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New Evidence of Asymmetric Dependence Structures in International Equity Markets

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Abstract

A number of recent studies finds two asymmetries in dependence structures in international equity markets; specifically, dependence tends to be high in both highly volatile markets and in bear markets. In this paper, a further investigation of asymmetric dependence structures in international equity markets is performed by using the Markov switching model and copula theory. Combining these two theories enables me to model dependence structures with sufficient flexibility. Using this flexible framework, I indeed find that there are two distinct regimes in the U.S.-U.K. market. I also show that for the U.S.-U.K. market the bear regime is better described by an asymmetric copula with lower tail dependence with clear rejection of the Markov switching multivariate normal model. In addition, I show that ignorance of this further asymmetry in bear markets is very costly for risk management. Lastly, I conduct a similar analysis for other G7 countries, where I find other cases in which the use of a Markov switching multivariate normal model would be inappropriate.

I. Introduction

The study of dependence structures in international equity markets has recently attracted increasing attention among theorists, empirical researchers, and practitioners. For instance, to control for risks that they face, portfolio managers and regulators have to take into account dependence between international equity markets when studying the returns across international financial markets. Therefore, the issue of asymmetric dependence structures, such as high dependence in a certain period or market, is particularly important for risk control and policy management. In addition, benefits from international diversification of asset allocation could be considerably affected by asymmetric dependence structures.

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A number of recent studies investigate asymmetry in dependence structures in international equity markets and observe the following two interesting asymmetries: dependence tends to be high in both i) highly volatile markets and ii) bear markets. For example, Hamao, Masulis, and Ng (1990) investigate the relations among equity markets across Japan, the U.K., and the U.S. using daily stock indices data. They estimate the GARCH-M model and find volatility spillover effects from the U.S. and U.K. stock markets to the Japanese market. Also, Longin and Solnik (1995) analyze monthly stock indices data for several industrial countries and conclude that the correlations between major stock markets increases in periods of high volatility based on a multivariate GARCH model. Similarly using a multivariate SWARCH model, Ramchand and Susmel (1998) report that monthly equity markets in the U.K., Germany, and Canada become more highly correlated with the U.S. equity market during periods of high U.S. variance. See also King, Sentana, and Wadhwani (1994), Ball and Torous (2000), Bekaert and Wu (2000), Ang and Bekaert (2002), and Das and Uppal (2004).

While these studies find evidence of the first asymmetry, several other studies recognize the second asymmetry. For instance, King and Wadhwani (1990) construct a contagion mechanism model and find evidence of contagion effects. They also find an increase in volatility raised its size using high frequency data from the stock markets in Japan, the U.K., and the U.S. Similarly, Lin, Engle, and Ito (1994) analyze two international transmission mechanism models based on daily stock indices in Japan and the U.S. to support the findings of King and Wadhwani (1990). Erb, Harvey, and Viskanta (1994) document that monthly cross-equity correlations among the G7 countries are highest when any two countries are in a common recession and also show that they are much higher in bear markets. Also, following Davison and Smith (1990) and Ledford and Tawn (1997), Longin and Solnik (2001) formally describe a method to measure the extreme correlation by the conditional tail correlation based on extreme value theory. They find that it is only in bear markets that conditional correlation between the U.S. and other G5 countries strongly increases, while conditional correlation does not seem to increase in bull markets. Other recent studies include Campbell, Koedijk, and Kofman (2002), Ang and Bekaert (2002), Das and Uppal (2004), Patton (2004), and Poon, Rockinger, and Tawn (2004).

Those findings suggest that there are two regimes in international equity markets: a high dependence regime with low and volatile returns and a low dependence regime with high and stable returns. Following this conjecture, Ang and Bekaert (2002) estimate a Markov switching multivariate normal (MSMVN) model using the U.S., the U.K., and German monthly stock indices. They find weak evidence of a bear regime characterized by low expected returns, high volatility, and high correlation, and a normal regime associated with high expected returns, low volatility, and low correlation. They also show that the MSMVN model can fairly successfully replicate Longin and Solnik's (2001) results. In addition, following Ang and Chen (2001), they confirm that an asymmetric bivariate GARCH model, which has been used as a main tool to analyze the international equity markets, cannot replicate them. Lastly, they consider the international asset allocation problem and evaluate economic significance measured by the utility cost of ignoring the regime switching dependence structure.

The contribution of this paper is to provide further evidence of asymmetric dependence structures in international equity markets. More precisely, in this paper I explicitly assume that two regimes exist in international equity markets. Then, I investigate characteristics of each regime, paying close attention to the asymmetries of the dependence structure. For this purpose, I use copula theory, which can be regarded as an explicit representation of dependence structure, in a Markov switching framework. Combining these two modern econometric techniques allows me to model asymmetric dependence structures with sufficient flexibility without losing any tractability of the models. These two theories enable me to introduce further asymmetries in a very natural way. Rodriguez (2007) considers similar models by using copulas with Markov switching parameters. Note that in this paper I consider more general models with Markov switching copulas.

Following Ang and Bekaert (2002), I first estimate the MSMVN model as my benchmark model for the U.S.-U.K. markets. I successfully replicate Ang and Bekaert's main findings of weak evidence for two distinct regimes: a bear regime characterized by high dependence with low and volatile expected returns and a normal regime identified by low dependence with high and stable expected returns.

One concern about the MSMVN model is that the conditional dependence structure in each regime must be symmetric without tail dependence,[1] even though the unconditional dependence structure could be asymmetric and generate greater "near tail" dependence.[2] Obviously, there is little reason to restrict the study to models exhibiting conditional symmetry on each regime.[3] The growing contagion literature also favors the models with conditional lower tail dependence.[4] I therefore use a family of asymmetric copula models with lower tail dependence for one regime, presuming that asymmetric copula models are suitable only for bear markets. Specifically, I use a multivariate normal model for describing one regime, while I adopt a family of asymmetric copula models with lower tail dependence for another regime. Using this Markov switching semi-asymmetric copula (MSSAC) model, I confirm that asymmetric copula models closely match bear markets for all models. More importantly, all MSSAC models uniformly improve log-likelihood values compared with the MSMVN model. In addition, I find stronger evidence of two distinct regimes. Furthermore, the values of implied lower tail dependences from the MSSAC models are at least 0.64 and are highly significant. With these lower tail dependences, I also verify that the MSSAC model can reproduce exceedance correlations observed in data better than the MSMVN model. Thus, these findings suggest that bear markets are better characterized by asymmetric copula models than the multivariate normal model.

To further investigate the appropriateness of the MSSAC model, I also consider another type of asymmetric model. I use a family of asymmetric copulas for both regimes, instead of only one regime. Using this type of asymmetric model, I can examine the possibility of asymmetric dependence in the normal regime. With

[1] For the definition of tail dependence, see the next section.

[2] I acknowledge the referee for clarifying this point.

[3] In what follows, I use the term "conditional" to mean conditional on each regime.

[4] See Pericoli and Sbracia (2003) for an excellent survey of recent contagion literature.

this Markov switching asymmetric copula (MSAC) model, I find clear evidence of two distinct regimes as given by the MSSAC model. The results, however, show that the MSAC models have uniformly lower log-likelihoods than those of MSSAC models. In addition, I confirm that the MSAC model cannot reproduce exceedance correlations observed in data as well as the MSSAC model. These results suggest that the multivariate normal model is more appropriate for normal markets than asymmetric copula models. Combining this result with the above finding indicates that bear markets are better characterized by the asymmetric dependence, while the symmetric dependence is more appropriate for normal markets, hence, the MSSAC model is the most appropriate among the three models for the U.S.-U.K. markets.

Following these empirical findings, I formally conduct the likelihood ratio tests against the MSMVN model. Using the MSSAC models, I can uniformly reject the MSMVN model. This finding clearly demonstrates that the MSMVN model is not enough to describe the asymmetric dependence in the U.S.-U.K. markets, specifically the asymmetric dependence in bear markets. Thus, my results establish that two types of asymmetric dependence exist in the U.S.-U.K. markets: asymmetric dependence between bear markets and normal markets, and further asymmetric dependence with lower tail dependence in bear markets, which the MSMVN model cannot capture well.

Next I investigate the economic significance of my empirical findings from a risk management point of view because ignoring the further asymmetry in bear markets could be costly when risk measures are evaluated. Following Ball and Torous (2000) and Guidolin and Timmermann (2006), I investigate this possibility by concentrating on the value at risk (VaR) and expected shortfall. According to my calculation, ignoring such an asymmetry in bear markets does indeed affect risk measures. I find that when this further asymmetry is ignored, the 99% VaR is undervalued by about 10%, while the expected shortfall is undervalued by about 5% to 10% consistently over the whole significance level between 90% to 99%, which is crucial for risk management.

Lastly, I examine whether commensurate asymmetric dependence can be found in other G7 countries through the adaptation of Longin and Solnik's (2001) pair-wise analysis. My analysis shows the existence of normal and bear regimes in other markets as well. I, however, find different asymmetric dependence structures. Although no asymmetric dependence is observed for the U.S.-JP pair, evidence of asymmetric dependence across regimes is found in the U.S.-CA market. Moreover, even though I do not find any strong evidence of asymmetric dependence across regimes in the U.S.-GE and U.S.-FR markets, I show that both markets are best characterized by the MSAC models but clearly not by the MSMVN model. In other words, asymmetric dependence exists within each regime in these markets. This result provides other cases in which the use of the MSMVN model is inappropriate.

The remainder of this paper is organized as follows. Section II briefly discusses the idea behind my methodology and the estimation method. Section III provides the empirical results for the U.S.-U.K. market, while Section IV evaluates the economic significance of my empirical findings in Section III. The results for other G7 countries are given in Section V. Lastly, Section VI concludes.

II. Methodology

The basic idea behind this paper is to combine copula theory with the Markov switching model to characterize two regimes in international equity markets. While the former can give flexibility in describing asymmetric interdependence across observations, the latter provides natural and tractable models for processes with switching regimes. Therefore, combining these two theories enables me to model regime switching dependence structures with sufficient flexibility.

A. Copula Theory and Measures of Dependence

The purpose of this paper is to investigate the dependence structures in international equity markets. It is, therefore, desirable to model dependence structure directly, which could be done by using the notion of a copula. Copula theory is a classical statistical theory based on Sklar's (1959) theorem. Despite the fact that the concept of the copula was introduced over 40 years ago, only recently has it been used in financial and econometric literature.[5] In what follows, I briefly explain the concept of a copula and related measures of dependence for the two-dimensional continuous case.[6]

Let H be a continuous two-dimensional distribution function and let F_X, F_Y be corresponding continuous marginal distributions. Then, copula theory claims that there exists a unique distribution function C, which I refer to as a copula, of a random vector in $\mathbb{R}^2$, with uniform (0,1) margins, such that for all $(x,y)' \in \mathbb{R}^2$,

$$H(x,y\,;\boldsymbol{\theta}) \quad = \quad C\left(F_X(x\,;\boldsymbol{\theta}_X), F_Y(y\,;\boldsymbol{\theta}_Y)\,;\boldsymbol{\delta}\right). \tag{1}$$

Here $\boldsymbol{\delta}$ is a parameter vector for the copula, $\boldsymbol{\theta}_X, \boldsymbol{\theta}_Y$ are parameter vectors for each margin, and $\boldsymbol{\theta} = (\boldsymbol{\delta}', \boldsymbol{\theta}_X', \boldsymbol{\theta}_Y')'$ is a parameter vector for the joint distribution. Thus, according to copula theory, the joint distribution can be decomposed into two parts: marginal distributions F_X and F_Y, which describe the marginal behavior of each variable, and a copula C, which represents the dependence structure between X and Y. This decomposition allows me to model marginal distributions and dependence structure separately. Hence, I can model the dependence structure explicitly. As a result, I can model the joint distribution with sufficient flexibility to investigate dependence structures in international equity markets.

One of my main interests is to detect the difference between dependence structures across regimes. Although cross-regime comparison of the entire copula forms works for this purpose, it is often helpful to focus on some scalar measures of dependence. In particular, in the finance world linear correlation ρ has been most widely used as a measure of dependence for several reasons. First, it is very easy to compute since it depends only on the second moments of joint distributions. Second, some basic theories in finance, such as the capital asset pricing model and the arbitrage pricing theory, are based on linear correlation between different financial instruments. Third, it is a natural measure of dependence in

[5]For instance, see Mashal and Zeevi (2002), Breymann, Dias, and Embrechts (2003), Jondeaua and Rockinger (2006), Patton (2006), and Rodriguez (2007) for recent applications of copula theory.

[6]For a more general discussion, see Joe (1997) and Nelsen (1999).

elliptical distributions,[7] such as the multivariate normal distribution and multivariate t-distribution, as discussed below. In general, however, linear correlation is not a good measure of dependence, as emphasized by Embrechts, McNeil, and Straumann (2002), and Embrechts, Lindskog, and McNeil (2003). Since it can capture only symmetric linear dependence, it does not have one of the desired properties for measures of dependence: invariance under nonlinear strictly increasing transformation. In addition, it is possible to construct random variables that are perfectly dependent in the sense of comonotonicity or countermonotonicity,[8] but their linear correlation is arbitrarily close to 0. It is, therefore, sometimes very misleading to use linear correlation as a measure of dependence for non-elliptical distributions such as the distributions employed in this paper. The main reason why linear correlation fails to provide a reasonable measure in general is that it is not a copula-based measure. As I have shown, the entire dependence structure is described by a copula, hence measures of dependence must be based on copulas. Since linear correlation can be viewed as the common determinant of all elliptical copulas,[9] it can be considered as a reasonable measure of dependence within the elliptical distributions, but it is not always the case. For general non-elliptical distributions, Joe (1997) introduced three copula-based measures of dependence[10]—Kendall's tau τ_K, Spearman's rho ρ_S, and (lower) tail dependence λ_L. Using these three measures, I can compare the degree of dependence from three different aspects, which can be very helpful for the paper's purpose. I will use all three as measures of dependence in the paper. In what follows, I briefly discuss definitions, basic characteristics, and formulas to calculate each measure. For more details, see Joe (1997) and Nelsen (1999).

Both Kendall's tau and Spearman's rho are sometimes called rank correlations since they can be interpreted as the linear correlation between some "ranks" of the data. They are very reasonable alternatives to linear correlation as measures of dependence for non-elliptical distributions. Like linear correlation they can take values between -1 and 1, and independence implies both measures are 0. On the other hand, unlike linear correlation, comonotonicity is equivalent to $\tau_K = 1$ and $\rho_S = 1$, while countermonotonicity is completely characterized by $\tau_K = -1$ and $\rho_S = -1$. They can also be considered as measures of a particular dependence called concordance since they satisfy all conditions for a measure of concordance as proposed by Scarsini (1984).

Kendall's tau is defined as the difference of the probability of two random concordant pairs and the probability of two random discordant pairs for two iid vectors (X_1, Y_1) and (X_2, Y_2), which can be calculated from copulas by the following formula:

[7]See Fang, Kotz, and Ng (1987) about the elliptical distributions in detail.

[8]For continuous random variables (X, Y), comonotonicity can be defined as the existence of strictly increasing transformation T such that $Y = T(X)$ with probability 1 and similarly countermonotonicity can be defined as the existence of strictly decreasing function. See Embrechts et al. (2002) for more detail and other dependence concepts that appeared in this section.

[9]Copulas for elliptical distributions are called elliptical copulas.

[10]For other possible measures, see Nelsen (1999).

$$
\begin{aligned}
(2)\quad \tau_K &= \text{Prob}[(X_1 - X_2)(Y_1 - Y_2) > 0] - \text{Prob}[(X_1 - X_2)(Y_1 - Y_2) < 0] \\
&= 4\int_0^1 \int_0^1 C(u,v)dC(u,v) - 1.
\end{aligned}
$$

Similarly, Spearman's rho is defined to be proportional to the probability of concordance minus the probability of discordance for the two vectors (X_1, Y_1) and (X_2, Y_3), i.e., a pair of vectors with the same margins, but one vector has a distribution function H, while the components of the other are independent. It can also be considered as linear correlation between $F_X(X)$ and $F_Y(Y)$, which can be calculated from copulas as follows:

$$
\begin{aligned}
(3)\quad \rho_S &= 3\{\text{Prob}[(X_1 - X_2)(Y_1 - Y_3) > 0] \\
&\quad - \text{Prob}[(X_1 - X_2)(Y_1 - Y_3) < 0]\} \\
&= \frac{\text{Cov}(F_X(X), F_Y(Y))}{\sqrt{\text{Var}(F_X(X) \cdot \text{Var}(F_Y(Y))}} \\
&= 12\int_0^1 \int_0^1 C(u,v)dudv - 3.
\end{aligned}
$$

Tail dependence measures the dependence in the upper right quadrant or the lower left quadrant tail of a bivariate distribution. It is a concept that is relevant to dependence in extreme values. In particular, lower tail dependence is closely related to the concept of contagion in the sense that the existence of contagion effects naturally implies positive lower tail dependence. Hence, the growing contagion literature indicates that lower tail dependence is a very important measure of dependence in international equity markets. Therefore, I use lower tail dependence as one of my measures of dependence. The definition of lower tail dependence is the probability that one variable takes an extremely large negative value, given that the other variable took an extremely large negative value, which can be equivalently defined in terms of copulas as follows:

$$
\begin{aligned}
(4)\quad \lambda_L &= \lim_{u \downarrow 0} \text{Prob}[X \le F_X^{-1}(u) | Y \le F_Y^{-1}(u)] \\
&= \lim_{u \downarrow 0} \text{Prob}[Y \le F_Y^{-1}(u) | X \le F_X^{-1}(u)] \quad = \quad \lim_{u \downarrow 0} \frac{C(u,u)}{u},
\end{aligned}
$$

provided the limit exists. A bivariate copula C has lower tail dependence if $\lambda_L \in (0,1]$ and no lower tail dependence if $\lambda_L = 0$.

B. Markov Switching Model

The Markov switching model was originally developed by Hamilton (1989) for describing the process influenced by an unobserved random variable s_t, which is usually called the state or regime. In this subsection, I describe his idea using my stock market framework.

Let $\mathbf{r}_t$ be a (2×1) vector consisting of returns from two countries' stock markets at date t. The starting point of the Markov switching model is to assume that stock returns are generated from

$$\mathbf{r}_t = \boldsymbol{\mu}(s_t) + \boldsymbol{\Sigma}_1^{1/2}(s_t)\boldsymbol{\varepsilon}_t(s_t),$$

where s_t is an unobserved latent variable taking a value of either 1 or 2 that reflects the state of stock markets. That is, the model allows returns to follow one of two different equations, depending on the state of stock markets. In my model, $\boldsymbol{\mu}(s_t) = (\mu_X(s_t), \mu_Y(s_t))'$ is a vector of each variable's marginal mean in regime s_t, $\boldsymbol{\Sigma}_1(s_t)$ is a diagonal matrix with each variable's marginal variance in regime s_t along the diagonal, namely

$$\boldsymbol{\Sigma}_1(s_t) = \begin{pmatrix} \sigma_X^2(s_t) & 0 \\ 0 & \sigma_Y^2(s_t) \end{pmatrix},$$

and $\boldsymbol{\varepsilon}_t(s_t)$ follows a joint distribution with copula $C(s_t)$ in regime s_t. Since $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}_1$ describe the mean and variance of margins, $\boldsymbol{\varepsilon}_t(s_t)$ is assumed to have mean $\mathbf{0}$ and correlation matrix $\mathbf{R}(s_t)$:

$$\mathbf{R}(s_t) = \begin{pmatrix} 1 & \rho_{XY}(s_t) \\ \rho_{XY}(s_t) & 1 \end{pmatrix}.$$

Thus, $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}_1$ govern the mean and variance of each marginal distribution, respectively, while C describes the dependence between each variable. I will discuss models of the marginal distributions and a copula of $\boldsymbol{\varepsilon}_t$, C in the next subsection.

Following Ang and Bekaert (2002), I assume $\boldsymbol{\mu}$, $\boldsymbol{\Sigma}_1$, C are only state dependent although they could depend on other economic variables such as their own past values and interest rates in general.[11]

The model also requires specifying a stochastic process for s_t, which governs the behavior of the state. Hamilton (1989) proposed using the Markov chain and showed how to calculate maximum likelihood estimates. This Markov switching model is very realistic to describe economic behavior, since the current economic state is typically the most important factor in determining next period's economic state. In addition, as emphasized in Hamilton (1990), the EM algorithm, developed by Dempster, Laird, and Rubin (1977), is easy to apply to get relatively robust maximum likelihood estimates with respect to poorly chosen starting values. After Hamilton's pioneering work, several studies have suggested more complicated specifications,[12] but I use this simplest form to keep my estimation robust and easy to interpret. More specifically, based on two asymmetries found by previous studies I assume that s_t follows a two-state Markov chain with transition probability $\mathbf{P}$ of the form

$$\mathbf{P} \quad = \quad \begin{pmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{pmatrix}.$$

[11]Actually, I tried to use more complicated models with AR mean and GARCH variance. I, however, could not find important evidence for such models.

[12]For example, Diebold, Lee, and Weinbach (1994) and Filardo (1994) considered time-varying transition probabilities evolving as logistic functions of some other economic exogenous variables. Durland and McCurdy (1994) also argued that the transition probabilities should depend on the duration of the state.

C. Models for Margins and Copulas

As emphasized in the previous subsections, copula theory enables me to model marginal distributions and a copula separately to specify joint distributions of $\boldsymbol{\varepsilon}$, which I will discuss now.

For the marginal distributions, I use three distributions: the normal distribution, the t-distribution, and the generalized error distribution (GED). All three distributions are normalized to have mean 0 and variance 1, hence the densities of t-distribution and the GED are given as:[13]

$$
\begin{aligned}
f_t(x) &= \frac{\Gamma[(1+\nu)/2\nu]\sqrt{\nu}}{\Gamma(1/2\nu)\sqrt{\pi(1-2\nu)}}\left[1+\frac{\nu x^2}{(1-2\nu)}\right]^{-(1+\nu)/2\nu} \\
f_{GED}(x) &= \frac{\nu}{2^{[(\nu+1)/\nu]}\Gamma(1/\nu)\lambda}\exp\left(-\left|\frac{x}{\lambda}\right|^{\nu}\right), \text{ where } \lambda = \sqrt{\frac{2^{(-2/\nu)}\Gamma(1/\nu)}{\Gamma(3/\nu)}}.
\end{aligned}
$$

These three distributions are the most widely used in modeling financial data as well as in all econometric applications. In addition, using these distributions in the framework of the Markov switching model enables me to consider the mixture of these distributions. As a consequence, I can express important features of marginal equity returns such as fat tails and stochastic persistent volatility and conditional higher moments.[14] The t-distribution and the GED distribution have an additional parameter ν, which governs the thickness of tails. I refer to ν as the tail parameter for the GED distribution. Note that both the t-distribution and the GED contain the normal distribution as a special case: the t-distribution with $\nu = 0$ and the GED with $\nu = 2$ correspond with the normal distribution.

To specify the form of the copula, I continue to assume that $d = 2$; however, multivariate or partial multivariate extensions are straightforward.[15] My benchmark copula is a normal copula.

Normal Copula:

$$
C^{nor}(u, v\,; \delta) = \int_{-\infty}^{\Phi^{-1}(u)}\int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\delta^2}}\exp\left\{-\frac{s^2-2\delta st+t^2}{2(1-\delta^2)}\right\} dsdt, \quad -1 < \delta < 1,
$$

where Φ is a cumulative distribution function of a standard normal distribution. Notice that δ is equivalent to the usual linear correlation parameter between two variables. Note also that joint distributions with a normal copula and normal margins reduce to multivariate normal distributions, which are used in almost all Markov switching multivariate applications. One concern about normal copulas is that they describe only symmetric dependence since their dependence parameter δ corresponds to linear correlation. Unconditionally this is not a problem

[13]Note that in this paper the degree of freedom parameter for the t-distribution ν is parameterized as the reciprocal of the usual degree of freedom parameter for the t-distribution. This is because it is more convenient to use the reciprocal of the usual degree of freedom to estimate models since the degree of freedom could take on large values ($=\infty$ in the case of the normal distribution, for example).

[14]See Timmermann (2000).

[15]See Joe (1997) section 5.3.

because the Markov switching framework allows the unconditional joint distribution to have asymmetric dependence. It is, however, very doubtful whether the conditional joint distribution given each regime is characterized by the distribution with symmetric dependence.

Another drawback of normal copulas is that they cannot capture dependence in the tail part of joint distributions in the sense that their tail dependence is 0 unless $\delta = \rho = 1$.[16] In other words, if I use normal copulas, I implicitly assume that there is no tail dependence, which is relevant to the concept of contagion naturally. Again, this is problematic in describing the dependence structure given each regime since the growing contagion literature suggests using asymmetric copulas with lower tail dependence. Other problems associated with normal copulas can be found in Embrechts et al. (2003).

To mitigate these concerns, I also use copulas with both lower and upper tail dependence. However my preliminary empirical results showed that the evidence of upper tail dependence is very weak or nonexistent as can be imagined easily. Therefore, I choose to use a family of asymmetric copulas with only lower tail dependence: the Kimeldorf and Sampson (KS) copula,[17] the Joe copula, the Gumbel copula, the Galambos copula, and the Hüsler and Reiss (HR) copula. Their forms are given as follows.[18]

KS Copula:

$$C^{ks}(u, v\,; \delta) \quad = \quad (u^{-\delta} + v^{-\delta} - 1)^{-1/\delta}, \quad 0 \le \delta < \infty.$$

Joe Copula:

$$C^{joe}(u, v\,; \delta) \quad = \quad u + v - (u^{\delta} + v^{\delta} - u^{\delta} v^{\delta})^{1/\delta}, \qquad 1 \le \delta < \infty.$$

Gumbel Copula:

$$\begin{aligned} C^{gum}(u, v\,; \delta) \ &= \ u + v - 1 \\ &\quad + \exp\left\{ - \left[\{-\log(1-u)\}^{\delta} + \{-\log(1-v)\}^{\delta} \right]^{1/\delta} \right\}, \quad 1 \le \delta < \infty. \end{aligned}$$

Galambos Copula:

$$\begin{aligned} C^{gal}(u, v\,; \delta) \ &= \ u + v - 1 + (1-u)(1-v) \\ &\quad \times \exp\left\{ - \left[\{-\log(1-u)\}^{-\delta} + \{-\log(1-v)\}^{-\delta} \right]^{-1/\delta} \right\}, \quad 0 \le \delta < \infty. \end{aligned}$$

[16] For a proof of this statement, see Embrechts et al. (2002).

[17] The KS copula is also called the Clayton copula in Nelsen (1999).

[18] They are adapted from Joe (1997). See their B4–B8 copulas. Note, however, that I transformed copulas with upper tail dependence using the formula,

$$C'(u, v) \quad = \quad u + v - 1 + C(1-u, 1-v),$$

so that they have lower tail dependence if necessary.

HR Copula:

$$\begin{aligned} C^{hr}(u, v\,; \delta) &= u + v - 1 \\ &\quad + \exp\left\{ \log(1-u)\Phi\left(\delta^{-1} + \frac{1}{2}\delta \log\left[\frac{\log(1-u)}{\log(1-v)}\right]\right) \right. \\ &\qquad \left. + \log(1-v)\Phi\left(\delta^{-1} - \frac{1}{2}\delta \log\left[\frac{\log(1-u)}{\log(1-v)}\right]\right)\right\}, \\ &\quad \delta \geq 0. \end{aligned}$$

Note that each of these copulas has one parameter δ, which governs the degree of dependence in the sense that the degree of dependence is strictly increasing in δ. Also all copulas have the property that they imply independence at the lower bound of δ and comonotonicity at the upper bound of δ. Hence, they can provide a parsimonious way of describing positive dependence, which includes the special cases of independence and comonotonicity.

D. Deriving the Likelihood Function and Simulating Data

To calculate the likelihood function, I need the density h of the joint distribution. I can derive an expression for h by differentiating equation (1) with respect to x and y:

$$\begin{aligned} h(x, y\,; \boldsymbol{\theta}) &= \frac{H(x, y\,; \boldsymbol{\theta})}{\partial x \partial y} \\ &= c\left(F_X(x\,; \boldsymbol{\theta}_X), F_Y(y\,; \boldsymbol{\theta}_Y)\,; \delta\right) \cdot f_X(x\,; \boldsymbol{\theta}_X) \cdot f_Y(y\,; \boldsymbol{\theta}_Y), \end{aligned}$$

where f_X and f_Y are marginal densities of X and Y, respectively, and c is the density of the copula. c can also be derived easily by differentiating the copula $C(u, v)$ with respect to variables u and v:

$$c(u, v) = \frac{C(u, v)}{\partial u \partial v}.$$

With the use of these two equations, it is straightforward to construct the likelihood function and maximize it to derive the MLEs of $\boldsymbol{\theta}$ for each regime, and p_{11} and p_{22}, which represent the probabilities of staying in the same regime in the next period.

Lastly, I discuss how to simulate $(X, Y)'$ based on decomposition (1) (for a more general case where dimensions are greater than two, see Embrechts et al. (2003)). For this purpose, I have to generate a random vector $(U, V)'$ from C. Once I obtain $(U, V)' \sim C$, $(X, Y)'$ can be produced by transforming U and V with F_X^{-1} and F_Y^{-1}, respectively. Thus, if $(U, V)' \sim C$, $(X, Y)' \equiv (F_X^{-1}(U), F_Y^{-1}(V))' \sim H$. The main difficulty in simulating $(X, Y)'$ is, therefore, to generate $(U, V)'$ from C. This is achieved by using a conditional distribution $C_{2|1}$ of a copula, which can be calculated as a partial derivative of C with respect to the first variable:

$$C_{2|1}(v|u) = \frac{C(u, v)}{\partial u}.$$

This can be seen easily from

$$c(u,v) = c_{2|1}(v|u) \cdot f_U(u) = c_{2|1}(v|u),$$

since $f_U(u) = 1$. By definition, if $U \sim U(0,1)$ and $V \sim C_{2|1}(\cdot|U)$, then $(U,V) \sim C$. Hence, if U and Q are independent $U(0,1)$ random variables and define $V \equiv C_{2|1}^{-1}(Q|U)$, then $(U,V) \sim C$. In sum, I can use the following algorithm to simulate data: i) simulate U and Q from $U(0,1)$ independently; ii) define $V \equiv C_{2|1}^{-1}(Q|U)$ so that $(U,V)' \sim C$; iii) define $(X,Y)'$ by $(X,Y)' \equiv (F_X^{-1}(U), F_Y^{-1}(V))'$; and iv) then $(X,Y)' \sim H$. As for $C_{2|1}^{-1}$, the analytical expression can be found in Joe (1997) for the KS copula. For other copulas, I use a root-finding routine to find $C_{2|1}^{-1}$ numerically.

III. Empirical Results

My study is based on monthly total market index data of the U.S. and U.K. markets obtained from Datastream with the sample period lasting from 1973:2 to 2003:8. I follow Longin and Solnik (2001) and Ang and Bekaert (2002) on the decision of choosing to use monthly data. Also, the use of monthly data is relevant in capturing directional dependence, which is the main purpose of my paper. I define stock returns for each country simply as 100 times the change in the natural logarithm of each country's stock index. In addition, I focus on the U.S. and the U.K. in order to investigate the dependence structures in international equity markets. To achieve this goal, the investigated markets should be representative of total markets and should be reasonably integrated during my sample period, which is definitely the case for the U.S. and U.K. markets. Moreover, for comparison purposes results from other G7 countries will be provided in a later section.

Using the copula theory enables me to select the best marginal model before estimating the joint distribution. By doing so, I can reduce the number of joint models to estimate and the possibility of misspecification of the models. Therefore, I estimate the marginal models of each country first. After choosing the best marginal model for each country, I then estimate several joint models to investigate the dependence structure between the U.S. and U.K. returns.

A. Results for Marginal Models

I estimate five Markov switching models for each country: each of three different distributions for both regimes, or the normal distribution for one regime and one of the other two distributions for the other regime.[19] As a result, I can consider 25 different possible combinations for the marginal models of two countries.

Estimation results of the marginal models for each country are shown in Tables 1 and 2, respectively. For the U.S., the normal-normal combination model is supported from several aspects. The estimates of the degree of freedom for the t-distribution and the tail parameter for the GED distribution indicate that they are

[19]The idea of mixtures of different distributions can also be found in Perez-Quiros and Timmermann (2001).

not significantly different from those of the normal distribution. In addition, the negligible improvement to the log-likelihoods from adding one extra parameter provides a strong evidence in favor of the normal distribution, according to the standard information criteria, such as SIC.

TABLE 1

Estimates of Marginal Models for the U.S.

Regime	Distribution		p	ν	μ	σ	Log-Likelihood
1	*t*-Dist	Estimate	0.900	0.080	−0.698	6.689	−1045.95
		Std Error	0.120	0.101	1.015	1.006	
2	*t*-Dist	Estimate	0.968	0.000	1.053	3.377	
		Std Error	0.029	0.074	0.232	0.271	
1	Normal	Estimate	0.849		−1.170	7.135	−1046.26
		Std Error	0.104		0.942	0.904	
2	Normal	Estimate	0.963		1.039	3.409	
		Std Error	0.021		0.224	0.202	
1	GED	Estimate	0.866	1.660	−0.870	6.827	−1045.81
		Std Error	0.091	0.558	0.895	0.853	
2	GED	Estimate	0.959	2.283	1.058	3.330	
		Std Error	0.022	0.436	0.228	0.204	
1	*t*-Dist	Estimate	0.898	0.080	−0.717	6.691	−1045.95
		Std Error	0.117	0.115	1.184	1.173	
2	Normal	Estimate	0.967		1.054	3.379	
		Std Error	0.024		0.223	0.218	
1	GED	Estimate	0.886	1.624	−0.905	6.790	−1046.04
		Std Error	0.093	0.460	1.120	0.926	
2	Normal	Estimate	0.966		1.064	3.396	
		Std Error	0.025		0.242	0.228	

TABLE 2

Estimates of Marginal Models for the U.K.

Regime	Distribution		p	ν	μ	σ	Log-Likelihood
1	*t*-Dist	Estimate	0.861	0.279	−3.922	13.531	−1112.51
		Std Error	0.074	0.173	1.758	3.104	
2	*t*-Dist	Estimate	0.986	0.081	1.087	4.416	
		Std Error	0.011	0.068	0.260	0.259	
1	Normal	Estimate	0.821		−2.268	12.715	−1115.11
		Std Error	0.085		1.207	1.618	
2	Normal	Estimate	0.980		1.040	4.266	
		Std Error	0.012		0.256	0.223	
1	GED	Estimate	0.844	1.176	−4.556	12.753	−1113.41
		Std Error	0.079	0.345	0.753	2.426	
2	GED	Estimate	0.984	1.693	1.138	4.368	
		Std Error	0.014	0.252	0.263	0.275	
1	*t*-Dist	Estimate	0.821	0.241	−3.505	12.174	−1113.18
		Std Error	0.072	0.117	1.806	2.395	
2	Normal	Estimate	0.976		1.109	4.219	
		Std Error	0.013		0.250	0.228	
1	GED	Estimate	0.819	1.228	−4.208	12.201	−1113.94
		Std Error	0.073	0.445	1.864	2.159	
2	Normal	Estimate	0.978		1.101	4.241	
		Std Error	0.013		0.275	0.201	

For the U.K. data, the evidence for a normal distribution is somewhat weaker. The estimate of the degree of freedom for the t-distribution from the t-normal model is significantly different from 0. The tail parameter estimate for the GED distribution in regime 1 from the GED-GED model is also significantly different from 2. The small magnitude of the improvement of log-likelihoods, however, still supports the normal distribution model by reason of parsimony. In fact, the SIC select the normal-normal model as the best model for the U.K. as well.

Table 3 reports the hypothesis testing results of the equality of expected returns and volatilities across regimes. My best model, the normal-normal model indicates that there are significant differences in both expected returns and volatilities across the regimes for each country. All differences are statistically significant at the 5% level. Thus, the best model provides the favorable evidence of two distinct regimes for both countries: a bear regime (regime 1) with negative expected returns and high volatility and a normal regime (regime 2) with high and stable expected returns. Most of the other models also confirm two distinct regimes with at least a 10% significance level. This provides further motivation to use the normal-normal model for both countries. Therefore, I will use the normal distribution as margins for both regimes and both countries throughout the following bivariate analysis.

TABLE 3

Hypotheses Testing Results of the Equality of Expected Returns and Volatilities across Regimes

Distribution		Expected Return of U.S.	Expected Return of U.K.	Volatility of U.S.	Volatility of U.K.
t-Dist	Wald Stat	0.979	4.879	4.135	9.924
	p-Value	0.323	0.027	0.042	0.002
Normal	Wald Stat	4.522	5.227	19.665	29.948
	p-Value	0.033	0.022	0.000	0.000
GED	Wald Stat	4.477	53.420	16.284	25.193
	p-Value	0.034	0.000	0.000	0.000
t-Dist and Normal	Wald Stat	1.566	5.655	9.495	10.978
	p-Value	0.211	0.017	0.002	0.001
GED and Normal	Wald Stat	2.760	3.291	14.232	22.458
	p-Value	0.097	0.070	0.000	0.000

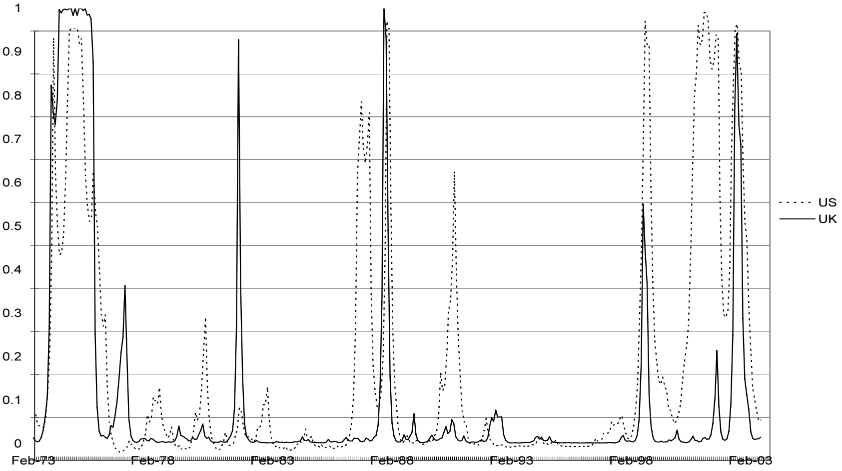
B. The Common Regime Classification Assumption and the Markov Switching Test

To estimate joint models, I assume that the regime classification in the U.S. and U.K. markets is identical. To check the plausibility of this assumption, Figure 1 shows smoothed probabilities of a normal regime under normal-normal models for both countries. As can be seen, most of the time regimes are identified as a normal regime and both markets experienced several common turmoil periods. As a result, their regime classifications appear to be very similar to each other, indicating the acceptability of my common regime classification assumption. The validity of this assumption is further provided by my inability to find evidence to support the use of four-state models assuming unsynchronized regimes for the

U.S. and U.K. markets. For the background of this assumption, see Ang and Bekaert (2002).

FIGURE 1

Smoothed Probabilities of a Bear Regime for U.S. and U.K. Markets



There is, however, one concern associated with Figure 1. Figure 1 reveals that there are only a few bear regime periods over my sample period. By the nature of a bear regime, it is very natural but doubtful whether I really need a Markov switching regime framework to distinguish such rare events. To explore this possibility, I conduct a test of the null hypothesis being there is no Markov switching as recently proposed by Carrasco, Hu, and Ploberger (2004). They have found the asymptotic optimal test against the alternative of Markov switching based on the second Bartlett identity. The test statistics turn out to be 5.889 for the U.S. and 6.077 for the U.K, whereas the 5% critical values are 4.153 and 4.150, respectively. These results indicate bear regimes have considerable impact on both countries' markets, as well as the importance of modeling bear markets correctly.

To summarize, my empirical results of marginal models suggest the plausibility of estimating two-state Markov switching models with normal marginal distributions as joint models, and I will pursue this topic in the subsections that follow.

C. Results for the Benchmark Model

Following Ang and Bekaert (2002), I estimate the MSMVN model (normal copula model) as my benchmark model. The parameter estimates are shown in Table 4, which also gives estimates and standard errors of Kendall's tau and Spearman's rho implied by the parameter estimates and formulas (2) and (3). Note that as discussed in the previous section, tail dependence of multivariate normal distributions is 0 by nature so there is no need to estimate tail dependence. Note also that all standard errors of the dependence measures are calculated using the delta method. This is achieved by viewing each measure of dependence as a monotonic continuous transformation of a copula parameter δ. I can construct those transformations explicitly based on formulas (2), (3), and (4) and use them to evaluate gradients numerically.

TABLE 4

Estimation Results for the MSMVN Model

Regime		p	δ	μ_{US}	μ_{UK}	σ_{US}	σ_{UK}	Log-Likelihood
1	Estimates	0.770	0.743	−2.379	−1.745	7.451	12.393	−2063.57
	Std Error	0.092	0.070	1.261	1.945	0.818	1.116	
2	Estimates	0.970	0.608	0.992	1.020	3.780	4.155	
	Std Error	0.017	0.042	0.238	0.247	0.213	0.214	

Regime		Kendall's Tau	Spearman's Rho
1	Estimates	0.534	0.727
	Std Error	0.067	0.072
2	Estimates	0.416	0.590
	Std Error	0.033	0.042

Table 5 also reports hypothesis testing results for equality of expected returns, volatilities, and two dependence measures across regimes. Not surprisingly these results are essentially the same as those documented by Ang and Bekaert (2002). I find a bear regime characterized by negative expected returns, high volatility, and high dependence, and a normal regime associated with high expected returns, low volatility, and low dependence. This evidence, however, is not strong in the sense that I fail to reject the null hypotheses of the equality of the U.K. expected return and both measures of dependence across regimes at the 10% significance level.

TABLE 5

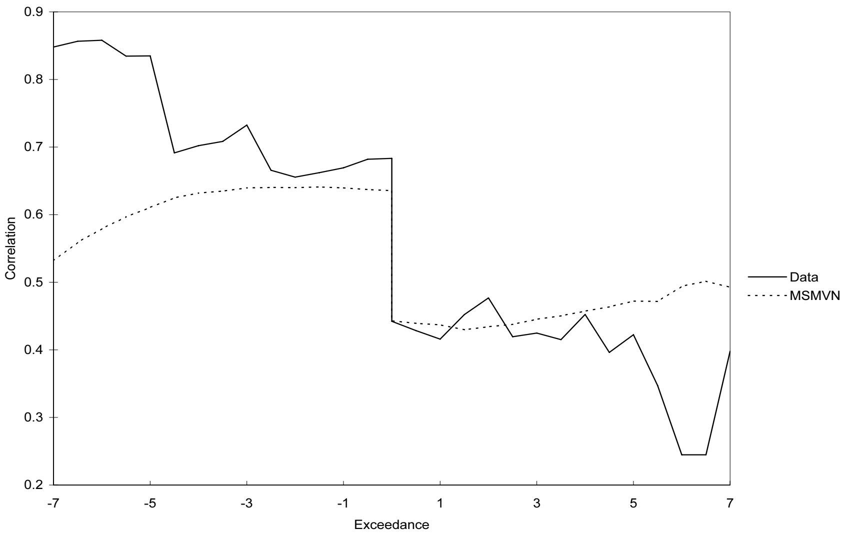
Hypothesis Testing Results of the Equality of Expected Returns, Volatilities, and Dependence Measures across Regimes for the MSMVN Model

	Expected Return of U.S.	Expected Return of U.K.	Volatility of U.S.	Volatility of U.K.	Kendall's Tau	Spearman's Rho
Wald Stat	6.789	1.981	17.615	40.554	2.258	2.460
p-Value	0.009	0.159	0.000	0.000	0.133	0.117

In Figure 2, I repeat the exceedance correlation analysis of Longin and Solnik (2001). I plot the exceedance correlations defined by the correlations between the subset of observations $\{(x_i, y_i)'\}$, where values of x_i and y_i are greater (or less) than the positive (or negative) exceedance level. The solid line in Figure 2 shows the exceedance correlations of the U.S. and U.K. returns observed in real data, while the dashed line represents the exceedance correlations of simulated data with size 100,000 using the estimated MSMVN model. As shown by Ang and Chen (2001) and Ang and Bekaert (2002), the MSMVN model can replicate fairly successfully the observed asymmetric pattern with negative exceedance correlations being higher than positive exceedance correlations. It is, however, clear that the negative exceedance correlations from the MSMVN model are consistently lower than those observed from actual data. In addition, the discrepancy tends to be more striking as the exceedance level becomes more extreme. This observation suggests that there might be further asymmetry in the U.S.-U.K. equity markets, which the MSMVN model may not capture.

FIGURE 2

U.S.-U.K. Exceedance Correlations from the MSMVN Models



D. Results of the MSSAC Models

One concern associated with the normal copula (or the multivariate normal model) is that it describes only symmetric dependence. In addition, by its very nature, the normal copula cannot capture tail dependence. These concerns provide possible explanations of the poor performance of the MSMVN model in reproducing the extreme negative exceedance correlations observed in the data. Therefore, I explore this possibility by using a family of asymmetric copulas with lower tail dependence, introduced in the previous section, to mitigate these concerns.

The crucial difference between normal and these asymmetric copulas is the existence of lower tail dependence, which captures the co-movements of extreme negative values. One motivation for introducing lower tail dependence is contagion effects discovered in previous studies. The nature of contagion effects, however, also suggests that lower tail dependence might be weaker or nonexistent in a normal regime. Also, the relatively good performance of the MSMVN model in reproducing positive exceedance correlations supports this possibility. I therefore decide to estimate the MSSAC models, which use a multivariate normal model for describing one regime and a family of asymmetric copula models with lower tail dependence for another regime, presuming that asymmetric copula models are suitable only for bear markets. Then I check this presumption by estimating the MSAC models, which use asymmetric copula for both regimes, in the next subsection.

Table 6 reports estimation results of five MSSAC models. The implied estimates and standard errors of three dependence measures are also given in Table 7. As expected, the asymmetric copulas are associated with a bear regime for all models. In addition, the parameter estimates of marginal distributions and transition probabilities are quite close to the results of the MSMVN model. More remarkably, the log-likelihood values increase uniformly for all copula models, even though they have the same number of parameters as the MSMVN model. In other words, these results indicate that bear markets are better characterized by any of the proposed asymmetric copula models when compared with the multivariate normal model.

TABLE 6

Estimation Results of the MSSAC Models

Regime	Copula		p	δ	μ_{US}	μ_{UK}	σ_{US}	σ_{UK}	Log-Likelihood
1	KS	Estimate	0.754	2.473	−2.252	−1.654	8.483	12.010	−2061.75
		Std Error	0.089	0.562	1.327	1.876	0.744	0.975	
2	Normal	Estimate	0.965	0.592	1.017	1.054	3.689	4.102	
		Std Error	0.018	0.039	0.223	0.241	0.204	0.195	
1	Joe	Estimate	0.753	3.296	−2.330	−1.704	8.566	12.082	−2062.08
		Std Error	0.099	0.537	0.910	1.027	0.698	0.942	
2	Normal	Estimate	0.966	0.592	1.016	1.053	3.694	4.106	
		Std Error	0.019	0.042	0.235	0.260	0.222	0.213	
1	Gumbel	Estimate	0.753	2.263	−2.514	−1.490	7.769	11.459	−2061.33
		Std Error	0.090	0.340	1.250	1.702	0.800	1.195	
2	Normal	Estimate	0.963	0.592	1.056	1.068	3.670	4.079	
		Std Error	0.016	0.044	0.241	0.256	0.213	0.204	
1	Galambos	Estimate	0.754	1.554	−2.445	−1.526	7.737	11.461	−2061.13
		Std Error	0.092	0.321	0.875	0.957	0.762	0.997	
2	Normal	Estimate	0.963	0.592	1.052	1.066	3.671	4.078	
		Std Error	0.019	0.040	0.235	0.258	0.196	0.200	
1	HR	Estimate	0.755	2.150	−2.320	−1.613	7.710	11.483	−2060.79
		Std Error	0.085	0.378	1.156	1.463	0.845	1.298	
2	Normal	Estimate	0.963	0.593	1.047	1.067	3.672	4.077	
		Std Error	0.014	0.043	0.239	0.247	0.202	0.192	

The results of hypothesis tests based on the MSSAC models are shown in Tables 7 and 8. As before, I test the equality of three dependence measures and each country's expected return and volatility across regimes. The KS and Gumbel copula models fail to reject the equality of the U.K. expected return across regimes at a 10% significance level. However, all other hypothesis testing results indicate that there are two distinct regimes. Thus, the MSSAC models provide strong evidence in favor of two distinct regimes.

Interestingly, the implied estimates of Kendall's tau and Spearman's rho are very similar for each regime regardless of copula forms for a bear regime. Thus, both the MSMVN and MSSAC models capture almost the same amount of global dependence suggesting the stability of my models. There are, however, large differences between the magnitude of lower tail dependence captured by the MSMVN model and the MSSAC models for a bear regime. For the MSMVN

TABLE 7

Implied Estimates and Hypothesis Testing Results
for Three Dependence Measures from the MSSAC Models

Copula for Regime 1			Kendall's Tau	Spearman's Rho	Lower Tail Dependence
KS	Regime 1	Estimate	0.553	0.739	0.756
		Std Error	0.056	0.058	0.048
	Regime 2	Estimate	0.403	0.574	0
		Std Error	0.031	0.039	N/A
	Testing of Equality	Wald Stat	5.003	5.053	246.4
		p-Value	0.025	0.025	0.000
Joe	Regime 1	Estimate	0.551	0.736	0.766
		Std Error	0.056	0.058	0.042
	Regime 2	Estimate	0.403	0.574	0
		Std Error	0.033	0.042	N/A
	Testing of Equality	Wald Stat	4.843	4.690	329.1
		p-Value	0.028	0.030	0.000
Gumbel	Regime 1	Estimate	0.558	0.745	0.642
		Std Error	0.066	0.069	0.063
	Regime 2	Estimate	0.403	0.574	0
		Std Error	0.034	0.044	N/A
	Testing of Equality	Wald Stat	3.908	3.971	105.1
		p-Value	0.048	0.046	0.000
Galambos	Regime 1	Estimate	0.559	0.748	0.640
		Std Error	0.062	0.065	0.059
	Regime 2	Estimate	0.403	0.574	0
		Std Error	0.032	0.040	N/A
	Testing of Equality	Wald Stat	4.607	4.817	117.8
		p-Value	0.032	0.028	0.000
HR	Regime 1	Estimate	0.564	0.757	0.642
		Std Error	0.062	0.064	0.059
	Regime 2	Estimate	0.404	0.575	0
		Std Error	0.034	0.043	N/A
	Testing of Equality	Wald Stat	5.007	5.426	119.9
		p-Value	0.025	0.020	0.000

model, the lower tail dependence is 0 by nature, but for the MSSAC models it is at least 0.64. In addition, all estimates of lower tail dependence from the MSSAC models are highly significant. To see the effect of these lower tail dependences, I repeat the exceedance correlation analysis again. Figure 3 shows the exceedance correlations of simulated data with size 100,000 using the five estimated MSSAC models along with those of the actual and the simulated MSMVN data. As can be seen, the simulated MSSAC data yield higher negative exceedance correlations and replicate extreme negative exceedance correlations better than the simulated MSMVN data. Also, the simulated MSSAC data reproduce positive exceedance correlation equally as well as the simulated MSMVN data. Therefore, the MSSAC models are again preferred to the MSMVN model in terms of describing exceedance correlations observed in the data.

In sum, the results of the MSSAC models show MSSAC models perform significantly better than the MSMVN model and provide strong evidence in favor of two distinct regimes. However, it is too early to conclude that I have obtained a clear indication of the appropriateness of the MSSAC models since I cannot rule out the possibility of further improvement by introducing asymmetry into normal markets. More importantly, it is not clear that the magnitude of the improve-

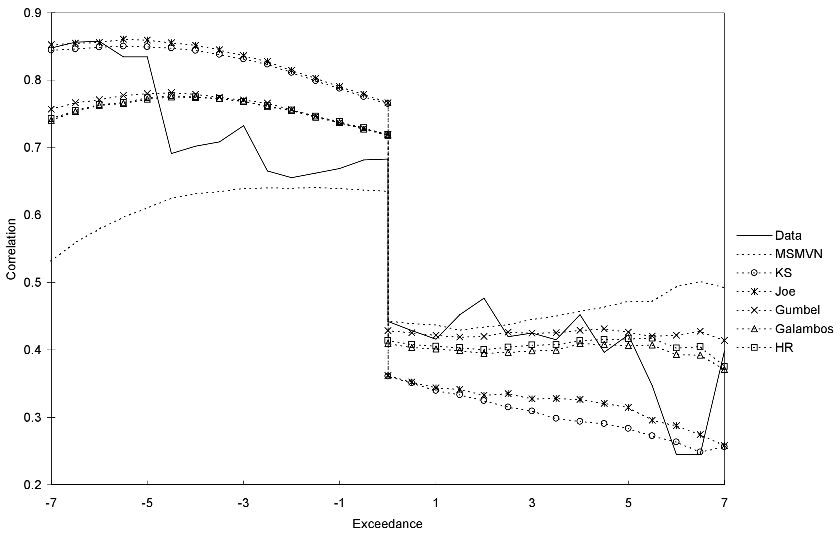
TABLE 8

Hypothesis Testing Results of the Equality of Expected Returns and Volatilities across Regimes for the MSSAC Models

Copula for Regime 1		Expected Return of U.S.	Expected Return of U.K.	Volatility of U.S.	Volatility of U.K.
KS	Wald Stat	5.713	2.009	40.113	33.441
	p-Value	0.017	0.156	0.000	0.000
Joe	Wald Stat	12.041	6.297	44.975	68.435
	p-Value	0.001	0.012	0.000	0.000
Gumbel	Wald Stat	7.656	2.187	25.745	32.732
	p-Value	0.006	0.139	0.000	0.000
Galambos	Wald Stat	14.142	6.372	27.669	63.038
	p-Value	0.000	0.012	0.000	0.000
HR	Wald Stat	8.051	3.253	22.908	38.940
	p-Value	0.005	0.071	0.000	0.000

FIGURE 3

U.S.-U.K. Exceedance Correlations from the MSSAC Models



ment of log-likelihoods is large enough to reject the MSMVN model based on the MSSAC model. These are the main themes in the following subsections.

E. Further Investigation of the Asymmetric Dependence Structure

The MSSAC models introduced in the previous subsection are very attractive since they significantly outperform the MSMVN model and reproduce exceedance correlations observed in the data very well. However, one concern about the MSSAC model is the assumption of symmetric dependence in a normal regime. In other words, I might improve the MSSAC model by introducing asymmetric dependence in the normal regime. I investigate this possibility by introducing the MSAC models.

I estimate five MSAC models that assume five different asymmetric copulas for both regimes. Estimations of these MSAC models are presented in Table 9. As expected, the parameter estimates of marginal distributions and transition probabilities are reasonably similar to the results of the MSMVN and MSSAC models. Also, almost all hypothesis tests for equality of three dependence measures and each country's expected return and volatility among regimes indicate that they are significantly different across regimes.[20] Thus, the MSAC models can provide a clear evidence of distinction across regimes similar to the MSSAC model. In addition, the implied estimates of Kendall's tau and Spearman's rho are also very close for all models, but particularly for the bear regime suggesting that the stability of the MSAC models is similar to the MSSAC models.

TABLE 9

Estimation Results of the MSAC Models

Copula	Regime		p	δ	μ_{US}	μ_{UK}	σ_{US}	σ_{UK}	Log-Likelihood
KS	1	Estimate	0.775	2.427	−1.984	−1.625	8.598	11.835	−2070.80
		Std Error	0.096	0.572	0.904	1.048	0.835	1.108	
	2	Estimate	0.967	0.892	1.057	1.119	3.635	4.153	
		Std Error	0.014	0.134	0.257	0.286	0.196	0.209	
Joe	1	Estimate	0.774	3.252	−2.036	−1.669	8.663	11.865	−2075.54
		Std Error	0.082	0.691	1.356	1.681	1.038	1.214	
	2	Estimate	0.966	1.758	1.037	1.100	3.666	4.196	
		Std Error	0.013	0.131	0.228	0.251	0.196	0.200	
Gumbel	1	Estimate	0.788	2.234	−2.451	−1.749	8.005	11.623	−2064.80
		Std Error	0.086	0.358	1.214	1.483	0.909	1.342	
	2	Estimate	0.971	1.606	0.991	1.003	3.745	4.203	
		Std Error	0.013	0.100	0.253	0.258	0.206	0.203	
Galambos	1	Estimate	0.789	1.525	−2.365	−1.688	7.952	11.613	−2063.18
		Std Error	0.080	0.288	0.876	0.986	0.734	0.990	
	2	Estimate	0.971	0.882	1.000	1.018	3.724	4.192	
		Std Error	0.016	0.091	0.223	0.261	0.197	0.192	
HR	1	Estimate	0.789	2.119	−2.279	−1.704	7.877	11.591	−2061.59
		Std Error	0.087	0.395	1.146	1.316	0.920	1.407	
	2	Estimate	0.970	1.340	1.004	1.026	3.710	4.187	
		Std Error	0.016	0.121	0.224	0.254	0.204	0.215	

There are, however, important differences between the MSAC models and the MSSAC models. The MSAC models have uniformly lower log-likelihood values than those of the MSSAC models. In particular, the log-likelihood values of the KS and Joe copula models decrease dramatically compared with the MSSAC models. One possible explanation for this observation is that these copulas are too asymmetric to describe a normal regime. As a result, the copulas with lower tail dependencies that are too large worsen the fit of the models. These results indicate that the multivariate normal model is more desirable to describe normal markets than any of the asymmetric copula models considered. Note that this does not contradict with the highly significant results of implied lower tail dependence for a normal regime. All MSAC models have only one parameter for the copula, which governs the degree of dependence. Therefore, high global dependence can produce apparent tail dependence in the MSAC models even if

[20] To save space, the results of the hypothesis tests are not reported here, but are available from the author.

there is no lower tail dependence. Because of this fact, the rejection of no tail dependence based on the asymmetric copula models does not necessarily imply the rejection of the multivariate normal model.

In sum, combining the MSAC model results with the MSSAC model demonstrates that asymmetric copula models can characterize bear markets better than the multivariate normal model, while the multivariate normal model can describe normal markets better than the asymmetric copula models. Thus, the MSSAC model is the most appropriate among the three models.

F. Rejection of the MSMVN Model

To conclude the empirical section, I establish that my findings significantly reject the use of the MSMVN model. To this end, I conduct the likelihood ratio (LR) test of the MSMVN model based on my results. The test statistics are simply the difference between the log-likelihood values, or log-likelihood ratios, across the MSMVN and each MSSAC model, which are 1.5, 1.9, 2.3, 2.5, and 2.8 for the KS, Joe, Gumbel, Galambos, and HR copula models, respectively. To find finite sample distributions of these log-likelihood ratios, I first simulate 3,000 samples with the same size as my sample (366) from the estimated MSMVN model. Then, I fit the MSMVN and MSSAC models and calculate the log-likelihood ratios for each simulated sample in order to construct the finite distributions. The p-values of these LR tests of the MSMVN model are 0.046, 0.032, 0.021, 0.015, and 0.007, respectively. For all of the MSSAC models, the MSMVN model is uniformly rejected at a 5% significance level. Thus, these results clearly indicate that the MSMVN model is not enough to characterize the asymmetric dependence in international equity markets; indeed, there exists a further asymmetric dependence with lower tail dependence in bear markets.

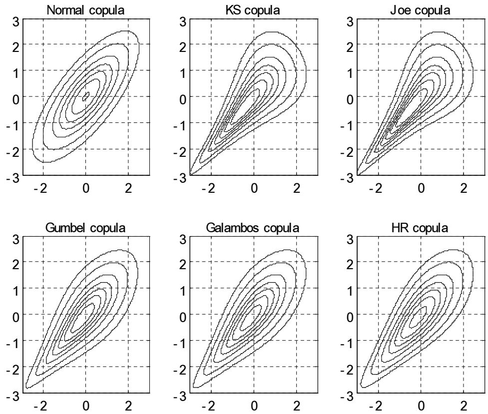
This asymmetric dependence structure can be seen from the contour plots presented in Figure 4. It draws the contours of the estimated copula densities for bear markets from the MSMVN and MSSAC models with standard normal margins. All estimated copulas from the MSSAC model are skewed to the lower left and have fat lower left tails. Note that the HR copula, which gives the highest likelihood value, has the mildest asymmetry among those copulas. However, note also that the KS and Joe copula models, which are highly asymmetric, give better descriptions for a bear regime than the multivariate normal model.

IV. Economic Significance of Further Asymmetry

In the previous section, I documented further asymmetry in bear markets that the MSMVN model cannot capture completely. In this section, I evaluate the economic significance of ignoring this further asymmetry from a risk management point of view. Following Ball and Torous (2000) and Guidolin and Timmermann (2006), I assess the economic significance based on VaR and expected shortfall ratios. This is relevant because disregarding this further asymmetry affects mostly the evaluation of the left lower tail of the joint distribution. Therefore, it could disturb the calculation of $100 \cdot \alpha\%$ VaR, VaR(α), which is usually defined as the $100 \cdot \alpha$ percentile point of a portfolio loss distribution, and $100 \cdot \alpha\%$ expected

FIGURE 4

Contours of Densities of the Estimated Copulas for a Bear Regime from the MSMVN and MSSAC Models with Standard Normal Margins



shortfall, ES(α), which is defined as the expected loss conditional on the loss exceeding the VaR(α).[21] Consider a risk manager required to invest 10 million dollars into a portfolio consisting of the U.S. and U.K. stock indices with minimum risk. One typical way for doing this is to minimize the VaR(α) or ES(α) exposed by the portfolio. In what follows, I will use VaR(α) as an example. In this case, the risk manager solves the following asset allocation problem: $\min_w$ VaR(α, w), where w is the weight of the U.S. stock index, with the constraint of the portfolio market value at 10 million dollars. Suppose that the risk manager finds the optimal weight w^* using the estimated MSMVN model and evaluates the VaR$_{nor}(\alpha, w^*)$, i.e., the VaR based on w^* and the estimated MSMVN model. Suppose further that the true model is one of the estimated MSSAC models. I then calculate the VaR ratio of the true VaR to the risk manager's evaluated misspecified VaR, namely, VaR$(\alpha, w^*)/$VaR$_{nor}(\alpha, w^*)$.

The results for the VaR ratio are presented in Figure 5 with each graph showing the computed VaR ratios assuming each of the MSSAC models is true.[22] Interestingly, the effect of ignoring the further asymmetry is negligible if the significance level range of the VaR is $(0.9 \leq \alpha \leq 0.95)$. The effect is relatively small and more importantly it causes overvaluation of the VaR, which is not influential for risk management. On the contrary, the effect is notable at the VaR significance level range $0.95 \leq \alpha \leq 0.99$, which is more commonly used in practice.

[21] The formal definitions of VaR(α) and ES(α) are given as follows. Let X be a loss from a certain portfolio. Then,

$$\text{VaR}(\alpha) \quad = \quad \text{argmax}\,\{x : \text{Prob}(X \geq x) \geq 1 - \alpha\}$$

and

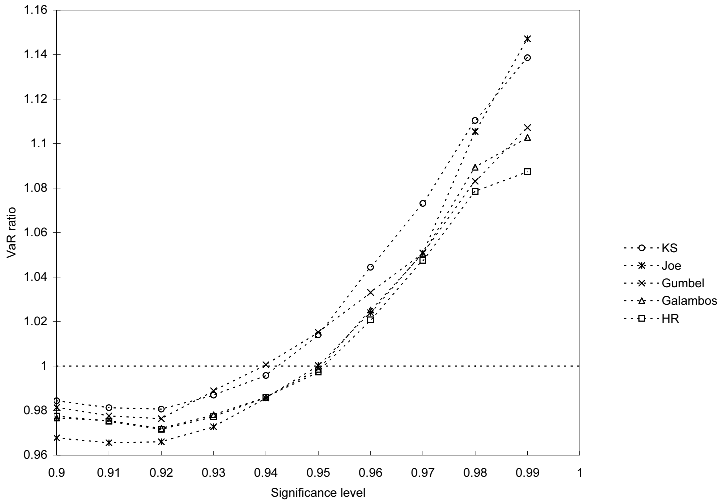
$$\text{ES}(\alpha) \quad = \quad E\left[X \middle| X \geq \text{VaR}(\alpha)\right].$$

[22] An equally weighted portfolio, i.e., $w = ½$, was also examined and yielded results similar to those presented here.

The misspecification produces undervaluation of VaR instead of overestimation and the magnitude of undervaluation increases rapidly as the significance level of VaR becomes larger. As a result, all VaR ratios are about 1.1 or more at the 99% significance level implying 10% undervaluation of VaR(0.99). For example, at the 99% significance level, the risk manager evaluates the VaR(0.99) at 1.229 million dollars, while the true VaR(0.99) is 1.410 million dollars for the worst case of the Joe copula model and 1.336 million dollars for the best case of the HR copula model. Similarly, Figure 6 shows the computed expected shortfall ratios assuming each of the MSSAC model is true. In contrast to the VaR case, expected shortfall is undervalued by about 5% to 10% consistently over the whole significance level between 90% to 99%. For example, at the 99% significance level the risk manager evaluates the ES(0.99) at 1.345 million dollars, while the true VaR(0.99) is 1.553 million dollars for the worst case of the Joe copula model and 1.439 million dollars for the least worst case of the HR copula model. Obviously these undervaluations of risk measures are a tremendous danger for risk management.

FIGURE 5

VaR Ratios between the MSSAC Models and the MSMVN Models



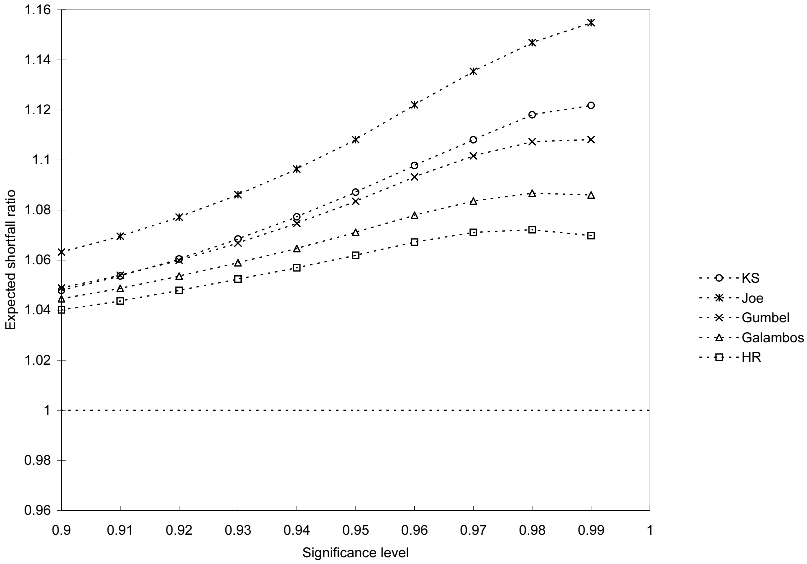
V. Results for Other G7 Countries

So far I have focused on the dependence structure between the U.S. and U.K markets, where I found the asymmetric dependence that the MSMVN model cannot capture. It would be very instructive to examine whether I can find commensurate asymmetric dependence in other G7 countries.

To this end, I adapt the pairwise analysis given in Longin and Solnik (2001). More precisely, I estimate the MSMVN, MSSAC, and MSAC models using the

FIGURE 6

Expected Shortfall Ratios between the MSSAC Models and the MSMVN Models



U.S. and other G7 country pairs, then conduct the same hypothesis tests of asymmetric dependence. Table 10 summarizes those results based on the best model for each pair.[23] The second column of the table shows the best combination of copulas for each pair, while the third column gives the log-likelihood value of the best model. As can be seen, the best copula combinations are different across each pair. For the U.S.-JP and U.S.-CA pairs, the normal copula gives the best fit for both regimes. Thus, for these pairs no asymmetry is needed to describe each regime. In other words, the mixture of normal distributions seems to be enough to capture symmetric dependence for these pairs. On the other hand, for the U.S.-GE pair, the Gumbel-Gumbel copula model captures dependence structures best, while the U.S.-FR pair's dependence is best characterized by the Galambos-Galambos copula model. Thus, for these two pairs I find further evidence of asymmetric dependence, i.e., asymmetric copulas are more suitable in describing both regimes. This asymmetry is different from the U.S.-U.K. pairs, where the asymmetric copulas give better descriptions only for a bear regime. To test whether this asymmetry can result in rejection of the MSMVN model, I conduct the same LR test as above. The values of the LR statistic are given in the fourth column of Table 10 with their p-values reported in the fifth column. The essentially 0 p-values indicate a clear rejection of the MSMVN model. Hence, these two pairs provide other important cases where the use of a MSMVN model becomes inappropriate.

[23] Since I cannot obtain reasonable evidence of two regimes in the U.S. and Italy pair, its associated result is not reported. Also, to save space all estimates and test statistics are not provided, but are available from the author.

TABLE 10

Summary of the Results of Each Pair

			Tests of MSMVN		p-Values of Tests for Equalities across Regimes				
						Expected Returns		Volatilities	
Paired Country	Copulas	Log-Likelihood	LR Statistic	p-Value	Dependence	U.S.	Other	U.S.	Other
Japan	Nor-Nor	−2134.36	N.A.	N.A.	0.993	0.029	0.031	0.000	0.000
Germany	Gum-Gum	−2084.87	4.53	0.000	0.147	0.087	0.012	0.000	0.000
France	Gal-Gal	−2157.66	4.50	0.000	0.375	0.020	0.010	0.000	0.000
Canada	Nor-Nor	−1966.90	N.A.	N.A.	0.080	0.030	0.044	0.000	0.000

The last five columns of Table 10 provide the p-values of hypothesis tests for equality of dependence, expected returns, and volatilities across regimes. For expected returns and volatilities, the first value indicates the p-value for the U.S. market, while the second value is for the other country. The differences in volatilities are highly significant with essentially 0 p-values for all pairs, while the differences in expected returns are slightly weaker, but still significant at a 10% level. On the contrary, dependence differences are not significant at a 10% level for three pairs. In particular, for the U.S.-JP pair, the degrees of dependence are virtually identical in both regimes. The only exception is in the U.S.-CA pair for which the dependence difference is significant at a 10% level. Those results indicate that there are indeed two distinct regimes for all pairs. However, evidence of asymmetric dependence across regimes is not uniform.

In sum, there are two distinct regimes and many types of asymmetric dependence structures in international markets. My analysis demonstrates the usefulness of my framework, which combines the MS model and copula theory, to describe these two distinct regimes and asymmetric dependence structures together.

VI. Conclusions

This paper examines asymmetric dependence structures in international equity markets. I argue that by combining the Markov switching model with copula theory, I can model asymmetric dependence structures with sufficient flexibility. Then, after observing two asymmetries in previous studies I estimated three two-state Markov switching models: the MS multivariate normal (MSMVN), the MS semi-asymmetric copula (MSSAC), and the MS asymmetric copula (MSAC) models. Both the MSSAC and MSAC models indicate that there are two distinct regimes in U.S.-U.K. markets: a bear regime characterized by high asymmetric dependence with lower tail dependence, and low and volatile expected returns, and a normal regime identified by low symmetric dependence with no tail dependence, and high and stable expected returns. The MSSAC models also provide the best replication of Longin and Solnik's (2001) asymmetric exceedance correlations. More importantly, the MSSAC models uniformly reject the MSMVN model with a highly significant statistical level. Thus, I conclude that there exist two types of asymmetric dependence in the U.S.-U.K. markets: asymmetric dependence between bear markets and normal markets, and further asymmetric dependence with lower tail dependence in bear markets that the MSMVN model cannot capture well. My conclusion is consistent with recent studies about asymmetric movements in international equity markets, but is distinct from other studies in recognizing two types of asymmetries. Moreover, my conclusion can be considered as part of the evidence for contagion effects that are documented by the growing contagion literature as well.

This paper also evaluates the economic significance of ignoring the second type of asymmetric dependence found in the U.S.-U.K. markets from a risk management point of view. The VaR and expected shortfall ratio analysis indicate that ignoring the second type of asymmetric dependence causes about 10% undervaluation of VaR at a 99% significance level and expected shortfall over the whole significance level between 90% to 99%, which is crucial for risk management. Therefore, I conclude that recognizing the second type of asymmetric dependence is extremely important, particularly for risk management.

As a final contribution of the paper, I investigate dependence structures in the other G7 countries except Italy. My analysis shows the existence of normal and bear regimes in other markets as well. I, however, find different asymmetric dependence structures. For the U.S.-J.P. pair, no asymmetry is observed, while there is evidence of asymmetric dependence across regimes in the U.S.-CA markets. Although I did not find any strong evidence of asymmetric dependence across regimes in the U.S.-GE and U.S.-FR markets, my results indicate that both markets are characterized by the MSAC models but clearly not by the MSMVN model. In other words, asymmetric dependence exists within each regime in these markets. Hence, I conclude that there are two distinct regimes and many types of asymmetric dependence structures in international equity markets, and my framework provides sufficient flexibility to describe these two distinct regimes and asymmetric dependence structures.

These conclusions also raise several questions for future investigation. One of these is to pursue the economic factors behind asymmetric dependence in international equity markets. This paper assumes that unobserved state s_t follows a first-order Markov chain with constant transition probabilities. As a result, my models do not have much forecastability of s_t, which is essential for forecasting future co-movements of stock returns and, hence, portfolio choice. Also, this paper treats conditional means and volatilities as if they were only state dependent. Therefore, modeling time-varying transition probabilities, conditional means, and volatilities in some other economic exogenous explanatory variables would be interesting future work.

Another future topic is to evaluate the economic significance of ignoring asymmetric dependence structures more generally. I evaluate this only from a risk management point of view, which is probably most affected by the second type of asymmetric dependence found in the U.S.-U.K. markets. But evaluating the economic significance from a more general view point is an attractive topic. For example, the method of international asset allocation with regime shifts developed by Ang and Bekaert (2002) can be adapted for this purpose.

The final topic is to examine dependence structures across more countries. This paper focuses on the dependence structures in G7 countries because these countries are considered to be representative of total markets and reasonably integrated during my sample period. However, recent developments of emerging markets cannot be unheeded. Therefore, investigating dependence structures among those countries is an important topic. For emerging markets, I might need a more complicated model for describing the dynamics of dependence structures. This is relevant because, for instance, the dependence between the U.S. and emerging markets should become stronger and stronger as emerging markets become more developed. In addition, since emerging markets' economies tend to be unstable, it is suspected that their dependence structures are unsteady as well. Thus, there is a need for a more sophisticated model to consider these possibilities. To this end, I can model the dependence parameters for copulas with other economic exogenous explanatory variables. I also model the dynamics of dependence parameters for copulas as in Patton (2006) in which the DCC framework performed in Engle (2002) and Tse and Tsui (2002) is utilized.

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