(X\bar{Q})^{F-1}). \tag{2.11}$$

The second components of $\mathcal{B}, \bar{\mathcal{B}}$ are $B, \bar{B}$ of (2.4), and we will argue that the first components vanish due to the F-flatness conditions. This is because X has at most $F-1$ non-vanishing elements (otherwise X^2 would not be vanishing, since X is a $2F-1$ by $2F-1$ matrix; see Eq. (2.8)), and the color index contraction in $Q^{F-1}(XQ)^F$ yields a vanishing result due to the antisymmetry in color. This also explains why the baryons formed this way are not part of the confining spectrum. Thus

$$\mathcal{B} = (0, B), \qquad \bar{\mathcal{B}} = (0, \bar{B}). \tag{2.12}$$

We know that in the enlarged flavor space the classical constraints are given by

$$\mathcal{M}_{ij}\mathcal{B}^j = 0, \quad \bar{\mathcal{B}}^i\mathcal{M}_{ij} = 0, \quad \bar{\mathcal{B}}^i\mathcal{B}^j = \text{cof}\mathcal{M}^{ij}, \tag{2.13}$$

where the cofactor of a p by p matrix A is defined as

$$(\text{cof } A)^{ij} = \frac{1}{(p-1)!}\epsilon^{ii_2i_3\cdots i_p}\epsilon^{jj_2j_3\cdots j_p}A_{i_2j_2}A_{i_3j_3}\cdots A_{i_pj_p} = \frac{\partial \det A}{\partial A_{ij}}. \tag{2.14}$$

[†]This theory is s-confining. Note, however, that this analysis is strictly classical and it may or may not be a coincidence that both of these theories are s-confining.

Written in terms of the confined variables M_1, M_2, B and $\bar{B}$ these constraints read:

$$M_{2ij}B^j = 0,$$

$$\bar{B}^i M_{2ij} = 0,$$

$$\begin{pmatrix} 0 & 0 \\ 0 & \bar{B}^i B^j \end{pmatrix} = \begin{pmatrix} 0 & \det M_2 \text{cof} M_2^{ij} \\ \det M_2 \text{cof} M_2^{ij} & (M_1 \text{cof} M_2) \text{cof} M_2^{ij} \end{pmatrix}. \qquad (2.15)$$

The superpotential which reproduces these classical constraints is given by

$$W = \frac{1}{h^{2F-1}\Lambda^{6F-4}} \left(\bar{B} M_2 B - \det M_2 (M_1 \text{cof} M_2) \right), \qquad (2.16)$$

where Λ is the dynamical scale of the original confining $SU(2F-1)$ gauge group. Note that, unlike in the usual s-confining theories [1, 6], one has the two-instanton factor appearing in the confining superpotential, rather than the 1-instanton factor Λ^{3F-2}.

One should ask the question whether the fact that (2.16) reproduces the classical constraints in itself is enough evidence for it being the full confining superpotential. The answer is no for the following reason: one wants to obtain the classical limit when the expectation values of the fields are big, $\langle \Phi \rangle \gg \Lambda$. This means that the highest powers in $1/\Lambda$ in the superpotential have to reproduce the classical constraints, thus (2.16) can not contain terms of higher order in $1/\Lambda$. However, since (2.16) has only a term containing the 2-instanton factor, it is in principle possible that an additional term proportional to an integer power of the one-instanton factor $1/\Lambda^{3F-2}$ is present in the superpotential. We show, however, that this is not possible, if all fields appear with positive powers, and thus (2.16) is indeed the full superpotential. As explained above, the only possible additional term should be proportional to $1/\Lambda^{3F-2}$. Then the form of the extra piece in the superpotential in terms of the high energy fields is fixed by the global symmetries to be

$$\frac{1}{\Lambda^{3F-2}} (Q^F X^{1+F} \bar{Q}^F). \qquad (2.17)$$

However, it is impossible to write this combination of fields in terms of the confining spectrum (2.4). The reason is that due to the $U(1)$ baryon number B and $\bar{B}$ would have to appear with the same power. Thus we would have to use a combination of $(B\bar{B})$, M_1 and M_2 to obtain (2.17). This is however impossible, since $(B\bar{B})$, M_1 and M_2 contain more or equal number of $(\bar{Q}Q)$'s than X's, while (2.17) contains more X's than $(\bar{Q}Q)$'s. Thus we conclude that (2.16) is indeed the full confining superpotential.

This conclusion however might change if additional tree-level superpotential terms (other than M_1) are added to the theory, for example a term proportional to M_2. The reason is that in this case the classical constraints arising from the F-terms are modified, and the analysis presented above has to be changed, which invalidates the form of the dressed meson matrix $\mathcal{M}$ and the dressed baryons $\mathcal{B}, \bar{\mathcal{B}}$ of (2.10). The most general confining superpotential

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for this theory would incorporate the dependence on the coupling constants of all possible tree-level superpotential terms, and reduce to (2.16) in the limit where all couplings other than $\text{Tr}\,X^3$ are turned off. We leave the determination of the most general superpotential for future work, and note only that (2.16) can be used when a superpotential proportional to M_1 is added to the theory. This is because in this case the F-term equation for X is not affected by the presence of the additional tree-level superpotential term, and therefore the form of the dressed meson $\mathcal{M}$ remains unchanged. This perturbation is what we will use later to integrate out a flavor from the theory.

2.1.2 The superpotential from the Magnetic Theory

We will now argue, that the superpotential (2.16) is indeed generated in the dual magnetic theory, when integrating out a flavor. We start with a theory which has one more flavors $(F+1)$ than the s-confining case, and N is still given by $2F-1$. The theory thus has a magnetic dual in terms of an $SU(3)$ gauge group. The dual theory is given by

	$SU(3)$	$SU(F+1)$	$SU(F+1)$	$U(1)$	$U(1)_R$
Y	Adj	1	1	0	$2/3$
q	$\square$	$\overline{\square}$	1	1	$1-\frac{2}{F+1}$
$\bar{q}$	$\overline{\square}$	1	$\overline{\square}$	-1	$1-\frac{2}{F+1}$
M_1	1	$\square$	$\square$	0	$2-\frac{4(2F-1)}{3(F+1)}$
M_2	1	$\square$	$\square$	0	$\frac{8}{3}-\frac{4(2F-1)}{3(F+1)}$

$$W_{magn} = -h\text{Tr}Y^3 + \frac{h}{\mu^2}\left(M_1 \bar{q} Y q + M_2 \bar{q} q\right). \tag{2.19}$$

When integrating out a flavor in order to arrive at the s-confining case, we add a term $mQ_{F+1}\bar{Q}_{F+1}$ to the electric theory, which modifies the magnetic superpotential to

$$W_{magn} = -h\text{Tr}Y^3 + \frac{h}{\mu^2}\left(M_1 \bar{q} Y q + M_2 \bar{q} q\right) + mM_{1F+1,F+1}. \tag{2.20}$$

The equation of motion with respect to $M_{1F+1,F+1}$ forces an expectation value to $\bar{q}_{F+1}Y q_{F+1}$, breaking the gauge group completely. The expectation values which satisfy D- and F-flatness are given by:

$$\langle \bar{q} \rangle = (v,0,0), \quad \langle Y \rangle = \begin{pmatrix} 0 & v & \\ & 0 & \\ & & 0 \end{pmatrix}, \quad \langle q \rangle = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}. \tag{2.21}$$

These VEV's, while breaking $SU(3)$ completely, give masses either through the superpotential or through the D-terms to all components of Y, the elements of the last row and

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column of M_1 and M_2, q_{F+1} and $\bar{q}_{F+1}$, and the first and second components of all other q's and $\bar{q}$'s. The remaining components of q and $\bar{q}$ can be identified with B and $\bar{B}$ of the confining theory. The tree-level superpotential $M_2 \bar{q} q$ will then result in the term $\bar{B} M_2 B$. The remaining question is how to obtain the term $\det M_2 (M_1 \operatorname{cof} M_2)$. This will be generated by a 2-instanton effect in the completely broken magnetic group. The 't Hooft vertex for the 2-instanton is given by:

$$\lambda^{12} \tilde{q}^{2F+2} \tilde{\bar{q}}^{2F+2} \tilde{Y}^{12} \Lambda_{magn}^{10-2F}, \tag{2.22}$$

where λ is the gaugino, and the other fields denote the fermionic components of the given chiral superfields. We will use the tree-level superpotential couplings and the gaugino-fermion-scalar vertices to convert this to a term in the superpotential [17]. First we use the $q^* \lambda \tilde{q}$ and $\bar{q}^* \lambda \tilde{\bar{q}}$ vertices three times and the $Y^* \lambda \tilde{Y}$ vertex once to convert the 't Hooft vertex to

$$\Lambda_{magn}^{10-2F} \tilde{Y}^{11} Y^* \lambda^5 \tilde{q}^{2F-1} \tilde{\bar{q}}^{2F-1} q^{*\,3} \bar{q}^{*\,3}. \tag{2.23}$$

Next we use $\frac{h}{\mu^2} M_1 \bar{q} Y q$ superpotential coupling to obtain

$$\Lambda_{magn}^{10-2F} \tilde{Y}^{11} \lambda^5 \tilde{q}^{2F-2} \tilde{\bar{q}}^{2F-2} q^{*\,3} \bar{q}^{*\,3} M_1 \frac{h}{\mu^2}. \tag{2.24}$$

Then we use the $\frac{h}{\mu^2} M_2 \bar{q} q$ term twice to obtain

$$\Lambda_{magn}^{10-2F} \tilde{Y}^{11} \lambda^5 \tilde{q}^{2F-3} \tilde{\bar{q}}^{2F-3} q^{*\,2} \bar{q}^{*\,2} M_1 \tilde{M}_2^2 \left(\frac{h}{\mu^2} \right)^3. \tag{2.25}$$

Finally we use the $\frac{h}{\mu^2} M_2 \bar{q} q$ superpotential term $2F - 3$ times, the $-h Y^3$ term 3 times and the $Y^* \lambda \tilde{Y}$ coupling five times to obtain the term

$$\Lambda_{magn}^{10-2F} M_1 \tilde{M}_2^2 M_2^{2F-3} \langle Y \rangle^3 \langle q^* \rangle^2 \langle \bar{q}^* \rangle^2 \langle Y^* \rangle^5 \frac{h^{2F+3}}{\mu^{4F}}. \tag{2.26}$$

Now we substitute the expectation values v for q, $\bar{q}$ and Y to obtain

$$\Lambda_{magn}^{10-2F} M_1 \tilde{M}_2^2 M_2^{2F-3} v^3 v^{*\,9} \frac{h^{2F+3}}{\mu^{4F}}. \tag{2.27}$$

In order for the superpotential to be holomorphic, the additional factors of vv^* appearing from the integral over the instanton size have to cancel the dependence on v^*[17]. Therefore one expects that the instanton integral results in an additional factor of $(vv^*)^{-9}$. Thus, we obtain that the two-instanton in the completely broken $SU(3)$ group gives a contribution to the superpotential of the form

$$\frac{h^{2F+3} \Lambda_{magn}^{10-2F}}{\mu^{4F} v^6} \det \hat{M}_2 (\hat{M}_1 \operatorname{cof} \hat{M}_2), \tag{2.28}$$

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where $\hat{M}_1$ and $\hat{M}_2$ are the meson operators for the theory with one less flavor. Let us check that the coefficient is indeed the two-instanton factor of the electric theory as expected from (2.16). The matching of scales between the electric and magnetic theories is given by

$$\Lambda_{el}^{3F-3}\Lambda_{magn}^{5-F} = \left(\frac{\mu}{h}\right)^{2F+2}.$$
(2.29)

The expectation value v is given by $v^3 = \mu^2 m/h$, thus we obtain that the superpotential term is (leaving the hats off)

$$\frac{1}{h^{2F-1}m^2\Lambda_{el}^{6F-6}}\det M_2(M_1 \mathrm{cof}\, M_2).$$
(2.30)

Taking into account the scale matching in the electric theory $\Lambda_{el}^{3F-3}m = \tilde{\Lambda}_{el}^{3F-2}$ we obtain exactly the second term in (2.16) from this two-instanton effect. Thus we conclude that the $2F - N = 1$ theory is described by the superpotential (2.16), which correctly reproduces the classical constraints of the theory, and which can be shown to arise from the dual magnetic theory when integrating out a flavor.

2.1.3 Integrating out Flavors

Using the results from the previous section we can obtain results for theories with fewer number of flavors. Contrary to SUSY QCD and all other s-confining theories which confine without the presence of a tree-level superpotential, the theories with one less flavor does not yield a theory with a quantum modified constraint, instead it will result in a theory with a dynamically generated Affleck-Dine-Seiberg-type superpotential [12]. One can expect this by realizing, that the dual gauge group is $SU(2F - N)$, thus integrating out a single flavor will result in breaking two colors instead of just one (or k colors for a superpotential $\mathrm{Tr}\, X^{k+1}$). Here we show how to integrate out a single flavor from the confining theory presented in the previous section.

Adding a mass term to one flavor results in the superpotential

$$W = \frac{1}{h^{2F-1}\Lambda^{6F-4}}\left(\bar{B}M_2 B - \det M_2(M_1 \mathrm{cof}\, M_2)\right) + m(M_1)_{FF}.$$
(2.31)

The $\bar{B}$, B equations of motion just set the baryons to zero. The $(M_1)_{FF}$ equation of motion gives

$$(M_2)_{FF} = -\frac{mh^{2F-1}\Lambda^{6F-4}}{(\det \tilde{M}_2)^2},$$
(2.32)

where $\tilde{M}_2$ is the M_2 meson matrix for the theory with one less flavors. The $\bar{B}M_2 B$ piece and the pieces which contain $(M_1)_{FF}$ of the superpotential are set to zero, so the only remaining

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piece can be written as

$$W_{eff} = \frac{1}{h^{2F-1}\Lambda^{6F-4}}(M_2)_{FF}(\det \tilde{M}_2)\tilde{M}_1(\mathrm{cof}\,\tilde{M}_2)(M_2)_{FF} = m^2 h^{2F-1}\Lambda^{6F-4}\frac{\tilde{M}_1\mathrm{cof}\,\tilde{M}_2}{(\det \tilde{M}_2)^3}.$$
(2.33)

Using the scale matching relation $m\Lambda^{3F-2} = \tilde{\Lambda}^{3F-1}$, we obtain that the dynamically generated superpotential is given by

$$W_{ADS} = \frac{h^{2F-1}\Lambda^{6F-2}(M_1\mathrm{cof}\,M_2)}{(\det M_2)^3}.$$
(2.34)

This has the right quantum numbers to be a two-instanton effect in the $SU(2F-1)$ theory with $F-1$ flavors and an adjoint. Note, that this superpotential (contrary to the confining superpotentials) does vanish in the $h \to 0$ limit. Similarly, one can integrate out further flavors to obtain the dynamically generated superpotentials for the theories with fewer flavors, which we leave as an exercise to the reader.

2.2 The SU theories for $k > 2$

Next we discuss the theories with $k > 2$, with the superpotential $W = h\mathrm{Tr}X^{k+1}$.[‡] First of all, one can obtain the confining spectrum similar to (2.4) which satisfies the 't Hooft anomaly matching conditions for arbitrary k. This spectrum is given in the table below for $N = kF - 1$.

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$	$Z_{(k+1)F}$
X	Adj	1	1	0	$\frac{2}{k+1}$	F
Q	$\square$	$\square$	1	1	$1-\frac{2(kF-1)}{(k+1)F}$	$-(kF-1)$
$\bar{Q}$	$\bar{\square}$	1	$\square$	-1	$1-\frac{2(kF-1)}{(k+1)F}$	$-(kF-1)$
M_i		$\square$	$\square$	0	$\frac{2(2+F(1+i-k))}{F(1+k)}$	$2+F(i+2)$
B		$\bar{\square}$	1	$kF-1$	$\frac{F+kF-2}{F+Fk}$	$-1-\frac{k(k+1)F^2}{2}$
$\bar{B}$		1	$\bar{\square}$	$-kF+1$	$\frac{F+kF-2}{F+Fk}$	$-1-\frac{k(k+1)F^2}{2}$

(2.35)

where $i = 1, \cdots, k$ and the generalized mesons and baryons are defined by

$$M_i = (\bar{Q}X^{i-1}Q),$$
$$B = (Q^F(XQ)^F\cdots(X^{k-1}Q)^{F-1}),$$
$$\bar{B} = (\bar{Q}^F(X\bar{Q})^F\cdots(X^{k-1}\bar{Q})^{F-1}).$$
(2.36)

[‡]Now the coupling h is dimensionful.

This spectrum satisfies all the 't Hooft anomaly matching conditions for $SU(F)^3$, $SU(F)^2U(1)$, $SU(F)^2U(1)_R$, $U(1)^3$, $U(1)^2U(1)_R$, $U(1)U(1)_R^2$, $U(1)_R^3$, $U(1)$, $U(1)_R$, $SU(F)^2Z_{(k+1)F}$, $Z_{(k+1)F}$, $Z_{(k+1)F}^3$, $U(1)^2Z_{(k+1)F}$, $U(1)Z_{(k+1)F}^2$, $U(1)_R^2Z_{(k+1)F}$, $U(1)_RZ_{(k+1)F}^2$, $U(1)U(1)_RZ_{(k+1)F}$. For $k = 1$, the spectrum correctly reproduces the s-confining spectrum of the $SU(F-1)$ theory with F flavors. However, in order to establish that the theory is in the confining phase, one has to show that these are the only flat directions of the theory. We will show that this is indeed the case in a rather non-trivial manner; all invariants of the form $\mathrm{Tr}\, X^p$ are lifted either by the F and D-flatness conditions or by non-perturbative quantum effects.

The F-flatness condition for the tree-level superpotential $\mathrm{Tr}\, X^{k+1}$ is

$$X^k - \frac{1}{N}\mathrm{Tr}\, X^k = 0. \tag{2.37}$$

Thus $X^k \propto 1$, and the diagonal elements of X in the Jordan normal form must be k-th roots of unity. On the other hand, the sum of the diagonal elements has to vanish since $\mathrm{Tr}\, X = 0$. If k is prime, we cannot have $kF - 1$ eigenvalues (all k-th root of unity) summing up to zero, and all the eigenvalues must vanish. This proves that the operators $\mathrm{Tr}\, X^p$ are all lifted.
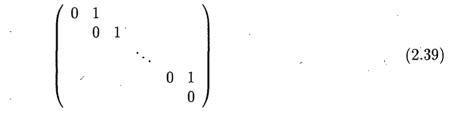
However, one can find classical flat directions where only the field X has an expectation value when k is not prime. This would lead to additional branches of the theory and to invariants of the form $\mathrm{Tr}\, X^p$ in addition to those listed above. For example, in the case $N = 5$, $k = 6$, $F = 1$, there is a direction

$$X = v \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & -1 & & \\ & & & \omega & \\ & & & & \omega^2 \end{pmatrix}, \tag{2.38}$$

where $\omega = \frac{-1+i\sqrt{3}}{2}$. This direction satisfies the F-term condition $X^6 - \frac{1}{5}\mathrm{Tr}\, X^6 = 0$ since all elements are sixth roots of unity, and also $\mathrm{Tr}\, X = 0$. This would mean that in addition to the operators listed above one would need to include $\mathrm{Tr}\, X^2$, $\mathrm{Tr}\, X^3$, $\mathrm{Tr}\, X^4$, and $\mathrm{Tr}\, X^6$ into the spectrum, which would give rise to a Coulomb branch and the theory would likely not be confining at the origin. This is however not the case. Under the unbroken $SU(2)$ gauge group left by the above flat direction, we have one flavor of quarks, which leads to the Affleck–Dine–Seiberg superpotential. Therefore, this classical flat direction is lifted quantum mechanically and is removed from the quantum moduli space.

The same mechanism lifts all classical flat directions where only X has an expectation value, as can be proven below. If we have p_1 diagonal entries in X which are the first k-th root of unity, p_2 of the second one etc., the gauge group is broken to $SU(p_1) \times SU(p_2) \times \cdots \times SU(p_k) \times U(1)^{k-1}$, with each $SU(p_i)$ factor having F flavors. We will show that such directions are lifted by quantum effects. This happens if there is an ADS-type superpotential

in any of the $SU(p_i)$ factors, that is if $p_i > F$ for some i. Assume the contrary, that is $p_i \leq F$ for all i. This is only possible if for example $p_1 = p_2 = \cdots = p_{k-1} = F, p_k = F - 1$ or its permutations, since the size of the gauge group is $kF - 1$. However, in this case the adjoint is not traceless, which is a contradiction, and hence $p_i > F$ for at least one i. Therefore configurations of the X which are F-flat and not lifted by quantum effects have only vanishing diagonal elements. All dangerous classical flat directions leading to Coulomb branches are lifted by quantum effects (all operators $\text{Tr} X^p = 0$), and the theory is indeed confining for any value of k. The most general X configuration, which can be made D-flat together with Q and $\bar{Q}$, is then given in Jordan normal form with blocks of the form

$$
\begin{pmatrix}
0 & 1 & & & \\
 & 0 & 1 & & \\
 & & \ddots & & \\
 & & & 0 & 1 \\
 & & & & 0
\end{pmatrix}
\tag{2.39}
$$

where each of the blocks is at most $k \times k$ to satisfy the F-flatness condition $X^k = 0$.

The confining superpotential can be fixed similarly to the case of $k = 2$. One again considers the dressed flavors $Q = (Q, XQ, X^2 Q, \cdots, X^{k-1} Q)$ and $\bar{Q} = (\bar{Q}, X\bar{Q}, X^2 \bar{Q}, \cdots, X^{k-1} \bar{Q})$. Then we can construct the mesons $\mathcal{M} = \bar{Q} Q$ and baryons $\mathcal{B}$ for these dressed flavors:

$$
\mathcal{M} = \begin{pmatrix}
M_1 & M_2 & M_3 & \cdots & M_k \\
M_2 & M_3 & \cdots & M_k & 0 \\
M_3 & \cdots & M_k & 0 & 0 \\
\vdots & & & & \vdots \\
M_k & 0 & \cdots & & 0
\end{pmatrix},
$$

$$
\mathcal{B} = (0, 0, \cdots, B),
$$

$$
\bar{\mathcal{B}} = (0, 0, \cdots, \bar{B}).
\tag{2.40}
$$

The fact that other components in $\mathcal{B}, \bar{\mathcal{B}}$ vanish can be shown based on the same argument for the $k = 2$ case in the previous section. We then rewrite the classical constraints $\mathcal{B}\bar{\mathcal{B}} = \text{cof}\,\mathcal{M}$ and $\mathcal{B}\mathcal{M} = \bar{\mathcal{B}}\mathcal{M} = 0$ in terms of M_i, B and $\bar{B}$. The latter two conditions are satisfied if the term $B M_k \bar{B}$ is present in the superpotential, while the $\mathcal{B}\bar{\mathcal{B}} = \text{cof}\,\mathcal{M}$ implies that the full superpotential is of the form

$$
\begin{aligned}
W = {} & \frac{1}{h^{kF-1} \Lambda^{k((2k-1)F-2)}} \Big[B M_k \bar{B} + (\det M_k)^{k-1}(M_1 \text{cof}\, M_k) + \\
& (\det M_k)^{k-2}(M_2 \text{cof}\, M_k)(M_{k-1} \text{cof}\, M_k) + (\det M_k)^{k-2}(M_3 \text{cof}\, M_k)(M_{k-2} \text{cof}\, M_k) + \cdots \\
& + \cdots + \\
& \cdots + (\det M_k)(M_{k-1} \text{cof}\, M_k)^{k-1} \Big],
\end{aligned}
\tag{2.41}
$$

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where we have not fixed the relative coefficients in the above superpotential. The structure of the terms in the above superpotential is such that $\det M_k$ has to appear at least once in every term (except the first term $BM_k\bar{B}$) at least once. After the power of $\det M_k$ in a given term is fixed one has to add all possible terms containing the appropriate number of X and $Q, \bar{Q}$ fields to find the most general superpotential.

Note that the overall dependence on the instanton-factor $\Lambda^{(2k-1)F-2}$ is that of the k-instanton. For example, in the case $k = 3$, the superpotential (again without fixing the relative coefficients) has the form

$$\frac{1}{h^{3F-1}\Lambda^{15F-6}} \left[BM_3\bar{B} + (\det M_3)^2(M_1 \mathrm{cof}\, M_3) + (\det M_3)(M_2 \mathrm{cof}\, M_3)^2\right]. \qquad (2.42)$$

3 Other Models

In this section we present other examples which confine in the presence of a suitable tree-level superpotential, similarly to the theory presented in the previous section. These examples are based on the dualities presented in Refs. [18, 19, 20]. For the first example we present both the confining spectrum and the confining superpotential, and show how it is obtained by a three-instanton effect by integrating out a flavor in the dual theory. For the remaining examples we give only the confining spectrum satisfying the 't Hooft anomaly matching conditions, leaving the determination of the superpotentials for future work.

3.1 Sp with an adjoint and fundamentals

This confining theory is based on the duality of Ref. [19], where it is shown that an $Sp(2N)$ theory with an adjoint X and $2F$ fundamentals Q, and a superpotential $W_{tree} = h\mathrm{Tr}X^{2(k+1)}$ is dual to $Sp(2\tilde{N})$ with an adjoint Y, $2F$ fundamentals q, and gauge singlet mesons M_i, $i = 0, ..., 2k$, with $\tilde{N} = (2k+1)F - N - 2$. The confining case is obtained when $\tilde{N} = 0$, that is for $N = (2k+1)F - 2$. The field content, symmetries and the confining spectrum are given in the table below:

	$Sp((2k+1)2F - 4)$	$SU(2F)$	$U(1)_R$	$Z_{2(k+1)F}$
Q	$\square$	$\square$	$\frac{1-Fk}{F(k+1)}$	$-((2k+1)F - 1)$
X	$\square\square$	1	$\frac{1}{k+1}$	F
$M_{2i} = QX^{2i}Q$		$\boxminus$	$\frac{2F+4Fi+4k-2Fk}{F+Fk}$	$2(1 + F(i+1))$
$M_{2j+1} = QX^{2j+1}Q$		$\square\square$	$\frac{4F+4Fj+4k-2Fk}{F+Fk}$	$2 + 3F + 2Fj$

$$(3.1)$$

where $i = 0, ..., k$ and $j = 0, ..., k-1$. It is straightforward to check that this particle content saturates all anomaly matching conditions, including the discrete ones ($SU(2F)^3$, $SU(2F)^2U(1)_R$, $U(1)_R^3$, $U(1)_R$, $SU(2F)^2Z_{2(k+1)F}$, $U(1)_R^2Z_{2(k+1)F}$, $U(1)_RZ_{2(k+1)F}^2$, $Z_{2(k+1)F}$,

$Z^3_{2(k+1)F}$). In addition, for $k = 0$ it reproduces the s-confining $Sp(2F - 4)$ theory with $2F$ fundamentals.

Next we determine the confining superpotential for $k = 1$. We need to first determine the classical constraints in this theory, taking into account the F-flatness conditions. Just like in the previous section, we consider an $Sp(6F-4)$ with $6F$ dressed flavors $\mathcal{Q} = (Q, XQ, X^2Q)$.[*] The classical constraints were given in Ref. [2]: $\epsilon_{a_1 a_2 \cdots a_{6F}} \mathcal{M}^{a_3 a_4} \cdots \mathcal{M}^{a_{6F-1} a_{6F}} = 0$, $a_1, a_2 = 1, \cdots, 6F$. In our case, the meson matrix $\mathcal{M}$ is given by

$$\mathcal{M} = \begin{pmatrix} M_0 & M_1 & M_2 \\ -M_1 & M_2 & 0 \\ M_2 & 0 & 0 \end{pmatrix}, \tag{3.2}$$

which already incorporates the F-flatness condition. In the following, we will only consider the case of $F = 1$, that is $SU(2)$ with an adjoint and a single flavor, and a superpotential $h\mathrm{Tr}X^4$. In this case one can easily see that the tree-level superpotential indeed lifts the Coulomb branch of the theory. This is because the gauge group is just $SU(2)$, and the $h\mathrm{Tr}X^4$ superpotential is nothing but $(\mathrm{Tr}X^2)^2$. Then the equation of motion yields $(\mathrm{Tr}X^2)X = 0$, from which $\mathrm{Tr}X^2 = 0$, thus the mesons of (3.1) are indeed sufficient to describe the moduli space of the $F = 1, k = 1$ theory. We believe that the $\mathrm{Tr}X^{2p}$ operators are lifted by the F-flatness conditions for any k and F, even though we have not proven it.

The classical constraints for the $k = 1, F = 1$ theory are given by

$$\mathrm{Pf}\, M_2 = 0, \quad \mathrm{cof}\, M_1 \mathrm{Pf}\, M_2 = 0, \quad \det M_1 - \mathrm{Pf}\, M_2 \mathrm{Pf}\, M_0 = 0, \tag{3.3}$$

where of an antisymmetric $2N$ by $2N$ matrix A is defined by

$$\mathrm{Pf}\, A = \frac{1}{N!2^N} \epsilon^{i_1 i_2 \cdots i_{2N-1} i_{2N}} A_{i_1 i_2} \cdots A_{i_{2N-1} i_{2N}} = \sqrt{\det A}. \tag{3.4}$$

Note that for the case considered ($F = 1$) $\mathrm{Pf}\, M_0 = (M_0)_{12}$, $\mathrm{Pf}\, M_2 = (M_2)_{12}$. These constraints can be derived from the superpotential

$$\mathrm{Pf}\, M_2 \left(\det M_1 - \frac{1}{2}\mathrm{Pf}\, M_2 \mathrm{Pf}\, M_0 \right). \tag{3.5}$$

Note, however, that simply by dimensional reasons one can not have the instanton factor Λ^3 to be the only constant appearing in the superpotential, and the extra mass scale h^{-1} of the tree-level superpotential must appear in the confining superpotential as well. The correct form of the superpotential for $F = 1$ is given by

$$\frac{1}{h^2\Lambda^9}\mathrm{Pf}\, M_2 \left(\det M_1 - \frac{1}{2}\mathrm{Pf}\, M_2 \mathrm{Pf}\, M_0 \right). \tag{3.6}$$

---

[*]The indices are contracted as $X^{ij}J_{jk}Q^k$ etc with the symplectic matrix J.

One can check that this superpotential is invariant under all the global symmetries of the theory, including the anomalous ones if appropriate charges are assigned to h and to Λ^3. Note that this superpotential is indeed a three-instanton effect, and that it diverges if the tree-level superpotential is turned off, signaling that the description is valid only if a tree-level superpotential is present. Next we explain how this superpotential is generated by a three-instanton effect of the dual theory. Consider the theory with one more flavors, that is $SU(2)$ with an adjoint X, four fundamentals Q, and a superpotential $h\,\mathrm{Tr}\,X^4$. The duality is given in the table below.

	$SU(2)$	$SU(4)$
Q	$\square$	$\square$
X	$\square\square$	1

	$Sp(6)$	$SU(4)$
q	$\square$	$\square$
Y	$\square\square$	1
M_0	1	$\boxvert$
M_1	1	$\square\square$
M_2	1	$\boxvert$

(3.7)

The magnetic superpotential is given by

$$W_{magn} = -h\,\mathrm{Tr}\,Y^4 + \frac{h}{\mu^2}\left(M_0 qY^2 q + M_1 qYq + M_2 q^2\right).$$

(3.8)

The matching of scales is given by

$$\Lambda_{el}^2 \Lambda_{magn}^6 = \left(\frac{\mu}{h}\right)^4.$$

(3.9)

Integrating out a flavor from the electric theory corresponds to adding the linear term mM_0 to the magnetic superpotential (3.8), which forces a non-vanishing expectation value for $qY^2 q$, completely breaking the magnetic $Sp(6)$ gauge group. In order to obtain the superpotential (3.6), we consider the 3-instanton 't Hooft vertex of the broken magnetic $Sp(6)$ theory:

$$\tilde{Y}^{24}\lambda^{24}\tilde{q}^{12}\frac{\mu^{12}}{h^{12}\Lambda_{el}^6},$$

(3.10)

where we have already used the scale matching (3.9). We can use the $Y^*\lambda\tilde{Y}$ vertex twice to convert this to

$$\tilde{Y}^{22}\lambda^{22}Y^{*2}\tilde{q}^{12}\frac{\mu^{12}}{h^{12}\Lambda_{el}^6}.$$

(3.11)

Next we use the $\frac{h}{\mu^2}qY^2 qM_0$ superpotential term once to get

$$\tilde{Y}^{22}\lambda^{22}M_0\tilde{q}^{10}\frac{\mu^{10}}{h^{11}\Lambda_{el}^6}.$$

(3.12)

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Now we use the $q^*\lambda\tilde{q}$ vertex twice to get

$$\tilde{Y}^{22}\lambda^{20}M_0\tilde{q}^8 q^{*2}\frac{\mu^{10}}{h^{11}\Lambda_{el}^6},\tag{3.13}$$

and then the $\frac{h}{\mu^2}M_2 q^2$ superpotential coupling twice to get

$$\tilde{Y}^{22}\lambda^{20}M_0\tilde{q}^6\tilde{M}_2^2\frac{\mu^6}{h^9\Lambda_{el}^6}.\tag{3.14}$$

Next we use the $-hY^4$ superpotential coupling four times to get

$$\langle Y\rangle^8\tilde{Y}^{14}\lambda^{20}\tilde{q}^6 M_0\tilde{M}_2^2\frac{\mu^6}{h^5\Lambda_{el}^6}.\tag{3.15}$$

Now we use the remaining fermionic components to convert them to VEV's:

$$\langle Y\rangle^8\langle Y^*\rangle^{14}\langle q^*\rangle^6 M_0\tilde{M}_2^2\frac{\mu^6}{h^5\Lambda_{el}^6}.\tag{3.16}$$

The expectation values for q and Y are given by $v = (\mu^2 m/h)^{\frac{1}{4}}$. Because the superpotential has to be holomorphic in v, the integral over the instanton size has to give an additional factor of $(vv^*)^{20}$ in the denominator. Thus the superpotential term will be

$$\frac{\mu^6 M_0\tilde{M}_2^2}{v^{12}h^5\Lambda_{el}^6}.\tag{3.17}$$

Substituting $v^{12} = \mu^6 m^3/h^3$ and using the scale matching of the electric theory $m^3\Lambda_{el}^6 = \tilde{\Lambda}_{el}^9$ we get the instanton generated superpotential term

$$\frac{1}{h^2\tilde{\Lambda}_{el}^9}M_0 M_2^2,\tag{3.18}$$

which is one of the terms of (3.6). The other term can be presumably generated from the same three-instanton vertex by closing up the instanton legs with different vertices.

3.2 Sp with a traceless antisymmetric tensor and fundamentals

We present only the confining spectrum which satisfies the 't Hooft anomaly matching conditions $SU(2F)^3$, $SU(2F)^2U(1)_R$, $U(1)_R^3$, $U(1)_R$, $SU(2F)^2 Z_{(k+1)F}$, $U(1)_R^2 Z_{(k+1)F}$, $U(1)_R Z_{(k+1)F}^2$,

$Z^3_{(k+1)F}$ and $Z_{(k+1)F}$ for $Sp(2k(F-2))$ with a traceless antisymmetric tensor X ($\mathrm{Tr}JX = 0$), $2F$ fundamental fields Q and a tree-level superpotential $\mathrm{Tr}X^{k+1}$.

	$Sp(2k(F-2))$	$SU(2F)$	$U(1)_R$	$Z_{(k+1)F}$
X	$\boxminus$	1	$\frac{2}{k+1}$	F
Q	$\Box$	$\Box$	$-\frac{F\frac{k-2k-F}{}}{F(1+k)}$	$1-(-2+F)k$
$M_i = QX^iQ$		$\boxminus$	$\frac{2i}{1+k} - 2\frac{Fk-2k-F}{F(1+k)}$	$2+Fi+4k-2Fk$
$T_j = X^j$		1	$\frac{2j}{k+1}$	Fj

$$\text{(3.19)}$$

where $i = 0, \cdots, k-1$ and $j = 2, \cdots, k$. For $k = 1$ this theory reduces to the s-confining $Sp(2F-4)$ theory with $2F$ fundamentals described in [2].

It is interesting to note that some of the T_j operators may be missing from the classical flat directions. For instance, for $k = 3$, $F = 3$, one can easily show that T_2 vanishes due to the F-flatness conditions. There is however no contradiction, since for this particular case, T_2 has $U(1)_R$ charge one and Z_{12} charge 6. Therefore a mass term $m(T_2)^2$ is allowed in the superpotential, and it is removed from the moduli space.

One consistency check for confinement in this theory is to consider the classical flat direction of the form

$$X = i\sigma_2 \otimes \begin{pmatrix} 1 & & & \\ & \omega & & \\ & & \ddots & \\ & & & \omega^{k-1} \end{pmatrix}, \qquad \text{(3.20)}$$

where $\omega = e^{2i\pi/k}$, and every diagonal element in X is multiplied by the $F - 2$ dimensional unit matrix. This direction indeed satisfies the F-flatness condition $(JX)^k \propto 1$, where $J = i\sigma_2 \otimes 1$ is the symplectic matrix, and corresponds to the polynomial $T_k = \mathrm{Tr}X^{2k}$. The theory is indeed s-confining along this direction because it leaves an $(Sp(2(F-2)))^k$ gauge group unbroken with F flavors for each $Sp(2(F-2))$ factor, and it confines [2]. If k is non-prime, $k = lm$, however, one may worry about the following direction

$$X = i\sigma_2 \otimes \begin{pmatrix} 1 & & & \\ & \omega^l & & \\ & & \ddots & \\ & & & \omega^{l(k-1)} \end{pmatrix}, \qquad \text{(3.21)}$$

where again every diagonal element is multiplied by the $F - 2$ dimensional unit matrix. This direction is lifted quantum mechanically because it leaves an $(Sp(2l(F-2)))^m$ gauge group unbroken, and each of the $Sp(2l(F-2))$ factor develops the Affleck–Dine–Seiberg superpotential.

3.3 SO with an adjoint and vectors

The spectrum which satisfies the anomaly matching conditions $SU(F)^3$, $SU(F)^2U(1)_R$, $U(1)^3_R$, $U(1)_R$, $SU(F)^2Z_{2(k+1)F}$, $U(1)^2_RZ_{2(k+1)F}$, $U(1)_RZ^2_{2(k+1)F}$, $Z^3_{2(k+1)F}$, $Z_{2(k+1)F}$, $SU(F)^2Z_{2F}$, $U(1)^2_RZ_{2F}$, $U(1)_RZ^2_{2F}$, Z^3_{2F}, Z_{2F}, $U(1)_RZ_{2F}Z_{2(k+1)F}$, $Z^2_{2F}Z_{2(k+1)F}$ and $Z_{2F}Z^2_{2(k+1)F}$ is given by

	$SO(N)$	$SU(F)$	$U(1)_R$	$Z_{2(k+1)F}$	Z_{2F}
X	⊟	1	$\frac{1}{k+1}$	F	0
Q	□	□	$-\frac{1+Fk}{F(1+k)}$	$-1-F-2Fk$	1
$M_{2i}=QX^{2i}Q$		⊞	$\frac{2i}{1+k}-\frac{2(1+Fk)}{F(1+k)}$	$-2+2F(-1+i-2k)$	2
$M_{2j+1}=QX^{2j+1}Q$		⊟	$\frac{1+2j}{1+k}-\frac{2(1+Fk)}{F(1+k)}$	$-2+F(-1+2j-4k)$	2
$B=Q^{F-1}X^{\frac{N-F+1}{2}}$		⊡	$\frac{1+F+Fk}{F(1+k)}$	$1+2F(1+k)-F^2(1+k)$	$F-1$

where $i=0,\cdots,k$, $j=1,\cdots,k-1$, $N=(2k+1)F+3$, and the tree-level superpotential of the $SO(N)$ theory is $\mathrm{Tr}\,X^{2(k+1)}$.

For $k=0$ this spectrum reproduces that of the $SO(F+3)$ theory with F flavor, which has a confining branch (of which this is the generalization for $k>0$), and a branch with a dynamically generated superpotential. We assume that this multiple branch structure persists for the case with $k>0$, therefore the above spectrum describes only one of the possible branches of the theory. Naively, for $k=0$ this spectrum does not exactly agree with the spectrum of [4], since the baryon operator is $B=Q^{F-1}X^2$ here, while the baryon in [4] is $W^2_\alpha Q^{F-1}$. However, in the presence of the tree-level mass term in the superpotential $M_X\mathrm{Tr}\,X^2$, these two operators can be identified using the chiral anomaly equation [15]. The argument is the following. We start from the part $X^{ij}X^{kl}$ in the operator B, where none of the indices i,j,k,l are the same because they are contracted with an epsilon tensor. We contract two X fields via a one-loop triangle diagram with two external gauge fields. This is the same calculation as the contribution of the Pauli–Villars field in the Konishi anomaly [21], except that the gauge indices are not contracted between two fields. The result is proportional to $\frac{1}{32\pi^2M_X}$. The two gauge vertices in the triangle diagram must transform the indices of X^{ij} to those of X^{kl}, and hence require $SO(N)$ generators M^{ik} and M^{jl}, or M^{il} and M^{jk}. Therefore the resulting gauge fields have indices $W^{ik}_\alpha W^{\alpha jl}-W^{il}_\alpha W^{\alpha jk}$. Now recall that all these indices were contracted with the epsilon tensor, and the above two terms give identical contributions as $-W^{ij}_\alpha W^{\alpha kl}$. Therefore the net result is to replace $X^{ij}X^{kl}$ by $-W^{ij}_\alpha W^{\alpha kl}$ apart from numerical factors and $1/M_X$.

Note that in the spectrum the operator B could have been substituted by the operator $W^{2(k+1)}_\alpha Q^{(k+1)F-1}X^{k(F(1+k)-2k-3)}$ without the modification of any of the continuous global anomalies, and even the $k=0$ limit of this operator is correct. The only way to distinguish between this operator and the operator B which is the correct confined degree of freedom is by considering the discrete anomaly matching conditions [15], which is only satisfied if one uses the operator B.

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3.4 SO with a traceless symmetric tensor and vectors

The spectrum which satisfies the anomaly matching conditions $SU(F)^3$, $SU(F)^2 U(1)_R$, $U(1)_R^3$, $U(1)_R$, $SU(F)^2 Z_{(k+1)F}$, $U(1)_R^2 Z_{(k+1)F}$, $U(1)_R Z_{(k+1)F}^2$, $Z_{(k+1)F}^3$, $Z_{(k+1)F}$, $SU(F)^2 Z_{2F}$, $U(1)_R^2 Z_{2F}$, $U(1)_R Z_{2F}^2$, Z_{2F}^3, Z_{2F}, $U(1)_R Z_{2F} Z_{(k+1)F}$, $Z_{2F}^2 Z_{(k+1)F}$ and $Z_{2F} Z_{(k+1)F}^2$ is given by

	$SO(N)$	$SU(F)$	$U(1)_R$	$Z_{(k+1)F}$	Z_{2F}
X	⊟	1	$\frac{2}{k+1}$	F	0
Q	□	□	$-\frac{(-2-F+4k+Fk)}{F(1+k)}$	$-(N-2)$	1
$M_j = QX^jQ$		⊟	$\frac{2jF-2(-2-F+4k+Fk)}{F(1+k)}$	$jF - 2(N-2)$	2
B		$\bar{□}$	$\frac{-2+F+4k+Fk}{F(1+k)}$	$\frac{2+8k-F^2k(1+k)+F(2-4k-6k^2)}{2}$	$(kF-1)$

$$(3.22)$$

where $j = 0, \cdots, k-1$, $N = k(F+4) - 1$ and the tree-level superpotential of the $SO(N)$ theory is $\operatorname{Tr} X^{k+1}$, and the field content of B is $W_\alpha^{2k} Q^{kF-1} X^{\frac{Fk(k-1)}{2}+(k+1)^2}$. The contraction of the gauge indices in B is presumably

$$B = Q^F (XQ)^F (X^2 Q)^F \cdots (X^{k-2}Q)^F (X^{k-1}Q)^{F-1} (W_\alpha)^2 (XW_\alpha)^2 \cdots (X^{k-1}W_\alpha)^2. \quad (3.23)$$

For $k = 1$ this theory reproduces again the $SO(F+3)$ theory with F vectors [4], and therefore similar multi-branch structure is expected in this case as well.

One may worry about a possible Coulomb branch along the direction

$$X = v \operatorname{diag}(x_1, x_2, \cdots, x_{k(F+4)-1}). \quad (3.24)$$

To satisfy the F-flatness condition $X^k \propto 1$, all the eigenvalues x_i must be k-th root of unity. The tracelessness of X also imposes the condition $\sum_i x_i = 0$. If k is prime, one cannot satisfy the tracelessness condition with $k(F+4) - 1$ eigenvalues which are all k-th roots of unity. Therefore, there is no classical flat direction of this form. If k is not prime, one may find divisors of k which sum up to $k(F+4) - 1$; i.e., $p_1, \cdots, p_m$ are all divisors of k and $\sum_j p_j = k(F+4) - 1$.[†] Then using the quotients $q_m = k/p_m$, one can satisfy both the tracelessness and F-flatness with

$$X = v\operatorname{diag}(1, \omega^{q_1}, \cdots, \omega^{(p_1-1)q_1}, 1, \omega^{q_2}, \cdots, \omega^{(p_2-1)q_2}, \cdots, 1, \omega^{q_m}, \cdots, \omega^{(p_m-1)q_m}), \quad (3.25)$$

where $\omega = e^{2i\pi/k}$. This direction corresponds to the polynomials $\operatorname{Tr} X^{p_j}$. It is, however, lifted quantum mechanically. The proof is by contradiction. Suppose the direction is not lifted by quantum effects. Since there are F vectors, the unbroken group should not contain a factor larger than $SO(F+4)$ to avoid Affleck–Dine–Seiberg superpotential. On the other hand,

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[†]For instance, for $k = 6$ and $F = 1$, $k(F+4) - 1 = 29$, X can be given by repeating $(1, \omega, \omega^2, \omega^3, \omega^4, \omega^5)$ four times, and the remaining five eigenvalues by $(1, \omega^3, 1, \omega^2, \omega^4)$.

there are $k(F+4)-1$ eigenvalues with only k possibilities $1, \omega, \cdots, \omega^{k-1}$, and hence the only allowed case is repeating all of the above k-th roots of unity $F+4$ times except one of them repeated $F+3$ times. Then, however, X is not traceless, and hence the assumption is not correct.

3.5 SU with an antisymmetric flavor and fundamental flavors

The spectrum which satisfies the anomaly matching conditions $SU(F)^3$, $SU(F)^2U(1)_X$, $SU(F)^2U(1)_B$, $SU(F)^2U(1)_R$, $U(1)_X^3$, $U(1)_X$, $U(1)_B^3$, $U(1)_B$, $U(1)_R^3$, $U(1)_R$, $U(1)_X^2U(1)_B$, $U(1)_XU(1)_B^2$, $U(1)_X^2U(1)_R$, $U(1)_XU(1)_R^2$, $U(1)_B^2U(1)_R$, $U(1)_BU(1)_R^2$, $U(1)_XU(1)_BU(1)_R$, $SU(F)^2Z_{2(k+1)F}$, $Z_{2(k+1)F}$, $Z_{2(k+1)F}^3$, $U(1)_X^2Z_{2(k+1)F}$, $U(1)_XZ_{2(k+1)F}^2$, $U(1)_B^2Z_{2(k+1)F}$, $U(1)_BZ_{2(k+1)F}^2$, $U(1)_R^2Z_{2(k+1)F}$, $U(1)_RZ_{2(k+1)F}^2$, $U(1)_XU(1)_BZ_{2(k+1)F}$, $U(1)_XU(1)_RZ_{2(k+1)F}$ and $U(1)_BU(1)_RZ_{2(k+1)F}$ is given by

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)_X$	$U(1)_B$	$U(1)_R$	$Z_{2(k+1)F}$
X	⊟	1	1	1	0	$\frac{1}{k+1}$	F
$\bar{X}$	⊟	1	1	-1	0	$\frac{1}{k+1}$	F
Q	□	□	1	0	1	$1 - \frac{N+2k}{F(k+1)}$	$3+4k+F$
$\bar{Q}$	$\bar{□}$	1	□	0	-1	$1 - \frac{N+2k}{F(k+1)}$	$3+4k+F$
M_j		□	□	0	0	$\frac{2+2Fj+4k-2Fk}{F(k+1)}$	$2jF-2(N-2)$
P_r		⊟	1	-1	2	$\frac{2+F+4k-2Fk+2Fr}{F(1+k)}$	$(2r+1)F-2(N-2)$
$\bar{P}_r$		1	⊟	1	-2	$\frac{2+F+4k-2Fk+2Fr}{F(1+k)}$	$(2r+1)F-2(N-2)$
B		$\bar{□}$	1	$\frac{N-F+1}{2}$	$F-1$	$\frac{-1+F-2k+Fk}{F(1+k)}$	$-3-4k-F^2(1+k)$
$\bar{B}$		1	$\bar{□}$	$-\frac{N-F+1}{2}$	$-F+1$	$\frac{-1+F-2k+Fk}{F(1+k)}$	$-3-4k-F^2(1+k)$
T_i		1	1	0	0	$\frac{2i}{k+1}$	$2Fi$

$$\tag{3.26}$$

where $j = 0, \cdots, k$, $r = 0, \cdots, k-1$, $i = 1, \cdots, k$, $N = (2k+1)F - 4k - 1$, and the tree-level superpotential of the $SU(N)$ theory is $\text{Tr}(X\bar{X})^{k+1}$, and the composite operators are given by

$$M_j = Q(X\bar{X})^j\bar{Q},$$
$$P_r = Q(X\bar{X})^r\bar{X}Q,$$
$$\bar{P}_r = \bar{Q}(X\bar{X})^rX\bar{Q},$$
$$B = X^{\frac{N-F+1}{2}}Q^{F-1},$$
$$\bar{B} = \bar{X}^{\frac{N-F+1}{2}}\bar{Q}^{F-1},$$
$$T_i = (X\bar{X})^i. \tag{3.27}$$

For $k = 0$ this theory reproduces the s-confining $SU(F-1)$ theory with F flavors. Note that there are no P or T operators for $k = 0$.

When $N = (2k+1)F - 4k - 1$ is even (*i.e.,* F odd), there is an additional classical flat direction:

$$X = i\sigma_2 \otimes \text{diag}(x_1, \cdots, x_{N/2})v, \qquad \bar{X} = 0. \tag{3.28}$$

This direction obviously satisfies the F-flatness $\bar{X}(X\bar{X})^k = (X\bar{X})^k X = 0$ for $k > 0$, and corresponds to the gauge-invariant polynomial PfX. D-flatness requires $X^\dagger X \propto 1$ and hence all the eigenvalues have the same absolute value. Moreover, a general $SU(N)$ gauge transformation can make all the eigenvalues to be equal, and hence $X = i\sigma_2 \otimes 1_{N/2}v$, which leaves an $Sp(N)$ subgroup unbroken. The low-energy theory then is an $Sp(N)$ gauge theory with an anti-symmetric tensor $\bar{X}$ with superpotential $\text{Tr}\bar{X}^{k+1}$. The particle content is precisely the same as the model discussed in Section 3.2 except for the trace part of $\bar{X}$. The gauge group $N = (2k+1)F - 4k - 1$ is larger than the confining case for which $N = 2k(F-2)$ discussed in Section 3.2 by $\Delta N = F - 1 \geq 2$, because asymptotic freedom of the original $SU(N)$ theory requires $F \geq 3$. Then the theory is expected to develop an Affleck–Dine–Seiberg type superpotential and the direction is removed from the quantum moduli space for $F \geq 4$.[‡]

3.6 *SU* with a symmetric flavor and fundamental flavors

The spectrum which satisfies the anomaly matching conditions $SU(F)^3$, $SU(F)^2U(1)_X$, $SU(F)^2U(1)_B$, $SU(F)^2U(1)_R$, $U(1)_X^3$, $U(1)_X$, $U(1)_B^3$, $U(1)_B$, $U(1)_R^3$, $U(1)_R$, $U(1)_X^2U(1)_B$, $U(1)_XU(1)_B^2$, $U(1)_X^2U(1)_R$, $U(1)_XU(1)_R^2$, $U(1)_B^2U(1)_R$, $U(1)_BU(1)_R^2$, $U(1)_XU(1)_BU(1)_R$, $SU(F)^2Z_{2(k+1)F}$, $Z_{2(k+1)F}$, $Z_{2(k+1)F}^3$, $U(1)_X^2Z_{2(k+1)F}$, $U(1)_XZ_{2(k+1)F}^2$, $U(1)_B^2Z_{2(k+1)F}$, $U(1)_BZ_{2(k+1)F}^2$, $U(1)_R^2Z_{2(k+1)F}$, $U(1)_RZ_{2(k+1)F}^2$, $U(1)_XU(1)_BZ_{2(k+1)F}$, $U(1)_XU(1)_RZ_{2(k+1)F}$

[‡]The case $F = 3$ is not fully understood. The low-energy $Sp(N)$ dynamics is expected to give a quantum modified moduli space, while the trace part of $\bar{X}$ interacts with the traceless part of $\bar{X}$ via the tree-level superpotential. It is suggestive that the PfX and Pf$\bar{X}$ operators can be added to the confining spectrum without spoiling the anomaly matching conditions only when $F = 3$. It is likely that the theory is still confining together with these operators.

and $U(1)_B U(1)_R Z_{2(k+1)F}$ is given by

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)_X$	$U(1)_B$	$U(1)_R$	$Z_{2F(k+1)}$
X	$\square$	1	1	1	0	$\frac{1}{k+1}$	F
$\bar{X}$	$\overline{\square}$	1	1	-1	0	$\frac{1}{k+1}$	F
Q	$\square$	$\square$	1	0	1	$1-\frac{F(1+2k)+2k-1}{F(1+k)}$	$-(N+2)$
$\bar{Q}$	$\overline{\square}$	1	$\square$	0	-1	$1-\frac{F(1+2k)+2k-1}{F(1+k)}$	$-(N+2)$
M_j		$\square$	$\square$	0	0	$\frac{2+2Fj-4k-2Fk}{F(1+k)}$	$2jF-2(N+2)$
P_r		$\square\square$	1	-1	2	$\frac{2+F-4k-2Fk+2Fr}{F(1+k)}$	$(2r+1)F-2(N+2)$
$\bar{P}_r$		1	$\square\square$	1	-2	$\frac{2+F-4k-2Fk+2Fr}{F(1+k)}$	$(2r+1)F-2(N+2)$
B		$\overline{\square}$	1	$\frac{N-F+1}{2}$	$F-1$	$\frac{-1+F+2k+Fk}{F(1+k)}$	$1+F^2(1+k)+4k$
$\bar{B}$		1	$\overline{\square}$	$-\frac{N-F+1}{2}$	$1-F$	$\frac{-1+F+2k+Fk}{F(1+k)}$	$1+F^2(1+k)+4k$
T_i		1	1	0	0	$\frac{2i}{k+1}$	$2iF$
b		1	1	$N-F$	$2F$	$\frac{1}{1+k}$	$-F(N+F+4)$
$\bar{b}$		1	1	$F-N$	$-2F$	$\frac{1}{1+k}$	$-F(N+F+4)$

$$\text{(3.29)}$$

where $j=0,\cdots,k$, $r=0,\cdots,k-1$, $i=1,\cdots,k$, $N=(2k+1)F+4k-1$, and the tree-level superpotential of the $SU(N)$ theory is $\text{Tr}(X\bar{X})^{k+1}$. The composite operators are given by

$$M_j = Q(X\bar{X})^j \bar{Q},$$
$$P_r = Q(X\bar{X})^r \bar{X} Q,$$
$$\bar{P}_r = \bar{Q}(X\bar{X})^r X \bar{Q},$$
$$B = W_\alpha^{2k} Q^{-1+F+Fk} \bar{Q}^{Fk} X^{k(F+k+Fk)} \bar{X}^{k(-2+k+Fk)},$$
$$\bar{B} = W_\alpha^{2k} \bar{Q}^{-1+F+Fk} Q^{Fk} \bar{X}^{k(F+k+Fk)} X^{k(-2+k+Fk)},$$
$$T_i = (X\bar{X})^i,$$
$$b = Q^{2F} X^{N-F},$$
$$\bar{b} = \bar{Q}^{2F} \bar{X}^{N-F}.$$

$$\text{(3.30)}$$

The color indices κ, λ and flavor indices i, j in b are contracted by two epsilon tensors each: $b = \epsilon_{\kappa_1 \cdots \kappa_N} \epsilon_{\lambda_1 \cdots \lambda_N} \epsilon^{i_1 \cdots i_F} \epsilon^{j_1 \cdots j_F} Q_{i_1}^{\kappa_1} \cdots Q_{i_F}^{\kappa_F} Q_{j_1}^{\lambda_1} \cdots Q_{j_F}^{\lambda_F} X^{\kappa_{F+1} \lambda_{F+1}} \cdots X^{\kappa_N \lambda_N}$. The gauge contraction in the operator B is presumably given by

$$\begin{aligned} B = \ & (XW_\alpha)^2 (X(X\bar{X})W_\alpha)^2 \cdots (X(X\bar{X})^{k-1}W_\alpha)^2 Q^F ((X\bar{X})Q)^F \cdots ((X\bar{X})^k Q)^{F-1} \\ & (X\bar{Q})^F (X(X\bar{X})\bar{Q})^F \cdots (X(X\bar{X})^{k-1}\bar{Q})^F, \end{aligned}$$

with one epsilon tensor and similarly for the operator $\bar{B}$ with $Q \leftrightarrow \bar{Q}$ and $X \leftrightarrow \bar{X}$. For $k=0$ this theory again reproduces the s-confining $SU(F-1)$ theory with F flavors. Note that in the $k=0$ case there are no P_i and T_i operators, B and $\bar{B}$ are just the usual baryons Q^F and

$\bar{Q}^F$, and the operators b and $\bar{b}$ are complete singlets under all symmetries except the $U(1)_R$ under which they carry R charge one for $k = 0$. Thus presumably there is a mass term $b\bar{b}$ in the confining superpotential for $k = 0$, which eliminates these fields from the low-energy spectrum. It would be very interesting to see this explicitly happening by examining the actual form of the confining superpotential for arbitrary values of k.

In addition to the above operators which describe the quantum moduli space, there is a classical flat direction

$$X = v \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix}, \qquad \bar{X} = 0. \tag{3.31}$$

This direction satisfies both the F-flatness $\bar{X}(X\bar{X})^k = (X\bar{X})^k X = 0$ and the D-flatness $X^\dagger X \propto 1$ conditions, and corresponds to the operator $\det X$. This flat direction, however, is removed from the moduli space quantum mechanically. It leaves an $SO(N)$ gauge group unbroken, with $2F$ vectors and a symmetric tensor $\bar{X}$ with the superpotential $\mathrm{Tr}\bar{X}^{k+1}$. This is precisely the particle content of the model discussed in Section 3.4, except for the trace part of $\bar{X}$. The gauge group, however, is larger, $N = (2k+1)F + 4k - 1 = [k(2F+4) - 1] + F$. Therefore the $SO(N)$ dynamics is expected to produce an Affleck–Dine–Seiberg type superpotential and the flat direction is lifted quantum mechanically for $F > 1$.[§]

4 Conclusions

In this paper we have examined $N = 1$ supersymmetric gauge theories which become confining after a suitable tree-level superpotential is added. These theories are obtained by examining the cases when the dual gauge groups of Ref. [13, 14, 11, 18, 19, 20] become trivial. We find that in all cases when the dual gauge group reduces to the trivial group the theory is confining at the origin of the moduli space with a set of composites satisfying the 't Hooft anomaly matching conditions. A confining superpotential for these composites, which is necessary in order to reproduce the classical constraints, is generated by the strong dynamics. This superpotential can be fixed in most cases by considering "dressed quarks", and examine the classical constraints of the s-confining theory of these dressed flavors. We

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[§] The case $F = 1$ is not fully understood. From the analogy to the $SO(N)$ theory with $N - 4$ vectors [4], the low-energy $SO(6k)$ dynamics is expected to give two branches, one confining with spontaneous discrete symmetry breakdown [15] and no confining superpotential, and the other with run-away behavior. Unlike in the previous Section, the $\det X$ and $\det \bar{X}$ operators cannot be added to the spectrum without spoiling the anomaly matching conditions. A likely possibility is that low-energy $SO(6k)$ dynamics is forced to choose the branch with the run-away behavior and the flat direction is removed from the quantum moduli space. Another possibility is that the Z_2 instanton effect in $SU(6k)/SO(6k)$ [17] induces a term in the superpotential which leads to a run-away behavior. We leave this issue for future investigation.

have shown several examples of such theories, and in some cases we have completely fixed the confining superpotential, and showed how this superpotential is generated in the dual gauge group by instantons.

An interesting question is how to classify the sort of confining theories examined in this paper. Most confining theories can be simply found, because the matter content obeys an index condition $\mu_{matter} = \mu_{adjoint} + 2$ for s-confining theories and $\mu_{matter} = \mu_{adjoint}$ for theories with a quantum modified constraint [6], where μ is the Dynkin index. However, in the theories presented in this paper the size of the confining gauge group also depends on the form of the tree-level superpotential, therefore a simple index constraint does not seem to be possible. This might be an advantage to these models compared to the ordinary s-confining ones, since the index constraint restricted the possible s-confining theories to a rather small set, with limited sizes and varieties of global symmetries. It would be very interesting to find a general way of analyzing these new confining theories without having to refer to the dualities of the theories with a bigger matter content, and to establish which confining spectra could be obtained this way.

Acknowledgements

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