NEW DESIGN METHODS FOR FIR FILTERS WITH EQUIRIPPLE STOPBANDS AND PRESCRIBED DEGREES OF PASSBAND FLATNESS

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ABSTRACT

A technique is presented for designing linear-phase digital FIR filters, with a prescribed degree of flatness in the passband, and a prescribed (equiripple) attenuation in the stopband. The design is based entirely on appropriate use of the McClellan-Parks algorithm along with certain maximally flat building blocks.

1. INTRODUCTION

The purpose of this paper is to advance a new technique for the design of linear-phase FIR filters with equiripple stopbands and with a prescribed degree of flatness in the passbands. Darlington [1] has considered certain general transformation techniques for handling these problems. Steiglitz [2] employs a linear programming approach for the design of such FIR filters, by imposing constraints on the derivatives of the frequency response. In [3] Kaiser and Steiglitz point out the existence of numerical difficulties in the design of such FIR filters based on linear programming.

The technique we propose here is based on the well-known McClellan-Parks algorithm [4] for FIR filter design. No other optimization programs are involved. The design technique directly leads to a filter structure that has very low "passband sensitivity" (which is crucial in the implementation of filters with very flat passbands). In Section 2 the method is introduced along with numerical examples. For the design of narrow passband filters, improved methods are described in Section 3.

2. THE PROPOSED APPROACH

Recall that, in the McClellan-Parks method [6], a weighted error function is first formulated:

$$E(\omega) = W(\omega)[D(\omega) - P(\omega)] \tag{1}$$

where $P(\omega)$ is a sum of cosines. $D(\omega)$ is the desired frequency response and $W(\omega)$ is the "weight" of the approximation error. The algorithm [6] essentially finds $P(\omega)$, such that $E(\omega)$ is equiripple, thus minimizing the peak weighted error for a given filter order. All the techniques to be presented in this paper are based on appropriate choices of the functions $W(\omega)$ and $D(\omega)$.

Consider the design of a lowpass linear-phase FIR transfer function $G(z)$ such that $G(e^{j\omega})$ has a tangency of $M-1$ at $\omega = 0$ (*i.e.*, the first $M-1$ derivatives of $G(e^{j\omega})$ are zero at $\omega = 0$). Let the specifications δ_1, δ_2 and ω_p be as in Fig. 1. In order to design $G(z)$, we first design a lowpass filter $H(z)$ with "complementary" specifications, as shown in Fig. 2. The desired tangency at $\omega = \pi$ is easily forced by decomposing $H(z)$ as

$$H(z) = H_1(z)H_2(z) \tag{2}$$

where

$$H_2(z) = \left(\frac{1+z^{-1}}{2}\right)^M \tag{3}$$

The transfer function $H_1(z)$ can now be designed by using the McClellan- Parks algorithm with the following specifications:

Desired response:

$$D(\omega) = \begin{cases} |1/H_2(e^{j\omega})|, & 0 \le \omega \le \pi - \omega_s \\ 0 & \pi - \omega_p \le \omega \le \pi \end{cases} \tag{4}$$

Weighting function $W(\omega)$ for the weighted equiripple error:

$$W(\omega) = \begin{cases} |H_2(e^{j\omega})| & 0 \le \omega \le \pi - \omega_s \\ \delta_2/\delta_1 \;\; |H_2(e^{j(\pi-\omega_p)})| & \pi - \omega_p \le \omega \le \pi \end{cases} \tag{5}$$

The choice of the desired function and the weighting function as in Eqns. (4) and (5) ensures that (a) the cascaded filter of Eqn. (2) has equiripple response in the passband, and that (b) the ratio of passband error to the peak stopband error is δ_2/δ_1. Assuming that $H(z)$ has been designed as above, if we now construct the function, $\hat{G}(z) = z^{-N/2} - H(z)$, the resulting response is as shown in Fig. 3. If each delay unit in the circuit for $\hat{G}(z)$ is replaced with $(-z^{-1})$, the required transfer function is thus obtained:

$$G(z) = (-z)^{-N/2} - H(-z) \tag{6}$$

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Figure 4 shows the overall implementation of $G(z)$. (N is assumed even in Eqn. (6).) In summary, the filter section $H_2(z)$ takes care of the flatness of the passband of $G(z)$, whereas the section $H_1(z)$ takes care of the equiripple stopband of $G(z)$.

Letting N_1 denote the order of $H_1(z)$, the overall filter order $N = N_1 + M$ should be even, so that the "complementation" of Eqn. (6) can be realized. Given a certain "degree of tangency" equal to $M - 1$ at $\omega = 0$, and given the tolerances δ_1 and δ_2, it only remains to find N_1. An estimate of N_1 can be found as [8]:

$$N_1 = \frac{-10 \log_{10} \delta_1 \delta_2 - 13}{14 \cdot 6 \, \Delta f} \tag{7}$$

where $\Delta f = (\omega_s - \omega_p)/2\pi$. Notice that, even though the overall filter order is $N_1 + M$, the number of multiplications involved is only about $N_1/2$.

Example 2.1

Consider the following lowpass specifications: $\omega_p = (0 \cdot 3)2\pi$, $\omega_s = (0 \cdot 35)2\pi$; $\delta_1 = 0 \cdot 016$, $\delta_2 = 0 \cdot 2\delta_1$ (corresponding to -50 dB); order of tangency $(M-1)$ at zero frequency $= 15$. The estimated order of a conventional equiripple design $G_e(z)$ is 42 from Eqn. (7). An order, $N_e = 44$ was actually required for $G_e(z)$. Assuming that the required order N_1 of $H_1(z)$ is also equal to 44, and taking $M = 16$, the filter $G(z)$ was designed as described earlier. Figure 5 shows the frequency response magnitudes of the new design $|G(e^{j\omega})|$ and the equiripple design $|G_e(e^{j\omega})|$. Note that $|G(e^{j\omega})|$ is extremely flat at $\omega = 0$, as expected, and furthermore that the tolerance specifications are met.

Example 2.2. Narrowband Design.

Let us assume that, starting from certain specifications, we have arrived at the following parameters: $\omega_p = (0 \cdot 1)2\pi$, $\omega_s = (0 \cdot 14)2\pi$, $N_1 = 44$, $M - 1 =$ order of tangency $= 7$, $\delta_2 = 0 \cdot 2\ \delta_1$. The value of δ_1 is automatically fixed because all the remaining parameters have been specified. The narrowband nature of $G(z)$ implies that $H(z)$ is a wideband lowpass filter (see Fig. 2). The desired response $D(\omega)$ which is input to the "McClellan-Parks" program (Eqn. 4) is $D(\omega) = 1/\cos^8 \frac{\omega}{2}$ in the passband of $H(z)$, and spans a huge dynamic range. So, the coefficients of $H_1(z)$ are numbers of large magnitude. However, at $\omega = 0$ these "large" numbers add up to approximately unity. Thus, the sensitivy of $|H(e^{j\omega})|$ in its passband, with respect to the coefficients of $H_1(z)$, tends to be severe. Consequently, the stopband sensitivity of $|G(e^{j\omega})|$ is high.

The above sensitivity problem has its root in the fact that $G(z)$ is a narrowband function. A simple means of

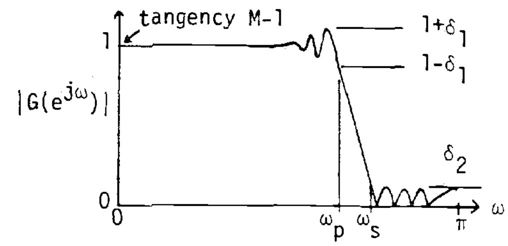

Fig. 1. The desired specifications

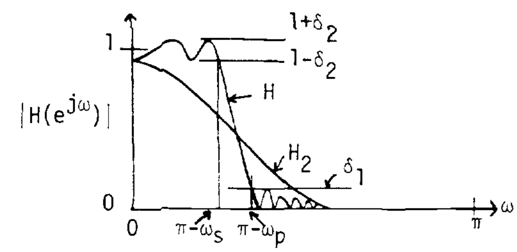

Fig. 2. Complementary specifications

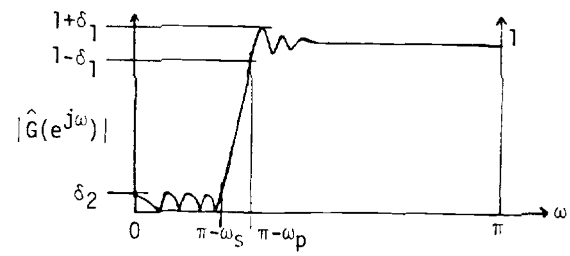

Fig. 3. The response of Ĝ(z)

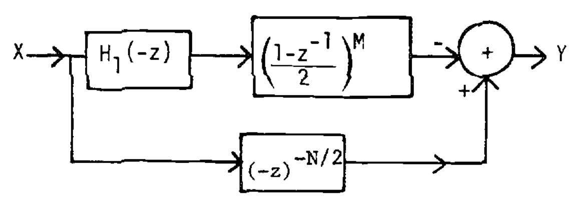

Fig. 4. The overall implementation

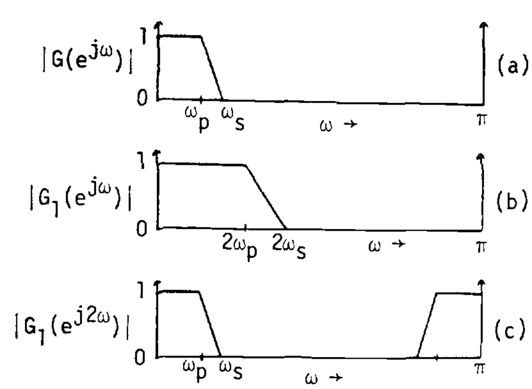

Fig. 6. The IFIR approach

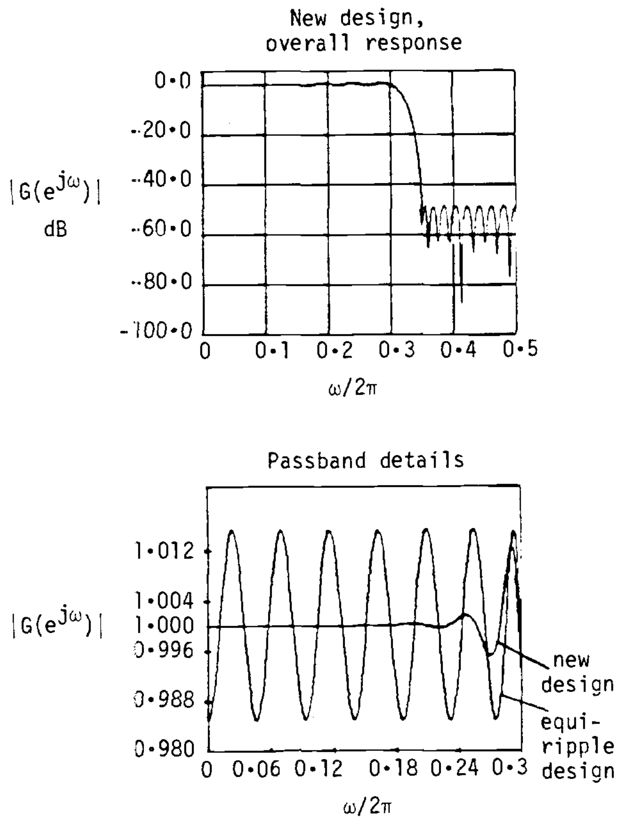

Fig. 5. Example 2.1

overcoming this problem is based on the interpolated FIR (IFIR) approach [5], and is described in the next section. Essentially, a narrowband filter can be designed by designing a wideband filter and then manipulating it.

3. NARROWBAND LOWPASS DESIGN

In order to understand the IFIR techniques [5], consider a narrowband specification for $G(e^{j\omega})$ as shown in Fig. 6(a). Instead of designing $G(z)$ directly, one can first design a filter with specifications as shown in Fig. 6(b) where the frequency axis has been "stretched" by a factor of 2. In the resulting transfer function $G_1(z)$, if each delay element is replaced by two units of delay, then the response is as shown in Fig. 6(c). If now the passband around $\omega = \pi$ is suppressed *without affecting the passband around* $\omega = 0$, then the specifications of Fig. 6(a) are met. This suppression is done by cascading an "interpolator" $G_2(z)$ with $G_1(z^2)$. As $G_1(z)$ has a wider transition band, it requires a lower order. $G_2(z)$ is usually multiplierless, hence the overall implementation is less complex than a conventional design.

In this section we use the IFIR approach for a different reason: we wish to convert the narrowband design problem ($G(z)$) to a wideband problem ($G_1(z)$), so that the function $\widehat{H}(z)$, now defined as $\widehat{H}(z) = z^{-N/2} - G_1(-z)$ has a narrower passband (and *a wider transition band, equal to that of* $G_1(z)$) as compared to the function $H(z)$ defined according to Eqn. (6). $\widehat{H}(z)$ is then designed as the product $\widehat{H}_1(z)\widehat{H}_2(z)$, where $\widehat{H}_2(z)$ is given by the righthand side of Eqn. (3). Thus, the IFIR technique has the usual advantage of reducing the total number of multipliers (*because the transition band of* $\widehat{H}(z)$ *is now wider*) and the advantage of requiring only well-conditioned coefficients in $\widehat{H}_1(z)$.

In order to suppress the unwanted passband at $\omega = \pi$ without affecting the flatness at $\omega = 0$, we employ a new class of interpolators for $G_2(z)$, called the "maximally-flat" interpolators. An FIR lowpass frequency response magnitude with degree of tangency $2K-1$ at $\omega = \pi$ and $2L-1$ at $\omega = 0$ is given by [9],[10],

$$|I_{K,L}(e^{j\omega})| = \cos^{2K}\frac{\omega}{2}\sum_{n=0}^{L-1} d(n)\sin^{2n}\frac{\omega}{2} \qquad (8)$$

where

$$d(n) = \frac{(K-1+n)!}{(K-1)!\,n!} \qquad (9)$$

Notice that $d(n)$ are integers, and for small K and L, $I_{K,L}(z)$ can be implemented very efficiently with few additions and no multiplications [11,12]. If $G_2(z)$ is chosen to be $I_{K,L}(z)$ with $L = M/2$ where $(M-1)$ is the degree of tangency of $|G_1(e^{j\omega})|$ at $\omega = 0$, then the order of tangency of $G_1(z^2)G_2(z)$ is equal to $M-1$, at $\omega = 0$. Furthermore, the flatness of $|I_{K,L}(e^{j\omega})|$ at $\omega = \pi$ ensures that the unwanted passband of $G_1(z^2)$ is satisfactorily suppressed. For example, consider once again the narrowband specifications of Example 2.2. Let us "stretch" the specified ω_p and ω_s and obtain the new specifications as follows: $\omega_p' = (0\cdot 2)2\pi$, $\omega_s' = (0\cdot 28)2\pi$; $N_1 = 44$, $M-1 = 7$, $\delta_2 = 0\cdot 2\ \delta_1$. The transfer function $G_1(z)$ meeting the modified specifications can be designed as $G_1(z) = (-z)^{-N/2} - \widehat{H}(-z)$ where

$$\widehat{H}(z) = \widehat{H}_1(z)\widehat{H}_2(z)\ ,\ \widehat{H}_2(z) = \left(\frac{1+z^{-1}}{2}\right)^8 \qquad (10)$$

and $\widehat{H}_1(z)$ is designed as described in Section 2. Finally, the desired original specifications are met by designing $G(z)$ as $G(z) = G_1(z^2)\ I_{4,4}(z)$.

The impulse response coefficients $\hat{h}_1(n)$ of the filter $\widehat{H}_1(z)$ have much smaller magnitudes than those in Example 2.2, even though the dynamic range is about the same. Consequently, the passband sensitivity of $H(z)$ and hence the stopband sensitivity of $G(z)$ are much better than in Example 2.2. (We have verified these claims by a formal sensitivity study and simultation examples. Details are omitted here for sake of brevity.)

Figure 7 shows the response $|G(e^{j\omega})|$ and also the response of a direct equiripple design of order 44, with same δ_1 and δ_2. The new design has a much sharper transition band, and in general, "overmeets" the specifications everywhere. This is primarily because, in view of the IFIR approach, $\widehat{H}_1(z)$ needs to have an order of only 22 rather than the actually employed 44. (*An order of 22 was actually found to be sufficient, by explicitly plotting out the responses.*)

4. CONCLUDING REMARKS

The methods introduced in this paper do not require any optimization program other than the McClellan-Parks algorithm. The passbands are "flat" to a prescribed degree, but not monotone. A restriction of our method is that the order N of the overall filter should be even, so that Eqn. (6) is physically realizable.

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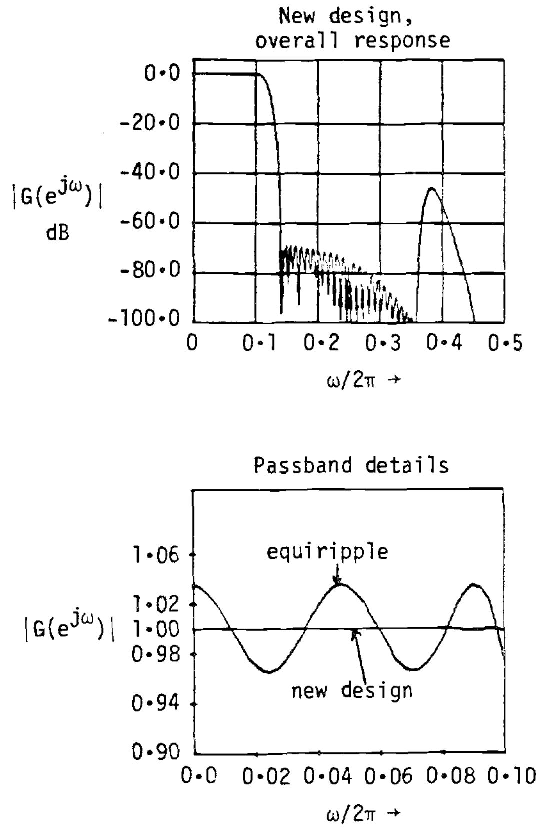

Fig. 7. Example 2.2, redone using the IFIR approach.

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