New design method of permanent magnets by using the finite element method

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NEW DESIGN METHOD OF PERMANENT MAGNETS BY USING THE FINITE ELEMENT METHOD

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ABSTRACT

A new method for determining the shapes and sizes of magnets which produce the prescribed flux densities by using the finite element method has been developed.

In this paper, the new technique is explained briefly, and then the finite element formulation for non-linear analysis is derived. Finally, the usefulness of the technique is shown by applying this method to the design of magnetic circuits.

1. INTRODUCTION

We have already reported the method for determining the lengths of magnets which produce prescribed flux densities [1],[2],[3]. A new technique for determining not only the lengths but also the widths and shapes of magnets has been developed. The non-linear magnetic circuit composed of several magnets can be designed numerically by using this method without any experience.

2. OUTLINE OF NEW METHOD

A magnetic field produced by permanent magnets is controlled by changing (1) the shapes, (2) sizes, (3) positions, (4) directions of magnetization or (5) materials of magnets. (3) to (5) are usually given in practical designs. Therefore, it is necessary to determine the shapes and sizes of magnets which produce the prescribed flux densities at specified points. When the shapes or sizes of magnets are unknown variables, the Rayleigh-Ritz matrix equation for the magnetic field becomes non-linear. Therefore, the equation cannot be easily solved. This difficulty is avoided by a newly developed method. The outline of the method is explained by the following two typical examples: (1) rectangular magnet which is simple and the most popular, (2) quadrangular magnet which is fundamental for an arbitrary shaped magnet.

2.1 Rectangular magnet

Let us consider to determine only one dimension, namely, the width W or the length L of a magnet shown in Fig.1. The flux density B_0 in the direction denoted by arrow at a point P is prescribed. Figure 1 (a) shows an example adjusting the width W so as to produce B_0 at point P. The length L is fixed. Figure 1 (b) shows a model adjusting the length L. The width W is fixed. The material of magnet is given and the direction of magnetization is chosen to be the same as that of the flux density B_0.

The width W or length L of the magnet is suitably presumed. The flux density B produced by the presumed magnet at point P is usually different from B_0. A rectangular additional magnet denoted by the hatched part in Fig.1 is introduced in order to adjust the difference $(B-B_0)$ of flux density.

The width Dwa or length DLa of the additional magnet, which is small enough compared with that of the presumed magnet, is assumed adequately. The magnetization Mwa or MLa of the additional magnet is calculated as the so-called inverse problem [1]. As the number of the given flux density is only one, the number of unknown variable, namely, the number of the magnetization of additional magnet must be only one. By the way, the values of magnetizations M in the presumed magnet are nearly all the same with the mean value $\overline{M}$ of them. As both the presumed magnet and the additional magnet should be essentially made of the same material, the magnetization Mwa or MLa must be nearly the same with $\overline{M}$. For this reason, the additional magnet is replaced by a rectangular equivalent magnet of which the magnetization is $\overline{M}$ and produces the same flux density at point P as the additional magnet. The width Dw or length DL of the equivalent magnet is calculated by the following equations:

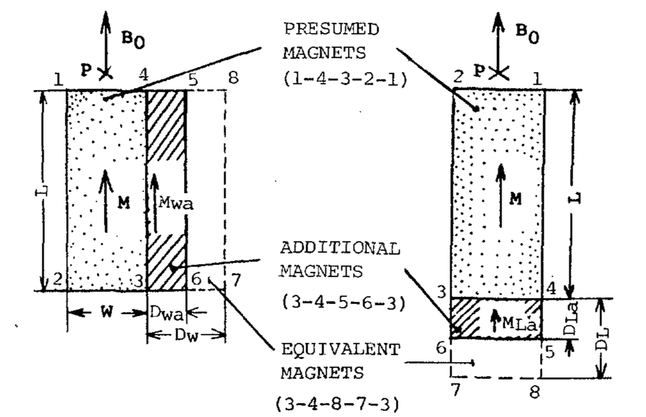
$$D_W=\frac{M_{wa}}{\overline{M}}D_{wa}\ ,\qquad D_L=\frac{M_{La}}{\overline{M}}D_{La} \tag{1}$$

These equations are derived on the assumption that the width or length of the magnet which produces the same flux density is inversely proportional to the magnetization as shown in Fig.2.

The modified size W* or L* of the magnet which produces B_0 is calculated from

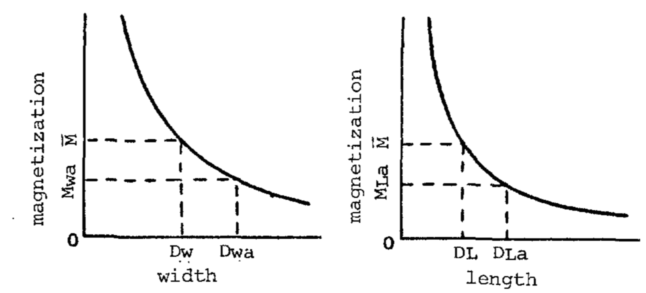
$$W^*=W+D_W\ ,\quad L^*=L+D_L \tag{2}$$

Because of the assumptions that the distribution of magnetization in the magnet is uniform and the dimension of the magnet is inversely proportional to the magnetization, the magnet obtained by the above procedure does not usually produce the prescribed flux density B_0 precisely. Therefore, iterations are necessary until the desired accuracy is obtained by



(a) model adjusting width (b) model adjusting length

Fig.1 Design of rectangular magnets.



(a) relationship between width and magnetization (b) relationship between length and magnetization

Fig.2 Relationships between size and magnetization which produce the same flux density.

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modifying the size of the magnet.

The procedure of determining the width of magnet shown in Fig.1(a) is denoted in Fig.3.

① : The width W of magnet is presumed taking into account the given flux density B_0. The width Da of the additional magnet is assumed corresponding to the width W.

② : The initial values of vector potentials{A}in the whole region and the magnetization Ma of the additional magnet are set.

③ : The increments{δA}and δMa can be calculated by the Newton-Raphson iteration scheme taking into account the non-linearity of the presumed magnet.

④ : The mean value $\overline{M}$ of the magnetizations is calculated from the vector potentials {A} in the presumed magnet.

⑤ : The width D of the equivalent magnet is calculated from (1). Some iterations are necessary until the accurate width which produces the given flux density can be obtained.

Though the number of unknown variable is only one in Fig.1, this method can also be applied when the number is increased. The details for plural cases will be explained in the following section.

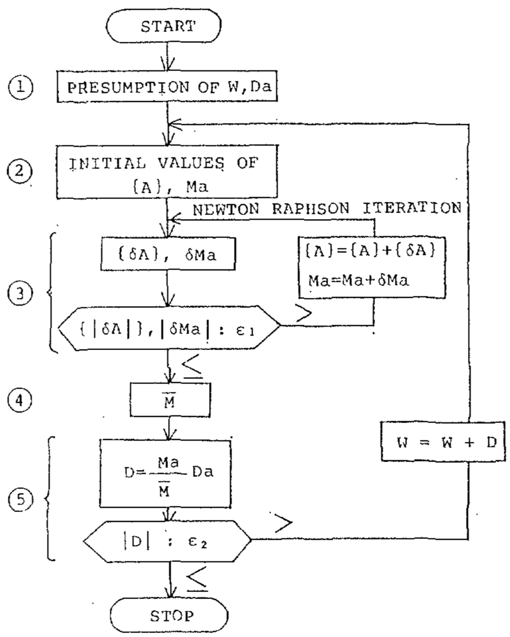

Fig.3 Flow chart.

2.2 Quadrangular magnet

By expanding the above-mentioned procedure, the shape and size of a quadrangle in Fig.4 ,which produces prescribed flux densities B_{10} to B_{30} at points P_1 to P_3, can be determined. In this case, the material and the direction of magnetization are given. The side 1-4 is fixed and the side 1-2-3-4 is adjustable. The shapes of the additional magnets denoted by the hatched parts in Fig.4 are trapezoids. The widths Da_1 to Da_3 of additional magnets are adequately assumed, and the widths D1 to D3 of the equivalent magnets are calculated in the same way as Section 2.1.

The shape of the modified magnet is uneven near the corners D_1 and D_3 of the presumed magnet. Therefore, for example, a new point 14 is defined at the center point between points 10 and 11. Thus a new shape (9-14-15-13-9) is obtained. Some iterations are necessary until the desired accuracy can be obtained by setting the additional magnets on the sides of the magnet. In this case, because of the mutual influences of magnets, the number of iteration is larger than that in Section 2.1.

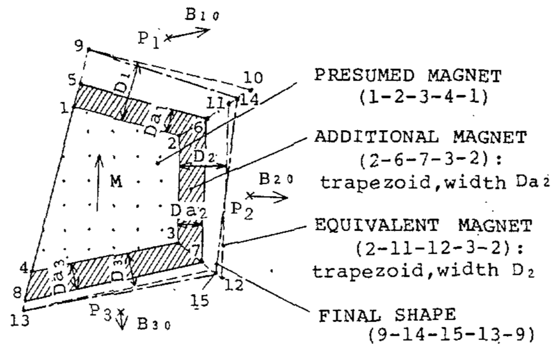

Fig.4 Design of an arbitrary shaped magnet.

It is also possible to determine the shape of the polygonal magnet by using the same method. If the surface of the magnet is curved, the shape of the magnet can be determined by approximating the curve as polygon.

The number of the additional magnets must be the same as the number of the prescribed flux densities. Here let us consider the number of flux densities. The flux density is a vector. Therefore, the value and the direction, or two components, for example, the x- and y- components are usually defined. Then, two unknown dimensions are determined corresponding to each point at which the flux density is prescribed. But sometimes we need specify only one component , for example, the x- component and need not control the other one. In such a case, one unknown dimension is determined corresponding to each point.

3. FINITE ELEMENT FORMULATION

3. 1 Rayleigh-Ritz matrix equation

The partial derivative of the energy χ of an element e in the additional magnet is given as [4]

$$\frac{\partial \chi}{\partial A_i} = \nu_0 U_i - J_{mi} \tag{3}$$

where Ai is the vector potential of node i. For the first-order finite element method, Ui and Jmi are written as follows:

$$U_i = \sum_{j=1}^{3} S_{ij} A_j \tag{4}$$

$$S_{ij} = \frac{1}{4\Delta^{(e)}}(c_i c_j + d_i d_j) \tag{5}$$

$$J_{mi} = \frac{\nu_0}{2} T_{\theta i} M a \tag{6}$$

$$T_{\theta i} = d_i \cos\theta - c_i \sin\theta \tag{7}$$

Where $\Delta^{(e)}$ and ν_0 denote the area of element e and the reluctivity of air, respectively. Ma is the magnetization of the additional magnet. θ is the angle which shows the direction of magnetization and is measured from the x-axis. ci,di are denoted as follows:

$$\left.\begin{aligned} c_i &= y_j - y_k \\ d_i &= x_k - x_j \end{aligned}\right\} \tag{8}$$

with the other coefficients obtained by a cyclic permutation of subscripts in the order i, j, k.

By replacing Ma in (6) with Mat and substituting (4) and (6) into (3) and expanding it in a multidimensional Taylor's series in increments of δA_j and δMat, the following equation for the Newton-Raphson iteration scheme is obtained.

$$\frac{\partial^2 \chi}{\partial A_i \partial A_j}\delta A_j + \frac{\partial^2 \chi}{\partial A_i \partial M_{at}}\delta M_{at} = -\frac{\partial \chi}{\partial A_i} \tag{9}$$

Where Mat is the magnetization of the t'th additional magnet. (9) can be rewritten as follows:

$$\nu_0 S_{ij}\delta A_j - \frac{\nu_0}{2} T_{\theta i}\delta M_{at} = -\nu_0 U_i + \frac{\nu_0}{2} T_{\theta i} M_{at} \tag{10}$$

If δM_{at} in (10) is treated as an unknown variable, the matrix equation for the whole region containing magnets is obtained as follows:

$$\left[\begin{bmatrix} H_{11} & \cdots & H_{1n} \\ \vdots & H_{ij} & \vdots \\ H_{n1} & \cdots & H_{nn} \end{bmatrix} \begin{bmatrix} C_{11} & \cdots & C_{1r} \\ \vdots & C_{it} & \vdots \\ C_{n1} & \cdots & C_{nr} \end{bmatrix}\right] \begin{Bmatrix} \begin{Bmatrix} \delta A_1 \\ \vdots \\ \delta A_j \\ \vdots \\ \delta A_n \end{Bmatrix} \\ \begin{Bmatrix} \delta M_{a1} \\ \vdots \\ \delta M_{at} \\ \vdots \\ \delta M_{ar} \end{Bmatrix} \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ \vdots \\ Q_j \\ \vdots \\ Q_n \end{Bmatrix} \tag{11}$$

where n is the number of nodes of which the vector potentials are unknown, and r is the number of unknown magnetizations of additional magnets. [H] is the so-called coefficient matrix. Its element H_{ij} is calculated by

$$H_{ij} = \begin{cases} \sum_{R_i} \nu_0 S_{ij} & \text{(additional magnet or air)} \\ \sum_{R_i} \nu_0 S_{ij} - \sum_{R_i} \dfrac{\partial J_{mi}}{\partial A_j} & \text{(presumed magnet)} \end{cases} \tag{12}$$

where $\sum_{R_i}$ means the summation for the total elements containing node i. [C] is the coefficient matrix of $\{\delta M_a\}$. For example, the coefficient C_{it} corresponding to δM_{at} is denoted by

$$C_{it} = -\sum_{R_i} \frac{\nu_0}{2} T_{\theta i} \cdot \delta i \tag{13}$$

where δi is unity when the node i is in the t'th additional magnet and zero when the node i is outside of the t'th additional magnet. Q is a function of equivalent magnetic current densities [5] of presumed magnet and the known vector potentials on the Dirichlet boundaries. Q_i is denoted by

$$Q_i = -\nu_0 U_i + \frac{\nu_0}{2} T_{\theta i} M_{at} \tag{14}$$

As the number of the equations is n, and the number of the unknown variables is (n+r), (11) cannot be solved. Therefore, the following relationships among the vector potentials and flux densities should be introduced.

3.2 Relationships among vector potentials and flux densities

In the design of permanent magnets the following flux densities are usually prescribed: (a) the x-and y-components B_{x0} and B_{y0} of the flux density at a specified point, (b) the component B_{k0} in an arbitrary direction of the flux density. The relationships among vector potentials and flux densities for the case (a) are already discussed in Reference [1]. The relationships for the latter case are investigated in this paper.

Figure 5 shows a first-order triangular element e. The flux density B_{k0} at point P in the element is prescribed. α is the angle measured from the x-axis.

As B_{k0} is the projection of the vector of flux density at point P, it is denoted by

$$B_{k0} = \frac{1}{2\Delta^{(e)}} \sum_{k=1}^{3} T_{\alpha k} A_{ke} \tag{15}$$

$T_{\alpha k}$ is obtained by replacing the subscript θ of $T_{\theta i}$ in (7) with α. Replacing the difference of the right- and left-hand sides of (15) by η, and expanding η in a multidimensional Taylor's series in the same way as (9), the following equation can be obtained:

$$\sum_{j=1}^{3} \frac{\partial \eta}{\partial A_j} \delta A_j = -\eta \tag{16}$$

By substituting η into (16), one obtains

$$\frac{T_{\alpha j}}{2\Delta^{(e)}} \delta A_j = -\frac{1}{2\Delta^{(e)}} \sum_{k=1}^{3} T_{\alpha k} A_k + B_{k0} \tag{17}$$

If there exist r independent prescribed flux densities, r relationships similar to (17) can be obtained as follows:

$$\begin{bmatrix} F_{11} & \cdots & F_{1n} \\ \vdots & F_{ij} & \vdots \\ F_{r1} & \cdots & F_{rn} \end{bmatrix} \begin{Bmatrix} \delta A_1 \\ \vdots \\ \delta A_j \\ \vdots \\ \delta A_n \end{Bmatrix} = \begin{Bmatrix} K_1 \\ \vdots \\ K_j \\ \vdots \\ K_r \end{Bmatrix} \tag{18}$$

For example, F_{ij} is the coefficient of δA_j and K_j is the right-hand side of (17).

3.3 Matrix equation for inverse problem

If (11) is combined with (18), the number of unknown variables becomes equal to the number of equations. Therefore, the increments of the unknown vector potentials and the magnetizations of the additional magnets can be directly calculated by the following equation obtained from (11) and (18):

$$\begin{bmatrix} [H] & [C] \\ [F] & [0] \end{bmatrix} \begin{Bmatrix} \{\delta A\} \\ \{\delta M_a\} \end{Bmatrix} = \begin{Bmatrix} \{Q\} \\ \{K\} \end{Bmatrix} \tag{19}$$

The values {D} of modification can be calculated from the obtained {Ma} by using (1).

4. EXAMPLES

4.1 Determination of magnet widths and lengths

Figure 6 shows a magnetic circuit with two magnets of which the widths and lengths are unknown. The magnets are uniaxially anisotropic and magnetized in the y- direction. They are made of ferrites (B_r=0.38(T), H_c=0.25x10^6(A/m)). The x- and y-directional flux densities B_{x10}, B_{y10}, B_{x20} and B_{y20} are specified to be -0.02, 0.1, 0.06 and 0.06(T), respectively.

The estimated widths are 15(mm) and estimated lengths are 20(mm). The widths and lengths of magnets ① and ②, which produce B_{x10}, B_{y10}, B_{x20} and B_{y20} at points P_1 and P_2 above the upper sides of magnets, are

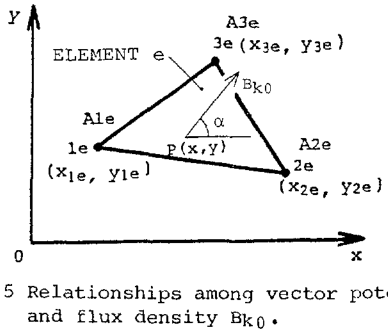

Fig.5 Relationships among vector potentials and flux density B_{k0}.

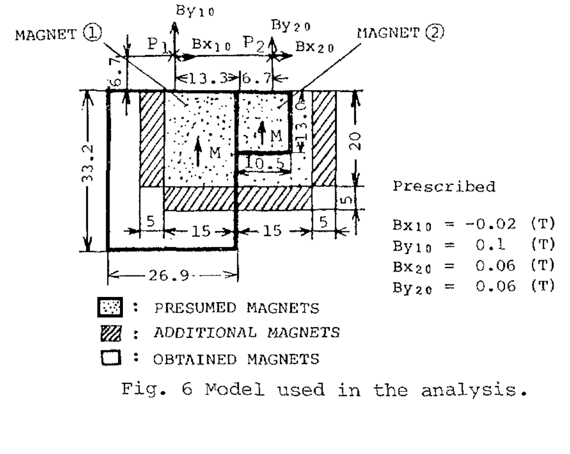

Fig. 6 Model used in the analysis.

calculated by setting the additional magnets denoted by the hatched parts on the sides of the presumed magnets denoted by the dotted parts in Fig.6. The dimension of each additional magnet is 5(mm) during the repetitions of modifying the widths and lengths of magnets ① and ②.

The calculated widths and lengths are 26.9, 10.5, 33.2 and 13.0(mm), respectively and are denoted by thick lines in Fig.6.

The error of the flux density at each position is listed in Table 1. The error ε is defined by the following equation:

$$\varepsilon = \frac{B_n - B_0}{B_0} \times 100(\%) \qquad (20)$$

where B_0 is the prescribed flux density and Bn is the obtained flux density at the n'th iteration. The errors after 4 iterations are within 2.0(%).

Table 1 Errors of flux densities.

NUMBER OF ITERATION	ERRORS OF FLUX DENSITIES (%) ε_{x1}	ε_{y1}	ε_{x2}	ε_{y2}
0	-220.0	-38.0	-65.2	30.5
1	19.0	-6.1	9.7	-66.7
2	8.0	-3.2	3.5	-17.8
3	3.2	-1.6	0.8	1.7
4	1.5	-0.7	0.2	2.0

$\varepsilon_{x1}, \varepsilon_{y1}$: errors of B_{x10} and B_{y10}
$\varepsilon_{x2}, \varepsilon_{y2}$: errors of B_{x20} and B_{y20}

4.2 Determination of the shape of a curvilinear magnet

An example determining the curved shape of a magnet which produces a prescribed flux distribution is analyzed. Figure 7 shows the analyzed model. The flux distribution on the circumference A-A' is prescribed to be spatially sinusoidal as shown in Fig.8. The magnet is made of the same material as that in Section 4.1 and is magnetized uniformly in the y- direction.

The presumed magnet is divided into seven quadrangles. The additional magnets are set on the upper side of the magnet.

The presumed shape of the magnet is denoted by dashed lines and the finally obtained one is denoted by thick line in Fig.7. The shape at each iteration is shown in Fig.9. The error of the flux density at each position is listed in Table 2. The desired shape and size of magnet of which the errors are within 2.0(%) is obtained after 18 iterations.

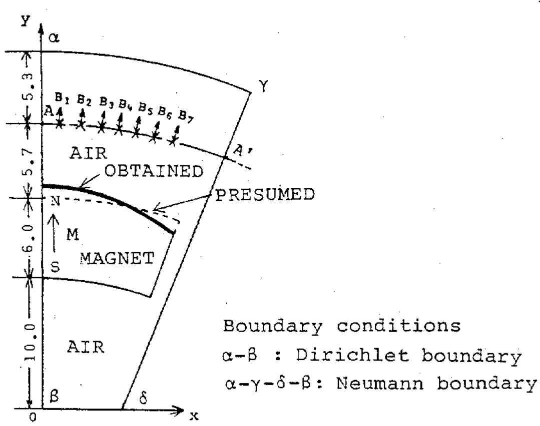

Fig.7 Analyzed model.

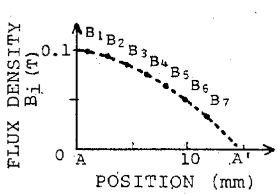

Fig.8 Prescribed flux distribution on the circumference A-A'.

It is clear that our method can be applied to determining the curved shape of magnet.

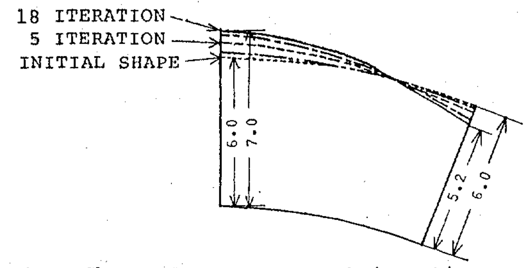

Fig.9 Shape of magnet at each iteration.

Table 2 Errors of flux densities.

NUMBER OF ITERATION	ERRORS OF FLUX DENSITIES (%) ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7
0	-19.4	-17.4	-13.3	-7.0	-1.6	12.8	21.5
5	-9.2	-8.4	-6.9	-3.6	-0.7	6.7	11.1
10	-4.4	-4.1	-3.2	-1.9	-0.0	3.5	5.7
15	-2.1	-2.0	-1.6	-1.0	-0.2	1.8	3.0
16	-1.8	-1.7	-1.4	-0.9	-0.3	1.6	2.6
17	-1.6	-1.5	-1.2	-0.8	-0.3	1.4	2.3
18	-1.3	-1.3	-1.0	-0.7	-0.3	1.3	2.0

ε_i: error of flux density at point i

5. CONCLUSIONS

The new method enables us to determine the shapes and sizes of magnets in a magnetic circuit.

The analysis of the following problems will be reported later:

(a) investigation of the case when the sizes are not obtained;
(b) determination of positions of magnets;
(c) acceleration of the convergence of sizes;
(d) the optimum directions of magnetizations;
(e) the optimum design of magnets having the minimum volume.

By expanding our method, many electrical machinery and apparatus such as a magnetizer can also be designed [6], [7].

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