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New directions in the analysis and interactive elicitation of personal construct systems

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The computer elicitation and analysis of personal construct systems has become a technique of great interest and wide application in recent years. This paper takes the current state of the art as a starting point and explores further developments that are natural extensions of it. The overall objective of the work described is to develop man–computer symbiotic systems in which the computer is a truly dialectical partner to the person in forming theories and making decisions. A logical model of constructs as predicates applying to elements is used to develop a logical analysis of construct structures and this is contrasted with various distance-based clustering techniques. A grid analysis program called ENTAIL is described based on these techniques which derives a network of entailments from a grid. This is compared and contrasted with various programs for repertory grid analysis such as INGRID, FOCUS and Q-Analysis. Entailment is discussed in relation to Kelly's superordination hierarchy over constructs and preference relations over elements. The entailment analysis is extended to rating-scale data using a fuzzy semantic model. The significance of Kelly's notion of the opposite to a construct as opposed to its negation is discussed and related to other epistemological models and the role of relevance. Finally, the interactive construct elicitation program PEGASUS is considered in terms of the psychological and philosophical importance of the dialectical processes of grid elicitation and analysis, and recommendations are made about its generalization and extension based on the logical foundations described. Links are established between the work on repertory grids and that on relational data bases and expert systems.

1. Introduction

It is now 25 years since Kelly (1955) published his seminal book on personal construct theory. It provides a remarkably far-reaching and well-structured foundation for epistemology. His work is anchored very firmly both in its close correspondence to the actual behaviour of people and in its coherent and consistent philosophy. This is not to say that Kelly fully worked out a logically, philosophically and psychologically complete model of knowledge acquisition. His attempts to link his work to other philosophical studies of epistemology, his attempt to present it axiomatically, and his embodiment of it as an empirical tool through the repertory grid, are all incomplete. They need much further development and modification to take them to levels of scholarship, science and technology which would allow them to stand critical comparison with related work.

However, there are now many who would endorse Kelly's intuition for what he proposed as a starting position: his model of the *personal scientist* acquiring a personal model of his world; and his idea of *constructs* as personally developed templets needed to filter perception in order to allow past experience to relate to future behaviour. Many would now agree that these provide adequate foundations for a true psychological

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epistemology well-grounded philosophically and capable of being developed into both theory and technology.

In the quarter of a century since the publication of Kelly's book there have been many developments that relate to it and form a basis for a fresh impetus during the next 25 years. In philosophy the balance has been struck between the extremely personal epistemologies of the existentialists and the extremely impersonal epistemologies of the logical positivists. Attempts, such as those of Ayer (1936), to define precisely the one acceptable method of legitimating belief have died down, and Kelly's constructive alternativism has become fashionable in conventionalist and pluralist positions such as those of Gellner (1974) and Feyerabend (1975). In Kelly's work we can find incorporated both Kuhn's (1962) emphasis on the importance of the paradigm and hence the possibility of revolutionary changes in viewpoint, and Popper's (1972) emphasis on falsifiability as the prime test of meaningful belief. Modern philosophy has swung the focus of attention from science to the scientist, a viewpoint which makes Kelly's work now appear central to the key issues.

Positivist science advanced as rapidly as it did because of its very close links with formal logic. Kelly himself was probably influenced by this in his attempt to present his own position axiomatically as a "fundamental postulate" and a set of 11 "corollaries". However, the possibility of forming logical foundations for his theories, let alone axiomatizing them, was not within the realms of the mathematics open to him at the time. His concept of a *construct* applied to *elements* and having a *range of convenience* requires a modal logic incorporating notions of relevance, and the theory underlying these was only formalized during the mid-1960s (Snyder, 1971; Anderson & Belnap, 1975). The formalization of modal logic has been very fruitful in establishing semantic foundations for natural language (Cresswell, 1973), and its basis in the concept of *possible words* (Lewis, 1973; Bradley & Swartz, 1979) seems very close to the model that Kelly needed for the dynamics of construct formation and modification. A related development in recent years has been that of multivalued logical foundations for set theory such as Zadeh's (1976) *fuzzy logic*, and the application of this also to modelling human semantic processes has much in common with Kelly's approach.

Neither the philosophical nor the logical developments would be of value unless interest in Kelly's work had been developed and sustained during the past 25 years. This has come about largely through its clinical applications (Slater, 1976) and its integration into the mainstream of work on personality (Bannister & Fransella, 1971; Hogan, 1976). Because of the experimental nature of much of this work the analysis of Kelly's repertory grids through computer programs has itself become a significant line of development (Shaw, 1980). The on-line application of computers to operationalize Kelly's construct theory and to reflect to an individual his role as a personal scientist adds a new dimension to the work. We can see the beginnings of the *man–machine symbiosis* (Licklider, 1960) promised in the early days of computing, in which the logical processing power of the computer is used to complement the creative imagination of the person.

Shaw's (1980) PEGASUS was one of the first available computer programs to elicit personal construct systems interactively whilst at the same time feeding back the results of analysis and directing further elicitation through this. It has been widely used in a variety of educational, clinical and managerial applications. In this paper we attempt to draw out of the current programs those features which seem of greatest value and project them to the next stage of development. This entails the use of recent developments in logic and semantics to give rigorous and operational foundations for Kelly's notions of the construct system of the personal scientist. New methods of analysis of repertory grid data are defined and the results compared with previous analyses. The extension of PEGASUS to be a truly dialectical partner to a person in forming theories and making decisions is proposed.

2. Construct structure and analysis

Kelly put forward "personal constructs" as filters through which we perceive events (Kelly, 1955, pp. 8–9):

> Man looks at his world through transparent templets which he creates and then attempts to fit over the realities of which the world is composed.

He continually emphasizes the epistemological status of these constructs in predicting and controlling the world and their ontological status as personal conjectures rather than reality-derived absolutes (Kelly, 1955, p. 14):

> Constructs are used for predictions of things to come, and the world keeps on rolling on and revealing these predictions to be either correct or misleading. This fact provides the basis for the revision of constructs and, eventually, of whole construct systems.

When it came to the formal and practical representation of constructs Kelly took them to be binary in nature such that each event construed was classified as belonging to one "pole" of a construct, or the other. In essence Kelly placed the same fundamental emphasis as did Spencer Brown in his seminal work, *Laws of Form*, on the human, creative operation of "making a distinction" (Spencer Brown, 1969, p. v):

> The theme of this book is that a universe comes into being when a space is severed or taken apart. . . . By tracing the way we represent such a severance, we can begin to reconstruct, with an accuracy and coverage that appear almost uncanny, the basic forms underlying linguistic, mathematical, physical and biological science, and can begin to see how the familiar laws of our own experience follow inexorably from the original act of severance.

It casts an interesting light on the further development of Kelly's work that Spencer Brown goes on to use the notion of a distinction to develop a logical "calculus of distinctions" with fewer primitives than the classical propositional calculus which he claims avoids the paradoxes of previous approaches. In his own practical development of a personal construct technology through the "repertory grid" and the extraction of "factors" from it Kelly treats constructs as if they gave a vector of measurements of the event rather than a logical representation of it. This approach seems to have been followed also by all later workers on the analysis of the repertory grid through a variety of methods such as principal components analysis. In the following sections we show that the analysis of construct systems as *logical structures* both encompasses many of the advantages of such methods and also leads to interesting new directions of analysis.

The central part of this paper deals with the analysis of the grid rather than its elicitation and it is worth emphasizing at this stage that our prime motivation for the form of logical analysis developed here was to extend techniques for the interactive elicitation of grids through feedback of the analysis. We have been aiming to develop a *conversational, dialectical* system of computer programs for the *self-reflective* study of one's role as a personal scientist. With this in mind it has been important to develop forms of analysis that can support a conversation by commenting upon its contents without introducing new constructs beyond those the user already employs. It is this which has led us to a logical analysis in which the *meta-language* used for comments on construct structures is essentially the same as the *object-language* in which the information defining these structures is given.

Section 3 reviews the main current distance-based grid analysis techniques INGRID and FOCUS together with the more recent Q-Analysis. Section 4 develops a logical model of a repertory grid and the notion of *entailment* between the poles of constructs. Section 5 describes a program, ENTAIL, that extracts such entailments from grids and gives a comparison of some results with those of the distance-based methods. Section 6 extends the analysis to consider the strength of entailment and section 7 relates it to the subordination/superordination hierarchy. Section 8 shows how a similar asymmetric analysis may be applied to the element structure, section 9 extends the analysis to grids with more than two values through a *fuzzy semantic* model, and section 10 further extends it to compound predicates. Section 11 introduces the special features of interactive grid elicitation, and section 12 shows how the dialetical nature of such a conversational process is related to the logical anlysis and the enhancement of the results obtained. Section 13 gives a series of recommendations for the direction of further work and section 14 concludes the paper.

3. Distance-based grid analysis

Figure 1 is a repertory grid from Shaw (1980, p. 79) showing Jane's allocation of 12 acquaintances to the poles of eight constructs. It is a particularly good illustrative example because Jane has given far more background explanation to the poles of her constructs than is usually available and this makes it easier to assess the prima facie *meaningfulness* of any analysis. The only difference between Fig. 1 here and Fig. 6.4 in the book is that Shaw uses the letter "X" for the assignment to the left-hand pole and the letter "O" for the assignment to the right-hand pole, whereas we have used the numbers "1" and "0", respectively. This change to numerals is deliberate because we wish to examine how the values in the grid may be viewed in two ways: firstly as numerical values; and then as logical values.

We will concentrate initially on the relations between the constructs in a grid such as that shown in Fig. 1. For any given construct we may regard the numbers in the grid as a *vector of values* giving the assignment of each element in turn to one or other of the poles of the construct. From this point of view each construct becomes represented as a point in a multi-dimensional space whose dimension is the number of elements involved. The natural relation to examine between constructs is then the *distance* between them in this space. Two constructs which are zero distance apart are such that all elements are construed in the same way in relation to them and hence we might infer that they are being used in the same way—in some way they are *equivalent* constructs. For constructs which are not equivalent we may analyse the entire constellation in space to determine a set of axes such that the projection of each construct onto the first axis accounts for most of the distance between them, the projection on the second axis

			ELEMENTS											
	1	0	1	2	3	4	5	6	7	8	9	10	11	12
C1	*Intensity*. They both are interested in other people. Concerned with world problems. Ambitious. Slightly detached.	*Humorous*. Creative. Unconventional approach to work & relationships. Exciteable.	1	1	0	0	1	0	0	1	0	1	0	0
C2	*Individualistic*. Musical. Calm (exteriorally). Unconventional. Non-aggressive. Loyal. Interested in myth & fantasy. Homely. Landloving. Tending toward introversion. Unusual humour.	*Self aware*. Controlled. Sporting. Experienced in relationships. Attracted to sophistication & the exotic. Extroverted. Light hearted.	0	0	0	0	1	1	0	0	0	0	1	0
C3	*Generous*. Interested in history. Slow living. Perfectionist in work. Unusual relationships.	*Direct*. Political. Super active. Strong integrity. Committed.	0	0	0	0	1	1	1	0	1	0	1	0
C4	*Ambitious*. Questioning. Quick minds. Confident. Interested in "societies ills."	*Artistic*. Capable. Gentle. Romantic. Exploratory.	1	1	1	1	0	0	0	1	0	1	0	1
C5	Outdoor enthusiasts. Anxious to succeed. Anxious about success with other sex. Active. *Enigmatic*. Need mental stimulation.	Creative. *Enjoys comfort*. Relaxed.	1	1	1	1	0	0	0	1	0	0	0	1
C6	Enjoy intellectual discussion. Difficult to understand initially. City livers. *Seek challenges*. Insecure backgrounds.	Affectionate. Humble. Sensitive. Musical. Involved with those immediately around. *Compassionate*. Philosophical.	0	1	1	1	0	0	1	1	1	1	0	1
C7	Energetic. Sociable. Politically concerned interests. *Dynamic*. Restless. Factual approach as opposed to interest in fantasy world.	Thorough. *Care for detail*. Extremely creative. Not concerned with social success. Gentle. Perceptive.	1	1	1	1	0	0	0	1	1	0	0	1
C8	Both need company. *Gregarious*. Prepared to compromise. Factual approach. Enjoy discussion.	Musical. Scientific but also keen on the "unreal" world. *Fantastical*.	1	1	1	1	0	0	0	1	1	1	0	1

FIG. 1. Jane's repertory grid.

accounts for most of the remaining distance, and so on. This is a *principal components analysis* of the construct space (Slater, 1977). We may also group constructs together that are close together in space using a variety of techniques. These are all some form of *cluster analysis* (Duran & Odell, 1974).

All of the techniques based on such a numerical spatial view of construct structures depend on the notion of constructs being equivalent if they are represented by the same point in space and somehow *nearly equivalent* if they are represented by points close to one another. Principal components analysis goes even further and assumes that the distances between points are themselves meaningful and that the distribution of points in space gives an indication of meaningful directions in that space. However, it is most often used just as a basis for clustering constructs according to their distance apart on the two principal dimensions so that the notion of the "meaning" of these dimensions does not necessarily arise.

The grid of Fig. 1 was analysed using Slater's (1977) INGRID program for determining the principal components. Figure 2 shows the twelve elements and the two poles of each of the two constructs plotted against the first two principal components. In this section we will concentrate on the construct analysis and treat the elements analysis later in section 8. It can be seen that the left-hand poles of constructs 4, 5, 7 and 8 form a fairly tight cluster together with the right-hand poles of constructs 2 and 3. The

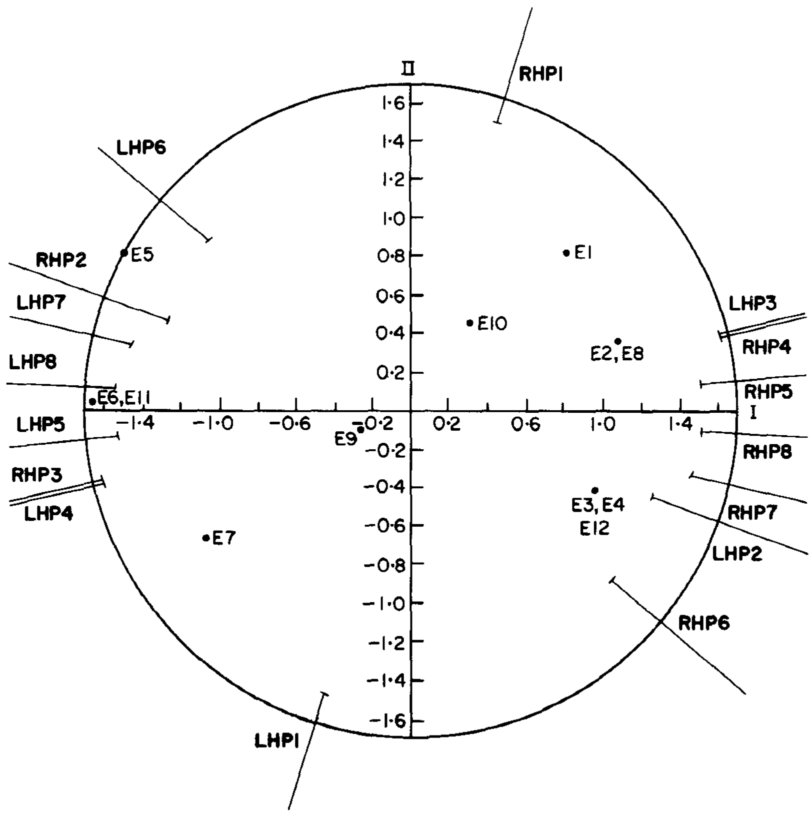

FIG. 2. Principal components analysis of Jane's grid by INGRID.

left-hand pole of construct 6 is associated more loosely with this but both poles of construct 1 are isolated well away from the cluster. There is a mirror image cluster of the right-hand poles of constructs 4, 5, 6, 7 and 8, together with the left-hand poles of constructs 2 and 3. Because the assignment of elements to poles is such that the vector of assignments to the left-hand pole is the reverse of that to the right-hand pole such a mirror image is bound to occur with conventionally elicited grids.

Atkin's (1974) *Q-Analysis* provides an alternative means of analysing the structure behind a matrix of data such as that of Fig. 1. In terms of the present discussion it is convenient to regard it as a form of hierarchical cluster analysis based on a distance-measure, although it is conventionally presented in combinatorial topology terms. The data of Fig. 1 was analysed using a program QARMS (*Q-A*nalysis of *R*elations in *M*ultilevel *S*tructures) that can also deal with grids using rating scales as well as the binary data shown. The results are shown in Fig. 3 with the connectivities also drawn out as a hierarchical cluster.

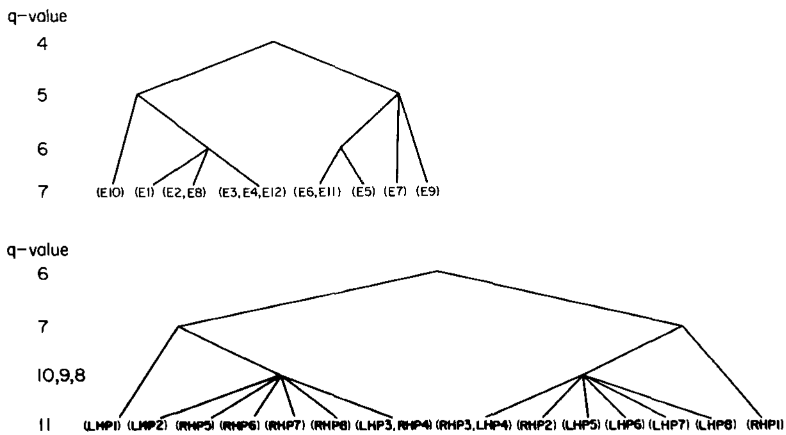

FIG. 3. Q-Analysis of Jane's grid by QARMS.

Shaw's (1980) FOCUS algorithm is another distance-based grid analysis technique that sorts the constructs into a linear order such that constructs closest together in the space are also closest together in the order. It has the advantage in presentation that the sorting is used only to represent the original grid reorganized by the "neighbourness" of constructs and elements. It is up to the user to construe meaning into the result and confirm this directly in terms of the original data. Figure 4 shows the grid of Fig. 1 as processed by FOCUS. Note that the letters "X" and "O" have been replaced by the numerals "1" and "2", respectively, as the normal FOCUS convention, rather than the "1" and "0" used above. In reorganizing the grid FOCUS has also reversed constructs 2 and 3. Concentrating on the construct analysis again, it can be seen that constructs 3 and 4 are equivalent and close neighbours of 5, 7 and 8, and that this cluster is itself a close neighbour of the cluster formed by 2 and 6. Construct 1 is not linked into the other constructs at a meaningful level.

Thus, for this example at least, the actual clusters produced by FOCUS, QARMS and INGRID do not differ in any meaningful way. In general, since all these techniques use

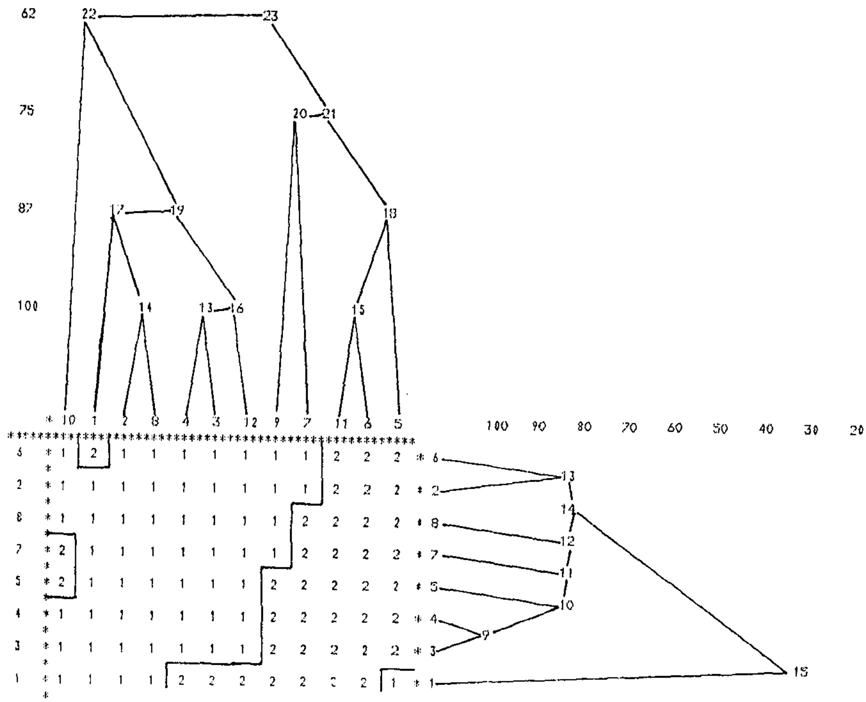

FIG. 4. Cluster analysis of Jane's grid by FOCUS.

distance measures to produce clustered data, one would expect the results to be similar. The objective of FOCUS is not to produce a different analysis in terms of clusters but rather to present the analysis in terms of the data that produced it. In this way those who produced the grid are able to see how the decisions they made in doing this affect the actual analysis.

The *reversal* performed by FOCUS is an important operation in analysing a repertory grid. Kelly (1955, p. 283) uses such a process of reversal (which he calls "reflection") in his analysis of repertory grids, and the need for it clearly arises from the artificiality of the assignment of "left-hand" and "right-hand" poles to a construct. Unless some special additional rationale is in operation, what are called the left-hand and right-hand poles of a construct may be reversed without distorting the grid providing the assignments of elements to those poles is also reversed. In principal components analysis and Q-Analysis such reversals show up in the clustering of left-hand poles together with right-hand poles, for example as in Figs 2 and 3.

It is convenient to make a point here that applies to all techniques for grid analysis. Any relation we infer between constructs from a given grid are derived from the set of elements used in eliciting that grid. Hence they should be qualified by a reference to that set: not "constructs 3 and 4 are equivalent", but rather "in relation to elements 1 through 12, constructs 3 and 4 are equivalent". To the extent that we drop this qualification we are proceeding inductively rather than deductively and our conclusions may be incorrect. This applies to any conclusion that extends the relation between constructs to elements that have not been tested, for example "in relation to your close acquaintances, constructs 3 and 4 are equivalent". The significance of such inductive steps in the conversational elicitation of constructs will be discussed in section 13.

In conclusion the various distance-based analyses of grids each provide related methods of clustering elements and constructs in such a way that one can provide feedback on possible *structures* underlying the construing. They have two factors in common that restrict their application in some contexts. Firstly, the structure exhibited is limited in its semantics to a symmetric relation of "neighbourness" between the items clustered. Secondly, the analyses produce results about distances, components, connections, geometrical relationships, and so on, which represent a different way of looking at the data. This may be valuable in itself and may be expressed through basic notions of *similarity*. However, for some applications such as interactive discussion in conversational grid elicitation it would be preferable to have an analysis that expresses relations in the data in terms more immediately meaningful and directly related to the data itself. It was these considerations that led us to the logical data analysis described in the next section.

4. Logical grid analysis

There is an alternative way of looking at the grid of Fig. 1 which views it not as a set of vectors in a space but instead as an assignment of *truth-values* to *logical predicates*. We may take the left-hand pole of each construct in Fig. 1 to be a logical predicate that may be applied to a person and take the assignment of the value to a particular element in the grid to mean that the predicate is *true* for that element. Conversely we may take the value of 0 assigned to an element for a construct to mean that the predicate represented by the left-hand pole of that construct is *false* for that element. It is convenient to use the abbreviation **LHPm** for the predicate that corresponds to the left-hand pole of construct *m*. Thus **LHP5** is the predicate for the left-hand pole of construct 5. If we then also adopt the convention that E*n* stands for the *n*th element then the notation **LHPm** E*n* may be used to denote the truth value of the predicate corresponding to the left-hand pole of construct *m* when applied to the logical constant corresponding to the *n*th element. A repertory grid, such as that of Fig. 1, is then the matrix of such truth values for the *m* constructs and *n* elements involved.

Because of the inverse relation already noted between assignments to the opposite poles of a construct in a conventional repertory grid, the predicate corresponding to the right-hand pole is logically related to that corresponding to the left-hand pole. We normally require that an element be assigned to one, and only one pole, so that if **LHP** E is true then **RHP** E must be false, and vice versa. Hence, **LHP** E is essentially the *logical negation* of **RHP** E. For the current discussion we shall accept that this relation exists as a constraint between the two predicates corresponding to the two poles. However, it is not an essential one for the theory and we discuss in section 11 the possibility of relaxing it and the consequences of doing so. For this reason we shall carry out most of the discussion in terms of the left-hand poles and associated predicates primarily, noting occasionally the corresponding phenomena for right-hand poles.

The logical analysis of construct systems in repertory grid form proposed here seems completely new. However, it is interesting to note that Slater (1977) has a section on

"Connections between personal construct theory, logic and probability theory". In this he states that (Slater, 1977, p. 34):

> The typical proposition in personal construct theory, *E may be construed as C*, paraphrases the typical proposition of Aristotelian logic, *S is P*, i.e. subject is predicate. It is difficult to conceive of any proposition that can be stated in one of these forms and not the other.

Thus, the point is made that it is possible to conceive of the assignment of an element to the pole of a construct as being similar to the assignment of the truth value *true* to the predicate applying to that element, but it is not developed. The rest of the book referenced develops the numerical, principal components approach to grid analysis. In this section we show that a purely logical analysis may also be developed.

First let us examine the previous relation of equivalence between constructs in logical terms. We can define two logical propositions to be equal if their truth-values are the same, and this also corresponds to their numerical truth-values being equal, e.g.

$$\mathbf{LHPm}\ \mathrm{E} = \mathbf{LHPn}\ \mathrm{E}. \qquad (1)$$

We can define two logical propositions involving the same free variable as being equivalent if they are equal for all values of that free variable, e.g.

$$\forall \mathrm{E} \quad \mathbf{LHPm}\ \mathrm{E} = \mathbf{LHPn}\ \mathrm{E}, \qquad (2)$$

and it is then convenient to drop the variable and write

$$\mathbf{LHPm} \equiv \mathbf{LHPn}. \qquad (3)$$

Now this equivalence between the poles of constructs clearly coincides with our previously discussed equivalence in terms of distance. If two propositions are logically equivalent in this way then the vectors of truth-values against elements are the same and hence they are zero distance apart. The converse may also be shown for any proper distance measures.

However, in terms of logical relations equality is only one of many possible relations. There are six binary logical operators between propositions that establish relations between them. Two of these relations are symmetrical and correspond to the two propositions being equal, or to one being equal to the negation of the other. This corresponds to the reversal or reflection of constructs discussed above. The other four operators are forms of *implication* between propositions, that one proposition being true implies that the other is also true. The four forms arise because of the possibilities of negation, that one being true implies the other is not, and so on. They may all be derived from the one operator, $\supset$, where

$$\mathbf{LHPm}\ \mathrm{E} \supset \mathbf{LHPn}\ \mathrm{E} \qquad (4)$$

means that the assignment of element E to the left-hand pole of construct *m* implies that it is also assigned to the left-hand pole of construct *n*.

In constrast to the equality relation, the implication relation is asymmetric. If we assert the implication given in (4) then we are only constraining the truth-value of **LHPn** E if **LHPm** E is true. If this is not so, and element E is not assigned to the left-hand pole of construct *m*, then we are saying nothing about its assignment to the left-hand pole of construct *n*. This constrasts to the equality relation asserted in (1) where the proposition **LHPm** E being false also leads to **LHPn** E being false in order to satisfy the equality.

One important property of the implication relation is its transitivity. From the way in which we have defined it we can see that if, as well as (4), we have

$$\mathbf{LHPn}\ \mathrm{E} \supset \mathbf{LHPo}\ \mathrm{E}, \tag{5}$$

then we can derive

$$\mathbf{LHPm}\ \mathrm{E} \supset \mathbf{LHPo}\ \mathrm{E}. \tag{6}$$

This is the normal transitivity of an implication relation in a logical calculus.

Asserting mutual implication between two propositions allows us to derive their equality. Thus adding the converse asymmetric assertion

$$\mathbf{LHPn}\ \mathrm{E} \supset \mathbf{LHPm}\ \mathrm{E} \tag{7}$$

to that of (4) does enable us to derive (1). From this we can see that the relation of implication is a weaker one than that of equality but closely related to it in that if we know the four implication relations between two propositions we may infer the two equivalence relations between them. These results from elementary propositional logic show that it is of interest to consider the implication relation in repertory grid analysis since the equality and equivalence relations normally analysed may be derived from it but not vice versa.

In the same way that we moved from the relation of equality between individual propositions in (1) to that of universal equivalence between them in (2), we may say that one proposition involving a free variable *entails* another proposition involving the same variable if it has an implication relation with it for all values of the free variable, e.g.

$$\forall \mathrm{E} \quad \mathbf{LHPm}\ \mathrm{E} \supset \mathbf{LHPn}\ \mathrm{E}, \tag{8}$$

and it is then convenient to drop the variable and write

$$\mathbf{LHPm} \rightarrow \mathbf{LHPn}. \tag{9}$$

We will read this as "the left-hand pole of construct m entails the left-hand pole of construct n". Clearly entailment, being derived from implication, is also asymmetric, and mutual entailment gives us equivalence in the same way as mutual implication gives us equality. Thus adding the converse entailment to (9):

$$\mathbf{LHPn} \rightarrow \mathbf{LHPm} \tag{10}$$

to (9) itself allows us to derive the equivalence of (3). Note similarly that the entailment relation is transitive like the implication relation so that from (9) and

$$\mathbf{LHPn} \rightarrow \mathbf{LHPo} \tag{11}$$

we may derive

$$\mathbf{LHPm} \rightarrow \mathbf{LHPo}. \tag{12}$$

We have linked the discussion of this section to personal construct theory. However we note that most of our definitions come directly from classical logic and are independent of personal construct theory. The formal mechanisms for defining entailment are rather more complex than those used here because the logic of entailment is concerned to avoid certain paradoxical results (Anderson & Belnap, 1975). The nature of these paradoxes does have some interest in personal construct theory because they are to do with *relevance* in entailment—does one proposition entail another in a relevant way or just through an artefact of the logical calculus? Similar but deeper questions arise when we consider the derivation of entailment from repertory grid data—is one construct relevant to another in the way in which it entails it or is the derived relation a fortuitous one? We consider such questions in sections 7 and 13.

It is also worth noting that our definitions of equivalence and entailment are also related to those in *modal* logics (Snyder, 1971). We can regard (2) and (8) as being definitions of *necessary* equality and *necessary* implication in a quantification model of a modal logic. In the context of personal constructs we can see this best by noting that two verbal interpretations of (8) are acceptable: "when you assign an element to the left-hand pole of construct *m* you *always* also assign it to the left-hand pole of construct *n*", or "when you assign an element to the left-hand pole of construct *m* you *necessarily* assign it to the left-hand pole of construct *n*". These links may be formalized through a *possible worlds* (Bradley & Swartz, 1979) model of modal expressions by noting that each element provides a possible world for construing. Entailments according to our definition then become logical implications that are true for all possible worlds currently under consideration. This is a useful and evocative viewpoint because it links personal construct theory with the linguistic semantics of counterfactuals and presuppositions (Lewis, 1973) which is very relevant to Kelly's concept of constructs being "used for predictions of things to come". It also provides useful technical links into the formal mechanisms for treating the topological structure (Lemmon, 1966) of possible worlds and its role in logic and semantics which seem equally applicable to personal construct theory.

To conclude the rather abstract discussion of this section and lead into the more concrete operational implementation of the next it is worth considering a specific example of what we mean by entailment, its asymmetry, and the derivation of equivalence from entailment but not vice versa. The poles of two constructs may be quite distinct in terms of equivalence yet closely related in terms of entailment. For example suppose that in construing people someone uses the two constructs *m*: *runs–doesn't run* and *n*: *energetic–passive*, then we might well expect to find that **LHPm** entails **LHPn** but that **LHPn** does not entail **LHPm**, that it is that being a *runner* entails being *energetic* but being *energetic* does not entail being a *runner*. If we analyse such a construct structure in terms of distance measures and hence of equivalence only then we shall not derive such asymmetrical relations between constructs even though they are meaningful and of practical interest.

5. ENTAIL: a program to derive entailments between constructs

It is simple to derive the entailment structure between the poles of constructs. We only have to check the truth of the four possible implications for all elements. Thus **LHPm** entails **LHPn** is checked by noting whether whenever an element is assigned to the left-hand pole of *m* it is also assigned to the left-hand pole of *n*. If so, then the entailment relation holds true, otherwise it is false. Clearly, as we noted above, it would also suffice to check that whenever an element is not assigned to the left-hand pole of *n* it is also not assigned to the left-hand pole *m*. We call the program that performs this analysis ENTAIL (*E*ntailment *N*ets *T*hrough *A*nalysing *I*mplicational *L*inks). Note again that the inference from a particular set of elements that one pole of construct *m* entails one pole of construct *n* is an inductive one if we assume that it applies to other elements in addition to those used in its derivation.

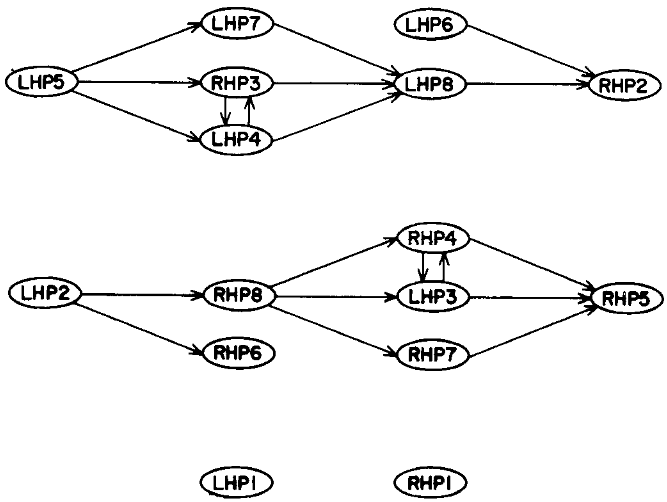

FIG. 5. Entailment analysis of Jane's constructs by ENTAIL.

Figure 5 shows the entailments between the poles of constructs derived by ENTAIL from the grid of Fig. 1; they are drawn out as a direct graph. There are effectively two main sub-graphs which are mirror images of one another plus two isolated poles. One of the sub-graphs shows the entailments for one set of poles, and the other the entailments for the opposite poles. Note that the "reversal" of constructs 2 and 3 apparent in the INGRID, QARMS and FOCUS analyses shows up as **LHP2** and **LHP3** appearing in the graph of the right-hand poles of the other constructs and vice versa. Because of the essential bipolarity assumed in the elicitation of the grid the two graphs are essentially the same with the arrows and poles reversed in one relative to the other. In section 11 we discuss extensions to the form of grids which would result in such pairs of graphs not necessarily having such a simple relation.

Note that we have taken advantage of the transitivity of the entailment relation not to draw in all the arrows strictly necessary. Thus we have not drawn an arrow from **LHP2** to **RHP7**, **RHP5**, **RHP4**, and **LHP3** because there is an arrow from **LHP2** to **RHP8** and then one from **RHP8** to **RHP7**, **RHP7** to **RHP5**, and so on. We can see from the figure that **LHP2** entails **RHP8**, **RHP7**, **RHP5**, **RHP4** and **LHP3** by tracing through the graph. Note that the equivalence between **LHP3** and **RHP4** now shows up as mutual entailment.

It is very interesting to compare Fig. 5 with the results of the INGRID clustering in Fig. 2, the QARMS clustering in Fig. 3 and the FOCUS clustering in Fig. 4. We can see that the same hierarchy of clusters has turned up: (3, 4); ((3, 4), 5, 7, 8); (2, 6); (((3, 4), 5, 7, 8), (2, 6)); with construct 1 unrelated to the others in all cases. Thus the ENTAIL analysis gives rise to the same basic clustering as did INGRID, QARMS and

FOCUS. At a more fundamental technical level we would expect all such distance-based clustering techniques to show such similarity with non-pathological data. Both the fundamental and empirical similarities are important in their own right since two of these programs are widely used for grid analysis and one would hope that any new technique would continue to provide at least the same basic analysis.

However, there is additional information in Fig. 5 that goes beyond that available in Figs 2, 3 and 4. This comes from the directed nature of the entailment links shown.

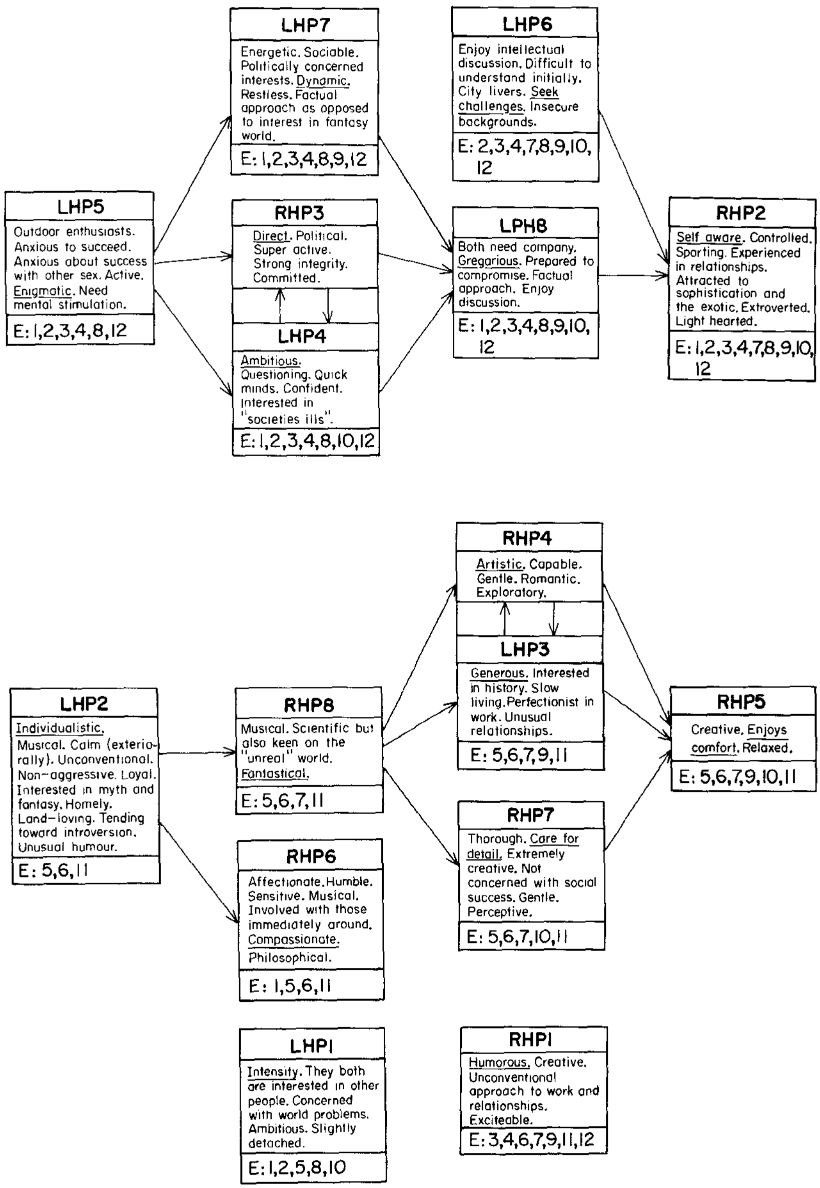

FIG. 6. Entailment analysis of Jane's grid by ENTAIL.

There is equivalence only between **LHP3** and **RHP4**—all the other relations are one way only. To show the significance of this we have drawn out Fig. 6 which gives the full text attached by Jane to each pole of the construct together with the elements assigned to that pole. The assymetry of the entailment relation may be seen by considering that from **RHP4** to **RHP5** for example. We see from the descriptions of the poles that Jane is saying that any one of her acquaintances who is *artistic* and so on is also *creative* and so on. However, the converse does not hold.

From the element data in Fig. 6 we can see the reason for this asymmetry. For example, from the elements assigned to **RHP4** and **RHP5** we can see that the entailment between them not being mutual is due to Jane's acquaintance element 10 being termed *creative* but not *artistic*. In this case only one element breaks down the equivalence. If we consider the entailment from **LHP8** to **LHP5**, that her acquaintances who are *musical* are also *artistic* then the converse is not true of two acquaintances, elements 9 and 10. And if we consider the entailment from **LHP2** to **RHP5**, that being *individualistic* entails being *creative* then the converse is not true for elements 7, 9 and 10. Thus the construct analysis produced by ENTAIL has reproduced the clusters shown by INGRID, QARMS and FOCUS but it has also shown up new features of the data not evident in these distance-based and essentially symmetric forms of analysis.

Note finally that the form of analysis produced by ENTAIL has some of the features of INGRID in that it is two-dimensional and some of the features of QARMS in that it provides hierarchic clusters. However, it also retains the key feature of FOCUS in that it represents the original data in a reorganized form. Like both INGRID and FOCUS it also shows the relation between elements and constructs, but unlike them it extends this relation to show the asymmetrical, directed entailment structure between constructs.

6. Strength of entailment

The program ENTAIL described in section 5 produces a list of entailments between constructs. The status of these entailments is best seen by noting that the question asked in putting an entailment on the list is effectively "does any assignment of elements to the poles of constructs show that this entailment does not hold"? If the answer to this question is "yes" then the entailment is not listed. Thus each entailment listed is *consistent* with the grid. We shall consider in section 13 the question of ascertaining whether the entailments listed are in some sense real determinants of the results or just artefacts. In this section we look at the other side of this question as to the significance of not listing entailments.

When we evaluate a graph of entailments such as that shown in Fig. 5, we are noting not only the arrows which are present but also those which are absent. There is an entailment from **LHP5** to **LHP4** but not one from **LHP4** to **LHP5**. Therefore **LHP4** is not equivalent to **LHP5**. There is an asymmetric relation between the two predicates which may be due to a variety of interesting phenomena (such as superordination—section 7). We are beginning to *interpret* the grid through the analysis produced by ENTAIL. However, how sure are we that entailments not shown are actually missing? How "near" to being equivalent are the two predicates? Section 13 examines one approach to answering such questions through interaction with the person from whom the grid was elicited. In this section we consider only the mathematical analysis of the actual grid data.

One possible approach to the "strength" of entailment is to relate it to *conditional probability* measures. We note that if, and only if, the entailment relation of (9) holds, then the conditional probability of **LHMn** being true for an element given that **LHPm** is true is 1, i.e.

$$p(\mathbf{LHPn}|\mathbf{LHPm}) = 1. \tag{13}$$

Hence it is natural to take this probability measure as one also of the strength of entailment. However, it has the defect of not dropping to zero when no relation holds between the two predicates. Indeed if **LHPn** and **LHPM** are independent of one another we have

$$p(\mathbf{LHPn}|\mathbf{LHPm}) = p(\mathbf{LHPn}), \tag{14}$$

so that a more descriptive measure of entailment can be obtained by subtracting this value and renormalizing to unity for the case of entailment:

$$m(\mathbf{LHPm} \to \mathbf{LHPn}) = (p(\mathbf{LHPn}|\mathbf{LHPm}) - p(\mathbf{LHPn}))/(1 - p(\mathbf{LHPn})). \tag{15}$$

This takes the value: 1 if **LHPm** entails **LHPn**; 0 if the two predicates are independent; and negative or intermediate values otherwise.

Such a measure is useful in giving more detail to the entailment analysis. However it does not satisfy our criterion of providing an analysis interpretable at the same level as the data—the measure itself introduces a new construction which will not be inherently meaningful to the person who generated the grid. An alternative approach to the grading of entailment was given in Shaw & Gaines (1980) which introduced the predicate *usually* in the analysis performed by ENTAIL. This predicate is a quantifier similar in nature to the "for all" used in defining entailment in (8), but qualified to allow for some disconfirming instances so that it may be read as "for all but N cases" where N is some small number, such as 1 or 2.

Such a quantifier allows a natural grading of entailment in terms that are immediately meaningful to the originator of the grid: "when you say someone runs you always also say they are energetic and when you say someone runs you usually also say they are energetic". Use of the quantifier *usually* to give a graded analysis gives a structure similar to the connectivity levels coming from Atkin's (1974) Q-Analysis. It is also readily extended to the multilevel case where rating scales rather than binary assignments are used in eliciting a grid (see section 9).

ENTAIL has facilities for calculating entailments under the quantifier *usually*. If we apply it to Jane's grid, then it condenses the construct structures shown in Fig. 5 into just: an equivalence between **LHP2, LHP3, RHP4, RHP5, RHP6, RHP7** and **RHP8**; a similar equivalence between the opposite poles to these; **LHP1**; and **RHP1**. With more complex grids, however, we have found the use of graded entailment through such a predicate an important feature of the analysis.

7. Entailment and the superordination/subordination hierarchy

The directed graph of entailment is reminiscent of the type of structure that we get when considering Kelly's concepts of "superordination" and "subordination" between constructs. He notes that there is a natural hierarchy amongst constructs (Kelly, 1955, p. 479):

> Constructs are construed by means of other constructs, and those, in turn, by still other constructs. It is thus a system is formed.

Entailment as defined here appears to treat constructs at the same level and yet to derive a hierarchical structure amongst them. We can see that this structure may have some relationship to Kelly's "system" through the example given previously: *runs* entails *energetic* but not vice versa because running is an energetic activity. Thus *energetic* is superordinate to *runs*. In logical terms we would normally expect predicates applicable to different categories to have different names and note that the predicate *energetic* applied to an activity is different from the predicate *energetic* applied to a person. In everyday language, however, ellipsis of various sorts is common and such distinctions are dropped, or implicit. The rationale seems to be that someone who undertakes an energetic activity will themselves be termed energetic.

We can formalize this argument by considering two constructs m and n such that n is superordinate to m and such that **LHPm** is construed as being assigned to **LHPn**. If we now assume that ellipsis occurs in statements such that any element assigned to **LHPm** of the subordinate construct is also stated to be assigned to **LHPn** of the superordinate construct, then we have the entailment

$$\mathbf{LHPm} \rightarrow \mathbf{LHPn}. \tag{16}$$

However, we do not have the converse entailment since it is possible for an element to be construed as assigned to **LHPn** without its being assigned to **LHPm**. This might happen, for example, through it being assigned to **LHPo** of an alternative subordinate construct o of construct n.

Thus we can see that the subordination/superordination hierarchical system defined by Kelly will show up as an entailment structure between the poles of constructs. However, can an entailment itself always be construed as arising from superordination/subordination? Again a simple model of some natural language phenomena suggests that the answer is yes. Korzybski (1933) has noted the wide ranging effects of the common phenomenon in natural language whereby we treat class-names as if they were those of individuals. If we have an entailment of the form of (16) then we may express this as **LHPm** "leads to" **LHPn**, meaning that any element assigned to **LHPm** is also assigned to **LHPn**. We may then through ellipsis treat **LHPm** itself as representing the class of elements assigned to it and hence itself being construed as an "element" assigned to **LHPn**. There then exists a relation between the constructs on Kelly's definition whereby m is subordinate to n and n is superordinate to m.

This relation between entailment and the subordination/superordination hierarchy raises many other questions: how does it relate to other approaches to eliciting the hierarchy such as Hinkle's (1965) "laddering" and Glanville's (1980) "circle of derivations"; how can we speak of a hierarchy of constructs when the converse entailment applies to the right-hand poles; does it throw light on the criticisms of the whole concept of a superordination/subordination hierarchy?

Firstly, the question of the relationship between implicitly derived structures in human rationality and explicitly verbalized ones is complex. Laddering derives the construct hierarchy directly by asking "why" questions to go up it and "how" questions to go down it: Q: why do you run? A: to be energetic; Q: how are you energetic? A: through running. We can infer from the first that running entails being energetic, and from the second being energetic is entailed by running. Thus, from a logical point of view the indirect elicitation of implicit entailments through ENTAIL and the direct elicitation of the construct hierarchy through laddering should correspond. There are two reasons in practice why this may not occur: that laddering tends to bring in additional constructs in that it is not only a structural analysis but also a different form of elicitation; and, more fundamentally, that the logical correspondence does not necessarily imply a psychological one—people's verbal expressions of the rationale behind their behaviour can be quite dissociated from their actual behaviour.

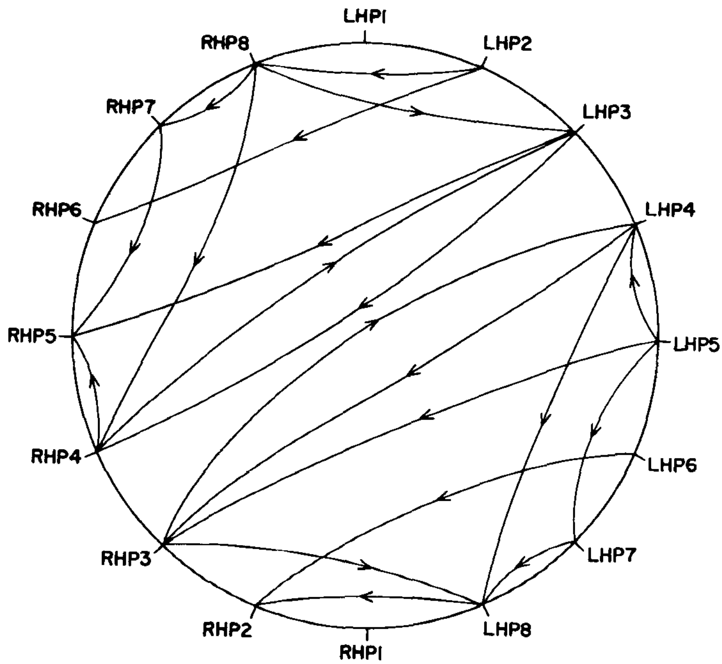

FIG. 7. Circle of derivations representation of ENTAIL analysis of Jane's constructs.

Figure 7 shows the "circle of derivations" corresponding to Fig. 5. This is what would be obtained directly from Jane using Glanville's technique if she agreed totally with the entailments derived by ENTAIL. It would be interesting in future studies to use both Hinkle's and Glanville's methods to obtain directly entailment structures and compare them with those from ENTAIL. Any irresolvable disagreements between the directly derived and the indirectly derived structures would be evidence of dissociation between verbal and actual behaviour. This dissociation can be very significant in attempts to extract from a person information about their skilled behaviour (Bainbridge, 1979). Examples occur in the literature on *expert systems* (Michie, 1979) which throw light on the difference between modelling the actual behaviour of people and accepting their own verbal models. Michalski & Chilausky (1980) have reported some interesting comparative results on a system for acquiring knowledge from experts on plant disease diagnosis where rules induced from the decision-making behaviour of an expert were far superior to those which the expert actually stated he was following.

The possibility of such dissociation between verbal and actual rationality does not affect just the relation between construct structures derived through ENTAIL and those derived through laddering. It is a general phenomenon whereby psychological and logical models of human rationality differ. For example, we might have someone who agrees that **LHPm** entails **LHPn** and also that **LHPn** entails **LHPm**, but does not agree that **LHPm** and **LHPn** are equivalent. Wason & Johnson-Laird (1972) have demonstrated that such pathology in the reasoning process is common in many human cognitive activities. Clearly there are many ways of resolving such conflicts. We can go back to the definitions of the terms, agree them and then point out the discrepancy, either in the general case or relative to the particular data. Such a "socratic" approach through explanation and example seems a natural extension of the interactive grid elicitation program PEGASUS (Shaw, 1980) that is often used in conjunction with FOCUS, and this is discussed in section 13. In concluding the discussion here we note that it is dangerous to assume that even basic logical relationships and results derived from them will always be obvious, or even accepted without debate, by people using personal construct structure analysis programs.

Our second question is on the direction of the construct hierarchy. We have already noted that in the conventional elicitation of constructs the entailment between two left-hand poles is inherently associated with a reverse entailment between the corresponding right-hand poles. Thus from (16) we can infer

$$\mathbf{RHPn} \rightarrow \mathbf{RHPm}. \tag{17}$$

This association is often a natural one but there seem to us no logical grounds why it should be a necessary one and in section 11 we discuss an extension of conventional grid methodology which avoids the direct derivability of (17) from (16). However, regardless of this, there will still be a tendency for the left-hand and right-hand poles of a construct to be at opposite ends of the order relation derived by ENTAIL. This might seem to imply that any particular construct may be at either end of the hierarchy according to which pole one considers, and this then conflicts with Kelly's definition of the hierarchy in terms of generality.

This problem can be resolved in major part by noting that the inverse relation exemplified by (16) and (17) causes the entailment graph for the poles to split into two subgraphs that are duals of each other. Figure 5 illustrates this for the particular example analysed. Either subgraph gives rise to the same construct hierarchy but with the arrows reversed. Whether the direction of the arrows indicates increasing subordination or increasing superordination is often obvious by inspection in looking at the relative generality of the two extremes. Thus the entailment hierarchy can be used to derive the structure of the subordination/superordination hierarchy but its direction needs to be determined by other considerations.

The example of Jane's grid used as an illustration is exceptionally simple and in general more complex and fragmentary structures may be found. For example, the isolation of construct 1 in Fig. 5 illustrates that there is no necessity for all the constructs to fit into the same hierarchy. Each of the dual subgraphs may fragment into subgraphs and then one may have several different systems of subordination/superordination. Also, as noted above, there is no reason for the entailments to be necessarily construed as a subordination/superordination hierarchy. This is only one way in which entailment can arise and provides one possible rationale for explaining the entailments.

It is probable that the discussion of this section and particularly these final notes throw light on our third question about criticism of the whole concept of a subordination/superordination hierarchy. Slater presents a number of argments that lead him to the conclusion that "the theory that construct systems are hierarchical appears questionable to some extent" (Slater, 1976, p. 45). In particular he quotes Kelly to the effect that "the ordinal relationship between constructs may reverse itself from time to time" (Kelly, 1955). If our discussion here relating entailment through linguistic ellipsis to subordination/superordination provides a model of the actual processes at work then they may be expected to be variable and subject to change. Constructs themselves are conventional and so are the entailments between them and hence so is the structure of the construct system. If our construct systems are used to guide our actions then as our goals change it may well be that the structure of our construct system itself changes. What is a "cause" in one context may become an "effect" in another.

Thus, viewing the basic ordinal structure of constructs as being one of logical entailment between poles does enable one to subsume other such structures and provide a basis for understanding their operation and dynamics.

8. Asymmetric element analysis

The logical analysis of construct structures through the asymmetrical implication relation makes sense both formally and intuitively. Is there a comparable analysis for the elements? At first sight the answer may appear to be negative. One element "entailing" another is not necessarily a natural concept, whereas one element being "near" another in construct space is much more so. We can interpret such "nearness" as *similitude* and have a natural interpretation of the two elements being similar. However, there are two factors which should be taken into account in analysing the element structure.

Firstly, if we look at the relation between elements in terms of a distance structure based on the vectors of values of elements on constructs then the *weighting* assigned to each construct dimension is very significant in determining the element clusters. This weighting determines the relative significance that we attach to dissimilarities between elements in relation to differing constructs. If we apply a uniform weighting then we are effectively assuming that each construct is equally important in determining the grouping of elements. This clearly depends on how the grid was elicited and the purpose of doing the grouping.

Secondly, if we look at an asymmetric implication relation between elements we are again making assumptions about constructs and their significance. The type of relation will be

$$\forall \mathbf{LHP} \quad \mathbf{LHP}\,\mathrm{E}m \supset \mathbf{LHP}\,\mathrm{E}n, \tag{18}$$

which we can abbreviate conveniently to

$$\mathrm{E}m \rightarrow \mathrm{E}n. \tag{19}$$

The quantification is now over the predicates so that what we are considering is not expressible in the first-order predicate calculus. The meaning of the expression is dependent on some assumptions about the coherence of the class of predicates over which quantification occurs.

One possible source of coherence amongst constructs is that they have a *preferred* pole and we might then interpret the "for all" in (19) as "for all the preferred poles". The arrow in (19) then defines a *preference order* on elements since E$m \rightarrow$ En then means precisely that "En is construed as preferred on at least every construct where Em is preferred". If we use ENTAIL to analyse Jane's grid of Fig. 1 for the preference relation between elements assuming for the purposes of this example that the preferred poles are the left-hand ones except for constructs 2 and 3, then we obtain the preference graph between elements shown in Fig. 8. The close resemblance of this in terms of clustering between elements to the INGRID (Fig. 2), QARMS (Fig. 3) and FOCUS (Fig. 4) analyses will be noted. However, Fig. 8 also contains additional information since it gives a direction of preference.

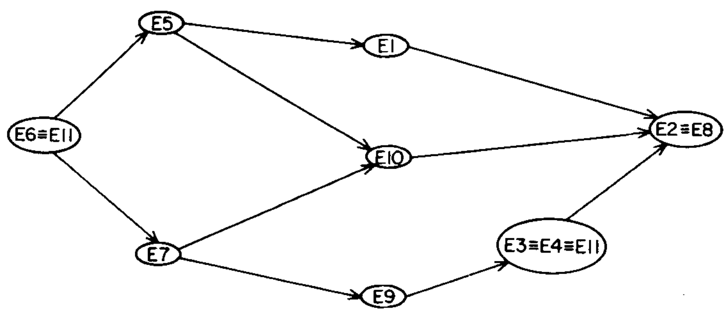

FIG. 8. Preferences analysis of Jane's elements by ENTAIL.

Thus the logical approach to grid analysis also leads to a comparable element analysis to other approaches. It can also provide additional information about asymmetric relations between elements when an appropriate interpretation such as a preference relation over constructs exists.

9. Extending entailment to rating scales—fuzzy semantics

So far in this paper we have analysed grids with binary assignments of elements to poles using a classical logic with two truth values. In this section we show how the logical analysis extends to the *multivalued logics* (Rescher, 1969) with which one can analyse grids based on rating scales. Kelly (1955) presented constructs as binary categories and based his own methodology for eliciting constructs on this. However, other workers found the need for "shades of grey" between the two poles of a construct and in a later work Kelly (1970, pp. 13–14) notes that this is consistent with his notion of a construct:

> The construct, of itself, is the kind of contrast one perceives . . . while constructs do not represent or symbolize events, they do enable us to cope with events, which is a statement of a quite different order . . . . They also enable us to put events into arrays or scales, if we wish.

It is common in many practical applications of repertory grids to use an N-point scale with 1 being an assignment to the left-hand pole and N being an assignment to the right-hand pole, and intermediate numbers representing some form of "intermediate" assignment. N is usually odd, 5 or 7, to allow a "neutral" mid-point to the scale.

The semantics of such rating scales presents a number of problems in its own right. Kelly's original binary assignments may be interpreted as the truth or falsity of predicates. Intermediate points on a rating scale are not so readily, or uniquely, interpretable. For example, the "neutral" point 3 on a 1 to 5 scale say may be interpreted as "this element lies half way between the poles", or as "this element should be assigned to both poles", or as "neither pole is appliable to this element", or "sometimes this element comes under one pole and sometimes another", or "I am not sure what pole to put this under", or "I do not wish to construe this element in this way", and so on. In logical terms we are attempting to use a single truth value to encompass many different modalities (White, 1975).

The extension of binary distinctions to multi-valued ones may be treated at a fundamental level. We have already noted in section 2 the close relation of Kelly's constructive alternativism to Spencer Brown's "calculus of distinctions". Varela (1979) has shown how Brown's calculus may be extended to the multi-valued case. Within a basic bipolar distinction may be interpolated others through logical operations that correspond to expressions that generate paradoxes of *self-reference* in classical logic. Varela (1975) shows that an essentially three-valued logical calculus arises from the use of a single self-referential form in Brown's calculus of distinctions. Gaines (1976) shows how such "primitive paradoxes" may be iterated to give an indefinite number of distinctions between the poles of the distinction originally made, and hence how the truth value of an arbitrary proposition may be approximated to any accuracy on a continuous scale through a Dedekind section.

This move from a binary basis for making distinctions to a multivalued one raises problems of a semantic nature even at a fundamental level, particularly those of interpreting intermediate "truth-values" (Haack, 1979). However, the need for rating scales in practice, and an appropriate underlying theory, does seem an essential one in terms of the human construct systems and their logic. In the physical sciences the expected and preferred source system in which to represent data is quantitative. We use a source system of physical quantities and their precise measurement. However, the underlying constructs of physics have been derived and refined over a very long period and are themselves of a peculiar, and perhaps unique, nature. The existence of continuous and limitless scales for physical variables of length, time, mass, charge and so on, is an important phenomenon that marks out the constructs involved as being different from those in many other sciences.

The existence of refined measuring schemes for some constructs should not blind us to their close relationships to other constructs for which no such physical measurement exists, for example, the concepts of "tallness" and "beauty" (Gaines, 1976). The concept, the perception, of "tallness" exists in a more primitive sense than does the measurement of "height". We are able to generate and follow arguments involving "tallness" without having any concept of inches, centimetres, or any other metric scales. Whilst a "scientific" analysis might conclude that there is a wide and ill-defined range of physical phenomena that combine in an extremely complex fashion to produce the subjective impression of "beauty", in everyday reasoning it is as primitive a term as "tallness". We certainly do not distinguish between them in arguments such as:

He likes girls that are tall and beautiful.
Mary is not very tall but very beautiful.
He will probably like Mary.

Such considerations led Zadeh (1965) to develop a theory of *fuzzy sets* that closely paralleled that of classical set theory but allowed for "shades of grey" in set membership. He extended the definition of the *characteristic function* of a set to include not just the binary values 0 and 1 but also the continuous interval between them. In classical set theory the characteristic function of a subset maps the elements of the universal set into 1 if they belong to the subset and into 0 if they do not. Zadeh allowed the elements to take the values in between also and called them *degrees of membership* to the subset. He showed that it was possible to extend the normal set-theoretic operations such as union, intersection and complementation, in a simple and natural way to fuzzy sets with continuous characteristic functions.

Since Zadeh's original study there has been a rapid growth in the literature on fuzzy sets and their application to system theory, control engineering, psychological modelling, linguistics, and so on (Gaines & Kohout, 1977). The related logical calculus derived from fuzzy set theory in the same way that the classical predicate calculus may be related to conventional set theory is of particular interest for this paper and has been presented as a system for *fuzzy reasoning*. This logic has been found to be one already studied by the Polish logician Łukasiewicz (Rescher, 1969) and of particular importance since White (1979) has shown recently that it avoids paradoxes such as that of Russell's "barber" (Hughes & Brecht, 1976) which arise from the unrestricted use of the axiom of comprehension in naive set theory. Since its inception fuzzy set theory has been used to model human verbal reasoning and concept processing. Goguen (1974) takes a formal axiomatic approach to the notion of a "concept" in natural and artificial languages and shows within a very general category-theoretic framework that one obtains generalized fuzzy sets.

These considerations led Shaw & Gaines (1979, 1980) to propose a fuzzy set semantics for personal constructs that could deal with the analysis of entailment in repertory grids using rating scales. In this paper the fuzzy sets and logic have been left deliberately until this late section so that they do not confuse the basic discussion of systems of entailment and their derivation from grid data. Suppose in the discussion of section 4 one now assumes that the predicates **LHP** and **RHP** are not just true or false, but also have the possibility of intermediate degrees of membership to being true (with false interpreted as a degree of membership of 0 to being true). Then the rest of the discussion of that section follows virtually without change but one now has a model of entailment in grids whose values are not binary. The implication and entailment operations are now those of Łukasiewicz multivalued logic and entailment holding between two poles is now not just true or false but can also take intermediate values.

The program ENTAIL described in section 5 has been written to take into account such multivalued data (as have INGRID, QARMS and FOCUS). The discussions of sections 6, 7 and 8 also generalize immediately to multivalued data and logics. Clearly the logic system itself now provides another measure of the "strength" of an entailment and we can see that what is discussed in section 6 differs from this in measuring the strength to which the entailment is verified as being present. Since Łukasiewicz logic defaults back to the standard propositional and predicate calculi when intermediate values are not used (Gaines, 1978) it is actually more convenient to develop the whole of the theory of construct structures and analysis described here directly in terms of fuzzy logic and this would seem appropriate for future studies.

One important feature of Zadeh's work has been its emphasis on the linguistic nature of human reasoning and the use of fuzzy set theory to model the use of *hedges* such as *very* and *rather* in human reasoning. This is similar to the interpretation of the points on a rating scale in terms of such hedges as "very", "slightly" and "quite" used in semantic differential techniques (Osgood, Suci & Tannenbaum, 1957). Thus there are natural verbal interpretations of the rating scale when values are input and these may also be applied to the equivalent values resulting from the ENTAIL analysis. One may say that there is a "quite strong" or a "very strong" entailment from one pole to another. Our requirement that the terminology and concepts of the analysis be those of the data thus continue to be satisfied in the extension to multivalued logics.

10. Extending entailment to compound predicates

The analysis of the entailment structure of a repertory grid given in section 4 was applied only to the atomic predicates and not to compounds such as **LHP1** OR **LHP2**, or **RHP3** AND **LHP5**. Since the truth values of all such compounds may be derived in any truth-functional logical calculus from the truth values of the components it is possible to extend the analysis to relations between components. There is no intrinsic technical problem except that the number of compound predicates that might be considered grows as a double exponentional of the number of atomic predicates. Thus a simple-minded extension to the techniques described in sections 4 and 5 produces an overwhelming mass of results.

Fortunately there are two properties of the entailment relation that greatly simplify the analysis. The first is that it is possible to represent the entailment from the disjunction of a number of predicates as the conjunction of a number of elementary entailments. We have:

$$(\mathbf{A} \text{ OR } \mathbf{B} \text{ OR } \mathbf{C} \text{ OR} \ldots) \rightarrow \mathbf{X} \equiv (\mathbf{A} \rightarrow \mathbf{X}) \text{ AND } (\mathbf{B} \rightarrow \mathbf{X}) \text{ AND } (\mathbf{C} \rightarrow \mathbf{X}) \text{ AND} \ldots. \tag{20}$$

So that it is possible to neglect such compounds as that on the left-hand side of (20) in the analysis and consider only the atomic forms on the right-hand side.

A similar consideration applies to the conjunction of propositions on the right-hand side of an entailment. We have

$$\mathbf{X} \rightarrow (\mathbf{A} \text{ AND } \mathbf{B} \text{ AND } \mathbf{C} \text{ AND} \ldots) = (\mathbf{X} \rightarrow \mathbf{A}) \text{ AND } (\mathbf{X} \rightarrow \mathbf{B}) \text{ AND } (\mathbf{X} \rightarrow \mathbf{C}) \text{ AND} \ldots. \tag{21}$$

So that it is possible to neglect such compounds as that on the left-hand side of (21) in the analysis and consider only the atomic forms on the right-hand side.

We also have that adding a further predicate conjunctively to the left-hand of an entailment or disjunctively to the right-hand leads to a derived entailment. That is, if we have

$$\mathbf{A} \rightarrow \mathbf{X}, \tag{22}$$

then we also have, for any **B**,

$$(\mathbf{A} \text{ AND } \mathbf{B}) \rightarrow \mathbf{X} \tag{23}$$

and

$$\mathbf{A} \rightarrow (\mathbf{X} \text{ OR } \mathbf{B}). \tag{24}$$

From these considerations and the transitivity of entailment it is possible to provide a set of entailments between compound propositions that serves as a base for deriving all others. This form of compound analysis comes closer than does the basic ENTAIL analysis of Fig. 5 to Pask's (1975) form of "entailment structure" analysis of subject matter for learning. If certain predicates are thought of as *outputs* to be derived from the others which are *inputs* then the entailment analysis can be seen to be closely related to the analysis of switching functions in both binary and fuzzy automata theory (Kandel & Lee, 1979).

11. Negation, opposites and relevance

A number of times in this paper we have noted that the role of the two poles of a construct as *opposites* has not been adequately treated. In our logical analysis the left-hand pole and the right-hand pole have been treated as distinct predicates of equal status. We have noted (sections 4 and 7) that the conventional elicitation of constructs leads to an inverse relation between the poles such that the predicate corresponding to one pole behaves as the logical *negation* of that corresponding to the other. This should perturb us since it appears to lead to precisely those defects of formal logic that Kelly warns against (Kelly, 1955, p. 106):

> Now conventional logic would say that black and white should be treated as separate concepts. Moreover, it would say that the opposite of black can only be stated as not black, and the opposite of white can only be stated as not white. Thus the person whose field we mentioned would have shoes which would be just as much not white as the time of day, and he would write on paper which would be just as not black as the distance to his office.

Part of the problem that Kelly is discussing here is one of *relevance*. "Not white" is a predicate relevant to shoes but not to the time of day. The standard predicate calculus fails to distinguish between "not" and "not relevant". We noted in section 1 that it is only in recent years that logics accounting for "relevance" in a very formal sense have been established (Anderson & Belnap, 1975). However, what even such logics do not encompass and Kelly brings out is the psychological role of the concept of *opposite* which has no logical counterpart—it is related to negation but not identical to it.

This introduction of the importance of modelling the role of opposites in human thinking is not peculiar to Kelly but is a continuing theme in philosophy from early times. The Pythagoreans used a table of opposites in analysing entities with ten constructs such as "*limited–unlimited*" and "*good–evil*". Mao Tsetung in his essay "On Contradiction" emphasizes the essential interdependence of opposites (Mao Tsetung, 1937, p. 61):

> no contradictory aspect can exist in isolation. Without its opposite aspect, each loses the condition for its existence.... Without life, there would be no death;

> without death there would be no life. Without above there would be no below; without below there would be no above.... It is so with all opposites; in given conditions, on the one hand they are opposed to each other, and on the other hand they are interconnected, interpenetrating, interpermeating and interdependent.

Mao also brings in the notion of relevance in defining opposites and uses the notions of contradiction yet identity amongst opposites in his exposition of an epistemology which closely mirrors Kelly's constructive alternativism.

This line of reasoning can be traced back through Lenin (1914) to Hegel whose basic logic of *thesis* and *antithesis* leading to a *synthesis* is founded on what seems to be the most careful distinction between opposite and negation in the philosophical literature. Hegel distinguishes between negation as an *absolute* difference and opposition as an *essential* difference, and Bogomolov singles this out as the foundation of dialectical logic (Bogomolov, 1977, p. 137):

> the investigation of the relation of two objects... begins with establishing the *difference* between them, expressed in the most general form, with their mutual negation (A and ~A). To put it differently the second object acts initially as the simple negation of the first and is naturally expressed in logic by its indefinite negation.... Describing this kind of development of the concept, Hegel saw in it the transition from absolute difference to essential difference (variety), and from this to opposition (antithesis), as one of the stages of the general path from identity through difference to *contradiction*.

Thus we may see that Hegel's dialectics is crucially dependent on the transition from the concept of general negation to that of opposition. An opposite is some basis for there being negation, some *reason* for it, and it is the underlying construct to which this opposition is *relevant* that Hegel regards as the "synthesis" of the opposition between thesis and antithesis. Thus there is a close relationship between the epistemology put forward by Kelly and that put forward by previous philosophers concerned with dialectics. However, neither Pythagoreans nor Hegelians justify in logical terms their assertion that opposites are fundamental to reasoning. Kelly does not himself do so except by quotations like that at the beginning of this section which point out by example the difference between the negation of a construct and an opposite to it. Indeed one may argue from the presentation so far of a classical logical analysis of the repertory grid that in its original form it has already lost the possibility of coping with either relevance or the distinction between negation and opposition.

If we start with essentially bipolar constructs such that an element must be assigned to one, and only one, pole then we cannot treat relevance within a uniform framework. Kelly has to introduce it separately in terms of constructs having a "range of convenience". However, by considering an element to have quite distinct assignments to the two poles of a construct, i.e. to a construct and its "opposite" we can also capture the concept of relevance. A construct is *irrelevant* to an element if the element is assigned to neither of its poles (or, in the context of fuzzy logic, if its degree of membership to both poles is zero). Thus, in terms of Kelly's example at the beginning of this section the construct "*white–black*" is irrelevant to the time of day because it is both *not white* and *not black*. Those who extended his bipolar notion to allow for multipoint rating scales also failed to allow for relevance when they made the scales a one-dimensional interpolation between the two poles of a construct. However, the approach taken here is readily extended to the multipoint case by allowing separate ratings on the two poles of the construct. It is clearly debatable still whether this explication of relevance captures all its psychological connotations. We would suggest only that it captures some key ones.

What we have proposed is a very simple extension of Kelly's repertory grid methodology that gives us a logic capable of dealing with relevance and Kelly's notion of a "range of convenience". The mechanism used is crucially dependent on every predicate having an "opposite" so that one can distinguish between the predicate being *not true* for an element (element assigned to opposite predicate) and its being *not relevant* for the element (element assigned to neither predicate nor opposite). This demonstrates the importance of the concept of an "opposite" emphasized by so many different philosophers and gives a formal model for the utility of opposites. In previous papers we have analysed the semantics of opposite predicates and developed various logical constraints upon them (Shaw & Gaines, 1979, 1980). However, in the present context of repertory grid analysis an opposite predicate is just whatever the person from whom the grid is elicited chooses it to be. The ENTAIL analysis will cope with assignments to the two poles of a construct that are completely unconstrained in their mutual relationships.

The possibility of making separate assignments to the two poles of a construct and of analysing such extended forms of the repertory grid seems significant for a number of applications already noted in the literature. Slater (1977, p. 46) points out that missing data creates major problems for distance-based grid analysis, and yet it is a common problem. Kelly states (Kelly, 1955, p. 271):

> The assumption which is specific to a grid form of the test is that all the figures fall within the range of convenience of the constructs.... This may not be a good assumption in all cases; it may be that the client has left a void at a certain intersect simply because the construct does not seem to apply one way or the other.

Landfield (1976, p. 97) gives an example of a grid elicited from a patient which goes beyond this and allows the two additional values "N" for neither pole applicable and "?" for either pole applicable. In terms of our discussion above his "N" corresponds to an assignment of false to both poles and his "?" corresponds to an assignment of true to both poles. Thus the grid he elicits is readily analysed by ENTAIL. Obviously when ENTAIL analyses a particular entailment between a pair of poles under these circumstances it is relative to the elements actually construed in relation to those poles. However, it is possible to provide an analysis which does draw as much as possible out of the data given and does not crucially depend on all elements being assigned to one pole of every construct.

It is interesting to note that the logic being used by ENTAIL to deal with Landfield's four "truth values" is precisely that proposed by Belnap (1976) to deal with the epistemology of database systems. He proposes to deal with both missing and contradictory information in a database by allowing four values: *Told True*; *Told False*; *Not Told*; *Told True and Told False*. Gaines (1979) shows that such a logical structure also avoids the possibility of paradoxes such as that of Russell's barber arising through the imposition of semantic constraints on a database, and suggests the extension of the logic to continuous values in order to avoid deeper paradoxes. Again in this one can see the significance of the separate treatment of the opposite of a predicate in establishing a logic that is pragmatically sound.

12. Interactive construct elicitation and analysis

In section 13 we shall discuss how some of the unresolved problems of the logic of personal constructs may be resolved through a "dialectical" approach in which an interactive computer is used to explore the results of the analysis. It would be easy to assume that such interactive programs are merely more convenient ways of eliciting construct systems through extensions of Kelly's repertory grid and do not themselves add anything qualitatively new to the process. However, such an assumption would be missing centain crucial psychological factors in the man–computer situation and its differences from the man–man situation. We have observed informally in making PEGASUS available to a wide range of people in a variety of situations that those coming to it for the first time often seem to find it a very dramatic experience. They react to it intensely and become gripped by the interactive process of construct elicitation. They also feel that they are learning something new from the process and are prepared to use this in determining their behaviour.

Probably such involvement is also significant in the elicitation of construct systems by a person rather than computer interaction. However, we believe there are certain quite fundamental differences when the elicitation is done in such a way that interpersonal interaction is clearly absent. In particular, when a *person* is feeding back comments and guidance it is a natural and ready assumption that the constructs are being *injected* rather than elicited. It is easy for the subject to believe that the elicited constructs do not come from himself but that a tutorial or debating situation with another person is taking place. It is necessary to *persuade* him that this is not so and the persuasion has to be stronger the more striking and significant the constructs elicited. However, when a computer is the tool by which his construct structure is being reflected or laid bare then such an assumption of outside injection and interference is far less tenable.

When constructs are being elicited by a computer program then it is more likely to be accepted that it is precisely and only oneself that is being portrayed. We "trust" a computer program to be doing just what it appears to be doing without deeper motivations and without attempting to persuade us to its point of view. No-one is telling the user anything. He is seeing in interacting with PEGASUS, possibly for the first time, the basis for his own thought processes. Very often extreme surprise is the first reaction. If another person were eliciting the construct structure then the surprise would be taken as an indication that he was incorrect and one would ignore him or argue with him. With computer elicitation it is more likely that one will accept the reflected structures as being self-generated and the surprise acts as motivation to know more.

That this knowledge can be totally private to oneself is another important feature of interaction with the computer. We do not like, as Kelly put it, to be "caught with our constructs down". When another person is involved we are more reluctant to expose and explore our constructs the more surprising they are; perhaps because the surprise is often the result of a conflict between our ostensive value judgements and the basis of our behaviour. Or it may just be sloppy verbal behaviour: that we are naming two distinct constructs with the same label. For example, in using PEGASUS a scientist found that he was using the word "time" to label several different constructs and generating confusion in his arguments because of this.

Another reason that we are reluctant to explore construct structures freely in interacting with another person, particularly a professional person, is that we are acutely aware of the possible "waste" of their time. This phenomenon has been noted (Card, Nicholson, Crean, Watkinson, Evans, Witson & Russell, 1974) as accounting for a major part of the preferences expressed by patients to be interviewed through an interactive computer program rather than their doctor. There are many pressures and artefacts of interpersonal relationships that can totally obscure and undermine such reflective processes as we require in the elicitation of personal constructs.

It is interesting to note that this argument has been put in reverse: Adams (1979) notes that children learn quickly to play games on a personal computer and conjectures that this is because of the lack of interpersonal complications. She suggests, however, that "one of the benefits of game-playing is that a child learns how to behave with and towards others, how to cope with success and failure, and what effect it has on others. In the human–computer relationship the child does not learn these valuable social skills." We are arguing conversely that the need to be deploying such "social skills" is a load that can seriously detract from the exploration of the self.

A notable technical feature of PEGASUS that profoundly affects human reactions to it is that relationships between constructs may be inferred instantly and queried with the user. This immediate analysis and feedback is a key factor in most applications of interactive computers and can go way beyond what any manual analysis can accomplish. Instant feedback whilst one remembers one's line of reasoning is very different from delay analysis that arrive at a later time when the entire context of the replies one has been giving may have been forgotten. Construct structures in particular have a high degree of context-dependence. It is often the relationship between the structures elicited and the *role* we are adopting in answering the questions that elicit them which is of prime interest to us. Using Wolff's (1976) terminology, we *surrender* ourselves to a particular role and become a "physicist", a "mathematician", a "manager", a "father", etc., and it is the analysis of our construal of the world in the specific role which we are attempting to *catch.*

These aspects of the computer elicitation of repertory grids with immediate feedback of the results of the analysis were those that led to our study of more powerful logical tools for analysing grids. In the next section we consider some of the implications of the discussion earlier in this paper for extending PEGASUS.

13. Database dialectics

This section is the most speculative of the paper since it represents work to be done rather than that already completed. We are presenting here the new directions in construct elicitation that follow from the discussion of this paper both in terms of how the analytical results can aid the elicitation and also in terms of how the availability of direct interaction can aid the analysis. To make the discussion of this section more pointed we present a number of specific recommendations for the further development of interactive construct elicitation systems.

In terms of the discussion of the preceding section it seems reasonable to suggest that one takes an existential view of the phenomenon of computer elicitation of personal constructs regardless of one's view of Kelly's theory and methodologies based on it. The computer interaction is in itself a meaningful and significant experience for many people and they gain from it. Perhaps it is only that introspection is not a skill developed by most current educational systems. We promote the "received view" of knowledge and act as teachers to bring the minds of students into conformity with our consensual models of reality. What you think does not matter in itself, only that it does not deviate from what it is "correct" to think. It is a novel experience for many people to realize that there are actually individualistic thought processes going on within them. It is even more novel for them to realize that these condition "reality" and that different approaches to life and different reactions to the same circumstances may be ascribed to different construals of reality.

Thus our objectives in developing PEGASUS are to set up a suite of interactive computer programs that enable people to explore their own "realities", singly or in groups, through an open-ended "discussion" of freely chosen elements and constructs and the relations between them. Currently a view of reality is expressed as a grid giving ratings of elements on a scale between the poles of constructs. A collection of such grids is precisely equivalent to a *relational database* (Codd, 1970) with constructs as field names and elements as objects in the database. Thus our first generalization from PEGASUS is to work with a general database that contains the grids as relational entities:

> *Recommendation 1*: Regard a construct elicitation program as building up a database in which construed elements are objects in the database and the constructs determine field names.

In section 11 we advanced reasons why one should allow ratings to be separately assigned to each of the poles of a construct and hence a second generalization is:

> *Recommendation 2*: Assign a separate field for each pole of a construct and allow a degree of membership to be assigned independently to each.

Note that this is not intended to preclude the conventional form of grid in which the rating on one pole is the complement of that on the other. It allows for the generalization and also for the conventional usage.

Gaines (1979) analyses some of the defects of current relational database implementations and notes the need for fuzzy predicates to be allowed even when apparently definite values are assignable. For example we may wish to say that someone is either in department X or department Y. This can be represented by giving a degree of membership of unity to both these departments and to no others. It seems useful to allow for such conventional data base fields in this extended form to be stored also even if they are not conventional examples of constructs:

> *Recommendation 3*: Allow conventional database items to be stored with a field for each value to which a degree of membership may be assigned.

One important feature of PEGASUS is its conversational mode of operation but this currently involves the use of rating scales which can seem somewhat artificial. We have already noted that it is possible to replace these with fuzzy hedges such as "slightly" and "very", and it seems desirable to incorporate this facility into any new system:

Recommendation 4: Allow for rating scale values or degrees of membership to be entered linguistically as fuzzy hedges.

The database itself should be accessible for interrogation, modification and deletion through any reasonable access path, and the presentation and modification of data should also be linguistic where appropriate:

Recommendation 5: Allow access to the database for interrogation, deletion and modification through normal database access mechanisms and present the data in the linguistic form in which it was entered.

The results of an ENTAIL analysis are essentially degrees of membership to equivalences and entailments or preferences. These may also be expressed linguistically through the use of fuzzy hedges, and other relevant features of the analysis such as the use of the quantifier "usually" may also be presented in this way:

Recommendation 6: Express the results of the database analysis in linguistic terms using the same hedges as those used in setting it up.

Another important feature of PEGASUS is the way in which it directs the dialogue in eliciting constructs by feeding back information about closely related constructs or elements and asking the user to provide further data to split them. The following dialogue with PEGASUS in the context of "exploring learning situations" demonstrates this process in action (Shaw, 1980, pp 61–62):

```
THE TWO CONSTRUCTS YOU CALLED
 2 FLEXIBLE-RIGID
 6 VARIABLE CONTENT-SPECIFIC CONTENT
ARE MATCHED AT THE 85 PERCENT LEVEL
THIS MEANS THAT MOST OF THE TIME YOU ARE SAYING
FLEXIBLE YOU ARE ALSO SAYING
VARIABLE CONTENT
AND MOST OF THE TIME YOU ARE SAYING
RIGID YOU ARE ALSO SAYING
SPECIFIC CONTENT
THINK OF ANOTHER ELEMENT WHICH IS EITHER FLEXIBLE AND
SPECIFIC CONTENT
OR VARIABLE CONTENT AND RIGID
IF YOU REALLY CANNOT DO THIS THEN JUST PRESS RETURN AFTER
THE FIRST QUESTION MARK. BUT PLEASE TRY. THEN YOU MUST GIVE
THIS ELEMENT A RATING VALUE ON EACH CONSTRUCT IN TURN. TYPE
A VALUE FROM 1 TO 5 AFTER EACH QUESTION MARK.

WHAT IS YOUR ELEMENT?VIDEO TAPE
RATINGS:
INVOLVEMENT-REMOTENESS?3
FLEXIBLE-RIGID?2
...........................
```

This type of feedback is readily generalized to other forms of analysis such as that produced by ENTAIL. A similar dialogue might contain:

```
WHENEVER YOU SAY "RUNS" OF A PERSON
YOU ALSO SAY "ENERGETIC"
CAN YOU THINK OF A PERSON WHO IS "RUNS"
BUT WHO IS NOT "ENERGETIC"
```

This leads to the recommendation:

Recommendation 7: Offer a mode of database elicitation in which significant features of the structural analysis are fed back to the user to encourage exploration of the data space.

Clearly, the feedback should follow recommendation 6 and use linguistic terms. The "85 PERCENT LEVEL" mentioned in the first example above has no equivalent in the user's vocabulary and requires a user to have some technical knowledge to interpret it.

We have commented a number of times in this paper on the problem of determining whether the results of an analysis are just artefacts of particular data or represent significant relations that are *necessary* in some sense. There are mathematical techniques for evaluating the *significance* of analyses but these all depend on fairly strong assumptions about some form of *distribution* from which the data is a sample. Such assumptions are singularly inappropriate for personal construct data where one is examining the data structure of an individual in restricted circumstances. In this context it seems more appropriate to ask the users themselves to verify the meaningfulness of the analyses for themselves:

Recommendation 8: Feed back the results of the analysis to the user and ask him to rate the meaningfulness or significance of each part of it.

The feedback of recommendation 7 is related to this process in that it gives the user the opportunity to change the analysis in a critical way by adding data that does not conform with it. This may be thought of as a "Popperian" mode of falsification of hypothesis through the search for confounding data, whereas recommendation 8 allows for this by command. A failure to agree with the analysis whilst at the same time being unable to produce a counter-instance might correspond to the dissociation between behaviour and verbalization discussed in section 7.

The converse failure to agree with the analysis is, for example, to feel that an entailment should exist which is not derived. In this case the user should be able to ask the system for the evidence against the supposed relation. For example:

```
WHY NOT ENERGETIC MEANS RUNS
BECAUSE YOU SAID JACK IS "ENERGETIC" BUT NOT "RUNS"
```

This leads to the recommendation:

Recommendation 9: Allow the user to propose possible analyses and reflect back to him evidence from the database which disconfirms these.

PEGASUS has no inbuilt knowledge of natural language and its "conversations" are somewhat stilted. This is even more apparent when one uses linguistic forms of analysis as in the example above. The success of elementary natural language conversational systems in recent years ranging from Weizenbaum's (1967) ELIZA through Winograd's (1972) SHRDLU to Harris's (1977) ROBOT leads one to believe that it may be possible to embed interactive construct elicitation within a framework of natural language conversational access to the database. For a while this will be possible only on fairly large machines rather than personal computers but the development seems worth undertaking:

> *Recommendation 10*: Use a simple natural language analysis and generation system to enhance the conversational flow of interactive construct elicitation and analysis systems.

Other apparently advanced enhancements are also possible by noting that the element and construct names are just arbitrary symbols created by the user and that the PEGASUS vocabulary is very small and defined in advance apart from these. There are now simple and effective speech recognizers available for personal computers that discriminate some 30 or more words. There are similarly low-cost speech synthesizers that can be used not only with a pre-defined vocabulary but also to record and replay words input interactively. Thus it is possible to develop a form of interactive construct elicitation system which operates completely in a speech mode and requires no keyboard input or display output. In the current state of the technology it is likely that such systems will be curiosities rather than practical tools and we cannot recommend them in a practical sense. However, computer speech technology is developing rapidly and in time such systems will become practically important.

We conclude this section by emphasizing that our concept of future personal construct elicitation and analysis systems is one of a suite of programs operating around a database. The programs will allow various forms of entry of data to the database coupled with a wide range of analysis techniques including all those compared in this paper.

14. Conclusions

In this paper we have shown how a repertory grid may be regarded as a logical structure in which the poles of constructs are predicates applying to the elements and have developed the foundations for a methodology of grid analysis based on this logical interpretation. We have given examples of such analysis produced by the computer program ENTAIL and compared it with other techniques such as INGRID, FOCUS and Q-Analysis. We have shown that the logical approach extends to grids using rating scales and also to grids in which there is independent rating on each pole.

We have attempted throughout the paper to present the new methodology in a way which clearly relates it to Kelly's original development of personal construct theory and demonstrates that it is a logical derivation from that theory. We have also linked the methodology to foundational work in logic which was not available to Kelly yet seems essential to sustain an accurate formalization of his work. We have emphasized also the peculiar significance of the interactive computer in allowing a dialectical, conversational approach to grid elicitation and analysis, and have shown how the logical approach using fuzzy linguistic semantics supports this approach.

This has been a fairly technical paper and it would be appropriate to end with a balancing reminder that the methodology and technology should not blind us to the problems of actually gaining knowledge of personal constructs and their structures. The repertory grid, no matter how it is enhanced, is only a tool for allowing us to gain some view of a person's construct space. It is a powerful tool but by no means a comprehensive one. It gives us a simplified, partial representation of the very much richer processes underlying human reasoning. *Logic* derives from these processes, not they from it, and we should beware of forcing human reasoning into a Procrustean bed of mathematical theory. The multivalued, modal logics used in this paper are a long way from the basic predicate calculus developed by Frege in setting up formal foundations for arithmetic. These modern developments in logic seem to provide adequate foundations for Kelly's personal construct psychology. However, we should always retain a suspicion that continuing development and refinement will always be necessary for any formal structure that purports to capture the processes of the human mind.

In the final section we have given a set of recommendations for the direction of future development of interactive construct systems which are those guiding our own work. In particular we see a convergence between work on relational databases, expert systems and personal construct elicitation. The personal computer systems of the future will be tools that complement the minds of their users and work together with them at a high level of mental symbiosis. The major use of computers to date has been "technical cognitive" to use Habermas' (1968) evocative phrase for the situation in which the technology dominates and controls the user. We see interactive construct elicitation and analysis systems as providing an "emancipatory cognitive" technology in Habermas' terms that encourages the user to comprehend, change and develop in his own fashion by reflecting back to him the essence of his own approach to various aspects of his life.

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