New Efficient Iterative Optimization Algorithm to Design the Two Channel QMF Bank

Ram Kumar Soni, Alok Jain, and Rajiv Saxena

Abstract—This paper proposes an efficient method for the design of two channel quadrature mirror filter (QMF) bank. To achieve minimum value of reconstruction error near to perfect reconstruction, a linear optimization process has been proposed. Prototype low pass filter has been designed using Kaiser window function. The modified algorithm has been developed to optimize the reconstruction error using linear objective function through iteration method. The result obtained, show that the performance of the proposed algorithm is better than that of the already exists methods.

Keywords—Filterbank, near perfect reconstruction, Kaiser window, QMF.

I. INTRODUCTION

THE theory and design of QMF bank was first introduced by Johnston [1]. These filter banks finds wide applications in many signal processing fields such as transmultiplexer [2] - [4], Equalization of wireless communication channel [5], sub-band coding of speech and image signals [6]-[11], sub-band acoustic echo cancellation [12]. Because of such wide application, many researchers giving a lot of attention in efficient design of such filter bank [13]-[14].

In QMF bank the input signal x (n) splits into two subband signals having equal bandwidth using the lowpass and highpass analysis filter H_0 (z) and H_1 (z) respectively. These sub-band signals are down sampled by factor of two to reduce processing complexity. At the output corresponding synthesis bank has two-fold interpolator for both sub band signal, followed by F_0 (z) and F_1 (z) synthesis filters. The outputs of the synthesis filters are combined to obtain the reconstructed signal y (n). This reconstruction of signal at output is not perfect replica of the input signal x (n), due to three types of errors: aliasing error, amplitude error and phase error [15]-[16].

Since the introduction of QMF bank, most of the researchers giving main stress on the elimination or minimization of these errors and obtain near perfect reconstruction (NPR). In several design methods [17]-[19], [20]-[23] aliasing and phase distortion has been eliminated completely by designing all the analysis and synthesis FIR linear phase filter by a single lowpass prototype even order symmetric FIR linear phase filter.

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Amplitude distortion is not possible to eliminate completely, but can be minimize using optimization techniques [15][16].
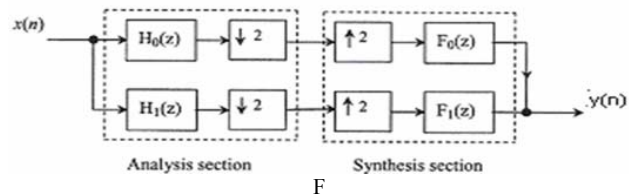

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Fig. 1 Two - channel quadrature mirror filter bank

Johnston [1] used Hanning window to design lowpass prototype FIR filter and used nonlinear optimization technique to minimize reconstruction error. Creusere and Mitra [14] used Park-McClellan algorithm to design the prototype lowpass FIR filter and minimize reconstruction error using linear optimization technique. Similarly many other researchers used different objective function which have linear/non-linear in nature and minimized the reconstruction error using different optimization technique. In continuation to previous work a modified algorithm has been proposed which has linear objective function and linear optimization algorithm has been proposed to minimize reconstruction error and obtained near perfect reconstruction. The iterative steps are required for optimization. Kaiser window function is used to design lowpass prototype even order symmetric FIR linear phase filter. Final results are compared with other existing publications.

II. FIR FILTER DESIGN USING WINDOW TECHNIQUE

Kaiser window approach has been used to design prototype lowpass filter. Impulse response of lowpass filter *h(n)* of length *(N+1)* designed through Kaiser window function [15], [23] is of the form

$$h(n) = h_i(n).w(n)$$

where

$$h_i(n) = \frac{\mathrm{Sin}(\omega_c(n - 0.5N))}{\pi(n - 0.5N)}$$

is the impulse response of the ideal filter with cutoff frequency ω_c.

The Kaiser window function $w(n)$ is defined as[15],[23]

$$w(n) = \frac{Io\left\{\beta\sqrt{1-\left(n/M\right)}\right\}}{Io\left(\beta\right)} \quad -M < n < M$$

where $Io(.)$ is the zeroth order modified Bessel function, β is the window shape parameter which control the stopband attenuation. The empirical design equation developed by Kaiser is given by-

$$\beta = \begin{cases} 0 & \text{for} As < 21, \\ 0.5842(As-21)+0.07886(As-21) & \text{for} 21 \le As \le 50, \\ 0.1102(As-8.7) & \text{for} As > 50, \end{cases}$$

The designed of prototype filter using Kaiser window can be specified by the three parameter ω_c, N and β. The order of the window N can be estimated by the following formula developed by Kaiser [15], [23]:

$$N = \frac{2 \times (As - 7.95)}{14.36 \Delta\omega}$$

where A_s is the stop band attenuation and $\Delta\omega$ is the normalized transition width.

III. ANALYSIS OF TWO CHANNEL QMF BANK

The z-transform of the output signal $y(n)$ of the filter bank can be written as [10], [17]-[19]

$$Y(z) = \frac{1}{2}\left\{H_0(z)F_0(z)+H_1(z)F_1(z)\right\}X(z)$$
$$+\frac{1}{2}\left\{H_0(-z)F_0(z)+H_1(-z)F_1(z)\right\}X(-z) \quad (1)$$

The second term in the above equation is due to alaising. The alaising can be removed completely by defining the synthesis filter as given below [6], [16], [19], [21]-[22]

$$F_0(z) = H_1(-z)$$
$$F_1(z) = -H_0(-z) \quad (2)$$

The above expression indicates that to eliminate alaising completely analysis and synthesis filters in the bank are essentially determined from one prototype lowpass filter. $H_0\left(e^{j\omega}\right)$ is mirror image of $H_1\left(e^{j\omega}\right)$ with respect to $\pi/2$.

The final expression for the alaise free reconstructed signal can be written as

$$Y(z) = \frac{1}{2}\left[H_0^2(z)-H_0^2(-z)\right]X(z) \quad (3)$$

$$\text{or } Y(z)=T(x)X(z). \quad (4)$$

Here $T(z)$ is the overall system function of the alaise free QMF bank given as:.

$$T(z) = \frac{1}{2}\left[H_0^2(z)-H_0^2(-z)\right] \quad (5)$$

To obtain perfect reconstruction amplitude and phase distortion also be eliminated. In this situation the overall function must satisfy the following equation

$$Y(z) = cz^{-n0}X(z)$$
$$y(n) = cx(n-n_0) \quad (6)$$

$y(n)$ is simply made equal to scaled and delayed version of input $x(n)$.

Above equation show that if the analysis filter $H_0(z)$ is selected to be linear phase FIR, then phase response of the overall transfer function of QMF bank also becomes linear. It implies that phase distortion of the QMF bank is also eliminated.

Let the prototype analysis filter be a linear phase FIR filter of order N with real coefficients transfer function $H_0(z)$ given by

$$H_0(z) = \sum_{n=0}^{N} h_0[n]z^{-n} \quad (7)$$

The corresponding frequency response can be written as

$$H_0\left(e^{j\omega}\right) = e^{-jN/2}H_R(\omega) \quad (8)$$

Where $H_R(\omega)$ is the amplitude function. For real $h_0(n)$, $\left|H_0\left(e^{j\omega}\right)\right|^2$ is an even function of ω. The overall frequency response of the QMF bank can be written as

$$\frac{e^{-jN\omega}}{2}\left\{H_0\left(e^{j\omega}\right)^2-(-1)^N\left|H_0\left(e^{j(\pi-\omega)}\right)\right|^2\right\} \quad (9)$$

From the above equation it can be seen that if order N is even, then $T(e^{j\omega})=0$ at $\omega = \pi/2$ implying severe amplitude distortion at the output of the bank. As a result N must be chosen to be odd, in which case above equation reduces to

$$T\left(e^{j\omega}\right) = \frac{e^{-jN\omega}}{2}\left\{\left|H_0\left(e^{j\omega}\right)\right|^2+\left|H_1\left(e^{j\omega}\right)\right|^2\right\} \quad (10)$$

It follows from above expression, the FIR two–channel filter bank with linear–phase analysis and synthesis filters will be perfect reconstruction [15]-[16] type if

$$\left\{\left|H_0\left(e^{j\omega}\right)\right|^2+\left|H_1\left(e^{j\omega}\right)\right|^2\right\} = 1 \quad (11)$$

The corresponding objective function can be written as

$$\Phi = \max\left\{\left|H_0\left(e^{j\omega}\right)\right|^2+\left|H_1\left(e^{j\omega}\right)\right|^2\right\}-1 \quad (12)$$

IV. DESIGN OF NPR FILTERBANK USING OPTIMIZATION ALGORITHM

As from Eq. (2) to eliminate the phase and alaising distortion all the four filters must be designed using one prototype lowpass linear phase FIR filter. For the perfect reconstruction Eq. (11) must be satisfied. Since Eq. (11) cannot satisfied exactly due to finite length of filter so it always exhibit some amplitude distortion unless $\left|T_0\left(e^{j\omega}\right)\right|$ is a constant for all value of ω. If $H_0(z)$ is a very good lowpass filter with $\left|H_0\left(e^{j\omega}\right)\right|=1$ in the passband and $\left|H_0\left(e^{j\omega}\right)\right|=0$ in the stopband, then $H_1(z)$ is a good highpass filter with its pass band coinciding with the stopband of $H_0(z)$ and vice versa. As a result, $\left|T\left(e^{j\omega}\right)\right| \cong \frac{1}{2}$ in the passband of $H_0(z)$ and $H_1(z)$. Thus the amplitude distortion mainly occurs in the transition band of these filters. The degree of overlap of $H_0(z)$ and $H_1(z)$ is very crucial in determining this distortion. This distortion can be minimized by controlling the overlap, which in turn can be controlled by appropriately choosing filter coefficients in such a way that the filters satisfy the condition of perfect reconstruction approximately. Systematic computer aided optimization technique is used which iteratively adjust the filter coefficients to satisfy the above condition as close as possible using objective function.

Johnston [1] used a nonlinear objective function for optimization process and obtained minimum reconstruction error. Creusere and Mitra [14] used linear objective function and optimized the process using Park–McClellan algorithm. Initially different parameters such as passband frequency, stopband frequency, stopband attenuation are assumed as per requirement of design and design the filter. Now passband frequency was adjusted to minimize the objective function under the optimization loop.

Similarly, Lin and Vaidyanathan [24] used linear optimization technique using window method and optimized the QMF bank up to the near perfection.

In proposed algorithm objective function (12) is minimize by adjusting the cutoff frequency and near perfection is achieved. Kaiser window has been used in design of prototype low pass filter [23].

Initially filter order N, window shape parameter and window coefficients of Kaiser window are determine using initial values of passband, stopband frequency and stopband attenuation before the optimization loop.
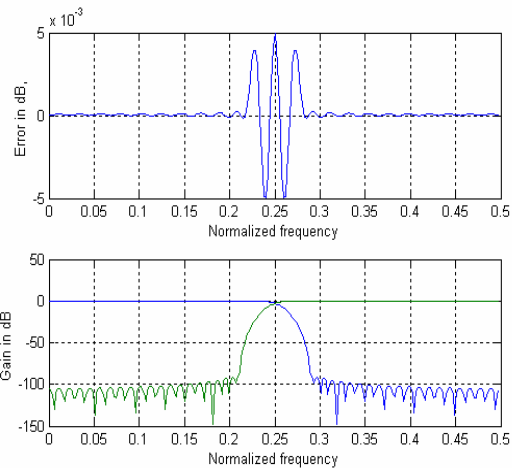

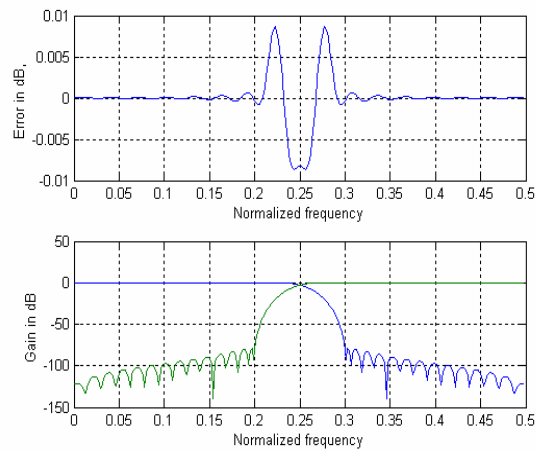Fig. 2 (a) Amplitude response and reconstruction error in dB for $\omega_s = 0.295\pi$

Fig. 2 (b) Amplitude response and reconstruction error in dB for $\omega_s = 0.3\pi$

V. ALGORITHM

i. Assume initial value of Sampling frequency, Passband frequency, Stop band frequency, Stopband attenuation, Passband ripple

ii. Calculate Normalized Passband and Stopband frequencies.

iii. Calculate filter order, window shape parameter, and Window coefficients.

iv. Initialize step value, search direction, flag, minimum absolute value and initial absolute value of objective function.

Set the While loop with flag = 0

1. Design prototype low pass filter, QMF filter, determine the reconstruction error and maximum absolute value of objective function using current

value of cutoff frequency, filter order and window coefficients.

2. Compare the current value of objective function with previous or initial value-

    a) If the current value greater than previous value step become half and change the search direction. Calculate modified value of cutoff frequency by ($\omega_c = \omega_c$ +dir×step) and go to step (3).
    b) If the current value smaller or equals to than minimum absolute value, than flag becomes set to '1'. Come out from the loop, and go to step (vi).
    c) If the current value equals to previous value, than flag set to '1' and control come out from the loop and go to step (vi).

3. Modified value of cutoff frequency as ($\omega = \omega_c$+dir×step) and now current maximum Absolute value of objective function of Step (x) become previous value. Go to step [1].

End of the loop

    v. Calculate the value of reconstruction error for given minimum value of objective function obtained above.
    vi. Plot the Reconstruction Error in log scale and draw graphically Reconstruction error.

VI. RESULT AND DISCUSSION

The proposed optimization technique for the design of QMF bank has been implemented in MATLAB. Following are the two examples, which give the best result using this algorithm.

Example 1

For $\omega_s = 0.3\pi$, A_s (Stopband attenuation) = 80dB, Filter coefficients are obtained. The corresponding normalized magnitude response plots and Reconstruction error plot are shown in Fig. 2b. The significant parameters obtained are: the reconstruction error in dB is 0.0086, length of filter (N+1)= 61, Stopband frequency is $\omega_s = 0.3\pi$.

Example 2

For $\omega_s = 0.295\pi$, A_s = 90dB, Filter coefficients are obtained.
The corresponding normalized magnitude response plots and Reconstruction error plot are shown in Fig. 2a. The significant parameters obtained are: the reconstruction error in dB is 0.0049, length of filter (N+1) = 87, Stopband frequency is ω_s = 0.295π.

The simulation results of the proposed method are compared with the method of Jain-Crochiere [9], Chen-Lee [23], General method [26] for $\omega_s = 0.3\pi$ and summarized in Table I. The obtained filter coefficients are given in Table II.

TABLE I
COMPARISON OF THE PROPOSED METHOD WITH THE OTHER EXISTING
METHOD BASED ON THE PARAMETER FOR $\omega_s = 0.3\pi$

Method	As (dB)	Reconstruction Error	Phase Response
Jain Crochiere	33	0.015	Linear
Chen –Lee	34	0.016	Linear
General purpose	49.2	0.009	Nonlinear
Proposed method	80	0.0086	Linear

VII. CONCLUSION

A new iterative method to design quadrature mirror filter (QMF) bank has been developed. The main attraction of proposed method is that it gives minimum attenuation error as compared with the already existing methods for given filter parameters.

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TABLE II
FILTER COEFFICIENTS FOR EXAMPLE-1 AND EXAMPLE-2

S.No	Filter coefficient	Example -1 (Length of filter = 61, A_s = 80dB, Reconstruction Error = 0.0086)	Example -2 (Length of filter = 87, A_s = 90dB, Reconstruction Error = 0.0049)
1	h (0)	0.5163	0.5122
2	h (1)	0.3166	0.3174
3	h (2)	-0.0161	-0.0121
4	h (3)	-0.1011	-0.1033
5	h (4)	0.0152	0.0117
6	h (5)	0.0556	0.0590
7	h (6)	-0.0139	-0.0112
8	h (7)	-0.0348	-0.0392
9	h (8)	0.0122	0.0104
10	h (9)	0.0226	0.0276
11	h (10)	-0.0103	-0.0095
12	h (11)	-0.0147	-0.0199
13	h (12)	0.0083	0.0084
14	h (13)	0.0093	0.0145
15	h (14)	-0.0064	-0.0074
16	h (15)	-0.0057	-0.0105
17	h (16)	0.0047	0.0063
18	h (17)	0.0033	0.0075
19	h (18)	-0.0033	-0.0052
20	h (19)	-0.0018	-0.0053
21	h (20)	0.0021	0.0042
22	h (21)	0.0009	0.0036
23	h (22)	-0.0013	-0.0033
24	h (23)	-0.0004	-0.0024
25	h (24)	0.0007	0.0025
26	h (25)	0.0001	0.0015
27	h (26)	-0.0003	-0.0019
28	h (27)	-0.0000	-0.0009
29	h (28)	0.0001	0.0013
30	h (29)	0.0000	0.0005
31	h (30)	-0.0000	-0.0009
32	h (31)		-0.0003
33	h (32)		0.0006
34	h (33)		0.0001
35	h (34)		-0.0004
36	h (35)		-0.0001
37	h (36)		0.0002
38	h (37)		0.0000
39	h (38)		-0.0001
40	h (39)		-0.0000
41	h (40)		0.0000
41	h (41)		-0.0000
43	h (42)		-0.0000
44	h (43)		0.0000